forests

MDPI

Article

Modeling the Effect of Stand Characteristics on Oak Volume Increment in Poland Using Generalized Additive Models

Hoang Duong Xo Viet [1,2,*], Luiza Tymińska-Czabańska [1] and Jarosław Socha [1]

[1] Faculty of Forestry, University of Agriculture in Krakow, Al. 29 Listopada 46, 31-425 Krakow, Poland
[2] Faculty of Forestry, University of Agriculture and Forestry, Hue University, Hue 530000, Vietnam
* Correspondence: hdxviet@hueuni.edu.vn

Abstract: Volume increment is one of the main concerns in forestry practice. The aim of our study was to examine the impact of factors influencing the periodic annual increment of oak. To meet our objective, we used measurement data from the national forest inventory in Poland from 2005 to 2019 for oak-dominated stands. Our study used data of 1464 sample plots with dominant oak species (*Quercus sessilis* Ehrh. ex Schur and *Quercus robur* L.) measured within the national forest inventory in Poland. We developed models explaining the dependence of the periodic annual volume increment on stand characteristics using the generalized additive model. The generalized additive model allows us to analyze each variable's effect on the dependent variable, with all other variables fixed. We documented the effect of age, height, basal area, and relative spacing index (RSI) on the periodic annual volume increment (PAIv) of oaks in Poland. The PAIv of oaks decreased gradually as the tree aged. The dependence of the PAIv on stand density was shown through its relationship with the basal area and RSI. The developed model explained about 64.6% of the periodic annual volume increment variance.

Keywords: periodic annual volume increment; GAM; stand density; basal area; RSI

Citation: Viet, H.D.X.; Tymińska-Czabańska, L.; Socha, J. Modeling the Effect of Stand Characteristics on Oak Volume Increment in Poland Using Generalized Additive Models. *Forests* **2023**, *14*, 123. https://doi.org/10.3390/f14010123

Academic Editors: Huaqiang Du, Dengsheng Lu, Huaguo Huang, Mingshi Li and Yanjun Su

Received: 15 December 2022
Revised: 3 January 2023
Accepted: 6 January 2023
Published: 10 January 2023

1. Introduction

The volume increment of stands is one of the most important indicators of forest dynamics. Knowledge of volume increment allows for forecasting and developing appropriate forest management plans [1,2]. It is also vital in the context of determining biomass production and the potential for CO_2 sequestration by forest ecosystems [3–6]. Identifying how individual stand factors influence volume increment can be useful in forestry practice and is of growing importance in sustainable forest management.

Essentially, increment describes the rate at which the tree or stand increases in weight or size over a given period of time [7]. The measurements on sample plots in national forest inventories (NFIs) can provide accurate and comprehensive information on the various components of the annual increment [8]. Instead of annual measurements, periodic surveys at n-year intervals are carried out; then, the recorded increment in height, diameter, and volume must be divided by n and is called the periodic annual increment [9]. The periodic annual increment is a more realistic indicator of a tree's capacity (or a stand's) to grow to a given age or size. The volume increment can be influenced by environmental factors and tree characteristics [7,10]. Toledo et al. [11] demonstrated that competition from neighboring trees is an essential biological factor limiting volume growth. Some studies have shown that stand volume increases with narrower plantation spacing. However, when a certain threshold is reached, the narrow plantation spacing can decrease the volume growth rate [12]. A study on Oriental beech (*Fagus orientalis* Lipsky) in Iran showed that stand volume at the beginning of the measurement period and tree diameter had the greatest impact on the variation in volume increment [13]. The influence of these on forest growth, productivity, and biodiversity can be important for sustainable forest management [14,15].

Periodic change in volume is the foundation of many forest growth and productivity models [8,10,13,16–18] and is necessary for determining sustainable harvests in unevenly aged forest management [2,19].

Regression analyses are often used in forest growth models to predict the response of a dependent variable to changes in the relationship with the independent variables [20]. However, due to the complex relationship between the dependent and independent variables, as well as the interaction between the independent variables in the environment, regression analysis may be limited [2,21]. Regression models often lose their ruggedness due to strong linear correlations between independent variables. In addition, regression models do not automatically take care of nonlinearities and do not work with categorical variables [22]. Among the many possible modeling methods, Aertsen et al. demonstrated the usefulness of generalized additive models (GAMs) for the prediction of a site index in Mediterranean mountain forests [23]. GAMs enable making estimates for multivariate variables using the additive approximation of the regression function by substituting the linear function of the explanatory variable with nonparametric functions. The use of a GAM allows us to analyze the effect of each individual variable on the dependent variable, with all other variables fixed. GAMs can model highly complex nonlinear relationships when the number of potential predictors is large, and it also works with categorical variables [24,25].

The *Quercus* genus belongs to the Fagaceae family; in Europe, 27 native species of *Quercus* genus have been found [26,27]. This study focused on *Quercus robur* L., known as pedunculate or English oak, and *Quercus petraea* (Matt.) Liebl., known as sessile oak. These two species occur in many sites as a major component of temperate deciduous mixed forests. The large ecological amplitude is responsible for the wide range of this species at different sites [28]. Oak is of great economic importance, and predicted changes in site conditions may increase their importance in forest ecosystems in Europe in the future [29,30].

Therefore, the aim of our study was to examine the impact of factors influencing the periodic annual increment of oak. To meet our objective, we used measurement data from the national forest inventory activities in Poland from 2005 to 2019 for oak-dominated stands. This extensive data set allowed us to analyze the relationship between the periodic annual increment and the features of oak stands. The results can extend our knowledge of how individual stand factors affect the periodic annual increment patterns for oak, which can be of significant operational and theoretical importance.

2. Materials and Methods

2.1. Sample Plot Data

The material used in this study was measurement data from the NFI activities in Poland from 2005 to 2019. The measurement period started in 2005, with a length of one inventory cycle being 5 years. The NFI measurements started in 2005. Every year, a fifth of the sample plots determined for the whole country were measured. Thus, for plots measured for the first time in 2005, the data covered three periods: The first covered 2005–2009, the next covered 2010–2014, and the third period was 2014–2019. For plots measured for the first time in subsequent years, the increment covered two incremental periods. The study used data collected from 1464 sample plots of three cycles with the dominant oak species (*Quercus sessilis* and *Quercus robur*) (Figure 1). The analyses did not distinguish between the two oak species as they are very similar in terms of growth and productivity [31]; moreover, hybrids that are difficult to assign unambiguously to either species are very common in Poland.

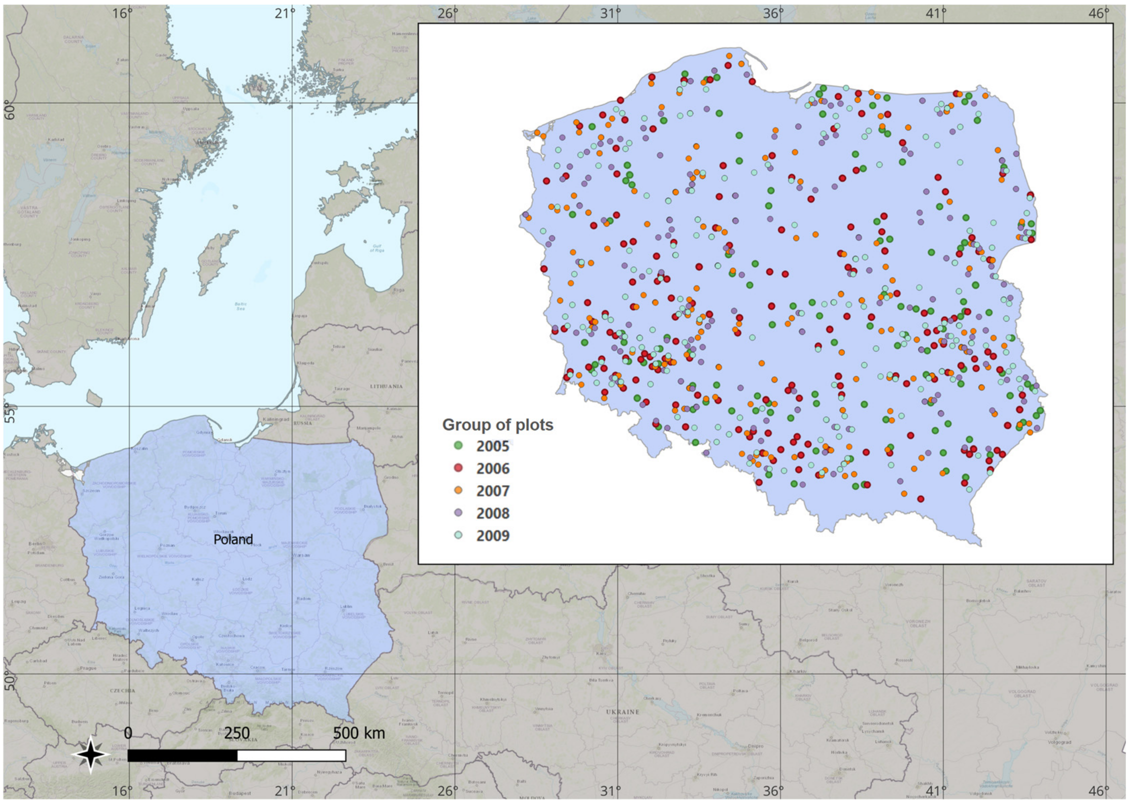

Figure 1. Location of 1464 NFI oak-dominated sample plots in Poland. The colors represent groups of plots measured for the first time in a given year.

The sample plots were set up with an area of 200 or 500 m^2. The basic properties were determined and calculated for each plot:

- Density (number of trees per hectare);
- Quadratic mean diameter at breast height (DBH), in centimeters;
- Top height (TH), calculated as the mean height of the 100 trees with the largest DBH per hectare, in meters;
- Total basal area (total cross-sectional area of trees at breast height):

$$G = \frac{\pi \times DBH^2}{40000} \tag{1}$$

- Stand volume (V):

$$V = g \times h \times f \tag{2}$$

where g is the basal area; h is the height of the tree; f is the form factor, which refers to the characteristic shape of the tree and is the reduction factor of the cylinder volume to the actual tree volume. For our study, we considered the form factor functions of Bruchwald et al. [32]:

$$f = 0.5441 \times DBH^{-0.0415} \left(\frac{DBH - 3}{0.9549 + 0.9439 \times (DBH - 3)} \right) \tag{3}$$

- The stand density index(SDI) was calculated by the average DBH and the number of trees per ha (N) using the Formula (4) [33]:

$$SDI = N \times \left(\frac{DBH}{25} \right)^{1.605} \tag{4}$$

- The relative spacing index (RSI) was calculated as the ratio, which was expressed as a percentage, between the average distance among trees and the top height of the stand, according to Formula (5) [34,35]:

$$RSI = \frac{AS}{TH} * 100 = \frac{10^4 \times \sqrt{\frac{2}{N \times \sqrt{3}}}}{TH} \tag{5}$$

where TH is the top height of the stand, N is the number of trees per hectare, and AS is the average spacing between trees. For the estimation of AS using N, trees were assumed to be positioned on a triangular grid.

- The periodic annual volume increment (PAIv) is the volume growth rate of tree or stand over some period of time and was calculated using Formula (6) [13,16]:

$$PAIv = \frac{V_E + V_H - V_B}{T_j - T_i} \tag{6}$$

where V_E is the volume at the end of the measurement period; V_H is the average volume that was harvested or died (cut and mortality) across all plots during the same period; V_B is the volume at the beginning of the measurement period; T_i is the year at the beginning of the measurement period; T_j is the year at the end of the measurement period. In this study, we calculated the PAIv of stands between each measurement, with the length of the period being five years.

- The stocking index (w_g) is the ratio of the actual volume of the stand to the model volume estimated using the yield tables (for the same tree species, with the same site index and age):

$$w_g = \frac{V_g}{V_t} \tag{7}$$

where V_g is the actual volume per 1 ha; V_t is the volume per 1 ha estimated using yield tables.

The sample plots were established in oak stands with age varying from 10 to 198 years and the number of trees per hectare ranged from 20 to 2275 (Table 1). The average volume on the sample plots is 218 m^3/ha and the average of PAIv is 6.90 m^3/ha/year (Table 1).

Table 1. Basic characteristics of the sample plots.

Variable	Mean	Minimum	Maximum	Standard Deviation
Predictor Variable				
Age (years)	77.22	10.00	198.00	37.04
Diameter (cm)	32.46	7.06	86.16	15.42
Height (m)	22.06	3.50	38.47	6.97
Density (trees/ha)	632.00	20.00	2275.00	359.79
Volume (m^3/ha)	217.86	0.10	780.90	152.53
Basal area (m^2/ha)	17.14	0.59	47.43	9.09
RSI (%)	20.17	10.33	43.106	5.64
Stocking index	0.84	0.00	2.93	0.55
SDI	718.64	6.48	2479.44	441.04
Dependent Variable				
PAIv (m^3/ha/year)	6.90	0.31	18.39	3.98

2.2. Model Development

This study aimed to develop models explaining the dependence of the PAIv on the characteristics of the stand using a GAM. GAMs provide good predictability and allow analysis of a wide range of data types (qualitative and quantitative) as well as allowing us to determine the importance of the variables and their suitability for the model [24,25,36]. GAMs strike a balance between an interpretable but unbiased linear model and highly flexible "black box" learning algorithms [24,25,36]. GAMs allow us to control the smoothness of prediction functions to prevent overfitting. We can directly solve the bias/variance trade-offs by controlling the swings of the prediction functions [24,25,36].

Variables that can cause multicollinearity were detected by calculating the variance inflation factor (VIF) with helper functions for using "mgcv package" in R (Version 4.2.2, Vienna, Austria) [37]. When the predictors have absolutely no absence of collinearity, the VIF value is 1. In practice, there is usually a collinearity among the predictors. A VIF value that exceeds 5 or 10 indicates a problematic amount of collinearity [38–40]. Variables are evaluated with the VIF function and removed one by one, starting with the highest VIF, until all parameter estimates are significant with VIF at around 5.

The VIF for each variable can be computed using the following formula:

$$VIF_{X_J} = \frac{1}{1 - R^2_{X_J|X_{-J}}} \tag{8}$$

where $R^2_{X_J|X_{-J}}$ is R^2 from a regression of X_J onto all of the other predictors.

The structure of the GAM is:

$$\mathrm{g(E(Y))} = \alpha + \mathrm{s_1(x_1)} + \ldots + \mathrm{s_p(x_p)}$$

where Y is the dependent variable (i.e., what we are trying to predict); E(Y) denotes the expected value; g(Y) denotes the link function that links the expected value to the predictor variables $\mathrm{x_1}, \ldots, \mathrm{x_p}$; $\mathrm{s_1(x_1)}, \ldots, \mathrm{s_p(x_p)}$ denote smooth, nonparametric functions.

We also used the variable importance plots (vip) function of the "vip package" in R to evaluate the significance of the variables participating in the GAM model [41]. This is a general framework for constructing variable importance plots from various types of machine learning models in R. With vip, there is one consistent interface for computing variable importance for many types of supervised learning models across a number of packages [41]. The selected variables were included in the GAM model for analysis.

In the process of building the GAM model, we used plots, coefficient tables, and the ANOVA function of the "mgcv package" in R to analyze the deviance for the GAM model to determine if any variable is a crucial term to include in the model. The model should only be as complex as necessary to describe the dataset. Therefore, to select the maximum complexity of the model and decide whether to include a given variable in the model, we used ANOVA. The ANOVA function takes the model objects as arguments and returns an ANOVA testing whether a more complex model including an additional variable is significantly better at capturing the data than a simpler model without that variable. If the resulting p-value was sufficiently low (we used the 0.05 level), we concluded that the more complex model is significantly better than the simpler model and thus favor the more complex model. If the p-value was not less than 0.05, we chose the simpler model without the additional variable.

The model performance and possible overfitting in calculating the adjusted R^2 were analyzed by the use of 10-fold cross-validation. In this method, the data were randomly divided into 10 parts. Then, 9 of those parts were used for training and 1 for testing. This procedure was repeated 10 times, each time reserving a different tenth for testing. This method uses all the data for training and validation and also for estimating the prediction error [42,43]. The procedure was performed using the R language packages gam and caret [42,43]. In the last step, we evaluated the performance of the model using:

Mean absolute error(MAE):

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \tag{9}$$

Root mean square error(RMSE):

$$RMSE = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n}} \quad (10)$$

Adjusted coefficient of determination:

$$R^2_{adj} = 1 - \left(1 - R^2\right)\frac{n-1}{n-p-1} \quad (11)$$

where y_i terms are the observed values, $\hat{y}_i$ terms are the model values, n is the number of errors, p denotes the number of parameters used in the model, and R^2 is the coefficient of determination.

3. Results

Incorporating all of the variables used to describe stand characteristics into the model (Table 2, variable set 1) resulted in high redundancy. The highest redundancy with the other variables was shown by the diameter. When the diameter was excluded (variable set 2), high redundancy was shown by the volume, which was excluded in the next step. In order to develop a model describing the increment, variables for which the VIF was at most around 5 were finally selected (Table 2, variable set 3).

Table 2. Variance inflation factor for each predictor variable considered in the volume increment modeling.

Covariate	Variance Inflation Factor		
	Variable Set 1	Variable Set 2	Variable Set 3
Age	5.70	4.50	4.29
Height	7.64	6.23	5.60
Diameter	8.21	x	x
Volume	6.82	6.55	x
Density	3.31	3.13	2.92
Basal area	1.86	1.84	1.72
Stocking index	5.31	5.02	1.14
RSI	2.03	1.92	1.89
SDI	2.43	2.06	2.03

Examining the significance of the predictor variables indicated their different influence on the PAIv of the oak stands (Figure 2).

Our results showed that the basal area is the most important variable determinant of the PAIv of oak stands. We found that with an increase in the basal area, the PAIv of oak stands substantially increased (Figure 3).

We also found clear overall effects of stand age on the PAIv. Stand age significantly decreased the PAIv of the oak stands. The average PAIv of the oak stands decreased by approximately 1.5 m^3/ha/year every 20 years (Figure 4). However, in stands older than 100 years, the decrease was not so pronounced.

The next important variable determining the PAIv was the height of the oak stands. As the height increased, an increase in the PAIv was observed. The greatest increase in the PAIv was in the range of 25–38 m (Figure 5).

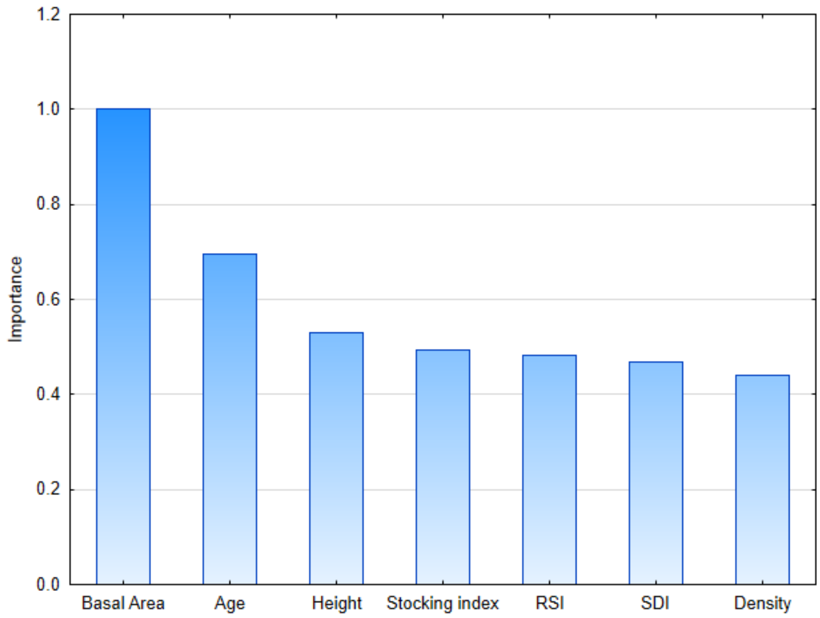

Figure 2. Significance of the predictor variables on the periodic annual volume increment of the oak stands estimated using the variable importance method.

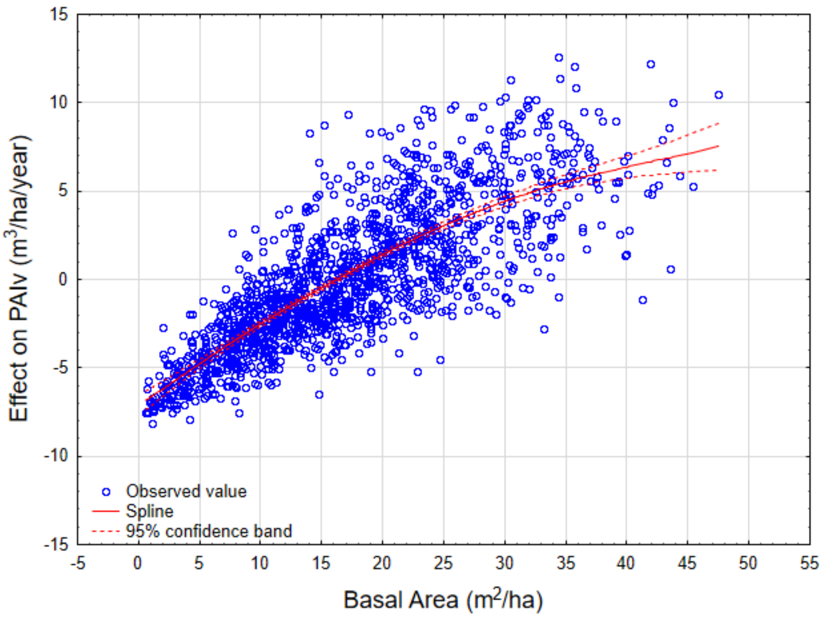

Figure 3. Partial effect of the basal area on the periodic annual volume increment of the oak stands.

Our results also showed slight effects of the stocking index, relative spacing index, stand density index, and density on the PAIv of the oak stands (Figure 6). We found that the fixed-effect stocking index, relative spacing index, stand density index, and density variables were of low significance in the model (Table 3).

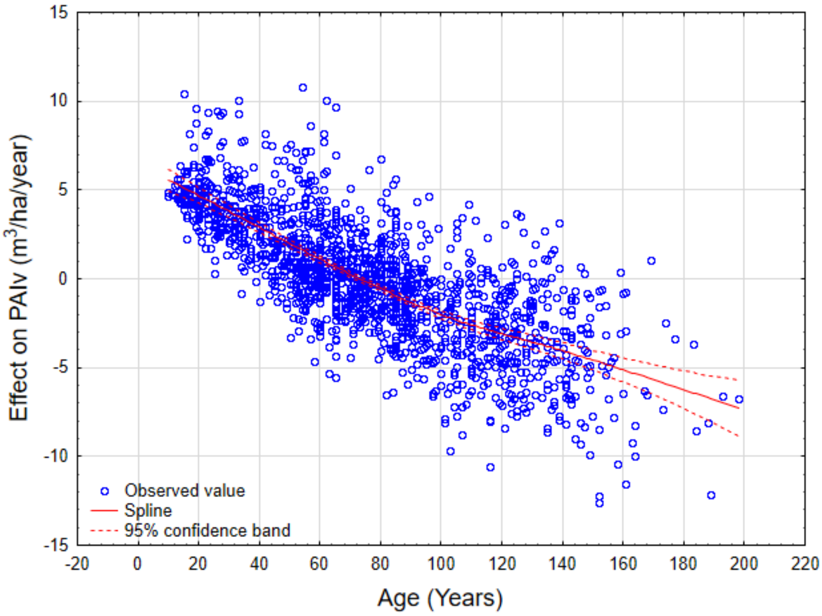

Figure 4. Partial effect of age on the periodic annual volume increment of the oak stands.

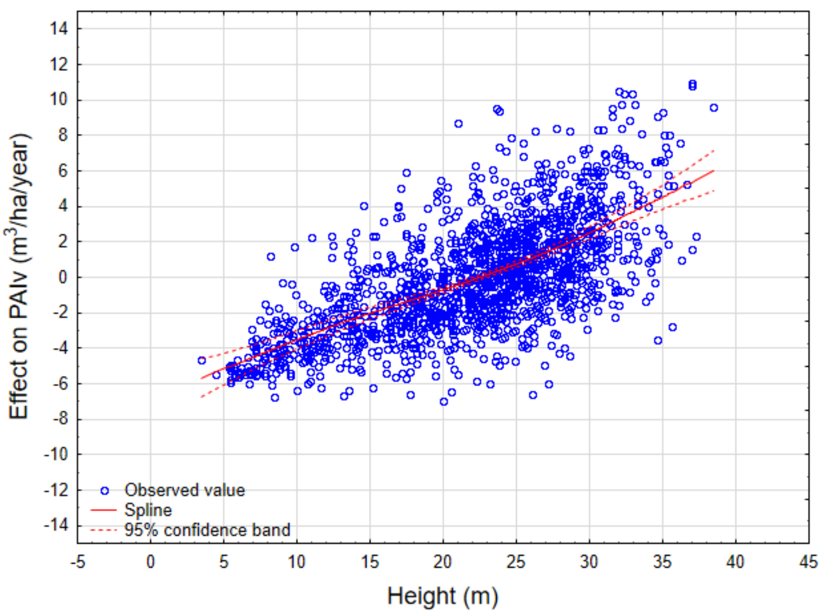

Figure 5. Partial effect of height on the periodic annual volume increment of the oak stands.

Table 3. Approximate significance of seven predictor variables on the periodic annual volume increment described using the GAM model.

Predictor Variable	Effective Degrees of Freedom	Reference Degrees of Freedom	*F*	*p*-Value
Age	4.441	5.511	70.657	<0.0001
Height	2.449	3.168	61.930	<0.0001
Basal area	4.652	5.746	266.957	<0.0001
Relative spacing index	3.904	4.869	2.834	0.0154
Density	1.001	1.002	3.526	0.0608
Stocking index	1.001	1.003	0.779	0.3774
Stand density index	1.796	2.304	0.100	0.8342

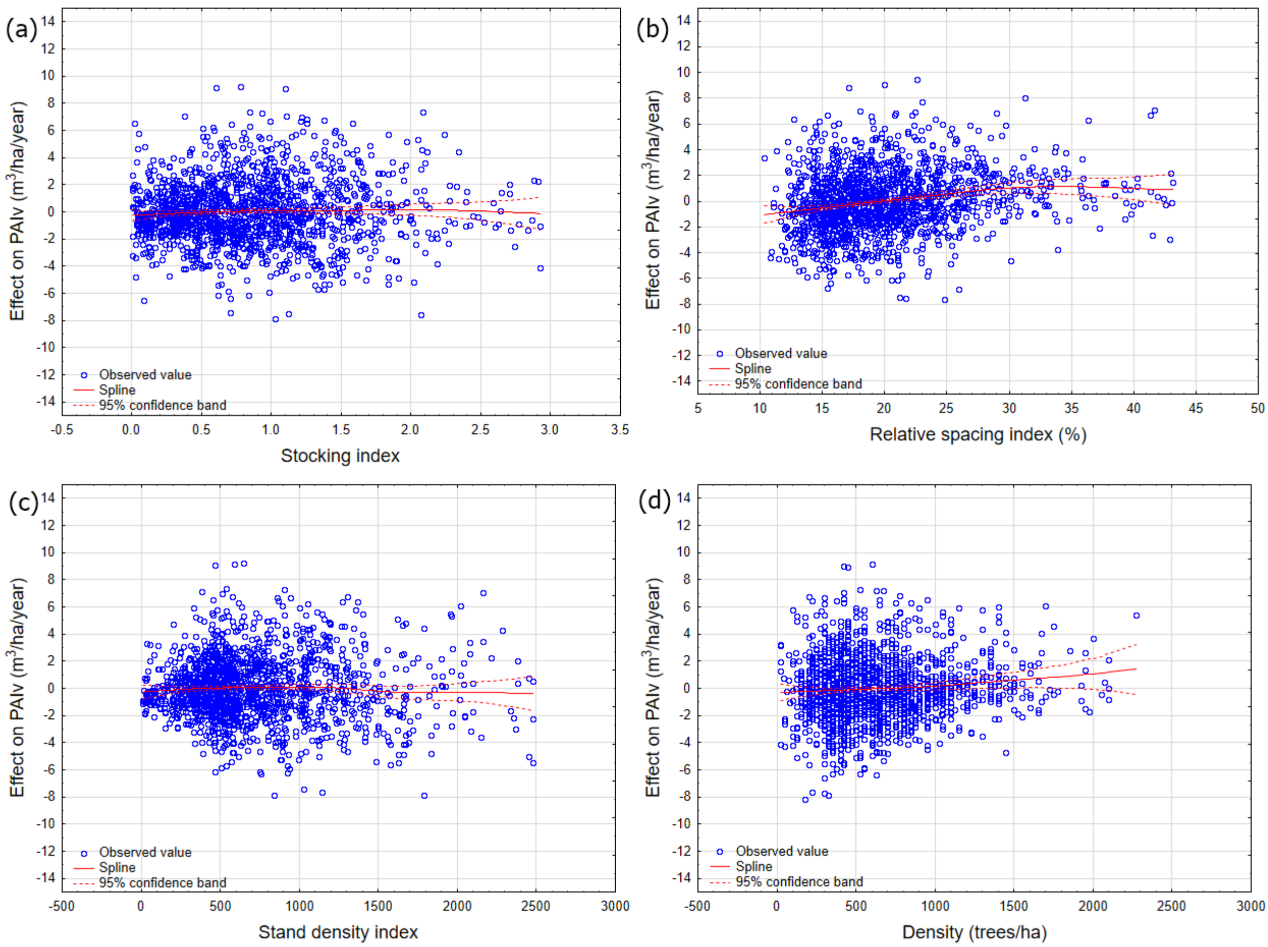

Figure 6. Partial effects of the stocking index (**a**), relative spacing index (**b**), stand density index (**c**), and density (**d**) on the periodic annual volume increment of the oak stands.

In order to test whether the inclusion of variables related to stand density affects growth, a comparison of the simple model with models augmented with the RSI, SDI, stocking index, and density was used. However, using ANOVA of the more complex models that included additional variables describing stand density and of the simpler models without this variable, it was found that the models with the SDI, stocking index, and density variables were not significantly better at capturing the data. Only the addition of the RSI variable significantly increased the predictive ability of the model (ANOVA, $p < 0.05$) (Table 4).

Table 4. Analysis of deviance for the periodic annual volume increment model.

Simple Model (Number)	Extended Model (Number)	*F*	*p*
(1) Age, height, basal area	(2) Age, height, basal area, density	2.56	0.0330
	(3) Age, height, basal area, stocking index	0.87	0.3355
	(4) Age, height, basal area, RSI	3.12	0.0079
	(5) Age, height, basal area, SDI	1.77	0.1352
(4) Age, height, basal area, RSI	(6) Age, height, basal area, RSI, density	2.90	0.0766

Our results demonstrated that the model developed with four predictor variables (age, height, basal area, and relative spacing index) can explain approximately 64.6% of the PAIv variability. The mean absolute error of the model (MAE) was 1.80 m^3/ha/year and the root mean square error (RMSE) was 2.35 m^3/ha/year (Table 5). R^2 adj calculated on the basis of 10-fold cross-validation was 61.4%, suggesting model overfitting was not a concern.

Also the distribution of residuals of the volume increment (Figure 7) indicates the good predictive ability of the developed model.

Table 5. Statistical indicators of the models.

Indicator	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
R^2-adjusted	0.643	0.645	0.643	0.646	0.644	0.646
Root mean square error (m^3/ha/year)	2.37	2.37	2.37	2.35	2.36	2.35
Mean absolute error of the model (m^3/ha/year)	1.82	1.82	1.82	1.80	1.81	1.80

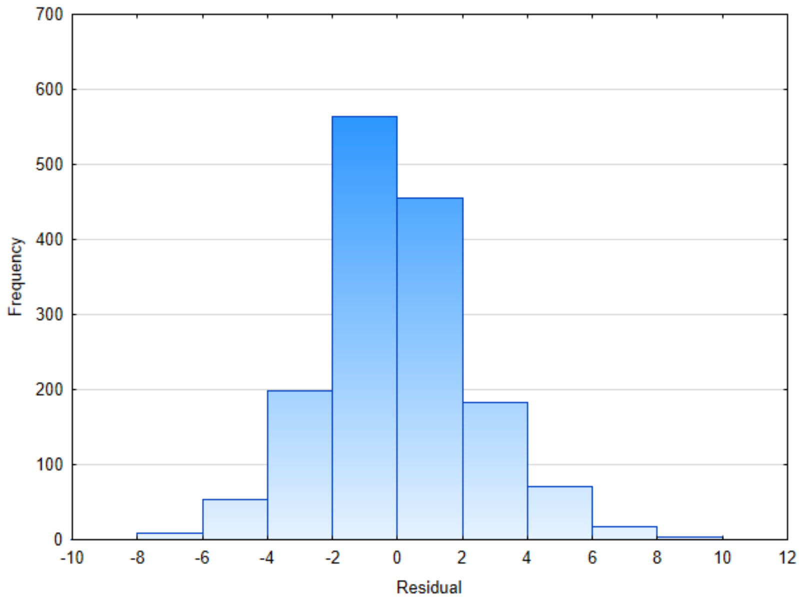

Figure 7. Histogram of the residual values of the model describing the volume increment of oak as the function age, height, basal area, and relative spacing index.

4. Discussion

We identified the most important factors determining the PAIv of oak. This study documented a relationship between the PAIv and the basal area, age, height, and RSI of oak stands. The developed model explained approximately 64.6% of the variance of the PAIv.

Our study showed a strong positive relationship between the basal area and the PAIv of oak stands. We also found that the PAIv of oak stands started to slow down when the basal area increased beyond 30 m^2/ha. This information can be used in forestry practice to determine thinning intensity. The effect of the basal area on the volume increment has been previously reported by other authors. Allen and Burkhart [44] conducted studies on thinning loblolly pine plantations in the southeastern United States. The results-based growth–density relationships suggested that thinned stands can exhibit increased growth at relatively lower densities compared to that of an un-thinned stand on a similar site [44]. Hamidi et al. [2] demonstrated that the basal area is the most important predictor value for estimating the annual volume increment in uneven-aged mixed forests. A study on spruce plantations in Norway showed that the volume increment increases with increasing basal area up to the maximum basal area of a given site [45]. Another study in the Boreal Forest Natural Region of Alberta, Canada, also indicated a significant positive relationship between the volume increment and the basal area of white spruce stands [46]. The basal area is a useful index to help forest managers take appropriate silvicultural measures. From the point of view of the intensification of timber production in the case of oak stands, it is therefore advantageous to maintain a relatively large (30 m^2/ha) basal area, which can be achieved by, among other things, less intensive silvicultural treatments. However, the problem of stand stability must be taken into account when planning treatments. This is because a high basal area leads to an increase in the slenderness and shortening of tree crowns, which can have an adverse effect on wind risk and the condition of individual trees.

Besides the basal area, we verified the significance of the RSI for the PAIv of oak stands. Saud et al. [47] used the RSI as a predictor in the growth model, showing that a quality nonlinear model with minimum information loss can be obtained. Our results showed a proportional relationship between the RSI and PAIv of oak stands; however, when the RSI value exceeded 30%, it hardly affected the PAIv anymore. Another study documented the effectiveness of using the RSI to determine thinning schedules and delineate indirectly derived survival patterns over time for young loblolly pine plantations [48]. A study by Socha et al. using NFI data for Scots pine in Poland also demonstrated a strong correlation between the RSI and volume of stands [35]. Relative spacing is therefore a measure of crowding and competition in stands, relating well to a stand's growth rate, canopy depth, and self-thinning capacity [34,35]. Relative spacing is an indicator that can be quantified and predicted in the future. Thus, it is essential for planning and determining when to take future management actions for stands.

Our results confirmed the high significance of age for the PAIv, which is in line with other studies on the effect of age on the growth increment. These results could be explained by trees undergoing physiological changes as they age, including lower rates of photosynthesis, reduced efficiency in transporting water, and shifting carbon sources to different parts of the tree [49–54]. Yang et al. [55], through regression analysis, proved that forest age is one of the most important factors affecting growth. Research conducted in a primary forest in Heilongjiang province, northeastern China, found that forests have a faster growth rate at a young age and decrease after reaching a maximum [56]. Another study in the eastern United States demonstrated that black oak in older age classes grows much more slowly than younger black oak throughout the lives of these older trees [54]. The results of a study on oak forests in the Eastern Carpathians also showed similar results for the decline in biomass with age [57]. A study in mixed stands (pine/oak) in the Netherlands noticed a decline in the PAIv with age in each species [58]. Moreover, Stimm et al. [59] showed that the stand age variable has a negative effect on the PAIv of oak in both monospecific and mixed oak stands (*Quercus petraea* and *Quercus robur*). The high importance of age for the PAIv should be an important guideline in forest management for optimization the potential of forests for timber production. However, the inhibition of the PAIv should also be an important signal in the context of exploiting the potential of forests for climate change mitigation and carbon sequestration in forest ecosystems.

Tree height is the most dynamic biometric features due to its sensitivity to the environmental changes and silvicultural treatments. The volume increment varies depending on the tree height and height increment [7,9,20,60,61]. Our results demonstrated a positive relationship between height and the PAIv. In forest management practice, information about stand height is often used due to the facility of data collection and high accuracy under application of modern methods [51,62–64]. Therefore, stand height can be commonly used as a good proxy for predicting the PAIv. RSI and stand height can be determined using remote sensing data such as airborne laser scanning point clouds [35]. Therefore, the relationship of volume increment with RSI and stand height documented in our study can be used in the remote sensing determination of volume increment of oak stands. In addition, the change in height growth trends is a good indicator of the effect of climatic and environmental changes on forest conditions. Thus, establishing a relationship between height and the PAIv can provide additional insight into the importance of climatic and environmental factors in shaping tree growth.

In addition to the basal area and RSI indicators, researchers also use other indicators to assess the competition level of the trees in a stand. A study on Norway spruce and European beech proved a relationship between the SDI and periodic annual increment. When the SDI is reduced in young stands, periodic annual increment follows a unimodal curve, while in older stands, it follows an increasing pattern [65]. Allen and Burkhart [44] showed that the relationship between the periodic annual increment and SDI increases at low to mid-densities, but the benefits of increasing density gradually decrease at higher densities. Another study using Spanish NFI data demonstrated that the maximum volume increment of oak occurs at the maximum stocking index [66]. In our study, we also evaluated the effect of density, SDI, and stocking index on the PAIv of oak stands. Our analyses showed that when the RSI is included, the SDI does not increase the significance of the model, so the SDI was excluded from the final version of the model.

The influence of tree characteristics on periodic annual tree increment trends is useful for understanding stand growth. Knowing the contribution of each factor to the size of the PAIv during tree development provides information for silvicultural work to stimulate the volume increment. In our study, we did not analyze factors related to site or climate, which can also significantly affect the PAIv. Toledo et al. [11] showed that climate is the most vital driver affecting volume growth and has significant consequences for forest productivity. Baribault et al. [67] demonstrated that volume growth is also affected by irradiance, soil fertility, and topography. Therefore, future research on oak volume increment should be expanded to include analyses that additionally take into account environmental factors and genetic variation.

5. Conclusions

We documented the effect of age, height, basal area, and RSI on the PAIv of oaks in Poland. The PAIv of oaks decreased gradually as the tree aged. The dependence of the PAIv on stand density was also shown through its relationship with the basal area and RSI. The results of the study may be helpful in determining the intensity and frequency of silvicultural treatments in oak stands in order to achieve the optimal level of volume increment through appropriate regulation of basal area and density in relation to stand height and age. The volume increment of stands is one of the most important indicators of forest dynamics. The possibility of modeling the volume increment allows for forecasting forest development and is important in determining wood and biomass production and the potential for CO_2 sequestration by forest ecosystems. Therefore, identifying how individual stand factors influence the volume increment is crucial in sustainable forest management.

Author Contributions: Conceptualization, J.S.; methodology, H.D.X.V. and J.S.; validation, H.D.X.V.; formal analysis, H.D.X.V.; investigation, H.D.X.V., J.S. and L.T.-C.; resources, J.S.; data curation H.D.X.V. and J.S.; writing—original draft preparation, H.D.X.V., L.T.-C. and J.S.; visualization, H.D.X.V.; funding acquisition, J.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by General Directorate of State Forests in Poland under the project "Volume increment models for the main forest-forming species of Poland", contract no. EZ.271.3.20.2021. H. D. X. Viet has received a bilateral cooperation scholarship between the Polish National Agency for Academic Exchange and the Vietnamese Government for the Ph.D. program at the Faculty of Forestry of the University of Agriculture in Krakow.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Miller, T.F.; Mladenoff, D.J.; Clayton, M.K. Old-Growth Northern Hardwood Forests: Spatial Autocorrelation and Patterns of Understory Vegetation. *Ecol. Monogr.* **2002**, *72*, 487–503. [CrossRef]
2. Hamidi, S.K.; Zenner, E.K.; Bayat, M.; Fallah, A. Analysis of Plot-Level Volume Increment Models Developed from Machine Learning Methods Applied to an Uneven-Aged Mixed Forest. *Ann. For. Sci.* **2021**, *78*, 4. [CrossRef]
3. Krug, J.H.A. How Can Forest Management Increase Biomass Accumulation and CO_2 Sequestration? A Case Study on Beech Forests in Hesse, Germany. *Carbon Balance Manag.* **2019**, *14*, 17. [CrossRef] [PubMed]
4. Fang, S.; Xue, J.; Tang, L. Biomass Production and Carbon Sequestration Potential in Poplar Plantations with Different Management Patterns. *J. Environ. Manag.* **2007**, *85*, 672–679. [CrossRef]
5. Clark, D.A.; Clark, D.B. Assessing the Growth of Tropical Rain Forest Trees: Issues for Forest Modeling and Management. *Ecol. Appl.* **1999**, *9*, 981–997. [CrossRef]

6. Purves, D.; Pacala, S. Predictive Models of Forest Dynamics. *Science* **2008**, *320*, 1452–1453. [CrossRef] [PubMed]
7. Vanclay, J.K. *Modelling Forest Growth and Yield: Applications to Mixed Tropical Forests*; CAB International: Wallingford, UK, 1994; ISBN 0851989136.
8. Tomter, S.M.; Kuliešis, A.; Gschwantner, T. Annual Volume Increment of the European Forests—Description and Evaluation of the National Methods Used. *Ann. For. Sci.* **2016**, *73*, 849–856. [CrossRef]
9. Pretzsch, H. *Forest Dynamics, Growth and Yield: From Measurement to Model*; Intergovernmental Panel on Climate Change, Ed.; Springer: Berlin/Heidelberg, Germany, 2010; Volume 53, ISBN 9783540883067.
10. Wang, W.; Chen, X.; Zeng, W.; Wang, J.; Meng, J. Development of a Mixed-Effects Individual-Tree Basal Area Increment Model for Oaks (*Quercus* Spp.) Considering Forest Structural Diversity. *Forests* **2019**, *10*, 474. [CrossRef]
11. Toledo, M.; Poorter, L.; Peña-Claros, M.; Alarcón, A.; Balcázar, J.; Leaño, C.; Licona, J.C.; Llanque, O.; Vroomans, V.; Zuidema, P.; et al. Climate Is a Stronger Driver of Tree and Forest Growth Rates than Soil and Disturbance. *J. Ecol.* **2011**, *99*, 254–264. [CrossRef]
12. Hébert, F.; Krause, C.; Plourde, P.Y.; Achim, A.; Prégent, G.; Ménétrier, J. Effect of Tree Spacing on Tree Level Volume Growth, Morphology, and Wood Properties in a 25-Year-Old Pinus Banksiana Plantation in the Boreal Forest of Quebec. *Forests* **2016**, *7*, 276. [CrossRef]
13. Bayat, M.; Bettinger, P.; Hassani, M.; Heidari, S. Ten-Year Estimation of Oriental Beech (*Fagus Orientalis* Lipsky) Volume Increment in Natural Forests: A Comparison of an Artificial Neural Networks Model, Multiple Linear Regression and Actual Increment. *For. Int. J. For. Res.* **2021**, *94*, 598–609. [CrossRef]
14. Tinkham, W.T.; Mahoney, P.R.; Hudak, A.T.; Domke, G.M.; Falkowski, M.J.; Woodall, C.W.; Smith, A.M.S. Applications of the United States Forest Inventory and Analysis Dataset: A Review and Future Directions. *Can. J. For. Res.* **2018**, *48*, 1251–1268. [CrossRef]
15. Hao, M.H.; Zhang, C.; Zhao, X.; von Gadow, K. Functional and Phylogenetic Diversity Determine Woody Productivity in a Temperate Forest. *Ecol. Evol.* **2018**, *8*, 2395–2406. [CrossRef] [PubMed]
16. Ábri, T.; Rédei, K. Analyses of Periodic Annual Increment by Diameter and Volume in Differently Aged Black Locust (*Robinia Pseudoacacia* L.) Stands: Case Study. *J. For. Sci.* **2022**, *68*, 213–219. [CrossRef]
17. Kaźmierczak, K. The Current Growth Increment of Pine Tree Stands Comprising Three Different Age Classes. *For. Res. Pap.* **2013**, *74*, 93–100. [CrossRef]
18. Gasparini, P.; di Cosmo, L.; Rizzo, M.; Giuliani, D. A Stand-Level Model Derived from National Forest Inventory Data to Predict Periodic Annual Volume Increment of Forests in Italy. *J. For. Res.* **2017**, *22*, 209–217. [CrossRef]
19. Bayat, M.; Pukkala, T.; Namiranian, M.; Zobeiri, M. Productivity and Optimal Management of the Uneven-Aged Hardwood Forests of Hyrcania. *Eur. J. For. Res.* **2013**, *132*, 851–864. [CrossRef]
20. Weiskittel, A.R.; Hann, D.W.; Kershaw, J.A.; Vanclay, J.K. *Forest Growth and Yield Modeling*; John Wiley & Sons Ltd.: Chichester, UK, 2011; ISBN 9780470665008.
21. Leite, R.V.; do Amaral, C.H.; de Paula Pires, R.; Silva, C.A.; Soares, C.P.B.; Macedo, R.P.; da Silva, A.A.L.; Broadbent, E.N.; Mohan, M.; Leite, H.G. Estimating Stem Volume in Eucalyptus Plantations Using Airborne LiDAR: A Comparison of Area- and Individual Tree-Based Approaches. *Remote Sens.* **2020**, *12*, 1513. [CrossRef]
22. Harrell, F.E. *Regression Modeling Strategies with Applications to Linear Models, Logistic and Ordinal Regression, and Survival Analysis*, 2nd ed.; Springer Series in Statistics; Springer International Publishing: Cham, Switzerland, 2015.
23. Aertsen, W.; Kint, V.; van Orshoven, J.; Özkan, K.; Muys, B. Comparison and Ranking of Different Modelling Techniques for Prediction of Site Index in Mediterranean Mountain Forests. *Ecol. Modell.* **2010**, *221*, 1119–1130. [CrossRef]
24. Hastie, T.J.; Tibshirani, R.J. *Generalized Additive Models*; Taylor & Francis Group: New York, NY, USA, 2017; pp. 1–335. [CrossRef]
25. Wood, S.N. *Generalized Additive Models*; Chapman and Hall/CRC: Boca Raton, FL, USA, 2017; ISBN 9781315370279.
26. Petritan, A.M.; Biris, I.A.; Merce, O.; Turcu, D.O.; Petritan, I.C. Structure and Diversity of a Natural Temperate Sessile Oak (*Quercus Petraea*, L.)—European Beech (*Fagus Sylvatica* L.) Forest. *For. Ecol. Manag.* **2012**, *280*, 140–149. [CrossRef]
27. Mabberley, D.J. *The Plant Book*; Cambridge University Press: Cambridge, UK, 1990.
28. Eaton, E.; Caudullo, G.; Oliveira, S.; de Rigo, D. *Quercus Robur and Quercus Petraea in Europe: Distribution, Habitat, Usage and Threats*; San-Miguel-Ayanz, J., de Rigo, D., Caudullo, G., Durrant, T.H., Mauri, A., Eds.; Publication Office of the European Union: Luxembourg, 2016.
29. Hanewinkel, M.; Cullmann, D.A.; Schelhaas, M.J.; Nabuurs, G.J.; Zimmermann, N.E. Climate Change May Cause Severe Loss in the Economic Value of European Forest Land. *Nat. Clim. Change* **2012**, *3*, 203–207. [CrossRef]
30. Nothdurft, A.; Wolf, T.; Ringeler, A.; Böhner, J.; Saborowski, J. Spatio-Temporal Prediction of Site Index Based on Forest Inventories and Climate Change Scenarios. *For. Ecol. Manag.* **2012**, *279*, 97–111. [CrossRef]
31. Tymińska-Czabańska, L.; Socha, J.; Maj, M.; Cywicka, D.; Hoang Duong, X.V. Environmental Drivers and Age Trends in Site Productivity for Oak in Southern Poland. *Forests* **2021**, *12*, 209. [CrossRef]
32. Bruchwald, A.; Dmyterko, E. *Lasy Beskidu Śląskiego i Żywieckiego—Zagrożenia, Nadzieja*; Instytut Badawczy Leśnictwa: Raszyn, Poland, 2010.
33. Reineke, L.H. Perfecting a Stand-Density Index for Even-Aged Forests. *J. Agric. Res.* **1933**, *46*, 627–638.
34. Meredieu, C.; Perret, S.; Dreyfus, P. Modelling Dominant Height Growth: Effect of Stand Density. In *Modelling Forest Systems*; Amaro, A., Reed, D., Soares, P., Eds.; Cabi Publishing: Cambridge, MA, USA, 2002; pp. 111–121.

35. Socha, J.; Hawryło, P.; Pierzchalski, M.; Stereńczak, K.; Krok, G.; Wężyk, P.; Tymińska-Czabańska, L. An Allometric Area-Based Approach—A Cost-Effective Method for Stand Volume Estimation Based on ALS and NFI Data. *For. Int. J. For. Res.* **2020**, *93*, 344–358. [CrossRef]
36. Larsen, A.K. GAM: The Predictive Modeling Silver Bullet. *Public* **2015**, *1986*, 1–27.
37. R: Variance Inflation Factor and Test for Multicollinearity. Available online: https://search.r-project.org/CRAN/refmans/usdm/html/vif.html (accessed on 28 February 2022).
38. James, G.; Witten, D.; Hastie, T.; Tibshirani, R. *An Introduction to Statistical Learning*; Springer Texts in Statistics; Springer New York: New York, NY, USA, 2013; Volume 103, ISBN 978-1-4614-7137-0.
39. Melnychuk, M.C.; Peterson, E.; Elliott, M.; Hilborn, R. Fisheries Management Impacts on Target Species Status. *Proc. Natl. Acad. Sci. USA* **2017**, *114*, 178–183. [CrossRef] [PubMed]
40. Mendes, H.; de Fátima Borges, M.; Scott, C.L.; Frid, C. *Climatic Impact on Hake Recruitment in Iberian Peninsula and Implications for Fisheries Management: An InExFish Project Study*; International Council for the Exploration of the Sea: Copenhagen, Denmark, 2008.
41. Greenwell, B.M.; Boehmke, B.C. Variable Importance Plots-An Introduction to the Vip Package. Available online: https://journal.r-project.org/archive/2020/RJ-2020-013/RJ-2020-013.pdf (accessed on 28 February 2022).
42. K-Fold Cross Validation in R (Step-by-Step). Available online: https://www.statology.org/k-fold-cross-validation-in-r/ (accessed on 28 February 2022).
43. Training Sets, Test Sets, and 10-Fold Cross-Validation. -KDnuggets. Available online: https://www.kdnuggets.com/2018/01/training-test-sets-cross-validation.html (accessed on 28 February 2022).
44. Allen, M.G.; Burkhart, H.E. Growth-Density Relationships in Loblolly Pine Plantations. *For. Sci.* **2019**, *65*, 250–264. [CrossRef]
45. Allen, M.; Brunner, A.; Antón-Fernández, C.; Astrup, R. The Relationship between Volume Increment and Stand Density in Norway Spruce Plantations. *For. Int. J. For. Res.* **2021**, *94*, 151–165. [CrossRef]
46. Reyes-Hernández, V.J.; Comeau, P.G. The Influence of Stocking and Stand Composition on Productivity of Boreal Trembling Aspen-White Spruce Stands. *Forests* **2015**, *6*, 4573–4587. [CrossRef]
47. Saud, P.; Lynch, T.B.; Anup, K.C.; Guldin, J.M. Using Quadratic Mean Diameter and Relative Spacing Index to Enhance Height–Diameter and Crown Ratio Models Fitted to Longitudinal Data. *For. Int. J. For. Res.* **2016**, *89*, 215–229. [CrossRef]
48. Zhao, D.; Kane, M.; Borders, B. Development and Applications of the Relative Spacing Models for Loblolly Pine Plantations Plantation Management Research Cooperative. *For. Ecol. Manag.* **2009**, *259*, 1922–1929. [CrossRef]
49. Kaufmann, M.R. To Live Fast or Not: Growth, Vigor and Longevity of Old-Growth Ponderosa Pine and Lodgepole Pine Trees. *Tree Physiol.* **1996**, *16*, 139–144. [CrossRef] [PubMed]
50. Martínez-Vilalta, J.; Vanderklein, D.; Mencuccini, M. Tree Height and Age-Related Decline in Growth in Scots Pine (*Pinus Sylvestris* L.). *Oecologia* **2007**, *150*, 529–544. [CrossRef]
51. Ryan, M.G.; Yoder, B.J. Hydraulic Limits to Tree Height and Tree Growth. *Bioscience* **1997**, *47*, 235–242. [CrossRef]
52. Carrer, M.; Urbinati, C. Age-Dependent Tree-Ring Growth Responses to Climate in Larix Decidua and Pinus Cembra. *Ecology* **2004**, *85*, 730–740. [CrossRef]
53. Mencuccini, M.; Martínez-Vilalta, J.; Vanderklein, D.; Hamid, H.A.; Korakaki, E.; Lee, S.; Michiels, B. Size-Mediated Ageing Reduces Vigour in Trees. *Ecol. Lett.* **2005**, *8*, 1183–1190. [CrossRef]
54. Johnson, S.E.; Abrams, M.D. Age Class, Longevity and Growth Rate Relationships: Protracted Growth Increases in Old Trees in the Eastern United States. *Tree Physiol.* **2009**, *29*, 1317–1328. [CrossRef]
55. Yang, Y.; Watanabe, M.; Li, F.; Zhang, J.; Zhang, W.; Zhai, J. Factors Affecting Forest Growth and Possible Effects of Climate Change in the Taihang Mountains, Northern China. *For. Int. J. For. Res.* **2006**, *79*, 135–147. [CrossRef]
56. Yu, Y.; Chen, J.M.; Yang, X.; Fan, W.; Li, M.; He, L. Influence of Site Index on the Relationship between Forest Net Primary Productivity and Stand Age. *PLoS ONE* **2017**, *12*, e0177084. [CrossRef]
57. Kozak, I.; Holubets, M. Biomass Production and Productivity in Oak Forests of the Eastern Carpathians in Relationship with Stands Age. *Int. J. Ecol. Probl. Biosph.* **2001**, *20*, 301–309.
58. Lu, H.; Mohren, G.M.J.; del Río, M.; Schelhaas, M.J.; Bouwman, M.; Sterck, F.J. Species Mixing Effects on Forest Productivity: A Case Study at Stand-, Species- and Tree-Level in the Netherlands. *Forests* **2018**, *9*, 713. [CrossRef]
59. Stimm, K.; Heym, M.; Nagel, R.V.; Uhl, E.; Pretzsch, H. Long-Term Productivity of Monospecific and Mixed Oak (*Quercus Petraea* [Matt.] Liebl. and *Quercus Robur* L.) Stands in Germany: Growth Dynamics and the Effect of Stand Structure. *Forests* **2022**, *13*, 724. [CrossRef]
60. West, P.W. *Tree and Forest Measurement*; Springer International Publishing: Cham, Switzerland, 2015; ISBN 978-3-319-14707-9.
61. West, P.W. *Growing Plantation Forests*; Springer International Publishing: Cham, Switzerland, 2014; ISBN 978-3-319-01826-3.
62. Carmean, W.H.; Lenthall, D.J. Height-Growth and Site-Index Curves for Jack Pine in North Central Ontario. *Can. J. For. Res.* **2011**, *19*, 215–224. [CrossRef]
63. Socha, J.; Tymińska-Czabańska, L. A Method for the Development of Dynamic Site Index Models Using Height–Age Data from Temporal Sample Plots. *Forests* **2019**, *10*, 542. [CrossRef]
64. Manso, R.; Davidson, R.; Mclean, J.P. Diameter, Height and Volume Increment Single Tree Models for Improved Sitka Spruce in Great Britain. *For. Int. J. For. Res. For.* **2022**, *95*, 391–404. [CrossRef]
65. Pretzsch, H. Stand Density and Growth of Norway Spruce (*Picea Abies* (L.) Karst.) and European Beech (*Fagus Sylvatica* L.): Evidence from Long-Term Experimental Plots. *Eur. J. For. Res.* **2005**, *124*, 193–205. [CrossRef]

66. del Río, M.; Sterba, H. Comparing Volume Growth in Pure and Mixed Stands of Pinus Sylvestris and Quercus Pyrenaica. *Ann. For. Sci.* **2009**, *66*, 502. [CrossRef]
67. Baribault, T.W.; Kobe, R.K.; Finley, A.O. Tropical Tree Growth Is Correlated with Soil Phosphorus, Potassium, and Calcium, Though Not for Legumes. *Ecol. Monogr.* **2012**, *82*, 189–203. [CrossRef]