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Modeling Free Branch Growth with the Competition Index for a *Larix principis-rupprechtii* Plantation

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Abstract: Competition among free branches in the tree canopy is an important factor influencing branch length growth. Therefore, there is a need to quantify this competition and to understand the impact of the regression technique on the predictive accuracy of the growth of free branch length (GFBL) model in a *Larix principis-rupprechtii* plantation. This study focused on an *L. principis-rupprechtii* plantation in Saihanba Mechanized Forest Farm. Five competition indices based on 2176-branch data points from 76 trees were used to quantify the branch competition, and three regression techniques (nonlinear least squares (NLS), nonlinear mixed-effects model (NLME), and nonlinear quantile regression (NQR)) were used to construct the GFBL model including the branch competition index. The results showed that the Chapman–Richards growth function, including the diameter at breast height (DBH) and depth of branch into crown (DINC), was the optimal equation for describing the GFBL in the studied *L. principis-rupprechtii* plantation. The branch competition index (CI) was found to be optimal for quantifying the branch competition when used with the maximum value parameter (a_0) of the Chapman–Richards growth function. The three parameter estimation methods were compared, and the NLME, which included the CI, was found to have the highest predictive accuracy. The results of this study can act as a reference for improving the management, assessing the management effectiveness, and enhancing the quality of *L. principis-rupprechtii* plantations.

Keywords: free branch length growth; competition index; nonlinear mixed-effects model; nonlinear quantile regression; *Larix principis-rupprechtii* plantation

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1. Introduction

Free branches are branches in the upper canopy of a tree that are not constrained or affected by the surrounding stand density. The growth and distribution of free branches directly affect the structure and function of the tree canopy [1–3]. However, quantification of the growth of free branch length (GFBL) is important for studies concerning the number and spatial structure of branches [4], tree leaf distribution and quantity [5,6], photosynthesis and nutrient cycling in trees [7,8], tree wood quality [3,9], and forest wildlife habitats [10]. In addition, estimates of the GFBL can compensate for the difference between photosynthesis and canopy allometry [11], and they can reflect the impact of varying stand densities on tree development [9,12]. Therefore, the study of the GFBL is of great significance for improving tree management practices [13], evaluating tree management effects [14], and enhancing stand quality [9].

Recent studies have shown that the main factors affecting the GFBL are the tree diameter at breast height (DBH) [15], tree height (TH) [16], depth of branch into crown (DINC) [11], crown length (CL) [17], and branch competition [18]. While branch competition has been shown to have a significant impact on branch development [19], Weiskittel et al. [12] determined that the factors regulating branch competition include nutrient transport [20] and canopy resource allocation [18]. Therefore, these factors ultimately impact the growth, survival, crown structure, and function of branches [12,21,22]. Currently, the commonly used indicators of branch competition include distance-independent indicators and distance-dependent indicators [23,24], with the former reflecting the competitive ability of branches [25] and the latter quantifying the degree of competition between branches by measuring their growth space [26–28]. Therefore, further research is required into the selection of competition indicators for quantifying the impact of branch competition on the GFBL within the canopy.

Competition indicators and different parameter estimation methods affect the accuracy of the GFBL model [15]. Currently, the commonly used parameter estimation methods for branch length growth models include the nonlinear least squares (NLS) [18,29], nonlinear mixed-effects model (NLME) [13,16,30], and nonlinear quantile regression (NRQ) [16] methods. While the NLS method requires data for independent observations that are random [31], free branch data within the canopy often exhibit time-dependence and spatial heterogeneity [30]. The NLME method effectively overcomes the above limitation of the NLS method [30] and has been widely applied in studies concerning the height–diameter equation [32,33], taper equations [31], and branch length growth prediction models [13,20]. The NQR method can describe the complete conditional distribution of data and can illustrate the relationships between independent and dependent variables at different quantile points [16]. As such, the NQR method allows for the extraction of a greater quantity of information from data and provides a more flexible means of describing the relationship between dependent and independent variables [34]. Consequently, the NQR method has been widely used in studies concerning the height–diameter equation [32], taper equations [35], and models of branch length and diameter growth [16]. Therefore, the selection of a suitable approach for estimating parameters is crucial for achieving the optimal accuracy of the GFBL model.

Larix principis-rupprechtii is commonly cultivated for afforestation in northern China due to its several advantages, including valued timber and tolerance of cold temperatures. The Saihanba Mechanized Forest Farm is situated in the warm temperate zone of northern China in which *L. principis-rupprechtii* is the primary species cultivated as part of afforestation efforts. The cultivation of this tree species offers significant economic, ecological, and social benefits to the Beijing–Tianjin–Hebei region [31]. Therefore, there is value in studying the growth and development of branches of an *L. principis-rupprechtii* plantation to improve forest structure, enhance forest productivity, and leverage multiple ecological and social benefits.

Competition factors [19] and parameter estimation methods [16] have an impact on the accuracy of predictions of the GFBL model. However, further work is needed in terms of the identification of suitable indices for quantifying the impact of branch competition within the canopy on the GFBL. The hypothesis of the present study is that characterization of branch competition and regression techniques have an impact on the accuracy of predictions of GFBL models. To validate the above hypothesis, the present study focused on free branches within the canopy of an *L. principis-rupprechtii* plantation. The aim of the present study was to characterize the impacts of regression techniques and branch competition on the estimates of the GFBL in an *L. principis-rupprechtii* plantation. The objectives of the present study were to: (1) evaluate the impacts of different competition indices on the GFBL; (2) construct a GFBL model that integrates the optimal branch competition index using the NLS, NLME, and NQR methods; and (3) compare and analyze the effects of different regression techniques on the accuracies of the GFBL models. The results of the present study can act as a reference for the accurate prediction of branch growth and development patterns, optimization of canopy structure, and improvement of forest management practices.

2. Materials and Methods

2.1. Study Area

The Saihanba Mechanized Forest Farm (41°22′–42°58′ N, 116°53′–118°31′ E) is in northern Hebei Province and has a total operational area of 9.4×10^4 hm^2, of which 7.5×10^4 hm^2

is forested land. The *L. principis-rupprechtii* plantation covers 5.3×10^4 hm^2, accounting for 70.6% of the forested land area. The study area falls within the northern China warm temperate zone and has elevations ranging between 1010 and 1940 m, average annual, average maximum, and average minimum temperatures of −1.2 °C, 33.4 °C, and −43.3 °C, respectively, and average annual sunshine hours, average annual precipitation, average annual snow cover days, and average annual potential evaporation of 2548.7 h, 452.2 mm, 169 d, and 1339.2 mm, respectively. The major soil types in the study area are brown forest soil, chestnut soil, sandy soil, and loam. The main tree species in the study area include *Larix principis-rupprechtii*, *Populus davidiana*, *Betula platyphylla*, and *Betual davurica*. The main shrubs include *Lonicera japonica*, *Spiraea salicifolia*, and *Prunus padus*. The main herbaceous plants include *Clematis florida Thunb*, *Campanula punctata*, and *Saussurea japonica*.

2.2. Data Collection

The present study collected data from 38 standardized plots (0.06 hm^2) of an *L. principis-rupprechtii* plantation in the Saihanba Mechanized Forest Farm between 2017 and 2022. Mature trees with a diameter at breast height (assumed to be 1.3 m) (DBH) ≥ 5 cm were measured for each plot. In addition, site factors (elevation, slope, aspect, etc.) and stand factors (stand age, average crown width, density, etc.) were measured and recorded. The stand age and density ranged from 14 to 55 years and from 240 to 3 600 tree·hm^{-2}, respectively. Two standard trees were selected from each plot for stem and branch analysis, and the tree height (TH), first branch height (FBH), and CL were measured for each tree, whereas the branch diameter (BD), branch length (BL), depth of branch into crown (DINC), branch insertion angle (BA), age (BAGE), and azimuthal orientation were measured for each free branch (Figure 1). The TH, CL, FBL, BL and DINC were measured using a tape measure; the BD was measured using a vernier caliper; the BA and azimuthal orientation were measured using a protractor; the DBH was measured using a diameter tape; and the BAGE was determined via branch analysis. The present study surveyed 76 trees and 2176 branches, and the data were divided into modeling data (1741 branch measurements from 56 sample trees) and testing data (435 branch measurements from 20 sample trees) based on a 3:1 ratio (Table 1).

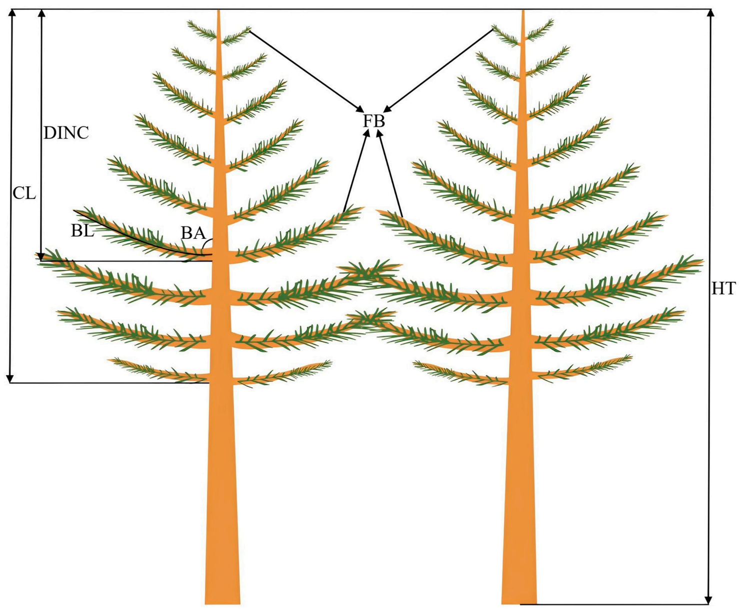

Figure 1. A graphical description of the characteristics of the tree and free branch variables. HT is total tree height (m), FB is free branch, CL is crown length (m), BL is branch length (m), DINC is depth of branch into crown (m), and BA is branch insertion angle (°).

Table 1. Summary of the results of the statistical analysis of the modeling and validation data.

Variables	Modeling Data (N = 1741)				Verification Data (N = 435)			
	Max	Min	Mean	Std	Max	Min	Mean	Std
TH (m)	23.60	6.50	14.93	4.08	19.30	10.90	15.84	2.82
DBH (cm)	29.80	7.80	17.88	5.43	32.20	13.90	19.98	5.19
CW (m)	3.36	0.94	1.82	0.63	3.55	1.24	1.92	0.49
FBH (m)	13.00	1.65	6.96	2.93	11.70	3.60	7.47	2.50
CL (m)	13.10	4.30	7.98	2.20	12.50	4.40	8.38	2.02
BAGE (year)	27.00	1.00	8.75	5.32	23.00	1.00	9.59	5.69
BD (cm)	50.02	1.60	18.05	8.94	45.41	2.11	18.60	9.20
BL (m)	4.32	0.06	1.46	0.77	3.47	0.14	1.49	0.76
DINC (m)	13.10	0.04	3.69	2.42	10.7	0.05	3.82	2.50
BA (°)	90.00	10.00	64.79	11.66	90.00	30.00	65.18	11.55

2.3. Competition Indices

Competition among free branches can impact the growth of branches [18]. The present study used different competition indices (Table 2) to quantify the effects of competition among free branches on the accuracy of predictions of the GFBL model. The positions of branches in the crown of the studied *L. principis-rupprechtii* plantation were determined based on the DINC and azimuthal orientation of each free branch [13]. Each free branch was considered a target branch, the four branches closest to the target branch were considered competing branches [36] and the competition indices were calculated based on the actual measurement data of the free branches.

Table 2. Selection of competition indices between free branches.

Type	Competition Index Name	Expression	Reference
Distance-dependent	Competition Index (CI)	$CI = \sum_{i=1}^{n} \frac{D_j}{D_i \times L_{ij}}$	[26]
	Competition Area Index (CA)	$CA = \sum_{i=1}^{n} \left(\frac{H_i}{L_{ij}} \frac{D_j}{D_i} \right)$	[27]
	Sum Line Length (SLL)	$SLL = \sum_{i=1}^{n} \frac{D_i}{D_i + D_j} \times L_{ij}$	[28]
Distance-independent	Competing Branch Diameter Ration (CD)	$CD = \sum_{i=1}^{n} \frac{D_j}{D_i}$	[25]
	Branch Diameter Ration (DR)	$DR = \frac{D_i}{\overline{D}}$	[25]

Note: D_j, D_i, L_{ij}, H_i, and $\overline{D}$ represent the diameter of the competing branch (mm), the diameter of the target branch (mm), the distance between branches (m), the branch length (m), and the mean value of the diameter of the branch (mm), respectively.

2.4. Methods

2.4.1. Base Model for the GFBL

Since the GFBL exhibits a nonlinear pattern with increasing age [14,28], the present study selected five biologically significant nonlinear growth equations as the fundamental models for the GFBL in an *L. principis-rupprechtii* plantation (Table 3).

Table 3. Theoretical equations of free branch length growth.

Equation Name	Expression	Reference
Chapman–Richards	$BL = a_0\left(1 - e^{-a_1 BAGE}\right)^{a_2}$	[37]
Schumacher	$BL = a_0 e^{-\frac{a_1}{BAGE}}$	[38]
Gompertz	$BL = a_0 e^{-a_1 e^{-a_2 BAGE}}$	[39]
Mitscherlich	$BL = a_0\left(1 - a_1 e^{-a_2 BAGE}\right)$	[16]
Logistic	$BL = \frac{a_0}{1 + a_1 e^{-a_2 BAGE}}$	[23]

Note: BL is free branch length (m); BAGE is free branch age (a); a_0, a_1 and a_2 are the model parameters to be estimated.

2.4.2. Nonlinear Mixed-Effects Model

The NLME model is a regression model representing the nonlinear relationship between the regression function and fixed and random effects [30]. Wang et al. [15] previously showed the NLME model based on tree-level effects to be the most effective for predicting the branch length growth of *Pinus koraiensis*. Therefore, the present study adopted the tree-level effects approach within the development of the NLME model. The general form of the NLME model can be expressed as:

$$\begin{cases} y_{ij} = f\left(\varphi_{ij}, x_{ij}\right) + \varepsilon_{ij} \\ i = 1, \ldots, M,\ j = 1, \ldots, N \\ \varphi_{ij} = A_{ij}\beta + B_{ij}u_i \\ u_i \sim N(0, D) \\ \varepsilon_{ij} \sim N\left(0, \sigma^2 G_i^{0.5} \Gamma_i G_i^{0.5}\right) \end{cases} \tag{1}$$

where y_{ij} is the observed length of the j branch of the i tree; M is the number of trees; N is the number of branches of the i tree; f is a nonlinear model of branch length growth containing a parameter vector φ_{ij} and a variable x_{ij}; ε_{ij} is the random error term; β is a fixed effects parameter vector; A_{ij} and B_{ij} are design matrices for β and u_i, respectively; u_i is a random effects parameter vector for the i tree; D is a generalized positive definite matrix structure representing the variance–covariance matrix of u_i [40]; σ^2 is the residual variance of the model; $G_i^{0.5}$ is an $n_i \times n_i$ diagonal matrix used to describe the heterogeneity of variation within groups; and Γ_i is an $n_i \times n_i$ diagonal matrix used to describe the structure of the error correlation within groups.

2.4.3. Nonlinear Quantile Regression Model

The NQR model can be used to estimate the complete conditional distribution of the dependent variable, to evaluate the differences in predictor variables at different quantiles [41], and to analyze the effects of specific quantiles on the distribution of the dependent variable [34]. In addition, the NQR model is insensitive to outliers and will continue to maintain good predictive accuracy under the influence of significant heteroskedasticity [35]. The parameters from the NQR model are obtained by minimizing Equation (2):

$$S = \sum_{y \geq \overline{y}_\tau} \tau(y - \overline{y}_\tau) + \sum_{y < \overline{y}_\tau} (1 - \tau)(\overline{y}_\tau - y) \tag{2}$$

where S represents the sum of weighted absolute residuals at the quantile; τ (0.1, 0.2, 0.3, . . . , 0.9); y is the measured branch length; and $\overline{y}_\tau$ is the predicted branch length at the corresponding quantile.

2.4.4. Model Fitting and Evaluation

The present study fitted the NLS, NLME, and NQR models for the GFBL using the Proc NLIN, Proc NLMIXED, and Proc NLP procedures in the SAS 9.4 statistical analysis software package. The accuracy of the model fitting was evaluated using the determination coefficient (R^2), adjusted determination coefficient (R^2_{adj}), mean square error (MSE), mean percentage of bias (MPB), root mean square error (RMSE), Akaike's information criterion (AIC), twice the negative log-likelihood (−2LL), and the Bayesian information criterion (BIC) ((Equation (3)) to (Equation (10))). These statistical methods are commonly used to evaluate the accuracy of model fitting.

$$R^2 = 1 - \left[\frac{\Sigma_{i=1}^{n}(y_i - \hat{y})^2}{\Sigma_{i=1}^{n}(y_i - \overline{y})^2}\right] \tag{3}$$

$$R^2_{adj} = 1 - \frac{n-1}{n-p}\left(1 - R^2\right) \tag{4}$$

$$\text{MES} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-1} \quad (5)$$

$$\text{RMES} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-1}} \quad (6)$$

$$\text{MPB} = 100 \times \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{\sum_{i=1}^{n} y_i} \quad (7)$$

$$\text{AIC} = -2\ln L + 2p \quad (8)$$

$$\text{BIC} = -2\ln(L) + p\ln(n) \quad (9)$$

$$-2\text{LL} = -2\ln(L) \quad (10)$$

In Equation (3) to Equation (10), y_i represents the observed branch length (m); $\overline{y_i}$ represents the mean branch length (m); $\hat{y}_i$ represents the predicted branch length (m); n is the number of branch samples; L is the maximum likelihood value; and p represents the number of model parameters.

3. Results

3.1. Development of the GFBL Model

The present study fitted five models describing the GFBL of the *L. principis-rupprechtii* plantation using the NLS method (Table 4). The Chapman–Richards growth function was found to be optimal for describing the GFBL of the *L. principis-rupprechtii* plantation, with model R^2_{adj}, MES, RMES, and MPB of 0.7716, 0.1172, 0.3424, and 1.1176, respectively. Re-parameterizing the maximum parameter of free branch length (a_0) in the Chapman–Richards growth function as a function of the DBH and DINC (regression coefficients of the DBH and DINC of b_0 and b_1, respectively) (Table 5) resulted in the highest model fitting accuracy, with R^2_{adj}, MES, RMES, and MPB of 0.8505, 0.0853, 0.2920 and 0.9500, respectively. Therefore, the GFBL base model developed in the present study was:

$$\text{BL} = \left(0.0161 \times \text{DBH} + \text{DINC}^{0.4002}\right)\left(1 - e^{-0.1289 \times \text{BAGE}}\right)^{0.6110} \quad (11)$$

where BL is free branch length (m), DBH is the diameter at breast height (cm), DINC is the depth of the branch into the crown (m) and BAGE is the age of the free branch.

Table 4. Parameter estimates and fitting statistics for the base model of growth of free branch length.

Equation	Parameter Estimates			Fitting Statistics				
	a_0	a_1	a_2	MSE	RMES	MPB	R^2	R^2_{adj}
Gomboze	2.8448 (0.0553)	2.2979 (0.0481)	0.1504 (0.0057)	0.1332	0.3650	1.1914	0.7580	0.7578
Logistic	2.6453 (0.0393)	5.8033 (0.2144)	0.2347 (0.0072)	0.1376	0.3709	1.2109	0.7501	0.7499
Schumacher	3.1633 (0.0385)	5.7226 (0.1155)		0.1474	0.3840	1.2533	0.7321	0.7320
Mitscherlich	3.5633 (0.1686)	0.9790 (0.0087)	0.0630 (0.0053)	0.1382	0.3718	1.2135	0.7424	0.7421
Chapman-Richards	3.9423 (0.3057)	0.0486 (0.0083)	0.8864 (0.0409)	0.1172	0.3424	1.1176	0.7718	0.7716

Note: In parentheses is the standard error.

Table 5. Statistical analysis of the fit of the diameter at breast height (DBH) and depth of branch into crown (DINC) to the parameters of the Chapman–Richards growth function.

Impact Factor Position	Fitting Statistics				
	MSE	**RMES**	**MPB**	**R^2**	**R^2_{adj}**
$BL = (b_0 \times DBH)\left(1 - e^{-a_1 \times BAGE}\right)^{a_2}$	0.1257	0.3546	1.1506	0.7827	0.7822
$BL = \left(DINC^{b_0}\right)\left(1 - e^{-a_1 \times BAGE}\right)^{a_2}$	0.1019	0.3193	1.036	0.8238	0.8234
$BL = a_0\left(1 - e^{-(b_0 \times DBH) \times BAGE}\right)^{a_2}$	0.1089	0.3299	1.0706	0.8119	0.8114
$BL = a_0\left(1 - e^{-(b_0 \times DINC) \times BAGE}\right)^{a_2}$	0.1117	0.3343	1.0847	0.8069	0.8064
$BL = \left(b_0 \times DBH + DINC^{b_1}\right)\left(1 - e^{-a_1 \times BAGE}\right)^{a_2}$	0.0853	0.2920	0.9500	0.8519	0.8505
$BL = (b_0 \times DBH)\left(1 - e^{-(b_1 \times DINC) \times BAGE}\right)^{a_2}$	0.2380	0.4879	1.5868	0.5868	0.5858
$BL = \left(DINC^{b_0}\right)\left(1 - e^{-(b_1 \times DBH) \times BAGE}\right)^{a_2}$	0.1019	0.3193	1.0383	0.8230	0.8226
$BL = a_0\left(1 - e^{-(b_0 \times DBH + b_1 \times DINC) \times BAGE}\right)^{a_2}$	0.2658	0.5156	1.6769	0.5384	0.5341

Note: DBH is the tree diameter at breast height (cm); DINC is the depth of branch into crown (m); a_0, a_1, a_2, b_0 and b_1 are the model parameters to be estimated.

3.2. Inclusion of the Competition Index into the GFBL Model

The present study considered the effect of branch competition on the maximum GFBL and growth rate by evaluating the effects of five branch competition indices on the maximum branch length parameter (a_0), growth rate parameter (a_1), maximum value parameter (a_0), and growth rate parameter (a_1) of the GFBL base model (Equation (11)) and the accuracy of the model fit (Table 6). The CI was shown to be the optimal competition index for explaining branch competition among free branches; the CI (the regression coefficient of the CI was b_2), along with the DBH and DINC, showed the highest performance in the model when acting on equation a_0. The R^2_{adj}, RMES, and MPB of the model were 0.8870, 0.2507, and 0.8151, respectively. Therefore, the present study constructed the model of GFBL including branch competition as:

$$BL = \left(0.0162 \times DBH + DINC^{0.4001} - 0.0004 \times CI\right)\left(1 - e^{-0.1290 \times BAGE}\right)^{0.6092} \quad (12)$$

where CI is the branch competition index among free branches.

Table 6. Fitting statistics of different branch competition indices acting on different parameters of branch growth.

Competition Index Position	Competition Index	Fitting Statistics			
		RMES	**MPB**	**R^2**	**R^2_{adj}**
$BL = \left(b_0 \times DBH + DINC^{b_1} + b_2 \times CI_0\right)\left(1 - e^{-a_1 \times BAGE}\right)^{a_2}$	CI	0.2507	0.8151	0.8881	0.8870
	CA	0.2710	0.8812	0.8754	0.8742
	SLL	0.2851	0.9274	0.8621	0.8607
	CD	0.2906	0.9449	0.8567	0.8553
	DR	0.3871	1.2589	0.7458	0.7433
$BL = \left(b_0 \times DBH + DINC^{b_1}\right)\left(1 - e^{-(b_2 \times CI_0) \times BAGE}\right)^{a_2}$	CI	0.2683	0.8721	0.8716	0.8707
	CA	0.3049	0.9919	0.8361	0.8345
	SLL	0.4313	1.4027	0.6776	0.6745
	CD	0.2971	0.9659	0.8434	0.8419
	DR	0.2783	0.9049	0.8667	0.8663
$BL = \left(b_0 \times DBH + DINC^{b_1} + b_2 \times CI_0\right)\left(1 - e^{-(b_3 \times CI_0) \times BAGE}\right)^{a_2}$	CI	0.2882	0.9198	0.8557	0.8547
	CA	0.3134	1.0192	0.8294	0.8278
	SLL	0.3084	1.0030	0.8347	0.8336
	CD	0.5935	1.9305	0.3882	0.3824
	DR	0.4020	1.3075	0.7193	0.7167

Note: b_0, b_1, b_2, b_3, a_1 and a_2 are the model parameters to be estimated; CI_0 is the branch competition index.

3.3. Nonlinear Mixed-Effects Model of the GFBL

The present study constructed a nonlinear mixed effect model of the GFBL in an *L. principis-rupprechtii* plantation, which included the branch competition index (CI) and 15 different forms of random effect parameters, with 9 forms converging (Table 7). An optimal equation performance was obtained by introducing random effect parameters into the DINC (b_1) and CI (b_2), with an AIC, BIC, and -2LL of 947.8, 966.7, and 929.8, respectively. Table 8 shows the parameter estimates of the nonlinear mixed-effects GFBL model. The R^2_{adj}, MES, RMES, and MPB of the model were 0.9076, 0.2327, 0.0542, and 0.7572, respectively.

Table 7. Fitting statistics of nonlinear mixed-effects models for different random combination forms. Abbreviations: Akaike's information criterion (AIC), Bayesian information criterion (BIC), and twice the negative log-likelihood (-2LL).

Random Effect Parameter Position	AIC	BIC	-2LL
b_0	1146.3	1160.9	1132.3
b_1	1173.3	1188.0	1159.3
b_2	1045.5	1060.1	1031.5
a_2	1041.4	1056.1	1027.4
b_0, b_1	1055.7	1074.5	1037.7
b_0, b_2	1024.3	1043.1	1006.3
b_0, a_2	954.8	973.6	936.8
b_1, b_2	947.8	966.7	929.8
b_1, a_2	968.1	987.0	950.1

Table 8. Parameter estimates for the branch growth equation including the branch competition based on the nonlinear mixed effects model.

Parameter	Estimate	Standard Error	95% Confidence Interval		Value of *p*
b_0	0.0208	0.0031	0.0270	0.0147	<0.001
b_1	0.3833	0.0152	0.4132	0.3533	<0.001
b_2	-0.0123	0.0034	-0.0057	-0.019	<0.001
a_1	0.1445	0.0208	0.1854	0.1037	<0.001
a_2	0.6545	0.6969	0.7912	0.5178	<0.001
$Var(u_1)$	0.0772	0.0305	0.0587	0.1016	0.003
$Var(u_2)$	0.0196	0.0119	0.0147	0.0260	0.041
$Cov(u_1, u_2)$	-0.1241	0.0008	-0.4489	-0.1241	0.004
σ^2	0.3110	0.0118	0.3001	0.3224	<0.001

Note: u_1 and u_2 are random parameter vectors; $Var(u_1)$ and $Var(u_2)$ are the variances of u_1 and u_2, respectively; $Cov(u_1, u_2)$ is the covariance between the random effects; σ^2 is the residual variance.

3.4. Nonlinear Quantile Regression Model of GFBL

The present study constructed a model of the GFBL at different quantile points in an *L. principis-rupprechtii* plantation based on the NRQ and including the branch competition index (CI) (Table 9). The highest accuracy of the branch length growth model was achieved for the quantile $\tau = 0.5$, with an R^2_{adj}, MES, RMES, and MPB of 0.8932, 0.2390, 0.0572, and 0.7775, respectively.

Table 9. Parameter estimates and fitting statistics for the branch growth equation including branch competition and based on nonlinear quantile regression.

Parameter	τ	τ	τ	τ	τ	τ	τ	τ	τ
b_0	0.0040 **	0.0053 ***	0.0100 ***	0.0106 ***	0.0084 ***	0.0156 **	0.0148 **	0.0143 **	0.0164 *
b_1	0.3311 **	0.3437 ***	0.3922 ***	0.3962 ***	0.4106 ***	0.4317 ***	0.4529 **	0.4714 **	0.4890 ***
b_2	-0.0006 ***	-0.0030 **	-0.0035 **	-0.0085 **	-0.0198 ***	-0.0212 ***	-0.0236 **	-0.0253 **	-0.0266 *
a_1	0.0822 ***	0.1438 **	0.1073 ***	0.1351 ***	0.0438 ***	0.1440 ***	0.1571 ***	0.1804 ***	0.2154 *
a_2	0.6019 ***	0.7460 *	0.6091 ***	0.6452 **	0.5243 ***	0.6310 ***	0.6125 ***	0.5954 ***	0.5738 ***

Table 9. *Cont.*

Parameter	τ	τ	τ	τ	τ	τ	τ	τ	τ
MSE	0.3223	0.1798	0.1127	0.0892	0.0572	0.0788	0.1353	0.1972	0.3286
RMSE	0.5678	0.4240	0.3356	0.2987	0.2390	0.2807	0.3678	0.4440	0.5732
MPB	1.8466	1.3792	1.0916	0.9716	0.7775	0.9133	1.1965	1.4443	1.8646
R^2	0.4403	0.6885	0.8044	0.8450	0.8942	0.8617	0.7650	0.6575	0.4294
R^2_{adj}	0.4350	0.6856	0.8025	0.8435	0.8932	0.8604	0.7627	0.6543	0.4240

Note: "***" < 0.001, "**" < 0.01, "*" < 0.05. τ is different quantiles.

3.5. Modeling Verification and Evaluation

The results of the statistical evaluation metrics, including the MSE, RMSE, MPB, R^2, and R^2_{adj} (Table 10), along with the results of the residual analysis (Figure 2), indicated that the NLME model incorporating branch competition (CI) was optimal for predicting the GFBL in an *L. principis-rupprechtii* plantation. The present study predicted and analyzed the GFBL using various modeling methods (Figure 3). Notably, the NLS and NQR models (τ = 0.5) showed good predictive accuracy for a branch age range of 1–10 a, whereas the performance of the NQR model slightly exceeded that of the NLS model for a branch age range of 11–23 a. However, the NLME model offered the most accurate prediction of the GFBL across different ages when compared to the NLS and NQR models (τ = 0.5).

Table 10. Evaluation of the predictive accuracy of different parameter estimation methods for the free branch length growth model.

Parameter	NLS	Include Branch Competition Index		
		NLS	NLME	NQR (τ = 0.5)
MSE	0.0853	0.0629	0.0542	0.0572
RMSE	0.2921	0.2507	0.2327	0.2390
MPB	0.9500	0.8151	0.7572	0.7775
R^2	0.8518	0.8881	0.9085	0.8942
R^2_{adj}	0.8504	0.8871	0.9076	0.8932

Note: NLS is nonlinear least squares model; NLME is nonlinear mixed-effects model; NQR is nonlinear quantile regression model; τ is different quantiles.

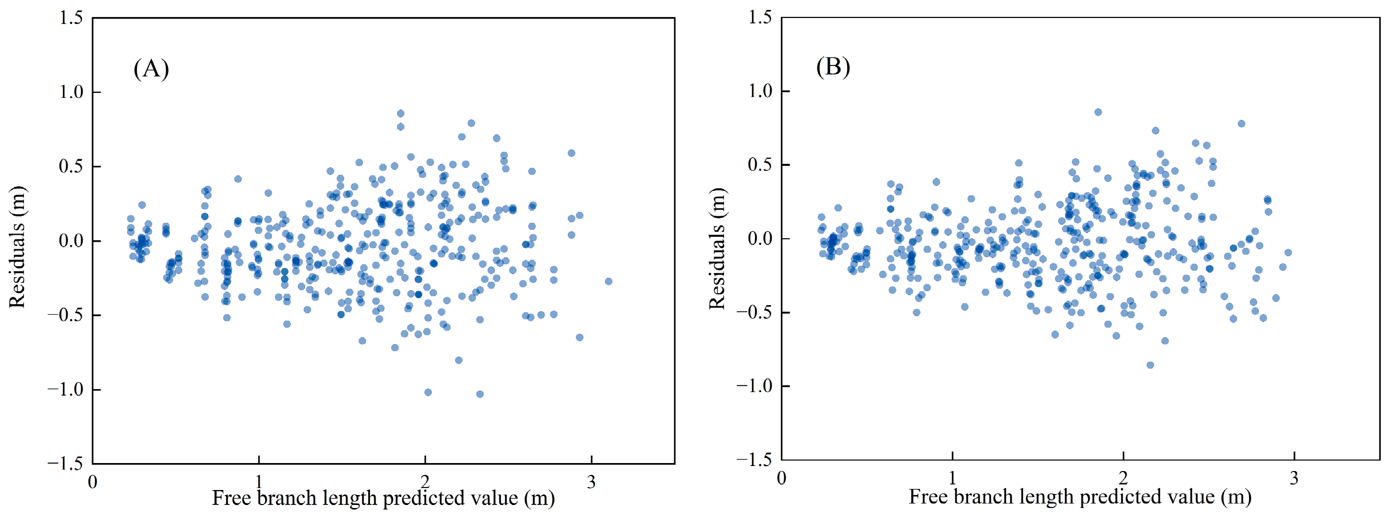

Figure 2. *Cont.*

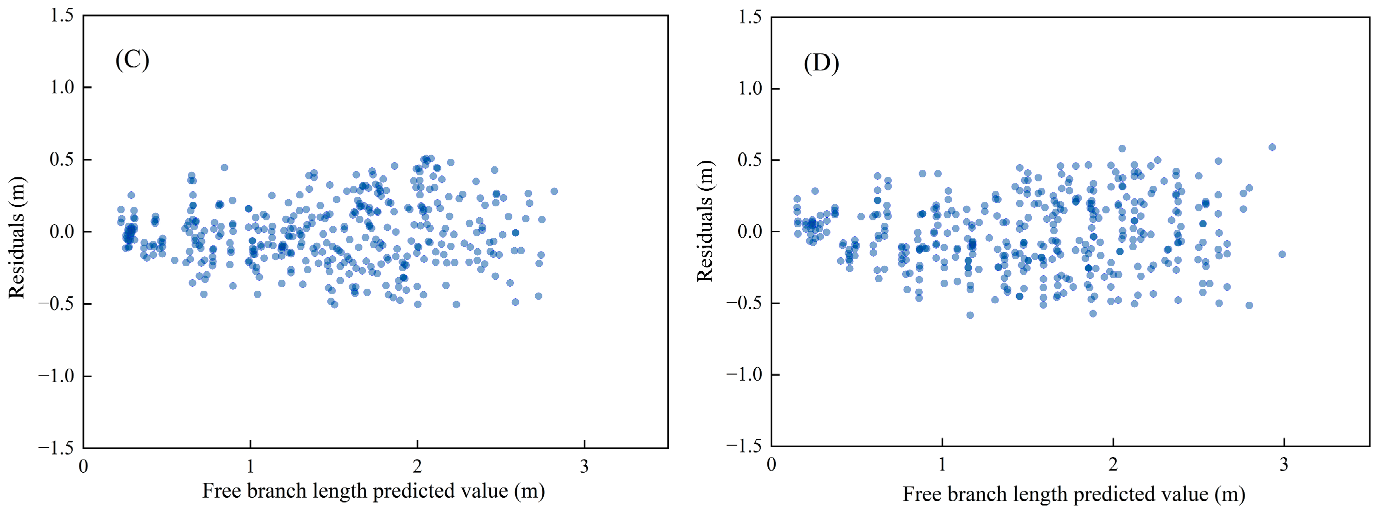

Figure 2. Residuals of the free branch length growth models with different parameter estimation methods. Nonlinear least squares (NLS) model without branch competition (**A**); NLS with branch competition (**B**); nonlinear mixed-effects (NLME) model with branch competition (**C**); and nonlinear quantile regression (NQR) model with branch competition (**D**).

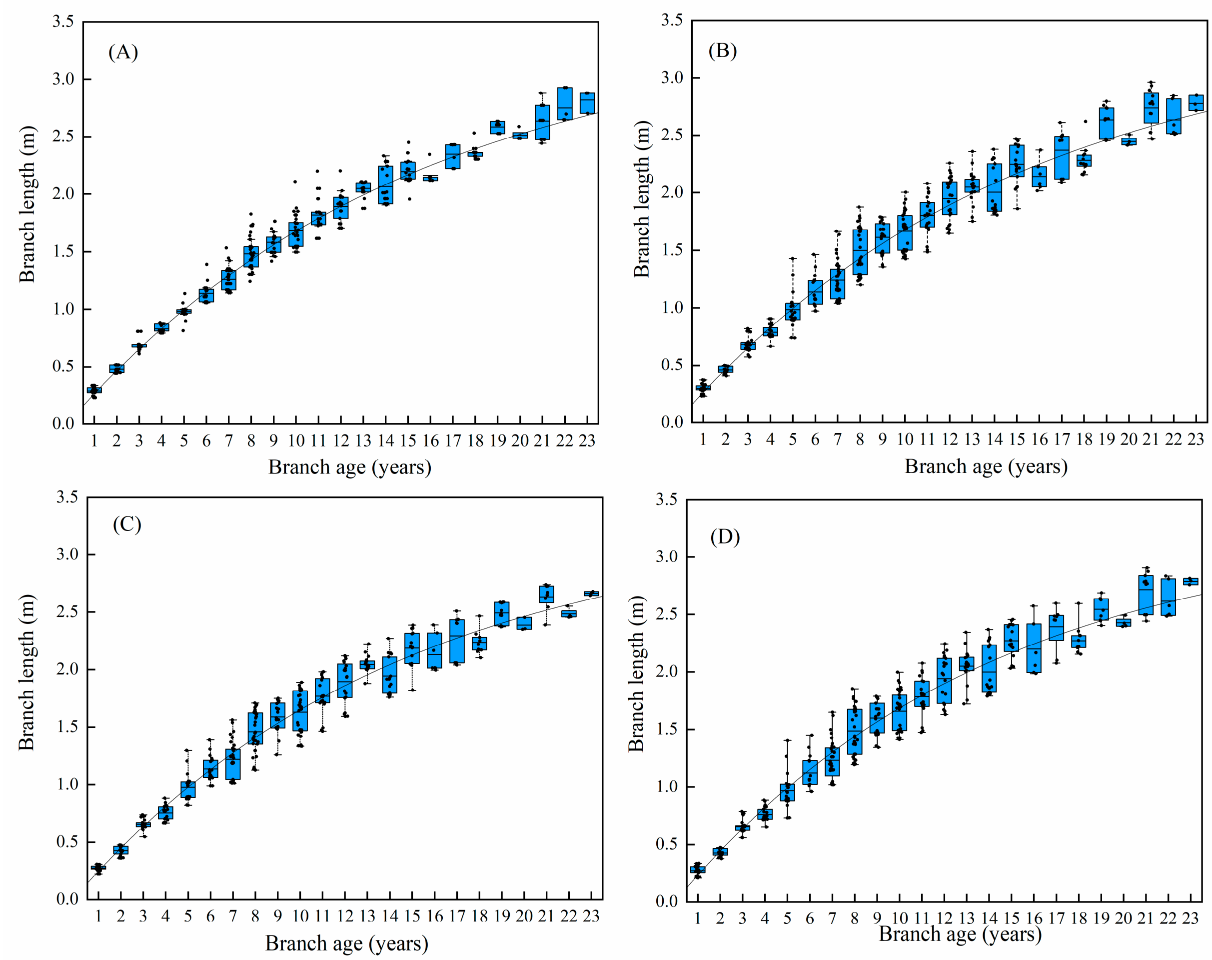

Figure 3. Estimates of the free branch length growth models based on different parameter estimation methods. Nonlinear least squares (NLS) model without branch competition index (**A**); NLS with branch competition index (**B**); nonlinear mixed-effects (NLME) model with branch competition (**C**); and nonlinear quantile regression (NQR) model with branch competition (**D**).

The present study applied the developed nonlinear mixed-effects GFBL model including branch competition to separately simulate the effects of the DBH, DINC and branch competition index (CI) on the GFBL (Figure 4). For trees with the same DBH at the same DINC, the GFBL decreased with an increasing CI (Figure 4A); for branches with the same DINC and CI, the GFBL increased with an increasing DBH (Figure 4B); and for branches with the same CI in trees with the same DBH, the GFBL increased with an increasing DINC (Figure 4C).

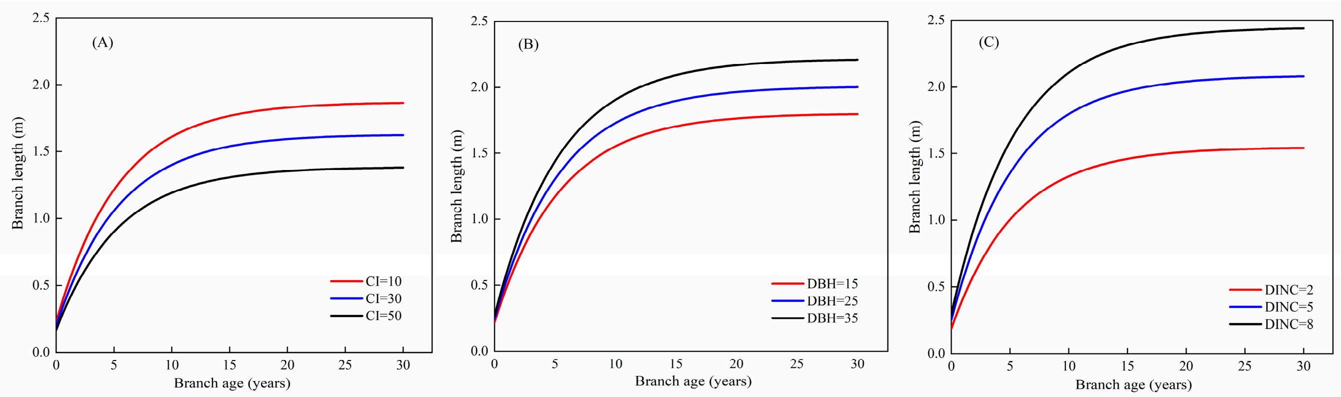

Figure 4. Effects of the branch competition index (CI) (**A**), diameter at breast height DBH (cm) (**B**), and depth of branch into crown DINC (m) (**C**) on the growth of free branch length.

4. Discussion

The present study aimed to investigate the effects of different branch competition indices and regression techniques on the accuracies of GFBL models. Based on the selection of an optimal base model from among five different theoretical models, the effects of five different branch competition indices on the branch growth patterns and model accuracy were compared, and the fitting accuracies of three regression techniques were evaluated.

4.1. Base Model for the GFBL

The present study showed that the Chapman–Richards growth function is optimal for describing the GFBL in an *L. principis-rupprechtii* plantation. In this study, the fitting accuracy of five commonly used growth equations with ecological significance was compared, and it was found that the Chapman–Richards growth function had the highest fitting accuracy (Table 5). Our results were consistent with previous similar studies conducted by Li et al. [29] and Wang et al. [15], who also used the Chapman–Richards growth function to study branch growth in *Larix olgensis* and *P. koraiensis* plantations. This could be attributed to the fact that the Chapman–Richards growth function is more suitable for describing the branch growth of coniferous tree species in plantation forests [42]. However, our study results differ from Dong et al. [30], who used the Weibull growth function to study the branch growth of *P. koraiensis* under mixed forests. The discrepancies between our findings and Dong et al. [30] may be attributed to the variations in the forest types and the biological characteristics of the tree species [13]. Therefore, models of branch growth under different tree species and forest types should be explored in future studies to increase our understanding of the complexity of branch growth.

4.2. Influence Factors of the GFBL

In this study, the DBH and DINC are identified as the influencing factors of the GFBL in the *L. principis-rupprechtii* plantation. Weiskittel et al. [12] observed that branches and the DBH have the same growth trend in a wide range of silviculture treatments. Therefore, in this study, the DBH was considered as the influencing factor of the GFBL, reflecting tree size and vitality and explaining the influence of stand conditions on the GFBL [8,12,16]. The DINC can reflect the growth state and environment of branches at different locations within the crown [42], and it is crucial in quantifying the competition effects of free branches [19,42]. Therefore, this study used the DBH and DINC as influencing factors of the GFBL. Except for the DBH and DINC, the TH was identified as an important variable for the branch growth model [13,16]. However, the TH has no significant effect on branch growth in plantation forests [15,42,43], because the variation in the TH within plantation forests usually is negligible [42] and the TH not significant for the GFBL [43]. Therefore, the present study did not examine the effect of the TH on the GFBL in the *L. principis-rupprechtii* plantation. Future studies should further analyze the impact of the TH on the GFBL.

The DBH and DINC simultaneously applied to the maximum parameter (a_0) and the GFBL model had the highest fitting accuracy. The results show that when the DBH and DINC are simultaneously associated with the maximum parameter value (a_0) (Table 5), the model achieves the highest accuracy, with an increase of 7.89% in the R^2_{adj} value. This finding is consistent with the finding of Wang et al. [15] in constructing a branch length growth model for *P. koraiensis*. The positive values of the parameters b_0 and b_1 in Equation (12), which are associated with the DBH and DINC, indicate that larger DBHs and DINCs are related to larger branch sizes (Figure 4). However, in Equation (11), the DINC is expressed as $DINC^{b_1}$, which showed that the contribution of different DINCs to the growth of the GFBL was different [16]. With the increase in the DINC, the branches experience a decrease in light exposure and nutrient availability [8], resulting in a proportional decrease in their contribution to the GFBL as the DINC increases [16].

4.3. Response of the GFBL to Competition

The CI was the optimal competition index to describe free branch competition. Competition indices that are distance-dependent are generally more effective than those that are distance-independent [23,24]. This finding was confirmed in the present study, where distance-dependent indices (CI, CA, SLL) were preferred over distance-independent ones (CR, CD) (Table 6). The CI also exhibited higher predictive accuracy compared to the CA, SLL, CR, and DR (Table 6). Long et al. [24] observed that the CI is the optimal index for measuring the degree of competition between the target and competing branch for shared resources, and for quantifying the growth space of branches to explain the degree of competition between the target and competing branch [23], due to its precision and effectiveness [44,45]. Therefore, the CI is used as a competition index to quantify free branch competition in this study. However, in other related studies, different competition indices have been used to quantify competition [11]. This may be because the effectiveness of different competition indices is often site-specific and varies depends on forest types and site conditions [46].

The CI was applied to the maximum parameter (a_0) and the GFBL model had the highest fitting accuracy. The results show that when the CI was associated with the maximum parameter value (a_0) (Table 6), the model achieved the highest accuracy, with an increase of 3.67% in the R^2_{adj} value. These findings of this study align with those reported by Gao et al. [13]. In Equation (12), the CI parameter b_2 is negative, indicating that increasing competitive pressure can lead to a slowing of the GFBL and even to death and defoliation (Figure 4A) [13,21,22,47], which is consistent with the conclusions of Hein et al. [8] and Weiskittel et al. [10]. In addition, unequal distribution of nutrients among branches [48] and the influence of hormones on branch growth [49] may be important factors leading to this phenomenon.

4.4. Regression Techniques Affect Accuracy of the GFBL

This study compares the predictive ability of the NLS, NQR and NLME regression techniques. Although the NLS method is the most widely used method to construct branch growth models [16,18,29,42], the NLS method requires data that must meet the assumption of independent error terms [30,50], which results in lower predictive accuracy than the NQR and NLME methods (Table 10). However, the NRQ and NLME methods can effectively solve these problems [16,31,35].

The NQR method has high flexibility because all the parameters of the regression based on various quantiles are different [35]. By comparing the predictive accuracy of different quantile points, it is found that the NQR method has the highest predictive accuracy when $\tau = 0.5$ (Table 9). This is the same result as obtained by Xu et al. [31] and Cao et al. [35]. However, the NQR method treats the data from each individual free branch as independent from each other [35], ignoring the hierarchical structure of the data (branches within tree) [31], resulting in the GFBL curves of different trees following the basic shape specified by the NQR method [16]. Despite this limitation, the NQR method still has its own advantages in analyzing the distribution of the dependent variable under specific quantiles [16,32].

The NLME method can solve the problem of the NQR method ignoring the hierarchical structure of the data [31]. In this study, the tree was used as the grouping variable for random effects in the NLME model. The fitting effects of different combinations of random effect parameters were compared using the AIC, BIC, and −2LL values (Table 7). It was observed that models with more random effect parameters yielded superior fitting effects, which aligns with previous findings by Wang et al. [15] and Dong et al. [42]. The most accurate fitting results were achieved when random effects parameters were applied to b_1 and b_2. Furthermore, by employing a variance–covariance structure to adjust the values of random effects parameters (Table 8), the NLME model was able to predict the GFBL of each tree, leading to more specific and realistic predictions [15,16,30,35].

Compared to the NLS and NQR models ($\tau = 0.5$), the NLME model showed the best fitting accuracy for modeling the GFBL in an *L. principis-rupprechtii* plantation (Table 10). The NLME model can significantly alleviate the residual of the model [15,30]. This finding was confirmed in the present study, in which the residual values were significantly reduced in the GFBL model after the inclusion of random parameters (Figure 2). Notably, the NLS and NQR models ($\tau = 0.5$) performed well for the BAGE between 1–10 a, with the NQR model slightly outperforming the NLS model for the BAGE 11–23 a. However, the NLME model provided the most accurate prediction of the GFBL across all the BAGE compared to the NLS and NQR models ($\tau = 0.5$) (Figure 3). These results were similar to findings obtained by Miao et al. [16]. However, there was an unexplained error of 9.92% remaining in this study, and in future research, the effects of factors such as genetics [42] and climate [18] can be considered in the model.

5. Conclusions

The present study aimed to investigate how competition between free branches and different regression techniques affect the predictive accuracy of the GFBL model for an *L. principis-rupprechtii* plantation. The Chapman–Richards growth function considering the DBH and DINC was the most effective for describing the GFBL in the *L. principis-rupprechtii* plantation. The CI was the most effective index for quantifying branch competition among the CA, SLL, DR, and CD competition indices, and the model that incorporated the CI in the maximum value parameter (a_0) of the Chapman–Richards growth function demonstrated higher predictive accuracy. Furthermore, the NLME model accounting for branch competition (CI) showed the best fitting accuracy for modeling the GFBL in an *L. principis-rupprechtii* plantation when compared to that of the NLS and NQR models. The results of the present study can provide a scientific basis for improving the management measures, assessing management effectiveness, and enhancing the quality of *L. principis-rupprechtii* plantations.

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