World Electric Vehicle Journal

MDPI

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The Prediction Method and Application of Off-Road Mobility for Ground Vehicles: A Review

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Abstract: With the rapid advancement of technologies related to unmanned ground systems, ground vehicles are being widely deployed across various domains. However, when operating in complex, soft terrain environments, the low bearing capacity of such terrains poses a significant challenge to vehicle mobility. This paper presents a comprehensive review of mobility prediction methods for ground vehicles in off-road environments. We begin by discussing the concept of vehicle mobility, followed by a systematic and thorough summary of the primary prediction methods, including empirical, semi-empirical, numerical simulation, and machine learning approaches. The strengths and weaknesses of these methods are compared and analyzed in detail. Subsequently, we explore the application scenarios of mobility prediction in military operations, subsea work, planetary exploration, and agricultural activities. Finally, we address several existing challenges in current mobility prediction methods and propose exploratory research directions focusing on key technologies and applications, such as real-time mobility prediction, terrain perception, path planning on deformable terrain, and autonomous mobility prediction for unmanned systems. These insights aim to provide valuable reference points for the future development of vehicle mobility prediction methods.

Keywords: soil terrain; terramechanics; mobility prediction; off-road; numerical simulation

Academic Editor: Grzegorz Sierpiński

Received: 19 December 2024
Revised: 14 January 2025
Accepted: 16 January 2025
Published: 19 January 2025

Citation: Hua, C.; Zhang, W.; Fu, H.; Zhang, Y.; Yu, B.; Jiang, C.; Wei, Y.; Chen, Z.; Kuang, X. The Prediction Method and Application of Off-Road Mobility for Ground Vehicles: A Review. *World Electr. Veh. J.* **2025**, *16*, 47. https://doi.org/10.3390/wevj16010047

1. Introduction

Research on ground vehicle mobility began in the 1950s, driven by the need to address vehicle terrain navigability in military operations. This demand led to extensive investigations by organizations such as the "Military Vehicles Experimental Mud Research Committee" in the United Kingdom, the "SAE War Emergency Committee" of the United States Army Ordnance Department, the "National Research Council" in Canada, and research institutes in the former Soviet Union [1]. In recent years, with the rapid advancements in military intelligence, agricultural mechanization, and deep space and deep-sea exploration, the requirements for ground vehicle mobility have become increasingly critical and urgent.

In the military domain, vehicles operating in the field often encounter various unknown and complex terrain environments. To ensure the mobility of military tactics, it is crucial to predict how these machines will perform in such areas. This need has become even more pressing with the rapid development of unmanned ground technologies in recent years, where effective prediction of autonomous mobility presents a significant challenge in military intelligence. In the agricultural sector, enhancing the intelligence of agricultural machinery to achieve precision operations and optimize driving strategies requires active research into mobility prediction for such equipment. In the resource extraction industry, significant reserves of oil, natural gas, and minerals are often located in deserts, swamps, mudflats, and permafrost regions, where highly mobile machinery is essential for transportation and operational support. In deep space exploration, the lunar and Martian surfaces are characterized by soft, sandy soils, making the prediction of planetary exploration vehicles' mobility a crucial element in ensuring the reliable operation of these systems during expensive space missions. Clearly, the study of mobility prediction methods is vital for advancing national defense, agriculture, the economy, and space exploration.

Current research has seen some scholars conducting review studies on methods for predicting ground vehicle mobility. Ref. [2] primarily focuses on practical applications and introduces empirical methods for predicting vehicle mobility, noting that numerical simulation-based prediction methods are likely to be the future development trend. Ref. [3] provides a detailed analysis of the cone index vehicle mobility prediction method based on empirical models. Ref. [4] offers an overview of prediction methods for military vehicle mobility, with a particular emphasis on "vehicle–terrain analysis" approaches. Ref. [5] reviews research on the ground mechanics of tracked robots, highlighting the characteristics of ground mechanical responses in semi-empirical models.

Based on an extensive literature review and related research efforts, this paper first provides a comprehensive and systematic explanation of the definition and scope of mobility, integrating the latest advancements in relevant fields. It then analyzes and summarizes the primary methods currently employed for predicting mobility. Following this, the paper discusses the applications of these prediction methods across various domains. Finally, the paper concludes with a summary and outlook, proposing key research directions for future work in vehicle mobility prediction, aiming to offer valuable references for further advancements in this research area.

2. The Concept of Mobility

Ground vehicle mobility primarily refers to the ability of a vehicle to travel rapidly across various potential road, surface, and terrain conditions [6]. The United States Army Corps of Engineers characterizes ground vehicle mobility by using the maximum achievable speed between two points within a given area as a fundamental metric [7]. This maximum achievable speed serves as a highly integrated parameter representing the complex interactions between the vehicle and its operating environment.

When a vehicle traverses a certain area, its maximum achievable speed may be influenced by one or more of the following factors [8]: (1) the driving force required to overcome resistances such as sinkage, slopes, obstacles, and vegetation, (2) the driver's tolerance limits for discomfort when traversing uneven terrain and for collisions with obstacles, (3) due to the limited visibility ahead on the road, the driver's reluctance to exceed the speed at which they can stop in time, and (4) reductions in speed due to acceleration, deceleration, and maneuvering to avoid obstacles.

By calculating and comparing the effects of each of these factors on speed, one can determine the maximum achievable speed of a vehicle within a specific area. The results of this analysis can be represented using a mobility map, as shown in Figure 1. In Figure 1, numbers indicate the maximum speed achievable by a particular vehicle across various sections of the area, while shaded regions represent areas that are impassable. This information provides a basis for selecting the optimal path, thereby maximizing the average speed of the vehicle through a given region [9].

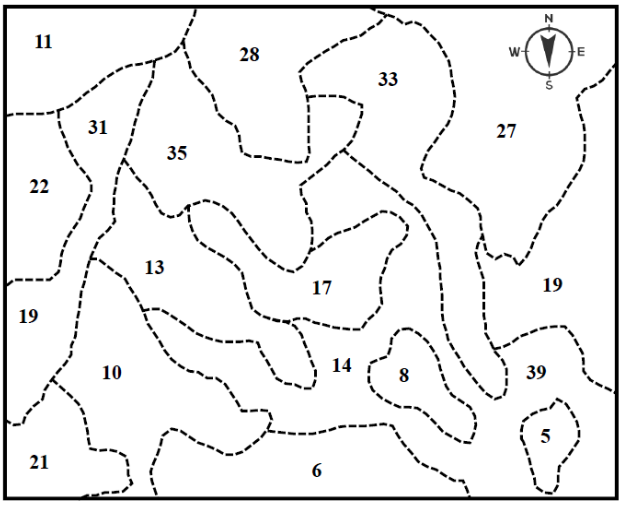

Figure 1. Mobility map; the numbers represent the vehicle speed, with the units in km/h.

3. Mobility Prediction Methods

Early mobility prediction methods were based on empirical data accumulated from long-term trials. These empirical data were used to derive empirical formulas for predicting a vehicle's performance in traversing a given area, representing a theoretical approach based on empirical models.

In the United States and some Northern European countries, the Vehicle Cone Index (VCI) is used as a mobility indicator. In the United Kingdom, the mean maximum pressure (MMP) is employed as a measure of tracked vehicle mobility. Both the VCI and MMP are theoretical methods based on empirical models.

With the advancement of research, in the 1950s, Bekker from the University of Michigan conducted mechanical studies on the interaction between vehicles and terrain. Based on field test data, Bekker developed approximate simplified formulas for vehicle–terrain interaction and identified the primary soil characteristics affecting vehicle mobility as soil resistance caused by vehicle settlement, and the driving force and slip rate during soil shear. This led to the development of a semi-empirical analytical method for describing ground vehicle mobility [10].

With the development of numerical computation methods and computer technologies, a new research approach emerged in vehicle mobility studies: numerical simulation methods. Utilizing satellite remote sensing data and incorporating soil mechanics, elastoplastic theory, and constitutive relations, this approach includes techniques such as finite element analysis and discrete element analysis. These methods are suited for solving nonlinear relationships between wheels/tracks and soil, addressing issues that traditional mechanics could not analyze, and thus have become increasingly widespread as an effective means for vehicle mobility prediction.

In recent years, with the rapid advancement of artificial intelligence, machine learning methods have effectively improved the efficiency of mobility prediction through training data and algorithms. Data-driven modeling approaches can replace the complex iterative processes of soil mechanics formulas, and are attracting growing attention in computational mechanics. Consequently, machine learning-based mobility prediction methods are expected to be a significant research direction in the future.

3.1. Mobility Prediction Methods Based on Empirical Models

The development of empirical model-based mobility prediction methods relies entirely on data obtained from experimental measurements. The process of establishing an empirical model typically involves the following steps: first, identifying the factors that influence vehicle mobility; second, conducting displacement tests based on these factors; and finally, applying curve-fitting techniques to model the data trends by selecting appropriate functions within a given input range, resulting in an empirical formula. The two most widely used empirical methods are the mobility prediction method based on the Vehicle Cone Index (VCI) and the method based on the mean maximum pressure (MMP).

3.1.1. Mobility Prediction Method Based on the Cone Index

The Cone Index (VCI) and the Reshaped Cone Index (Rating Cone Index, RCI) are used to evaluate soil passability [11]. The VCI represents the minimum strength of the soil in the critical layer that allows a given vehicle to successfully traverse a specified number of passes (typically 1 or 50 passes). The VCI values for 1 pass (VCI_1) and 50 passes (VCI_{50}) can be calculated using empirical formulas based on the Vehicle Mobility Index (MI).

The Vehicle Mobility Index (MI) is related to several factors, including the vehicle's ground pressure coefficient, axle load coefficient, tire/track coefficient, slip coefficient, engine coefficient, and drive coefficient. It reflects the overall mobility performance of the vehicle. The expression for the Vehicle Mobility Index (MI) is given by

$$\mathrm{MI} = \left[\frac{P_{FG} \cdot W}{T \cdot G} + L - H\right] \cdot E \cdot X \tag{1}$$

In the expression, P_{FG} represents the ground pressure coefficient, W is the vehicle weight, E denotes the engine coefficient, G is the track or tread coefficient, T refers to the track or tire coefficient, L indicates the load coefficient, H is the ground clearance, and X represents the drive coefficient.

For a tracked vehicle with a single pass:

$$\mathrm{VCI}_1 = 7 + 2\mathrm{MI} - \left(\frac{39.2}{\mathrm{MI} + 5.6}\right) \tag{2}$$

For a tracked vehicle with 50 passes:

$$\mathrm{VCI}_{50} = 19.27 + 0.43\mathrm{MI} - \left(\frac{125.79}{\mathrm{MI} + 7.08}\right) \tag{3}$$

For a wheeled vehicle with a single pass and $\mathrm{MI} \leq 115$:

$$\mathrm{VCI}_1 = 11.48 + 0.2\mathrm{MI} - \left(\frac{39.2}{\mathrm{MI} + 3.74}\right) \tag{4}$$

For a wheeled vehicle with a single pass and $\mathrm{MI} > 115$:

$$\mathrm{VCI}_1 = 4.1\mathrm{MI}^{0.446} \tag{5}$$

For a wheeled vehicle with 50 passes:

$$\mathrm{VCI}_{50} = 28.23 + 0.43\mathrm{MI} - \left(\frac{92.67}{\mathrm{MI} + 3.67}\right) \tag{6}$$

Once the VCI values (VCI_1 and VCI_{50}) are computed using the MI value, the criteria for predicting vehicle passability are as follows: (1) if $\mathrm{RCI} \geq \mathrm{VCI}_{50}$, the vehicle can pass, (2) if $\mathrm{RCI} \geq \mathrm{VCI}_1$, the vehicle can only pass once, (3) if $\mathrm{RCI} < \mathrm{VCI}_1$, the vehicle cannot pass. Here, the RCI is the ratio of the force required to vertically press a cone penetrometer into the soil at a certain depth to the base area of the cone [11]. The process, as illustrated in Figure 2, involves using input parameters such as vehicle weight, ground pressure, and tire/track parameters to calculate the Vehicle Mobility Index and correction factors. If the vehicle's cone index exceeds the soil cone index, the vehicle can pass. Curve-fitting techniques are then applied to output mobility parameters such as traction force, maximum slip ratio, and rolling resistance.

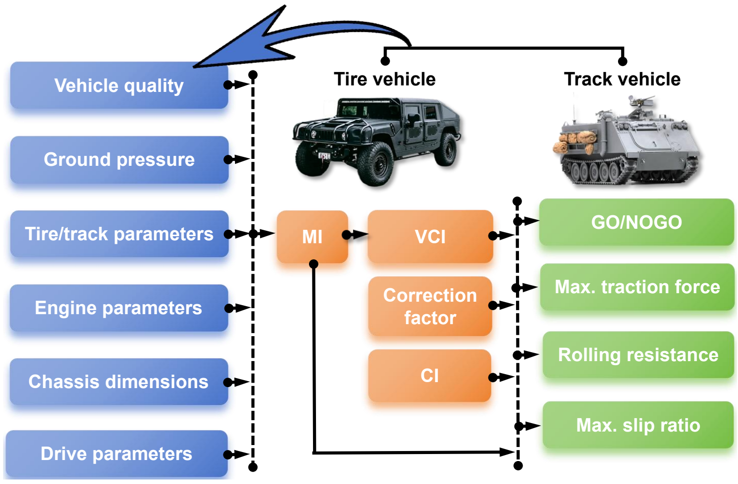

Figure 2. Framework of mobility prediction method based on CI.

3.1.2. Mobility Prediction Method Based on Mean Maximum Pressure

Rowland, in the United Kingdom [12], proposed a mobility prediction method based on the mean maximum pressure (MMP) beneath the road wheels of tracked vehicles. This method calculates the MMP value using tracked vehicle parameters and compares it with experimentally obtained MMP values suitable for tracked vehicle passage on different types of terrain, thereby obtaining the mobility prediction results.

Below is the empirical formula used to predict the MMP value for tracked vehicles with different structures. For vehicles with link tracks and rigid road wheels, the formula is as follows:

$$\mathrm{MMP} = \frac{1.26W}{2n_r \cdot A_1 \cdot b\sqrt{t_1 \cdot D}} \tag{7}$$

For vehicles with band tracks and pneumatic road wheels, the predictive formula is as follows:

$$\mathrm{MMP} = \frac{0.5W}{2n_r \cdot b\sqrt{f_1 \cdot D}} \tag{8}$$

In the formula, n_r is the number of road wheels on a single track, A_1 is the rigid area of the link (or band track), proportional to $b \times t_1$, where b is the track or tire width, and t_1 is the track pitch, D is the diameter of the track's road wheel or tire, and f_1 represents the radial deformation of the tire under load.

To predict whether a specific vehicle with a given MMP value has sufficient mobility on a particular terrain, researchers have compiled the expected mean maximum pressure values required under specific surface conditions, as shown in Table 1.

For certain types of terrain, the impact of vehicle design parameters on the MMP value may not be accurately represented by empirical formulas. Additionally, this method can only be used to predict vehicle performance on soft ground in terms of "GO/NOGO" criteria and cannot quantitatively predict other vehicle performance metrics, such as rolling resistance, driving force, traction force, and traction efficiency under given operational conditions.

Table 1. Large-scale soil modeling technology -level evaluation.

Surface Conditions	MMP ($kN \cdot m^{-2}$)		
	Expected Value (Multi-Pass)	Good Value	Max. Value (Single Pass)
Temperate infiltrated clay soils	150	200	300
Tropical infiltrated clay soils	90	140	240
Marshland	30	50	60
Marshland flow layer	5	10	15
Snowfields	10	25–30	40

In summary, empirical methods are simple and effective for predicting vehicle mobility on soils similar to those used in the tests and are still widely used today. However, these methods only correlate vehicle mobility with indicators like the cone index, simplifying the interaction between the vehicle's mobility systems and the ground, which introduces significant limitations. Ref. [13] noted that the applicability of empirical methods is confined to test scenarios similar to those used to derive the formulas, making these models unsuitable for investigating new mobility system designs and vehicle performance on untested soil conditions. Furthermore, empirical methods are only feasible when the number of variables involved is relatively small.

3.2. Mobility Prediction Method Based on Semi-Empirical Models

Due to the limitations of the aforementioned empirical methods, Bekker developed a parameterized semi-empirical analysis method for ground vehicle performance. This method is based on measurements of ground response characteristics under simulated vehicle loads and the mechanical principles of vehicle–ground interaction. It derives a mechanical model of vehicle–soil interaction that incorporates both vehicle and soil parameters [10]. The method identifies the key soil characteristics affecting vehicle mobility as soil resistance caused by soil settlement under vehicle loads and the driving force and slip rate provided by the soil during shear. In vehicle mobility research, the Bekker apparatus (see Figure 3) is used to measure soil response to loads [14]. This apparatus includes both plate penetration tests and shear tests. In penetration tests, a plate with dimensions similar to the contact area of tracks or wheels is used to measure the relationship between pressure and settlement. In shear tests, a shear ring or shear plate simulates the shear effect of the vehicle's mobility system. By determining the relationship between shear stress and displacement, it provides data required for predicting shear stress at the vehicle–ground interface and the relationship between vehicle traction force and slip ratio.

Based on the results from the Bishop ring shear test, the data are fitted to establish the typical relationships between pressure–settlement and shear stress–displacement as follows:

$$P = \left(\frac{k_c}{b} + k_\varphi\right) \cdot Z^n \tag{9}$$

$$\tau = \tau_{\max} \frac{e^{\left[\left(-k_2+\sqrt{k_2^2-1}\right)\cdot k_1 \cdot j\right]} - e^{\left[\left(-k_2-\sqrt{k_2^2-1}\right)\cdot k_1 \cdot j\right]}}{e^{\left[\left(-k_2+\sqrt{k_2^2-1}\right)\cdot k_1 \cdot j_0\right]} - e^{\left[\left(-k_2-\sqrt{k_2^2-1}\right)\cdot k_1 \cdot j_0\right]}} \tag{10}$$

where P represents the ground pressure, b denotes the shorter dimension of the wheel–ground contact area, which is the width of the rectangular contact patch or the radius of a circular contact patch, Z is the sinkage depth, n is the soil deformation index, k_c and k_φ are the soil cohesion and friction deformation moduli, respectively, τ is the shear stress, $\tau_{\max}$ is the maximum shear stress, k_1 and k_2 are empirical constants, and j and j_0 represent the shear displacement.

Based on Equations (9) and (10), the relationship between running resistance, driving force, and slip ratio, as well as the tractive force and maximum speed of the vehicle, can be derived. Subsequently, researchers have introduced various modifications and improvements to the semi-empirical model originally proposed by Bekker to enhance the predictive accuracy of the mechanical model.

Figure 3. Bevameter.

The semi-empirical approach, rooted in classical soil mechanics, involves the development of a series of semi-empirical formulas based on extensive simulation experiments. Although the parameters within these formulas are derived from experimental data, they can effectively guide the design of off-road vehicles. This approach benefits from controlled experimental conditions, yielding results with strong repeatability and comparability, making it a valuable research method even in the future [15]. However, the current pressure–sinkage and shear stress–displacement models rely heavily on in situ soil measurement data, and the measurement methods are predominantly based on axisymmetric vertical loading in a planar model. These models neglect the effects of uneven load distribution on the ground during vehicle roll and do not account for vehicle turning performance or lateral dynamics. Therefore, developing a spatial vehicle–ground interaction model is more advantageous for predicting and analyzing vehicle–ground interactions during operation. Additionally, with the continuous evolution of vehicle configurations, further research is needed to establish mechanical models that capture the interactions between new vehicle designs and the ground.

3.3. Mobility Prediction Methods Based on Numerical Simulation

To predict the mobility of a vehicle in a specific area, researchers evaluate and analyze the geographic information of the region, combining empirical and semi-empirical methods to obtain mobility prediction results. Currently, to more accurately capture the nonlinear interaction between the vehicle and soil, and to achieve more precise mobility predictions than those provided by empirical and semi-empirical approaches, researchers employ numerical simulation techniques for soil modeling. By coupling vehicle multibody dynamics models with soil models, they can simulate and predict the mobility performance of ground vehicles. Typical soil modeling methods include finite element analysis (FEA) based on mesh discretization and particle-based modeling approaches.

3.3.1. High-Precision Geographic Information Acquisition

Geographic information is typically obtained from remote sensing sources, such as remote sensing technology. However, remote sensing topographic data may contain errors relative to actual spatial locations. Additionally, high-resolution topographic data reconstructed through interpolation methods also exhibit errors. Variability in soil physical properties, such as soil cohesion and the internal friction angle, further introduces uncertainty into vehicle–ground interactions, leading to unreliable prediction results [16–18]. To address the uncertainty in terrain elevation, [19] proposed a method for predicting vehicle mobility over large areas (26 × 40 km^2). To enhance map accuracy, they applied kriging interpolation methods from geostatistics to the initial terrain data. Additionally, to quantify terrain reconstruction uncertainty, they performed Monte Carlo simulations to generate random samples and conducted statistical analysis to classify interpolated terrain points as passable or impassable. Finally, by combining terrain slope, soil distribution data, and vehicle kinematic models, they generated passable and impassable maps, which were then used for path planning, as illustrated in Figure 4.

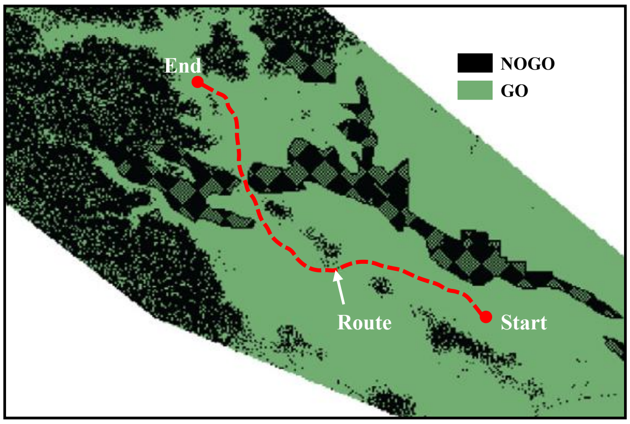

Figure 4. GO/NOGO map and path planning result.

Due to the large number of terrain points involved in reconstructing extensive terrain areas, computer processing capabilities are significantly challenged. To address this, ref. [20] proposed a down-sampling method to obtain a reduced-order representation of the terrain elevation model, facilitating rapid interpolation of terrain data. On the other hand, soil is a crucial factor affecting vehicle mobility, with properties such as particle composition, liquid limit, and plasticity influencing soil passability. To determine the impact of soil moisture on vehicle mobility, [21] proposed a multi-source data-based method for predicting soil off-road passability. This method integrates rainfall data from meteorological stations and satellite rainfall measurements to construct high-precision rainfall data, and evaluates vehicle passability based on empirical methods (data download from http://www.soilinfo.cn/map/index.aspx, accessed on 20 April 2017).

3.3.2. High-Fidelity Soil Model Construction

High-fidelity soil models are crucial for vehicle mobility prediction methods based on numerical simulation and modeling. However, the stress–strain relationships in granular soil models exhibit significant nonlinearity. Currently, most soil models employ grid-based finite element methods, particle-based modeling approaches, or layered multi-scale modeling techniques.

(1) Grid-based finite element soil modeling methods are described as follows: in finite element methods (FEMs), soil is typically approximated as a continuum and its mechanical properties are commonly described using elastoplastic constitutive models [22], such as

the Drucker–Prager yield model, Mohr–Coulomb yield model, and Cam-Clay critical state plasticity model [23,24]. In this context, commercial finite element software such as ABAQUS, PAM-CRASH, and LS-DYNA have been developed and are utilized to simulate soil–wheel interactions. Ref. [25] employed ABAQUS to develop a soil finite element model and investigated the impact of armored vehicle loads on mobility in off-road environments. Ref. [26] used PAM-CRASH to study the mechanical relationships between single and multi-wheeled vehicles and soft soil interactions.

Most FEM soil models use the Lagrangian finite element method (Lagrangian FEM) to describe the movement of soil finite element nodes. When soil undergoes significant deformation, Lagrangian FEM requires mesh updates, which involve re-interpolating the solution domain (including plastic and elastic deformations) onto a new mesh. This process demands substantial computational resources and can reduce solution accuracy. To address this, ref. [27] employed the Arbitrary Lagrangian–Eulerian finite element method (ALE FEM) to simulate the interaction between wheels and soil, as illustrated in Figure 5. This model accounts for soil deformation effects through the force–displacement transmission between the wheel and the mesh. However, despite using ALE FEM, effects such as soil separation and adhesion still require reprocessing.

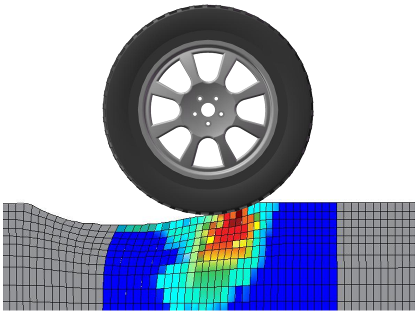

Figure 5. Wheel and soil interaction simulation.

Additionally, in Eulerian finite element models (Eulerian FEMs), Eulerian formulations can handle soil flow, separation, and adhesion. Ref. [28] utilized the Eulerian formulation in LS-DYNA to simulate interactions between tires and non-cohesive soil. The main advantage of the Eulerian FEM is its ability to model large soil deformations and flow while incorporating the effects of material flow, fracture, plasticity, friction, and cohesion on soil properties. However, accurately modeling the friction at solid boundaries remains challenging in Eulerian formulations. Therefore, precise construction of mechanical constitutive models for cohesive soils continues to be an active research area.

The primary advantage of grid-based FEM soil models is their flexibility in adjusting element sizes. Smaller elements can be used in high-deformation areas near the surface and tires, while larger elements are suitable for low deformation regions deeper in the soil and farther from the tires. Consequently, the number of degrees of freedom in finite element soil models is typically lower than in particle-based soil models, leading to reduced computation times, as illustrated in Figure 6. However, a major drawback is that significant soil deformations require remeshing, which increases computational effort and decreases the accuracy of Lagrangian FEM and ALE FEM solutions. Even with remeshing, simulating soil separation and adhesion remains challenging. Although Eulerian FEMs can handle soil flow and separation/adhesion effects, accurately modeling friction at solid boundaries in Eulerian formulations is difficult.

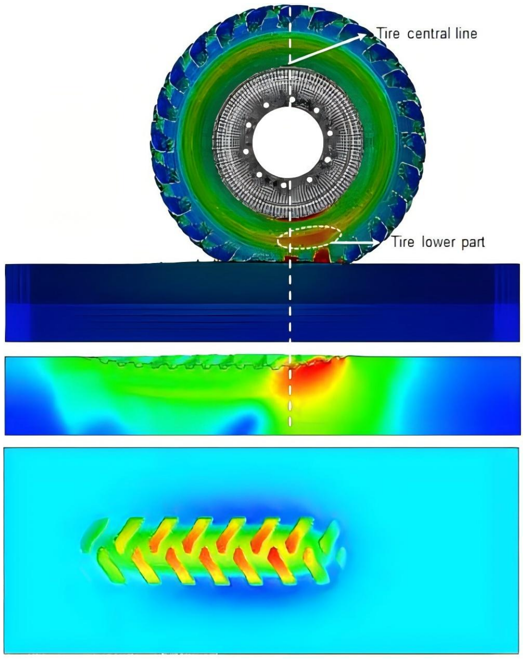

Figure 6. Different mesh sizes for simulating soil characteristics.

(2) Particle-based soil modeling methods are described as follows: Although continuum finite element models using phenomenological constitutive assumptions can simulate soil deformation, they struggle to explain the microscopic mechanical behavior between soil particles, leading to lower simulation accuracy. In contrast to finite element models, particle-based models are more effective at simulating material characteristics at the particle scale. These models capture material properties through particles with frictional contacts, allowing for a relatively direct simulation of particle-based soil characteristics.

In particle-based soil modeling methods, the mechanical behavior of soil is simulated using interaction force models between particles [29], as illustrated in Figure 7. In this model, normal force contact is based on Hertzian contact theory, while tangential force contact is governed by the Middlin–Deresiewicz theory. There are elastic forces F_s, damping forces F_d, and rolling friction forces F_d between particles i and j. Specifically, the elastic forces F_s and damping forces F_d are categorized into the normal elastic force F_s^n, tangential elastic force F_t^s, normal damping force F_d^n, and tangential damping force F_d^t. The parameters K_n, K_t, C_n, C_t, and μ represent the normal spring stiffness coefficient, tangential spring stiffness coefficient, normal damping coefficient, tangential damping coefficient, and static friction coefficient, respectively.

$$\begin{cases} F_s^n = \frac{4}{3}E^*\sqrt{R^*}\delta_n^{\frac{3}{2}} \\ F_s^t = -S_t\delta_t \\ F_d^n = -2\sqrt{\frac{5}{6}}\beta\sqrt{S_n m^*}\overrightarrow{v_n^{rel}} \\ F_d^t = -2\sqrt{\frac{5}{6}}\beta\sqrt{S_t m^*}\overrightarrow{v_t^{rel}} \end{cases} \tag{11}$$

where E^* denotes the equivalent elastic modulus, R^* represents the equivalent radius, δ_n and δ_t correspond to the normal and tangential overlap quantities, respectively, and S_n and S_t are the normal and tangential stiffness coefficients. m^* is the equivalent mass, and $\overrightarrow{v_n^{rel}}$ and $\overrightarrow{v_t^{rel}}$ are the normal and tangential components of the relative velocity. β is the damping ratio.

Due to the variability and complexity of soil, it is necessary to adjust the parameters within the soil model in simulation software during soil modeling [30]. The basis for adjustment is to conduct pressure–settlement and shear–displacement experiments on the soil within the simulation environment, fitting the obtained experimental curves with the measured curves to achieve an approximate real soil mechanical response in the simulation environment, as shown in Figure 8. The pressure–sinkage test (Figure 8a) is described as follows: in this test, a cylindrical object (usually a probe) is pressed vertically into a soil or surface material, and the resulting sinkage (or penetration depth) is measured under applied pressure. This test helps assess the compaction or resistance of the material when a vertical force is applied. The pressure increases as the object penetrates deeper into the material, and the resulting sinkage is observed to determine the material's response to vertical loading. The shear–displacement test (Figure 8b) is described as follows: this test involves applying shear forces to the material, typically by rotating or moving a cylindrical object laterally within the material. It measures how the material deforms under shear stress and quantifies the shear displacement (lateral movement) that occurs as the shear force is applied. This test is useful for assessing the material's shear strength, friction, and its ability to resist sliding or shearing forces. After establishing an accurate soil model, a co-simulation scheme centered on tires, vehicles, and terrain is adopted, employing a force–displacement co-simulation strategy for simulation [31–35], to predict the mobility of the vehicle.

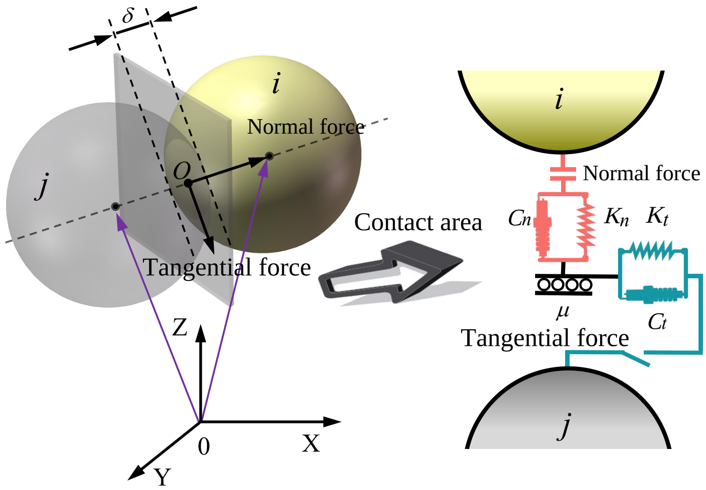

Figure 7. Model of the particle interaction force.

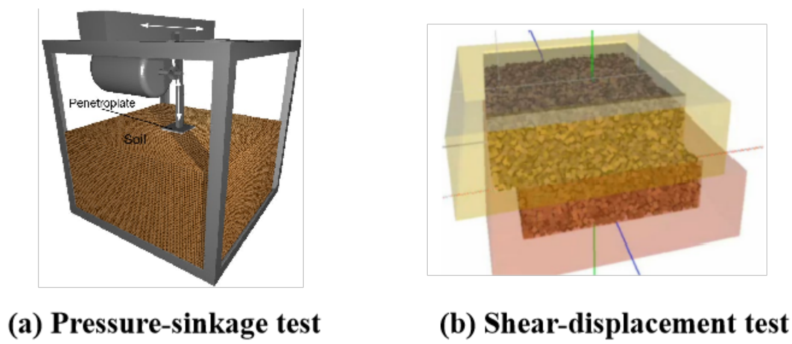

Figure 8. Simulation tests of the soil parameters calibration.

The granular model is the one that most closely approximates the actual physical properties of soil. Numerous methods based on granular soil models have been employed to simulate vehicle–soil interactions, including the discrete element method, smooth particle hydrodynamics, and the material point method.

In the discrete element method (DEM) [36,37], the mechanical behavior of soil is modeled through the inter-particle forces, which include the normal contact forces, attractive forces, tangential contact forces (including frictional and viscous forces), and forces related to the distance between particles (gravity, electrostatic forces, and magnetic forces). Smith et al. [38] developed a discrete element soil model for simulating the interaction between rigid wheels and cohesive soil. Ref. [39] developed an Implicit Differential Variational Inequality (DVI) solver for ground vehicle mobility simulation, which includes soil cohesion, friction, viscosity, and elastic effects, but does not account for plastic deformation and consolidation effects. Ref. [40] proposed a DEM model for cohesive soils that can explain the effects of normal and consolidation stresses on soil plasticity, density, and cohesion. Additionally, this model includes normal elastic forces, damping forces, tangential frictional forces, and viscous forces, and has been validated in applications for ground vehicle mobility. Subsequently, refs. [41,42] extended this model using a relaxation model for soil plastic deformation, introducing the loss of cohesive strength due to tension; at the same time, to enhance simulation efficiency, they proposed a mobile soil patch technique, enabling the simulation of vehicle travel on extended soft terrains, as shown in Figure 9.

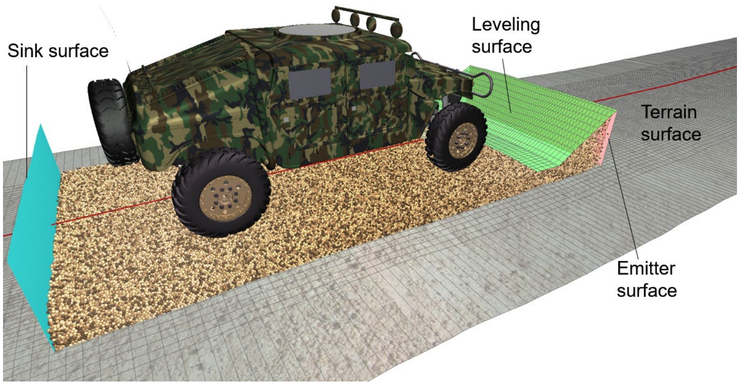

Figure 9. Multibody dynamics simulation of wheeled vehicles driving on the DEM terrain.

Smoothed Particle Hydrodynamics (SPH) [43] is also a granular modeling method that fundamentally employs interacting particles to represent continuous soil fluids. Each particle carries various physical quantities, including mass and velocity. By solving the dynamic equations of the particles and their trajectories, the mechanical behavior of the entire system is obtained. SPH involves the concept of a smoothing kernel, where the properties of a particle are diffused to the surrounding area, with the influence diminishing as the distance increases. This function, which decreases with increasing distance, is referred to as the smoothing kernel function, and its maximum influence radius is known as the smoothing kernel radius.

Refs. [44,45] utilized PAM-CRASH to create a finite element tire and SPH soil simulation coupling model (see Figure 10), which is employed to simulate the rolling of a flexible tire on soft soil. The soil is modeled using the SPH model, which, due to its excessive viscosity and difficulty in simulating soil compressibility, necessitates further refinement.

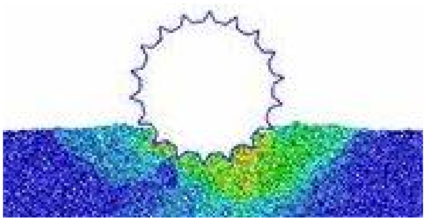

Figure 10. Simulation of tire interacting using SPH soil model.

Similar to other granular modeling techniques, the primary advantage of SPH lies in its capability to simulate significant soil deformations, including soil flow, separation, adhesion, and the adherence of soil to the tire surface. The main disadvantage of the SPH soil model in vehicle mobility applications is that each particle interacts not only with its neighboring particles but also with all particles within its kernel radius, resulting in slower computational speeds.

The material point method (MPM), introduced by Sulsky in 1996 [46], is a numerical simulation technique that employs a dual description of fluid mechanics characteristics using material points and an Eulerian mesh. This method involves discretizing the material into a set of material points that carry only mass and position information to describe material properties, while the corresponding physical quantities are computed on the Eulerian mesh. Interpolation functions facilitate the exchange of information between the material points and the Eulerian mesh, thereby avoiding mesh distortion and the treatment of advective terms. The MPM combines the advantages of both Eulerian and Lagrangian methods, making it highly suitable for simulating problems involving large deformations, impacts, and fracture fragmentation [47]. It effectively captures the characteristics of snow, such as its stickiness, fragmentation, and compression. Consequently, in [48], researchers applied the MPM to simulate the motion of vehicles on real snow, as depicted in Figure 11.

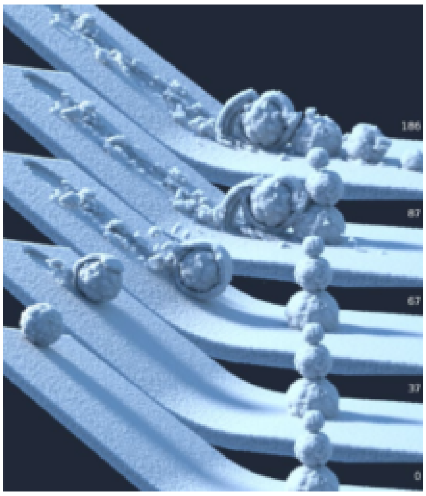

Figure 11. Simulation of the vehicle driving on the snow.

Currently, the evaluation criteria for macro-scale soil modeling technology levels [49] are presented in Table 2, with a scale ranging from 1 to 10, where a higher score indicates better performance.

Table 2. Large-scale soil modeling technology -level evaluation.

Evaluation Metrics	Lagrangian/ ALE FEM	Eulerian FEM	DEM	SPH	MPM
Soil Deformation Range	4	9	9	9	9
Embedding Obstacle Capability	3	7	9	9	9
Vehicle Interaction Fidelity	5	6	8	8	8
Simulation Computational Speed	5	7	6	5	9
Experimental Verification Accuracy	5	3	7	5	3
Current Application Trends	5	4	8	6	5
Total Score	27	36	47	42	40

From Table 2 and the discussions presented in this section, it can be concluded that particle-based soil modeling methods generally outperform grid-based finite element soil modeling methods, with the DEM being the current optimal soil modeling technology.

The Lagrangian/ALE FEM is recommended only for situations where soil deformation is minimal and no other media (rocks, other soils, obstacles, etc.) are embedded within the soil model.

(3) Soil modeling methods based on hierarchical multi-scale approaches are described below.

To combine the advantages of finite element and discrete element soil models, in [50], researchers proposed a hierarchical multi-scale soil model. In this model, the DEM is employed to describe the soil surface to simulate the dynamic interaction between the soil and the rolling tire, while the FEM is used to describe the underlying layers, thereby reducing the dimensionality of the overall model. Results indicate that the multi-scale soil model significantly reduces computational costs compared to a single-scale DEM while maintaining similar fidelity [51,52]. To further enhance simulation efficiency, soil modeling using the Representative Volume Element (RVE) model is utilized, where the FEM is employed to predict macroscopic soil deformation. At the integration points of the FEM, the granular-scale DEM is introduced in place of phenomenological constitutive models [53,54] to predict the complex granular-scale material behavior under soil strain response, as shown in Figure 12. Additionally, this method can be accelerated through large-scale parallel processing on high-performance computers.

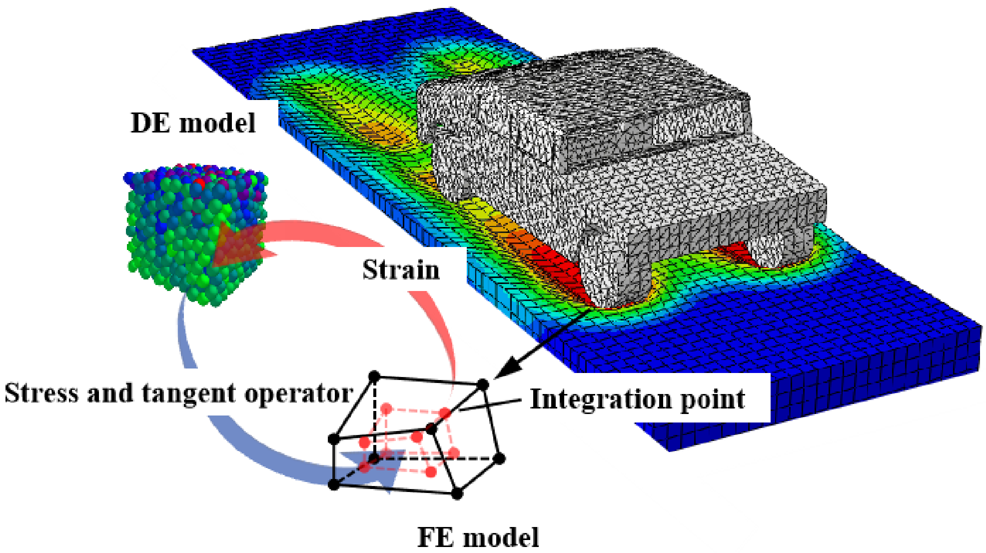

Figure 12. Interaction between tire and soil using RVE model.

3.4. Machine Learning-Based Mobility Prediction Methods

Numerical simulation-based methods for predicting vehicle mobility on soft terrain rely on high-fidelity soil models. Despite the introduction of numerous soil modeling techniques, the phenomenological constitutive assumptions in FEM soil models pose significant challenges in capturing the intricate granular mechanical behavior. Although the DEM soil models excel at simulating the complex mechanics of granular materials, their computational cost becomes excessively high when simulating large-scale terrains (>5 × 5 km^2) with millions of soil particles. Generating mobility distribution maps using these models can take weeks or even months [55]. With the rapid evolution of artificial intelligence technology, researchers have introduced machine learning-based mobility prediction techniques, significantly enhancing the efficiency of mobility simulation.

In [56], researchers introduced a machine learning method to efficiently predict vehicle mobility, encapsulating the terrain factors influencing vehicle mobility into soil strength and slope values. These parameters serve as inputs for the artificial neural network (ANN) algorithm, with the predicted output being the maximum attainable speed of the vehicle on the given terrain. The training dataset is generated by sampling a range of soil strength and slope combinations and conducting vehicle mobility simulations using numerical simulation techniques to determine the maximum speed of the vehicle on each terrain configuration. Thereafter, the ANN algorithm is employed to generate a comprehensive distribution of mobility predictions.

Although this method effectively enhances simulation efficiency, the generalization of mobility results is poor. The simulation of training data still requires a substantial amount of time, and the mobility results obtained only reflect the distribution of the vehicle's maximum speed, without providing real-time simulation data such as vehicle posture, traction force, and slip ratio. To obtain real-time vehicle driving data, data-driven modeling methods have garnered increasing attention in the field of computational mechanics [57], including the use of artificial neural networks to output stress–strain curves related to the elastoplastic and viscoelastic material behavior under laboratory test conditions [58–60], and the utilization of recurrent neural networks with long short-term memory to describe material behavior [61]. Furthermore, in the context of multi-scale modeling, in [62], researchers developed a surrogate model based on Gaussian process regression to accelerate the calculation of state equations in material multi-scale models. Refs. [63,64] proposed a neural network-based hierarchical multi-scale off-road mobility model. This method, based on the RVE model, takes as input data the instantaneous load force exerted by the vehicle on the ground and the terrain deformation during the simulation driving process, using neural networks to replace the solution process of inter-particle state changes; the output results are the subsequent terrain deformation and the forces transmitted to the vehicle by the ground. This method offers simulation accuracy comparable to the DEM and significantly reduces simulation time (70% to 80%), enabling the acquisition of real-time vehicle simulation data. However, obtaining its training data is more complex, requiring the construction of test scenarios in the simulation environment to test the mechanical effects of vehicle–ground interaction under various driving conditions, as shown in Figure 13.

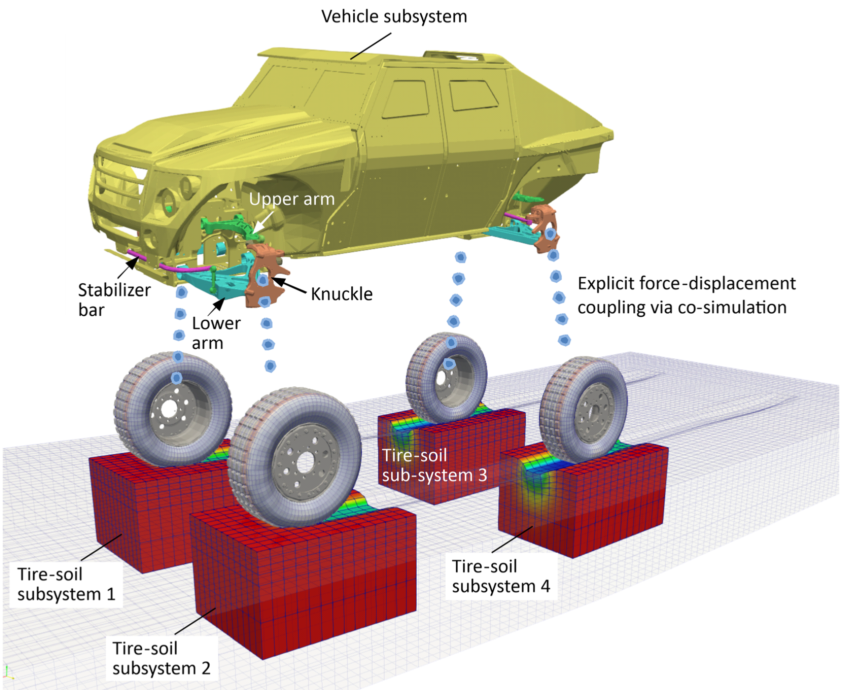

Figure 13. Test scenario in simulation environment.

In this section, the aforementioned mobility prediction methods are compared across various aspects, including the prediction methodologies, subjects of prediction, methodological limitations, and potential applications, with the results summarized in Table 3.

Table 3. Large-scale soil modeling technology -level evaluation.

Method	Technique	Target	Limitation	Application
Empirical	Field Test	Common Wheeled/ Tracked Vehicles	Poor generalizability	Poor
Semi-Empirical	Field Test/ Theoretical Derivation	Unrestricted	Model simplifications/ assumptions	Average
Numerical Simulation	Theoretical Derivation/ Computer Simulation	Unrestricted	Long simulation time/ high modeling complexity	Good
Machine Learning	Training Data/ Algorithm	Unrestricted	Challenges in data acquisition/ poor generalizability	Excellent

4. Applications of Mobility Prediction Methods

4.1. Military Vehicle Mobility Prediction

In the military domain, for strategic decision-makers, the application significance of ground vehicle mobility prediction encompasses several key aspects: (1) selecting passable areas based on vehicle types, (2) identifying vehicle types suitable for specific terrain scenarios, (3) optimizing vehicle design or developing new types of vehicles by predicting the mobility of existing vehicles in specific terrain scenarios. Consequently, since 1979, the United States and several NATO countries have developed the NATO Reference Mobility Model (NRMM) for predicting the mobility of military ground vehicles [65]. The NRMM utilizes the VCI as a metric for evaluating the mobility of military ground vehicles on soil. By determining the relationship between the VCI and the RCI, it predicts vehicle passability. The NRMM analyzes terrain into two-dimensional profiles and incorporates an obstacle module (OBSDP), employing a half-vehicle dynamics model to traverse the terrain, thereby ascertaining the minimum ground clearance and traction force required for vehicles under various terrain and obstacle conditions. To simulate the impact of driving shocks on speed, the NRMM integrates a ride dynamics module (VEHDYN), which adjusts the maximum vehicle speed achievable based on the endurance limits of the occupants, serving as a consideration for speed limitation. Ultimately, the data obtained from the aforementioned modules are compiled into the NRMM dataset. Users only need to input vehicle data, scenario data, terrain data, etc., and by leveraging historical vehicle test data, empirical models, and semi-empirical formulas, the mobility distribution of an area can be outputted [66–69].

Due to the limitations of vehicle mobility prediction methods based on empirical and semi-empirical models, the NATO Applied Vehicle Technology (AVT) group, under the support of the NATO Science and Technology Organization (STO), established the AVT-248 research working group to develop the Next Generation NATO Reference Mobility Model (NG-NRMM) [70]. The specific process of vehicle mobility prediction using NG-NRMM is achieved through the integrated simulation of Geographic Information Systems (GISs), FEM/DEM soil models, and multibody dynamics models of vehicles to evaluate the mobility of vehicles in a certain area [71–77], as shown in Figure 14.

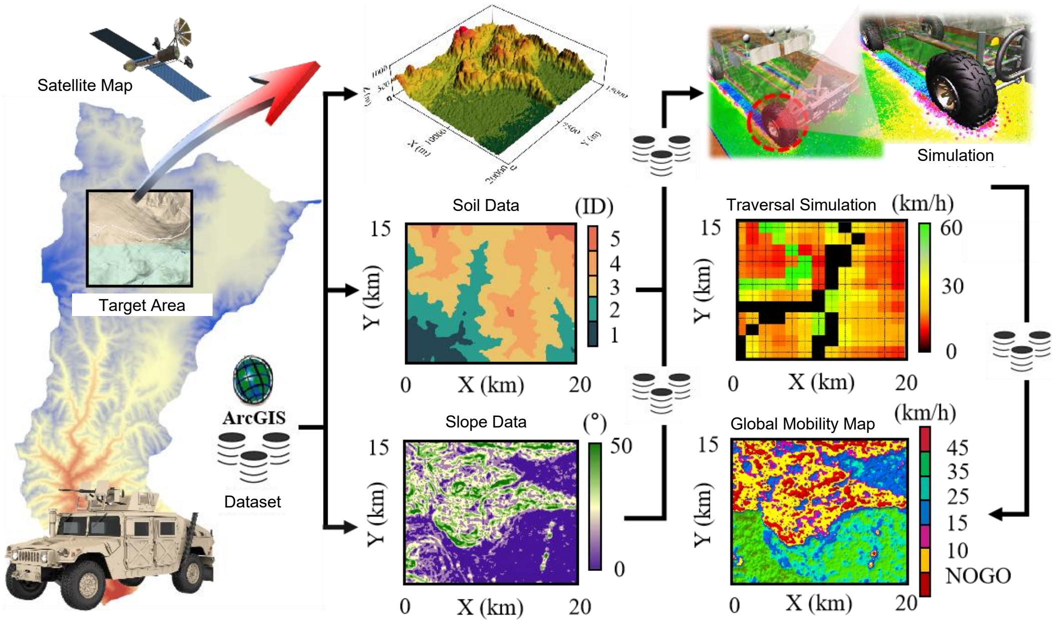

Figure 14. NG-NRMM flowchart.

4.2. Underwater Vehicle Mobility Prediction

In the realm of underwater operations, [78] conducted mechanical interaction tests between simulated track plates and simulated seabed sediments, based on the physical and mechanical properties of deep-sea mining areas. They obtained functional relationships between pressure–settlement and shear–displacement. Furthermore, leveraging the multibody dynamics software RecurDyn, they performed dynamic simulations of various walking conditions of tracked mining machines under special mechanical loads on the seabed, laying the groundwork for the structural design optimization, performance evaluation, and walking control research of underwater tracked mining machines. Subsequently, scholars completed shear–displacement tests on soft seabed sediments and analyzed the impact of track parameters on the traction performance of deep-sea tracked mining machines [79–81].

4.3. Planetary Rover Mobility Prediction

To mitigate the risks of sinking and slipping during Mars rover travel, ref. [82] employed semi-empirical formulas from vehicle ground mechanics to develop an Adams-based Mars rover ground mechanics and mobility interaction simulator. This simulator predicts terrain traversability and has become a part of path planning and navigation for Mars and lunar rovers. To determine the relationship between the normal force distribution of Mars rover wheels and mobility, ref. [83] used numerical simulation methods to conclude the impact of load distribution between Mars rover wheels on mobility.

4.4. Agricultural Vehicle Mobility Prediction

In agriculture, with the widespread and large-scale use of agricultural vehicles, the compaction and destruction of farmland soil have become increasingly severe. Excessive compaction affects the physical and chemical properties of the soil [84–86]. Ref. [87] used experimental testing and semi-empirical mechanical models to comparatively study the magnitude of vertical and horizontal stresses in soil under the compaction of wheeled and tracked agricultural machinery. They also analyzed the impact of vehicle travel speed on stress magnitude. The research results can provide references for the selection and use of agricultural vehicle walking mechanisms to reduce soil compaction.

5. Conclusions

Empirical methods are products of the early development of terramechanics and have a certain application value, but they cannot deeply explain the mechanism of vehicle–soil interaction and have significant limitations. Semi-empirical methods, based on classical soil mechanics theory and a large number of simulation experiments, have proposed a series of semi-empirical calculation formulas. This method has controllable experimental conditions, providing repeatable and comparable results. Compared to the above two methods, numerical simulation not only shortens the experimental cycle and improves the accuracy of mobility prediction results in vehicle ground mechanics research but also achieves certain trend predictions. In particular, multi-scale modeling that combines the advantages of the finite element method and discrete element method will become an important research method for analyzing vehicle–soil interaction relationships. Using machine learning methods to replace cumbersome mechanical solution formulas significantly improves efficiency compared to numerical simulation methods; however, their results have poor generalizability, and when used to test terrains that differ significantly from the training data, the accuracy of the results is difficult to ensure.

6. Future Development Trends

6.1. Real-Time Mobility Prediction and Terrain Perception

The efficiency of vehicle mobility prediction is crucial for mission decision-making. Current numerical simulation-based mobility prediction methods incur significant computational costs due to the force–displacement transfer between soils, which involves extensive iterative calculations, making real-time simulation of vehicle mobility challenging. Machine learning methods are being applied to enhance efficiency; however, their training data still requires substantial numerical simulation, and they suffer from poor generalizability. Therefore, achieving real-time prediction of vehicle mobility will be a future research direction. Additionally, to obtain vehicle mobility prediction results, most current methods evaluate and analyze known global terrains and combine empirical, semi-empirical, numerical simulation, and machine learning methods to predict vehicle mobility in a given area. However, the spatial variability of soil physical properties leads to uncertainty in vehicle mobility prediction results, increasing the risk of vehicle obstruction. Thus, real-time perception of soil mechanics parameters affecting vehicle mobility is necessary. Although some methods using on-vehicle sensors to identify in real time local terrain have been proposed [88–90], these perception methods are generally based on vehicles with symmetrical and uniformly distributed loads along the axis, neglecting the impact of vehicle tilt on non-uniformly distributed loads on the ground. To enable vehicles to avoid the impact of uncertain factors in mobility prediction during actual travel, further research in terrain real-time perception is required.

6.2. Path Planning for Deformable Terrain

Mobility prediction results determine the strategy for vehicle route planning (e.g., military operations, planetary exploration missions) [91], and a significant amount of research on path planning has been conducted [92,93], mostly focusing on terrain undulation factors and obstacle information. However, soil deformation caused by vehicle loads on soft terrain, as well as the aforementioned uncertain factors, can affect vehicle posture, speed, and slippage, which are essential for travel. Therefore, quantifying these factors, establishing appropriate cost functions for path planning algorithms, and developing suitable decision-making methods will also be future research directions.

6.3. Autonomous Mobility Prediction for Unmanned Vehicles

Current vehicle mobility prediction methods are based on manned vehicles, and how to predict the autonomous mobility of unmanned vehicles is also one of the hot topics in current research [94,95]. For unmanned autonomous vehicles, the impact of driving smoothness on drivers is not very significant (except for vehicle component durability), but it can reduce sensor performance and affect obstacle detection. Meanwhile, vehicle speed affects sensor update rates and the time window for on-board algorithms, and vehicle slippage may also affect the accuracy of vehicle state estimation. Therefore, the coupling relationships between sensor performance, scene information, unmanned control algorithms, and vehicle dynamics are an important factor affecting the autonomous mobility of unmanned vehicles, bringing new challenges to the modeling of complex sensor suites, environmental information, autonomous algorithms, and the closed-loop dynamics of the vehicle system. On the other hand, to help mission decision-makers set driving modes (manned driving, remote control, fully autonomous driving) for unmanned autonomous vehicles, it is necessary to generate autonomy distribution maps based on mobility distribution maps, with vehicle models, geographic information, scene information, sensor types, and autonomous algorithms as inputs. These maps will ultimately display the level of autonomy in different areas.

Author Contributions: Conceptualization, C.H.; methodology, C.H. and W.Z.; software, H.F. and C.J.; formal analysis, Y.Z.; resources, B.Y.; writing—original draft preparation, C.H. and Y.W.; writing—review and editing, Z.C.; supervision, X.K.; project administration, C.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Changzhou Institute of Technology's Initial Scientific Research Startup Projects for Newly Introduced High-Level Talents of funder number E3-6207-24-028.

Data Availability Statement: The original contributions presented in the study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Liu, Y.; Barthlow, D.; Mourelatos, Z.P. Mobility prediction of off-road ground vehicles using a dynamic ensemble of NARX models. *J. Mech. Des.* **2022**, *144*, 091709. [CrossRef]
2. Wong, J.Y.; Jayakumar, P.; Toma, E. A review of mobility metrics for next generation vehicle mobility models. *J. Terramech.* 2020, *87*, 11–20. [CrossRef]
3. Hao, L.; Xin-quan, L. Summary of evaluating the mobility of vehicle based on cone index. *Agric. Equip. Veh. Eng.* **2011**, *49*, 16–20.
4. Ning, J.; Li, J.; Li, H. Military vehicle mobility evaluation methods. *J. Ordnance Equip. Eng.* **2009**, *30*, 49–51.
5. Yi-dong, B.; Ling-yu, S.; Ming-lu, Z. Progress of research on terramechanics for tracked mobile robots. *J. Mach. Des.* **2020**, *37*, 1–13.
6. Verma, V.; Maimone, M.W.; Gaines, D.M. Autonomous robotics is driving Perseverance rover's progress on Mars. *Sci. Robot.* **2023**, *8*, 3099. [CrossRef] [PubMed]
7. Ahlvin, R.B.; Haley, P.W. *NATO Reference Mobility Model Edition II*; USA Army Corps of Engineers Waterways Experiment Station: Vicksburg, MI, USA, 1992.
8. Sohne, W. Four-wheel drive or rear-wheel drive for high power farm tractors. *J. Terramech.* **1968**, *5*, 9–28. [CrossRef]
9. Zhang, K.J. *Vehicle Ground Mechanics*; National Defense Industry Press: Beijing, China, 2002.
10. Bekker, M.G. *Introduction to Terrain-Vehicle Systems*; University of Michigan Press: Ann Arbor, MI, USA, 1969.
11. Rula, A.A.; Nuttall, C.J. *An Analysis of Ground Mobility Models (ANAMOB)*; USA Army Corps of Engineers Waterways Experiment Station: Vicksburg, MI, USA, 1971.
12. Rowland, D. A Review of Vehicle Design for Soft Ground Operation. In Proceedings of the International Conference of the International Society for Terrain-Vehicle Systems, Detroit, MI, USA, 15 June 1975; pp.179–219.
13. Wong, J.Y.; MGarber, M.; Preston, Thomas, J. Theoretical prediction and experimental substantiation of the ground pressure distribution and tractive performance of tracked Vehicles. *J. Antomobile Eng.* **1984**, *198*, 265–285. [CrossRef]
14. Wong, J.Y. *Theory of Ground Vehicles*; Wiley-Inter Science: New York, NY, USA, 2001.

15. Lessem, A.; Mason, G.; Ahlvin, R. Stochastic vehicle mobility forecasts using the NATO reference mobility model. *J. Terramech.* **1996**, *33*, 273–280. [CrossRef]
16. Hua, C.; Niu, R.; Yu, B. A Global Path Planning Method for Unmanned Ground Vehicles in Off-Road Environments Based on Mobility Prediction. *Machines* **2022**, *10*, 375. [CrossRef]
17. Ishigami, G.; Kewlani, G.; Iagnemma, K. Predictable mobility: a statistical approach for planetary surface exploration rovers in uncertain terrain. *IEEE Robot. Autom.* **2009**, *16*, 61–70. [CrossRef]
18. Peynot, T.; Lui, S.; McAllister, R. Learned stochastic mobility prediction for planning with control uncertainty on unstruc-tured terrain. *J. Field Robot.* **2014**, *31*, 969–995. [CrossRef]
19. Gonzalez, R.; Jayakumar, P.; Iagnemma, K. Stochastic mobility prediction of ground vehicles over large spatial regions: A geo-statistical approach. *Auton. Robot.* **2017**, *41*, 311–331. [CrossRef]
20. Gonzalez, R.; Jayakumar, P.; Iagnemma, K. Generation of stochastic mobility maps for large-scale route planning of ground ve-hicles: A case study. *J. Terramechanic* **2017**, *69*, 1–11. [CrossRef]
21. Kun-wei, L.; Xiong, Y.; Xin, Z. Evaluation of soil trafficability based on multi-Source data. *J. Geomat. Sci. Technol.* **2018**, *35*, 206–210.
22. Kok, S.; Bujang, B.K.H. A review of basic soil constitutive models for geotechnical application. *Electron. J. Geotech. Eng.* **2009**, *14*, 1–18.
23. Drucker, P. Soil mechanics and plastic analysis for limit design. *Q. Appl. Math.* 1952, *10*, 157–165. [CrossRef]
24. Sandler, I.S.; Rubin, D. An algorithm and a modular subroutine for the cap model. *J. Numer. Anal. Methods Geomech.* **1979**, *3*, 173–186. [CrossRef]
25. Grujicic, M.; Bell, W.C.; Arakere, G. Finite element analysis of the effect of up-armouring on the Off-Road Braking and sharp-turn performance of a high-mobility multi-purpose wheeled vehicle. *J. Auto Mob. Eng.* **2009**, *223*, 1419-1434. [CrossRef]
26. Ragheb, H.; EI Gindy, M.; Kishawy, H.A. Multi-wheeled combat vehicle modeling on rigid and soft terrain. In Proceedings of the NDIA Ground Vehicle Systems Engineering and Technology Symposum, Modeling & Simulation, Testing and Validation (MSTV), Troy, MI, USA, 25 June 2013; pp. 325–336.
27. Shoop, S.A.; Richmond, P.W.; Lacombe, J. Overview of cold regions mobility modeling at CRREL. *J. Terramech.* **2006**, *43*, 1–26. [CrossRef]
28. Wright, A. *Tire/Soil Interaction Modelling Withing a Virtual Proving Ground Environment*; Institute of Defence and Security, University of Cranfield: Berkeley, CA, USA, 2012.
29. Ravula, P.; Acar, G.; Balachandran, B. Discrete element method-based studies on dynamic interactions of a lugged wheel with granular media. *J. Terramech.* **2021**, *94*, 49–62. [CrossRef]
30. Rui, Z.; Jian-qiao, L.; Yin-wu, L. DEM macroscopic and mesoscopic analysis in disturbed behavior of soil acted by part with complex surface. *J. Jilin Univ. Eng. Technol. Ed.* **2009**, *39*, 1218–1223.
31. Dasch, J.; Jayakumar, P.; Bradbury, M. *Next-Generation NATO Reference Mobility Model (NG-NRMM)*; Aarhus: The NATO Science & Technology Organization: Brussels, Belgium, 2020.
32. Melanz, D.; Jayakumar, P.; Negrut, D. Experimental validation of a differential variational inequality-based approach for handling friction and contact in vehicle/granular-terrain interaction. *J. Terramech.* **2016**, *65*, 1–13. [CrossRef]
33. Tasora, A.; Magnoni, D.; Negrut, D. Deformable soil with adaptive level of detail for tracked and wheeled vehicles. *Int. J. Veh. Perform.* **2019**, *5*, 60–75. [CrossRef]
34. Serban, R.; Taylor, M.; Negrut, D. Template-based ground vehicle modeling and simulation. *Int. J. Veh. Perform.* **2019**, *5*, 2–17.
35. Recuero, A.; Serban, R.; Peterson, B. A high-finitely approach for vehicle mobility simulation: Nonlinear finite element tires operating on granular material. *J. Terramech.* **2017**, *74*, 39–54. [CrossRef]
36. Cundall, P.A.; Strack, O.D.L. A discrete numerical model for granular assemblies. *Geotechnique* **1979**, *29*, 47–65. [CrossRef]
37. Cundall, P.A. A computer model for simulating progressive large-scale movements in block rock mechanics. In Proceedings of the International Symposium on Visual Computing, Nevada, NV, USA, 28 July 1971; pp. 8–11.
38. Smith, W.; Peng, H. Modeling of wheel-soil interaction over rough terrain using the discrete element method. *J. Terramech.* **2013**, *50*, 277–287. [CrossRef]
39. Negrut, D.; Melanz, D.; Mazhar, H. Investigating through simulation the mobility of light tracked vehicles operating on discrete granular terrain. *SAE Int. J. Passeng. Cars Mech. Syst.* **2013**, *6*, 369–381. [CrossRef]
40. Wasfy, T.M.; Wasfy, H.M.; Peters, J.M. Coupled multibody dynamics and discrete element modeling of vehicle mobility on cohesive granular terrains. In Proceedings of the International Conference on Multibody Systems, Nonlinear Dynamics, and Control, Buffalo, NY, USA, 17–20 August 2014; pp. 8–13.
41. Wasfy, T.M.; Wasfy, H.M.; Peters, J.M. High-fidelity multi-body dynamics vehicle model coupled with a cohesive soil discrete element model for predicting vehicle mobility. In Proceedings of the International Conference on Multi-Body Systems, Boston, MA, USA, 29 June–2 July 2015; pp. 202–208.
42. Wasfy, T.M.; PJayakumar, M.D. Prediction of vehicle mobility on large-scale soft-soil terrain maps using physics-based simulation. *Int. J. Veh. Perform.* **2018**, *4*, 347–381. [CrossRef]

43. Akinci, N.; Cornelis, J. Coupling elastic solids with smoothed particle hydrodynamics fluids. *Comput. Animat. Virtual Worlds* 2013, *24*, 1–9. [CrossRef]
44. Lescoe, R.; El Gindy, M.; Koudela, K. Tire-soil modeling using finite element analysis and smooth particle hydrodynamics techniques. In Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering, Montreal, QC, Canada, 15–18 August 2010; pp. 161–166.
45. Lescoe, R.M.; Koudela, K. Improvement of soil modeling in a tire-soil interaction using finite element analysis and smoothed particle hydrodynamics. *Mech. Eng.* **2010**, *10*, 3–18.
46. Sulsky, D.; Zhou, S.J.; Schreyer, H. Application of particle-in-cell method to solid mechanics. *Comput. Phys. Commun.* **1996**, *87*, 236–252. [CrossRef]
47. Yu-xin, W.; Yuan-xian, G.; Ming, S. Application of material point method to shock load problems. *Eng. Mech.* **2006**, *23*, 46–51.
48. Stomakhin, A.; Schroedery, C.; Chai, L.; Teran, J.; Selle, A. A material point method for snow simulation. *ACM Trans. Graph.* **2013**, *32*, 102–114. [CrossRef]
49. Letherwood, M.; Gerth, R.; Jayakumar, O. *Cooperative Demonstration of Technology Next-Generation NATO Reference Mobility Model (NG-NRMM)*; Collaboration Support Office Applied Vehicle Technology Pane: Washington, DC, USA, 2018.
50. Yamashita, H.; Chen, G.; Ruan, Y. Hierarchical multiscale modeling of tire-soil interaction for off-road mobility simulation. *ASME J. Comput. Nonlinear Dyn.* **2019**, *14*, 1–11. [CrossRef]
51. Nishiyama, K.; Nakashima, H.; Shimizu, H. 2D FE-DEM analysis of contact stress and tractive performance of a tire driven on dry sand. *J. Terramech.* **2017**, *74*, 25–33. [CrossRef]
52. Nishiyama, K.; Kashima, H.; Shida, T. FEM-DEM with interchangeable modeling for off-road tire traction analysis. *J. Terramech.* **2018**, *78*, 15–25. [CrossRef]
53. Guo, N.; Zhao, J. A Coupled FEM/DEM approach for hierarchical multiscale modeling of granular media. *Int. J. Numer. Methods Eng.* **2014**, *99*, 789–818. [CrossRef]
54. Liu, Y.; Sun, W.C.; Yuan, Z. A nonlocal multiscale discrete-continuum model for predicting mechanical behavior of granular materials. *Int. J. Numer. Methods Eng.* **2016**, *106*, 129–160. [CrossRef]
55. Nakashima, H.; Oida, A. Algorithm and implementation of soil-tire contact analysis code based on dynamic FE-DE method. *J. Terramech.* **2004**, *41*, 127–137. [CrossRef]
56. Hua, C.; Jiang, C.; Niu, R. Double Neural Networks Enhanced Global Mobility Prediction Model for Unmanned Ground Vehicles in Off-Road Environments. *IEEE Trans. Veh. Technol.* **2024**, *73*, 7547–7560. [CrossRef]
57. Kirchdoerfer, T.; Ortiz, M. Data-driven computational mechanics. *Comput. Methods Appl. Mech. Eng.* **2016**, *304*, 81–101. [CrossRef]
58. Ghaboussi, J.; Garrett, J.H.; Wu, X. Knowledge-based modeling of material behavior with neural networks. *J. Eng. Mech.* **1991**, *117*, 132–153. [CrossRef]
59. Ellis, G.W.; Yao, C.; Zhao, R. Stress-strain modeling of sands using artificial neural networks. *J. Geotech. Geoenviron. Eng.* **1995**, *121*, 429–435. [CrossRef]
60. Furukawa, T.; Yagawa, G. Implicit constitutive modelling for vis-coplasticity using neural networks. *J. Eng. Mech.* **1998**, *43*, 195–219.
61. Wang, K.; Sun, W.C. A multiscale multi-permeability poroplasticity model linked by recursive homogenizations and deep learning. *Comput. Methods Appl. Mech. Eng.* **2018**, *334*, 337–380. [CrossRef]
62. Leiter, K.W.; Barnes, B.C.; Becker, R. Accelerated scale-bridging through adaptive surrogate model evaluation. *J. Sci. Comput.* **2018**, *27*, 91–106. [CrossRef]
63. Yamashita, H.; Chen, G.C.; Ruan, Y.F. Parallelized multiscale off-road vehicle mobility aimulation algorithm and full-scale vehicle validation. *J. Comput. Nonlinear Dyn.* **2020**, *15*, 367–380.
64. Chen, G.C.; Yamashita, H.; Ruan, Y.F. Enhancing hierarchical multiscale off-road mobility model by neural network sur-rogate model. *J. Comput. Nonlinear Dyn.* **2021**, *16*, 104–121.
65. Tw, A.; Pj, B. Next-generation NATO reference mobility model complex terramechanics: Definition and literature review. *J. Terramech.* **2021**, *47*, 21–31.
66. Priddy, J.D. *Stochastic Vehicle Mobility Forecasts Using the NRMM*; US Army TARDEC: Warren, MI, USA, 1995.
67. Ma, R.; Chemistruck, H.; Ferris, J.B. State-of-the-art of terrain profile characterization models. *Int. J. Veh. Des.* **2013**, *61*, 285–304. [CrossRef]
68. Vong, T.T.; Haas, G.A.; Henry, C.L. *NATO Reference Mobility Model (NRMM) Modeling of the DEMO II Experimental Unmanned Ground Vehicle*; Army Research Lab: Adelphi, MD, USA, 1999.
69. Willoughby, W.E.; Jones, R.A.; Mason, G.L. Application of historical mobility testing to sensor-based robotic performance. *Soc. Photo-Opt. Instrum. Eng.* **2006**, *6230*, 25–39.
70. Choi, K.K.; Jayakumar, P.; Funk, M. Framework of reliability-based stochastic mobility map for next generation NATO ref-erence mobility model. *J. Comput. Nonlinear Dyn.* **2020**, *14*, 83–95.

71. Homer, C.; Dewitz, J.; Yang, L.M. National land cover database for the conterminous United States representing a decade of land cover change information. *Photogramm. Eng. Remote Sens.* **2015**, *81*, 345–354.
72. Chemistruck, H.M.; Ferris, J.B. Developing compact models of terrain surfaces. *J. Dyn. Syst. Meas. Control.* **2013**, *135*, 1–9. [CrossRef]
73. Cressie, N.; Johannesson, G. Fixed rank kriging for very large spatial data sets. *J. R. Stat. Soc.* **2008**, *70*, 209–226. [CrossRef]
74. Hosseini, S.Z.; Kappas, M.; Bodaghabadi, M.B. Comparison of different geostatistical methods for soil mapping using remote sensing and environment variables in Poshtkouh rangelands, Iran. *Pol. J. Environ. Stud.* **2014**, *23*, 737–751.
75. Yamashita, H.; Jayakumar, P.; Alsaleh, M. Physics-based deformable tire-soil interaction model for off-road mobility simulation and experimental validation. *J. Comput. Nonlinear Dyn.* **2018**, *13*, 11–26. [CrossRef]
76. Reid, A.A.; Shoop, S.; Jones, R. High-fidelity ground platform and terrain mechanics modeling for military applications involving vehicle dynamics and mobility analysis. In Proceedings of the Asia-Pacific ISTVS Conference and Annual Meeting of Japanese Society for Terramechanics, Fairbanks, AL, USA, 27 July 2007; pp. 23–26.
77. Raymond, J.B.; Jayakumar, P. The shearing edge of tracked vehicle-soil interactions in path clearing applications utilizing mul-ti-body dynamics modeling and simulation. *J. Terramech.* **2015**, *58*, 39–50. [CrossRef]
78. Tao, Z.; Yu, D.; Shao-jun, L. Multi-body dynamic modeling and mobility simulation analysis of deep ocean tracked miner. *J. Mech. Eng.* **2015**, *51*, 173–180.
79. Ma W.B.; Rao, Q.H.; Feng, K. Experimental research on grouser traction of deep-sea mining machine. *Appl. Math. Mech.* **2015**, *36*, 1243–1252. [CrossRef]
80. Li, J.; Liu, S.; Dai, Y. Effect of grouser height on tractive performance of tracked mining Vehicle. *J. Braz. Soc. Mech. Sci. Eng.* **2017**, *39*, 2459–2466. [CrossRef]
81. Xu, F.; Rao, Q.H.; Ma, W.B. Turning traction force of tracked mining vehicle based on rheological property of deep-sea sediment. *Trans. Nonferrous Met. Soc. China* **2018**, *28*, 1233–1240. [CrossRef]
82. Trease, B.; Arvidson, R.; Lindemann, R. Dynamic modeling and soil mechanics for path planning of the Mars exploration rovers. In Proceedings of the ASME International Design Engineering Technical Conferences, Washington DC, USA, 28–31 August 2011; pp. 96–105.
83. Ghotbi, B.; Gonzalez, F.; Jozsef, K.J. Mobility evaluation of wheeled robots on soft terrain. *Mech. Mach. Theory* **2016**, *100*, 259–282. [CrossRef]
84. Xian-liang, W.; Qing-jie, W.; Hong-wen, L. Current research status of soil compaction by agriculture machinery. *Chin. J. Trop. Agric.* **2015**, *35*, 72–76.
85. Xiao-juan, Y.; Chun-jian, L. Impacts of mechanical compaction on soil properties, growth of crops, soil-borne organisms and environment. *Sci. Agric. Sin.* **2008**, *41*, 2008–2015.
86. Keller, T.; Arvidsson, J. A model for prediction of vertical stress distribution near the soil surface below rubber-tracked under carriage systems fitted on agricultural vehicles. *Soil Tillage Res.* **2016**, *155*, 116–123. [CrossRef]
87. Zhao, D.; Yao-ming, L.; Zhong, T. Compaction effects of wheeled vehicles and tracked on farmland soil. *Trans. Chin. Soc. Agric. Eng.* **2020**, *36*, 10–18.
88. Liang, W.; Xian-biao, D.; He-hua, J. Homography based visual measurement of wheel sinkage for a lunar rover. *J. Astronaut.* **2011**, *32*, 1701–1707.
89. Hegde, G.M.; Ye, C. Computer-vision-based wheel sinkage estimation for robot navigation on lunar terrain. *IEEE/ASME Trans. Mechatron.* **2013**, *18*, 1346–1356. [CrossRef]
90. Shirai, T.; Ishigami, G. Development of in-wheel sensor system for accurate measurement of wheel terrain interaction characteristics. *J. Terramech.* **2015**, *62*, 51–61. [CrossRef]
91. Zhan, H.; Li, L.; Fu, Y. Applying Large Language Model to a Control System for Multi-Robot Task Assignment. *Drones* **2024**, *8*, 728. [CrossRef]
92. Pedroche D.S.; García, J.; Amigo, D.; Zubasti, P.; Molina, J.M. Drone Swarm for Distributed Video Surveillance of Roads and Car Tracking. *Drones* **2024**, *8*, 695. [CrossRef]
93. Lee, S.; Kim, H.S.; Hwang, S. T–S Fuzzy Observer-based Output Feedback Lateral Control of UGVs Using a Disturbance Observer. *Drones* **2024**, *8*, 685. [CrossRef]
94. Chen, S.; Miao, Z.; Chen, H.; Mukherjee, M.; Zhang, Y. Point attention Net: A graph attention convolution network for point cloud segmentation. *Appl. Intell.* **2023**, *52*, 11344–11356. [CrossRef]
95. Alaba, S.; Ball, J. Deep Learning-based Image 3D Object Detection for Autonomous Driving: Review. *IEEE Sens. J.* **2023**, *23*, 3378–3394. [CrossRef]