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## **Lateral Handling Improvement with Dynamic Curvature Control for an Independent Rear Wheel Drive EV**

Young-Jin Jang<sup>1</sup>, Min-Young Lee<sup>2</sup>, In-Soo Suh<sup>3</sup>, Kwang Hee Nam<sup>4</sup>

<sup>1</sup> Research Engineer, Hyundai Motor, Korea, [youngjin530@naver.com](mailto:youngjin530@naver.com)

<sup>2</sup> PhD Student, KAIST, Korea, [lighton@kaist.ac.kr](mailto:lighton@kaist.ac.kr)

<sup>3</sup> Professor, KAIST, Korea, [insoo.suh@kaist.ac.kr](mailto:insoo.suh@kaist.ac.kr) (corresponding author)

<sup>4</sup> Professor, POSTECH, Korea, [kwnam@postech.ac.kr](mailto:kwnam@postech.ac.kr)

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### **Abstract**

The integrated longitudinal and lateral dynamic motion control is important for four wheel independent drive (4WID) electric vehicles. Under critical driving conditions, direct yaw moment control (DYC) has been proved as effective for vehicle handling stability and maneuverability by implementing optimized torque distribution of each wheel, especially with independent wheel drive electric vehicles. The intended vehicle path upon driver steering input is heavily depending on the instantaneous vehicle speed, body side slip and yaw rate of a vehicle, which can directly affect the steering effort of driver. In this paper, we propose a dynamic curvature controller (DCC) by applying a newly-defined parameter, the dynamic curvature of the path, derived from vehicle dynamic state variables; yaw rate, side slip angle, and speed of a vehicle. The proposed controller, combined with DYC and wheel longitudinal slip control, is to utilize the dynamic curvature as a target control parameter for a feedback, avoiding estimating the vehicle side-slip angle. The effectiveness of the proposed controller, in view of stability and improved handling, has been validated with numerical simulations and a series of experiments during cornering engaging a disturbance torque driven by two rear independent in-wheel motors of a 4WD micro electric vehicle.

*Keywords: direct yaw-moment control, dynamic curvature, stability, handling*

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### **1 Introduction**

Vehicle lateral dynamic control techniques such as direct yaw-moment control (DYC) and active front- and rear-wheel steering can improve the lateral dynamic performances by directly regulating yaw rate and tire slip angles and thus lateral tire forces, and/or by generating speed differentials or torque distributions, which are important in vehicle stability and maneuverability control under critical driving circumstances, such as aggressive cornering or near-limit driving performance conditions [1].

The fundamental DYC adopts the yaw moment observer by estimating the yaw rate, which is difficult to be measured. Using the yaw moment observer, the controller compensates the disturbance applied to the EV to keep the vehicle follow the reference trajectory. When the yaw rate is not the same as the desired yaw rate for cornering, the yaw moment observer finds the differences between actual and desired value of yaw rate. The perspective of DYC control is to provide a feedback of the difference of yaw rate for desired cornering regardless of disturbances.

Numerous control methods applying DYC have been proposed and verified in vehicle lateral dynamic performance improvement to assist drivers in enhancing cornering and straight-line stability on slippery roads and in unfavorable road conditions [2]. For micro electric vehicles (MEVs) with in-wheel motor application, an electric differential system is widely used [3]. Combined with independently controlled four in-wheel motored vehicles, the stability of electric vehicles (EVs) especially can be improved by controlling the driving/braking forces from generated speed differential between left and right wheels, thus the yaw moment is controlled to minimize the side slip angle at a vehicle center of gravity (c.g.) [3]. Recently numerous studies are reported to improve the vehicle stability and maneuverability by applying body slip angle fuzzy observer, parameter uncertainties for robust yaw moment control [4], sliding mode control [5], and comprehensive unified approach or vehicle full-state estimation [6].

However, due to the difficulties on estimating the side slip angle or lateral velocity of the vehicle, current production vehicles are not equipped to measure the side slip angle directly. Thus, the side slip angles are estimated for vehicle control purposes for lateral dynamic stability and steer-by-wire or brake-by-wire system development, which require full vehicle dynamic state feedback [7]. Common techniques for estimating side slip angle are integrating on-board inertial sensors or using a physical model as an observer. While these approach have applied to actual vehicle dynamics control, both has some fundamental errors, such as accumulated sensor errors and unrealistic measurements from road grade or bank angle, etc., while estimating with a physical vehicle model can be sensitive to changes in the vehicle parameters and inaccuracy on slippery road surfaces, etc.

We propose a new parameter, called as a dynamic curvature, to be applied as a main control target parameter. By introducing the dynamic curvature as a main control parameter in the proposed dynamic curvature control (DCC) system, it will eliminate the necessity of measuring the vehicle side slip angle, which can be a unique approach from conventional DYC systems. In this case, the measured dynamic curvature can be obtained from an on-board IMU (inertial motion unit), while the reference dynamic curvature is set as the curvature of the vehicle path of the neutral steering case. An IMU is the basic on-board equipment for proposed

DCC system. The dynamic curvature can be measured relatively easily, compared with estimating the vehicle sideslip angle, which will be described in this paper.

## 2 Vehicle Lateral Dynamics for DCC during Cornering

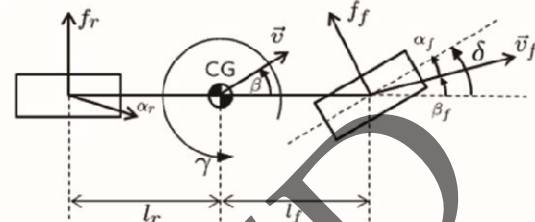


Fig. 1. The bicycle model of the vehicle

### 2.1 Lateral Dynamics during Cornering

As shown in Fig.1, the lateral motion dynamics of a vehicle can be simplified as a bicycle model with two degrees of freedom by neglecting the force differences between left and right wheels [8]. From the model, the equations of lateral motion can be derived as:

$$mv \left( \frac{d\beta}{dt} + \gamma \right) = 2(f_f + f_r) \quad (1)$$

$$I \frac{d\gamma}{dt} = M_z + M_t + M_d \quad (2)$$

where  $f_r$  and  $f_f$  are the lateral forces of front and rear tires, respectively,  $\beta$  is the vehicle side slip angle at the vehicle center of gravity (c.g. or CG),  $\gamma$  is the yaw rate,  $M_z$  is the motor torque moment from differential force,  $M_t$  is the torque moment from the lateral force of tire,  $M_d$  is a disturbance torque moment, and  $I$  is the yaw moment of inertia of the vehicle. For simplicity of analysis and verification, the longitudinal slip of tires or roll-over motions are not included in this paper. It is also assumed that the lateral force on the tire is generally proportional to tire slip angle for small slip angles [8]. Side slip angles of each tire are approximated with the side slip angle of vehicle body. Using the above equations, the lateral dynamics of side slip angle and yaw rate can be derived as follows [8]:

$$\dot{x} = A(v_x)x + B(v_x)u, \quad (3)$$

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, u = \begin{bmatrix} \delta \\ M_z \end{bmatrix},$$

$$A(v_x) = \begin{bmatrix} \frac{2(C_f + C_r)}{mv_x} & -1 - \frac{2(l_f C_f - l_r C_r)}{mv_x^2} \\ \frac{2(l_f C_f - l_r C_r)}{I} & \frac{2(l_f^2 C_f + l_r^2 C_r)}{Iv_x} \end{bmatrix},$$

$$B(v_x) = \begin{bmatrix} \frac{2C_f}{mv_x} & 0 \\ \frac{2l_f C_f}{I} & \frac{1}{I} \end{bmatrix}$$

It follows from (3) that the transfer functions of steering angle and state variables are obtained such that

$$\frac{\beta(s)}{\delta(s)} = \frac{g_{b1}s + g_{bo}}{s^2 + a_1s + a_2}, \quad (4)$$

$$\frac{\gamma(s)}{\delta(s)} = \frac{g_{f1}s + g_{f0}}{s^2 + a_1s + a_2}, \quad (5)$$

$$\frac{\gamma(s)}{M_z(s)} = \frac{g_{m1}s + g_{m0}}{s^2 + a_1s + a_2}, \quad (6)$$

where  $g_{b1} = \frac{2C_f}{mv_x}$ ,  $g_{bo}(v_x) = -\frac{2l_f C_f}{I} + \frac{4l_r L C_f C_r}{mlv_x^2}$ ,

$g_{f1} = \frac{2l_f C_f}{I}$ ,  $g_{f0}(v_x) = \frac{4L C_f C_r}{mlv_x}$ ,  $g_{m1} = \frac{1}{I}$ ,

$g_{f0}(v_x) = \frac{4L C_f C_r}{mlv_x}$ ,  $g_{m0}(v_x) = \frac{2(C_f + C_r)}{mlv_x}$ ,

$a_1 = \frac{2(C_f + C_r)}{mv_x} + \frac{2C_f}{Iv_x}$  and

$a_2 = \frac{4L^2 C_f C_r}{mlv_x^2} - \frac{2(l_f C_f - l_r C_r)}{I}$ .

$\alpha_f, \alpha_r$ : tire slip angle at front and rear

$\beta$ : body side slip angle at vehicle c.g.

$\beta_f, \beta_r$ : tire side slip angle at front and rear

$\delta$ : steering angle input by driver

$\vec{v}$ : velocity vector at vehicle c.g.

$v_x$ : longitudinal velocity

$m$ : mass of the vehicle

$l_f, l_r$ : distances between the vehicle c.g. and front or rear wheel axle

$L$ : wheelbase of the vehicle ( $=l_f + l_r$ )

$C_f, C_r$ : stiffness at rear and front tire

With the simplified 2-d.o.f. model, the lateral dynamic equations can be described with two state variables and two inputs. Specifically, the yaw rate,  $\gamma$  and side slip angle,  $\beta$  can be controlled by steering angle,  $\delta$  and motor torque moment  $M_z$ . The objective of a DYC in general is to maintain vehicle stability via regulating the vehicle yaw rate. It is emphasized here that the differential torque of two rear motors is utilized for DYC aside from steering angle change. It is believed that the differential torque makes the vehicle's cornering effort more less than relying just on side slip angle of the front wheel. Sub-headings are numbered as above. Subsections are numbered sequentially.

## 2.2 Dynamic Control for Neutral Steering

Vehicle handling conditions during cornering will be different depending on vehicle speeds and difference in tire stiffness, etc. They are classified as under-steering, neutral steering, and over-steering, as shown in Fig. 2. A normal vehicle tends to be under-steering making the radius of curvature larger, if it is accelerated with a constant steering angle. The side slip angle in case of under-steering is illustrated in Fig.3.

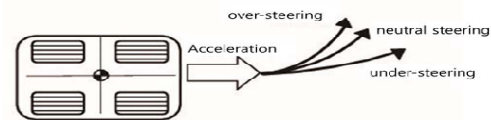
The vehicle dynamic motions during a neutral steering can be described from (3) by letting  $v_x$  a constant,  $v_c$ .

$$\dot{\bar{x}} = A(v_c)\bar{x} + B(v_c)u \quad (7)$$

$$\bar{x} = \begin{bmatrix} \bar{\beta} \\ \bar{\gamma} \end{bmatrix}, u = \begin{bmatrix} \delta \\ M_z \end{bmatrix},$$

where  $v_c$  is a constant longitudinal velocity,  $\delta$  is a steering angle for neutral steering, and the variables with ' $\bar{\cdot}$ ' indicate the variables while the vehicle is under neutral steering for a constant radius.

(a)



(b)

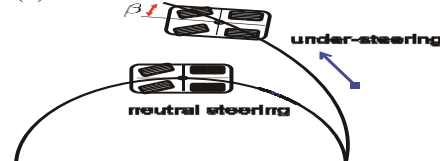


Fig. 2. Vehicle dynamic motions during cornering (a) Vehicle handling conditions, (b) Side slip angle during under-steering

### 3 Curvature Determination and Dynamic Curvature for Control

#### 3.1 Curvature of Vehicle Driving Path

There are many ways to measure the curvature during vehicle motion. Firstly, the curvature can be derived from the dynamic steady state model with the following assumptions, that the sum of the lateral forces at front and rear tires is the same as the centrifugal force, and that the torque moment by the lateral force is zero. When the vehicle drives following a curve with a constant radius, the curvature should be equal to [8]:

$$\frac{1}{R} = \frac{\delta}{L + mv_x^2 \left( \frac{l_r C_r - l_f C_f}{2C_f C_r L} \right)} \quad (8)$$

where the steering angle and longitudinal speed can be measured by on-board sensors in a vehicle. However, a drawback of this approach is that the tire stiffness,  $C_r$  and  $C_f$  are not readily available, so they need to be estimated by other methods. Also, there can be possible errors because this curvature formula is derived from the steady state model.

#### 3.2 A Dynamic Curvature

In this work, we introduce a dynamic curvature variable as a main control parameter within a DCC system. When the radius of vehicle cornering path is  $R$ , the instantaneous curvature of the vehicle driving path,  $k$  can be expressed, as introduced by Okajima et al. in [9]:

$$k = \frac{\beta + \gamma}{v_x} \quad (9)$$

The variable  $k$  can be considered as a measure of the instantaneous dynamic curvature of vehicle path, so we name it here as a ‘dynamic curvature’. Note that  $k$  is conveniently measured in practice since yaw rate,  $\gamma$  is directly available from a yaw rate sensor and time-derivative of the side slip angle,  $\dot{\beta}$  can be calculated from an on-board IMU sensor. The dynamic curvature is a characteristic variable of vehicle dynamics during cornering. Let a total lateral vehicle movement angle defined

$$\text{by } \varphi = \beta + \int_0^t \gamma d\tau .$$

### 4 Proposed Dynamic Curvature Control (DCC) System

#### 4.1 Proposed DCC Control with Reference Neutral Steering

The control algorithm and resultant performances of the direct yaw moment control method based on the yaw moment observer have been presented in [1-6]. The typical DYC algorithm utilizes the yaw rate as a feedback variable, to stabilize the vehicle lateral dynamic motion. Note that the typical DYC’s focus on regulating the yaw rate so that they may fail to obtain neutral steering trajectory when the lateral tire slip conditions are not even or changing. More specifically, DYC controls the yaw rate only, the vehicle maneuverability or the dynamic curvature control is not directly reflected during steering motion.

In this work, the dynamic curvature is a target variable to be controlled instead of yaw rate. It should be emphasized that the side slip angle change, being a major influential element to determine a circular motion, is included in the dynamic curvature. Therefore, the DCC enables the vehicle to track a neutral trajectory better than DYC. Note that the lateral acceleration which is equal to  $(\gamma + \dot{\beta})v_x$  directly obtainable from an IMU sensor. Therefore, the dynamics curvature is measured as

$$k = \frac{\gamma + \dot{\beta}}{v_x} = \frac{A_y}{v_x^2} \quad (10)$$

Note that the DCC is advantageous in implementation since the dynamic curvature,  $k$  is relatively easily measured for feedback control without monitoring or estimating the side slip angle,  $\beta$  separately.

Fig. 3 shows the proposed DCC block diagram based on measured dynamics curvature, which will be described in below. A steering reference value,  $\delta^*$  is applied as a command value.

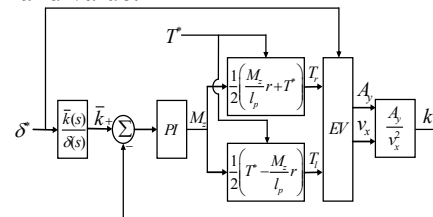


Fig. 3. Block diagram of the proposed DCC control algorithm

## 5 Simulation and Test Results

### 5.1 Simulation Results

TABLE I. VEHICLE PARAMETERS FOR SIMULATION

Parameter	Description	Value
$C_f$	Front tire stiffness	22000 N/rad
$C_r$	Rear tire stiffness	26000 N/rad
$m$	Vehicle mass	650 kg
$l_f$	Distance bet. CG and front tire	0.68 m
$l_r$	Distance bet. CG and rear tire	0.59 m

An EV with two sets of independent motors at rear wheels is accelerated from 18 km/h to 30 km/h for 20 seconds for a simulation purpose. The velocity of the reference model is chosen as  $v_c = 10$  km/h. Initial position of the vehicle is set as the coordinate of (0, 0) for plotting purpose. For the simulation and experimental purpose, the disturbance torque is applied at the front left tire. The disturbance torque moment applied from 7.5 second to 12.5 second with a value of 50 Nm at the front left tire, which simulates the vehicle velocity is abruptly changing during the cornering, possibly caused by strong wind and road friction difference between left and right, etc. By using the reference law, the reference dynamic curvature,  $\bar{k}$  is determined to keep the vehicle's desired trajectory with a constant speed, as shown in Fig. 4 (a). We can see that the vehicle's radius of curvature with the proposed control follows the reference trajectory, i.e. the neutral steering case, while the radius of the vehicle path without the control becomes larger as the vehicle velocity is increased, as shown in Fig. 4 (b).

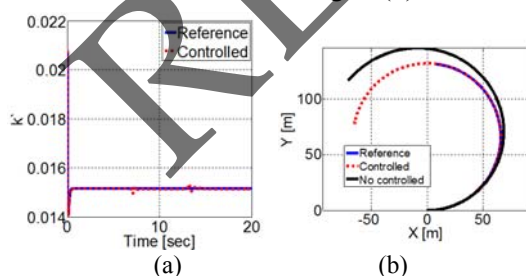


Fig. 4. Simulation results: (a) the dynamic curvature  $k$ , (b) the vehicle trajectory

### 5.2 Test Results

Fig.9 shows the experimental environment, such as the test track, the micro electric vehicle with two rear wheel independent control, driving test scene, and the on-board GPS receiver. The

specification of the test track can be explained with the vehicle trajectory as shown in Fig. 5. The vehicle, as described in [10], of which specifications are shown in Table 1, is driven along the line in Fig. 5 (a).



Fig. 5. Experimental environment: (a) test track for driving test (b) subject vehicle for testing with two independent motor control at rear wheels

The actually engaged steering angle input and torque input during this experiment are shown in Fig. 5. Fig. 6 shows the measured vehicle trajectory with the dynamic curvature controller with the dynamic curvature controller turned on. Differently from no curvature control case, the trajectory keeps the constant curvature, as if the vehicle maneuvers under neutral steering. As the longitudinal speed increases, the lateral acceleration increases also due to the centrifugal forces during cornering. But the dynamic curvature controller provides a feedback signal of the lateral acceleration to stay at the constant curvature radius.

## 6 Conclusion

The direct yaw rate control (DYC) system introducing newly defined parameter of the dynamic curvature,  $k$ , which is called in dynamic curvature control (DCC) system, is proposed and verified the effectiveness with the simulation and a series of experiment in this paper. The instantaneous dynamic curvature can be obtained from the measured lateral acceleration and longitudinal speed by an on-board IMU easily.

The measurable dynamic curvature,  $k$ , is utilized as a feedback parameter during vehicle cornering motion especially when the speed of the vehicle is varied, emphasizing the dynamic handling as well as the vehicle dynamic stability.

The dynamic curvature of the vehicle path with the control system maintained the constant radius of curvature, as the reference model of neutral steering during the cornering, even with the disturbance engaged, which proved the effectiveness of vehicle handling as well as the dynamic stability.

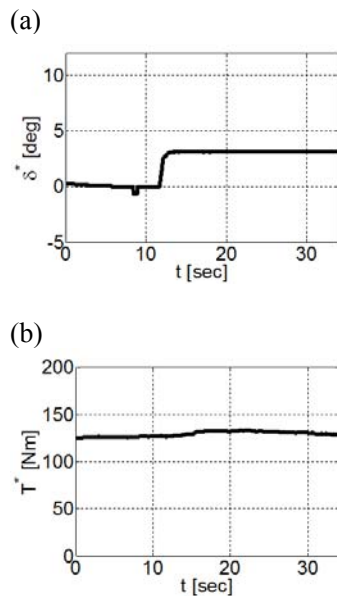


Fig. 6. Vehicle control input during the experiment when the controller is turned on: (a) steering angle, (b) torque at rear wheels

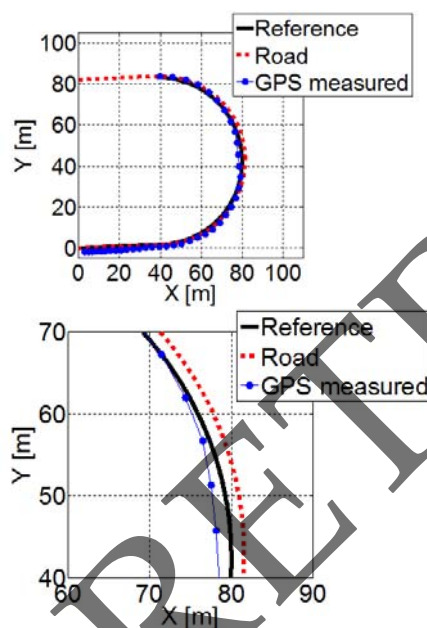


Fig. 7. Experimental results when the controller is turned on: measured vehicle trajectory

## References

- [1] D. Li, S. Du, and F. Yu, "Integrated Vehicle Chassis Control based on Direct Yaw Moment, Active Steering and Active Stabiliser," *Vehicle System Dynamics*, Vol. 46, Supplement, pp. 341-351, 2008.
- [2] P. Raksincharoensak, T. Mizushima, and M. Nagai, "Direct Yaw Moment Control System Based on Driver Behavior Recognition," *Vehicle*

*System Dynamics*, Vol. 46, Supplement, pp. 911-921, 2008.

- [3] H. Okjima, S. Yonaha, N. Matunaga and S. Kawaji, "Direct Yaw-Moment Control Method for Electric Vehicles to Follow the Desired Path by Driver", *Proceedings of SICE Annual conference*, pp. 642-647, 2010.
- [4] H. Du, N. Zhang, and G. Dong, "Stabilizing Vehicle Lateral Dynamics with Considerations of Parameter Uncertainties and Control Saturation through Robust Yaw Control," *IEEE Trans. Veh. Tech.*, Vol. 59, no. 5, June 2010.
- [5] H. Zhou and Z. Liu, "Vehicle Yaw Stability-Control System Design Based on Sliding Mode and Backstepping Control Approach," *IEEE Trans. Veh. Tech.*, Vol. 59, no.7, September 2010.
- [6] W. Cho, J. Choi, C. Kim, S. Choi, and K. Yi, "Unified Chassis Control for the Improvement of Agility, Maneuverability, and Lateral Stability," *IEEE Trans. Veh. Tech.*, Vol. 61, no. 3, March 2012.
- [7] O. Mokhiamar and M. Abe, "Active Wheel Steering and Yaw Moment Control Combination to Maximize Stability as well as Vehicle Responsiveness during Quick Lane Change for Active Vehicle Handling Safety," *Proc. Instn. Mech. Engrs.*, Vol. 216, Part D: J. Automobile Engineering, pp. 115-124, 2002.
- [8] R. Rajamani, "Vehicle Dynamics and Control", Springer, 2012. J.J. Romm, *The hype about Hydrogen*, ISBN 1-55963-704-8, Washington, Island Press, 2005.
- [9] H. Okajima, S. Yonaha, N. Matsunaga, and S. Kawaji, "Direct Yaw-Moment Control Method for Electric Vehicles to Follow the Desired Path by Driver", in *Proceedings of SICE Annual conference*, pp 642 - 647, 2010.
- [10] I. Suh, K. Hwang, M. Lee, and J. Kim, "In-wheel motor application in a 4WD electric vehicle with foldable body concept," *IEEE Int. Electric Machines & Drives Conf.*, pp. 1307-1312, May 2013.