sustainability

MDPI

Article

Fuzzy Measures of Multidimensional Poverty in the Mediterranean Area: A Focus on Financial Dimension

Martina Ciani, Francesca Gagliardi, Samuele Riccarelli and Gianni Betti *

Department of Economics and Statistics, University of Siena, 53100 Siena, Italy; ciani20@student.unisi.it (M.C.); gagliardi10@unisi.it (F.G.); samuele.riccarelli@unisi.it (S.R.)

* Correspondence: gianni.betti@unisi.it; Tel.: +39-0577-233084

Received: 16 October 2018; Accepted: 21 December 2018; Published: 28 December 2018

Abstract: The main scope of the paper is to adopt a fuzzy sets approach for the measurement of multidimensional poverty over a period of eight years, from 2007 to 2015, which takes into account the effect of the 2008 economic and financial crisis. In particular, the focus is on the financial dimension of poverty, and its effects on citizens in the EU Mediterranean Area. The empirical analysis, based on the European Union—Statistics on Income and Living Conditions survey (EU-SILC), covers eight Mediterranean Countries.

Keywords: multidimensional poverty; financial poverty; Mediterranean area; fuzzy sets

1. Introduction

It is already well established in the literature of the last thirty years, that to understand poverty and social exclusion, it is necessary to consider deprivation aspects simultaneously in its multiple dimensions. In fact, the need to adopt a multidimensional approach has been underlined, among others, by References [1–5]. In that literature, the many attempts to study the poverty phenomenon under a multidimensional perspective appear to fall into two broad categories: non-axiomatic versus axiomatic.

A non-axiomatic approach implicitly considers indicators to be perfectly substitutable. In this approach different indicators or *items* are combined to obtain a multidimensional index; this combination can be done at a macro level [6] or at an individual level [7–9]. By contrast, References [10–13] have contributed to the development of an axiomatic approach.

In the present work, we follow a non-axiomatic approach and go beyond the conventional study of poverty based simply on the poor/non-poor dichotomy defined in relation to some chosen poverty line. Instead, multidimensional poverty is treated as matters of degree determined in terms of the individual's position in the distribution of some aspects of their living conditions. The state of deprivation is, thus, seen in the form of "fuzzy sets" to which all members of the population belong but to varying degrees. In this way, we are able to clarify and propose a solution for one of the problems raised in the literature [14,15], and especially in Reference [16]: "How can different attributes be aggregated? . . . A distinction may be drowning between those who adopt a union approach and those who use an intersection measure".

In particular, this paper has the peculiar scope of adopting a fuzzy approach for the measurement of the multidimensional poverty in the Mediterranean Area, over a period of eight years, from 2007 to 2015, which takes into account the effect of the 2008 economic and financial crisis. A particular focus is given to the financial dimension of poverty since it represents some peculiar characteristics described later in Section 3. It is often based on somewhat subjective questions collected in a sample survey, and whose answers could be affected by measurement errors determined by the emotional status of respondents. From this point of view, sociological studies (see the miler stone [17]) have proven that

fuzzy sets and fuzzy logic provide useful tools to address unclear and imprecise data, and can help researchers to treat them accordingly.

The paper is composed of six sections; after the present introduction, Section 2 describes the wide literature review on the fuzzy sets approach to poverty measurement, starting from the seminal paper of Reference [18], until the very new contributions of recent years. Section 3 constitutes background information on the pre-crisis and post-crisis financial situation of households in the Mediterranean Area. Next, Section 4 explains the methodology proposed, while the empirical analysis is reported in Section 5. Finally, Section 6 concludes the paper.

2. Literature review

A first general treatment concerning the poverty of the households of the Mediterranean countries is the indicator At-Risk-of-Poverty Rate (ARPR)—corresponding to the percentage of people with an equivalised disposable income (after social transfer) below the at risk of poverty threshold, which is set at 60% of the national median equivalised disposable income after social transfer. As said, the main scope of this paper is to go beyond this conventional study of poverty based simply on the poor/non-poor dichotomy. Instead, monetary and multidimensional poverty are treated as matters of degree determined in terms of the individual's position in the distribution of some aspects of their living conditions, by means of fuzzy sets.

A number of authors have evoked the concepts of fuzzy sets in the analysis of poverty and living conditions. In this section, we review the literature on constructing fuzzy measures of poverty, deprivation, social exclusion, and other social phenomena.

The very first and seminal paper proposing fuzzy sets in poverty analysis is Reference [18]; it is based on monetary variables only (income or consumption expenditure), and it defined a grey area in which there is no absolute certainty about the predicate "being poor" or "being deprived". Rather than dichotomizing, the fuzzy set membership function would range from zero to one: When the result is zero or one, we have complete certainty that the individual is non-poor or poor, respectively; differently, any other value between zero and one indicates a partial degree of certainty in the predicate "being poor" or "being deprived". Reference [18] proposed a very simple linear function of the monetary variable to model the membership function between one and zero.

Later, References [19–21], improved such monetary index cross-sectionally, by proposing a non-linear function of the monetary variable, while References [22,23] developed a monetary fuzzy approach in a longitudinal perspective, by defining a set of persistent fuzzy poverty measures.

The Totally Fuzzy and Relative (TFR) approach of Reference [24] was the first paper to introduce fuzzy sets in the estimation of multidimensional poverty, by introducing the cumulative distribution function, F, in the definition of the fuzzy and multidimensional indices. Reference [25], later improved in Reference [26], developed the TFR approach by complementizing the F function with the Lorenz curve, L, and, thus, introducing a multidimensional index which could be seen as a Generalized Gini index.

The duality of poverty and welfare was firstly represented by Reference [27], since both concepts are diffuse and then nothing other than a fuzzy set approach can fit with them. Instead, References [28–30] gave assessments of the multidimensional fuzzy approach in the light of Sen's Functioning Approach [5,31].

An empirical comparison of the fuzzy sets approach versus other various approaches in measuring multidimensional poverty was performed by Reference [32], while Reference [33] edited the first book on philosophy [34], mathematics, economics, empirical applications worldwide [35], of the fuzzy set approach to poverty measurement. More recently, several other studies have further developed the methodology; these include References [36–46].

Finally, in the last decade, the fuzzy sets approach has been successfully applied in other social phenomena, such as marital disruption [47], quality of life [48–51], educational mismatch of

graduates [52], employment [53], and sustainable development [54]. In this way, this multidimensional methodology has proved to be robust and applicable also to other fields besides poverty.

3. Background: Financial Situation in the Mediterranean Area

A very rich source of information for observing the financial situation of households in the EU consists in the set of indicators collected every year by the European Union—Statistics on Income and Living Conditions survey (EU-SILC) survey and is related to arrears in the payment of utility bills, mortgages or rents, and consumer loans. The indicators based on arrears are, in fact, very useful to assess the financial situation of households and moreover, they allow to anticipate with good precision their future insolvencies. Reference [55] also identified the common definition of "over-indebtedness" of households based exactly on households which encounter difficulties in respecting their commitments, whether guaranteed or unsecured loan services. The economic crisis—which started in August 2007 and continued until the middle of 2011—affected European households differently. For some countries, over-indebtedness was already a consolidated problem even before the crisis. For others it was largely triggered by the crisis itself and by the consequent responses to it.

As shown in Table 1, the high percentages of Cyprus and Greece show how over-indebtedness in these two countries was already a widespread problem even before the crisis itself. From EU-SILC data relating to 2007, it can be observed that 23% and 26.4% of households of Cyprus and Greece, respectively, were in arrears in the payment of at least one financial commitment (mortgage or rent, utility bills or consumer loans).

Table 1. Household with arrears.

Country	Households with Arrears (%)								Inability to Face Unexpected Expenses (%)	
	Arrears on Key Commitments		Arrears on Utility Bills		Arrears on Mortgage or Rent Payments		Arrears on Consumer Loans			
	2007	2013	2007	2013	2007	2013	2007	2013	2008	2013
Cyprus	23.0	33.6	10.0	21.9	6.1	8.8	15.3	22.1	40.0	55.0
Greece	26.4	39.0	15.7	31.8	7.4	12.9	12.2	15.0	27.5	48.0
Spain	7.6	11.9	4.6	8.3	3.5	6.4	2.2	3.1	30.0	42.0
France	9.8	9.2	6.4	6.2	5.8	5.5	2.7	2.3	35	35.0
Italy	12.5	14.2	10.4	12.0	3.8	4.9	3.2	2.0	31.5	40.0
Malta	8.0	12.2	7.2	11.4	1.2	2.3	0.8	1.0	33.0	22.5
Portugal	7.0	11.8	5.2	8.2	3.1	5.7	1.8	2.9	27.0	43.0
Slovenia	14.2	21.2	11.4	19.7	2.3	3.7	5.1	4.1	45.0	45.0
Croatia	-	29.7	-	28.2	-	1.3	-	6.0	-	-
Average EU	10	12	7.3	10.1	3.4	4.1	2.8	2.9	34.3	39.4

However, the situation had been drastically worse, reaching very high percentages in the post-crisis period, amounting to 33.6% for Cyprus and 39% for Greece (estimated values refer to EU-SILC data of 2013). The restructuring of the Greek economy and the measures taken by the government to reduce structural budget deficits have, in fact, weakened a situation already in itself weak, and reduced the real incomes of households from 18,623 euros in 2008 to 15,177 in 2012. Cuts in the public sector and in pensions contributed to this decline, too. Further, there was a growth in unemployment.

Overall, in the all European countries, the increase in the level of over-indebtedness recorded since the start of the economic crisis to 2013 was rather modest, with an increase average of only two percentage points of households in arrears in payment of one or more financial commitments, from 10% to 12%. Paying attention to other Mediterranean countries, high levels of over-indebtedness were found in Slovenia (from 14.2% in 2007 to 21.2% in 2013), in Croatia (in 2013 the share estimated was equal to 29.7%), and to a lesser, extent in Italy, where the increase was relatively small (from 12.5% to 14.2%). Malta, Spain, and Portugal showed an increase of approximately 4/5 percentage

points, and remained below the EU average, along with France for which the situation tended to be even better, albeit slightly (from 9.8% in 2007 to 9.2% in 2013). Most of the increase in the level of European over-indebtedness was determined by arrears in the payment of utility bills, which went from 7% in 2007 to 10% in 2013. For the Mediterranean Area, the percentage tended to increase and rose from an average of 8.9% in 2007 to 14.9% in 2013. In fact, only a minority of households tended to have consumer credit commitments, a good part had a mortgage or a rent, and almost all of them were forced to pay their bills. The arrears in mortgages or rents were in fact increased by only 1%, while those relating to consumer loans remained almost the same. At a European level, it was also demonstrated that when household budgets were under pressure, the utility bills were one of the first commitments not to be remunerated.

Greece is the background of the most critical situation: if already in 2007, there was 16% of over-indebted households which struggled to pay utility bills, the crisis further worsened their situation, by more than doubling that percentage in 2013. Cyprus and Slovenia also suffered greatly from the effects of the economic crisis, going from a value of about 10–11% to shares of 20–21% in the same period. The growth of the trend of this type of debt by European households was not the same for all countries: for Greece and Cyprus, a small increase occurred from 2007 to 2010 (+ 2/3%) followed by a strong increase in the following three years. For other countries including Slovenia, the increase was greater from 2007 to 2010, going from 11.4% of over-indebted households in payment of utility bills to 18%, and then of only two percentage points three years later. In this case, the growth was caused mostly by a strong decline in employment and the consequent reduction of family incomes. High levels were registered in all countries of south-eastern Europe, including Croatia. Italy and Malta moved away from the lowest percentages of France, Spain and Portugal (6.2%, 8.3%, and 8.2%, respectively) with shares equal to 11–12% on EU-SILC data of 2013. Italy in 2007 already had a percentage of 10% of over-indebted households.

The estimated percentages in the payment of mortgages or rents tended to be slightly more for Spain, France, Italy, and Portugal, amounting to 5–6% in 2013 and a little lower in 2007 and for Malta where the percentage estimated for 2013 data was 2.3%. Slovenia showed a share of 3.7%, that was higher than that estimated in 2007, equal to 2.3%. Even in this case, age, education, and family income were among the most influential factors. Moreover, among the households' subject to this type of indebtedness we found those households whose head of the family was in poor health had double the probability of falling into arrears. Health problems can, in fact, affect both work and income and increase the outgoings due to medical expenses. According to recent investigations, it seems that only a minority of people attribute their financial difficulties to the excess of indebtedness. The request for a loan can, however, be a direct consequence of a difficult financial situation, because households often rely on consumer credit to cope with a sudden drop in their income. If this is a temporary phase, then the loan will be useful, but otherwise, it may represent the beginning of a state of over-indebtedness for the family from which they find it difficult to extricate themselves.

A last but fundamental indicator to assess the conditions of the family financial situation is the inability to face unexpected expenses. An unexpected expense may refer to an unscheduled surgery, a funeral, a large repair for home, the purchase of durable goods, such as a new car, a washing machine and so on. The data are collected through the EU-SILC survey, which asked the respondent expressly if their family can afford the payment of unplanned expenses by relying on their own resources. This means that the family can cope with possible money outflows without asking for economic aid, by charging their account in the requested period so that the situation regarding potential debts does not worsen. According to data collected by Eurostat, the percentage of households in the European population unable to face with unexpected expenditure amounted to 34.3% in 2008. The situation went on, in general, to worsen over the next few years, and topped a value average equal to 39.4% in 2013. The countries where there was a strong increase in shares were the ones most affected by the economic crisis: among them Greece with an increase of about 20.5 percentage points. Among the countries of the Mediterranean Area, the worst situation was, however, in Cyprus where in 2013 the estimated

percentage was about 55% compared to 48% in Greece. France and Slovenia saw their percentages remain intact. However, they were high and amounted to 35% and 45%, respectively. Portugal, Spain, and Italy presented similar situations with 40% of households who declared not being able to deal with any unexpected expenses. A significant decrease occurred for Malta, equal to 10 percentage points, and reached an estimated value of just over 20% in 2013.

Considering the full picture of Table 1, one can observe how subjective or emotional answers to questions in EU-SILC may lead to a high increase in over-indebtedness in Mediterranean countries, while the increase in EU as a whole was quite limited (i.e., just +2% in "Arrears on key commitments"). So, at the conclusion of this background section, one may wonder "How reliable are questions which directly collect information on financial poverty?" One answer could come from Reference [17], in which Fuzzy logic provides a useful tool to deal with unclear and imprecise data. In fact, the "fuzzy set" approach introduces a classification of a measure in terms of "degree of", for the individual's position, and in this way, this measure can successfully include qualitative as well as quantitative aspects of a phenomenon. This is the reason why the application of the method can be valuable in this context. This is because data in social sciences always encounters imprecise soft social facts.

4. Methodology

In this section, we describe the basic fuzzy methodology for constructing monetary and non-monetary measures of deprivation. As introduced in Section 2, the fuzzy set approach [56] treats poverty as a matter of degree, replacing the classical {0, 1} dichotomy, non-poor/poor into which statistical units (individuals or households) are traditionally divided. In the fuzzy conceptualization, all individuals are subject to poverty but at different degrees, so that each individual has a certain *propensity* to poverty in the whole continuum range [0, 1]. There are several advantages of treating poverty as a matter of degree, applicable to all members of the population, rather than as simply "yes–no" state. These are summarized by Reference [57] as follows:

(1) Non-monetary poverty depends on forced non-access to various facilities or possessions determining the basic conditions of life. An individual may have access to some of those but not to others. Hence, clearly, non-monetary poverty is inherently a matter of degree, and some *quantitative approach* (such as the present one) is essential.

(2) The fuzzy approach provides more robust and stable indicators of poverty [58]. Apart from the various methodological choices involved in the construction of conventional poverty measures, the introduction of fuzzy measures carries additional factors on which choices have to be made. The fundamental factor concerns the choice of "membership functions", meaning a quantitative specification of the propensity to poverty of each statistical unit (household/person), given the level and distribution of income of the population.

4.1. Fuzzy Membership Function

Reference [26] proposed two fuzzy membership functions (m.f., one for monetary and the other for non-monetary deprivation), based on the basic contributions of References [18,24]. These m.f. have been further elaborated in Reference [44].

In the generalized form, such membership functions are defined for any statistical unit (household/individual) i as follows:

$$\mu_{i,K} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma | X_\gamma > X_i}{\sum_{\gamma=2}^{n} w_\gamma | X_\gamma > X_1} \right)^{\alpha_K - 1} \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma X_\gamma | X_\gamma > X_i}{\sum_{\gamma=2}^{n} w_\gamma X_\gamma | X_\gamma > X_1} \right) i : 1, \ldots, n-1;\ \mu_{n,k} = 0 \qquad (1)$$

where X is the equivalised income in the monetary poverty, or the overall score s in the non-monetary poverty (see Section 4.2 below), w_γ is the sample weight of unit of rank γ in the ascending distribution,

and α_K (k = 1, 2) are two parameters corresponding, respectively, to monetary and non-monetary dimensions of poverty. Reference [26] proved that this a variant of the α_K-Generalized Gini index. The two α_K parameters are computed so that the mean of the corresponding membership function is equal to the At-Risk-of-Poverty Rate (ARPR, [59]) computed on the basis of the official poverty line. Reference [26] termed the monetary-based indicator as Fuzzy Monetary (FM), and the non-monetary indicator as Fuzzy Supplementary (FS).

4.2. Construction of the FS Measure

The construction of the Fuzzy Supplementary index follows a step-by-step. First of all, the items to be included in the index or indices are identified, which should be the more meaningful and useful ones (see Reference [59]). In fact, it is desirable to avoid items where issues of choice in terms of possession versus non-possession cannot be satisfactorily resolved, where the possession is relatively rare (i.e., possession of a boat), or where the degree of comparability among regions or countries is not sufficient. Then, for each item, we determine a quantitative deprivation indicator in the range [0, 1]: when the item is constituted by a fixed number of categories, then it is transformed using the following procedure. For each item we determine a deprivation score as follows:

$$d_{j,i} = \frac{1 - F(c_{j,i})}{1 - F(1)};\ j = 1, 2, \ldots, k;\ i = 1, 2, \ldots, n \tag{2}$$

where $c_{j,i}$ is the value of the category of the j-th item for the i-th individual and $F(c_{j,i})$ is the value of the j-th item cumulation function for the i-th individual. We transform the deprivation score to a positive score as follows:

$$s_{j,i} = 1 - \frac{1 - F(c_{j,i})}{1 - F(1)} = \frac{F(c_{j,i}) - F(1)}{1 - F(1)};\ j = 1, 2, \ldots, k;\ i = 1, 2, \ldots, n \tag{3}$$

In the special, but also common case, where the variable is a dichotomy, the deprivation index d is 1 for deprivation and 0 otherwise, while the positive score s is 0 for deprivation and 1 otherwise. Such indicators are used in a first exploratory factor analysis to identify underlined "dimensions". By dimension, we intend a distinct group of items of non-monetary poverty, ideally independent from other dimensions, and which should describe a particular facet of living conditions. After this first exploratory factor analysis, we proceed to rearrange some items in the dimensions identified to create more meaningful groups. To test the goodness of fit of such final groupings, a confirmatory factor analysis is necessary. Then, the weights to be assigned to each item are determined within each dimension. They are based on two elements, namely the dispersion of the item (prevalence weights) and the correlation with other items in the same dimension (correlation weights). For a detailed description of the weight construction, see Reference [26].

The score within each dimension h, $s_{i,h}$, is calculated as the weighted mean of items in such dimension. The overall score s_i is defined as the simple average of the dimension scores $s_{i,h}$, thus, giving the same importance to all the dimensions, each of which represents a different facet of non-monetary (supplementary) poverty. Finally, as explained above, the membership function FS is defined in Equation (1), which could be rewritten for every dimension h as:

$$\mu_i = FS_{hi} = \left(1 - F_{(S),hi}\right)^{\alpha-1}\left(1 - L_{(S),hi}\right) = \left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma}\left|s_{h\gamma} > s_{hi}\right.}{\sum_{\gamma=2}^{n} w_{h\gamma}\left|s_{h\gamma} > s_{h1}\right.}\right]^{\alpha-1}\left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma}s_{h\gamma}\left|s_{h\gamma} > s_{hi}\right.}{\sum_{\gamma=2}^{n} w_{h\gamma}s_{h\gamma}\left|s_{h\gamma} > s_{h1}\right.}\right], \tag{4}$$

$$h = 1, 2, \ldots, m;\ i = 1, 2, \ldots, n;\ \mu_{hn} = 0$$

5. Empirical Analysis

The methodology described in Section 4 has been applied to the European Union—Statistics on Income and Living Condition Survey (EU-SILC) for years 2007, 2011, and 2015 for the Mediterranean area EU countries. The years have been chosen to be just before the crisis, during the crisis, and (hopefully) out of the crisis. The time gap of four years allows the four cross-sectional samples to be independent since the presence of the same individuals in SILC panels is four-years long (see Reference [60] for a full and detailed description of the SILC survey structure and sample design of the rotating panel).

Among the possible variables collected in the SILC surveys, 29 items have been selected according to Reference [44], which are reported in the central column of Table 2.

Table 2. Dimensions identified by exploratory and confirmatory factor analysis.

Dimension	Items of Deprivation	Weights
1 Basic lifestyle (FS1)	Meals with meat, fish or chicken	1.90
	Household adequately warm	2.18
	Holiday away from home	0.68
	Ability to make ends meet	0.39
2 Consumer durables (FS2)	Car	2.22
	PC	2.87
	Telephone	8.43
	Washing Machine	8.05
	TV	10.68
3 Housing amenities (FS3)	Bath or Shower	4.85
	Indoor flushing toilet	5.17
	Leaking roof and damp	1.80
	Rooms too dark	2.95
4 Financial situation (FS4)	Inability to cope with unexpected expenses	0.80
	Arrears on mortgage or rent payments	3.69
	Arrears on utility bills	1.99
	Arrears on hire purchase instalments	4.06
5 Environment (FS5)	Crime, Violence, vandalism	1.57
	Pollution	1.77
	Noise	1.34
6 Work and Education (FS6)	Early school leavers	5.45
	Low education	1.35
	Worklessness	0.51
	Duration of unemployment	1.79
7 Health related (FS7)	General health	0.97
	Chronic illness	0.46
	Mobility restriction	0.51
	Unmet need for medical exam	1.79
	Unmet need for dental exam	1.69

Exploratory and confirmatory factor analyses have been performed using such items. Seven dimensions of non-monetary poverty have been identified, as presented in the left-hand column of Table 2. The goodness of fit of confirmatory factor analysis is excellent, as reported in Table 3. The goodness of fit index (GFI), which is based on the ratio of the sum of squared discrepancies to the observed variances, is equal to 0.927. It ranges from 0 to 1 with higher values indicating a good fit. The fit is considered really good if the root mean square residual (RMR) is equal or below 0.06 (0.065 in our case). The root mean squared error of approximation (RMSEA) is based on the analysis of residuals, with small values indicating a good fit. In addition, confirmatory factor analysis performed well, with an RMSEA equal to 0.52.

To aggregate individual items into dimensions, prevalence-correlation weights *a là* Reference [26] have been calculated. These are reported in the right-hand column of Table 1. According to the prevalence principle, items with are very much diffused in the population receive a high weight, so that the few statistical units which are deprived receive a high risk of poverty. For instance, individuals who cannot afford a TV or a telephone are assigned a high membership function. On the other hand, items which are highly correlated with others in the dimension, receive smaller weights.

For any individual, aggregation of items into a particular dimension h (h = 1, 2, ... , m) is performed by a weighted mean over j items: $s_{hi} = \sum w_{hi} \cdot s_{hj,i} / \sum w_{hi}$ where w_{hj} is the weight of the j-th deprivation variable in the h-th dimension, as reported in the right-hand column of Table 2. The overall multidimensional poverty score for the i-th individual is calculated as the unweighted mean over dimensions:

$$s_i = \frac{\sum_{h=1}^{m} s_{hi}}{m} \quad (5)$$

Table 3. Confirmatory factor analysis results.

Index	Value
Goodness of fit (GFI)	0.927
Adjusted GFI	0.914
Parsimonious GFI	0.845
Root Mean Square Residual	0.065
RMSEA	0.052

The values s_i are then used in Equation (1) to determine parameter α_2, so that the mean of the FS values is equal to the conventional at-risk-of-poverty rate (ARPR).

Finally, this unique estimated parameter α_2 is used in Equation (4) to calculate the indicator for each dimension of deprivation separately (FS1-FS7). The results are reported in Table 4 for the three years considered in the analysis, namely 2007, 2011, and 2015.

Table 4. Fuzzy Supplementary (FS) and FS1–FS7 for 2007, 2011, and 2015.

2007	FS	FS1	FS2	FS3	FS4	FS5	FS6	FS7
Cyprus	0.1555	0.1396	0.0585	0.0749	0.1166	0.1455	0.1284	0.1425
Greece	0.2027	0.1653	0.1089	0.1129	0.1517	0.1689	0.1600	0.1651
Spain	0.1975	0.1452	0.0732	0.1034	0.0950	0.1721	0.1628	0.1427
France	0.1313	0.1006	0.0585	0.0777	0.0775	0.1262	0.1106	0.1068
Italy	0.1983	0.1641	0.0641	0.0996	0.1170	0.1919	0.1554	0.1687
Portugal	0.1815	0.1296	0.1147	0.1187	0.0965	0.1583	0.1511	0.1538
Slovenia	0.1086	0.0936	0.0516	0.0656	0.0751	0.0999	0.0929	
2011	**FS**	**FS1**	**FS2**	**FS3**	**FS4**	**FS5**	**FS6**	**FS7**
Cyprus	0.1453	0.1252	0.0370	0.0684	0.1153	0.1382	0.1230	0.1333
Greece	0.2137	0.1853	0.0939	0.1083	0.1713	0.2126	0.1662	0.1796
Spain	0.2176	0.1537	0.0635	0.0921	0.1023	0.1436	0.1795	0.1704
France	0.1406	0.1142	0.0459	0.0815	0.0870	0.1221	0.1184	0.1174
Italy	0.1957	0.1929	0.0454	0.1010	0.1160	0.1696	0.1564	0.1715
Malta	0.1542	0.1442	0.0326	0.0805	0.0837	0.1432	0.1259	0.1136
Portugal	0.1797	0.1356	0.0944	0.0977	0.1080	0.1477	0.1511	0.1279
Slovenia	0.1365	0.1181	0.0478	0.0736	0.0909	0.1158	0.1163	
2015	**FS**	**FS1**	**FS2**	**FS3**	**FS4**	**FS5**	**FS6**	**FS7**
Cyprus	0.1622	0.1243	0.0455	0.0769	0.1299	0.1292	0.1389	0.1127
Greece	0.2139	0.2078	0.1010	0.0962	0.1779	0.1762	0.1691	0.2070
Spain	0.2213	0.1635	0.0789	0.0921	0.1183	0.1501	0.1839	0.1463
France	0.1362	0.1115	0.0369	0.0798	0.0799	0.1168	0.1152	0.1063
Croatia	0.1999	0.1703	0.0813	0.1011	0.1174	0.0923	0.1631	0.1323
Italy	0.1994	0.1864	0.0362	0.0974	0.1162	0.1776	0.1601	0.1777
Malta	0.1634	0.1507	0.0426	0.0821	0.0984	0.1494	0.1078	0.1072
Portugal	0.1946	0.1568	0.0872	0.1003	0.0976	0.1498	0.1608	0.1629
Slovenia	0.1435	0.1211	0.0462	0.0678	0.0965	0.1122	0.1217	

The right-hand panel of Table 4 shows dimension-specific fuzzy supplementary deprivation rates (FS1-FS7). As stated, these have been calculated by using the country-specific α_{FS}-Generalized Gini, which is the same for all dimensions in a given country. Hence numerically, these deprivation rates for each dimension are not scaled to automatically equal the overall poverty rate of that country (At-risk-of-poverty rate ARPR). Instead, they take into account inequalities of individuals in the different items the specific dimension is composed of. In this way, the indices are totally fuzzy and relative, in the terminology of Reference [24], introduced in Section 2. Observing Table 4, at least three aspects should be emphasized:

(1) First, most dimension-specific supplementary poverty rates are lower than the multidimensional poverty rate FS. This reflects the tendency for poverty in different dimensions to be concentrated on the same individuals, increasing the intensity of the multidimensional poverty rate FS, compared to the single-dimension poverty rates.

(2). The seven specific-dimensions indices appear to fall into two groups with rather different levels of poverty rates, compared to the overall multidimensional index FS. A first group, composed of four dimensions, where the indices are in general slightly lower than FS; these dimensions are:

FS1—basic lifestyle
FS5—environment
FS6—work and education
FS7—health related.

Another group of three dimensions, for which the poverty rates are sensibly lower than FS, namely:

FS2—consumer durables
FS3—housing amenities
FS4—financial situation.

(3). We may observe the presence of a fairly strong correlation between the ranking of Mediterranean countries according to the overall and dimension-specific indices of deprivation. The differences in correlations among dimensions are much more evident when observing individuals in the micro-data. The correlation with FS is particularly high for dimensions FS1, FS3, and FS6.

Taking into account these three aspects, dimension FS4—financial situation, seems to be quite peculiar, since it is not very highly correlated with FS or other dimensions, in this way describing a particular aspect or side of multidimensional poverty in the Mediterranean Area. Moreover, poverty rates in this dimension were lower than those in other dimensions and showed a lower level of inequalities or concentration of poverty in some groups in the countries.

Focusing on the financial dimension (FS4), it is possible to observe how Greece, Cyprus, and Italy have the highest financial deprivation for all the three considered years. Spain had a lower value in the first two years, but it increased to the value of Italy in 2015. The country with the best situation regarding financial deprivation is France. Generally, the level of financial deprivation increased along the years, apart for France and Portugal, for which we found a decreased level in 2015 from 2011. These figures, based on fuzzy set theory, can help us to understand how the financial situation changed from 2007 to 2011 and 2015, because of the crisis and because of the changed attitude to answer somewhat subjective questions. Recalling the background information reported in Table 1, a greater percentage of citizens in Cyprus and Greece, from 2008 to 2013, answered that they were in "Arrears on key commitments" and faced "Inability to face unexpected expenses". This has been greatly mitigated by the adoption of fuzzy sets, as shown in Table 4. In fact, Cyprus even experienced a small decrease in the FS4 index from 2007 to 2011 and a slight increase in 2015. It seems that here the effect of the economic and financial crisis influenced citizens later than in Greece. In this latter country, the small increase in FS4 occurred in 2011, while it was practically stable in the following four years. In conclusion, the economic and financial crisis clearly had a great effect on this financial dimension. Only France, Italy and Portugal were able to recover in 2015.

6. Concluding Remarks

In this paper, we have presented and adopted a fuzzy sets approach for the measurement of multidimensional poverty in the Mediterranean area, over a period of eight years, from 2007 to 2015, which takes into account the effect of the 2008 economic and financial crisis.

This was not the first paper to implement fuzzy sets. However, its novelty consisted in proving how the use of fuzzy sets theory has been very useful in some treating sociological aspects of answering questionnaires in a period of crisis. In particular, we have highlighted how, for some countries, in particular, Cyprus and Greece, answers to somewhat subjective questions were clearly affected by emotional aspects. In this regard, the proposed approach has been very useful in mitigating such problems.

The empirical analysis highlighted at least three main results that were not previously reported in the literature: (1) The tendency for poverty in different dimensions to be concentrated on the same individuals, increasing the intensity of the multidimensional poverty rate FS, compared to the single-dimension poverty rates. (2) The seven specific-dimensions indices appeared to fall into two groups with rather different levels of poverty rates, compared to the overall multidimensional index FS. A first group where the indices are, in general, slightly lower than FS; and another well-distinguished group, for which the poverty rates are sensibly lower than FS. (3) the presence of a strong correlation between the ranking of Mediterranean countries according to the overall and dimension-specific indices of deprivation. The differences in correlations among dimensions are much more evident when observing individuals in the micro-data.

Another novelty of this paper was that observing these three aspects, dimension FS4—financial situation, seems to be quite peculiar since it is not very highly correlated with FS or other dimensions. In this way, it described a particular aspect or side of multidimensional poverty in the Mediterranean Area. Moreover, poverty rates in this dimension were lower than those in other dimensions, showing a lower level of inequalities or concentration of poverty in some groups in the countries.

The main limitation of the paper surely consists in a lack of adequate analysis of the implications of the results, especially from a policy-oriented perspective. We are willing to undertake this further research once the new 2019 EU-SILC data is available, and the results could be useful for the new European Commission.

Author Contributions: Conceptualization, G.B. and F.G.; investigation, M.C.; software, F.G.; validation, S.R., writing—original draft preparation, M.C. and F.G.; writing—review and editing, G.B.; supervision, S.R.

Funding: This research received no external funding.

Acknowledgments: We thank three anonymous Referees for their contribution, and an English Lecturer for her language proofreading.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kolm, S.C. Multidimensional egalitarism. *Q. J. Econ.* **1977**, *91*, 1–13. [CrossRef]
2. Atkinson, A.B.; Bourguignon, F. The comparison of multidimensional distributions of economic status. *Rev. Econ. Stud.* **1982**, *49*, 183–201. [CrossRef]
3. Tsui, K. Multidimensional generalisation of the relative and absolute inequality indices: The Atkinson-Kolm-Sen approach. *J. Econ. Theory* **1985**, *67*, 251–265. [CrossRef]
4. Maasoumi, E. The measurement and decomposition of multidimensional inequality. *Econometrica* **1986**, *54*, 771–779. [CrossRef]
5. Sen, A.K. *Development as Freedom*; Oxford University Press: Oxford, UK, 1999.
6. Anand, S.; Sen, A.K. *Concepts of Human Development and Poverty: A Multidimensional Perspective*; Human Development Papers; United Nations Development Programme (UNDP): New York, NY, USA, 1997.
7. Townsend, P. *Poverty in the United Kingdom*; Allen Lane: Harmondsworth, UK, 1979.

8. Smeeding, T.; Saunders, P.; Coder, J.; Jenkins, S.P.; Fritzell, J.; Hagenaars, A.; Hauser, R.; Wolfson, M. Poverty inequality and family living standards impacts across seven nations: The effect of non-cash subsidies for health, education and housing. *Rev. Income Wealth* **1993**, *39*, 229–256. [CrossRef]
9. Klasen, S. Measuring poverty and deprivation in South Africa. *Rev. Income Wealth* **2000**, *46*, 33–58. [CrossRef]
10. Chakravarty, S.R.; Mukherjee, D.; Ranade, R. On the family of subgroup and factor decomposable measures of multidimensional poverty. *Res. Econ. Inequal.* **1998**, *8*, 175–194.
11. Miceli, D. *Measuring poverty using fuzzy sets. Canberra: National Centre for Social and Economic Modelling*; University of Canberra: Canberra, Australia, 1998.
12. Bourguignon, F.; Chakravarty, S.R. The measurement of multidimensional poverty. *J. Econ. Inequal.* **2003**, *1*, 25–49. [CrossRef]
13. Caramuta, D.M.; Contiggiani, F. A Fuzzy Set Approach to Poverty Measurement. *Fuzzy Econ. Rev.* **2006**, *11*, 37–55. [CrossRef]
14. Atkinson, A.B.; Cantillon, B.; Marlier, E.; Nolan, B. *Social Indicators: The EU and Social Inclusion*; Oxford University Press: Oxford, UK, 2002.
15. Duclos, J.-Y.; Sahn, D.; Younger, S.D. *Robust Multidimensional Poverty Comparisons*; Université Laval: Quebec, QC, Canada, 2001.
16. Atkinson, A.B. Multidimensional deprivation: Contrasting social welfare and counting approaches. *J. Econ. Inequal.* **2003**, *1*, 51–65. [CrossRef]
17. Pipino, I.L.; van Gigch, J.P. Potential Impact of Fuzzy Sets on the Social Sciences. *Cybern. Syst.* **1981**, *12*, 21–35. [CrossRef]
18. Cerioli, A.; Zani, S. A fuzzy approach to the measurement of poverty. In *Income and Wealth Distribution, Inequality and Poverty*; Dagum, C., Zenga, M., Eds.; Springer: Berlin, Germany, 1990; pp. 272–284.
19. Chiappero Martinetti, E. A New Approach to Evaluation of Well-Being and Poverty by Fuzzy Set Theory. *G. Econ. Ann. Econ.* **1994**, *53*, 367–387.
20. Cheli, B. Totally Fuzzy and Relative Measures of Poverty in Dynamics Context. *Metron* **1995**, *53*, 183–205.
21. Vero, J.; Werquin, P. Re-examining the Measurement of Poverty: How Do Young People in the Stage of Being Integrated in the Labour Force Manage. *Econ. Stat.* **1997**, *8–10*, 143–156. (In French)
22. Cheli, B.; Betti, G. Fuzzy analysis of poverty dynamics on an Italian pseudo panel, 1985–1994. *Metron* **1999**, *57*, 83–104.
23. Betti, G.; Cheli, B.; Cambini, R. A statistical model for the dynamics between two fuzzy states: Theory and application to poverty analysis. *Metron* **2004**, *62*, 391–411.
24. Cheli, B.; Lemmi, A. A Totally Fuzzy and Relative Approach to the Multidimensional Analysis of Poverty. *Econ. Notes* **1995**, *24*, 115–134.
25. Betti, G.; Verma, V. Measuring the degree of poverty in a dynamic and comparative context: A multi-dimensional approach using fuzzy set theory. In Proceedings of the ICCS-VI, Lahore, Pakistan, 27–31 August 1999; Volume 11, pp. 289–301.
26. Betti, G.; Verma, V. Fuzzy measures of the incidence of relative poverty and deprivation: A multi-dimensional perspective. *Stat. Methods Appl.* **2008**, *17*, 225–250. [CrossRef]
27. Baràn, B.; Rojas, A.; Britez, D.; Baràn, L. Measurement and Analysis of Poverty and Welfare Using Fuzzy Sets. In Proceedings of the International Conference on Systemics, Cybernetics and Informatics SCI'99, Orlando, FL, USA, 31 July–4 August 1999.
28. Garcia, P.; Lazzari, L.; Machado, E. Una Propuesta Fuzzy para Definir Indicadores de Pobreza. In Proceedings of the V SIGEF Congress, Lausanne, Switzerland, 24–27 November 1998.
29. Chiappero Martinetti, E. A Multidimensional Assessment of Well-Being Based on Sen's Functioning Approach. *Riv. Int. Sci. Soc.* **2000**, *2*, 207–239.
30. Lelli, S. Factor Analysis vs. Fuzzy Set Theory: Assessing the Influence of Different Techniques on Sen's Functioning Approach. 2001. Available online: http://www.econ.kuleuven.ac.be/ew/academic/econover/Papers/DPS0121.pdf (accessed on 5 October 2018).
31. Sen, A.K. Capability and Well-Being. In *The Quality of Life*; Nussbaum, M., Sen, A., Eds.; Clarendon Press: Oxford, UK, 1993.
32. Deutsch, J.; Silber, J. Measuring Multidimensional Poverty: An Empirical Comparison of Various Approaches. *Rev. Income Wealth* **2005**, *51*, 145–174. [CrossRef]

33. Lemmi, A.; Betti, G. *Fuzzy Set Approach to Multidimensional Poverty Measurement*; Series: Economic Studies in Inequality, Social Exclusion and Well-Being; Springer: New York, NY, USA, 2006.
34. Qizilbash, M. Philosophical Accounts of Vagueness, Fuzzy Poverty Measures and Multidimensionality. In *Fuzzy Set Approach to Multidimensional Poverty Measurement*; Lemmi, A., Betti, G., Eds.; Series: Economic Studies in Inequality, Social Exclusion and Well-Being; Springer: New York, NY, USA, 2006.
35. Vero, J. A Comparison of Poverty According to Primary Goods, Capabilities, and Outcomes. Evidence from French School Leaver's Surveys. In *Fuzzy Set Approach to Multidimensional Poverty Measurement*; Lemmi, A., Betti, G., Eds.; Series: Economic Studies in Inequality, Social Exclusion and Well-Being; Springer: New York, NY, USA, 2006.
36. Betti, G.; Cheli, B.; Lemmi, A.; Verma, V. The fuzzy set approach to multidimensional poverty: The case of Italy in the 1990s. In *Quantitative Approaches to Multidimensional Poverty Measurement*; Kakwani, N., Silber, J., Eds.; Palgrave Macmillan: Basingstoke, UK; New York, NY, USA, 2008; pp. 30–48.
37. Costa, M.; De Angelis, L. The multidimensional measurement of poverty: A fuzzy set approach. *Statistica* **2008**, *68*, 303–319.
38. Giordani, P.; Giorgi, G.M. A fuzzy logic approach to poverty analysis based on the Gini and Bonferroni inequality indices. *Stat. Methods Appl.* **2010**, *19*, 587–607. [CrossRef]
39. Belhadj, B. A new fuzzy unidimensional poverty index from an information theory perspective. *Empir. Econ.* **2011**, *40*, 687–704. [CrossRef]
40. Belhadj, B.; Limam, M. Unidimensional and Multidimensional Fuzzy Poverty Measures: New Approach. *Econ. Model.* **2012**, *29*, 995–1002. [CrossRef]
41. Betti, G.; Gagliardi, F.; Lemmi, A.; Verma, V. Sub-national indicators of poverty and deprivation in Europe: Methodology and applications. *Camb. J. Reg. Econ. Soc.* **2012**, *5*, 149–162. [CrossRef]
42. Belhadj, B. New fuzzy indices for multidimensional poverty. *J. Intell. Fuzzy Syst.* **2013**, *24*, 587–591.
43. Neff, D. Fuzzy set theoretic applications in poverty research. *Policy Soc.* **2013**, *32*, 319–331. [CrossRef]
44. Betti, G.; Gagliardi, F.; Lemmi, A.; Verma, V. Comparative measures of multidimensional deprivation in the European Union. *Empir. Econ.* **2015**, *49*, 1071–1100. [CrossRef]
45. Leu, C.H.; Chen, K.M.; Chen, H.H. A Multidimensional Approach to Child Poverty in Taiwan. *Child. Youth Serv. Rev.* **2016**, *66*, 35–44. [CrossRef]
46. Aristondo, O.; Ciommi, M.T. The Orness Value for Rank-Dependent Welfare Functions and Rank-Dependent Poverty Measures. *Fuzzy Sets Syst.* **2017**, *325*, 114–136. [CrossRef]
47. Aassve, A.; Betti, G.; Mazzuco, S.; Mencarini, L. Marital Disruption and Economic Well-being; A Comparative Analysis. *J. R. Stat. Soc. Ser. A* **2007**, *170*, 781–799. [CrossRef]
48. Imanov, G. The Fuzzy Approach to Estimation of the Index of Population Life Quality. *Fuzzy Econ. Rev.* **2007**, *12*, 85–93. [CrossRef]
49. Betti, G. Fuzzy measures of quality of life: A multidimensional and comparative approach. *Int. J. Uncertain. Fuzz. Knowl.-Based Syst.* **2016**, *24* (Suppl. 1), 25–37. [CrossRef]
50. Betti, G.; Soldi, R.; Talev, I. Fuzzy Multidimensional Indicators of Quality of Life: The Empirical Case of Macedonia. *Soc. Indic. Res.* **2016**, *127*, 39–53. [CrossRef]
51. Betti, G. What impact has the economic crisis had on Quality of Life in Europe? A multidimensional and fuzzy approach. *Q. Q.* **2017**, *51*, 351–364. [CrossRef]
52. Betti, G.; D'Agostino, A.; Neri, L. Educational mismatch of graduates: A multidimensional and fuzzy indicator. *Soc. Indic. Res.* **2011**, *103*, 465–480. [CrossRef]
53. Belhadj, B. Employment measure in development countries via minimum wage and poverty: New Fuzzy approach. *Opsearch* **2015**, *52*, 329–339.
54. Casini, M.; Bastianoni, S.; Gagliardi, F.; Gigliotti, M.; Riccaboni, A.; Betti, G. Sustainable Development Goals indicators: A methodological proposal for a Multidimensional Fuzzy Index in the Mediterranean area. *Sustainability* **2018**, *11*, 1–20.
55. European Commission. *Towards a Common Operational European Definition of Overindebtedness*; European Commission: Directorate-General for Employment, Social Affairs and Equal Opportunities: Brussels, Belgium, 2008.
56. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
57. Verma, V.; Betti, G.; Gagliardi, F. Fuzzy Measures of Longitudinal Poverty in a Comparative Perspective. *Soc. Indic. Res.* **2017**, *130*, 435–454. [CrossRef]

58. Betti, G.; Gagliardi, F.; Verma, V. Simplified Jackknife variance estimates for fuzzy measures of multidimensional poverty. *Int. Stat. Rev.* **2018**, *86*, 68–86. [CrossRef]
59. Eurostat. *Income, Poverty and Social Exclusion: 2nd Report*; Office for Official Publications of the European Communities: Luxembourg, Luxembourg, 2002.
60. Verma, V.; Betti, G. EU Statistics on Income and Living Conditions (EU-SILC): Choosing the survey structure and sample design. *Stat. Trans.* **2006**, *7*, 935–970.