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Joint Advertisement and Trade-In Marketing Strategy in Closed-Loop Supply Chain

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Abstract: With environmental problems becoming severe, many firms, including HP, Huawei, and Apple are simultaneously implementing trade-in programs and advertising to stimulate market demand. Offering trade-in service by the manufacturer is a method of price discrimination by providing replacement consumers with a rebate when they purchase new products. With the recycled, used products, the manufacturer can benefit through a strict series—via a remanufacturing process. Although numerous literatures have investigated the pricing strategy and advertising decisions in the closed-loop supply chain (CLSC), to the best of our knowledge, there is little research that analyzes and compares the economic and social performance of these two marketing strategies. To fill this gap, we establish two supply chain models with two periods, namely, an advertisement model and a joint model, while the equilibrium purchasing behavior of the replacement consumers can be characterized under three conditions: (1) all of the replacement consumers purchasing new products (ATA); (2) partial replacement consumers purchasing (PTA); (3) no replacement consumers purchasing (NTA). These three conditions are decided by the numerous relationships of the parameters. Solving the optimal decisions of the manufacturer in both models, the critical value in the joint model is higher in the advertisement model, which indicates that developing the trade-in program can enhance the robustness of the business model. Furthermore, through numerical example, we find that the market demand in the joint model is higher than in the advertisement model, and the cost of marketing strategy in the joint model is lower in the advertisement model, which means that the efficiency of the marketing strategy is higher than the single marketing strategy. As a result, comparing the economic and social performance between the two models, we conclude that the advertisement elasticity of the market demand is the key factor of the manufacturer's profits and total social welfare.

Keywords: advertisement; trade-in; closed-loop supply chain; social welfare; economic performance; marketing strategy

1. Introduction

With the depletion of resources and environmental degradation worldwide, the circular economy movement has been gaining significant momentum in practice. In the circular economy, the products are designed to enable cycles of disassembly and reuse; thus, reducing or eliminating waste. Closing the loop is an important block of the circular economy building [1,2]. Many companies, such as HP, Apple, Huawei, and so on, have been successfully realized the closed-loop business model. They developed the trade-in program through the company platform, which contains returning used products, repairing, recycling, and remanufacturing activities. This business model can simultaneously

provide economic and social benefits. McKinsey suggests that the economic value with the circular ideas can be as high as trillions of dollars [3].

Apple provided the trade-in program for its consumers, where they can purchase new products with the trade-in rebate after detection, reverse logistic, and process. This marketing strategy can simulate the market demand, which can be seen as a means of achieving price discrimination for holders. For example, on Apple's official website in China, consumers with used products can purchase an iPhone 11 with at least 3999 RMB through a trade-in program. Consumers without used products can purchase an iPhone 11 with at least 5499 RMB. Many companies, such as Huawei, Xerox, and so on, also adopt this pricing and recycling approach on their trade-in platforms. These companies face fierce market competition and they have to utilize advertisements and pricing marketing strategies to improve market demand. Through advertising, manufacturers can improve the consumer's purchasing potential willingness and brand awareness. The trade-in program set by manufacturers can increase the likelihood that consumers who hold the last generation of products will buy the next generation of products. The business model of trade-in is a sales strategy to improve market demand, though it can reduce the cost of production and save resources and raw materials for the manufacturers.

Literature closely related to our work can be classified into three broad categories: closed loop supply chain (CLSC) decision-making problem, trade-in, and advertising decision. First, the literature related to CLSC investigates the reverse channel choice and coordinating mechanism. Savaskan et al. [4] consider a Stackelberg game between the manufacturer and the recycler; the manufacturer has three methods for collecting used products that corresponds to three collecting models: the retailer collecting model, the third-party collecting model, and the direct collecting model, which is consistent with Ferrer and Swaminathan [5]. They found that collection efficiency is highest in the direct collecting model. Considering the potential market share cannibalization between the remanufactured products and new products, Liu et al. [6] investigate the optimal production and pricing strategies of the manufacturer, and identify how the manufacturer executes remanufacturing and manufacturing strategies. He et al. [7] developed the dual-channel CLSC model, considering three possible sales channel structures, no direct channel structures, selling new products through direct channels while selling remanufactured products through the third-party firm sales channel structure, and selling remanufactured products through the direct channel while selling new products through the retailer sales channel structure. They derived the government's optimal subsidy level under three possible sales channels. Dan et al. [8] investigate how retail services and consumer loyalty affects the pricing strategies of the manufacturer and retailer in a centralized and decentralized dual-channel supply chain based on the Stackelberg game.

The second category of research refers to the trade-in decision, which is a type of closed-loop business model. For example, a manufacturer, such as Apple, provides the consumers with a trade-in rebate only when they purchase new products. Agrawal et al. [9] investigate that an original equipment manufacturer (OEM) can achieve price discrimination by providing the trade-in program under a generalized Nash bargaining framework. Moreover, they evaluate the effect of providing the trade-in service on the performance of the supply chain system. Huang [10] took the retailer competition into consideration and studied the effect of the trade-in strategy on the optimal pricing decisions, trade-in rebate, and profits of the supply chain members. Rao et al. [11] developed a game model, concluding the cannibalization problem between the new and used products, and the problem of asymmetric information between consumers and the manufacturer about product quality. Cao et al. [12] investigated the optimal trade-in rebate and strategies, and they considered two forms of trade-in rebate, providing consumers with a gift card and cash coupon, which correspond to different game models, respectively. Desai et al. [13] studied the influence of waste aversion of consumers and dual types of trade-in promotions on the equilibrium results between the two competitive manufacturers, considering the trade-in program as a marketing promotion strategy. They conclude that the strategy consideration can lead to a dilemma where neither manufacturer will provide the trade-in service. Miao et al. [14] argued three collection strategies, namely, no consumers, partial consumers, and all

consumers participating in the trade-in program. Moreover, they found that the residual value of used products is one of the key factors to the trade-in strategy pricing decisions, and profits of the whole supply chain system. Based on their research, Li et al. [15] developed a two-period closed-loop supply chain model, considering the influence of a secondary market, where the OEM can sell its remanufactured products. The literature about the trade-in program mainly discuss the role of a trade-in strategy stimulating market demand, reducing resource waste, and improving the total social welfare [16–20].

Our studies also relate to research on advertising decisions and closed-loop supply chain. Yue et al. [21] found that the supply chain system offers a price discount and makes advertisements stimulate market demand and profits in the Nash and Stackelberg game equilibrium. Hong et al. [22] constructed an CLSC model based on the Stackelberg game to investigate the optimal advertisement investment and pricing decisions and they analyzed the influence of advertising on the whole supply chain system. Xie et al. [23] developed the revenue-sharing mechanism of the collection activity and achieved optimal prices and advertisement investment in the centralized and decentralized decisions models, respectively. Li et al. [24] studied the optimal cooperative advertisement strategies in the O2O supply chain, concluding a manufacturer and an online platform agent. Zhang et al. [25] proposed an advertisement cost-sharing mechanism between the manufacturer and retailer, considering the effect of advertising to the reference price of the consumers. Jena et al. [26] considered that the advertisement can stimulate the market demand in the CLSC system and analyzed the effect of the coordinate mechanism in cost sharing on the decisions of the supply chain members.

The above literature studied a lot about the role of advertising and trade-in in stimulating market demand. However, they ignore three aspects, because they explore the trade-in decision problems from a CLSC perspective, and the advertising problem from a marketing science perspective. First, they do not aim the trade-in program as a marketing strategy to achieve price discrimination, similar with advertisements to increase demand, though it can reduce the waste of resources and energy. Furthermore, previous literature does not analyze the impact of the two marketing strategies to the supply chain system, and their difference in efficiency and cost. Lastly, they do not consider application conditions and limitations of two marketing strategies. In fact, there exists the trade-in and advertising in many companies, and the previous models do not capture two strategies simultaneously. To fill this gap, we develop a two-period closed-loop supply chain and provide the industry with practical advice and guidelines.

Therefore, our work aims to address the following problems:

(1) What is the optimal trade-in rebate and advertising strategy of the manufacturer? Moreover, how can developing a trade-in program influence the business model, advertisement investment, and pricing decisions of the supply chain members?

(2) If the manufacturer chooses to develop the trade-in program, what are the key factors that are significant to the optimal decisions of the manufacturer, and how to distinguish the different applications of the marketing strategy?

(3) If the manufacturer chooses to adopt the joint trade-in program and advertising marketing strategy to increase the market demand, how to evaluate the economic and social benefits of the whole supply chain system, which contains profits of supply chain, consumer surplus, and social welfare?

In our research, we develop a two-period supply chain model where the manufacturer sells new products in the first period and decides whether to introduce a trade-in program in the second period. The manufacturer can attract the primary consumers, who do not have any products of the firm, to purchase new products through advertising and attract the replacement consumers, who have a product of the firm to purchase a new next generation product through a trade-in program. Under our proposed framework, we explore the manufacturer's profit-maximizing problem with respect to advertisement efforts and sales price.

Our main contributions are as follows. To the best of our knowledge, we first develop the joint trade-in and advertisement model to examine the optimal trade-in rebate, advertising effort, and pricing

strategy. Although some studies consider pricing discount and advertisement to stimulate market demand, they do not consider that offering a trade-in program can implement price discrimination for primary consumers, and replacement consumers, based on market segmentation, and analyze the difference in cost and efficiency between the two marketing strategies.

2. Model Description

In our investigation, we develop two two-period supply chain models, an advertisement model, and joint trade-in program and advertisement model. In the advertisement model, the manufacturer implements advertisements to stimulate primary consumers to purchase new products. In the joint model, the sales channel implements a trade-in program and advertisement marketing strategy to increase market demand by the manufacturer. The manufacturer produces new products at unit cost of C_n , and remanufactures products with the collected used products at unit cost of C_r . We assume that there is no difference in the sales channel between the new products and remanufactured products. We summarize the related symbols in Table 1.

Table 1. Summary of the related notations and variables.

Symbol	Definition
θ	Consumer's willing to pay in the market
l_1	The advertisement elasticity of the market demand
ρ	The discount factor of consumers' evaluation of old products
C_n	The unit cost of manufacturing with new material
C_r	The unit cost of remanufacturing with used products
p_1	Price set by the manufacturer in first period
p_2	Price set by the manufacturer in second period
q_1	The market demand in first period
q_{2n}	The market demand of new products in second period
q_{2r}	The market demand of remanufactured product in second period
a_1	The advertising effort supported by the manufacturer in first period
a_2	The advertising effort supported by the manufacturer in second period
π^{MT}	The profit of manufacturer in the joint model
π^{MA}	The profit of manufacturer in the benchmark model

2.1. Consumer Choice in the Advertisement Model

We consider a two-period advertisement supply chain model, based on θ , consumer's willingness to pay. Furthermore, θ is assumed to be uniform distributed between 0 and 1. In period 1, consumers can decide whether to purchase the new products. In period 2, consumers are divided into two types, replacement consumers and primary consumers. Both of them decide whether to buy the new products in period 2. We assume that the consumers are heterogeneous in θ , the willingness to pay for new products. The advertisement can increase the consumer's willingness to pay. The manufacturer's advertisement activities can effectively increase the consumer's willingness to purchase the new products [27,28].

The manufacturer set the price p_1 and the advertisement effort a_1 during period 1. The advertisement can improve the consumer's willingness to purchase, and the utility surplus of primary consumers is $\theta - p_1 + l_1 a_1$. The cost of implementing the advertisement is $\frac{1}{2} a_1^2$, which is widely used in the literature. Therefore, the purchasing behaviors of consumers occur only if $\theta - p_1 + l_1 a_1 > 0$. Moreover, the total demand for new products in period 1 is expressed as: $q_1 = 1 - p_1 + l_1 a_1$. We consider that the used products exist remaining utility of the replacement consumers. Based on consumer value perception, whether the replacement consumers purchase the new products is decided by the remaining utility of the used products. We assume that the discount factor of an old product is ρ , and ρ satisfies $0 \leq \rho \leq 1$. To some extent, the discount factor is due to the technological advancement of the new generation of products and richer functional experience.

If the replacement consumers choose to buy new products, the net utility they obtain is $\theta - p_2$; and if they choose to continue to hold the old product, the net utility is $\theta\rho$. In period 2, the replacement consumers purchase new products only if $\theta - p_2 > \theta\rho$. Therefore, the market demand of replacement consumers is expressed as: $q_{2r} = 1 - \frac{p_2}{1-\rho}$. Similarly, if the primary consumer chooses to purchase a new product, his net utility is $\theta - p_2 + l_2a_1 + l_1a_2$ and zero, otherwise. Moreover, the primary consumer will purchase a new product only if $\theta - p_2 + l_2a_1 + l_1a_2 \geq 0$. Therefore, the market demand of primary consumers is expressed as: $p_1 - p_2 + l_2a_1 + l_1a_2$.

In order to simplify the models and model solutions, we set $a_1 = 0$. Because our focus is on the joint advertising and trade-in marketing strategy, we omit the manufacturer’s advertising decisions in period 1 and its aggregation effect on the market demand in period 2. To check the robustness of this assumption, we relaxed the assumption in Appendix A Figure A1, but it provided similar results with the condition of $a_1 = 0$. For more details, please see Appendix A.

2.2. Consumer Choice in the Joint Model

We also consider a two-period advertisement supply chain model, based on θ , consumer’s willingness to pay. As shown in Figure 1, when the manufacturer has set the price p_1 and the advertisement effort during period 1, the utility surplus of consumers is $\theta - p_1$. Therefore, the purchasing behaviors of consumers occur only if $\theta - p_1 > 0$. Moreover, the total demand for new products in period 1 is expressed as: $q_1 = 1 - p_1$. If the replacement consumers choose to buy the new products and get the trade-in rebate through the trade-in program in the second period, the net utility they obtain is $\theta - p_2 + p_0$; and if they choose to continue to hold the old product, their net utility is $\theta\rho$. In period 2, the replacement consumers purchase new products only if $\theta - p_2 + p_0 > \theta\rho$. Therefore, the market demand of replacement consumers is expressed as: $q_{2r} = 1 - \frac{p_2 - p_0}{1-\rho}$. Similarly, if the primary consumer chooses to purchase a new product, the net utility is $\theta - p_2 + l_1a_2$ and zero otherwise. Moreover, the primary consumer will purchase a new product only if $\theta - p_2 + l_1a_2 \geq 0$. Therefore, the market demand of primary consumers is expressed as: $p_1 - p_2 + l_1a_2$.

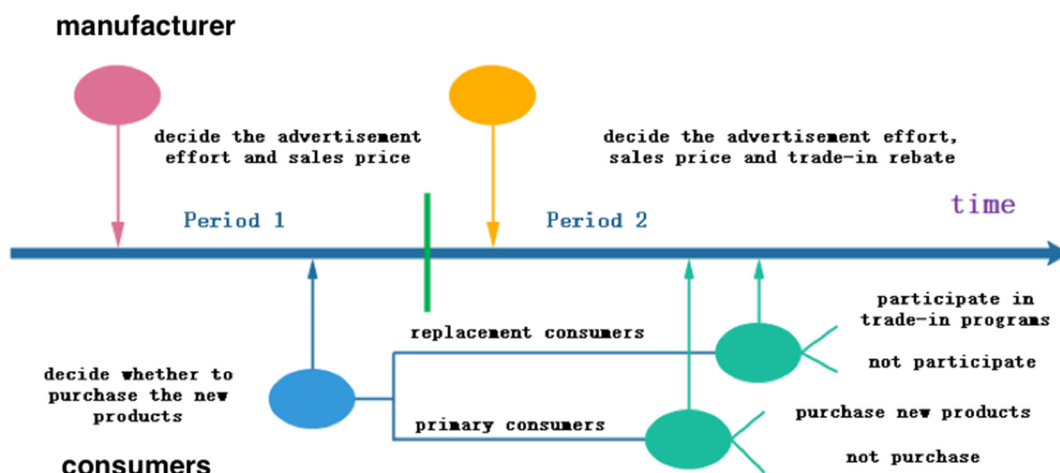


Figure 1. The joint trade-in and advertising model.

3. Model

3.1. Advertisement Supply Chain Model Without Trade-In Strategy

Under the single-marketing strategy, the manufacturer implements the advertisement marketing strategy to incentive market demand, and sells its products to the primary consumers. In the second period, the manufacturer also implements the advertisement marketing strategy and sells to the

primary consumers and the replacement consumers who have purchased the products in period 1. The profit-maximizing problem of the manufacturer can be expressed by:

$$\begin{aligned} \max_{(p_1, p_2, a_1, a_2)} \pi_m &= (p_1 - C_n)q_1 + (p_2 - C_n)q_2 - \frac{1}{2}(a_1^2 + a_2^2) \\ \text{s.t. } q_1 &\geq 1 - mp_2 \geq 0 \end{aligned} \tag{1}$$

The first term of the function indicates the profits of selling new products in period 1 and the second term of the function represents the profits of selling new products in period 2. The third term corresponds to the advertisement cost. In order to simplify the model and computing complexity, we let $a_1 = 0$ and substitute m to the expression $\frac{1}{1-p}$.

Proposition 1. *In the single-marketing strategy supply chain model, the model's conditions vary with the range of m as follows:*

- (1) If $m < \frac{3-l_1^2-m}{C_n(1+2m)} + l_1^2 - 1$, $q_{2r} = q_1$. All the replacement consumers purchase the new products, which we denoted as ATA.
- (2) If $\frac{3-2l_1^2+4m}{m+C_n(2-l_1^2+2m)} < m \leq \frac{3-l_1^2-m}{C_n(1+2m)} + l_1^2 - 1$, $0 < q_{2r} < q_1$. The partial replacement consumers purchase the new products, which we denoted as PTA.
- (3) If $m < \frac{3-2l_1^2+4m}{m+C_n(2-l_1^2+2m)}$, $q_{2r} = 0$. No replacement consumers purchase the new products, which we denoted as NTA.

The optimal price, trade-in rebate, and advertising investment effort are as shown in Table 2. More details are provided in Appendix B.

Table 2. The optimal decisions of manufacturer in each condition.

	p_1	p_2	a_2
NTA	$\frac{1+m}{2m}$	$\frac{1}{m}$	$l_1(-C_n + \frac{1}{m})$
PTA	$\frac{3+C_n-l_1^2-C_n l_1^2+2m+C_n m}{3-2l_1^2+4m}$	$\frac{3+C_n(2-2l_1^2+2m)}{3-2l_1^2+4m}$	$\frac{l_1(-3+C_n+2C_n m)}{-3+2l_1^2-4m}$
ATA	$\frac{m(1+m+C_n(1-l_1^2+m))}{2-l_1^2+2m^2}$	$\frac{1+C_n-C_n l_1^2+m+C_n m}{2-l_1^2+2m^2}$	$\frac{l_1(-1+-m-C_n m+2C_n m^2)}{l_1^2-2(1+m^2)}$

Proposition 1 indicates the optimal manufacturer’s decisions in the single-marketing strategy supply chain model. If the manufacturer only chooses the advertisement marketing strategy to incentive market demand, the behaviors of replacement consumers are varied with the value of m , which indicate the residual perception to the used products in period 2 of replacement consumers, there exists three types of models (namely ATA, PTA, and NTA). From Figure 2, we can observe the optimal advertisement effort, price in period 2, and the market demand in period 2 with respect to m . The sales price and advertisement effort decrease in m , because the marketing efficiency of the pricing is prior to the advertising to the replacement consumer’s, as the consumer’s satisfaction with the old products increases, which is decided by the optimal pricing mechanism between the manufacturer and the consumers. Interestingly, we find the market demand in period 2 reaches the peak at the entrance of ATA and PTA, which means it is the boundary of the marketing strategies from replacement consumers to the primary consumers, because the marketing strategy of the manufacturer to stimulate the replacement and primary consumers is simultaneously pricing strategy in period 2. Furthermore, when $m > \frac{1}{C_n}$, the business model of single-marketing strategy is not economic and meaningless, because the residual perception to the used products is too big for the replacement consumers to exchange it for a new one.

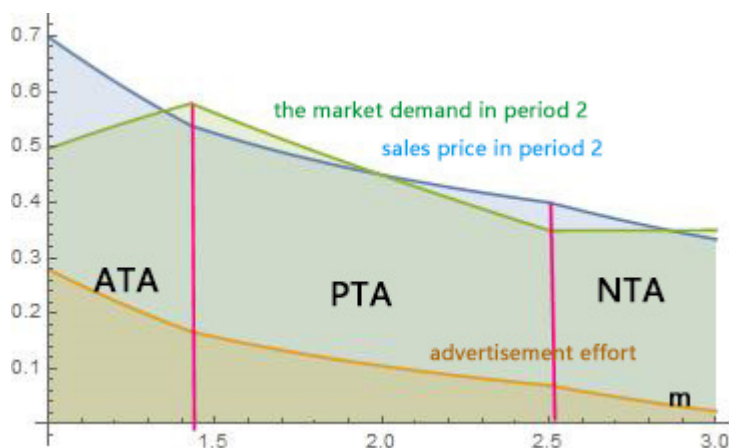


Figure 2. The optimal $p_2, q_2,$ and a_1 with respect to m .

3.2. Joint Advertisement and “Trade-in” Supply Chain Model under the Dynamic Strategy

If the manufacturer implements the “trade-in” and advertisement marketing strategies, the manufacturer sells new products and determines the advertising investment effort in period 1. Moreover, develop a “trade-in” program in period 2 to collect used products, which provides rebates for consumers who have purchased the products in period only when they are willing to replace it with the new generation products. Meanwhile, the manufacturer also determines the advertising investment effort in period 2. Therefore, the profit-maximizing problem of the manufacturer can be expressed as:

$$\begin{aligned} \max_{(p_1, p_2, p_0, a_1, a_2)} \pi_m &= (p_1 - C_n)q_1 + (p_2 - C_r - p_0)q_{2r} + (p_1 - C_n)q_{2n} - \frac{1}{2}(a_1^2 + a_2^2) \\ \text{s.t.} & 0 \leq q_{2r} \leq q_1 \end{aligned} \tag{2}$$

The first term of the function indicates the profit of selling new products in period 1, and the second term represents the profit of selling new products to primary consumers in period 2. The third term corresponds to the profits of selling remanufactured products through a “trade-in” program. The first constraint ensures that the quantity of replacement consumers in period 2 is less than the whole market demand in period 1. The second constraint ensures that the quantity of market demand of primary consumers in period is non-negative.

Proposition 2. Under the dynamic strategy, the manufacturer’s optimal trade-in and advertising investigate effort are follows:

- (1) If $m \geq \frac{1}{C_r}, q_{2r} = 0$. No replacement consumers participate in trade-in program, which we denoted as JNTA.
- (2) If $\frac{1-C_n(-1+\lambda^2)}{C_r(3-2\lambda^2)} < m < \frac{1}{C_r}, 0 \leq q_{2r} \leq q_1$. The partial replacement consumers participate in trade-in program, which we denoted as JPJA.
- (3) If $m < \frac{1-C_n(-1+\lambda^2)}{C_r(3-2\lambda^2)}, q_{2r} = q_1$. All the replacement consumers participate in trade-in program, which we denoted as JATA.

The optimal price, trade-in rebate, and advertising investment effort are as shown in Table 3. More details are provided in Appendix C.

Table 3. The optimal decisions of manufacturer in each condition.

	p_1	p_2	a_2	p_0
JNTA	$\frac{1+C_n-l_1^2-C_n l_1^2+C_r m}{3-2l_1^2}$	$\frac{1+2C_n(-1+l_1^2)-2C_r m}{-3+2l_1^2}$	$\frac{l_1(1+C_n-2C_r m)}{-3+2l_1^2}$	C
JPTA	$\frac{-2+l_1^2+C_n(-1+l_1^2)}{-3+2l_1^2}$	$\frac{1+2C_n-2C_n l_1^2}{3-2l_1^2}$	$\frac{(-1+C_n)l_1}{-3+2l_1^2}$	B
JATA	$\frac{-2-(2+C_n+2C_r)m+l_1^2(1+(1+C_n+C_r)m)}{-4-3m+2l_1^2(1+m)}$	$\frac{-1-(1+C_r)m+2C_n(-1+l_1^2)(1+m)}{-4-3m+2l_1^2(1+m)}$	$\frac{l_1(-1-(1+C_r)m+C_n(2+m))}{-4-3m+2l_1^2(1+m)}$	A

Where:

$$A = \frac{2 + (1 - C_n + 2C_r)m - (1 + 2C_n + C_r)m^2 + l_1^2(-1 + (-1 + C_n - C_r)m + 2C_n m^2)}{m(-4 - 3m + 2l_1^2(1 + m))} \quad (3)$$

$$B = \frac{3 + (-2 - 4C_n + 3C_r)m + l_1^2(-2 + 4C_n m - 2C_r m)}{2(-3 + 2l_1^2)m} \quad (4)$$

$$C = \frac{3 + m - 2C_n m - 2C_r m^2 + 2l_1^2(-1 + C_n m)}{(-3 + 2l_1^2)m} \quad (5)$$

Proposition 2 indicates the manufacturer's optimal pricing strategies, advertising investment effort, and trade-in rebate under different conditions, which corresponds to different models. We consider the trade-in and advertisement as marketing strategies, and the performance of the trade-in marketing strategy. Meanwhile, we also show whether the manufacturer should develop a trade-in program to incentive the market demand. From Proposition 1, relying on the relationship among parameters, the discount coefficient of the consumer's perception of old products, the advertisement effect of the market demand, the residual advertisement effect, the cost of manufacturing with new material, and the cost of remanufacturing with used products, there exists three types of conditions (namely JATA, JPTA and JNTA). The formulation is similar to Li et al. [15] and Miao et al [14].

Furthermore, in the section JATA, $q_{2r} = q_1$, which means all the replacement consumers participate in trade-in program to buy new products, and the market decreases as m increases, which is caused by the decrease of the holders at the beginning of the period 2. In the section JPTA, $0 \leq q_{2r} \leq q_1$, where the partial replacement consumers participate in a trade-in program to buy new products, and the market demand of replacement consumers decreases, as m and m does not affect the market demand of primary consumers, which is caused by the value perception to the used products of replacement consumers. In the section JNTA, $q_{2r} = 0$, where no replacement consumers participate in a trade-in program to buy new products and market demand of replacement.

As shown in Figure 3, the sales price and trade-in rebate increase monotonically in m , while the market demand replacement consumers decrease monotonically in m . We find that the increasing rate of trade-in rebate provided for the consumers, with respect to m in the JATA section, is higher than in the JPTA section. This means the manufacturer can achieve the goals of pricing strategy to the replacement consumers by providing the trade-in program, and the manufacturer can provide more rebates to stimulate them to participate in the trade-in program if their satisfaction with the old products is higher. Meanwhile, the trade-in program substitution effect of the pricing strategy in period 2 will make that m has little effect on the market demand of the primary consumers compared to the single marketing strategy model.

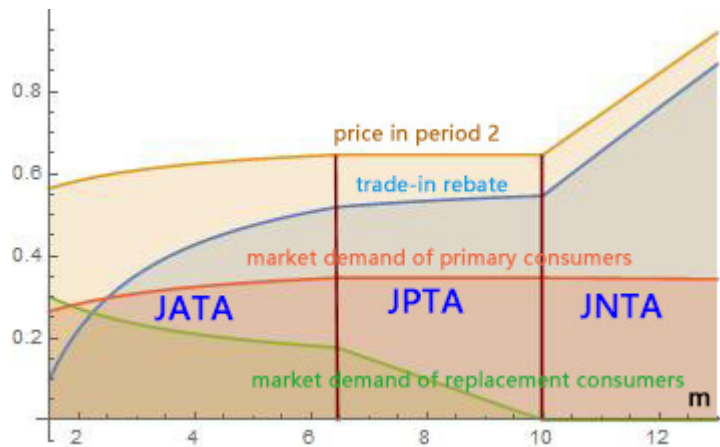


Figure 3. The optimal q_{2n} , q_{2r} , p_2 and p_0 with respect to m ($l_1 = 0.7$, $C_r = 0.1$ and $C_n = 0.3$).

To be specific, we find that the advertisement cost increases with m as shown in Figure 4, because when the residual perception value of the replacement consumers to the used products increases, the market demand incentive is more dependent on advertisement. In addition, trade-in cost increases with m in section *JATA* and decreases in section *JPTA*, which is caused by the decrease of replacement consumers market demand. Interestingly, in the section *JNTA*, the advertisement cost increases quickly with m , because the market demand incentive is completely dependent on advertisement strategy to the primary consumers. Furthermore, we can observe the complementary effects of the trade-in marketing strategy and the advertising strategy on different sections, which provide an important managerial insight for the manufacturer that the manufacturer should arrange the approximate investment between the two marketing strategies, according to the range of m in Figure 4.

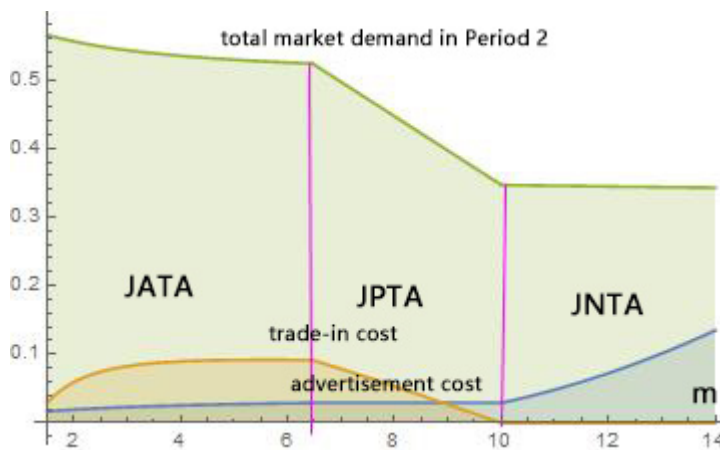


Figure 4. The optimal cost of marketing strategies and market demand in period 2 with respect to m ($l_1 = 0.7$, $C_r = 0.1$ and $C_n = 0.3$).

4. Discussion

4.1. The Boundary of the Different Sections

In this section, we compare and analyze the variations of the boundary of the different sections between the joint marketing strategy and the single marketing strategy. Figure 5a illustrates the boundary between the condition that all the replacement consumers purchase the new products and that the partial consumers purchase the new products in period 2. The top part of the curve represents the condition that the partial consumers purchase the new products in period 2; meanwhile, the bottom part of the curve represents the condition that all the replacement consumers purchase the new products. The curve at the top represents the condition of the joint marketing strategy; and the curve at the bottom represents the condition of the single marketing strategy.

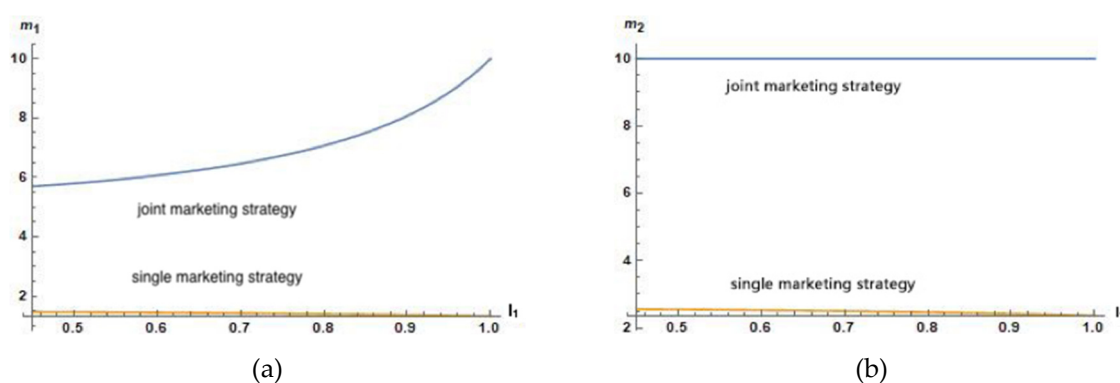


Figure 5. The critical value of m with respect to l_1 . (a) The value of m_1 with respect to l_1 ; (b) the value of m_2 with respect to l_1 .

We develop the curve according to the relationship between the value of the discount factor and the consumer elasticity coefficient to the advertisement. We find that no matter how the value of advertisement coefficient varies, the critical value of m between the ATA and PTA under the joint marketing strategy is higher than under the single marketing strategy, which means that the consumer can still purchase the new products, though the used products are less attractive under the joint marketing strategy. The joint marketing strategy can extremely improve the business model's replacement consumers. Moreover, we also find that the value of critical m increases as l_1 increases under the joint marketing strategy; and the value of critical m decreases as l_1 increases under the single strategy.

Figure 5b illustrates the boundary condition between that of the partial replacement consumers (PTA) who purchase new products and no replacement consumers (NTA) who purchase new products in period 2. The top part of each curve represents the condition of the partial consumers who purchase the new products (NTA) in period 2; meanwhile, the bottom part of each curve represents the condition of the partial replacement consumers who purchase the new products (PTA). The curve at the top represents the condition of the joint marketing strategy; and the curve at the bottom represents the condition of the single marketing strategy. We develop the curve according to the relationship between the value of m_2 , the boundary of the discount factor between PTA and NTA, and the consumer elasticity coefficient to the advertisement. We find that no matter how the value of advertisement coefficient varies, the value of between m_2 under the joint marketing strategy is higher than under the single marketing strategy, which means that the consumer can still purchase the new products, though the used products are less attractive under the joint marketing strategy. The joint marketing strategy can extremely improve the business model's replacement consumers. Moreover, we also find that the value of critical m_2 is static under the joint marketing strategy; and the value of critical m_2 decreases as l_1 increases under the single strategy.

4.2. The Profits of Manufacturer

In order to achieve deeper insight, we investigate the economic performance between the two models with numerical examples in this subsection. We present the relationship between the optimal profits of manufacturer and parameter m and l_1 . From Figure 6, we find that the optimal profit of the manufacturer decreases generally as the value of m increases, which can be thought of the durability of products. The consumers are still satisfied with the product and its function if the value of m is high, and they are less likely to purchase new products in period 2, which decreases the profits of manufacturer. Furthermore, we also find that the profit of the manufacturer in the joint model is higher than in the advertisement model. Simultaneously, we can conclude that the rate at which the manufacturer's profit is failing in the joint model is lower than in the advertisement model, because the trade-in program provided by the manufacturer in the joint model can decrease the effect of m on economic profit performance. Furthermore, we conclude that m is the main factor of the economic performance in the advertisement model, while l_1 , the advertising elasticity coefficient to the primary market demand, is the key factor in the advertisement, which indicates that the manufacturer should pay attention to these values of parameters.

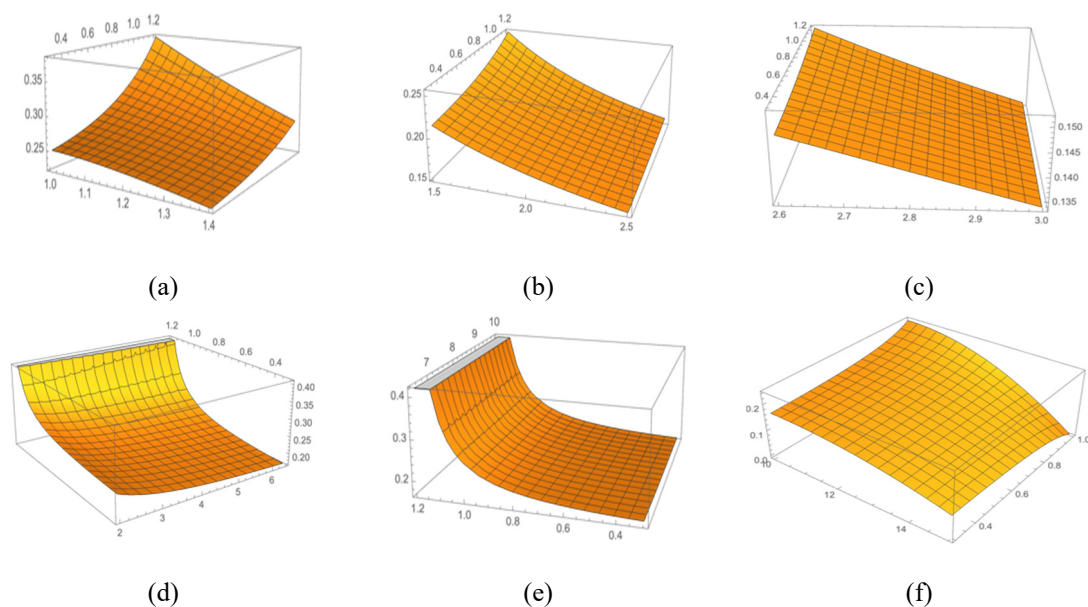


Figure 6. The profits of manufacturer in each condition with respect to l_1 and m . (a) The profits of the manufacturer with respect to l_1 and m of ATA conditions in the advertisement model; (b) the profits of manufacturer with respect to l_1 and m of PTA conditions in the advertisement model; (c) the profits of manufacturer with respect to l_1 and m of NTA conditions in the advertisement model; (d) the profits of manufacturer with respect to l_1 and m of ATA conditions in the joint model; (e) the profits of manufacturer with respect to l_1 and m of PTA conditions in the joint model; (f) the profits of manufacturer with respect to l_1 and m of NTA conditions in the joint model.

4.3. The Social Welfare

In this subsection, we analyze and compare the difference in the social welfare between the advertisement and the joint model with the numerical example. From the social benefits perspective, we investigate the total social welfare, including profits of the manufacturer, consumer surplus, and environmental impact.

In order to evaluate the benefits of consumers, we calculate the optimal values of consumer surplus between the advertisement model and the joint model, including consumer willingness to purchase new products in the first period and the second period. We denote CS^j the values of consumer surplus in j model, where $j \in \{A, J\}$. A represents the advertisement model and J represents the joint model. The calculation processes of optimal consumer surplus are as follows:

$$CS^A = \frac{1}{2}(1-p_1)q_1 + \frac{1}{2}(p_1-p_2)(q_1+q_{2r}) + \frac{1}{2}(1-p_2)q_{2n} \quad (6)$$

$$CS^J = \frac{1}{2}(1-p_1)q_1 + \frac{1}{2}(p_1-p_2+p_0)(q_1+q_{2r}) + \frac{1}{2}(1-p_2)q_{2n} \quad (7)$$

The first term represents the value of consumer surplus in the first period; the second term indicates the value of consumer surplus of the replacement consumers in the second period; and the third term corresponds to the value of consumer surplus of the primary consumers in the second period.

In addition, we investigate the environmental impact of the two models; we denote E^j the values of consumer surplus in j model, where $j \in \{A, J\}$. The calculation process of environmental impact are as follows:

$$E^A = e(q_1 + q_2) \quad (8)$$

$$E^J = e(q_1 + q_{2n}) \quad (9)$$

We denote e the environmental impact coefficient of the products. q_{2r} is derived from q_1 . Therefore, we only should include the q_{2n} in period 2. Then we can get the total social welfare, SW^j , which is decided by the value of profits of the manufacturer, consumer surplus, and environmental impact.

$$SW^j = CS^j + \pi_m^j - E^A \quad (10)$$

Finally, we analyze and compare the total social welfare between the two models through numerical example, as shown in Figure 7. We set parameter: $e = 0.08$. Generally, there exists a lot of difference in variation in social welfare with parameters m and l_1 . In the condition ATA of the advertisement model, we find that the social welfare increases as m increases when the value of l_1 is low, and decreases as m increases when the value of l_1 is high, which is contrary to profits of the manufacturer. This is because the high value of l_1 indicates the larger primary consumers in the second period and higher environmental impact caused by the market demand, and the consumer pays more for the advertisement cost by the manufacturer, which will decrease the consumer surplus; in other words, the reserve surplus will be occupied by the advertisement effect. Interestingly, in the conditions PTA and NTA of the advertisement model, the total social welfare goes down and then goes up as the value of m increases, which is caused by the decrease of replacement consumers and the environmental impact in the second period. The total social welfare decreases as l_1 increases in the condition PTA and increases as l_1 increases in the condition NTA of the advertisement. In the joint model, the total social welfare decreases as the value of m increases and the rate of decline is lower. In each condition of the joint model, we find that the social welfare also decreases as l_1 increases, which is also contrary to the profits of the manufacturer, while the rate of decline is higher. It indicates that the parameter is the key factor of the social welfare and benefits in the joint model.

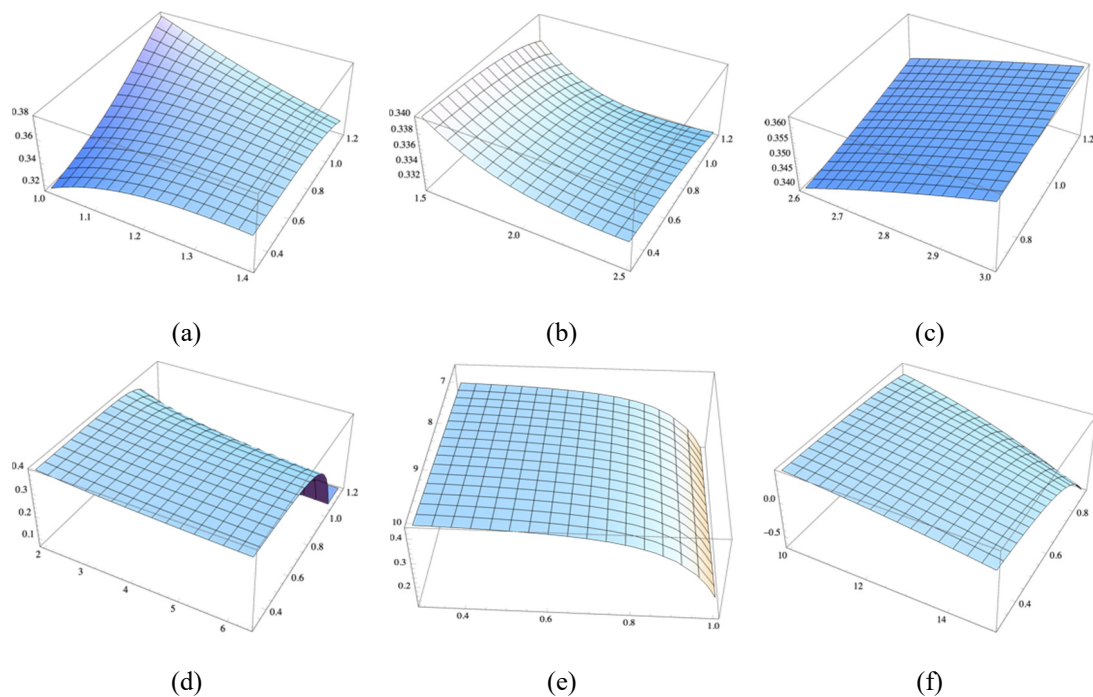


Figure 7. The social welfare with respect to l_1 and m . (a) the social welfare with respect to l_1 and m of ATA conditions in the advertisement model; (b) the social welfare with respect to l_1 and m of PTA conditions in the advertisement model; (c) the social welfare with respect to l_1 and m of NTA conditions in the advertisement model; (d) the social welfare with respect to l_1 and m of ATA conditions in the joint model; (e) the social welfare with respect to l_1 and m of PTA conditions in the joint model; (f) the social welfare with respect to l_1 and m of NTA conditions in the joint model.

5. Conclusions

Many manufacturers, such as HP, Huawei, and so on, implement the advertising and trade-in marketing strategy simultaneously to stimulate the market demand, and they are in a difficult situation to understand the difference and optimal decisions between the marketing strategies. Although numerous researchers have investigated the problems of pricing strategy, advertising, and remanufacturing operations, there is little that analyzes the optimal trade-in rebate and advertising decisions of the manufacturer from the perspective of marketing strategy. To fill this gap, we develop two two-period supply chain model, namely, the advertisement model and joint trade-in and advertisement model, and we achieve and compare the optimal decisions of the two models. Each model can be characterized under three scenarios, with the numerical relationship of the value of parameters C_n , C_r , l_1 and m : (1) all the replacement consumers purchasing the new products in the second period; (2) partial replacement consumers purchasing the new products in the second period; (3) no replacement consumers purchasing the new products in the second period. We achieve the optimal advertisement effort and pricing strategy respectively in these three scenarios, where we use the trade-in program as a means of price discrimination to the replacement consumers.

Based on the above comparing and numerical example analysis, we provide four main insights for the sustainability supply chain literatures. First, compared to the single marketing strategy of advertisements, the profit of the manufacturer is higher through developing the trade-in program to stimulate the market demand of replacement consumers. Meanwhile, the parameter m is the key factor of the manufacturer's profits in the advertisement model, and the parameter l_1 is the key factor of the manufacturer's profits in the joint model. Furthermore, we also find that the cost of advertisements in the joint model is lower than in the advertisement model, which indicates that implementing the trade-in program can save marketing costs and bring more benefits to the manufacturer. Furthermore, we conclude that the boundary values of m between the different conditions in the joint model are higher

than in the advertisement model, which means that the trade-in business model has stronger power to accommodate consumer boredom with old products. In other words, the trade-in business model is more robust for the business environment. Lastly, considering environmental impact, consumer surplus, and profit of the manufacturer, we find that the two models have the big difference with respect with to m and l_1 . Moreover, l_1 is also the key factor of social welfare in the joint model.

Our paper can be extended into four directions. First, we only consider the direct sales channel of the manufacturer in both models, and ignore the sales channel of the retailer and the third-party platform, which means that we do not consider the power of the different supply chain members. Therefore, future researchers can analyze the coordinate mechanism and optimal advertisement, trade-in rebate, and pricing strategy in the supply chain structure under the leadership of different supply chain members. Second, future models can consider the heterogeneity of consumer preferences to the new remanufactured products, which can be extended to develop the secondary market for the remanufacturer to sell the remanufactured products. Third, the advertising also can stimulate the replacement consumers to participate the trade-in program to purchase new products. Therefore, future researchers can analyze the effect of advertising to the reverse channel and its equilibrium results. Fourth, in reality, there exists three choices of reverse channels for the remanufacturer to recycle the used products from consumers, namely, outsourcing third-party firms, entrusting retailers, and developing a direct platform. The future models can compare these three models and achieve the optimal selection for the manufacturer in advertising and recycling decisions.

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Appendix A The Analysis of the Model Considering a_1 as a Decision Variable

We have developed the numerical example of the model, considering a_1 as a decision variable of the manufacturer in period 1, as shown in Figure 1a,b, which respectively corresponds to Figures 3 and 4 in the article. In reality, the main findings and managerial insights proposed above are similar to Figure 1a,b. Therefore, the assumption of $a_1 = 0$ has very little influence for us to get managerial insights; it is necessary to make this assumption to reduce the complexity of the models.

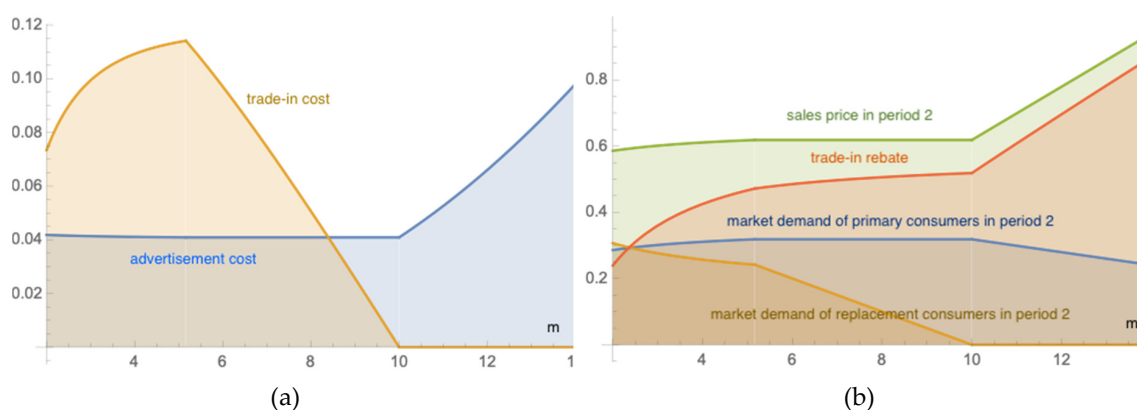


Figure A1. The numerical example considering a_1 as a decision variable. (a) The marketing strategy cost with respect to m ; (b) the optimal sales price in period 2, trade-in rebate, and market demand of primary consumers and replacement consumers in period 2 with respect to m .

Appendix B Proofs of Proposition 1

In the advertisement model, we divided the manufacturer's optimization problem into three conditions, according to the range of q_{2r} , namely, $q_{2r} = 0$ (NTA), $0 < q_{2r} < q_1$ (PTA) and $q_{2r} = q_1$ (ATA).

(1) First, solving the optimization function of the first condition, the Hessian matrix of the manufacturer's profit function in the condition PTA is:

$$H^A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2(1+m) & l_1 \\ 0 & l_1 & -1 \end{bmatrix}$$

The determinant is $|H^A| > 0$. Therefore, the manufacturer's optimization is a concave problem. Solving the first order of the π_m with respect to p_1, p_2 and a_1 . We can get the optimal value p_1^{PTA*}, p_2^{PTA*} and a_1^{PTA*} .

(2) Second, we consider the constraint condition $q_{2r} < q_1$ into the manufacture's profits function. With the above analysis in condition PTA, the Karush-Kuhn-Tucker (KKT) conditions are satisfied with the optimization problem. Thus, we define the Lagrangian for the manufacturer's profits function as $L_M^{ATA} = \pi_m + l(q_{2r} - q_1)$. We can obtain from the optimal values p_1^{ATA*}, p_2^{ATA*} and a_1^{ATA*} from the equation.

(3) Third, we consider the constraint condition $q_{2r} > 0$ into the manufacture's profits function. With the above analysis in condition PTA, the KKT conditions are satisfied with the optimization problem. Thus, we define the Lagrangian for the manufacturer's profits function as $L_M^{NTA} = \pi_m + lq_{2r}$. We can obtain from the optimal values p_1^{NTA*}, p_2^{NTA*} and a_1^{NTA*} from the equation.

Appendix C Proofs of Proposition 2

In the joint model, we also divided the manufacturer's optimization problem into three conditions, according to the range of q_{2r} , namely, $q_{2r} = 0$ (NTA), $0 < q_{2r} < q_1$ (PTA) and $q_{2r} = q_1$ (ATA).

Solving the optimization function of the first condition, the Hessian matrix of the manufacturer's profit function in the condition PTA is:

$$H^J = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2(1+m) & 2m & l_1 \\ 0 & 1 & -2m & 0 \\ 0 & l_1 & 0 & -1 \end{bmatrix}$$

The determinant is $|H^A| > 0$. Therefore, the manufacturer's optimization is concave problem. Solving the first order of the π_m with respect to p_1, p_2 and a_1 . We can get the optimal value $p_1^{PTA*}, p_2^{PTA*}, p_0^{PTA*}$ and a_1^{PTA*} .

(2) Second, we consider the constraint condition $q_{2r} < q_1$ into the manufacture's profits function. With the above analysis in condition PTA, the KKT conditions are satisfied with the optimization problem. Thus, we define the Lagrangian for the manufacturer's profits function as $L_M^{ATA} = \pi_m + l(q_{2r} - q_1)$. We can obtain from the optimal values $p_1^{ATA*}, p_2^{ATA*}, p_0^{ATA*}$ and a_1^{ATA*} from the equation.

(3) Third, we consider the constraint condition $q_{2r} > 0$ into the manufacture's profits function. With the above analysis in condition PTA, the KKT conditions are satisfied with the optimization problem. Thus, we define the Lagrangian for the manufacturer's profits function as $L_M^{NTA} = \pi_m + lq_{2r}$. We can obtain from the optimal values $p_1^{NTA*}, p_2^{NTA*}, p_0^{NTA*}$ and a_1^{NTA*} from the equation.

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