

A new sustainable and novel hybrid solar chimney power plant design for power generation and seawater desalination

Detailed equations of the mathematical model for the HSCPP. Refer to section 2.2 and Figure 2 in the manuscript. The energy balance equations for the different zones of the HSCPP are shown below:

2.2.1. Zone 1: Solar Air Heating

The energy balance equations are as follows:

Airflow:

$$q_{c,gl\text{--}air} + q_{c,ab\text{--}air} = - \frac{c_{p,air} \dot{m}_{air}}{2\pi r} \frac{dT_{air}}{dr} \quad (1)$$

where $\omega_1 = \omega_2$, from air mass balance equation

Absorber:

$$q_{r,ab\text{--}gl\text{--}s} + q_{c,ab\text{--}air} + q_{kabs} = \alpha_{abs} \tau_{gl\text{--}s} I \quad (2)$$

Collector:

$$q_{c,gl\text{--}out} + q_{c,gl\text{--}air} + q_{r,gl\text{--}spc} = \alpha_{gl\text{--}s} I + q_{r,ab\text{--}gl\text{--}s} \quad (3)$$

The convective heat transfer rate between the collector and the air flowing inside it is as follows:

$$q_{c,gl\text{--}air} = h_{c,gl\text{--}air} (T_{gl\text{--}s} - T_{air}) \quad (4)$$

The convective heat transfer coefficient between the air under the collector and the glass is given according to the following equations:

$$h_{c,gl\text{--}air} = \frac{0.2106 + 0.0026 V_{in} \left(\frac{T_m \rho_a}{\mu g (T_{gl\text{--}s} - T_{air})} \right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{gl\text{--}s} - T_{air}) g \rho_a^2 C_p k^2} \right)^{\frac{1}{3}}} \quad (5)$$

Where, T_m is the average temperature of $T_{gl\text{--}s}$ and T_{air} $\left[T_m = \left(\frac{T_{gl\text{--}s} + T_{air}}{2} \right) \right]$

$$h_{c,gl\text{--}air} = \frac{\left(\frac{f}{8} \right) (Re - 1000) Pr}{1 - 12.7 \left(\frac{f}{8} \right)^{\frac{1}{2}} \left(Pr^{\frac{2}{3}} - 1 \right)} \left(\frac{k}{d_h} \right) \quad (6)$$

When $T_{air} > T_{gl\text{--}s}$, then the value of $h_{c,gl\text{--}air}$ is based on the higher value produced from Eqs. (5) and Eqs. (6). However, if $T_{gl\text{--}s} > T_{air}$, Eqs. (26) is used.

The convective heat transfer rate between the base and the air under the collector is as follows:

$$q_{c,ab\text{--}air} = h_{c,ab\text{--}air} (T_{abs} - T_{air}) \quad (7)$$

The convective heat transfer coefficient between the air under the collector and the base is given according to the two following equations:

$$h_{c,ab\text{--}air} = \frac{0.2106 + 0.0026 V_{in} \left(\frac{T_m \rho_a}{\mu g (T_{abs} - T_{air})} \right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{abs} - T_{air}) g \rho_a^2 C_p k^2} \right)^{\frac{1}{3}}} \quad (8)$$

Where T_m is the mean temperature of T_{abs} and T_{air} .

$$h_{c,abs-air} = \frac{\left(\frac{f}{8}\right)(Re - 1000)Pr}{1 - 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}\left(\frac{k}{d_h}\right) \quad (9)$$

When $T_{air} > T_{abs}$, then the value of $h_{c,abs-air}$ is based on the higher value produced from Eqs. (8) and Eqs. (9). However, if $T_{abs} > T_{air}$, Eqs. (6) is used.

The radiative heat transfer rate between the absorber and the solar collector is given as follows:

$$q_{r,abs-gls} = h_{r,abs-gls} (T_{abs} - T_{gls}) \quad (10)$$

The radiative heat transfer coefficient is given follows:

$$h_{r,abs-gls} = \frac{\sigma (T_{gls}^2 + T_{abs}^2)(T_{gls} + T_{abs})}{\frac{1}{\varepsilon_{gls}} + \frac{1}{\varepsilon_{abs}} - 1} \quad (11)$$

Where $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K}$.

The convective heat transfer rate the between the collector and the outside environment is as follows:

$$q_{c,gls-spc} = h_{c,gls-spc} (T_{gls} - T_{spc}) \quad (12)$$

The convective heat transfer coefficient ($h_{c,gls-spc}$) is given by:

$$h_{c,gls-spc} = 2.8 + 3.0v_0 \quad (13)$$

Where, v_0 is the wind speed above the horizontal glass of the collector.

The sky temperature is given by:

$$T_{spc} = T_{out} - 6 \quad (14)$$

The radiation heat transfer rate between the collector and the and the outside environment is as follows:

$$q_{r,gls-spc} = h_{r,gls-spc} (T_{gls} - T_{spc}) \quad (15)$$

The heat transfer coefficient ($h_{r,gls-spc}$) is given as follows:

$$h_{r,gls-spc} = \sigma \varepsilon_{gls} \left(\frac{T_{gls}^4 - T_{spc}^4}{T_{gls} - T_{spc}} \right) \quad (16)$$

2.2.2. Zone 2: Water Evaporation

$$q_{ewtr} + q_{r,wtr-air} + q_{c,wtr-air} + \frac{c_{p,wtr} \bar{m}_{wtr}}{2\pi r} = q_{c,abs-wtr} + \alpha_{wtr} \tau_{gls} I \quad (17)$$

The energy balance equations for Airflow, Absorber, and Collector respectively are:

$$q_{c,gls-air} + q_{c,wtr-air} = - \frac{c_{p,air} \bar{m}_{air}}{2\pi r} \frac{dT_{air}}{dr} \quad (18)$$

$$q_{c,abs-wtr} + q_{kabs} = \alpha_{gls} \tau_{wtr} \tau_{gls} I \quad (19)$$

$$q_{r,gls-spc} + q_{c,gls-out} = q_{c,gls-air} + q_{r,wtr-gls} + \alpha_{gls} I \quad (20)$$

The convective heat transfer rate between the absorber plate and the seawater is as follows:

$$q_{c,abs-wtr} = h_{c,abs-wtr} (T_{abs} - T_{wtr}) \quad (21)$$

Where, heat transfer coefficient between the water and the base is given as:

$$h_{c,abs-wtr} = 135 \frac{W}{m^2} \quad (22)$$

The evaporative heat transfer rate between the seawater and the air under the collector is as follows:

$$\dot{q}_{ew} = \dot{m}_{evab} h_{fg} \quad (23)$$

The heat transfer coefficient (\dot{m}_{evab}) is as follows:

$$\dot{m}_{evab} = h_m A_c \Delta \rho \quad (24)$$

Where, $\Delta \rho$ can be found from:

$$\Delta \rho = \frac{\rho_{sat,Tair in} - \rho_{sat,Tair out}}{\ln \left(\frac{\rho_{sat,Twtr} - \rho_{sat,Tair out}}{\rho_{sat,Twtr} - \rho_{sat,Tair in}} \right)} \quad (25)$$

$$\rho_{sat,Tair out} = \rho_{sat,Twtr} + (\rho_{sat,Tair in} - \rho_{sat,Twtr}) e^{-\frac{h_m \rho_{air} A_c}{\dot{m}_{air}}} \quad (26)$$

To calculate the humidity ratio at the entrance of the chimney, the following can be used:

$$w_3 = \frac{\rho_{sat,Tao}}{\rho_a} \quad (27)$$

h_m can be found using Sherwood number as follows:

$$Sh_D = \frac{h_m d_h}{D_{AB}} \quad (28)$$

Where, $D_{AB} = 0.26 \times 10^{-4} \frac{m^2}{s}$.

$$Sh_D = \frac{\left(\frac{f}{8}\right) (Re_D - 1000) S_c}{1 + 12.7 \left(\frac{f}{8}\right)^{\frac{1}{2}} \left(S_c^{\frac{2}{3}} - 1\right)} \quad (29)$$

Given that $S_c = \frac{\nu}{D_{AB}}$.

$$Nu_D = \frac{\left(\frac{f}{8}\right) (Re_D - 1000) Pr}{1 + 12.7 \left(\frac{f}{8}\right)^{\frac{1}{2}} \left(Pr^{\frac{2}{3}} - 1\right)} \quad (30)$$

$$\overline{Nu_D} = Nu_D \left(\frac{C}{\frac{L}{d_h}} \right) \quad (31)$$

$$f = (0.79 \ln Re - 1.64)^{-2} \quad (32)$$

The radiation heat transfer rate between the water surface and the collector glass is as follows:

$$q_{r,wtr-gls} = h_{r,wtr-gls} (T_{wtr} - T_{gl}) \quad (33)$$

The heat transfer coefficient ($h_{r,wtr-gls}$) can be found from:

$$h_{r,wtr-gls} = \varepsilon_{eff} \sigma [T_{wtr}^2 + T_{gl}^2] (T_{wtr} + T_{gl}) \quad (34)$$

Where, ε_{eff} is as follows:

$$\varepsilon_{eff} = \left(\frac{1}{\varepsilon_{wtr}} + \frac{1}{\varepsilon_{gl}} - 1 \right)^{-1} \quad (35)$$

The convective heat transfer rate between the water and the air under the collector is as follows:

$$q_{c,wtr-air} = h_{c,wtr-air} (T_{wtr} - T_{air}) \quad (36)$$

The heat transfer coefficient ($h_{c,wtr-air}$) is as follows:

$$h_{c,wtr-air} = \frac{0.2106 + 0.0026 V_{in} \left(\frac{T_m \rho_{air}}{\mu g (T_{wtr} - T_{air})} \right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{wtr} - T_{air}) g \rho_{air}^2 C_p k^2} \right)^{\frac{1}{3}}} \quad (37)$$

Where, $T_m = \left(\frac{T_{wtr} + T_{air}}{2} \right)$.

Humidification mass balance equation are as follows:

$$\frac{q_{ew}}{h_{fg}} = \dot{m}_v = \dot{m}_a (w_3 - w_2) \quad (38)$$

The density of the air entering the chimney is given as:

$$\rho_{ent,ch} = \frac{\rho_{da}(1 + w_3)}{(1 + 1.609 w_3)} \quad (39)$$

2.2.2. Zone 3: Solar Chimney

The mass balance equation for the air in and out of the chimney is as follows:

$$\rho_{ch,ent} V_{ch} A_{in} = \rho_{out} V_{ch,out} A_{out} \quad (40)$$

For the IC zone, the energy balance equations when the SDCPP is operating in SCPP mode are as follows:

$$P_{elc} + Q_{out} = \bar{m}_{air} \left[\left(\frac{v_{ch,ent}^2}{2} + g z_{ch,ent} + i_{ch,ent} \right) - \left(\frac{v_{ch,out}^2}{2} + g z_{ch,out} + i_{ch,out} \right) \right] \quad (41)$$

Where:

$$Q_{out} = \bar{m}_{air} [(i_{ch,ent} - i_{ch,out}) - (\omega_4 - \omega_3) i_{wtr}] \quad (42)$$

The flow rate of condensed water was estimated by:

$$\bar{m}_{wtr} = \bar{m}_{air} (\omega_4 - \omega_3) \quad (43)$$

To calculate the enthalpy [42] of the air entering the IC ($i_{ch,ent}$):

$$i_{ch,ent} = T_{air} + \omega_2 (2501.3 + 1.86 T_{air}) \quad (44)$$

To calculate the velocity of the air as it enters the IC, the following equation was used[47]:

$$V_{ch} = \sqrt{2 g H_{ch} \frac{T_{ch,ent} - T_{out}}{T_{out}}} \quad (45)$$

The output power produced by the turbine at the bottom of the IC was calculated as follows[48]:

$$P_{elc} = \frac{1}{2} \rho_{en,ch} C_f A_{ch} V_{ch}^3 \quad (46)$$

where, C_f is the turbine efficiency, set at 0.42.

2.2.3. Cooling Tower

To calculate the enthalpy of the vapor the following was used:

$$i_{vap} = i_{air} + \omega_{vap} i_{wtr} \quad (47)$$

$$i_{air} = c_{p,air} T_{out} \quad (48)$$

The water enthalpy can be calculated as follows:

$$i_{wtr} = c_{p,wtr} T_{wtr} \quad (49)$$

The inlet enthalpy and exit enthalpy of the cooled air remains the same, because of this natural evaporation process. However, the temperature of the vapor decreases, due to the latent heat of vaporization. The change in the temperature can be calculated as follows:

$$\begin{aligned} c_{p,air} T_{out} + (\omega_{air} 2501.3 + T_{out} 1.86) \\ = c_{p,air} T_{vap} + (\omega_{vap} 2501.3 + T_{vap} 1.86) \end{aligned} \quad (50)$$

The velocity of the air and power generated from the CT can be calculated from equations 45 and 46 respectively.

To calculate the system efficiency for electricity production, the following was used:

$$\eta = \frac{P_{elc}}{\frac{1}{4} \pi (D_{col}^2 - D_{ch}^2) I} \quad (51)$$