

## Article

# An Order-Up-to Inventory Model with Sustainability Consideration

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**Abstract:** Along with growing interest in environmental concerns these days, significant academic efforts have been exerted to incorporate sustainability issues into the existing inventory models except for fixed-review interval (i.e., order-up-to models). In this study, we develop an order-up-to model considering environment-related costs and investigate the value of this new policy over the naïve one. Results of an extensive simulation study reveal that sustainability consideration reduces the total costs and that its value is higher when the mean demand is higher, when demand is more variable, when the costs of transshipment or inventory holding are lower, or when an ordering setup cost or an additional indirect cost of having inventory are higher. These findings fill the research gap in existing literature and contribute to managerial implications for periodic inventory control in practice.

**Keywords:** inventory control; order-up-to models; fixed-time interval inventory policy; environmental concerns



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## 1. Introduction

We live in a rapidly developing world, receiving benefit from advanced technology, whereas we humans have caused immeasurable harm to our environment. Lately, the climate change and global warming due to industrial pollution threaten the whole civilization. A recent outbreak of the COVID-19 pandemic is also recognized due to environmental damage. Nowadays, the importance of the natural environment is receiving more attention, and we must consider it when operating organizations. In business management fields over the past few decades, researchers have shown interests for a topic called “sustainability” that refers to maintaining balance in a system pursuing environmental stewardship and social responsibility in addition to economic prosperity [1]. Also in industry, an increasing number of companies become aware that sustainable operations are crucial to eventually survive in the market and gain a competitive advantage by acquiring a good impression through their efforts for the environment and society.

Firms can address environmental concerns in their operations in many ways. Scholars in the inventory management field have also raised some research questions on incorporating sustainability issues into the existing inventory models [2–17]. Among the well-known inventory models, the newsvendor model helps make a single-period inventory decision generally for perishable products. For multiple periods, there are fixed-order quantity models (e.g., economic order quantity model) and fixed-order interval models (e.g., the order-up-to model). There have been relatively much literature considering sustainability concepts for a newsvendor model [2–5] and an EOQ model [6–10]. However, research on order-up-to model has not been conducted up to date. This study contributes to filling this gap so that we can obtain managerial insights into how we can investigate sustainability issues for a fixed-order interval inventory policy and determine what factors affect the inventory replenishment decisions. The main research question is how we present environmental concerns in an order-up-to model. This study aims to develop an inventory-control optimization model, following one of the important research streams in inventory management searching for practical optimization models e.g., [18–21].

For model development, we must first quantify the inventory-related costs associated with the environment. These costs have been introduced in many articles and categorized into three kinds: (1) cost related to transshipment due to regulations on carbon emission, etc., (2) cost related to storage due to a waste of energy and resources for holding inventory, and (3) cost related to disposal of leftover inventory. Leftover inventories can be transferred to the next period in multi-period inventory models; therefore, we here deal with the first two kinds of costs. The costs related to order movement to reflect influence on the environment are defined as transportation cost, and the costs quantified for additionally affecting the environment due to inventory storage are defined as an indirect cost.

In this study, we conduct an extensive simulation study to explore the value of sustainability consideration for inventory decisions and the effect of variables on possible cost savings. Given specific parameters for the mean demand, demand variability, lead time of replenishment, inventory holding and backorder costs, a fixed setup cost of ordering, and the sustainability-related costs (e.g., transportation and indirect costs), we compare the total costs of the two cases, with and without sustainability consideration. Hence, we found that considering sustainability for order-up-to models indeed reduces the total costs. Additionally, we observe that the value of sustainability consideration in fixed-order interval inventory decisions increases as the mean demand increases, as the demand variability increases, as the lead time decreases, as the ordering cost increases, as the transportation cost decreases, as the unit holding cost decreases, and as the unit indirect cost increases. These results are not intuitive, thus providing helpful guidelines to inventory professionals, especially on when the sustainability consideration can significantly reduce costs in periodic inventory management.

The remainder of this article is structured as follows. Section 2 introduces the models used for simulation. We develop an order-up-to model with sustainability consideration and compare the costs of the two models with and without the sustainability consideration via simulation. Section 3 presents the simulation results to investigate the value of sustainability consideration in order-up-to models and explore the factors affecting the inventory decisions. Finally, Section 4 concludes the paper with remarks on future research directions.

## 2. Models

The order-up-to level,  $O$ , is determined by the following formula.

$$O = \bar{d}(T + L) + Z\sigma_{T+L}$$

where  $\bar{d}$  is the mean demand in one period,

$T$  is the review interval,

$L$  is the lead time,

$Z$  is the number of standard deviations for a specified service probability, and

$\sigma_{T+L}$  is the standard deviation of demand over the review and lead time.

An appropriate value of  $Z$  equals the critical ratio of a newsvendor model [22], which is the ratio of an underage cost to the sum of an underage cost and an overage cost. An underage cost means the opportunity cost of not ordering a unit that could have been sold in that period, and an overage cost means the loss incurred when a unit is ordered but unsold in that period. If we allow backorders for stock-outs, the  $Z$  value or the critical ratio would be

$$Z = \frac{b}{h + b}$$

where  $b$  is the unit backorder cost, and

$h$  is the unit holding cost.

Usually, the inventory holding cost includes the opportunity cost of capital, the cost of spoilage, obsolescence, insurance, and storage. In general, in order-up-to models, the target service probability tends to be high (mostly above 99%). The order-up-to models are often being used in practice for their convenience of ordering at fixed-time intervals.

Every  $T$  period, we review the inventory levels and place an order to make the inventory position up to the order-up-to level  $O$ . The order amount may vary time to time, but the order interval is constant.

To determine the optimal order-up-to level, we must decide the order interval beforehand. When an ordering cost exists, that is, a fixed cost associated with placing an order, the frequency of order placement significantly affects the total inventory cost. According to [22], we might be able to take advantage from the EOQ (Economic Order Quantity) model regarding appropriate order intervals. Though the EOQ model is for deterministic demand rate, they suggest using the model to search for the period length by dividing the EOQ quantity by the mean demand in one period. Given the combinations of variable parameters in this study's simulation model, we first derive the optimal EOQ and determine the order interval as the integer number we obtain from rounding off the EOQ divided by the mean demand in one period. Then, we decide the order-up-to level for each given setting.

To incorporate the sustainability concepts into the simulation model, let us consider two kinds of sustainability costs: a transportation cost and an indirect cost. Here, we quantify any harm to environment due to the transshipment of orders as the transportation cost. Therefore, a fixed cost occurs every time we place an order similar to the ordering cost. We pay additionally for transshipment due to regulations on carbon emission, and quantify the unit transportation cost as the cost for one transaction. The indirect cost is defined as the cost quantified for any environmental loss due to having an inventory on hold. For example, having more inventory in warehouses takes more energy, such as electricity or fuel for storage. This indirect cost is defined in addition to the existing holding cost to reflect the influence on the environment.

In our simulation study, given some specific parameters, we compute the two order-up-to levels with and without sustainability consideration and compare the costs. The order-up-to level without sustainability consideration is denoted by  $O_N$  ( $N$  stands for a naïve policy) and computed with the review interval  $T_N$  and the number of standard deviations  $Z_N = \frac{b}{h+b}$ . The order-up-to level with sustainability consideration is denoted by  $O_{SC}$  ( $SC$  stands for sustainability consideration) and computed with  $T_{SC}$  and  $Z_{SC} = \frac{b}{h+i+b}$  where  $i$  is the indirect cost. The review interval  $T_{SC}$  is affected by the transportation cost because the EOQ with sustainability consideration is calculated as  $\sqrt{\frac{2(S+TR)\bar{d}}{h+i}}$ , whereas the EOQ of the naïve policy is  $\sqrt{\frac{2S\bar{d}}{h}}$  where  $S$  is the ordering cost and  $TR$  is the transportation cost. Under the simulation setup in this study,  $T_{SC}$  is shorter than  $T_N$ , indicating that having fewer stocks on hand would save more costs in terms of sustainability.

The total cost in period  $t$  in this study is computed as follows.

$$C_t = SI(Q_t > 0) + TI(Q_t > 0) + hInv_t I(Q_t > 0) - bInv_t I(Q_t < 0) + iInv_t I(Q_t > 0)$$

where  $I(\cdot)$  is indicator function,

$Q_t$  is the order quantity in period  $t$ , and

$Inv_t$  is the ending inventory in period  $t$ .

Every  $T_N$  or  $T_{SC}$  period depending on policies, the order quantities are decided to make the inventory position up to  $O_N$  or  $O_{SC}$  and these orders arrive after the lead time. There is zero-order quantity at other times.

### 3. Analysis of Simulation Results

To investigate how we might save costs in order-up-to models by considering sustainability concerns, we specify the following parameters for the simulation setup.

$$\bar{d} = \{100, 200, 300\}$$

$$\text{SD rate} = \{0.1, 0.2, 0.3\}$$

$$L = \{1, 2, 3\}$$

$$S = \{200, 250, 300\}$$

$$TR = \{25, 50, 75\}$$

$$\begin{aligned}
 h &= \{0.01, 0.03, 0.05\} \\
 i &= \{0.02, 0.04, 0.06\} \\
 b &= 75
 \end{aligned}$$

The unit backorder cost is fixed at 75. We chose this number assuming the price is 100, and the gross margin is 75%; thus, the backorder cost can be gross margin because it could be regarded as a lost sale. This parameter value also appeared in [22]. We chose the parameters for the unit holding and indirect costs significantly lower than the unit backorder cost as they usually are. SD rate determines the demand variability making the standard deviation of demand as  $\bar{d}$  multiplied by SD rate. We assume Normal distribution for demand generation. We assign three levels (low, medium, high) for each parameter to investigate how the changes in each parameter value affect the simulation performance.

The performance measure in the study is defined as Cost savings (%), which show us how much cost we could save by considering sustainability concerns.

$$\text{Cost savings} = \frac{C_N - C_{SC}}{C_N} \times 100(\%)$$

where  $C_N$  is the average cost of the naïve order-up-to policy, and

$C_{SC}$  is the average cost of the order-up-to policy with sustainability consideration.

For every scenario with the aforementioned parameters, we run simulations for 10,000 periods and compare the average costs of both policies. A total of 2187 scenarios exist under the setup, and the average cost savings turn out to be 12.20% with a minimum of 0.25%, a maximum of 37.82%, and a median of 9.29%. Incorporating additional costs of transportation and inventory storage into the inventory decision indeed reduces the total cost.

For each scenario, if we only consider inventory holding, backorder, and ordering cost, the cost of the naïve policy is less than that of the policy with sustainability consideration because the naïve policy is optimal. If we compare the inventory costs, except for sustainability-related costs, the naïve policy can save more by 6.55% on average. However, for environmental costs only (transportation and indirect costs), the policy with sustainability consideration can save more than the naïve policy by 23.92% on average. One of the interesting and important findings is that we can reduce the total cost by using the policy with sustainability consideration for a fixed-review interval inventory model. Cost savings in sustainability-related costs exceed the cost increase in inventory holding, backorder and ordering costs.

The saving mechanism is not explicit, but it seems that holding less inventory helps reduce the total cost. Under our simulation setup, in most cases (93.00%), the order intervals of the policy with sustainability consideration are shorter than those of the naïve policy. Shorter order intervals imply more frequent order placements with less amount of inventory on hold. More frequent orders incur higher transportation costs, but the reduction in indirect costs appears to be more beneficial leading to lower total costs.

To observe the relationship between cost savings and order frequency, we define the following measure:

$$T \text{ differences} = T_N - T_{SC}.$$

The  $T$  differences in our simulation range between 0 and 14 with the average value 4.03 and the median 3.

Figure 1 illustrates the average cost savings and average  $T$  differences concerning the mean demand. As the mean demand increases, the cost savings also tend to increase, but the  $T$  differences tend to decrease. Thus, the increase in order frequency may not be the direct source of benefit in using a new policy with sustainability consideration.



Figure 1. Simulation results for mean demand.

Figure 2 shows that the average  $T$  differences are constant regardless of changes in the SD rate or in the lead time. The EOQ formula does not contain these variables; thus, the differences in the order intervals between the two policies are not affected by them. However, the cost savings of sustainability consideration increase as the demand variability increases or as the lead time decreases. In other words, we can expect higher cost savings from using the new policy with sustainability consideration when the demand is more variable or the lead time is relatively shorter.

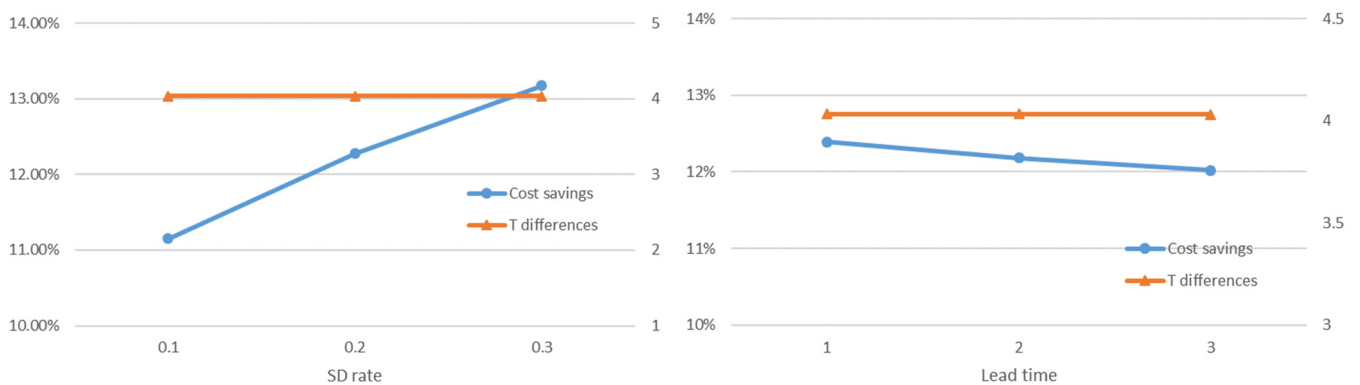


Figure 2. Simulation results for SD rate (left) and lead time (right).

Figure 3 indicates that the average cost savings increase as the ordering cost increases or as the transportation cost decreases. These two kinds of costs are both linked to order transshipment, but their relations with cost savings are opposite. Average  $T$  differences show the same pattern as that of average cost savings, implying that the decrease in order intervals may be closely related to the total cost reduction. We can infer that considering environment-related costs in the order-up-to model is more beneficial especially when the ordering setup cost is relatively higher or when the additional transportation cost is relatively lower.



Figure 3. Simulation results for ordering cost (left) and transportation cost (right).

Figure 4 shows that the average cost savings are lower when the unit holding cost is higher or when the unit indirect cost is lower. The two variables' relations with cost savings are opposite, too, and we can conjecture that the total cost savings are differently affected by different types of cost parameters—either environment-related costs or the original inventory-related costs. The experimental results imply that the new order-up-to policy is more useful in cost reduction when the unit inventory holding cost is comparatively lower or when the additional unit indirect cost is higher.

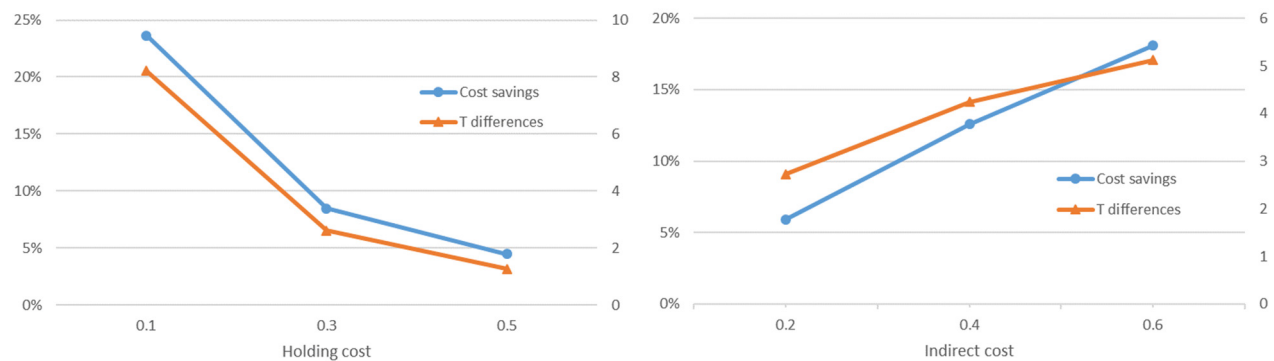


Figure 4. Simulation results for holding cost (left) and indirect cost (right).

These simulation results are not intuitive, so these findings give us meaningful insights on what factors we must consider when we implement order-up-to inventory policies in practice.

#### 4. Conclusions

We have developed an order-up-to model with sustainability consideration by quantifying environmental concerns as a fixed cost of order transshipment (in addition to ordering setup cost) and a unit variable cost of wasting resources for storing inventory (in addition to a general unit inventory holding cost). The new order-up-to policy has shorter order intervals than the naïve policy, and it reduces total costs.

Results of comprehensive simulation experiments reveal that the decrease in environment-related costs exceeds the increase in the original inventory holding and ordering costs when the new model is applied. We also observe that the benefit of using this new order-up-to policy is higher when the mean demand is higher, demand is more variable, the transportation or unit holding costs are lower, or the ordering or unit indirect costs are higher. These findings provide valuable implications in practice by understanding particularly under what circumstances is more beneficial to periodic inventory management, considering environmental costs. For example, products with highly variable demand require more sustainability consideration than so-called commodities. The results provide valuable insights on practice, but as they are based on simulation, there are some limitations of this

study such as in the choice of cost parameters or the assumptions. Still, there is no problem analyzing the effect of parameters on the benefit of concerning environments.

Although order-up-to models are well-known and have practical advantages due to their constant order intervals, most existing literature studying sustainability issues in inventory management has mainly focused on fixed-order quantity (EOQ) models for multi-period inventory decisions. This is the first study incorporating environmental costs into an order-up-to model, and more various extensions starting from this research are anticipated. For example, a future study may investigate the mechanism of how the new order-up-to policy leads to cost reduction. Alternatively, we may incorporate sustainability concerns into other inventory control systems, such as an (s, S) policy. We might consider social responsibility and environmental issues for sustainable inventory management in future research.

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