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Review

A Critical Review of the Definition and Estimation of Carbon Efficiency

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Abstract: The concept of carbon efficiency is closely related to energy efficiency but embraces a broader range of carbon emission sources. Many studies have covered carbon efficiency, investigating the climate crisis, economic growth, and a sustainable future; however, it is hard to agree that there is a consensus on the definition of carbon efficiency. To fill this gap, we reviewed the literature on carbon efficiency, especially the empirical studies that quantitatively measured carbon efficiency. As a result, we have categorized the articles into three groups based on defined criteria of carbon efficiency. We have also classified the methodology to measure carbon efficiency and to discuss misleading definitions in the empirical studies. Lastly, we suggest a desirable direction to define and measure carbon efficiency along with discussion points. Carbon efficiency is different from energy efficiency and our review will help build the carbon efficiency concept in a proper direction.

Keywords: carbon efficiency; carbon emission; carbon intensity; energy efficiency; sustainable development

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1. Introduction

In response to the climate crisis, many national and international activities have been explored and global commitments and actions are also increasing; however, even through these efforts, massive greenhouse gas emissions still exacerbate climate deterioration [1]. According to the IPCC [2], global warming will exceed 1.5 °C and 2 °C in the 21st century unless there are significant reductions in greenhouse gas emissions. According to the report, it is necessary to restrain the cumulative carbon emissions to at least net zero by implementing substantial, rapid, and sustained reductions in greenhouse gas emissions [3].

However, for sustainable development, we must consider both environmental and economic aspects together. Carbon emissions reduction with economic growth, also known as "green growth," is one of the main concerns in this context. Along with these circumstances, the interest in carbon emissions efficiency is also growing. Unlike energy efficiency, carbon efficiency is a field that has received relatively less attention or is sometimes mixed with energy efficiency. It is, however, evident that energy efficiency cannot encompass carbon efficiency, although energy consumption is the primary source of carbon emissions.

As interest in whether activities for energy transition, such as the development of green technology, are substantially efficient in carbon emission increases, the importance of carbon efficiency as an indicator is highlighted [4–6]. There are cases of energy efficiency and carbon efficiency that may exhibit opposite aspects [7]. Without the proper distinction of carbon efficiency, it could become the culprit of climate crisis exacerbation and a misguided policy and this means it is necessary for studies to focus on carbon efficiency.

Recent carbon efficiency studies have become very diverse not only in regard to region boundaries but also in the types of industry. Carbon-related issues are dealt with at the national level along with political methods such as the national basic plan. In other words, carbon efficiency research can be utilized as the basis for establishing policies. The studies are conducted at a national level [8,9] and in particular fields such as trade [10] or technology [11]. At the same time, it also provides information on whether the current policy is working effectively, and whether the direction should be to improve it [12,13]. In addition, carbon efficiency is the ecological efficiency that can directly determine overall carbon emissions; therefore, because it is related to the carbon reduction potential, it can also be one of the reasons for carbon efficiency study development.

Although various studies have been conducted on carbon efficiency, there is still no consensus on the definition of carbon efficiency [14]. Existing papers mainly focus on methodological points such as new models or applications and are less concerned with the fundamental meaning. Additionally, the definition of carbon efficiency is varied in the studies and because there are many variables related to carbon efficiency, the proposed definition of carbon efficiency becomes confusing due to additional modifications coming from such variables. Despite many developments in the carbon efficiency field, therefore, the considerations of the foundational issue of the definition are insufficient.

The basis of carbon efficiency is significant both academically and empirically. As mentioned above, policymakers can consider the research in this field as a basis for establishing a political direction. In this process, it is also necessary to compare the results of various studies. For example, if the results estimated using different methods are used in a comparison, it cannot be a correct comparison. When comparing the results of a study, a common definition should be used, and understanding the differences is essential even in unavoidable cases due to data limitations; thus, the importance of an apple-to-apple comparison ultimately results in a review of the establishment of the definition of carbon efficiency and a consideration of the fundamental meaning of carbon efficiency.

To fill this research gap, we reviewed the existing studies related to carbon efficiency focusing on how they define and measure carbon efficiency. According to our investigation, we can divide the literature into three groups. Within each group, there are also variations in the concept of a definition. At the same time, there are misused points when modifying an indicator or analyzing by using an indicator. We summarized all these studies along with the methodology that they used. Since there are many calculation methods depending on the combination of various approaches, we only summarized well-developed and representative methodologies measuring carbon efficiency. We also propose discussion points that must be considered in carbon efficiency research.

The implication of this article is threefold. First, we review the carbon efficiency research and organize the literature by definition and methodology. Even though they have been discussed it together, the carbon efficiency field receives less attention than the energy efficiency field does. The existing review paper on carbon efficiency also focused on the specific calculation methodology [14]. This paper can fill such a gap, focusing on carbon efficiency. Second, we suggest the criteria for a carbon efficiency definition. Research using a proxy indicator, such as carbon intensity, must confirm the conditions necessary for becoming a carbon efficiency study. Through this, we have corrected the misused terminology and prevent the confusing use of carbon efficiency terms. Finally, we present the direction to define and measure carbon efficiency by identifying the validity of each definition. We also provide a challenging point to expand the scope of carbon efficiency research. For the development of further carbon efficiency study, a future study has to take into account the fundamental definition that we have composed. This also serves as the basis for policymakers to evaluate the current policies and design future policies.

The paper is organized as follows. Section 2 provides a review of the literature focusing on the definition of carbon emission. Section 3 explains the model and methodology used for measuring carbon efficiency. Section 4 provides considerations for future studies in the carbon efficiency field, and this section is followed by the conclusion.

2. Definitions of Carbon Efficiency

As an increasing number of studies measuring carbon efficiency, the definitions of carbon efficiency presented in each study have also diversified, but still, there is no consensus about the meaning of carbon efficiency and several representative approaches exist. In this section, we examine the criteria of carbon efficiency used in the existing studies and the scope of the definition of carbon efficiency is separated into three groups. First, several studies estimate carbon efficiency using an indicator such as carbon intensity. The second group measures direct efficiency using an empirical approach on a theoretical basis, while the last group calculates direct and indirect emissions together.

2.1. Single-Factor Indicators

With increasing concern about the global climate crisis, the interest in carbon emissions also has increased. Toward a low-carbon economy, the study about energy-related carbon emissions draws the attention of many researchers and stakeholders. In the early stage, the direction of studies was focused on the form and mechanism of carbon emissions by production activities in-country or in industry; therefore, the perspective of carbon efficiency studies focused only on specific activities related to an emissions reduction or an absolute emission amount. For example, what level of carbon emissions come from production activities [15] or by much carbon emissions can be reduced under the given changes in technology and economic structures [16]. Thus, carbon efficiency has been interpreted in the context of the performance of carbon emissions.

Carbon intensity is one of the indicators that reflect carbon emissions performance [17–19]. In the early stage of the carbon efficiency research field, it was often referred to alongside energy efficiency; thus, like energy intensity, the carbon intensity, measured as CO_2 emissions per unit of output, was also treated as an indicator of carbon efficiency. In a study that dealt with carbon efficiency as a proxy to confirm the energy efficiency or emissions compared to production activities, the use of an index indicating the carbon emissions per unit of output, such as carbon intensity, was appropriate for the research purpose. In addition, studies using carbon efficiency from a similar perspective sometimes have used terms such as "carbon performance" [15] and "carbon productivity" [20] instead of carbon efficiency.

The indicators which measured carbon efficiency via carbon intensity or carbon productivity calculated by carbon emissions to the GDP (gross domestic product) are called single-factor indicators. Many pieces of research have been conducted with single-factor indicators amid defined indicators according to their research purpose. Greening et al. [17] used aggregate carbon intensity from the freight industry to analyze the development of freight energy consumption in OECD countries. For comparing freight transport modes and different modal energy intensities, they used total freight tonne-kilometers hauled in a specific year as the denominator [21,22]. In other words, the meaning of carbon intensity here was the carbon emissions from the freight sector when producing one unit of a freight tonne-kilometer.

Sun [23] set the carbon dioxide intensity of primary energy, termed the carbonization index, and calculated it by dividing the carbon emission by energy consumption. The unit of carbon emission intensity was kg of carbon per USD at 1995 prices. Fan et al. [18] also defined carbon intensity using the GDP as the denominator, but they defined two types of carbon intensity, namely, the primary energy-related carbon intensity and the material production sectors' final energy-related carbon intensity. Both used the real GDP of 1990 constant CNY as the denominator, but the contained energy and their emissions data were different.

Zhu et al. [19] investigated the carbon intensity of 89 countries to analyze the potential of their difference. Their definition of carbon intensity was the carbon dioxide emissions per unit of GDP in 1980, but the GDP converted to the value of USD constant of 2005. Wang et al. [24] used carbon intensity as obtained by the carbon emissions per unit of GDP and the unit of carbon intensity was a ton of emissions per CNY 10,000. Dong et al. [25] defined the carbon emissions per unit of GDP as the carbon emission intensity and utilized this to measure the regional carbon performance. According to the research purpose, the GDP and data for the indicator were expressed at the year 2000 constant prices. Su and Ang [26] also used the same definition with the ratio of carbon emissions to GDP as an aggregate carbon intensity, but they used an additional concept, namely, an "embodied" emission. We will cover this concept in detail later in this section.

On the other hand, when the purpose of the study is more concentrated on the carbon dioxide emitted regardless of the type of production, sometimes an absolute emission value is also used as a carbon performance indicator [27–30]. Tian and Zhou [31] adopted both an absolute carbon emissions value and carbon intensity as the carbon emissions efficiency indicators. They adopted four indicators: carbon emissions per capita, carbon emission intensity, residential carbon emissions per capita, and industrial carbon emissions per unit area. The last indicator is defined as the industrial carbon emissions divided by land area, which can denote both the emission density and intensity at the same time.

However, because they used the absolute carbon emissions as the indicators, the analysis was more focused on the static value, such as the actual emission amount rather than the efficiency that has variable characteristics. Considering that even carbon intensity only takes into account the carbon emissions and output, it is a leap-forward approach that views the absolute carbon emissions value as one of the general standards of carbon efficiency.

The use of single-factor indicators in efficiency studies has several advantages. First, because of the ease with calculating and understanding, single-factor indicators are applied widely [23,25]. Additionally, the number of data used for calculations are small. When measuring indicators, if they require a large amount of data, difficulties may be experienced in the analysis due to data limitations, apart from the difficulties of the calculation itself. Moreover, if large amounts of data are used, the sources may vary, which adversely affects the reliability of the analysis; however, the International Energy Agency has incorporated carbon intensity into its statistics to assess carbon emissions and sustainability targets. Through this data, reliability is not only secured, but also a unity for analyzing multiple countries.

However, there is an opinion that views it as unreasonable to regard a single-factor indicator as an indicator of carbon efficiency because it does not take into account either the contribution of other production factors or their related substitutions [13]. As we know through the defined form of carbon intensity, they only focus on a carbon emissions reduction and output growth, but other factors are out of consideration, such as capital and labor and structural changes in production [32,33], or changes in technical efficiency [34]. These deficiencies of single-factor indicators make the calculations of carbon efficiency under a total factor framework more convincing [35].

2.2. Total-Factor Indicators

In economics, we define the most efficient use of limited resources to satisfy people's needs under a given technology as efficiency [36]. Theoretically, efficiency is an economic state of reallocation to make one individual better off from a resource that cannot be achieved without making at least one individual worse off. Because this theoretical efficiency is unknown, we then use the relative efficiency based on empirically available data instead. In a broad sense, it can be said that the single-factor indicators identified above belong to this relative efficiency; however, it is also true that single-factor indicators have deficiencies.

The total-factor indicators refer to a method of calculating carbon efficiency by considering the various inputs and outputs that may affect it. Because it can fully take into account the influence of input factors and the interaction between factors on the carbon emissions efficiency, total-factor indicators have more advantages than by comparing single-factor indicators [37,38]. Additionally, total-factor indicators are used because they can better capture the contribution and estimate the technical efficiency of carbon inputs, as well as the interaction between different input factors [13]. Especially, total-factor indicators are more useful to capture the technical efficiency of carbon inputs which is, nowadays, a trajectory for low carbon development.

As aforementioned, the definition of efficiency in total-factor indicators is related to the economic theory. Based on the Pareto efficiency theory, Farrell extended the relative efficiency concept to apply an efficiency measurement using fewer assumptions [39]. It is known as the Farrell efficiency, which can estimate the amount of input that can be saved or output that can be increased without worsening other inputs or outputs. The empirical study using this efficiency was restricted to the case of a single output and is not feasible for applying efficiencies related to both inputs and outputs.

Charnes et al. (1978) developed a dual pair of linear programming models for data envelopment analysis (DEA) to identify the best-practice frontier and to estimate relative efficiency [40]. Their method can measure the technical efficiency of the minimum efficiency value under each input and output and because this method can reflect the overall performance of each decision-making unit, it is usually used to measure environmental or ecological efficiency such as carbon efficiency. We will explain this in detail in the methodology section.

Ramanathan [41] was the first to propose the concept of using this DEA to measure the number of carbon emissions that can be reduced, but he did not measure carbon efficiency. The studies used total-factor indicators called carbon emissions efficiency as the total-factor carbon emissions efficiency (TFCE). The TFCE is the ratio of theoretic carbon emissions to actual emissions and can be defined as the ratio of actual carbon emissions to target carbon emissions. Even though there are some newly proposed indicators such as the carbon performance index [42,43], measured as the ratio of the target carbon intensity to actual carbon intensity, the most dominant measurement indicator is the former.

On the other hand, because the definition of carbon efficiency is the ratio of actual carbon emissions to target carbon emissions, another measurement method is also used in many studies in addition to DEA. The stochastic frontier analysis (SFA) method has been extensively used as a methodology to estimate the TFCE, and we will also introduce this method in detail in the methodology section.

Research on carbon efficiency has been conducted for various purposes. The purpose of a study performed at the most aggregated level is to estimate the carbon efficiency of one country and compare it with other countries [4,7,44]; however, they treat the entire industry the same in country-level studies. This means that they are less focused on the intrinsic features that differ in the sector. Thus, to consider the heterogeneity stemming from the difference between each industry, the carbon efficiency of a specific industry [45] or city [46] is also analyzed. To a more detailed extent, some studies estimate carbon efficiency at the individual firm level for comparison [15,47]. On the other hand, a study focusing on efficiency itself have also been undertaken to analyze what factors would affect the carbon efficiency [13,48].

Herrala and Goel [44] examined carbon efficiency in 170 countries between 1997 and 2007. They tried to link environmental efficiency with policy by defining the TFCE as the distance from the policy objective emissions to the realized emissions. They used carbon emissions, GDP, population, and land area data as their variables. Jin and Kim [7] examined energy efficiency in the views of both economic and ecological aspects during 1995–2016 for 21 emerging countries. The entities were selected from Morgan Stanley Capital International. This study used the energy consumption, economic complexity index, and other factors of production based on the Cobb–Douglas production function. Dong et al. [4] explored the impact of green technology innovation on carbon emission efficiency to provide a policy basis for developed countries to mitigate carbon emissions and achieve carbon neutrality goals. They also used labor, capital stock, and energy consumption as the input variables and regional GDP and carbon emissions as the desirable and undesirable outputs, respectively.

Zhou et al. [45] measured the total-factor carbon emissions efficiency of the construction industry from 2003 to 2016 to explore the interior and exterior dynamic transmission mechanism of the carbon emission efficiency of the construction industry. They used three inputs, labor, capital, and energy consumption, and two outputs, the industrial economic output and carbon emissions. Zhang et al. [46] used the same variables to analyze the TFCE of 64 prefecture-level cities in four major urban agglomerations in China including the Pearl River Delta, Beijing–Tianjin–Hebei, the Yangtze River Delta, and Chengdu–Chongqing.

Trinks et al. [47] measured the carbon efficiency to evaluate firms' carbon emissions levels relative to those of their more efficient peers. They used capital, labor, and energy variables and treated carbon emissions as "bad output." According to the results, they could analyze the potential carbon reduction amount of each firm relative to the most efficient firm. Wang et al. [15] also analyzed the carbon efficiency of Chinese firms, but their purpose was to study the relationship between carbon efficiency and the financial performance of Chinese firms. Their input variables were their employees, net fixed assets, and total energy consumption. The output variables were the desirable output measured by the firm's operating income and the undesirable outputs measured by the firm's absolute carbon emissions.

Sun and Huang [48] evaluated carbon emissions efficiency to analyze the impact of urbanization on that emissions efficiency. They used the input variables including capital, labor, and carbon dioxide emissions, and the GDP was the output variable. Meanwhile, Tan et al. [13] estimated the total factor of carbon efficiency to investigate the impact of China's carbon regulatory policy and their input variables were the carbon emissions, capital stock, and labor and the output variable was the sectoral value-added output.

In addition to the above indicators, there are other indicators for measuring the carbon emissions efficiency. Zhou et al. [37] proposed a total-factor carbon emissions performance index to measure the relative carbon emissions performance. The carbon emissions performance index is measured by the ratio of target carbon intensity to the actual carbon intensity. They proposed a new index basis on the Shephard input distance function presented by Tyteca [49] and the Malmquist productivity index developed by Caves et al. [50], as a ratio of two distance functions for the measurement of productivity; however, they expressed this new indicator as the carbon emissions performance and not the carbon emissions efficiency. The other studies using indicators based on the Malmquist index also mentioned that indicators as the total-factor carbon emissions performance have even used different methodologies [51–53]. Usually, they also used capital, labor, and energy consumption as the input variables and the GDP and carbon emissions as the output variables.

Meanwhile, there is an indicator that coordinates the concepts of carbon intensity and TFCE. Zhou et al. [43] redefined the TFCE as the ratio of the carbon intensity target to the actual carbon intensity and they applied it to evaluate the carbon emissions performance of 126 countries in their electricity generation. When they first presented this concept, it was named the carbon performance index; however, the subsequent studies which used the same indicator defined that concept as the total-factor carbon emissions efficiency even though they used the same abbreviation [42,54]. The research using the additional indicators used the carbon efficiency and carbon emissions performance without distinction as usual.

2.3. Extended Concept of Carbon Efficiency

Most carbon efficiency studies refer to two types of carbon efficiency measurement indicators [13,14,48,54]. Some studies have stated that carbon efficiency studies consider only direct emissions as a limitation, but with the solutions to the climate crisis, such as the European carbon border adjustment mechanism and the corresponding frameworks such as consumption-based accounts, the concept of an "embodied" carbon emission has been presented in recent studies [26,55].

The embodied emission implies the emissions indirectly generated within the entire economic system in addition to the direct emissions. It is also treated as the indirect emissions because of this concept and because the products of one industry are used not only for final consumption but also as intermediates in the manufacturing processes of other sectors, it is necessary to examine the carbon emissions from intermediate production and consumption [56,57].

The environmental analysis uses an embodied framework proposed by Isard et al. [58] and Leontief [59]. Several studies under this framework have addressed analyzing the embodied energy or emission flows [60–62], while Su and Ang [63] especially, proposed the carbon intensity concept to consider embodied emissions. Additionally, the application of the embodied framework in the efficiency field was undertaken by Gao et al. [55]. That study argued that indirect emissions should be considered under the same total-factor efficiency framework and they estimated the direct carbon efficiency and the indirect carbon emissions efficiency of 28 industry sectors under the condition of trade openness.

Embodied carbon efficiency research is a concept recently proposed, and there are relatively few related studies. Since the embodied framework was first presented, 34 studies have referred to it according to Google Scholar. There are only four studies on carbon emissions efficiency among them [4,64–66], and even those studies used other definitions for an actual measurement; thus, we can know that the embodied framework is less developed than the single-factor indicator and total-factor indicator methods, and therefore, it requires careful consideration to address it for carbon efficiency research.

3. Methodologies to Measure Carbon Efficiency

There are several methodologies used for the measurement of carbon efficiency in the carbon efficiency field. In this section, we summarize the several calculation methods used in the previous studies discussed. We have only organized the methodology used for carbon efficiency measurement in practice. The composition of this section is based on the methodology type, because there are various methods of calculating carbon efficiency even though it is the same indicator.

3.1. Calculation with Given Data

This method does not require a specific theoretical background because it involves simple calculations using given data. It is the most common methodology used in the case of single-factor indicators such as carbon intensity. In this case, the only thing to consider is the reliability of the data. Usually, the data used for this comes from a credible institution such as the World Bank or International Energy Agency; therefore, the reliability also does not matter in most cases. As the calculation of carbon efficiency is simple, this type of study often conducts additional analysis.

The literature using this simple calculation often focuses on the additional analysis and we can classify the studies into three categories by the method or the purpose of measuring the carbon efficiency. The first type is the study to analyze the sources or structures of carbon emissions from the perspective of an industrial structure or demand structure [25]. The second category employs decomposition analysis to investigate the impacts of industrial or technological change and carbon intensity change in different departments [17,18]. The third category uses econometric analysis for a similar purpose to the other category, which is to reveal the effect of the changes in influencing factors on carbon efficiency [15,24].

For example, Dong et al. [25] tried to measure the carbon emission intensity to reveal the effect of influencing factors on the carbon emissions intensity. They decomposed the carbon emissions intensity using an input–output structural decomposition analysis because their purpose was to find the driving force. Since the driving force is not our current interest, the details of the methodology used in that study are not covered here. Additionally, the focus of Greening et al. [17] was to confirm the attribution factors contained in the aggregate carbon intensity. They used the adaptive weighting Divisia index method, but we do not cover this for the previous reason. Wang et al. [24] also examined carbon intensity using stochastic impacts by a regression on the population, affluence, and technology model [67]. As mentioned above, detailed descriptions of the additional methodologies beyond the purpose of our study are omitted.

3.2. Data Envelopment Analysis

The data envelopment analysis (DEA) is a well-established and the most commonly used method that was first proposed by Farrell [39]. It is appropriate to evaluate the relative efficiency of a set of comparable decision-making units (DMUs) with multiple inputs and outputs [68]. The traditional DEA models are the CCR (Charnes, Cooper, and Rhode) model [40] and the BCC (Banker, Charnes, and Cooper) model [69]. The CCR-DEA method measures the ratio of optimal input to actual input, which means an input-oriented model based on a constant return to scale (CRS). The technology was defined as $T = \{(x, q) : q \leq Q\lambda,\ x \geq X\lambda\}$. Assume that there are I DMUs, and each DMU has N inputs and M outputs. Then, the efficiency of the ith DMU is the solution to the following problem:

$$\begin{array}{c} \min_{\theta,\lambda} \theta \\ \text{s.t.} - \mathrm{q_i} + \mathrm{Q}\lambda \ \geq 0 \\ \theta \mathrm{x_i} - \mathrm{X}\lambda \geq 0 \\ \lambda \geq 0 \end{array} \tag{1}$$

where θ is a scalar, and λ is a $I \times 1$ vector of constant. The x_i and q_i are the column vectors of ith DMU's input and output, respectively. The matrix X is $N \times 1$ input matrix and the matrix Q is $M \times 1$ output matrix. They represent the data for all I DMUs. The solution value of θ is the efficiency score for the ith DMU. If the efficiency score is equal to 1, it means that the DMU is on the frontier, the efficient DMU.

On the other hand, the BCC-DEA method is an output-orientated DEA model proposed to improve the CCR-DEA model. The CCR-DEA model, also known as the CRS-DEA model, is based on the CRS assumption that is not valid in the case of imperfect competition, government regulation, financial constraints, etc.; therefore, Banker, Charnes, and Cooper [69] added the convexity constraint to the CRS-DEA model to explain the variable return to scale (VRS). The BCC-DEA model, also known as the VRS-DEA model, is expressed as a modified form of the CRS linear programming problem. All other equations are the same as Equation (1), and the convexity constraint $I1'\lambda = 1$ is added.

In the CCR and BCC model, all the inputs reduce or all the outputs expand in the same proportion to achieve efficient decision-making. Thus, they are called the radial model; however, when the DMU is inefficient, there is a distance consisting of the sum of the radial improvement and the slack improvement, but the radial model only considers the radial improvement. Consequently, it has a slacks problem. Tone [70] proposed the slacks-based measure integrating the DEA to solve this problem, named the SBM-DEA model. The SBM-DEA model has advantages in performance comparisons and is also effective for understanding the economic-environmental index [71].

The SBM model provides an effective solution to the relaxation problem by avoiding the influence and error caused by radial and angle differences. Suppose that there are n DMUs, with m inputs and s outputs. The production possibility set is defined as $P = \{(x, y) : y \leq Y\lambda,\ x \geq X\lambda,\ \lambda \geq 0\}$. The λ is a $n \times 1$ nonnegative vector such as $\lambda = (\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_n)$, the X is a $m \times n$ matrix of input vectors such as $X = [x_1,\ x_2,\ \cdots,\ x_n]$, and the $Y = [y_1,\ y_2,\ \cdots,\ y_n]$ is a $s \times n$ matrix of output vectors. The efficiency of the ith DMU is the solution to the following problem:

$$\begin{array}{c} \min \rho = \frac{1 - \frac{1}{\mathrm{m}} \sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{\mathrm{s_i^-}}{\mathrm{x_{i0}}}}{1 + \frac{1}{\mathrm{s}} \sum_{\mathrm{r}=1}^{\mathrm{s}} \frac{\mathrm{s_r^+}}{\mathrm{y_{r0}}}} \\ \text{s.t.} \mathrm{x_0} = \mathrm{X}\lambda + \mathrm{s^-} \\ \mathrm{y_0} = \mathrm{Y}\lambda - \mathrm{s^+} \\ \lambda \geq 0,\ \mathrm{s^+} \geq 0,\ \mathrm{s^-} \geq 0 \end{array} \tag{2}$$

where ρ is the technical efficiency, x_0 and y_0 are the input vector and output vector of each DMU, respectively, s_i^- means the input excess, and s_r^+ means the output shortfall.

Most studies that adopted DEA to evaluate the TFCE treat the carbon emissions as undesirable outputs or bad output; thus, the SBM model with an undesirable output can better reflect the essence of an efficiency evaluation [72]. If there are n DMUs, each DMU_j $(j = 1,\ 2,\ \cdots,\ n)$ has m inputs $x_j = (x_{1j},\ x_{2j},\ \cdots,\ x_{mj})'$, s_1 desirable outputs $y_j^g = \left(y_{1j}^g,\ y_{2j}^g,\ \cdots,\ y_{s1j}^g\right)'$, and s_2 undesirable outputs $y_j^g = \left(y_{1j}^g,\ y_{2j}^g,\ \cdots,\ y_{s1j}^g\right)'$. The production possibility set is defined as $P = \left\{\left(x,\ y^g,\ y^b\right) \,\middle|\, x \geq X\lambda,\ y^g \leq Y^g\lambda,\ y^b \leq Y^b\lambda,\ \lambda \geq 0\right\}$. The efficiency of this model is the solution to the following problem:

$$\begin{gathered} \min \rho = \frac{1-\frac{1}{\mathrm{m}}\sum_{\mathrm{i}=1}^{\mathrm{m}}\frac{\mathrm{s}_{\mathrm{i}}^{-}}{\mathrm{x}_{\mathrm{i}0}}}{1+\frac{1}{\mathrm{s}_1+\mathrm{s}_2}\left(\sum_{\mathrm{r}=1}^{\mathrm{s}1}\frac{\mathrm{s}_{\mathrm{r}}^{g}}{\mathrm{y}_{\mathrm{r}0}^{g}}+\sum_{\mathrm{r}=1}^{\mathrm{s}2}\frac{\mathrm{s}_{\mathrm{r}}^{b}}{\mathrm{y}_{\mathrm{r}0}^{b}}\right)} \\ \mathrm{s.t.x}_0 = \mathrm{X}\lambda + \mathrm{s}^- \\ \mathrm{y}_{ro}^{g} = Y^g\lambda - \mathrm{s}^g \\ \mathrm{y}_{ro}^{b} = Y^b\lambda - \mathrm{s}^b \\ \lambda \geq 0,\ \mathrm{s}^- \geq 0, \mathrm{s}^g \geq 0,\ \mathrm{s}^b \geq 0 \end{gathered} \tag{3}$$

where $s^-, s^b,\ s^g$ are all slacks. The boundary of ρ is $0 < \rho\ \leq\ 1$, when $\rho = 1$ this means the DMU is on the production frontier. In the carbon efficiency study, it becomes the target emission.

DEA does not require a specific form of a production function or strict assumptions and has fewer data constraints; therefore, many applications have developed in addition to the aforementioned model. We can identify three groups under this approach. First, several studies have applied the DEA method to make redefined indicators such as the Malmquist index [37,53]. The second approach is the meta-frontier DEA, developed to consider the heterogeneity problem, which would lead to biased estimates [51,73,74]. Third, using the directional distance function (DDF) introduced by Chambers et al. [75], researchers select the direction in which an inefficient DMU is projected onto the efficient frontier. This method provides a very flexible tool to evaluate efficiency and is called a super-efficiency DEA model [47,76]. In addition, some studies estimate by applying multiple approaches explained at the same time, and the basis of each approach is similar.

3.3. Stochastic Frontier Analysis

The frontier analysis is a common method to measure technical efficiency. DEA is one of the representative methods to estimate the technical efficiency, and another one is the stochastic frontier analysis (SFA). While DEA is the non-parametric form, SFA is the parametric form analysis; thus, in the view of the econometric approach, SFA has more statistical power than DEA by considering the measurement error and statistical random walk noise. SFA estimates the production or cost frontier from the data. The core concept of SFA suggested by Greene [77] is as the following:

$$\begin{gathered} y_{it} = \alpha_i + x_{it}\beta + e_{it} \\ e_{it} = v_{it} - u_{it} \\ v_{it} \sim \mathrm{N}\left(0, \sigma_v^2\right) \\ u_{it} \sim \mathrm{N}\left(0, \sigma_u^{2,+}\right) \end{gathered} \tag{4}$$

First, estimate the theoretic frontier, and decompose the error term into two components: idiosyncratic (random) error (v_{it}) and technical inefficiency (u_{it}). By doing this, SFA can avoid ascribing random errors to technical inefficiencies. The model assumes that the technical inefficiency term follows the half-normal distribution. Second, this technical efficiency is measured to the ratio of the predicted dependent variable when the technical inefficiency is zero, which refers to the frontier ($\hat{y}_{it}^{f}$) and observed dependent variable (y_{it}). Thus, the efficiency form is as the following:

$$efficiency_{it} = \frac{\hat{y}_{it}^{f}}{y_{it}} = exp(-u_{it}) \quad (5)$$

The equation can have various forms according to the function and production technology. For example, Sun and Huang [48] used the basic SFA model proposed by Battese and Coelli [78]. The other study proposed a method using the radial directional distance function followed by SFA techniques [79]. It also can be applied to estimate a parametric Malmquist index based on the fixed-effect panel SFA to analyze the dynamics of TFCE [80]. Like DEA, SFA can be varied by an econometric approach or the functional difference, but the core concept is the two-step approach explained above.

3.4. The Input–Output Method

The environmentally extended input-output model introduced by Miller and Blair [81] and Gao et al. [55] suggested new approaches to the carbon efficiency measurement. According to the studies, the direct carbon emissions (DCE) of the industry sector can be expressed as follows:

$$C_i = \sum_{k=1}^{n} C_{ik} = \sum_{k=1}^{n} (\theta_{ik} \times \varnothing_k)\ (i, k = 1,\ 2,\ \cdots,\ n) \quad (6)$$

where C_i is the DCE of the ith industry sector, C_{ik} is the total amount of DCE of the ith industry sector produced using $k = n$ the species of energy, θ_{ik} is the consumption of kth energy in the ith industry sector, and $\varnothing_k$ is the carbon emission coefficient of the kth energy.

Assume that the coefficient of DCE is E_i, which means the direct carbon emissions to obtain the output X_i of the ith industry sector. Then, the coefficient can be calculated by Equation (7):

$$E_i = C_i / X_i = \sum_{k=1}^{n} C_{ik} / X_i = \sum_{k=1}^{n} (\theta_{ik} \times \varnothing_k) / X_i \quad (7)$$

The coefficient matrix E is measured by extending the ith industry sector to all sectors. Then, according to the input–output method, the matrix is obtained as in the following:

$$\begin{gathered} C = EX = E(I - A)^{-1}Y \\ C/Y = E(I - A)^{-1} \\ F = E(I - A)^{-1} \end{gathered} \quad (8)$$

The left side of the third one in Equation (8) means the sum of the direct and indirect carbon emissions coefficients; therefore, if we set the embodied carbon emissions (ECE) as F, this means the coefficient matrix of all industry sectors.

Meanwhile, Gao et al. [55] used a non-competitive input–output model to distinguish the differences between a country's production and imported goods. In this case, the coefficient matrix of direct consumption is divided into two terms, namely, domestic goods and imported goods; thus, the domestic direct consumption coefficient matrix can be expressed as follows:

$$\begin{gathered} F = E\left(I - A^{d}\right)^{-1} \\ A^{d} = (I - M)A \\ M = \begin{bmatrix} m_{11} & 0 & \cdots & 0 \\ 0 & m_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{nn} \end{bmatrix} \\ m_{ij} = \frac{IM_i}{(X_i + IM_i - EX_i)},\ \left(i, j = 1, 2, \cdots, n;\ and\ i \neq j,\ m_{ij} = 0\right) \end{gathered} \quad (9)$$

where IM_i and EX_i are the imports and exports of the ith industry sector. In this Equation (8), $\left(I - A^d\right)^{-1}$ represents the Leontief inverse matrix excluding imports.

Then, we treat the industry sectors in an input–output table as a DMU, and each DMU has f inputs and m outputs. Suppose that the input variables are labor input (XL), capital stock input (XK), and energy input (XE), and the output variables are the industry added value (YG), and the ECE or DCE (YC). As we have already seen in the DEA model, the set of production possibilities p can be defined as $\mathrm{p} = \{(\mathrm{xl}, \mathrm{xk}, \mathrm{xe}, \mathrm{yg}, \mathrm{yc}) \mid \mathrm{xl} \geq \mathrm{XL}\lambda,\ \mathrm{xk} \geq \mathrm{XK}\lambda,\ \mathrm{xe} \geq \mathrm{XE}\lambda,\ \mathrm{yg} \leq \mathrm{YG}\lambda,\ \mathrm{yc} \geq \mathrm{YC}\lambda,\ \lambda \geq 0\}$. According to the DEA model, the frontier efficiency is differed.

As we can see from the above process, this approach is a mixture of input–output analysis and the existing frontier approach. The embodied concepts are significant in the view of intermediate production and consumption; however, the robustness of the mix of two different methodologies is not studied much yet and further study is needed in this field to become one of the representative methods to measure carbon efficiency.

4. Discussion

We have investigated the definitions and methodologies of carbon efficiency used in previous studies. The purpose of our study was to confirm the definition of carbon efficiency and solve the "no consensus problem" in this field. In this section, we discuss the carbon efficiency concepts that are misleading in various ways.

4.1. Various Definitions of Carbon Efficiency

In our study, we divided the definition of carbon efficiency used in the existing studies into three categories; however, we must consider that all the studies we investigated could not be considered a carbon efficiency study because of the carbon emissions efficiency concept that they defined within each study. The TFCE is the most common definition of efficiency from the theoretical background. The actual emissions compared to the best (frontier) emissions is the most consistent with the definition of carbon efficiency. Although the incidental problems that arise due to the input and output variables considered in the calculation of each emission, they fall into different categories as methodological problems, and not the definition itself; therefore, in research that analyzes the carbon efficiency, the definition should be considered a top priority rather than a methodology.

This does not mean that the use of other indicators cannot be considered a study of carbon efficiency. Depending on the purpose of the study, the use of relatively simplified indicators, such as carbon intensity, as a proxy of carbon efficiency also falls under the category of carbon efficiency research; however, since many factors must be considered but not reflected, the researcher must specify that it was used as a proxy. On the other hand, if the study uses a secondarily derived factor, such as the absolute emission value, or modifies the formula based on carbon intensity, it is necessary to distinguish it from the original carbon efficiency study.

One of the most common misuses of carbon efficiency is the use of terms such as carbon performance or carbon productivity. In most cases, these expressions are used when the equation of carbon intensity is modified according to the research purposes. For example, some studies have used the term "carbon intensity", even though they used a physical value such as the energy consumption for the denominator instead of a monetary value such as the GDP. This is not only against the definition of carbon intensity, but it is also difficult to say that it is a study of carbon efficiency. If researchers modify the indicator for a research purpose, they should mention it within a study with reasonable evidence. If necessary, they should use a different keyword instead of carbon efficiency to prevent confusion.

Meanwhile, when conducting a carbon efficiency study using the concept of embodied carbon efficiency, it is applied carefully. According to the proposed concept, the definition is consistent with the original carbon efficiency; however, in this case, it should be considered that different methodologies are used at the same time. In other words, in the view of the defined concept, it should be regarded as an extended type of TFCE; however, along with the methodological considerations, it is a relatively recently proposed method, and further research is needed to adopt it as a major calculation method.

The considerations for the definition of carbon efficiency are summarized as follows. Whether to use a single-factor indicator or a total-factor indicator as a concept of carbon efficiency corresponds to the question of how to define efficiency. In the cases of using a single-factor indicator such as intensity, the main purpose of the studies is to confirm the overall flow and form of carbon emissions in the economic system; however, this should be specified because it corresponds to the proxy of carbon efficiency rather than efficiency itself. In addition, it is necessary to prove its validity to use other indicators derived from the original definition.

In the case of the total-factor indicator, it was widely used to look at efficiency in consideration of changing technologies, relationships between other input and output variables, the effect of policy, or the comparison of entities. Because this defined concept is most appropriate to the meaning of efficiency, a study using this concept is the basis for a carbon efficiency study. Additionally, for additional transformation, reliable evidence must be presented. The input and output variables also have to be considered, but this needs to be addressed in a further study because it is related to methodology.

The extended concept presented is connected to a question related to the boundary of carbon emissions. Because it is a matter of determining whether to include the indirect emissions from intermediate goods in the carbon emissions efficiency, this can be a matter of another dimension; however, this is also a suggested concept, and should not be omitted for further research.

4.2. Research Directions for a Future Carbon Efficiency Study

In this study, we divided the definition of carbon efficiency into a single factor and a total factor. A single indicator uses one variable, namely, carbon intensity. The calculation of a single indicator is simple, and the number of data used for the calculation are small; therefore, the analysis is concise and intuitive, and easy to understand. In particular, carbon intensity has an advantage in comparing multiple countries based on reliable data published by international organizations such as IEA; however, carbon intensity only considers carbon emissions over economic production. In other words, it is a concept that does not take into account changes in both capital and labor, technology, or a change in industrial structure. A single factor indicator, such as the energy intensity, can be the proxy of carbon efficiency, rather than a comprehensive measurement of carbon efficiency. In studies that require the aforementioned advantages, if carbon emissions and production are the main concerns, studies using a single indicator are also meaningful, but in a strict sense, such studies cannot be considered as carbon efficiency studies.

The TFCE has strength in that it considers various variables simultaneously and takes into account the interaction between the variables. Because it is based on economic theory, the TFCF is also more persuasive and logically valid; however, the computation is complicated, and if the source of each variable is different, the reliability of the research may be lower. Nevertheless, the approach using the total factor corresponds to the definition of carbon efficiency. In the case of studying carbon efficiency in terms of policy, multiple variables are the basis for evaluation and estimation, although the variables used may vary depending on the purpose of the study; therefore, the researchers using TFCE must fully understand the complexity of the calculation method and secure the uniformity of the data sources.

Meanwhile, the methods mainly used in the measurement of carbon efficiency using the total-factor definition are the DEA and SFA. DEA has the strength that it does not require a specific production function. Additionally, it does not require the processing of dimensionless data before model construction [10]. On the other hand, SFA has more statistical power than DEA since DEA is a non-parametric methodology. Additionally, SFA has no assumption on the model but just a statistical part, whereas DEA has an assumption of the linear projection. SFA can take into account the change along with the time unlike DEA applied to cross-section data [7]. The researcher can then choose either DEA or SFA according to the purpose of the study.

The early literature on carbon efficiency focused on the measurement and confirmation of carbon efficiency itself. Recent carbon efficiency studies are developing in the direction of applying them to confirm the impact of specific industries, technologies, or environmental policies. The application method of carbon efficiency can be divided into two approaches. First, researchers try to make developments within the methodology or model. For example, in the DEA method, we can modify the constraints to overcome the inherent scale problem of carbon efficiency [45]. Being a non-parametric approach, a study using DEA can modify the constraints to adjust the relationship between variables. This means researchers can consider a method of changing the constraints according to the research purpose.

On the other hand, SFA can utilize the influencing factor existing in the equation. The relationship between the carbon efficiency and analysis target can be confirmed by putting variables related to the purpose of the study in the influencing factor. For example, to see the impact of urbanization on carbon efficiency, you can input the urbanization level as a variable in the influencing factor [48]. In another example, you can utilize the carbon regulation indicators variables to observe the effect of the regulation policy [13]. In addition, while using the existing measurement method, this approach using the forecasted value for input variables is also possible [6].

The second application method is to estimate the carbon efficiency by constructing a separate related model after measurement. For example, Zhou et al. [45] used the industry GVAR (global vector autoregressive) model to confirm the relationship between carbon efficiency and the construction industry. They used carbon efficiency as a variable for the industry GVAR model. Additionally, Pan et al. [82] built a dynamic model to judge the effect of environmental policy on the carbon emissions efficiency. Consequently, even if it is not necessarily a quantitative model, a model with an index is also possible [83]. In this way, researchers can develop their own research approach using multiple steps, but it would be challenging to build a straightforward research question and scope. Researchers should, then, carefully secure (or verify) a method's validity when applying a multiple-step approach.

4.3. Difference between Energy Efficiency and Carbon Efficiency

Even though there is a growing number of studies about carbon efficiency, it is often confused with energy efficiency and has not received enough research attention in its own right. According to the keyword clustering study, the result showed that the carbon emissions efficiency's cluster #2 was "energy efficiency" and that there was not enough differentiation between the two terms [4,14]. This confusing problem was investigated in the case of using single-factor indicators. For example, carbon intensity is the ratio of carbon emissions to GDP. We can replace the carbon emissions by the multiplication of the energy and the average carbon emissions. Then, the carbon intensity equation can be transformed into the multiplying form of the energy intensity and the average carbon emissions.

When using carbon intensity as a proxy of carbon efficiency, the energy intensity can represent most of the information in the carbon intensity according to the equation [24]; however, this is a contradiction of "using a proxy." Because they are connected on a formula and both intensities are the proxy of efficiencies, sometimes it is considered that this relationship also exists between the energy efficiency and carbon efficiency. What we should keep in mind is that this does not indicate the relationship between the efficiencies.

The improvement in energy efficiency may not lead to an increase in carbon efficiency and several studies prove this empirically using the TFCE indicator measured by the SFA method [7]. The results showed that the most energy-efficient country has inefficient carbon emissions, which means that the country could not be classified as being really energy efficient. Intuitively, energy efficiency has increased due to advances in technology, but the amount of carbon emitted in this process may increase or vice versa. That is, the energy efficiency and the carbon efficiency may increase or decrease in the same direction or may move in the opposite, and in this case, the efficiency needs to be calculated in its original equation, not in a proxy.

In addition, energy efficiency and carbon efficiency have different characteristics from the perspective of sustainable development. Assume the situation that the improvement of both efficiencies happens. An increasing energy efficiency can produce the same amount of output in a decreased input; however, this aspect may occur as an adverse effect, namely, the rebound effect, which means that the amount of energy used is increasing. Consequently, this effect should also be considered when achieving an energy efficiency goal for sustainable development.

On the other hand, an improvement in carbon efficiency means the carbon emissions reduction occurs under the same production. If this improvement occurs under all other conditions equally, there is no reason for a separate counter-benefit. This means that an improvement in energy efficiency does not directly lead to sustainable development, but that carbon efficiency can. Although the definition of efficiency can make it confusing, it is necessary to analyze each efficiency according to their characteristic. In this context, it is necessary to develop research focusing on the independent feature of carbon efficiency.

5. Conclusions

With the increasing concern about sustainable development under the climate crisis, the interest in carbon reduction has also increased. The environment and economics are two main dimensions that we must consider in achieving sustainable development. In line with this background, interest in study related to carbon emissions efficiency is also growing; however, most existing studies have focused on methodological applications in measuring the carbon emissions efficiency. Because of the lack of concern about the definition of carbon efficiency, several studies have misused the terminology and those misleading studies cause confusion in this field. This study was constructed to fill this research gap by conducting a review of the existing literature and making suggestions about the defined boundaries of carbon efficiency.

After investigating the previous study, we can divide the definition into three groups. The first involves single-factor indicators and the most common indicator used in this group is carbon intensity. The indicator is defined as the ratio of carbon emissions to GDP. Several studies changed the denominator from a monetary value to a physical value, but this type of study is difficult to define as a carbon efficiency study. Because the first group corresponds to a proxy of carbon efficiency on a strict basis, the variation of this proxy indicator can be the secondary proxy. Additionally, the variation usually focusses on the change of variables, which means the indicator has a different meaning from carbon intensity. The second group involves total-factor indicators which are the most commonly used definition. With the theoretical background, the definition of an indicator, namely, the ratio of target emissions to actual emissions, falls under the strictest definition. Lastly, the embodied concept is a relatively new approach. Additionally, it falls under the total-factor indicators category because the definition of this indicator is also the ratio of the target emissions to actual emissions. Their focus is on the actual emissions which contain the carbon emissions from intermediate consumption.

The novelty of this paper is the improvement of existing literature. First, we investigate the existing studies related to carbon efficiency. Additionally, we organized the literature using the definition and methodology. Through this process, we found the overall trend of extensive carbon efficiency studies. Second, we suggested the definition of carbon efficiency according to the purpose of the study. For example, if the purpose of the study is related to the derived point from carbon efficiency, we can use the single-factor indicator as a proxy for carbon efficiency, but if the study focuses on a strict carbon efficiency, such as a comparison with energy efficiency, then a strict definition of carbon efficiency must be used. Unless using the original definition of efficiency, the reliability of the analysis is debased. Finally, we confirmed each definition and example, and this can give direction to future studies. We also discussed a field that has research potential, such as embodied or extended concepts. At the same time, we provided a significant point related to the definition; therefore, even though the purpose of future research might be an improvement in the methodology, this study can form the basis.

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