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Implementing the Maximum Likelihood Method for Critical Gap Estimation under Heterogeneous Traffic Conditions

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Abstract: Gap acceptance analysis is crucial for determining capacity and delay at uncontrolled intersections. The probability of a driver accepting an adequate gap changes over time, and in different intersection types and traffic circumstances. The majority of previous studies in this regard have assumed homogeneous traffic conditions, and applying them directly to heterogeneous traffic conditions may produce biased results. Moreover, driver behavior concerning critical gap acceptance or rejection in traffic also varies from one location to another. The current research focused on the estimation of critical gaps considering different vehicle types (cars, and two- and three-wheelers) under heterogenous traffic conditions at uncontrolled crossings in the city of Peshawar, Pakistan. A four-legged uncontrolled intersection in the study area was used to investigate drivers' gap acceptance behavior. The gaps were investigated for various vehicle types: two-wheelers, three-wheelers, and cars. For data collection, a video recording method was used, and Avidemux video editing software was used for data investigation. The study investigated the applicability of the maximum likelihood (MLM) method to analyzing a vehicle's critical gap. MLM estimation results indicate that the essential critical gap values for car drivers are in the range from 7.45 to 4.6 s; for two-wheelers, the critical gap was in the range from 6.78 to 4.7 s; and for three-wheelers, the values were in the range from 6.3 to 4.9 s. At an uncontrolled intersection, the proposed method's results can assist in distinguishing between different road user groups. This study's findings are intended to be useful to both researchers and practitioners, particularly in developing countries with similar traffic patterns and vehicle adherence patterns at unsignalized intersections.

Keywords: traffic congestion; driver behavior; surveys; highway safety; critical gap estimation; risk assessment; maximum likelihood method

1. Introduction

Traffic congestion has become a threatening socioeconomic concern worldwide [1–3]. The hotspots for traffic congestion are usually urban road junctions, and effective control and management of traffic in these locations is essential for relieving congestion and improving safety [4]. Analysis of critical gap estimation is vital for capacity analysis of traffic networks. A plethora of studies have investigated the acceptance of critical gaps between pedestrians and vehicles at junctions having a stop or yield, or without controls. A specific motorist or pedestrian may accept or reject an open space under specific circumstances. Numerous research studies describe the critical gap as a value that determines whether a gap is acceptable. Instead of the critical gap, another study used the term critical headway, which is well-defined as "the duration in seconds below which a pedestrian or vehicle will not attempt to start crossing the roadway." Since the gap acceptance decisions of vehicles at unsignalized intersections, and those of pedestrians at unsignalized midblock crossings, are similar [5], it is important to review literature about both vehicle and pedestrian critical gap acceptance.

The critical gap is the most significant factor in the gap acceptance process. A responsible driver will accept all gaps that are larger than his crucial gap and reject all gaps that are smaller than it. The crucial gap is used to determine capacity at unsignalized intersections. The Highway Capacity Manual of the United States (HCM, 2000) defines the critical gap as the shortest interval between minor street vehicle arrivals during which a minor street vehicle can enter the intersection [6]. HCM (2010) defined critical headway as the minimum headway in a major traffic stream that will permit one small street vehicle to enter the intersection [7]. The phrase critical headway is often applicable to situations of uniform traffic; however, when there is a considerable variety in the operating traffic, as in the case of heterogeneous conditions, it is more appropriate to discuss critical gap rather than critical headway. Critical gap is a factor that cannot be measured directly in the field but is estimated based on rejected and accepted gaps. It can be assumed with confidence to lie between the maximum accepted gap and the rejected gap. When applied to heterogeneous traffic situations, such as those marked by a lack of lane discipline, a lack of movement priority, the forced entry of lower priority movements, zigzag crossings of the intersection area, etc., this estimating procedure becomes problematic. The varying static and dynamic properties of the operating traffic make estimation more difficult. The MLM approach, according to Miller (1974), better integrates realistic characteristics than the Ashworth Method, although it requires more effort with minimum efficiency advantages [8]. In a similar study of heterogeneous traffic conditions in India, Ashalatha and Chandra (2011) discovered a wide range in the critical gap values estimated using the prevalent approaches, with some of them giving unreasonably low values [9].

The majority of critical gap estimation methods proposed previously were created for homogeneous traffic conditions, and applying them directly to heterogenous traffic conditions may produce insignificant results [9]. Patil and Powar (2014) also reported that the values of the critical gap are smaller for Indian conditions than those of developed countries due to the aggressive driver behavior [10]. The analysis of the relevant literature showed that the MLM is the best technique for estimating crucial gaps. To be consistent with the criterion of having an acceptable and rejected gap as a pair for each driver, this solution, however, presupposes that drivers are homogeneous. This assumption greatly differs from actual driving situations and, due to aggressive driving style that is prevalent in developing nations such as Pakistan, the rejected gap is frequently absent. Conditions on local, provincial highways are more complicated due to a lack of lane discipline, disregard for traffic and priority laws, various vehicle characteristics, improper parking, inadequate geometric design of junctions, and improper road markings. An evaluation of critical gap estimation at uncontrolled four-legged intersections under heterogeneous traffic conditions has rarely been addressed in the local context. The critical gap plays an important role from a safety point of view at unsignalized intersections, particularly under heterogeneous traffic scenarios. Hence, it is worth studying the critical gap estimation for three different important modes of transportation in Pakistan using the maximum likelihood estimation method. The current research focused on estimation of critical gaps considering different vehicle types (cars, and two- and three-wheelers) under heterogenous traffic conditions at uncontrolled crossings in the city of Peshawar, Pakistan. Estimation of critical gap analysis was undertaken using the maximum likelihood estimation method. The critical gap so determined can be used to more realistically estimate the capacity of such a junction.

The remainder of the paper is organized in different sections as given below. Section 2 describes the related works, including the importance of critical gap estimation, methods previously used in this regard, and a summary of recent studies related to the topic. Section 3 describes the data collection and study area. Sections 4 and 5 present the proposed MLM method for critical gap estimation, and the results and discussion, respectively. Finally, Section 6 of this paper includes the conclusion and recommendations for future studies.

2. Related Works

2.1. Estimating the Critical Gap

The critical gap "*tc*" is the shortest time interval between major stream vehicles that is required for one minor stream vehicle to perform a move (see Figure 1). Critical gaps have varying values for potential drivers (some are excessively fast or unsafe, while others are slow or cautious), and they are based on the different kinds of movements, intersection geometry characteristics, and traffic scenarios. Because of this variability, the critical gap acceptance procedure is regarded as a stochastic process, with critical gaps acting as random variables. The estimation of critical gaps aims to determine values for the different variables and the parameters of their distributions that indicate normal driving behavior at the intersection under investigation.

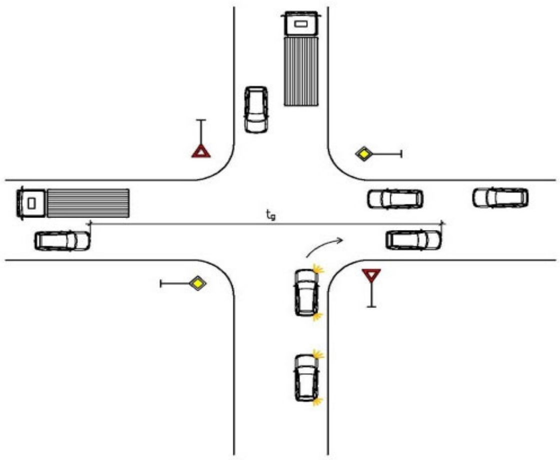


Figure 1. Typical schematic for understanding the critical gap for a right-turning vehicle from a minor road onto the main traffic stream.

In unsignalized intersection theory, drivers are required to be both homogeneous and consistent, and drivers are intended to behave in the same way in all of these conditions. This means that a driver with a specific "*tc*" value will never accept a gap lower than "*tc*" and will accept any significant stream gap more than *tc*. Within a population of numerous drivers who all behave consistently, different drivers may have different *tc* values. These *tc* values are then treated as random variables. *Ftc(t)* represents the cumulative distribution function and *ftc(t)* represents the statistical density function. The population of drivers is homogeneous if each sub-group of drivers in the population has the same functions *ftc(t)* and *Ftc(t).*

The problem is that critical gaps are difficult to quantify explicitly. Only the rejected and approved gaps of each minor stream vehicle can be measured at the intersection. The critical gaps can be predicted from such input data using statistical approaches or procedures. The maximum likelihood method (MLM) proposed by Troubeck et al. [11] was utilized to estimate critical gaps from vehicle sightings at unsignalized junctions in this study.

In developed countries, the traffic is homogeneous, with little distinction between the vehicles on the highways, and with passenger automobiles forming much of the traffic. These conditions are distinguished by the orderly movement of traffic, with users adhering to traffic laws. Road users adhere to the priority regulations at unsignalized crossings, which provide that lower priority movements must make way for higher-priority traffic.

By comparison, heterogeneous traffic conditions are typical in developing nations where slow- and fast-moving cars share the same road space [12]. The static and dynamic properties of the vehicles on the roadways vary greatly, making it difficult to predict how traffic will behave on the road. These are characterized by the lack of lane discipline and frequent violations of the law unless they are strictly enforced by a traffic officer [13,14]. At unsignalized intersections, drivers do not follow any priority norms and arbitrarily use the right of way. Minor street cars enter the region of conflict because drivers are so aggressive, which forces major street vehicles to slow down and provide a gap for minor street vehicles. As a result, the low mean allowed gap for the same vehicle type will be smaller under diverse traffic conditions than under homogeneous traffic conditions. Figure 2 shows the traffic composition for entering flow at a typical Pakistani crossroads.

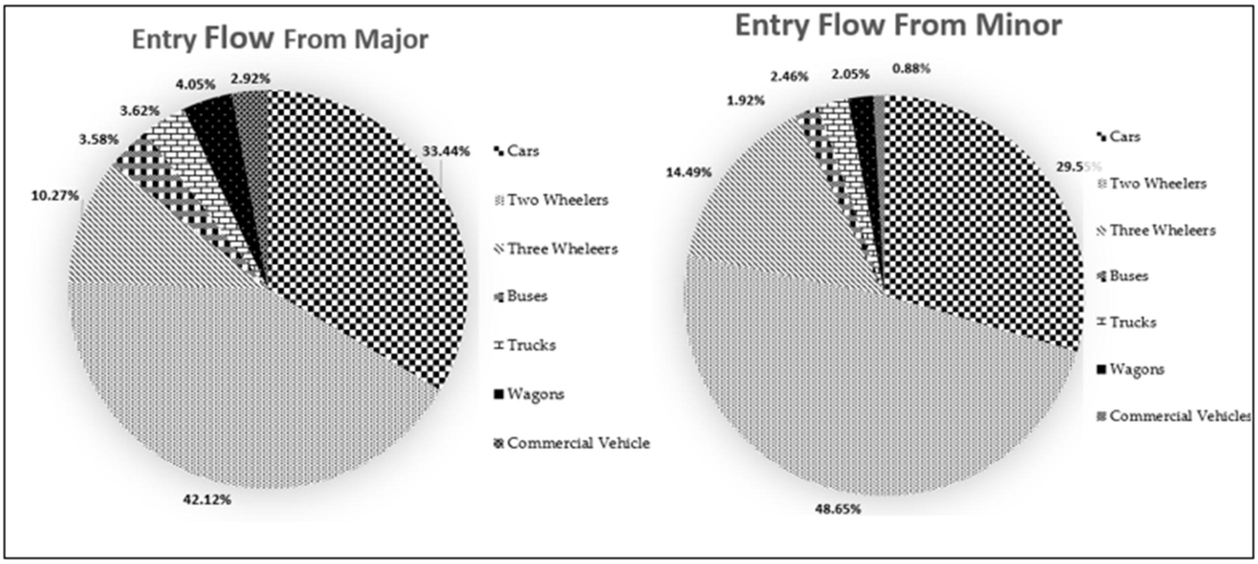


Figure 2. Percentage distribution of traffic mix in intersection entry flow from the study area.

2.2. Critical Gap Estimation Methods

In the literature, several methods for determining critical gaps have been proposed. The majority of these methods assume that driver's behavior is homogeneous and consistent. For example, one study [15] defined the critical gap as the distance for which the number of approved gaps shorter than it is equal to the number of rejected gaps longer than it. Other recent studies [16,17] calculated the average critical gap by combining the principal stream traffic volume, with the mean and standard deviation of accepted gaps. The authors of [18] and [19] proposed probit models. Recently, two research studies ([20] and [21]) employed binary logit and neural networks to investigate vehicle gap acceptance behavior at stop-controlled junctions. Logit and probit models are frequently used for modeling the relationships between a dependent variable Y and a set of independent variables X. In another paper [22], the authors developed a technique for estimating the probability distribution of critical gaps by combining accepted and rejected gaps. Tian et al. [23] employed the maximum likelihood method (MLM) to calculate a driver's crucial gap.

Hagrig (2000) employed MLM to estimate the critical gap at unsignalized intersections [24]. Tian et al. (2000) utilized a stepwise linear regression to determine the variables influencing the crucial gap and the follow-up duration at intersections [25]. Wu (2006) employed the probability equilibrium method based on the accepted and rejected gaps to estimate the critical gap [26]. To determine the likelihood of accepting or rejecting a gap or lag, Devarasetty et al. (2012) employed a binary logit model [27]. McGowen and Stanley (2012) suggested modifying MLM such that the process may be applied to data sets with only rejected gaps [28]. Wu (2012) used the probability equilibrium method to analyze the critical gap distribution and discovered that the Weibull distribution was superior at simulating the critical gap distribution than the log-normal distribution specified in MLM [29]. Miller (1972) compared numerous techniques for estimating crucial gaps and discovered that the MLM and Ashworth method provided appropriate results [30]. In another study, the researchers reported that the MLM method better combines realistic aspects than the Ashworth Method but requires more work with minimal efficiency gain [8]. Troutbeck (2014) examined MLM and PEM's capacity to forecast the mean and standard deviation of the crucial gap, and concluded that MLM was more accurate than PEM [11]. The studies in [31] analyzed many approaches, such as Lag, Raff, Ashworth, the logit technique, probit, and the maximum likelihood method (MLM).

Many other approaches estimate the critical gap based on the actual gap acceptance; however, HCM estimates are based on the adequate gap that is essential for pedestrians crossing. The Manual on Uniform Traffic Control Devices (MUTCD) [32] proposes the term "adequate gap", assuming it means the same thing as the critical gap in HCM. Recently, a study [33] determined the necessary gap based on walking speed, crossing length, and safety factors, which measures pedestrian confidence in crossing the street. Several approaches for studying pedestrian gap acceptance have been developed. In another study, the authors [34] developed a logistic regression model to investigate the impact of traffic gaps and several variables on pedestrians' decisions to cross, or not cross, the street. The findings revealed that the distance between approaching vehicles and pedestrian waiting times influence this decision. Another paper [35] proposed pedestrian gap acceptance (PGA) and motorist yield (MOY), which were modeled using different methodologies. They discovered that a pedestrian's decision is influenced by the vehicle's distance from the crossing and the vehicle's speed while modeling PGA. Shorter gaps were allowed by groups of pedestrians rather than individuals, according to research by [36], and the smallest acceptable gap in a single stream of vehicles was found to be 3.0 s or 75 feet.

2.3. Previous Studies

Mohan and Chandra presented a review of critical gap assessment techniques at two-way stop-controlled intersections [37]. It was reported that most of the included studies used the mean gap value, while a few also focused on the entire distribution of the critical gap. Further, the authors concluded that estimation methods for critical gap analysis are dependent on conflicting traffic volumes. In their study, Abhigna et al. investigated the effect of major street vehicle types on the gap acceptance behavior of minor stream drivers and calculated the capacity of uncontrolled urban intersections in Warangal city, India [38]. The authors also took into account the influence of right turning vehicles during the intersection capacity analysis. As a result, the total effect of traffic volume on the result was found to be negligible. Generally, the critical gap values are often employed in capacity and delay model estimation at intersections. Abhishek et al. adopted a queuing model incorporating driver impatience behavior and multiple classes of gap acceptance for estimating the critical gap distribution at unsignalized intersections [39]. The proposed method yielded useful results for determining the service time and, consequently, the capacity estimation on the minor road. The authors assumed that the arrival process on the major road is a Poisson process. In practice, nevertheless, a platoon may form on this road, and multiple articles have shown that this vehicle clustering would affect the capacity of the minor road. It would be interesting to integrate the framework presented in this study with Markov platooning.

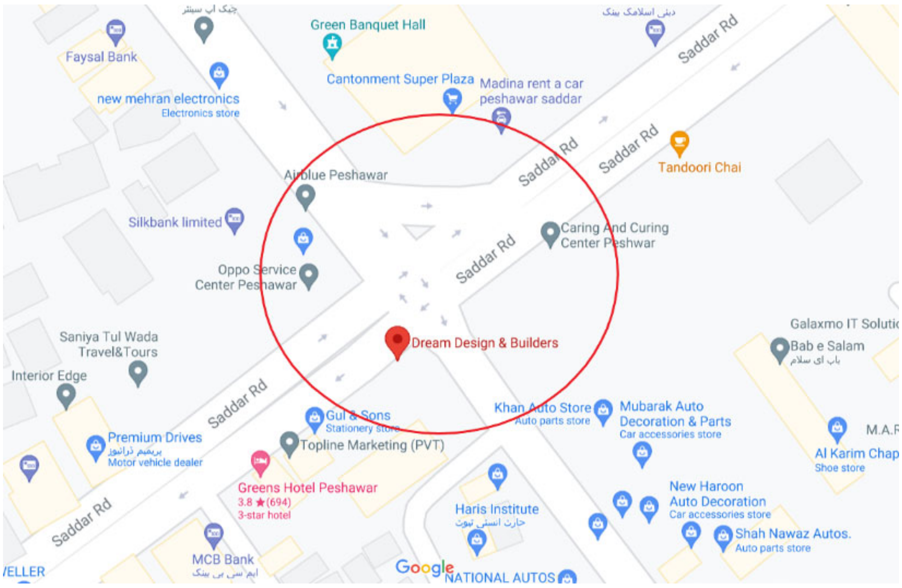
In another study, Adrian Barchański highlighted the importance of follow-up times and critical gap times for capacity analysis of unsignalized T-controlled intersections in an Upper Silesian agglomeration in Poland [40]. The findings for the specified object were unique and distinct from those of other intersecting types. Each object of the tested type managed a distinct type of traffic, and the drivers that utilized it exhibited distinct behaviors. To determine the values of critical gaps and follow-up periods that describe a particular type of intersection, considerably stricter conditions must be defined. In another study, Barchański and Zochowska attempted to evaluate the follow-up times and critical gaps at a median uncontrolled T-intersection (MUT) with major two-lane roadways in an Upper Silesian agglomeration area in Poland [41]. The empirical results obtained suggested that values for both the metrics (gap times and follow-up times) do not comply with corresponding values used in the HCM and Polish manual for capacity analysis of examined intersection types. Arasan and Koshy proposed a simulation-based methodology for modeling heterogenous traffic flow (comprising vehicles with a wide range of dynamic characteristics) in the absence of lane discipline under a mixed traffic flow [42]. Model validation using headway distribution and speeds was accomplished by data collection from the field. The findings of model validation indicate that the simulation model replicates the observed traffic flow characteristics in the field. Using a simulation model, it was determined that the new notion of the area occupancy is a legitimate metric that can be used to depict the concentration of road traffic under homogeneous traffic conditions.

Dutta and Ahmed also attempted to analyze and model minor street drivers' gap acceptance behavior, considering their aggressive nature [43]. The authors used data for an uncontrolled T-intersection in a northeast region in India. The authors argued that considering aggressive behavior and clearing time will yield more realistic gap acceptance behavior. In this study, only modest street right-turning movements at T-intersections were analyzed; the approach might be expanded to examine the gap acceptance behavior of major street right-turning vehicles. To obtain a deeper understanding of traffic behavior at uncontrolled crossings, it is possible to undertake research on four-legged intersections and the effects of other elements, such as geometric features, side friction, and driver characteristics. A recent study [44] determined the pedestrian critical gap using different methods. Study results revealed that the logit technique is the most appropriate for predicting critical gaps since it simultaneously analyses both pedestrian and vehicular factors. Significant changes were required to be made to projected critical gap values if pedestrian behavioral characteristics were considered. According to the latest study by [45], the pedestrian gap acceptance during midblock street crossings was found to be heavily influenced by the oncoming vehicle's speed and distance [46]. Pedestrians also accept lesser spacing when the opposing vehicle is smaller, such as a two-wheeler or an auto-rickshaw. This research can be used to evaluate the safety and performance of uncontrolled midblock traffic crossing in developing countries.

3. Study Area and Data Collection

3.1. Study Area

For analysis of the critical gap, a four-legged unsignalized intersection in the province of Khyber Pakhtunkhwa, Pakistan, was used. This four-legged non-signalized intersection on main Saddar Road Peshawar was selected to estimate the critical gap behavior of vehicles. This intersection was in an open field with reasonable visibility for all directions of travel. At intersections, there is no side friction, such as vehicle parking or bus stops. The selected four-legged unsignalized intersection is composed of a minor 14 m divided road, and a major 22 m divided road. Figure 3 illustrates the location and traffic composition at the study site. Because the chosen site is located on a major arterial road, the percentage of people using cars, two-wheelers, and three-wheelers was significantly higher than the percentage of people using other modes. Further, several distribution families were examined and it was discovered that the normal distribution best fit the measured speed data.



(**a**)

(**b**)

Figure 3. Plan view (**a**) and camera view (**b**) of the unsignalized intersection study area.

3.2. Data Collection

All the essential traffic and other supplementary data were collected from field surveys. The total traffic volume on main roads is between 1200 and 2500 vehicles per hour, while minor streams range between 600 and 1000 vehicles per hour. The average speed was discovered to be 52 km/h, with minimum and maximum speeds at the site measured to be 33 and 80 km/h, respectively. A video graphics survey was conducted to observe the movement of vehicles at the four-legged unsignalized intersection. An adjacent high-rise structure was used to mount two high-resolution video cameras. Data were obtained on December 2020. The information was gathered during the peak hours of a typical weekday (8 a.m to 11 a.m). Data on more than 350 trips were collected during the mentioned time frame for the selection of 200 random drivers, including both accepted and rejected gap criteria considered for this study The video camera was put up in such a way that it could record every movement of the cars. Because bicycles and heavy vehicles (trucks and buses) were rarely spotted approaching crossings on modest approaches, they were excluded from the analysis. Table 1 shows the descriptive statistics of all the study settings.

Table 1. Study setting and variable description.

Parameter/Variable	Description
Initial sample selection (total = 356)	2-wheelers (=133); 3-wheelers (=105); cars (=118)
Sample selected for analysis (satisfying threshold for accepted and rejected gaps, total = 200)	2-wheelers (=82); 3-wheelers (=71); cars (=47)
Survey period	Morning (including peak period)
AADT (major stream)	1200–2500 vph
AADT (minor stream)	600–1000 vph
Average stream speed	52 km/h
AVS video editor software	25 frames/s

The gap distribution was extracted and evaluated from the recorded data, and shown on a large screen using AVS video editor software (version 9.4.3) capable of processing videos at a frame rate of 25 frames per second. All major road vehicles were separated into three types (cars, two-wheelers, and three-wheelers), and only gap data for the right turn from a minor road was extracted. Among more than 350 trips, data was collected for a random sample of 200 drivers (for both accepted and rejected gaps) riding on different modes such as two-wheelers (82), three-wheelers (71), and passenger cars (47).

4. The Proposed MLM Method for Critical Gap Estimation

The maximum likelihood method (MLM) was adopted to achieve the research objective, i.e., the estimation of vehicle critical gaps under heterogeneous traffic conditions. The MLM method for estimating the critical gap is based on the driving population's critical gap values being distributed in a probabilistic distribution. Because different drivers will have varying perceptions of gap acceptance, a critical gap distribution is more appropriate than a single value. As a result, a single critical gap value will not be sufficient to capture the behavior of the whole driver population. Driver behavior varies stochastically with time, traffic circumstances, and intersection type while accepting the available gap [47]. The MLM can calculate the critical gap distribution, which ranges from the greatest rejected gap to the smallest accepted gap [48]. The MLM approach presupposes that the critical gap of a driver is larger than the largest rejected gap and smaller than the acceptable gap. Assuming a probability distribution for the crucial gaps is the first step, which is typically believed to be lognormal. For MLM, it is assumed that:

a_i = accepted gap by the ith driver in queue;

r_i = largest rejected gap rejected by the ith driver (r_i is assumed to be zero in the case where there is no rejected gap); and

$f(x)$ and $F(x)$ = probability density and cumulative distribution functions.

Critical gap distribution using MLM is defined by real positive values with one tail at either end. The log-normal distribution is typically assumed for modeling the critical gap distribution due to its simplicity and ease of calculations [49]. The total area under the curve must be 1 for a valid critical gap distribution. For the current study, data on the maximum rejected gap r_i (highest value amongst the rejected gaps) and the accepted gap a_i were observed for specific driver i (i = 1,2,3 ... n) riding on different modes and waiting in a queue on the minor leg/roadway to merge with the main arterial traffic. It was assumed that all the drivers will accept the gap value greater than the critical gap and vice versa. The MLM was then used to estimate the probability of critical gaps being between accepted gaps (a_d) rejected gaps and the largest rejected gap (r_d) for all drivers. The likelihood of vehicles' critical gap being between a_d and r_d was then computed using the following expression (Equation (1)):

$$LL^* = \prod_{i=1}^{n}[F(a_i) - F(r_i)] \tag{1}$$

The logarithmic of the likelihood is then calculated as (Equation (2)):

$$L = \sum_{i=1}^{n} \ln[(F(a_i) - F(r_i)] \tag{2}$$

Equation (2) can be used to obtain the mean and variance of the critical gap using the optimal solution. The two essential critical gap distribution parameters, mean (μ) and variance (σ^2), were determined by maximizing the above likelihood relation. *F()* represents the cumulative distribution function. After obtaining the values of μ and σ^2, critical gap values (t_c) were estimated using the relations presented in Equations (3) and (4).

$$t_c = e^{\mu + 0.5\sigma^2} \tag{3}$$

$$D(t_c) = E(t_c)^2.(e^{\sigma^2}) \tag{4}$$

5. Results and Discussions

Measurements were taken at an unsignalized intersection in Peshawar, Pakistan. All traffic flows at this crossroads were videotaped. Following that, video recordings were processed using the software. Each vehicle's time at a certain point (a specified line across the road) was recorded in an MS Excel spreadsheet. The passing time of a designated line across the road, the vehicle type, and the direction were all recorded for each mainstream vehicle. All the data were processed. For each minor stream vehicle, the accepted and greatest rejected gaps were identified (for each minor stream, separately). The biggest rejected time gap was equal to 0 s if a driver from the minor stream accepted the lag and did not reject any time gaps.

This study examined the use of the maximum likelihood technique (MLM) to predict critical gaps of vehicles (cars, and two and three-wheelers) to a minor right-turn stream at an unsignalized intersection in Peshawar (see Figure 3). First, data collection for 200 drivers was undertaken that included the maximum size of rejected gaps and the size of accepted gaps. For each driver, the maximum size of the rejected gap is always less than the size of the accepted gap. According to studies, gaps of more than 6 s are frequently accepted [9,50] (Figure 4).

Similarly, the least accepted gap is expected to be greater than 3 s, as the minimum perceptual reaction time is commonly assumed to be 2.5 s. In addition, the initial guesses for a and b were 0.5 s and 100 s, respectively. This overview is because a gap of less than 0.5 s cannot be accepted, and a gap of more than 100 s cannot be denied. The likelihood parameters, the mean μ and variance σ^2, were derived using an iteration procedure as a solution of two equations. The Excel application was used to program the iteration process. The values μ and σ^2 here were gradually modified until the functions in Equation (4) tended to zero. Equations (2) and (3) were used to calculate the critical gap's mean $E(tc)$ and variance D values. Using a data set of 200 drivers, the ideal values of parameters b and a for the critical gap distribution were found to be 7.45 and 4.6 s for cars, 6.78 and 4.7 s for two-wheelers, and 6.3 and 4.9 s for three-wheelers, respectively. As a result, the critical gap range was determined, and it can be inferred that a time gap of less than 4.6 s for a car is always rejected, while a gap of more than 7.45 s is always allowed. For two-wheelers, it can be inferred that a time gap of less than 4.7 s is always rejected, while a gap of more than 6.78 s is always accepted, whereas for three-wheelers it can be inferred that a time gap of less than 4.9 s is always rejected, while a gap of more than 6.3 s is always accepted. Furthermore, the parameters can be derived using the optimum values of *a* and *b* to determine the crucial gap distribution. Table 2 presents the mean and standard deviation of mean critical gap values for different road user groups. As shown in the table, drivers of two-wheelers require much smaller gaps to merge into the mainstream traffic compared to the other two groups. Table 2 also presents a comparative analysis of the obtained mean critical gap values with other methods widely used in the literature.

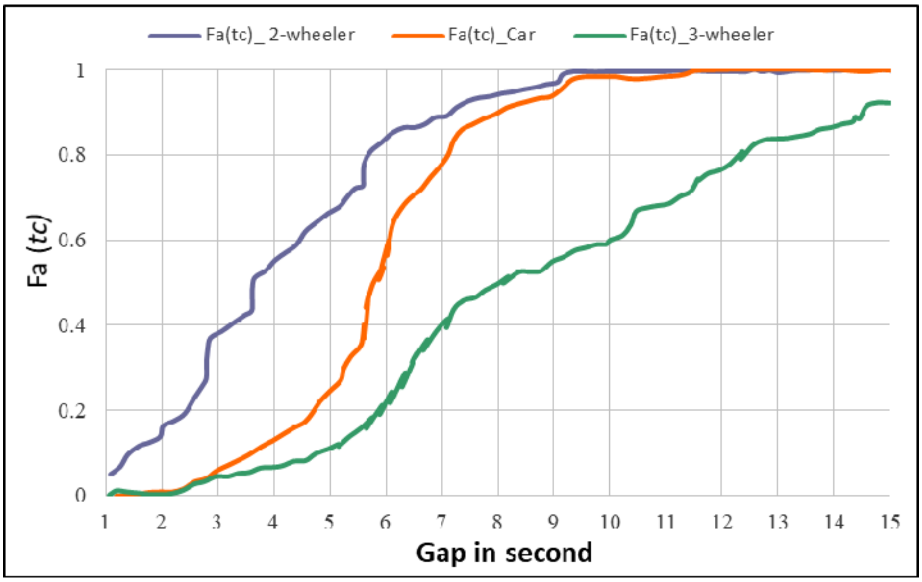


Figure 4. Distribution function of the critical gaps for different road user groups (two-wheelers, three-wheelers, and cars).

These values are much larger compared to the critical gaps estimated by other researchers. Maurya et al. estimated the critical gap for two- and three-wheelers and cars using different conventional methods [51]. Targeting the MLM method, X estimated the critical gap value for two- and three-wheelers and cars to be 2.65, 2.70, and 3.05 s respectively. In another recent study, Amin and Maurya compared the critical gap values for two- and three-wheelers and cars using nine differentmethods for both through movements and right-turn movements [52]. For through movements, the critical gap values vary between 2.30 and 4.80 s for two-wheelers, 2.25 and 4.65 s for three-wheelers, and 2.65 and 5.00 s for cars. By comparison, for right-turn movements, these values range between 2.20 and 4.70 s for two-wheelers, 2.20 and 4.50 s for three-wheelers, and 2.55 and 4.55 s for cars, respectively. The results obtained by these nine approaches are erroneous and thus cannot be applied under the conditions relevant to India and Pakistan, where the traffic is heterogeneous. In another study, the researchers used the Raff and clearing behavior methods for critical gap estimation of different vehicle types. Results revealed that the clearing behavior method yielded higher values for critical gaps compared to the Raff method. The critical gap values estimated using the clearing behavior method were 4.03, 6.53, and 7.69 (s) for two-wheelers, three-wheelers, and cars, respectively, which are close to those obtained by the MLM method in the current study for the corresponding vehicle types. A study conducted by Troutbeck et al. reported a mean critical gap value of 5.40 (s) for drivers merging from a minor stream, which is slightly lower than the average critical gap value of the road user groups considered in this study. A critical comparative analysis with previous studies (shown in Table 2) suggests that critical gap values of the current study are slightly larger those reported in the literature.

Table 2. Comparison of mean critical gap values (in seconds) for different road user groups from the literature.

Gap Estimation Method Used	Road User Groups		
	2-Wheelers	3-Wheelers	Cars
MLM (this study)	5.18 (1.55) *	6.39 (1.35) *	7.04 (2.13) *
Raff [52]	2.25	2.50	2.50
Logit [52]	2.70	2.90	3.10
Greenshield [52]	2.30	2.25	2.65
MLM [51]	2.65	2.70	3.05
Ashworth [51]	2.45	2.55	2.90
Clearing Behavior [51]	4.80	4.65	5.00
Raff [38]	2.75	4.05	4.80
Clearing Behavior [38]	4.03	6.53	7.69

* values in parenthesis shows standard deviation (SD).

6. Conclusions

This study investigated the possibility of applying the MLM to analyze the vehicle critical gap estimation for three different vehicle types for a case study of an unsignalized intersection in Peshawar, Pakistan. The MLM used was proposed by Troutbeck (2014), which is one of the essential models for calculating critical gaps, and yields realistic results. With the use of a video camera, a video graphic assessment was conducted in Peshawar city at a four-legged uncontrolled intersection. By studying the data, it was found that drivers act violently because they do not observe traffic rules, rather than because they have lost patience due to the lack of an appropriate space. For car drivers, the model found the essential critical gap to be in the range from 7.45 to 4.6 s; for two-wheelers the essential critical gap was in the range from 6.78 to 4.7 s; and for three-wheelers, the values were in the range from 6.3 to 4.9 s. At an uncontrolled intersection, the proposed method's results were proven to assist in distinguishing between cars, and two- and three-wheeled vehicles. According to the findings, city demographics and geographical characteristics play a significant effect in vehicle crossing maneuverers from minor to major streams. This study's findings are intended to be useful to both researchers and practitioners, particularly in Asian nations with similar traffic conditions and vehicle compliance patterns at unsignalized intersections. The future scope of research includes estimating the impact of various parameters (such as occupancy, age of the driver, sex of the driver, speed of oncoming traffic, size of oncoming vehicles, and number of rejections) on the critical gap parameter, particularly in the case of heterogenous traffic conditions. Furthermore, the performance of the MLM method for critical gap estimation may be compared with other widely used approaches, such as the Raff, logit, and clearing behavior methods in future studies.

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