



Article Does a Buyback Contract Coordinate a Reverse Supply Chain Facing Remanufacturing Capacity Disruption and Returned Product Quality Uncertainty?

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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). Abstract: This paper studies a two-echelon reverse supply chain (RSC) involving a remanufacturer and a collector, in which the collector receives the used products by paying a reward to consumers. The reward amount given to customers is crucial for encouraging them to exchange used products. An exchanged item is accepted if it meets the minimum acceptable quality level (AQL). Both the remanufacturing capacity and the quality of exchanged products present uncertainties. Under the buyback contract, the remanufacturer purchases used products at a higher price than in the decentralized and centralized cases from the collector. In return, the collector undertakes to repurchase a certain number of used products sold to the remanufacturer, but not remanufactured due to capacity shortages. Based on the aforementioned uncertainties, this study analyses channel coordination using buyback contracts and optimizes its parameters. By conducting a numerical analysis, we first ensure that under this contract, the risk of uncertainty is divided among the members, and that each party's profit is higher than when decisions are made individually. Therefore, a buyback contract would guarantee a win-win situation for both of the parties, and coordination for the RSC. A range of percentages of extra items purchased by collectors is derived, as well as the amount the collector pays for each item and the effect of increasing or decreasing these values is examined.

Keywords: reverse supply chain coordination; buyback contract; the remanufacturer's capacity uncertainty; quality uncertainty

1. Introduction

The reverse supply chain (RSC) involves collecting used products from consumers and returning them to the original manufacturers for remanufacturing [1–4]. RSCs have attracted the attention of many industries and researchers in recent years. Reusing used items not only saves money but also protects the environment, since remanufacturing these items reduces the consumption of natural resources [1,5,6]. Reverse logistics can also lead to a green network by valuing returned products, or destroying them properly [7].

Companies such as Dell (Round Rock, TX, USA) and Apple Inc. (Cupertino, CA, USA) provide the opportunity for consumers to exchange their used products while some other companies, such as Hewlett–Packard, offer a coupon for further purchases or free services to consumers in exchange for a used product [8,9]. Another example is Canon's Tone Cartridge Recycling program, which reduces the consumption of natural resources in the production of their product [10]. Moreover, the RSC as an eco-friendly process has been accepted in the plastic industry [11].

Implementing an effective RSC could be challenging, since most consumers are unwilling to return their used products since it takes a lot of time and they gain no benefit from bringing in their used items. As RSC implementation depends on this matter; there is a need to entice consumers to bring back their used products [12]. Therefore, the collector must consider a monetary reward. However, finding the optimal value of this reward, which is one of the most important decision variables in this study, can be challenging. A small reward leads to consumers not being willing to return their used products, and a large reward leads to less profit for the collector and the remanufacturer. Furthermore, when there are multiple sources of uncertainty in the remanufacturing process, decision making is more complicated since traditional capacity planning and optimization cannot be directly applied to RSC operations, and there is a need to modify existing models based on the new uncertainties. Each consumer behaves differently, so their usage of products and the quality of returned products vary. A number of companies offer gift cards as rewards for the exchange of used products. Before exchanging a product, some companies inspect it, while others accept any product. IKEA has implemented its Buy Back & Resell program in stores in nearly a dozen countries, as well as many U.S. states. Once the item is accepted by IKEA inspectors, store credit is awarded to the customer. REI has a trade-in program called Good & Used, in partnership with Yerdle. Customers receive a gift card after their gear, which they sent back to the company, is assessed. Considering that the quality of each returned item varies, the number of returned items and the reward paid to customers are uncertain parameters, and some items may not meet the minimum acceptable quality level (AQL). Therefore, the quality of the returned products is considered as an important factor [13]. Also, the remanufacturer's capacity is not deterministic, and some of the return items may not be remanufactured despite having the minimum AQL. A remanufacturer's capacity can be uncertain for a number of reasons, such as when a machine breaks down. This uncertainty becomes critical if added to the two major sources of uncertainty for remanufacturers (stochastic routing files and material replacement factors). [14]. When employees are absent or injured, it have a negative impact on their productivity, disrupting the production process. It is very common in the automotive and electronic industries to deal with these kinds of uncertainties [15].

The aforementioned uncertainties are reduced by using the contracts that link the RSC members and coordinate the RSC. Supply chain coordination is used as an effective approach to maximize supply chain profit and is achieved when independent entities of the SC work together and share resources and information [13]. Among all contracts, revenue sharing and buyback contracts are the most popular in the case of determining ordering policy and inventory management. For example, the revenue sharing contract is used by IBM's and HP's direct selling channels and their resellers [15]. Another case is Kawasaki motorcycle company, which shares its revenue with the retailers when it sells their accessories in the direct channel [16]. Previous research has shown that buyback and revenue sharing contracts can similarly effectively coordinate supply chain (SC) [16]. However, when the total revenue of the SC is not revealed for SC's members, implementing revenue sharing contract is not possible, and buyback contract is the better choice. Moreover, the payment method between members in these contracts is not the same. Although they can make the same profit for the chain, the sequence of payments within these two contracts are completely different. Specifically, they differ in the relative magnitude of the expected loss and gain (even though the sum of the loss and gain is equivalent) and the sequence of the potential loss (before or after demand is realized). These differences may stimulate a company to use a buyback contract instead of a revenue sharing contract. In previous studies, a revenue sharing contract was established to optimize the incentive given to consumers considering different uncertainties, such as uncertainty in the capacity of the remanufacturer and the quality of used products [5,14,17–19]. However, in the literature, using a buyback contract for this purpose is not evaluated. Furthermore, buyback contracts are far more used in many industries such as publishing, fashion, and high-tech; also, using buyback contracts is frequent when industries produce products with short life cycles such

as computers, books, and CDs [20]. Moreover, the buyback contract causes retailers to return unsold items at a typically lower price than the wholesale price [21]. For example, Wahmpreneur Books, which sells books to retailers and wholesalers, buys back books within 30 days of their purchase. In another case, McKesson, a health and beauty products distributor, offers retailers a return program [22]. Due to the applications of this contract in similar frameworks, its effectiveness in this problem is examined.

A two-echelon RSC involving a remanufacturer and a collector is considered in this study. Consumers are rewarded for returning used items that meet the minimum AQL by the collector. There are two sources of uncertainty in this RSC: the remanufacturing capacity and the quality of the exchanged product. Thus, to reduce the impact of these sources of uncertainty on RSC, an incentive will be offered to consumers and a buyback contract will be used to optimize this value. In some cases, products do not meet the minimum quality to be remanufactured and cannot be remanufactured. Consumers receive the incentive only if the exchanged products meet the minimum requirements. Furthermore, since the manufacturer's capacity for remanufacturing items is stochastic and limited, extra products cannot be remanufactured if the exchanged items exceed the manufacturer's capacity. In exchange for a specified price, the collector repurchases a certain number of extra products beyond the capacity of the remanufacturer. In return, the remanufacturer purchases the products at a higher price than the decentralized and centralized case from the collector. This study seeks to find a price policy that persuades consumers to return their used products and evaluate the impact of the buyback contract on the coordination of the studied RSC due to uncertainty regarding the remanufacturer's capacity and the quality of exchanged products.

This research aims to answer the following questions:

- 1. Considering the aforementioned uncertainties, what will be the optimal incentive to persuade consumers to return their used products?
- 2. Given the uncertainties, how is it possible to guarantee a win-win situation for the parties and a higher profit for the RSC and each party?
- 3. What is the optimal quantity and value of the repurchased items under the proposed buyback contract?

Three decision making scenarios are considered in order to answer these questions (i.e., centralized decision making, decentralized decision making, and coordination using buyback contracts). A decentralized RSC makes decisions independently, while a centralized RSC considers the total profit of the whole organization when making decisions. Finally, the model is formulated, and the optimal parameters of the buyback contract proposed from the collector to the remanufacturer are determined.

The proposed buyback contract enables the RSC to achieve its maximum potential profit, which is higher in the centralized case than in the decentralized case. Also, when the minimum AQL of the exchanged used products increases, the collector should use a buyback contract to compensate for the rise in its costs and increase the incentive offered to the consumers. Furthermore, the collector's decision is independent of the remanufacturer's capacity, and it is detrimental to the total profit of the RSC; however, the buy-back contract could be used to link members and increase total profit. On the other hand, when the remanufacturing capacity is high, the collector should propose a higher reward to consumers to convince more percentage of them to return their used products.

This paper is structured as follows. The literature on this subject is discussed in Section 2. The structure of the model, the assumptions, and the symbols are described in Section 3. Section 4 presents a model of the problem. An example is provided in Section 5. Section 6 explains managerial insights. Section 7 concludes by laying out future research directions and conclusions.

2. Literature Review

Buyback contracts, SC model uncertainty, and RSC coordination are all closely related to this study. We begin by examining the previous literature on coordination in RSC.

Afterwards, a literature review is conducted on buyback contracts. In conclusion, we primarily focus on papers that examine RSC models with uncertain parameters in order to demonstrate this work's relevance among other studies.

2.1. Coordination in RSC

Research on channel coordination in an RSC is explored in the first related research stream. If the SC is considered as a cooperative system, coordination across the SC provides joint success that allows the overall benefits of the SC to increase, since coordination can help establish smoother operations activities, and minimize total costs [23]. The literature widely refers to different types of contracts used in RSC coordination and their popularity [23–25]. Each firm uses a specific type of contract depending on its SC structure [23,26]. Govindan et al. [8] studied a revenue sharing contract to achieve coordination in a twochannel and three-channel RSC. In that study, the retailer sells returned products with a discounted fee to upstream and in return the total revenue of selling refurbished products is divided among RSC's parties. The retailer's discount influenced consumers' willingness to return used items. Khalafi et al. [27] considered a revenue sharing contract to maximize the profit for green perishable products. In another study, a combination of revenue sharing and expense-sharing contract were used to coordinate an SC, including a manufacturer, a remanufacturer, and a retailer [28]. Also, Bakhshi et al. [29] examined an RSC considering the remanufacturer's capacity as an uncertain parameter and coordinated the RSC using an option contract. In that study, the collector as the leader of the RSC, proposed an option contract that would allow two members to share the risk of the uncertain remanufacturing capacity and encourage the remanufacturer to order more while reinforcing the RSC surplus. Zhang et al. [30] investigated the effect of information asymmetry in an SC, on the contracts that are used in closed-loop supply chain (CLSC) coordination. Asl-Najafi et al. [31] considered a hybrid contract inspired by two-tariff and joint collection cost mechanisms to coordinate CLSC. Wu et al. [32] studied a dual channel RSC coordination problem, and they presented a revenue sharing contract to coordinate the recycling centers and third party recycler. Su et al. [33] considered a CLSC which is composed of a manufacturer, retailer and third party and they presented a cost-profit sharing contract to achieve coordination. Ding et al. [34] examined a three-stage, one-period SC and RSC that included a supplier, a manufacturer, and a retailer. By using appropriate contracts, a three-stage SC could be coordinated. Finally, Dutta et al. [35] used a buyback contract to coordinate a CLSC with uncertain demand and capacity. To encourage customers to return and sell their used products, the retailer offers a buyback price based on the quality and age of the returned items. Unlike previous studies investigating revenue sharing in RSCs [5,8,32], this paper considers an RSC involving a remanufacturer and a collector coordinated by using a buyback contract. A pricing and coordination strategy for GSC under two manufacturing modes was studied by [36].

2.2. Buyback Contracts

The second research stream examines buyback contracts and their application in various RSC links. In buyback contracts, the RSC member who is selling products to another member agrees to buy a certain amount of remaining products from another party at a specified price to share the risk of uncertain capacity. Most of the literature focused on buyback and return contracts, and investigated the returns implemented between manufacturers and retailers. Specifically, they assumed that the manufacturer exchanged both customer returned and leftover items [20,37–39]. In [38], the manufacturer has limited salvage capacity, while in [20], the manufacturer needs accurate information about customer returns. In [37], the manufacturer would buy back products only within a specified period, and the manufacturer in [39] deals with a demand which depends on refunds. In [40], the unsold products are returned to a manufacturer to overcome the impact of price sensitivity. On the other hand, investigation of buyback contracts in other links is limited. Huang et al. [41] explored returns in the retailer-collector links. Return contracts in the

remanufacturer-collector link were examined by [42], in which a remanufacturer would only buy back the products that can be remanufactured and hence guaranteeing its profit. Also, the supplier-manufacturer link was investigated by [43]. Furthermore, some papers compare buyback contracts with other contracts in reverse logistics. Su et al. [44] and Huang et al. [41] compared buyback contracts with rebate contracts. A comparison between buyback contracts and markdown contracts is conducted by [45]. Finally, a comparison between buyback contracts and wholesale pricing contracts is investigated by [46]. In this paper, a buyback contact in the remanufacturer-collector link is considered. Under this contract, the remanufacturer purchases the products at a higher price from the collector. In return, the collector guarantees to repurchase a specified amount of items that exceed the remanufacturer's capacity. Zokaee et al. [47] studied an inventory system with coordination among retailers and manufacturers considering buyback contract, carbon footprint and vertical integration.

2.3. Uncertainty in RSC

Almost every supply chain management problem involves some level of uncertainty [48–53]. Similarly, there is a higher risk associated with RSCs when there is uncertainty. Therefore, we are interested in classifying different uncertainty factors [54,55]. Chan et al. [56] developed a coordination mechanism in a distributed SC with quantity flexibility considering uncertainties in demand and supply. In a study, uncertainties in demand, supply, and technology are listed as the main sources of SC uncertainties [54]. Whereas, Liao et al. (2019) consider "parameter uncertainty", "background uncertainty", "CLSC model structure uncertainty" and "CLSC model result uncertainty" as the main sources. Researchers have mostly focused on simplifying the model using deterministic parameters, although, to get more accurate results, non-deterministic parameters have been used [55]. Factors such as demand, price, and cost are listed as common uncertain parameters which are studied in both RSC and CLSC models [55,57]. Pal et al. [58] considered uncertainty in demand and yield in both supplier and manufacturer, assuming that the manufacturer faces both the under-stocking and over-stocking risk. In another study, demand and yield were considered as uncertain parameters and the optimal shipment policy was determined by maximizing the average expected profit of the whole supply chain [59]. Jun et al. [60] considered recycling cost as an uncertain parameter in an RSC. Nativi et al. [61] considered uncertainties in demand, returns, and collection lead time in RSC. Furthermore, a revenue sharing contract under the remanufacturer's capacity uncertainty was used to coordinate an RSC [14]. Li et al. [62] considered uncertainty in remanufacturing yield and also in demand for remanufactured products to make remanufacturing and pricing decisions. Heydari et al. [5] considered a two-channel RSC in which the remanufacturer's capacity and the quality of the returned items are uncertain. Using a revenue sharing contract; they coordinated the SC. They concluded that the investigated revenue sharing scheme can be applied efficiently in the case of limited remanufacturers' capacity. Most of the previous literature has only considered the remanufacturer's capacity or quality of the returned products as an uncertain parameter [14,29,41]. However, in this paper, a buyback contract in the remanufacturer-collector link is examined considering the uncertainty in the remanufacturer's capacity and the quality of the exchanged used items.

2.4. Research Gap

In Table 1, a summary of the related literature is presented and compared with the current study. The classification is based on crucial topics in RSC coordination, and each one has a profound impact on it. In remanufacturing centers, different qualities of return products cause some problems. Managing remanufacturing centers is more challenging when product quality varies since remanufacturing processes, their costs, and their allocation of capacity are dependent on the quality of returns. If the quality of returned products is low, it will require a longer remanufacturing process; thus, the capacity of the remanufacturing centers should be increased in order to handle all returns. While

when returned products are of high quality, remanufacturing centers with less capacity are required to process them. There is, therefore, uncertainty in the quality of the return products, which affects the capacity of the remanufacturing centers. Several papers have examined the two above mentioned uncertainties [14], while other papers have examined RSC coordination with buyback contracts. There are differences between revenue sharing and buyback contracts in terms of payments, loss, and gain; therefore, buyback is preferred in some cases and many industries prefer buyback. In light of these uncertainties, RSC coordination using a buyback contract and determining the optimal parameters for the buyback is of utmost importance. While coordinating the RSC using a buyback contract, this research differs from existing studies by considering the aforementioned uncertainties. It is important to optimize the reward that is paid to customers in exchange for their products because it has a large impact on the RSC, since changing the reward amount changes the number of returned products and revenues. To the best of the authors' knowledge, there has been no previous research on optimizing this decision variable using a buyback contract under the aforementioned uncertainty parameters.

Table 1. Related literature on RSC.

Ref.	Demand	Recycling Price	Return Rate	Remanufacturer's capacity	Quality of returned products	Coordination Contract	Channel Coordination	Structure	Decision Variables
[60]		~				-	-	Manufacturer, Retailer and Recycler	Recycling price
[61]			~			Information- Sharing	~	Manufacturer-Supplier	Order from recycled-material or raw-material supplier
[39]	v					Buy-back	-	Retailer-Manufacturer	Refund amount, Order quantity
[42]	V					- Revenue sharing	~	Manufacturer-Distributor- Retailer	Optimal discount offered to the customers, discounted recycling fee Fraction of the total revenue shared
[41]					~	Buy-back	-	Retailer-Collector	Refund amount, Market price
[62]	v					- Dovonuo oborino	-	Collector Remanufacturer	Selling price Optimal reward offered to customers, Recycling fee,
[5]				v	v	Revenue sharing	v	Conector-Kemanulacturer	Share of revenue
[14]				~		Revenue- Sharing	~	Manufacturer-Supplier	Share of revenue
[32]						Revenue- Sharing	~		Offline recycling
[29]				~		Option Contract	•	Collector-Remanufacturer	Order quantity, Option amount, Option price, Exercise price
This Study				~	~	Buy-back	~	Collector-Remanufacturer	Optimal reward offered to customers, Fraction of products to buyback, buyback selling price

3. Problem Description

In this paper, a two-tier RSC involving a remanufacturer and a collector is examined. The remanufacturer remanufactures the used items collected by the collector, and sells the remanufactured products in the market. Consumers are encouraged to return their used items to the collector by gaining an incentive for each item. It is assumed that returned items have stochastic quality levels, and they follow a uniform probability distribution [14]; Therefore, a minimum AQL (*L*) is considered, and after investigating all of the returned used products, only the ones with the minimum AQL are accepted by the collector. These products are sold to the remanufacturer at a recycling fee w_r per unit. The remanufacturer once again inspects the products. Recyclable products are transferred to get remanufacturer's capacity is assumed to be stochastic due to machine breakdown, labour problems, etc. Due to this uncertainty, the remanufacturer sells some of the remanufactured items at a price *P* per unit, while other products that exceed the capacity are salvaged at a price *V* per unit in the secondary market. Figure 1 describes the model along with its specifications.



Figure 1. Structure of the examined RSC and material flow.

First, the optimal decision is analyzed for both centralized and decentralized scenarios. In the decentralized case, each member's decision is independent; therefore, the profit of RSC is not optimized, while the remanufacturer suffers from stochastic capacity's risk. On the other hand, in the centralized case, each member's decision is based on optimizing the total RSC profit; however, in the centralized case, more gain for both RSC participants in comparison to the decentralized case is not guaranteed. Hence, a buyback Contract is used to stimulate the collector to participate in the centralized case. In this contract, used products are sold to the remanufacturer at a higher price, and instead, a specified percentage of the used products that exceed the remanufacturer's capacity is repurchased by the collector at a specified price. In this manner, the risk of stochastic remanufacturer's capacity is shared between the RSC members.

Table 2 describes the notations as well as decision variables which are used in the proposed model.

Notations	Definition
Wr	Recycling price which is paid by the remanufacturer to the collector for each returned item (\$/unit)
C _{ci}	Inspection cost which is paid by the collector for each item (\$/unit)
C_{ch}	Holding cost which is paid by the collector for each item (\$/unit)
C _{cs}	Transportation cost which is paid by the collector to deliver each product to the remanufacturer (\$/unit)
α	A positive random variable, which indicates the quality of returned products with probability density function $g(x)$ and cumulative distribution function $G(x)$ in the range of [0,1]
L	Minimum AQL
а	Binary variable which shows if the collector accepts or rejects a returned product; if $L \le \alpha \le 1$ then $a = 1$, and if $0 \le \alpha \le L$ then $\alpha \le 0$.
Α	Total amount of used products that are accepted by the collector
Q	The total amount of used items which consumers have
φ	A variable which represents recovery rate, which is the ratio of exchanged items which are qualified to enter the process of the remanufacturing (if $(\varphi_0 + L < 1)$ then $\varphi = \varphi_0 + L$ and if $(\varphi_0 + L \ge 1)$ then $= 1$).

Table 2. Notations and definitions.

Table	2. Coi	nt.
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Notations	Definition
Р	The price which the remanufacturer sells each remanufactured item back to the consumers (\$/unit)
C _{ri}	Inspection cost which is paid by the remanufacturer for each item (\$/unit)
C_{rf}	Remanufacturing cost which is paid by the remanufacturer for each item (\$/unit)
C _{rd}	Disposal cost which is paid by the remanufacturer for each item (\$/unit)
С	A positive random variable, which indicates the remanufacturer's capacity; with probability density function function $f(.)$ and cumulative distribution function $F(.)$; mean = μ & standard deviation = σ
V	The salvage value at which each item is sold at the secondary market (\$/unit)
$ au_1$	A number indicating consumers' willingness to return their used items based on d_d . This number is a function of the incentive given by the collector to consumers; in the range of [0,1]
$ au_2$	A number indicating consumers' willingness to return their used items based on d_c . This number is a function of the incentive given by the collector to consumers; in the range of [0,1]
<i>d^{max}</i>	The maximum value of the monetary incentive at which all of the consumers are willing to return their used items
$E(\Pi_{sc})$	Expected profit of the RSC
$E(\Pi_r)$	Expected profit of the remanufacturer
$E(\Pi_c)$	Expected profit of the collector
Decision variables	Definition
d_d	The monetary incentive given to consumers from the collector for the returned products which have minimum AQL under decentralized case (\$/unit)
d_c	The monetary incentive given to consumers from the collector for the returned products which have minimum AQL under centralized case (\$/unit)
d_{BB}	The monetary incentive given to consumers from the collector for the returned products which have minimum AQL under buyback contract (\$/unit)
W _{BB}	A positive number indicating recycling fee of each used item having a quality equal or more than L under the buyback contract (μ)
Ŷ	Percentage of purchasing extra used items by the collector in the buyback contract
b	The amount paid by the collector for each extra used in the buyback contract (\$/unit)

To sum up, the proposed problem is to obtain the optimal values of the decision variables (monetary incentive) under three cases of decentralized, centralized and buyback contract.

4. Modeling

In this section, first the consumer's willingness to bring back products is determined and then the problem is modeled under the three aforementioned scenarios. The consumer's willingness to bring back products denoted as τ , is computed as below:

$$\tau_1 = f(d) = \begin{cases} \frac{d_d}{d^{max}}, & 0 < d_d < d^{max}, \\ 1, & d_d \ge d^{max}, \end{cases}$$
(1)

$$\tau_2 = f(d) = \begin{cases} \frac{d_c}{d^{max}}, & 0 < d_c < d^{max}, \\ 1, & d_c \ge d^{max}. \end{cases}$$
(2)

According to Equations (1) and (2), a linear relationship is assumed between willingness and reward. The value of d^{max} , which is the maximum reward given from the collector to the consumers is obtained using previous data. The brought-back products are either accepted or rejected based on their quality. According to Equation (2), if the quality of the product is more than an acceptable level of *L*, the collector accepts the product; Therefore, it will get a value of 1. Otherwise, the collector rejects the product, and it will be 0.

$$a = \begin{cases} 0 & 0 < \alpha < L, \\ 1 & L < \alpha < 1, \end{cases}$$
(3)

The expected total amount of used products that are accepted by the collector can be calculated based on τ_1 and τ_2 as:

$$E_1(A) = \tau_1 Q E(a) = \int_L^1 \tau_1 Q g(\alpha) d\alpha, \qquad (4)$$

$$E_2(A) = \tau_2 Q E(a) = \int_L^1 \tau_2 Q g(\alpha) d\alpha$$
(5)

As follows, Decentralized Decision Structure and Centralized Decision Structure which are analyzed. In the Decentralized case, members of the RSC make decisions without considering other members' interests, whereas in the Centralized case, the decision is made to maximize the RSC interest.

4.1. Decentralized Case

As mentioned, in this case, members decide by considering only their own interest. Therefore, the collector decides on the amount of the reward to maximize its profit, whereas the products are remanufactured based on the remanufacturing capacity. All returned used products are inspected by the collector at the cost C_{ci} per item. Among the inspected items, only the ones whose quality level is equal or more than L, are accepted. For these items, the reward d_d is paid to consumers per item. The returned products are sold to the remanufacturer at the price w_r per item. Hence, the collector's profit is a function of the reward d_d , the recycling fee w_r , the holding cost of C_{ch} , the inspection cost of C_{ci} , and the cost of C_{cs} . Therefore, the collector's profit function per unit of recycled product can be calculated as:

$$\Pi_{c}^{D} = \begin{cases} -C_{ci} & 0 < \alpha < L, \\ w_{r} - d_{d} - C_{ci} - C_{cs} & -C_{ch} & L < \alpha < 1, \end{cases}$$
(6)

And its expected profit can be formulated as:

$$E\left(\Pi_{C}^{D}\right) = \int_{L}^{1} \tau_{1}Q(w_{r} - d_{d} - C_{ci} - C_{ch} - C_{cs})g(\alpha)d\alpha - \int_{0}^{L} C_{ci}\tau_{1}Qg(\alpha)d\alpha,$$
(7)

where τ_1 is replaced from Equation (1) and expected profit can be calculated as:

$$E\left(\Pi_{C}^{D}\right) = \int_{L}^{1} \left(\frac{Qd_{d}}{d^{max}}(w_{r} - d_{d} - C_{ch} - C_{cs})\right) g(\alpha) d\alpha - (C_{ci}\frac{Qd_{d}}{d^{max}}).$$
(8)

Theorem 1. $E(\Pi_C^D)$ is concave in d_d . Therefore, the optimal amount of incentive that maximizes $E(\Pi_C^D)$ is:

$$d_d^* = \frac{-C_{ci} + \int_L^1 (w_r - C_{ch} - C_{cs})g(\alpha)d\alpha}{2\int_L^1 g(\alpha)d\alpha}.$$
(9)

Proof. From Equation (8) $\frac{\partial^2 E(\Pi_c^D)}{\partial d_d^2} = -\int_L^1 2 \frac{Q}{d^{max}} g(\alpha) d\alpha \leq 0$ is derived; hence, $E(\Pi_C^D)$ is concave in d_d . Therefore, the optimal amount of incentive that maximizes $E(\Pi_C^D)$ is calculated

from $\frac{\partial E(\Pi_{C}^{D})}{\partial d_{d}} = 0$, we have $\frac{\partial E(\Pi_{C}^{D})}{\partial d_{d}} = \frac{Q}{d^{max}} \left(-C_{ri} + \int_{L}^{1} (w_{r} - C_{ch} - C_{cs} - 2d_{d})g(\alpha)d\alpha \right) = 0$ and Equation (9) is obtained. This completes the proof. \Box

Corollary 1. d_d^* is strictly decreasing in C_{ci} and is strictly increasing in w_r .

Proof. From Equation (9) $\frac{\partial d_d^*}{\partial w_r} = \int_L^1 g(\alpha) d\alpha$ is derived which is ≥ 0 , and $\frac{\partial d_d^*}{\partial C_{ci}} = -1$, which is ≤ 0 . The proof is complete. \Box

When the cost of inspection and other costs of the collector increase, the profit of the collector will be lower; hence, the amount of reward paid to the consumer will be lower too. Also, when the w_r increases, the profit of the collector will be higher. Therefore, the amount of reward that is paid to the consumer will be higher as well. As mentioned, the products are remanufactured based on the remanufacturing capacity, but first, the returned products are once again inspected by the remanufacturer. Based on the item's quality, some of them are disposed of at the price C_{rd} , and the remainder are remanufactured. If the remanufacturer's capacity is less than the number of acceptable items ($\varphi E_1(A) > C$), C units will be remanufactured, and the rest will be bought in the secondary market at the price V. On the contrary, if the remanufacturer's capacity is enough for remanufacturing all of the acceptable items, ($\varphi E_1(A) \le C$), all of the items will be remanufactured. Based on the assumptions above, the remanufacturer's profit can be obtained as:

$$\Pi_{r}^{D} = \begin{cases} PC - C_{rf} - w_{r}E_{1}(A) - C_{rd}(1-\varphi)E_{1}(A) - C_{ri}E_{1}(A) + (\varphi E_{1}(A) - C)V & \varphi E(A) > C\\ P\varphi E(A) - C_{rf}\varphi E_{1}(A) - w_{r}E_{1}(A) - C_{rd}(1-\varphi)E_{1}(A) - C_{ri}E_{1}(A) & \varphi E(A) \le C \end{cases}$$
(10)

Moreover, the expected profit of the remanufacturer can be calculated assuming that the remanufacturer's capacity follows a normal probability distribution. Furthermore, we assume that the quality of used items follows a uniform probability distribution in the range of [0, 1].

$$E(\Pi_{r}^{D}) = \int_{0}^{\varphi E_{1}(A)} \left(\left(P - C_{rf} - V \right) C + \varphi E_{1}(A) V \right) f(c) dc + \int_{\varphi E_{1}(A)}^{\infty} \left(\left(P - C_{rf} \right) \varphi E_{1}(A) \right) f(c) dc - E_{1}(A) (w_{r} + C_{rd}(1 - \varphi) + C_{ri})$$
(11)

$$E(\Pi_{r}^{D}) = \int_{0}^{\varphi E_{1}(A)} \left(\left(P - C_{rf} - V \right) C + \varphi E_{1}(A) V \right) f(c) dc + \int_{\varphi E_{1}(A)}^{\infty} \left(\left(P - C_{rf} \right) \varphi E_{1}(A) \right) f(c) dc - E_{1}(A) (w_{r} + C_{rd}(1 - \varphi) + C_{ri}) \right) dc$$

In the decentralized case, the collector's decision is not affected by the remanufacturer. In fact, the collector will determine a load of returned used items by choosing the value of d_d .

4.2. Centralized Case

In this case, the RSC members try to maximize the total RSC profit. Under these conditions, the expected value of RSC's profit is written as:

$$E(\Pi_{sc}^{C}) = \int_{0}^{\varphi E_{2}(A)} \left(\left(P - C_{rf} - V \right) C + \varphi E_{2}(A) V \right) f(c) dc + \int_{\varphi E_{2}(A)}^{\infty} \left(\left(P - C_{rf} \right) \varphi E_{2}(A) \right) f(c) dc - E_{2}(A) (C_{cs} + C_{ch} + d_{c} + C_{rd}(1 - \varphi) + C_{ri}) - C_{mi} \tau_{2} Q.$$
(12)

Theorem 2. $E(\Pi_{SC}^{C})$ is concave in d_c . Hence, the optimal amount of incentive that maximizes $E(\Pi_{SC}^{C})$ is:

$$d_{c}^{*} = \frac{\int_{L}^{1} \left(C_{rf} + V - P\right) \varphi * F(\varphi E(A))g(\alpha)d\alpha - \int_{L}^{1} \left(C_{rd}(1 - \varphi) + C_{ri} - \left(p - C_{rf}\right)\varphi + C_{ch} + C_{cs}\right)g(\alpha)d\alpha) - C_{ci}}{\int_{L}^{1} g(\alpha)d\alpha}$$
(13)

Proof. From Equation (12), $\frac{\partial^2 E(\Pi_c^{BB})}{\partial d^2} = \left(\int_L^1 \frac{Q\varphi}{d^{max}}g(\alpha)d\alpha\right)^2 \left(V + C_{rf} - P\right) f(\varphi E_2(A)) - \int_L^1 \frac{2Q}{d^{max}}g(\alpha)d\alpha$ is calculated. Since $\frac{\partial^2 E(\Pi_c^{BB})}{\partial d^2} < 0$, $E(\Pi_{SC}^C)$ is concave in d_c , the optimal reward is calculated by solving $\frac{\partial E(\Pi_{SC}^C)}{\partial d} = 0$. We have $\frac{\partial E(\Pi_{SC}^C)}{\partial d} = \frac{Q}{d^{max}} \left(\int_L^1 (C_{rf} + V - P)\varphi\right) F\left(\varphi E_2(A)g(\alpha)d\alpha - \int_L^1 (C_{rd}(1-\varphi) + C_{ri} - (P - C_{rf})\varphi + C_{ch} + C_{cs} + 2d^C\right)g(\alpha)d\alpha - C_{ci}\right) = 0$. Therefore, Equation (13) is obtained. This completes the proof. \Box

Corollary 2. d_c^* is strictly decreasing in C_{ri} and is strictly increasing in *P*.

Proof. From Equation (12), we derive $\frac{\partial d_c^*}{\partial C_{ri}} = -\int_L^1 g(\alpha) d\alpha$, which is ≤ 0 , and $\frac{\partial d_c^*}{\partial P} = \int_L^1 \varphi g(\alpha) d\alpha - \int_L^1 \varphi * F(\varphi E_2(A)) g(\alpha) d\alpha$ which is ≥ 0 . This completes the proof.

In the centralized case, the purpose of the members is to increase the total profit of the RSC, while they are linked together. Therefore, if the cost of inspection by the remanufacturer increases, the collector should decrease d_c^* to compensate it, and also when *P* increases, the total profit of the RSC increases. Therefore, the collector can offer a higher incentive to the consumers. \Box

If d_c is used instead of d_d , the total profit of RSC will be higher, but using d_c cannot guarantee a higher profit for RSC members compared to the decentralized case. Therefore, proposing a contract to convince the RSC members to join the central decision making is vital.

4.3. Proposed Buyback Contract

If we want to persuade the RSC members to join in the centralized case, a contract that secures their profit should be used; therefore, a buyback contract is used for this purpose. Under this contract, the remanufacturer suggests a new recycling fee W_{BB} ; on the other side, the collector guarantee to buy *Y* percent of the remaining items at the price of *b* from the remanufacturer, which is higher than the selling price in the secondary market *V*. Therefore, the risk of the uncertainty of the remanufacturer's capacity as well as the risk of the quality of the exchanged items being uncertain is shared between the RSC members with this method.

The collector's expected profit function under the proposed contract is written as:

$$E(\Pi_{c}^{BB}) = \int_{L}^{1} \frac{Qd_{c}}{d^{max}} (w_{BB} - d_{c} - C_{ch} - C_{cs})g(\alpha)d\alpha - C_{ci}\frac{Qd_{c}}{d^{max}} - \int_{0}^{\varphi E_{2}(A)} (\varphi E_{2}(A) - C)Y(b - V)f(c)dc.$$
(14)

In the above formula, the first term shows collector's revenue from selling used items to the remanufacturer, and the next term indicates the cost of inspection paid by the collector for all of the received products, and the last term shows the collector's paying to the remanufacturer for remaining items under the buyback contract. The remanufacturer's expected profit function under the proposed contract is written as:

$$E(\Pi_{r}^{BB}) = \int_{0}^{\varphi E_{2}(A)} \left(P - C_{rf}\right) C + (\varphi E_{2}(A) - C)(1 - Y)v + Y(\varphi E_{2}(A) - C)b f(c)dc + \int_{\varphi E_{2}(A)}^{\infty} \left(P - C_{rf}\right) \varphi E_{2}(A)f(c)dc - E_{2}(A)(W_{BB} + C_{ri} + C_{rd}(1 - \varphi))$$
(15)

Theorem 3. Under buyback contract, the optimal recycling fee W_{BB} is formulated as:

$$W_{BB} = \frac{\int_{0}^{\varphi E_{2}(A)} (\int_{L}^{1} g(\alpha) d\alpha) Y(b - V) f(c) dc - \int_{L}^{1} (2d_{c} + C_{ch} + C_{cs}) g(\alpha) d\alpha}{\int_{L}^{1} g(\alpha) d\alpha}.$$
 (16)

Proof. From Equation (14), it is realized that $\frac{\partial^2 E(\Pi_c^{BB})}{\partial d^2} = -\int_L^1 2\frac{Q}{d^{max}}g(\alpha)d\alpha + \left[\int_1^L \frac{qQ}{d^{max}}g(\alpha)d\alpha\right] f(\varphi E_2(A))[Y(b-V)] \le 0$. Since we know that b > V. The RSC achieves coordination if the collector decides similar to the centralized scenario, i.e., d_c . $(d_c = d_{BB})$. Hence, $E(\Pi_c^{BB})$ is concave in d_{BB} , and the optimal value of d_{BB} is calculated using the below formula:

$$\frac{\partial E(\Pi_{c}^{BB})}{\partial d} = \int_{L}^{1} \frac{Q}{d^{max}} (W_{BB} - 2d_{BB} - C_{ch} - C_{rs})g(\alpha)d\alpha + \frac{C_{ci}Q}{d^{max}} - \int_{0}^{\varphi E_{2}(A)} \left(\int_{1}^{L} \frac{\varphi Q}{d^{max}}g(\alpha)d\alpha\right)Ybf(c)dc + \int_{0}^{\varphi (E_{2}A)} \left(\int_{1}^{L} \frac{\varphi Q}{d^{max}}g(\alpha)d\alpha\right)YVf(c)dc.$$
(17)

Hence, using Equations (14) and (17) a value for W_{BB} is found, which ensures that the collector decides like the centralized case under the proposed buyback contract. This completes the proof. \Box

We know that the remanufacturer engages in the proposed contract, if it gains more profit than the decentralized scenario. Hence, it is always required to have $E(\Pi_r^{BB}) \ge E(\Pi_r^D)$.

Theorem 4. $E(\Pi_r^{BB})$ is strictly increasing in Y and b.

Proof. W_{BB} is placed from Equation (16) in Equation (15) and $\frac{\partial E(\Pi_r^{BB})}{\partial Y} > 0$ is obtained, and also $\frac{\partial E(\Pi_r^{BB})}{\partial h} > 0$. \Box

We know that a decrease in *Y* and *b* leads to a decrease in the remanufacturer's profit. Hence, Y_{min} and b_{min} are formulated as:

Similarly, the collector participates in joint decision making, if its profit increases in comparison to the decentralized scenario. Hence, it is always required to have $E(\Pi_c^{BB}) \ge E(\Pi_c^D)$.

Theorem 5. $E(\Pi_c^{BB})$ is strictly decreasing in Y and b.

Proof. W_{BB} is placed from Equation (16) in Equation (14) and $\frac{\partial E(\Pi_c^{BB})}{\partial Y} < 0$ is obtained, and also $\frac{\partial E(\Pi_c^{BB})}{\partial h} < 0$. \Box

Obviously, by decreasing in *Y* and *b* the collector's profit decreases. Therefore, Y_{max} and b_{max} are formulated as:

$$Y_{max} = \frac{\int_{L}^{1} \frac{Qd_{d}}{d_{max}}(wr - d_{d} - C_{ch} - C_{cs})g(\alpha)d\alpha - C_{ci}\frac{Qd_{d}}{d_{max}} + C_{ci}\frac{Qd_{c}}{d_{max}}}{-\int_{0}^{\phi E_{2}(A)}(\phi E_{2}(A) - C)(b - v)f(C)dc}, -\frac{\int_{L}^{1} \frac{Qd_{c}}{d_{max}}(wbb - d_{c} - C_{ch} - C_{cs})g(\alpha)d\alpha}{-\int_{0}^{\phi E_{2}(A)}(\phi E_{2}(A) - C)(b - v)f(C)dc}$$
(20)

$$b_{max} = \frac{\int_{L}^{1} \frac{Qd_{d}}{d_{max}} (wr - d_{d} - C_{ch} - C_{cs})g(\alpha)d\alpha - C_{ci}\frac{Qd_{d}}{d_{max}} + C_{ci}\frac{Qd_{c}}{d_{max}}}{-\int_{0}^{\phi E_{2}(A)} (\phi E_{2}(A) - C)(Y)f(c)dc} - \frac{\int_{L}^{1} \frac{Qd_{c}}{d_{max}} (wbb - d_{c} - C_{ch} - C_{cs})g(\alpha)d\alpha + \int_{0}^{\phi E_{2}(A)} (\phi E_{2}(A) - C)(Yv)f(C)dc}{-\int_{0}^{\phi E_{2}(A)} (\phi E_{2}(A) - C)(Y)f(c)dc}.$$
(21)

Mathematically, it is not possible to prove $b_{min} \leq b_{max}$, and $Y_{min} \leq Y_{max}$. However, the numerical analysis indicates that these inequalities are always true. By using each value from the intervals $[Y_{min}, Y_{max}]$ and $[b_{min}, b_{max}]$ and by keeping the other two parameters constant, RSC can be coordinated. Furthermore, based on the selected values of *Y* and *b*, the optimal recycling fee under the buyback contract W_{BB} is obtained from Equation (16).

5. Numerical Analysis

The efficiency of the model is evaluated using numerical examples in this section. Table 3 specifies the values of variables and parameters of these numerical examples. These data are obtained from [8]. These values are scaled to be applicable to show the model's behavior and the proposed buyback contract. The datasets in the examined numerical example satisfy the assumptions required in the model and are consistent with previous literature.

Parameter	First Example	Second Example	Third Example
	500	400	450
C_{ci}	20	20	30
C_{ch}	30	50	40
C_{cs}	40	40	50
Q	1000	5000	3000
$arphi_0$	0.65	0.72	0.8
Р	1900	1500	1300
S	620	600	610
C_{rd}	20	20	30
C_{rf}	210	320	230
C_{ri}	200	260	210
d^{max}	210	200	180
L	0.3	0.2	0.1
$C \sim Normal(\mu, \sigma)$	Normal(Q/4, Q/12)	Normal(Q/3, Q/24)	Normal(Q/2, Q/12)

Table 3. Values of the parameters.

To calculate the parameters of the buyback contract, suitable values are selected for *b* and *Y* and then they are used to calculate the value of W_{bb} . The value of W_{bb} is used to obtain the minimum and maximum values of *Y* and *b*. By setting the initial *b* and the obtained value of W_{bb} a value from $[Y_{min}, Y_{max}]$ can be selected that guarantees the establishment of the contract. Also, by keeping the initial *Y* and W_{bb} , a value from $[b_{min}, b_{max}]$ can be selected, and the buyback contract can be established. In Table 4, the amount of profit of the collector and remanufacturer in decentralized and centralized mode and the proposed buyback contract, and the amount of variables such as d_c and d_d are specified. The results show that under decentralized decision making, the remanufacturer's profit is more than

the collector's profit $E(\Pi_r^D) > E(\Pi_c^D)$. Whereas in the centralized case, the remanufacturer's profit and the total profit of the RSC increases ($E(\Pi_r^C), E(\Pi_{sc}^C) \uparrow$), and the collector's profit decreases ($E(\Pi_c^C) \downarrow$). The remanufacturer's profit would be less than the centralized case by proposing the buyback contract $E(\Pi_r^C) > E(\Pi_r^{BB})$. However, it is still higher than the decentralized scenario $E(\Pi_r^D) < E(\Pi_r^{BB})$. The collector's profit under buyback contract is more than both decentralized and centralized cases $E(\Pi_c^{BB}) > E(\Pi_c^D), E(\Pi_c^C)$. Since d_{bb} (the reward given to consumers for their used products) is equal to d_c in centralized mode ($d_{bb} = d_c$), the total profit of the RSC under the buyback contract is equal to the centralized mode ($E(\Pi_{sc}^C) = E(\Pi_{sc}^{BB})$). This shows that coordination can be established in the RSC by establishing a buyback contract.

Table 4. Model performance under decentralized, centralized and buyback contract scenarios.

	$E(\Pi_c)$	$E(\Pi_r)$	$E(\Pi_{sc})$	d	W	W_{bb}	Y _{min}	Y_{max}	b_{min}	b _{max}	Ŷ	b
First example	-	-	-	-	-	-	-	-	-	-	-	-
decentralized	134,287.4	192,068.76	124,975.5	200.7	0.96	-	-	-	-	-	-	-
centralized	124,914.5	211,880.55	336,795.03	147.69	0.7	-	-	-	-	-	-	-
buyback	143,755.6	193,039.32	336,794.94	147.69	0.7	714.79	0.687	0.76	1193.54	1263.2	0.69	1200
Second example	-	-	-	-	-	-	-	-	-	-	-	-
decentralized	406,125	654,306.61	1,060,431.61	142.5	0.71	-	-	-	-	-	-	-
centralized	381,019.3	728,061.02	1,109,080.32	107.07	0.53	-	-	-	-	-	-	-
buyback	409,146.3	699,934.07	1,109,080.32	107.07	0.53	437.83	0.07	0.58	638.98	900.4	0.55	910
Third example	-	-	-	-	-	-	-	-	-	-	-	-
decentralized	400,166.7	410,700	810,866.67	163.33	0.91	-	-	-	-	-	-	-
centralized	376,772.8	476,003.04	852,775.82	123.84	0.69	-	-	-	-	-	-	-
buyback	433,532.8	419,243	852,775.81	123.84	0.69	506.19	0.45	0.93	1052.3	1526.69	0.55	1100

In Table 5, a sensitivity analysis was conducted on *L* the minimum AQL was changed since the quality of returned used items is a critical factor in this problem. The results show that the remanufacturer's profit increases by increasing the minimum AQL, since the products that are transferred from the collector to the remanufacturer have more quality. Therefore, the remanufacturer pays less for the disposal of non-recyclable items and can reproduce more. On the other hand, by increasing *L*, the amount of reward paid from the collector to the consumers in the decentralized mode decreases $(d^D \downarrow)$. By raising the minimum acceptable quality, the collector has to spend more to check the products whose quality is not confirmed. However, when the buyback contract is established, with the increase of *L*, the reward paid to the consumers also increases $(d^{BB} \uparrow)$, which guarantees more profit for the customer.

Another influential factor is the amount of the remanufacturer's capacity for reproduction, which was analyzed in Table 6 and examined the results. The results show that the change in the capacity of the remanufacturer does not affect the collector's decisions and profits in a decentralized mode. However, under the buyback contract scenario, when the remanufacturer's capacity increases ($C \uparrow$), the collector also increases the value of $d_c (= d_{bb})$ to buy more products from the consumers, which leads to more profit for the RSC. On the other hand, parameter *Y*, which represents the percentage of purchasing extra items by the collector, decreases, which leads to more profit for the collector reduces the reward paid to the consumers to decrease the number of products entering the RSC and prevents costs incurred by selling products beyond capacity in the second-hand market. Also, the results of Table 6 show that when the remanufacturer has limited capacity, the buyback contract is more effective than when there are no capacity shortages. _

Minimum AQL (L)	0.1	0.2	0.4	0.5
$E(\Pi_c^D)$	400,166.7	346,687.5	240,250	187,500
$E(\Pi_c^c)$	376,772.8	338,981.88	239,569.4	174,928.78
$E\left(\Pi_{c}^{bb}\right)$	433,532.8	349,729.15	242,528.6	192,500.68
$E(\Pi_r^D)$	410,700	582,499.99	599,249.8	512,011.78
$E(\Pi_r^c)$	476,003	598,451.58	604,107.2	592,060.81
$E\left(\Pi_r^{bb}\right)$	419,243	587,704.31	601,148	575,883.42
$E(\Pi_{sc}^D)$	810,866.7	929,187.49	839,499.8	699 <i>,</i> 511.78
$E(\Pi_{sc}^{c})$	852,775.8	937,433.46	843,676.5	766,989.59
$E\left(\Pi_{sc}^{bb}\right)$	852,775.8	937,433.46	843,676.6	768,384.1
d^D	163.33	161.25	155	150
$d^c = d^{bb}$	123.84	137.21	163.25	188.84
$ au^D$	0.91	0.895	0.86	0.83
$ au^c = au^{bb}$	0.69	0.76	0.9	1
w_{bb}	506.19	522.09	492.24	468.93
Y_{min}	0.45	0.71	0.92	0
Y_{max}	0.93	0.76	0.98	0.7
b_{min}	1052.3	1489.38	2117.34	0
b_{max} ,	1526.69	1551.91	2215.54	1991
Ŷ	0.55	0.75	0.95	0.5
b	1100	1100	1100	1100

Table 5. Model performance against the change in the AQL.

Table 6. Model performance against the change in the capacity of the remanufacturer.

$C \sim Normal(\mu, \sigma)$	Normal(Q/3, Q/9)	Normal(Q/3, Q/24)	Normal(Q/2, Q/9)	Normal(Q/2, Q/36)
$E(\Pi_{c}^{D})$	400,166.66	400,166.66	400,166.66	400,166.66
$E(\Pi_c^c)$	366,232.6	351,511.43	389,709.62	372,427.36
$E\left(\Pi_{c}^{bb}\right)$	413,808.96	422,904.85	415,627.78	416,836.35
$E(\Pi_r^D)$	178,937.27	180,699.99	409,744.06	410,700
$E(\Pi_r^c)$	257,922.57	278,086.68	444,143.73	483,095.41
$E\left(\Pi_r^{bb}\right)$	210,346.22	206,693.26	418,225.88	438,686.43
$E(\Pi_{sc}^D)$	579,103.93	580,866.65	809,910.72	810,866.66
$E(\Pi_{sc}^{c})$	624,155.17	629,598.11	833,853.35	855,522.77
$E\left(\Pi_{sc}^{bb}\right)$	624,155.18	629,598.11	833,853.66	855,522.78
d^D	163.3	163.3	163.3	163.3
$d^c = d^{bb}$	115.77	106.38	136.93	120.33
$ au^D$	0.9	0.9	0.9	0.9
$ au^c = au^{bb}$	0.64	0.59	0.76	0.67
w_{bb}	515.92	518.93	543.19	495.35
Y_{min}	0.1	0.05	0.71	0.12
Y_{max}	0.24	0.24	0.82	0.72
b_{min}	749.16	671.54	1503.29	796.51
b_{max}	925.75	909.27	1639.5	1676.02
Ŷ	0.2	0.15	0.75	0.5
b	1200	1200	1200	1200

In Table 7, the sales price factor was analyzed, and its effects on supply chain coordination and members' profits was examined. A case where the remanufacturer's capacity was limited was explored. The results show that a change in the selling price of the remanufactured product does not affect the collector's profit under the decentralized decision making and has little effect on the incentive given to the costumers in the contract mode. On the contrary, an increase in the selling price ($P \uparrow$) increases the remanufacturer's profit, but does not have a significant impact on the collector's profit, and the three parameters of the contract, W_{bb} , Y, and b) The high purchase percentage in P = 1900 is due to the high value of W_{bb} . If we set W_{bb} to the price range agreed in the contract at other selling prices, Y in P = 1900 decreases to a close percentage in other selling prices). Therefore, it can be concluded that in the case of capacity shortage, the remanufacturer can change the selling price of the remanufactured item, without worrying about its effects on the collector.

Price	p = 1700	p = 1800	p = 1900	p = 2000
$E(\Pi_{c}^{D})$	134,287.41	134,287.41	134,287.41	134,287.41
$E(\Pi_c^c)$	124,587.46	124,655.67	124,914.48	124,975.5
$E\left(\Pi_{c}^{bb}\right)$	138,565.25	136,497.15	143,755.62	138,184.9
$E(\Pi_r^D)$	142,062.39	167,065.57	192,068.76	217,071.93
$E(\Pi_r^c)$	162,230.51	187,152.02	211,880.55	236,809.85
$E\left(\Pi_r^{bb}\right)$	148,252.71	175,310.54	193,039.32	223,600.44
$E(\Pi_{sc}^{D})$	276,349.8	301,352.98	326,356.17	351,359.34
$E(\Pi_{sc}^{c})$	286,817.97	311,807.69	336,795.03	361,785.35
$E\left(\Pi_{sc}^{bb}\right)$	286,817.96	311,807.69	336,795.03	361,785.34
d^D	200.7	200.7	200.7	200.7
$d^c = d^{bb}$	146.77	146.96	147.69	147.86
$ au^D$	0.95	0.95	0.95	0.95
$ au^c = au^{bb}$	0.7	0.69	0.7	0.7
w_{bb}	553.97	555.74	714.79	557.54
Y_{min}	0.05	0.05	0.687	0.07
Y_{max} ,	0.13	0.14	0.76	0.15
b _{min,}	692.78	704.05	1193.54	718.88
b _{max} ,	814.94	825.19	1263.2	838.26
Y	0.1	0.4	0.69	0.12
b	1200	800	1200	1200

Table 7. Model performance against the change in the price of remanufactured products.

An analysis of different parameters, such as AQL, remanufacturer's capacity, and price, demonstrates that changing the value of the analyzed parameters could result in different outcomes and insights for decision-makers. The results indicate that the remanufacturer gains profit when the minimum AQL is raised, since the quality of the product entering the remanufacturing process is higher. As a result, the number of disposed of products decreases, which in turn reduces the remanufacturer's costs. Furthermore, by increasing the capacity of the remanufacturer, the collector increases the incentive paid to customers, thereby increasing the RSC's profit.

6. Discussion and Managerial Insights

Since most consumers are not willing to return used items without a monetary reward, finding the optimal monetary reward for RSCs is crucial. Additionally, decisions become more complex when the remanufacturer's capacity and the quality of the exchanged items are considered stochastic parameters. The last challenge is coordinating the RSC and convincing its members to participate in central decision making. A buyback contract in the manufacturer-collector link was examined to address these issues based on the uncertainty regarding the remanufacturer's capacity as well as the quality of the exchanged used items. Furthermore, the conditions for channel coordination were discussed. The results indicate that the risk of uncertainties between two members is fairly shared when using the optimal reward and buyback contract parameters. In industries such as fashion and electronics, where buyback contracts are preferred over other contracts when AQL is defined, and remanufacturer capacity is uncertain, these results can be helpful to decision-makers. Manufacturers and decision-makers can benefit from this research by following these tips:

- When the minimum quality of the exchanged used products increases, we advise the collector managers to increase the reward paid to the consumers and use a buyback contract to compensate for their costs.
- When the remanufacturer's capacity changes, the collector's profits, and its decisions do not change significantly. This could lead to a decrease in the total profit of the SC. In this case, the members of the RSC are advised to prevent this loss by establishing the buyback contract.
- When the remanufacturer's capacity is limited, the manufacturer can change the selling price of the products without worrying about its effects on the other member of the chain, i.e., the collector.
- When the remanufacturer's capacity is high, the collector should increase the reward paid to the consumers so that more used items can enter the SC and the total profit increases, for which a buyback contract can be used.
- In the decentralized case, the collector receives less profit than the remanufacturer. Therefore, the collector can specify a range of *Y* by offering a buyback contract to the remanufacturer for a specific *w*_{bb} and *b*, in order to achieve channel coordination conditions and increase his profit and the RSC compared to the decentralized case.
- Once the value of w_{bb} and b is specified, the more the Y value increases, the higher the remanufacturer's profit gets, and the more the Y value decreases, the higher the collector's profit gets.

7. Conclusions

Increasingly, industries are paying attention to RSC coordination and recycling products due to their environmental and economic benefits. This paper investigated an RSC with a remanufacturer and a collector by considering two sources of uncertainty: the remanufacturer's capacity and the quality of the exchanged used product. Two decision making scenarios were analyzed, including the case in which members make decisions independently and in which members make decisions centrally. To persuade members to participate in SC, a buyback agreement was used. In this contract, a percentage of extra items that exceed the remanufacturer's capacity would be repurchased by the collector at a specified price. The result of the numerical examples and the conducted sensitivity verified that the proposed buyback contract is helpful by creating a Pareto-improving situation, in which both members gain more profit.

We found that without the establishment of the buyback contract, when the minimum AQL of exchanged used products increases, the collector decreases the reward since it will have higher costs. As a result, consumers are less willing to return products. This can be avoided by increasing the reward paid to consumers, and the buyback contract can compensate the collector for its costs. It is also important to note that when the remanufacturer's capacity changes, collectors' profits and decisions do not change significantly. As a result, the SC's total profits can decrease, which can be prevented by offering a buyback contract. In addition, the collector should increase the incentives paid to consumers if the remanufacturer's capacity is high, so that more used items can enter the RSC and increase its profit.

To persuade the members of the RSC to join the central decision making, a buyback contract was considered. As an extension of this study, buyback contracts and other coordination contracts could be compared. It is also assumed that the w_{bb} is specified in the contract. The acceptable interval of the other parameter is obtained based on determining one of the two parameters of the contract. In future research, the acceptable interval for each parameter can be calculated first, and the value of the parameter will be calculated from these intervals, and the contract will be established. As another extension, the quality of the exchanged products could affect both the reward paid to the customer and the remanufacturing cost of the exchanged items. Furthermore, considering a dual-channel queueing-inventory model for collecting used items from consumers may be an interesting work in the future [63].

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