sustainability

MDPI

Article

Assessing the Sustainability of the Prepandemic Impact on Fuzzy Traveling Sellers Problem with a New Fermatean Fuzzy Scoring Function

Balasundaram Baranidharan [1], Ieva Meidute-Kavaliauskiene [2,*], Ghanshaym S. Mahapatra [1] and Renata Činčikaitė [2]

1 Department of Mathematics, National Institute of Technology Puducherry, Karaikal 609609, India
2 Research Group on Logistics and Defense Technology Management, General Jonas Žemaitis Military Academy of Lithuania, Silo St. 5A, 10332 Vilnius, Lithuania
* Correspondence: ieva.meidute@lka.lt

Abstract: This article focused on transportation sustainability during the COVID-19 pandemic situation under the Fermatean fuzzy environment. In both developed and developing nations, sustainability has risen to the top of the priority list for transportation policies and planning. We introduce a simplified presentation of the Fermatean fuzzy traveling seller problem solved by using a new computation approach. Several approaches for solving the traveling seller problem using fuzzy parameters have been described in the literature. Even so, all the current strategies use general fuzzy numbers as the parameters for the traveling salesman problems, but his study, focused on the new Fermatean fuzzy number, is more effective for representing real-life incidents. The Fermatean fuzzy scoring functions and numerical conditions in distinct models in the Fermatean fuzzy environment were described to construct the algorithm. New solution methodology developed through scoring functions to find the best solution to fulfill our goal of sustainable transportation for traveling sellers problem. Sustainable cost and the optimal path are obtained by this study.

Keywords: Fermatean fuzzy set; sustainable transportation; traveling salesman problem; score function; Hungarian method

check for updates

Citation: Baranidharan, B.; Meidute-Kavaliauskiene, I.; Mahapatra, G.S.; Činčikaitė, R. Assessing the Sustainability of the Prepandemic Impact on Fuzzy Traveling Sellers Problem with a New Fermatean Fuzzy Scoring Function. *Sustainability* **2022**, *14*, 16560. https://doi.org/10.3390/su142416560

Academic Editor: Bin Ji

Received: 26 September 2022
Accepted: 6 December 2022
Published: 9 December 2022

1. Introduction

The proposed study compares the effect of the pandemic on the employment of workers in other sectors and industries. Among the many factors that influence sustainable transportation, economic factors relate to business operations, work, and productivity; technical concerns relate to compliance with vehicle and traffic flow capacity; social relations relate to capital, global health and connectedness; and climate impact relate to pollution, climate change and habitat deterioration [1]. The traveling salesman problem (TSP) has piqued the interest of mathematicians and computer scientists owing to its simplicity in description and difficulty in solution. It was W.R. Hamilton, Mathematician from Ireland, who had first presented the TSP. This application is assumed that a numerical performance cost is assigned to moving a carrier from each old city to any new one, and that the best routing minimizes the total of the applicable expenses. The costs could, for example, be estimated times or lengths for various city-to-city trips, and the carriers could be tankers, pallets, freight cars, or other such vehicles. Many different approaches were developed to address the problem. This approach was developed in mathematics as a mathematical model of the TSP. The target of a problem is to determine the salesman's fastest route between a given city to each of the other cities, passing through every city just once and then back to that same starting city, as well as minimizing the cost of transportation. The assignment and transportation case problems are simultaneously marked differences from distribution problems [2]. Lancia & Serafini [3] provided a comprehensive historical overview of these and related problems in 1985. Zimmermann [4] has presented a linear programming problem optimal and efficient solution. Later, in their article on the generalized coverage of the salesman problem, Sayed et al. [5] proposed a new algorithm for fractional transshipment problem-solving. Quan et al. [6] proposed an insertion heuristic that includes the non-standard quality measure and crossing length percentage to resolve pickup and delivery problems with time windows. TSP is used to reduce the cost, travel distance, time, fuel, vehicle maintenance, and travel expenses for the salesman when he travels between locations. There are various methods for resolving problems with traveling salesmen, such as the Markov chain, simulated annealing, tabu search algorithm, and heuristic techniques such as cutting plane algorithms and the branch and bound method. Ant systems, neural networks, evolutionary computations, particle swarm optimization, artificial bee colony's and other algorithms are also available for solving the TSP.

2. Materials and Methods

Transportation problem (TP) is a worldwide phenomenon that has existed even before the introduction of the fuzzy set, and it has been a research interest for many researchers across the world. Still, its solutions are not suitable for real-life situations, because the conveyance systems always contain an uncertainty. As the problem of transportation (or) traveling person-related problems are solved under fuzziness, the solution is more accurate in representing the reality. Belman et al. [7] established the notation of decision-making problems with uncertainty in 1970. Transportation without accurate costs, supply, and demand is represented by Chanas et al. [8,9] through the fuzzy linear programming approach. The theory of fuzzy mathematical programming was proposed by Tanaka et al. [10]. Changdar et al. [11] proposed an efficient genetic algorithm for the multi-objective solid traveling salesman problem under fuzziness. The constrained solid TSP with a time window using an interval-valued parameter is solved using a genetic algorithm by Changdar et al. [12]. In the literature, many algorithms were there to find the solution for the fuzzy traveling salesman problem. A new approach for solving the TSP in purchasing concept is resolved in a quantum-inspired genetic algorithm by Pradhan et al. [13]. Feng et al. [14] proposed a hybrid evolutionary fuzzy learning approach that combines the benefits of adaptive fuzzy C-means, the short MAX-MIN merging idea, the simulated annealing learning algorithm, and a practical table transform-based particle swarm optimization. A fuzzy logic approach to solving the multi-objective multiple traveling salesman problems for multi-robot systems is resolved by Trigui et al. [15]. Many traveling salesman problems research have been conducted with and without sustainability. Sarkis et al. [16] presented a sustainable supply and production concept incorporating the COVID-19 pandemic situation.

The evolutionary transform-based algorithm are applied to optimize the traveling table, extracting the appropriate sequence codes for approaching the shorter traveling path. T–S fuzzy control of traveling wave ultrasonic motor is an extension of the traveling-based problem by Jingzhuo et al. [17]. This study suggests the suggested approach to handle the large-size TSP routing system. There are some extended traveling salesman problems applications in [18–22]. An effective revisiting algorithm for simultaneous localization and mapping using landmarks is presented by Hyejeong Ryu [23] to choose positions to revisit by taking into account both landmark visibility and sensor measurement uncertainty in TSP. Schiffer et al. [24] present integrated planning for electric commercial vehicle fleets: A case study for retail mid-haul logistics networks. Kazemzadeh et al. [25] proposed a new study during the pandemic time by electric bike (non) users' health and comfort concerns pre and peri world pandemic (COVID-19). Mainstreaming teleworking in a post-pandemic world is presented by Bojovic et al. [26]. Arteaga et al. [27] presented a credibility and strategic approach to hesitating multiple criteria decision-making with application to sustainable transportation. Considering the COVID-19 pandemic disruption, Mohammad et al. [28] presented a sustainable, resilient, and responsive mixed supply chain network design.

As an extension of Zadeh [29] concept of fuzzy sets, Atanassov [30] presented the idea of intuitionistic fuzzy sets in 1986. Since the situations have benefits and drawbacks, the membership and non-membership values of the intuitionistic fuzzy sets help define effectiveness, and it can be used to describe the situation's merits and shortcomings. Fischer et al. [31] suggested a dynamic programming solution for tackling the multi-objective traveling salesman problem. The time-dependent TSP using interval-valued intuitionistic fuzzy sets is optimized by Almahasneh et al. [32]. The definition of the intuitionistic fuzzy set is membership and non-membership values addition is always less than or equal one is the only possible solution, so we may not be able to present all the real-life problems by this intuitionistic fuzzy set. Therefore, Yager [33] introduced the Pythagorean fuzzy set.

The Pythagorean fuzzy set has a new class of non-standard fuzzy subsets. These non-standard fuzzy sets enable the specification of membership grades to account for uncertainty and imprecision: Pythagorean fuzzy set states the art and future directions presented by Peng et al. [34]. Different forms of fuzziness are presented in the literature [29,30,33], which are types of extensions of fuzzy sets. Therefore, we are not able to define some situational problems by the existing fuzzy sets. In 2018, Senapati et al. [35] presented a Fermatean fuzzy set (FFS). In this study, we are focusing on defining a new score function for the Fermatean fuzzy numbers (FFNs) and solving the traveling salesperson problem in a naturally existing problem. In literature, arithmetic operations, score functions, and some applications are already available [36–38]. FFSs have emerged as one of the most effective ways to address uncertainty and imprecision in various real-life concerns. As a result, the FFS environment is the main focus of the proposed work.

The significant contributions of this research work are as follows:

(i) In this study, we used the newly introduced FFS; in particular, all the parameters for the proposed model are considered FFS/FFNs.
(ii) In literature, Senapati et al. (2020) and Sahoo (2021) only represented score functions available for the Fermatean fuzzy defuzzification, so we propose a new score function and comparing with existing score functions.
(iii) We are framing a new model for solving the Fermatean fuzzy traveling seller problem (FFTSP); the model was used during the COVID-19 pandemic time to sustainably face traveling person problems.
(iv) We present a new methodology for solving the sustainable traveling person problem in Fermatean fuzzy environment based on pandemic impact.

This work contains five chapters. Section 1 deals with a detailed literature review of the traveling salesmen problem and their developments. Section 2 discusses the recent well-known fuzzy sets and their operations, then introduces a new score function and explained the existing score function. Section 3 describes a new model for the Fermatean traveling salesman problem and proposes a new mathematical procedure for solving the FFTSP. Section 4 deals with the case study and numerical solution of the FFTSP. Finally, a conclusion is presented in Section 5.

2.1. Basic Concepts

Atanassov's (1986) intuitionistic fuzzy set is a progression of the classic, efficient technique to handle uncertainty via a fuzzy set. It might mean the following:

Definition 1. *Intuitionistic fuzzy sets [30] is defined as objects with the form of a non-empty set Y.*

$$\widetilde{I} = \left(y,\ \widetilde{\mu}_I(y),\ \widetilde{\beta}_I(y)\ \middle|\ y \in Y\right) \tag{1}$$

where the functions $\widetilde{\mu}_I(y),\ \widetilde{\beta}_I(y)\colon Y \to [0,1]$ denote the degree of membership and non-membership of each element $y \in Y$ to the set $\widetilde{I}$ respectively, and $0 \le \widetilde{\mu}_I(y) + \widetilde{\beta}_I(y) \le 1$ for all $y \in Y$.

In 2013 Yager presented a new fuzzy set known as the Pythagorean fuzzy set. It might mean the following:

Definition 2. *A non-empty set Y, the Pythagorean fuzzy sets [34] is defined as objects of the form.*

$$\widetilde{\rho} = (y, \widetilde{\mu}_\rho(y), \widetilde{\beta}_\rho(y) \mid y \in Y) \tag{2}$$

where functions are $\widetilde{\mu}_\rho(y), \widetilde{\beta}_\rho(y) : Y \to [0,1]$ describe the degrees to which each element of membership and a non-membership in $y \in Y$ that set ρ, correspondingly, and 0, $\forall\, y \in Y$. For any Pythagorean Fuzzy set ρ and $y \in Y$,

$$\widetilde{\pi}_\rho(y) = \sqrt{1 - \widetilde{\mu}_\rho^2(y) - \widetilde{\beta}_\rho^2(y)} \tag{3}$$

It is known as the level of indeterminacy of y to ρ.

Currently, Senapati (2020) has introduced a new fuzzy set known as the Fermatean Fuzzy set, and it is shown by the following:

Definition 3. *Let Y be a discourse universe. A Fermatean fuzzy set $\widetilde{K}$ in Y is a form* [35] *of an item.*

$$\widetilde{K} = (y,\ \widetilde{\mu}_k(y), \widetilde{\beta}_k(y) \mid y \in Y) \tag{4}$$

with functions $\widetilde{\mu}_k(y), \widetilde{\beta}_k(y) : Y \to [0,1]$ indicate that condition $0 \leq \widetilde{\mu}_k^3(y) + \hat{\beta}_k^3(y) \leq 1$, $\forall\, y \in Y$. It's a number $\widetilde{\mu}_k(y)$ and $\widetilde{\beta}_k(y)$ indicates the degree of the element's membership and non-membership, respectively. y in the set k. Every Fermatean fuzzy sets $\widetilde{k}$ and $y \in Y$,

$$\widetilde{\pi}_k(y) = \sqrt[3]{\mathbf{1} - \widetilde{\mu}_k^3(y) - \widetilde{\beta}_k^3(y)} \tag{5}$$

It is known as the $\widetilde{\pi}_k(y)$ an Indeterminacy degree for the fuzzy set/number.

The symbol will be mentioned for the sake of simplicity $\widetilde{k} = \widetilde{\mu}_k(y), \widetilde{\beta}_k(y)$ due to the FFS $\widetilde{k} = \left(y,\ \widetilde{\mu}_k(y),\ \widetilde{\beta}_k(y)\right) : y \in Y$. We treat the Fermatean fuzzy numbers (FFNs) as the FFS's constituent parts for the sake of simplicity (Figure 1).

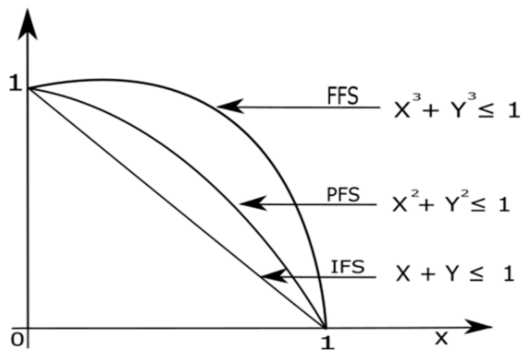

Figure 1. Representation of Intuitionistic, Pythagorean, and Fermatean Fuzzy sets.

Definition 4. *Let $\widetilde{\rho} = \left(\widetilde{\mu}_\rho(y), \widetilde{\beta}_\rho(y)\right)$ if there is a PFS $\widetilde{\rho}$. The scoring function is shown by $\widetilde{S}_{\rho(x)}$ is described as.*

$$\widetilde{S}_{\rho(x)} = \frac{1}{2} + \left(1 + \widetilde{\mu}_\rho^2 - \widetilde{\beta}_\rho^2\right) \tag{6}$$

Definition 5. *Let* $\widetilde{F} = \left(\widetilde{\mu}_f, \widetilde{\beta}_f\right)$, $\widetilde{F}_1 = \left(\widetilde{\mu}_{f_1}, \widetilde{\beta}_{f_1}\right)$ *and* $\widetilde{F}_2 = \left(\widetilde{\mu}_{f_2}, \widetilde{\beta}_{f_2}\right)$ *three FFSs across the universe* Y *and* $\partial > 0$*, then their fundamental operations in mathematics are predetermined to be* [35,37] *as follows:*

(i) Addition: $\widetilde{F}_1 \oplus \widetilde{F}_2 = \left(\sqrt[3]{\widetilde{\mu}^3_{f_1} + \widetilde{\mu}^3_{f_2} - \widetilde{\mu}^3_{f_1}\widetilde{\mu}^3_{f_2}}, \widetilde{\beta}_{f_1}\widetilde{\beta}_{f_2}\right)$

(ii) Multiplication: $\widetilde{F}_1 \otimes \widetilde{F}_2 = \left(\widetilde{\mu}_{f_1}\widetilde{\mu}_{f_2}, \sqrt[3]{\widetilde{\beta}^3_{f_1} + \widetilde{\beta}^3_{f_2} - \widetilde{\beta}^3_{f_1}\widetilde{\beta}^3_{f_2}}\right)$

(iii) Scalar multiplication: $\partial \odot \widetilde{F} = \left(\sqrt[3]{1 - \left(1 - \left(\widetilde{\mu}^3_f\right)\right)}, \left(\widetilde{\beta}_f\right)^{\partial}\right)$

(iv) Exponent: $\widetilde{F}^{\partial} = \left(\left(\widetilde{\mu}_f\right)^{\partial}, \sqrt[3]{1 - \left(1 - \left(\widetilde{\beta}^3_f\right)\right)}\right)$

(v) Subtraction: $\widetilde{F}_1 \ominus \widetilde{F}_2 = \left(\sqrt[3]{\frac{\widetilde{\mu}^3_{f_1} - \widetilde{\mu}^3_{f_2}}{1 - \widetilde{\mu}^3_{f_2}}}, \frac{\widetilde{\beta}_{f_1}}{\widetilde{\beta}_{f_2}}\right)$ .If $\widetilde{\mu}_{f_1} \geq \widetilde{\mu}_{f_2}, \widetilde{\beta}_{f_1} \leq \min\left\{\widetilde{\beta}_{f_2}, \frac{\widetilde{\beta}_{f_2}\pi_1}{\pi_2}\right\}$

(vi) Division: $\frac{\widetilde{F}_1}{\widetilde{F}_2} = \left(\frac{\widetilde{\mu}_{f_1}}{\widetilde{\mu}_{f_2}}, \sqrt[3]{\frac{\widetilde{\beta}^3_{f_1} - \widetilde{\beta}^3_{f_2}}{1 - \widetilde{\beta}_{f_2}}}\right)$ ·If $\widetilde{\mu}_{f_1} \leq \min\left\{\widetilde{\mu}_{f_2}, \frac{\widetilde{\mu}_{f_2}\pi_1}{\pi_2}\right\}, \widetilde{\beta}_{f_1} \geq \widetilde{\beta}_{f_2}$

2.2. Score Function

The score function is one of the tools for finding the appropriate crisp value from the fuzzified values. When real-life situations are defied through a fuzzy environment, it contains more imprecise data compared to a crisp environment. We divide the data into two: one focuses on the merits of the function, and another focuses on the part's demerits. The merits and demerits can define in the intuitionistic fuzzy set itself, but the lack of an intuitionistic fuzzy sets membership and non-membership functions additions is always $\leq$1. So, in the same way, the Pythagorean fuzzy set also has the same lacking, but the Fermatean fuzzy set did not have this disability. We consider the Fermatean fuzzy number for solving the fuzzy traveling salesman problem with a reliable background problem. Currently, two score functions are only available for the fuzzified value to convert into crispness. So, here, we explain the existing score function and propose here a new score function. Finlay, we compare those scores functions.

2.2.1. Existing Score Function

Let $\widetilde{F} = \widetilde{\mu}_f, \widetilde{\beta}_f$ if there is any FFS, then the scoring function for $\widetilde{F}$ is represented by $\widetilde{S}_f(x)$ is characterized as

$$\widetilde{S}_F(x) = \widetilde{\mu}^3_F - \widetilde{\beta}^3_F \tag{7}$$

The score value in the score function defined by Senapati et al. [37] lies between [−1, 1]. i.e., $\widetilde{S}_f(x) \in [-1, 1]$.

It should be highlighted that the function is positive when $\widetilde{S}_f(x) \in [0, 1]$. Likewise, when negative $\widetilde{S}_f(x) \in [1, 0]$. Most researchers have taken into account score functions whose score function values fall in the interval between 0 and 1 while rating FNs/FSs (either IFS or PFS). We have suggested a functioning mechanism for the score function of FFSs to keep everything the same.

See Sahoo et al. [36] for further information.

(i) (Type1)

$$\widetilde{S}_F(x) = \frac{1}{2}\left(1 + \widetilde{\mu}^3_F - \widetilde{\beta}^3_F\right) \tag{8}$$

(ii) (Type2)

$$\widetilde{S}_F(x) = \frac{1}{3}\left(1 + 2\widetilde{\mu}^3_F - \widetilde{\beta}^3_F\right) \tag{9}$$

(iii) (Type3)

$$\widetilde{S}_F(x) = \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3$$

The above score functions are available in the literature, so we are proposing a new score function for the Fermatean fuzzy numbers.

2.2.2. Proposed Score Function

An efficient score function takes the degrees of membership, non-membership, and hesitation into consideration simultaneously. For an FFN $\widetilde{F} = (\widetilde{\mu}_f, \widetilde{\beta}_f)$ We may use a voting model to explain the definition: $\widetilde{\mu}_f$ Encouragement, $\widetilde{\beta}_f$ Resistance and $\widetilde{\pi}_F$ Hesitate. The percentage of hesitant people who support and oppose is undetermined since they might be persuaded by supporters' and objectors' influence to support or object, respectively, but when individuals make decisions without thinking, it's simple to come out as having a herd mentality. In other words, when $\widetilde{\mu}_f > \widetilde{\beta}_f$, one group of individuals is more inclined to support when they hesitate; when $\widetilde{\mu}_f < \widetilde{\beta}_f$ A portion of those who are hesitant is more likely to be opposed. We thus consider the significance of the hesitating information when determining the score value. $\widetilde{S}_f(x) = \widetilde{\mu}_f^3 - \widetilde{\beta}_f^3$. The $\widetilde{\pi}_f$ positively impacts the score value $\widetilde{S}_f(x) = \widetilde{\mu}_f^3 - \widetilde{\beta}_f^3$ and makes $\widetilde{S}_f(\mathrm{x})$ increased when $\widetilde{\mu}_f > \widetilde{\beta}_f$. The $\widetilde{\pi}_f$ has an impact on the score value that is negative $\widetilde{S}_f(x) = \widetilde{\mu}_f^3 - \widetilde{\beta}_f^3$ and makes $\widetilde{S}_f(\mathrm{x})$ decreased when $\widetilde{\mu}_F < \widetilde{\beta}_F$. Following the S curve function's characteristics $f(x) = \frac{e^x}{e^x+1}$ and a new scoring function across the analysis-based and S curve function is defined as follows: For an FFN $\widetilde{F} = (\widetilde{\mu}_f, \widetilde{\beta}_f)$, its score function is defined as follows:

The proposed score function satisfies the score function (Senapati and Yager [37]) given below proposed score function is always suitable for all types of application problems because this function also provides a score within between $[-1, 1]$. The score function is,

$$\widetilde{S}_F(x) = \widetilde{x} = \left(\widetilde{\mu}_F^3 - \widetilde{\beta}_F^3\right) + \left(\left(\frac{e^{\widetilde{\mu}_F^3 - \widetilde{\beta}_F^3}}{e^{\widetilde{\mu}_F^3 - \widetilde{\beta}_F^3} + 1}\right) - \frac{1}{2}\right)(\pi)^3 \tag{10}$$

This research point of view, we frames the function in positive values in between [0, 1] $\widetilde{S}_F(x) = \sqrt{\widetilde{x}^2}$.

Definition 6. *Let* $\widetilde{F}_1 = (\widetilde{\mu}_{F_1}, \widetilde{\beta}_{F_1})$ *and* $\widetilde{F}_2 = (\widetilde{\mu}_{F_2}, \widetilde{\beta}_{F_2})$ *then*

(i) *If* $\widetilde{S}_{F_1}(x) > \widetilde{S}_{F_2}(x)$, *then* $\widetilde{F}_1 > \widetilde{F}_2$;
(ii) *If* $\widetilde{S}_{F_1}(x) < \widetilde{S}_{F_2}(x)$, *then* $\widetilde{F}_1 < \widetilde{F}_2$;
(iii) *If* $\widetilde{S}_{F_1}(x) = \widetilde{S}_{F_2}(x)$, *then*
  a. *If* $\widetilde{\pi}_{F_1} > \widetilde{\pi}_{F_2}$, *then* $\widetilde{F}_1 > \widetilde{F}_2$;
  b. *If* $\widetilde{\pi}_{F_1} < \widetilde{\pi}_{F_2}$, *then* $\widetilde{F}_1 = \widetilde{F}_2$;

Definition 7. *For any FFNs* $\widetilde{F}$ *is lies between scores*

(i) $-1 \le \widetilde{S_F}(x)(New) \le 1$
(ii) $\widetilde{S}_F(x) = 1$ *iff F = (0,1);* $\widetilde{S}_F(x) = -1$ *iff F = (0, 1).*

Example 1 ([36,37]). *If* $\widetilde{F}_1$ *= (0.90, 0.60) and* $\widetilde{F}_2$ *= (0.80, 0.75), then we have the following:*
For Type1: $\widetilde{S}_{F_1}$ *= 0.7565 and* $\widetilde{S}_{F_2}$ *= 0.5451 and hence,* $\widetilde{S}_{F_1}(x) > \widetilde{S}_{F_2}(x) \rightarrow \widetilde{F}_1 > \widetilde{F}_2$.
For Type2: $\widetilde{S}_{F_1}$ *= 0.7473 and* $\widetilde{S}_{F_2}$ *= 0.5340 and hence,* $\widetilde{S}_{F_1}(x) > \widetilde{S}_{F_2}(x) \rightarrow \widetilde{F}_1 > \widetilde{F}_2$.
For our Proposed score function: $\widetilde{S}_{F_1}$ *= 0.5741 and* $\widetilde{S}_{F_2}$ *= 0.1106 and hence,*

$$\widetilde{S}_{F_1}(x) > \widetilde{S}_{F_2}(x) \rightarrow \widetilde{F}_1 > \widetilde{F}_2$$

Therefore, we assert that the proposed score functions described here are reasonable because Senapati and Yager [36,37] *found that all scoring functions provide the same outcomes.*

Theorem 1. *The IFS is smaller than the PFS and FFS by the membership grades.*

Proof. Suppose that any point $\left(\widetilde{\mu},\ \widetilde{\beta}\right)$ in an FFS may or may not contain a PFS, and PFS having points may or may not be included in IFS. But all the number of $\widetilde{\mu},\ \widetilde{\beta} \in [0,1]$.

So, we get $\widetilde{\mu} \geq \widetilde{\mu}^2 \geq \widetilde{\mu}^3$ and $\widetilde{\beta} \geq \widetilde{\beta}^2 \geq \widetilde{\beta}^3$. Thus $\widetilde{\mu} + \widetilde{\beta} \leq 1, \widetilde{\mu}^2 + \widetilde{\beta}^2 \leq 1$ and $\widetilde{\mu}^3$
There are FFS not in a PFS and IFS.
Let us consider the $\widetilde{F}_1 = (0.9,\ 0.6)$ can able to define in FFS. But in the case of PFS and IFS,

$$\textbf{IFS}\widetilde{\mu}(x) + \widetilde{\beta}(x)\ 1 \to \textbf{PFS}\widetilde{\mu}^2(x) + \widetilde{\beta}^2(x) \nleq 1 \to \textbf{FFS}\widetilde{\mu}^3(x) + \widetilde{\beta}^3(x) \leq 1$$

$\widetilde{\mathrm{F}}_2 = (0.6,\ 0.5)$ it can able to define in FFS and as well as PFS but in the case of IFS,

$$\textbf{IFS}\widetilde{\mu}(x) + \widetilde{\beta}(x) \nleq 1 \to \textbf{PFS}\widetilde{\mu}^2(x) + \widetilde{\beta}^2(x) \leq 1 \to \textbf{FFS}\widetilde{\mu}^3(x) + \widetilde{\beta}^3(x) \leq 1$$

So, from the examples of the membership and non-membership grades, we conclude that IFS is smaller than the PFS and FFS. □

Theorem 2. *For a FFN $\widetilde{F} = (\widetilde{\mu}_f, \widetilde{\beta}_f)$, $\widetilde{S}_F(x)$ monotonically increases with respect to $\widetilde{\mu}$ and decreases with respect to $\widetilde{\beta}$.*

Proof. According to the Equation (10), we get the first partial derivative of $\widetilde{S}_F(x)$ with $\widetilde{\mu}$ and $\widetilde{\beta}$,

$$\frac{\partial \widetilde{S}_F(x)}{\partial \mu} = \frac{3\left(e^{2\widetilde{\mu}^3} + e^{2\widetilde{\beta}^3} + e^{\widetilde{\mu}^3+\widetilde{\beta}^3}\left(2+\pi^3\right)\right)\widetilde{\mu}^2}{\left(e^{\widetilde{\mu}^3} + e^{\widetilde{\beta}^3}\right)^2},\ \frac{\partial \widetilde{S}_F(x)}{\partial \beta} = -\frac{3\left(e^{2\widetilde{\mu}^3} + e^{2\widetilde{\beta}^3} + e^{\widetilde{\mu}^3+\widetilde{\beta}^3}\left(2+\pi^3\right)\right)\widetilde{\beta}^2}{\left(e^{\widetilde{\mu}^3} + e^{\widetilde{\beta}^3}\right)^2}$$

Since we have $\frac{\partial \widetilde{S}_F(x)}{\partial \mu} \geq 0,\ \frac{\partial \widetilde{S}_F(x)}{\partial \beta} \leq 0$.

Consequently, we can obtain that $\widetilde{S}_F(x)$ monotonically increases with respect to $\widetilde{\mu}$ and decreases with respect to $\widetilde{\beta}$. □

Theorem 3. *For a FFN $\widetilde{F} = (\widetilde{\mu}_f, \widetilde{\beta}_f)$, a new score function $\widetilde{S}_F(x)$ satisfies:*

(i) $-1 \leq \widetilde{S}_F(x) \leq 1$
(ii) $\widetilde{S}_F(x) = 1$ iff $F = (1,0)$; $\widetilde{S}_F(x) = -1$ iff $F = (0,1)$;

Proof. (i) According to the properties of S curve function, we can have

$$-\frac{1}{2} \leq \frac{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}}{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}+1} - \frac{1}{2} \leq \frac{1}{2}$$

Furthermore,

$$-\frac{1}{2}(\pi)^3 \leq \left(\frac{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}}{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}+1} - \frac{1}{2}\right)(\pi)^3 \leq \frac{1}{2}(\pi)^3$$

$$\widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 - \frac{1}{2}(\pi)^3 \leq \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 + \left(\frac{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}}{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}+1} - \frac{1}{2}\right)(\pi)^3 \leq \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 + \frac{1}{2}(\pi)^3$$

$$-1 = \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 - \frac{1}{2}(\pi)^3 \leq \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 + \left(\frac{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}}{e^{\widetilde{\mu}_F^3-\widetilde{\beta}_F^3}+1} - \frac{1}{2}\right)(\pi)^3 \leq \widetilde{\mu}_F^3 - \widetilde{\beta}_F^3 + \frac{1}{2}(\pi)^3 = 1$$

We can have $-1 \leq \widetilde{S}_F(x) \leq 1$.

(ii) According to Theorem 3, we can know that $\widetilde{S}_F(x)$ monotonically increases with respect to $\widetilde{\mu}$ and decreases with respect to $\widetilde{\beta}$.

Hence, $\widetilde{S}_F(x)$ can have a maximum value of 1 iff $\widetilde{\mu}_f = 1,\ \widetilde{\beta}_f = 0$ can have the minimum value -1 iff $\widetilde{\mu}_f = 0,\ \widetilde{\beta}_f = 1$.

That is to say, $\widetilde{S}_F(x) = 1$ iff F = (0,1); $\widetilde{S}_F(x) = -1$ iff F = (0,1). □

3. Development of Fuzzy Traveling Salesman Problem

In terms of friendly environmental transportation, the economic concerns are related to business operations, employment, and productivity. The technical problems associated with the adherence to flow and vehicle capacity standards on roads, the social concerns revolve along with environmental concerns, equality, public health, and inclusivity concerns could deal with mitigating pollution, degradation of the habitat, and climate change. Sustainability is considered in all factors of the transportation system. Therefore, this study focuses on the problems faced by the traveling sales person. During COVID-19, sellers suffered from selling their products. The seller travels from their home city (the city from where traveling started), simply making one trip to each place before returning economically to their home city (or) time used to cover the total distance. For example, given n cities and cost C_{st} (distances d_{st} or time t_{st}) from the city s to city t, the seller starts from city 1, then any permutation of 2,3,...,n represents the number of possible tour ways. Thus, there are (n-1)! possible ways of seller's time. Now the question is to select an optimal solution. Let us define the notations are,

- m: The total number of cities existing in the network,
- n: The total number of the destination city,
- s: The existing city index for all s,
- t: The destination city index for all t,
- x_{st}: The number of travels from one city to a designated city,
- $\widetilde{x}_{st}$: The fuzzy number of travels from one city to a designated city,
- $\widetilde{c}^f_{st}$: The Fermatean fuzzy cost for travel from s^{th} city to t^{th} city,
- c_{st}: The crisp cost of travel from s^{th} city to t^{th} city,

Mathematical formulation and how to solve this problem, let us define:

$$x_{st} = \begin{cases} 1 & if\ travels\ from\ \mathbf{s}\ to\ \mathbf{t}\ city, \\ 0 & otherwise. \end{cases} \tag{11}$$

Since each city can be visited only once, we have

$$\sum_{s=1}^{m} x_{st} = 1 \qquad t = 1, 2, \ldots n;\ s \neq t \tag{12}$$

Again, since the salesman has to leave each city except city n, we have

$$\sum_{t=1}^{n} x_{st} = 1, \qquad t = 1, 2, \ldots n;\ s \neq t \tag{13}$$

The objective function is this mathematical model is minimize the cost of transportation (c_{st}) with the prescribed conditions. Since $c_{st} = c_{ts}$ it is not required. It means that if a person visits one place, they no longer travel to the same city from the same city. Therefore $c_{st} = \infty$ for s = t. However, all c_{st} must be non-negative, i.e., $c_{st} \geq 0$ and $c_{st} + c_{tu} \geq c_{tu}$ for all s, t, u.

Let the price of travel to s^{th} the city t^{th} city be c_{st} and x_{st} = 1 when the salesperson travels straight from city s to city t, otherwise x_{st} = 0. Prior to finishing the tour of all cities, no city is visited again.They don't need to go from city s to s itself, in particular. By following the convention, this possibility might be prevented throughout the reduction procedure $c_{st} = \infty$ it guarantees x_{st} cannot ever be one.

Since the explained model can be expressed as

$$\text{Minimize } Z = \sum_{s=1}^{m} \sum_{t=1}^{n} c_{st}\ x_{st} \tag{14}$$

subject to constraints

$$\sum_{s=1}^{m} x_{st} = 1 \qquad t = 1, 2, \ldots .n;\ s \neq t\ \ s \neq\ m \tag{15}$$

$$\sum_{t=1}^{n} x_{st} = 1 \qquad s = 1, 2, \ldots .m;\ s \neq t\ \ s\ \neq\ n \tag{16}$$

$$x_{st} = 1 \ and\ \ x_{tt} = x_{ss} = 0.$$

The prescribed above model is the general model for the TSP. The objective of the problem is to determine the salesman's quickest route across each city, passing through it just once, from one city to all the others, and back to the starting city. Further, one of the included things was to minimize the traveling cost of whole transportation.

The problem stated from Equation (11) to Equation (16) is defined as a crisp environment that may not be suitable for all real-life problems of a changing nature and situations for the difficulties to give a convenient solution, but that is not sustainable for the environment and its factors. The following describes the preferred inputs and outputs to make the fuzzy TSP:

Flexibility in payments: Depending on the vehicle type, different vehicles have different availability, demand, and situation. Pay flexibility may make it easier to keep current clients and even draw in new ones.

Fuel availability: The marketability of new fuels is significantly impacted by the limited availability of energy. The switch to alternative fuels has not gotten much attention because petroleum fuels have dominated for a long time. It is the main thing for an average day in a pandemic; no petrol bunks are available.

Fuel economy: The rate of energy consumption provides a performance assessment of a vehicle that is more precise. Due to fuel availability issues, prices, and unpredictable fuel availability, transportation prioritizes fuel efficiency above all other factors because it provides the most significant driving force.

Providing goods in good condition: Ineffective logistics preparation may raise the rate of defective items, leading to a surplus of expenses, thus, improving operational effectiveness and cutting logistics costs are crucial factors.

Turnover of freight: Turnover of shipment is by dividing the travel distance by the weight of the package. The volume of the transportation load and the distance travelled affect the cargo turnover. Furthermore, the region's size, the geographic position of the resources, and businesses impact the freight shipping distance.

Suppose the model is considered in a fuzzy environment; it only represents the membership function based on a general fuzzy environment. It is also not more suitable for all real-life problems to give a suitable and sustainable solution. We are extending the TSP-based model for Fermatean fuzzy membership and non-membership factors to contribute to the environment.

Model-I: Fermatean fuzzy-based TSP

Let the Fermatean fuzzy cost of travel from $\widetilde{s}^{\,th}$ city to $\widetilde{t}^{\,th}$ city be $\widetilde{c}^{f}_{st}$ and $\widetilde{x}_{st} = 1$ when the salesperson travels straight from city s to city t and $\widetilde{x}_{st} = 0$ otherwise. Before finishing the tour of all cities, no city is revisited. They do not need to go from the city s to s itself, in particular. By following the convention, this possibility might be prevented throughout the reduction procedure $\widetilde{c}^{f}_{st} = \infty$ which ensures that $\widetilde{x}_{st}$ cannot ever be one.

The objective function is then

$$\text{Minimize Z} = \sum_{s=1}^{m}\sum_{t=1}^{n} \widetilde{c}^{f}_{st}\, \widetilde{x}_{st} \tag{17}$$

subject to constraints

$$\sum_{s=1}^{m} \widetilde{x}_{st} = 1 \qquad \text{t} = 1, 2, \ldots \text{n};\ \text{s} \neq \text{t}\ \ \text{s} \neq \text{m} \tag{18}$$

$$\sum_{t=1}^{n} \widetilde{x}_{st} = 1 \qquad \text{s} = 1, 2, \ldots \text{m};\ \text{s} \neq \text{t}\ \ \text{s} \neq \text{n} \tag{19}$$

$$\widetilde{x}_{st} = 1 \text{ and } \widetilde{x}_{ss} = \widetilde{x}_{tt} = 0.$$

The Proposed model is more appropriate for solving sustainable TSP in a pandemic. This model is economically sustainable for the economic sustainability input factors are flexibility in payments, fuel availability, and Infrastructure needs. The outputs for these factors are fuel economy, vehicle reliability, percentage of orders delivered without damage, and freight turnover.

Procedure for Solving the Fermatean Traveling Seller Problem

The considered situation is adopted into the mathematical problem, and the problem is modified into a Model: I. Then, the process for solving the formulated TSP is derived below:

Step-1: The traveling seller problem, which is created, requires that the cost or the duration be in Fermatean fuzzy numbers.

$$\begin{pmatrix} \infty & \left(\widetilde{\mu}_{c_{12}}, \widetilde{\beta}_{c_{12}}\right) & \left(\widetilde{\mu}_{c_{13}}, \widetilde{\beta}_{c_{13}}\right) & .. & .. & \left(\widetilde{\mu}_{c_{1n}}, \widetilde{\beta}_{c_{1n}}\right) \\ \left(\widetilde{\mu}_{c_{21}}, \widetilde{\beta}_{c_{21}}\right) & \infty & \left(\widetilde{\mu}_{c_{23}}, \widetilde{\beta}_{c_{23}}\right) & .. & .. & \left(\widetilde{\mu}_{c_{2n}}, \widetilde{\beta}_{c_{2n}}\right) \\ \left(\widetilde{\mu}_{c_{31}}, \widetilde{\beta}_{c_{31}}\right) & \left(\widetilde{\mu}_{c_{32}}, \widetilde{\beta}_{c_{32}}\right) & \infty & .. & .. & \left(\widetilde{\mu}_{c_{3n}}, \widetilde{\beta}_{c_{3n}}\right) \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ \left(\widetilde{\mu}_{c_{n1}}, \widetilde{\beta}_{c_{n1}}\right) & \left(\widetilde{\mu}_{c_{n2}}, \widetilde{\beta}_{c_{n2}}\right) & \left(\widetilde{\mu}_{c_{n3}}, \widetilde{\beta}_{c_{n3}}\right) & .. & .. & \infty \end{pmatrix} \tag{20}$$

Step-2: Calculate the score function for each generalized Fermatean fuzzy number using the above formulas one(type:1), two(type:2), three(type3) and four(proposed) For k = 1,2,3,4.

Calculate the following: $\text{S}_{kF}\left(\widetilde{\mu}_{c_{st}}, \widetilde{\beta}_{c_{st}}\right)$, s = 1,2,..m and t = 1,2,...n.

Step-3: The corresponding scoring function indices are used in place of the Fermatean fuzzy numbers.

Step-4: The Hungarian Method is used to resolve the ensuing problem [39] to search for the solution to the proposed model of the Fermatean traveling seller problem.

The proposed methodology for the FTSP is also explained graphically in a flowchart in Figure 2 It is constructive to understand the procedure for solving FTSP.

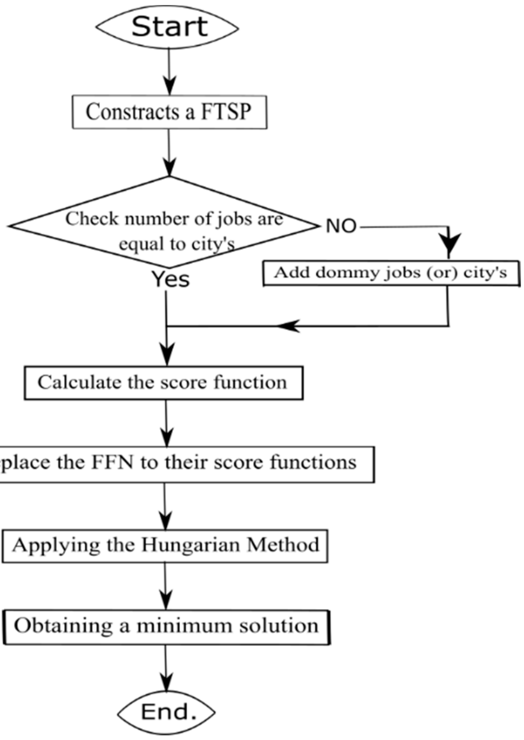

Figure 2. Algorithm for solving FTSP.

4. Case Study

This section presents a case study of choosing a sustainable transportation system in a COVID-19 pandemic scenario. We will look at the transportation system functioning from city to city, containing at least five cities. This case study will discuss how to sustainably minimize the cost of transportation and time of transportation.

4.1. Numerical Solution for Fermatean Commercial Traveller Problem

In this subsection, we discuss how the COVID-19 scenarios are converting into the mathematical formulation of TSP: consider five cities' five jobs; we can see the solution methodology elaborately in the example.

Example: The problem is naturally there in the transportation system of one zone to another at the time of the COVID-19 situation. The difficulty of transportation from one city to each city and the uncertainty of transportation time and cost, based on the realistic situation using the Fermatean fuzzy number, helps to define the uncertainty of the cost and time variations. In this problem, we consider the cost parameter almost dependent on the time factor. Suppose we spend more cost on transportation, it will take a minimum time. But our objective is minimizing the transportation cost as well as minimizing the time the environment has been changed due to the affection of coronavirus. How much was the cost variation there?

What was the sustainable reliability of transportation from one city to another city. The Fermatean fuzzy number contains two functions one is a membership function, and another is one non-membership function. Here the membership function represents the cost of one city to another city, and the non-membership function represents the rare and least transportation cost of one city to another city. These cities names are City-1 (C_1), City-2 (C_2), City-3 (C_3), City-4 (C_4), and City-5 (C_5). The mentioned Fermatean fuzzy number is used for this problem by framing an FFTSP based on the prescribed scenario. The solution to this problem is derived in the same section for finding the sustainable economy of the transportation of traveling seller problem.

The discussed background of the problem is the shipping agent working in a specific region. That staff wants to deliver the parcel from each city to other cities. Due to the lack of a shipping system during the lockdown, the agent offices deliver the medicines parcels. Here, the membership function represents the cost of one city to another during the pandemic, and the non-membership function represents the rare and least transportation cost of one city to another before the pandemic. Let us consider the travel from C_1 to C_2 no need for transportation, so it is considered infinite (∞). Another sense we can travel from C_1 to C_2 transit depending on the fuel availability and size of the vehicle is by making the flexibility of transportation. The Fermatean fuzzy cost of travel from the cities C_1, C_2, C_3, C_4, and C_5 to C_1, C_2, C_3, C_4, and C_5 is given below (Tables 1–5)

Table 1. Fermatean fuzzy supplier problem.

Job \ Work	C_1	C_2	C_3	C_4	C_5
C_1	∞	(0.2, 0.7)	(0.5, 0.1)	(0.1, 0.9)	(0.4, 0.2)
C_2	(0.30, 0.89)	∞	(0.2, 0.8)	(0.2, 0.7)	(0.7, 0.2)
C_3	(0.2, 0.7)	(0.4, 0.8)	∞	(0.1, 0.9)	(0.5, 0.4)
C_4	(0.4, 0.7)	(0.7, 0.3)	(0.8, 0.1)	∞	(0.4, 0.2)
C_5	(0.8, 0.3)	(0.5, 0.8)	(0.6, 0.4)	(0.6, 0.7)	∞

Table 2. Fermatean cost-based time of fuzzy seller problem using score function 1 (type 1).

Job \ Work	C_1	C_2	C_3	C_4	C_5
C_1	∞	0.3325	0.5610	0.1360	0.5280
C_2	0.1610	∞	0.2480	0.3325	0.6675
C_3	0.3325	0.2760	∞	0.1360	0.5305
C_4	0.3605	0.6580	0.7555	∞	0.8605
C_5	0.7425	0.3065	0.5760	0.4365	∞

Table 3. Fermatean cost-based time of traveling seller problem using score function 2 (type 2).

Job \ Work	C_1	C_2	C_3	C_4	C_5
C_1	∞	0.2243	0.4163	0.0910	0.3733
C_2	0.1163	∞	0.1680	0.2243	0.5593
C_3	0.2243	0.2053	∞	0.0910	0.3953
C_4	0.2617	0.5530	0.6743	∞	0.8167
C_5	0.6657	0.2460	0.4560	0.3630	∞

Table 4. Fermatean cost-based time of traveling seller problem using score function 3 (type 3).

Job \ Work	C_1	C_2	C_3	C_4	C_5
C_1	∞	0.3350	0.1240	0.7020	0.0560
C_2	0.6780	∞	0.5040	0.3350	0.3350
C_3	0.3350	0.4480	∞	0.7020	0.0610
C_4	0.2790	0.3160	0.5110	∞	00560
C_5	0.4850	0.3870	0.1520	0.1270	∞

Table 5. Fermatean cost-based time of traveling seller problem by proposed score function.

Job \ Work	C_1	C_2	C_3	C_4	C_5
C_1	∞	0.4458	0.1511	0.9890	0.0692
C_2	0.9520	∞	0.6896	0.4458	0.3902
C_3	0.4458	0.6075	∞	0.9890	0.0753
C_4	0.2676	0.3696	0.5721	∞	0.0692
C_5	0.5462	0.5195	0.1842	0.1627	∞

The Tables 2–5. Are the next step of the algorithm for solving the FFTSP. The Initial Table 1 was updating the fuzzified value into crip values through the respective score funtions.

Thus, the total Fermatean travel time based on the optimal (see Table 6.) Fermatean travel cost is (0.6162, 0.2822) using score functions 1, 2 are 0.4818, 0.6057.

Table 6. Solution of problem using type 1 and 2 score functions using the Hungarian method in the sequence form: City-1 → City-5, City-5 → City-2, City-2 → City-3, City-3 → City-4, City-4 → City-1.

Job \ Work	Defuzzied Cost by		Fermatean Cost
	Score Function 1 (Type 1)	Score Function 2 (Type 2)	
$C_1 \rightarrow C_5$	0.5280	0.3733	(0.4,0.2)
$C_5 \rightarrow C_2$	0.3605	0.2617	(0.5,0.8)
$C_2 \rightarrow C_3$	0.2480	0.1680	(0.2,0.8)
$C_3 \rightarrow C_4$	0.1360	0.0910	(0.1,0.9)
$C_4 \rightarrow C_1$	0.3605	0.2617	(0.4,0.7)
Fuzzy optimal time-based on cost	0.4818	0.6057	(0.6162,0.2822)

The total Fermatean travel time based on the optimal(see Table 7.) Fermatean travel cost is (0.7230,0.1568) using the existing and proposed score functions of 0.3741 and 0.4319. The result of the sustainable transportation of traveling seller problem explains a total minimized time by time -based cost symbolically telling the transport system's carbon emission. In the same way, we can see it shows the sustainability in economics and management for the traveling sellers.

Table 7. Solution of problem using type 3 and 4 score functions using the Hungarian method in the sequence form: City-1 → City-3, City-3 → City-5, City-5 → City-2, City-2 → City-4, City-4 → City-1.

Job \ Work	Defuzzied Cost by		Fermatean Cost
	Score Function 3 (Type 3)	Proposed Score Function	
$C_1 \rightarrow C_3$	0.1240	0.1511	(0.5,0.1)
$C_3 \rightarrow C_5$	0.0610	0.0753	(0.2,0.7)
$C_5 \rightarrow C_2$	0.3870	0.5195	(0.5,0.4)
$C_2 \rightarrow C_4$	0.3350	0.4458	(0.4,0.7)
$C_4 \rightarrow C_1$	0.2790	0.2676	(0.5,0.8)
Fuzzy optimal time-based on cost	0. 3741	0.4319	(0.7230,0.1568)

4.2. Result and Discussion

In the times of the COVID-19 pandemic, traveling sellers' economic sustainability is subjected to flexibility in payments, infrastructure needs, and fuel availability. Not only the transportation system but also the world itself underwent many changes. As a result of the government imposed restrictions, many businesses faced a crisis. The imports and exports of the products were reduced because of the limited fuel availability and the hike in fuel cost. Thus, the sale of products became confined to geographical coordinates. The infrastructure to contain maximum number of products as well as the efficiency of the vehicle are also essential to sell the products without failure. The solved transportation model suggested in this paper employs these factors as crucial for economic stability of a traveling supplier.

The proposed score function and (type 3) give the same results as the type 1,2 score function for finding the score value of the Fermatean fuzzy numbers. Table 8 shows that the minimum cost is obtained by score function type 3, but our proposed score function gives reliable fuzzy cost. Because we also get the same result using the Hungarian Fermatean fuzzy cost/time method, the allocated cells are also the same. The sustainable cost of transportation and the sustainable time for cost-based travel of seller problems also give a better solution to our proposed score function. Therefore, we claim that our proposed method also gives better results for solving the traveling seller problem in Fermatean fuzzy environment. The value of our proposed traveling seller problem model provides reliable and sustainable travel cost. Cost-based time is the imprecise value for the Fermatean fuzzy set and Fermatean fuzzy numbers. The solution represented by the graph will help to understand the solutions to the problem by using Figure 3. Reliable and sustainable solutions to the problem are represented in the above Figure 3. From the solution to the problem, a shipping agent starts his work from a city 'City-1' via 'City-5', 'City-2', 'City-3', and 'City-4' and then again reaches the starting city 'City-1', by type 1 and 2. But type 3 and the proposed score function give the shortest path and sustainable traveling time-based cost. The network is 'City-1' via 'City-3', 'City-5', 'City-2', and 'City-4' and then again reaches the starting city 'City-1'.

Table 8. Comparison of different score function solutions.

Score Function	Score Function Cost	Fermatean Cost/Time
Type 1	0.4818	(0.6162, 0.2822)
Type 2	0.6057	(0.6162, 0.2822)
Type 3	0.3741	(0.7230, 0.1568)
Proposed	0.4319	(0.7230, 0.1568)

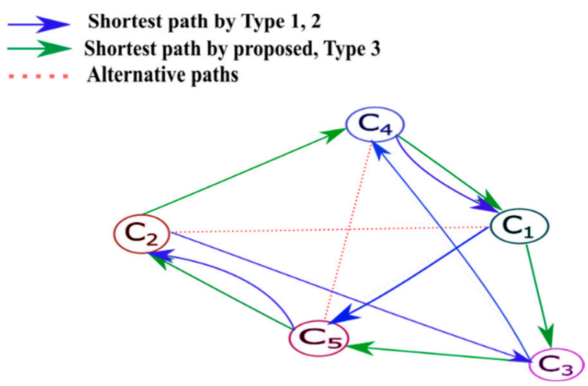

Figure 3. Solution and alternative network of the seller track problem.

4.3. Sensitivity Analysis

The solution to the sustainable fuzzy Fermatean traveling seller problem has been checked in this part by increasing the membership, non-membership, and both membership function values. Suppose the value of the membership function (pre-pandemic cost) increasing means that non-membership (post-pandemic cost) function values are decreasing.

The variations in cost from city to city have variations. Then, how the solution changed is analyzed through the network diagram and Fermatean score cost. The increasing and decreasing cost input variations are taken as the input values of the same problem to analyze how the result of the problem is analyzed in Table 9.

Table 9. Sensitivity analysis with changes of fuzziness of the problem.

Function	Increasing Rate %	Fermatean Fuzzy Total Traveling Cost and Network	
Membership value	10%	2.1751	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	15%	2.2125	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	20%	2.2788	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
Both membership value	10%	1.8879	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	15%	1.8319	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	20%	1.7883	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
Non-membership value	10%	1.8507	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	15%	1.7546	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$
	20%	1.6737	$C_1 \to C_3 \to C_5 \to C_2 \to C_4 \to C_1$

The proposed score function is better than the existing score function because the score functions can give one the least score value, and another one gives a higher score value. Let us consider the solved problem in the environment of Fermatean fuzziness; it increases the membership value of as in 10%, 15%, and 20%, then the value of total Fermatean sustainable cost also increases. In the same way, by increasing the non-membership value by 10, 15, and 20%, the total Fermatean sustainable cost will decrease. The values are increases in the value of the non-membership function; then the total Fermatean cost is 1.8507, 1.7546, 1.6737. As the quantity of both Fermatean membership and non-membership values increases, the value total Fermatean cost of the seller's problem also decreases. The score function values are 1.8879, 1.8319, 1.7883 (see Table 9). It will be helpful to understand the score function sense. So, our proposed score function will give an efficient score value other than the existing score function score value.

5. Conclusions

The sustainable fuzzy Fermatean traveling seller problem is investigated in this study. The traveling seller problem is one of the most critical problems in fuzzy decision-making. During the pandemic, the traveling supplier making this decision is more difficult, and it is rectified by the Fermatean fuzzy environment of a sustainable solution. We investigated various TSP models in the Fermatean fuzzy environment. Existing arithmetic operations were used to find the best solution to the Fermatean fuzzy set problem. We proposed a new scoring function for FFS, any Fermatean fuzzy set $\widetilde{F}$, where the score function values are within the unit interval, i.e., $-1 \leq \widetilde{S}_F(x) \leq 1$. According to the computational results, the FFSs are more suited than existing fuzzy sets and capable of managing more significant degrees of uncertainty. The presented algorithm represents a novel approach to dealing with uncertainty in a Fermatean in fuzzy environment. The Fermatean fuzzy set is more flexible and reliable than the Intuitionistic and Pythagorean fuzzy sets. The method suggested can be used to solve real-life problems.

For further future research work, handle transportation and traveling seller problems and display triangular and trapezoidal Fermatean fuzzy numbers with the associated ranking functions. This study has not considered vehicle payload, which can be construed in a new environment as a future study.

Author Contributions: Conceptualization, B.B., I.M.-K. and G.S.M.; methodology, B.B., R.Č. and G.S.M.; software, G.S.M. and B.B.; validation, I.M.-K.; formal analysis, G.S.M. and R.Č.; investigation, I.M.-K. and B.B.; resources, G.S.M. and R.Č.; writing—original draft preparation, B.B., G.S.M. and R.Č.; writing—review and editing, I.M.-K.; visualization G.S.M. and R.Č.; supervision, I.M.-K. and G.S.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data of this study are available from the authors upon request.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Jiang, L.; Chang, H.; Zhao, S.; Dong, J.; Lu, W. A Travelling Salesman Problem With Carbon Emission Reduction in the Last Mile Delivery. *IEEE Access* **2019**, *7*, 61620–61627. [CrossRef]
2. Balas, E. The prize collecting traveling salesman problem. *Networks* **1989**, *19*, 621–636. [CrossRef]
3. Lancia, G.; Serafini, P. Traveling Salesman Problems. *EURO Adv. Tutor. Oper. Res.* **2001**, *3*, 155–164. [CrossRef]
4. Zimmermann, H.-J. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst.* **1978**, *1*, 45–55. [CrossRef]
5. El Sayed, M.A.; El-Shorbagy, M.A.; Farahat, F.A.; Fareed, A.F.; Elsisy, M.A. Stability of Parametric Intuitionistic Fuzzy Multi-Objective Fractional Transportation Problem. *Fractal Fract.* **2021**, *5*, 233. [CrossRef]
6. Lu, Q.; Dessouky, M.M. A new insertion-based construction heuristic for solving the pickup and delivery problem with time windows. *Eur. J. Oper. Res.* **2006**, *175*, 672–687. [CrossRef]
7. Bellman, R.E.; Zadeh, L.A. Decision-Making in a Fuzzy Environment. *Manag. Sci.* **1970**, *17*, B-141–B-164. [CrossRef]
8. Chanas, S.; Kołodziejczyk, W.; Machaj, A. A fuzzy approach to the transportation problem. *Fuzzy Sets Syst.* **1984**, *13*, 211–221. [CrossRef]
9. Chanas, S.; Kuchta, D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. *Fuzzy Sets Syst.* **1996**, *82*, 299–305. [CrossRef]
10. Tanaka, H.; Asai, K. Fuzzy linear programming problems with fuzzy numbers. *Fuzzy Sets Syst.* **1984**, *13*, 1–10. [CrossRef]
11. Changdar, C.; Mahapatra, G.S.; Pal, R.K. An efficient genetic algorithm for multi-objective solid travelling salesman problem under fuzziness. *Swarm Evol. Comput.* **2014**, *15*, 27–37. [CrossRef]
12. Changdar, C.; Mahapatra, G.S. A modified genetic algorithm-based approach to solve constrained solid tsp with time window using interval valued parameter. *Int. J. Oper. Res.* **2016**, *26*, 398–421. [CrossRef]
13. Pradhan, K.; Basu, S.; Thakur, K.; Maity, S.; Maiti, M. Imprecise modified solid green traveling purchaser problem for substitute items using quantum-inspired genetic algorithm. *Comput. Ind. Eng.* **2020**, *147*, 106578. [CrossRef]
14. Feng, H.-M.; Liao, K.-L. Hybrid evolutionary fuzzy learning scheme in the applications of traveling salesman problems. *Inf. Sci. (Ny).* **2014**, *270*, 204–225. [CrossRef]
15. Trigui, S.; Cheikhrouhou, O.; Koubaa, A.; Baroudi, U.; Youssef, H. FL-MTSP: A fuzzy logic approach to solve the multi-objective multiple traveling salesman problem for multi-robot systems. *Soft Comput.* **2017**, *21*, 7351–7362. [CrossRef]
16. Sarkis, J.; Cohen, M.J.; Dewick, P.; Schröder, P. A brave new world: Lessons from the COVID-19 pandemic for transitioning to sustainable supply and production. *Resour. Conserv. Recycl.* **2020**, *159*, 104894. [CrossRef]
17. Jingzhuo, S.; Wenwen, H.; Ying, Z. T–S Fuzzy Control of Travelling-Wave Ultrasonic Motor. *J. Control Autom. Electr. Syst.* **2020**, *31*, 319–328. [CrossRef]
18. Marimuthu, D.; Mahapatra, G.S. Multi-criteria decision-making using a complete ranking of generalized trapezoidal fuzzy numbers. *Soft. Comput.* **2020**, *25*, 9859–9871. [CrossRef]
19. Bhavani, G.D.; Kavaliauskiene, I.M.; Mahapatra, G.S.; Renata, C. A Sustainable Green Inventory System with Novel Eco-Friendly Demand Incorporating Partial Backlogging under Fuzziness. *Sustainability* **2022**, *14*, 9155. [CrossRef]
20. Delgadillo, F.J.D.; Montiel, O.; Sepúlveda, R. Reducing the size of traveling salesman problems using vaccination by fuzzy selector. *Expert. Syst. Appl.* **2016**, *49*, 20–30. [CrossRef]
21. Hasheminejad, S.; Seifossadat, S.G.; Razaz, M.; Joorabian, M. Traveling-wave-based protection of parallel transmission lines using Teager energy operator and fuzzy systems. *IET Gener. Transm. Distrib.* **2016**, *10*, 1067–1074. [CrossRef]
22. Shi, J.; Zhao, J.; Cao, Z.; Liang, Y.; Yuan, L.; Sun, B. Self-tuning fuzzy speed controller of travelling wave ultrasonic motor. *Int. J. Smart Sens. Intell. Syst.* **2014**, *7*, 301–320. [CrossRef]
23. Ryu, H. A Revisiting Method Using a Covariance Traveling Salesman Problem Algorithm for Landmark-Based Simultaneous Localization and Mapping. *Sensors* **2019**, *19*, 4910. [CrossRef] [PubMed]
24. Schiffer, M.; Klein, P.S.; Laporte, G.; Walther, G. Integrated planning for electric commercial vehicle fleets: A case study for retail mid-haul logistics networks. *Eur. J. Oper. Res.* **2021**, *291*, 944–960. [CrossRef]

25. Kazemzadeh, K.; Koglin, T. Electric bike (non) users' health and comfort concerns pre and peri a world pandemic (COVID-19): A qualitative study. *J. Transp. Health* **2021**, *20*, 101014. [CrossRef] [PubMed]
26. Bojovic, D.; Benavides, J.; Soret, A. What we can learn from birdsong: Mainstreaming teleworking in a post-pandemic world. *Earth Syst. Gov.* **2020**, *5*, 100074. [CrossRef]
27. Santos-Arteaga, F.J.; di Caprio, D.; Tavana, M.; Tena, E.C. A Credibility and Strategic Behavior Approach in Hesitant Multiple Criteria Decision-Making with Application to Sustainable Transportation. *IEEE Trans. Fuzzy Syst.* **2022**, 1–15. [CrossRef]
28. Vali-Siar, M.M.; Roghanian, E. Sustainable, resilient and responsive mixed supply chain network design under hybrid uncertainty with considering COVID-19 pandemic disruption. *Sustain. Prod. Consum.* **2022**, *30*, 278–300. [CrossRef]
29. Zadeh, L.A. Fuzzy sets. *Fuzzy Sets Inf. Control.* **1965**, *8*, 338–353. [CrossRef]
30. Atanassov, K.T. *Intuitionistic Fuzzy Sets*; Studies in fuzziness and Soft computing; Springer Nature: Cham, Switzerland, 1999; Volume 35, pp. 1–137. [CrossRef]
31. Fischer, R.; Richter, K. Solving a multiobjective traveling salesman problem by dynamic programming. *Math. Oper. Stat. Ser. Optim.* **1982**, *13*, 247–252. [CrossRef]
32. Almahasneh, R.; Tüű-Szabó, B.; Kóczy, L.T.; Földesi, P. Optimization of the Time-Dependent Traveling Salesman Problem Using Interval-Valued Intuitionistic Fuzzy Sets. *Axioms* **2020**, *9*, 53. [CrossRef]
33. Yager, R.R.; Abbasov, A.M. Pythagorean Membership Grades, Complex Numbers, and Decision Making. *Int. J. Intell. Syst.* **2013**, *28*, 436–452. [CrossRef]
34. Peng, X.; Selvachandran, G. Pythagorean fuzzy set: State of the art and future directions. *Artif. Intell. Rev.* **2019**, *52*, 1873–1927. [CrossRef]
35. Senapati, T.; Yager, R.R. Fermatean fuzzy sets. *J. Ambient Intell. Humaniz. Comput.* **2020**, *11*, 663–674. [CrossRef]
36. Sahoo, L. Some Score Functions on Fermatean Fuzzy Sets and Its Application to Bride Selection Based on TOPSIS Method. *Int. J. Fuzzy Syst. Appl.* **2021**, *10*, 18–29. [CrossRef]
37. Senapati, T.; Yager, R.R. Some new operations over fermatean fuzzy numbers and application of fermatean fuzzy wpm in multiple criteria decisions making. *Informatica* **2018**, *30*, 391–412. [CrossRef]
38. Gül, S. Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. *Expert Syst.* **2021**, *38*, e12769. [CrossRef]
39. Kuhn, H.W. The Hungarian method for the assignment problem. *Nav. Res. Logist. Q.* **1955**, *2*, 83–97. [CrossRef]