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Durability Evaluation of Hydraulic Tunnel Lining Structure Based on Set Pair Analysis and Extension Coupling Model

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Abstract: A series of water diversion projects to address the uneven distribution of water resources in China have involved the construction of a large number of hydraulic tunnels. As the lining structure is there to maintain the stability and durability of the tunnels, durability damage can easily occur in the operation process, thus affecting the safety of water transmission and water supply capacity. Therefore, it is important to evaluate the durability of hydraulic tunnel lining structure. Considering the randomness and fuzziness of the factors affecting the durability of hydraulic tunnel lining structure, this paper proposes a comprehensive evaluation model based on the coupling of set pair analysis and extension. The G1 method and the simple correlation function method are used to determine the subjective and objective weights of the evaluation indexes, respectively, and the combination weight of them is assigned based on the principle of minimum entropy; next, the set pair analysis principle is used to establish the linkage affiliation function, which can calculate the comprehensive linkage affiliation of the object to be evaluated, and then the maximum affiliation principle is used to judge the durability level of the hydraulic tunnel lining structure. Finally, taking a section of hydraulic tunnel as an example, the model proposed in this paper is used to calculate its durability grade as Class III, with the set pair potential SHI(H) = 7.5856, which is consistent with the actual engineering practice, and a comparative study is done in combination with the AHP-Extenics method. It is verified that the evaluation model can scientifically and reasonably evaluate the durability of hydraulic tunnel lining structure, providing a basis for subsequent maintenance and reinforcement.

Keywords: set pair analysis; extension theory; lining structure; durability evaluation; G1 method; simple correlation function method

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1. Introduction

In order to solve the problem of water shortage and promote the development of local economy, China has invested in a series of water diversion projects, such as South to North Water Diversion, Diversion of Yellow River to Qingdao, Diversion of Yangtze River to Huaihe River, etc. These projects basically involve the construction of a large number of hydraulic tunnels. During the long-term use of these tunnels, due to the coupling effect of load, geology, environment, and various erosive substances, cracks, material deterioration, water leakage, and other diseases will appear in the lining structure, which seriously affects the safety condition and normal operation of hydraulic tunnels. Therefore, a reasonable evaluation of the durability of hydraulic tunnel lining structure and timely adoption of scientific and effective measures to maintain it are of great significance to prolong the service life of hydraulic tunnels [1].

In the area of tunnel evaluation, more research results have been achieved. Guo et al. [2] proposed an AHP-Extenics model based on the evaluation of tunnel lining structure damage, using hierarchical analysis to construct an index system for forming disease factors, determining the evaluation index weights through expert scoring, and carrying out topologizable operations on the evaluation indexes and weights to finally determine the tunnel lining structure damage levels. Zhu et al. [3] evaluated the durability of tunnel lining structure by variable fuzzy comprehensive assessment method, first establishing the affiliation matrix of evaluation index characteristic values to different levels, applying variable fuzzy preference model to determine the comprehensive affiliation degree, and conducting comprehensive assessment on the evaluation object, and finally determining the durability level of tunnel lining structure. Jin Chunling [4] used the PSR model to establish 31 evaluation indexes for the safety evaluation of the diversion tunnel and determined the index weight by the AHP method. Rao et al. [5] classified the structural safety of karst road tunnels into five levels, determined quantitative evaluation indexes through the normal affiliation function, and determined their weights based on the expert scoring method, and finally evaluated the tunnel safety through a three-level fuzzy comprehensive evaluation model.

Arends B J [6] proposed a method for evaluating tunnel safety based on probabilistic risk assessment based on three major aspects: economic risk, personal risk, and social risk, and applied it in a practical case of a Dutch tunnel project. Ye et al. [7] used transient electromagnetic radar to detect the lack of voids and lining thickness behind the tunnel composite lining to determine the contact state between the surrounding rock and the composite lining based on the change in apparent resistivity, and to evaluate the durability of the tunnel composite lining. Manchao et al. [8] concluded that large deformations can occur in tunnels during or after excavation due to adverse geology and design defects, and then proposed a method based on finite element software to simulate material deformations and a Bayesian neural network-based evaluation method for the condition of large tunnel deformations. Wang Y [9] combined triangular fuzzy numbers and exponential scaling, and then proposed an improved scale based on the fuzzy analysis network process, and applied it to the risk analysis of the Huma Ling tunnel to verify that the method could accurately reflect the actual engineering situation.

Zhang et al. [10] identified carbon infiltration and chloride as common problems in tunnel durability, developed a procedure for assessing the strength of tunnel lining concrete under corrosive conditions, thus indirectly evaluating the durability of the lining structure, and used relevant experimental data to verify the accuracy of the method. Hussain et al. [11] calculated the rock mass of an excavated diversion tunnel based on a combination of empirical and numerical methods, and analyzed the stability of the tunnel before and after excavation. Qiu W [12] constructed an evaluation index system for the sustainability of railway tunnels based on three main aspects: regional ecological environment, supporting structures and auxiliary facilities, and policy management. Li K [13] used experimental simulations of concrete-immersed tube tunnels exposed to seawater and non-destructive testing to evaluate the durability of immersed tube tunnels through a fully probabilistic approach. Akula P [14] evaluated the durability of the lime-treated broken concrete lining of the Friant-Kern canal from mineralogical and engineering aspects and preliminarily explored the effect of lime on the repair of hydraulic buildings.

Hydraulic tunnel lining structure mainly adopts reinforced concrete structure, which is exposed to high head pressure, high ground stress, and various chemical and physical effects when it is in a water environment for a long time. Eventually, the lining structure gradually deteriorates in performance due to erosive ions, acidic substances, carbonation, penetration pressure, etc., resulting in a continuous decline in durability. The durability of hydraulic tunnel lining structures is not only related to the normal operation of the tunnel, but also plays a vital role in its safety and service life. However, there are a variety of complex factors that jointly affect its durability, and the influencing factors themselves have certain characteristics such as randomness and ambiguity. At the same time, in the process of evaluating the durability status of hydraulic tunnel lining structures, it is necessary not only to resolve the contradictory issue between quantitative and qualitative changes, but also to reasonably depict the influence degree of different evaluation indexes on its durability. Set pair analysis can reasonably describe the nature of definite and uncertain connections, while extension theory can quantitatively describe the process of quantitative and qualitative changes through the correlation function, and use the matter element model to solve the objective contradiction problem [15]. The integration of set pair analysis and extension theory can effectively solve the above problems. In addition, there are two main types of calculation methods for weights, objective and subjective. Only using one of them will make the weight calculation more objective or subjective, which will lead to the failure of the final evaluation [16].

This paper applies the set pair analysis and extension coupling model to the evaluation of the durability of hydraulic tunnel lining structure. The G1 method and the simple correlation function method are used to calculate the subjective and objective weights of the evaluation indexes, and the minimum entropy (MIE) principle is used to calculate the comprehensive weight of the evaluation index. The paper then establishes a comprehensive evaluation model based on the combination weighting of the set pair analysis and extension coupling model. The relevant calculations are also carried out using a section of water diversion project hydraulic tunnel as the research background to verify the reasonableness and scientific validity of the model in the evaluation of the durability of hydraulic tunnel lining structures.

2. The Establishment of Comprehensive Evaluation Model

2.1. Determination of Index Weight

2.1.1. G1 Method to Determine the Subjective Weight

The hierarchical analysis method (AHP) requires a high degree of awareness among evaluators in the process of calculating the weight of evaluation index, and has disadvantages such as cumbersome calculations, the need to construct judgment matrices, and consistency tests. Therefore, Guo Yajun proposed the G1 method, which first ranks the importance of evaluation indexes and then determines the weights by comparing the importance between adjacent indexes, with the following calculation steps [17]:

(1) Ranking the relative importance of the evaluation indexes. Based on the subjective opinions of experts, one most important index is selected from the set of evaluation indexes $\{C_1, C_2 \ldots C_n\}$ and recorded as C_1' with the weight of α_1, and the most important index from the remaining $n - 1$ indexes is selected as C_2' with the weight of α_2, and so on, the relative importance ranking of each evaluation index can be derived: $C_1' > C_2' > \cdots C_{i-1}' > C_i' > C_{i+1}' > \cdots C_n'$.

(2) Calculate the relative importance between the neighboring indexes C_{i-1}' and C_i'. The importance ratio C_{i-1}' and C_i' of the ranked neighboring indexes r_i is reasonably assigned, as shown in Equation (1). The values of r_i are given in Table 1:

$$r_i = \frac{\alpha_{i-1}}{\alpha_i} \quad i = n, n-1, \cdots, 3, 2 \tag{1}$$

Table 1. r_k reference table of assignment values.

r_i	r_i Assignment Description
1.0	C_{i-1}' is equally important as C_i'
1.2	C_{i-1}' is slightly more important than C_i'
1.4	C_{i-1}' is obviously more important than C_i'
1.6	C_{i-1}' is more strongly important than C_i'
1.8	C_{i-1}' is more extremely important than C_i'
1.1, 1.3, 1.5, 1.7	The median of the above two adjacent judgments

(3) The weight of the n-th evaluation index is calculated. The calculation formula is Equation (2):

$$\alpha_n = (1 + \sum_{i=2}^{n} \left(\prod_{k=i}^{n} r_i \right))^{-1} \tag{2}$$

(4) Calculate the weights of other evaluation indexes. The calculation formula is Equation (3):

$$\alpha_{i-1} = r_i \alpha_i \; i = n, n-1, \cdots, 3, 2 \tag{3}$$

2.1.2. Simple Correlation Function Method to Determine Objective Weight

The simple correlation function method is an objective assignment method calculated based on the theory of matter–element extension, and the specific calculation procedure is shown in Equations (4)–(7) [18–20]

$$\text{Assume } h_{ij}(v_i, V_{ij}) = \begin{cases} \frac{2\left(v_i - c_{ij}^-\right)}{c_{ij}^+ - c_{ij}^-} & v_i \le \frac{c_{ij}^- + c_{ij}^+}{2} \\ \frac{2\left(c_{ij}^+ - v_i\right)}{c_{ij}^+ - c_{ij}^-} & v_i > \frac{c_{ij}^- + c_{ij}^+}{2} \end{cases} \tag{4}$$

$$(i = 1, 2 \cdots n \; ; \; j = 1, 2 \cdots m)$$

where h_{ij} is the correlation between the i-th evaluation index and the j-th evaluation level; v_i is the sample value of the i-th evaluation index; V_{ij} is the range of values of the i-th evaluation index corresponding to the j-th evaluation level, and its value is $V_{ij} = \left(c_{ij}^-, c_{ij}^+\right)$.

If $v_i \in V_{ip}$, then $h_{ijmax}(v_i, V_{ij}) = \underset{j \in (1,2,\cdots,m)}{max} \{h_{ij}(v_i, V_{ij})\}$.

If the evaluation index C_i of the object P to be evaluated belongs to a larger evaluation level j, the greater the weight assigned to this index, then h_i is shown in Equation (5):

$$h_i = \begin{cases} j_{max}\left[1 + h_{ijmax}(v_i, V_{ij})\right] & h_{ijmax}(v_i, V_{ij}) \ge -0.5 \\ 0.5 j_{max} & h_{ijmax}(v_i, V_{ij}) < -0.5 \end{cases} \tag{5}$$

where j_{max} is the evaluation level at which the measured sample value of evaluation index i in the object to be evaluated is placed, and the larger the value, the stronger the restriction on the object to be evaluated, when $h_{ijmax} = h_{im}$, $j_{max} = max\{m\}$.

If the evaluation index C_i of the object P to be evaluated belongs to a larger evaluation level j, the smaller the weight assigned to this index, then h_i is shown in Equation (6):

$$h_i = \begin{cases} (m - j_{max} + 1)\left[1 + h_{ijmax}(v_i, V_{ij})\right] & h_{ijmax}(v_i, V_{ij}) \ge -0.5 \\ 0.5(m - j_{max} + 1) & h_{ijmax}(v_i, V_{ij}) < -0.5 \end{cases} \tag{6}$$

where m is the number of categories into which each evaluation index is classified. When $h_{ijmax} = h_{im}$, $j_{max} = min\{m\}$.

Then, the objective weight of evaluation index i of the object P to be evaluated is Equation (7):

$$\beta_i = \frac{h_i}{\sum_{i=1}^{n} h_i} \tag{7}$$

where β_i is the i-th index weight normalized value.

2.1.3. Calculating Combination Weight

The evaluation index weights calculated based on the G1 method and the simple correlation function method are α_i and β_i, respectively, and in order to make the combination weight ω_i as close as possible to α_i and β_i, the combination weight ω_i is obtained based on the MIE principle, and the computational model is Equation (8) [21]:

$$\begin{cases} min\ J(\omega) = \sum_{i=1}^{n} \left(\omega_i \ln \frac{\omega_i}{\alpha_i} + \omega_i \ln \frac{\omega_i}{\beta_i} \right) \\ s.t. \sum_{i=1}^{n} \omega_i = 1, \omega_i \geq 0, i = 1, 2, \cdots, n \end{cases} \tag{8}$$

Solving this optimization model based on the Lagrangian algorithm, the combination weight can be obtained by Equation (9):

$$\omega_i = \frac{\sqrt{\alpha_i \beta_i}}{\sum_{i=1}^{n} \sqrt{\alpha_i \beta_i}} \tag{9}$$

2.2. *Set Pair Analysis and Extension Coupling Model*

2.2.1. Matter Element Model

The theory of matter–element extension usually represents the overall characteristics of the thing to be evaluated by the ordered triad $R = (N, C, V)$, where N is the thing to be evaluated, C is the feature of the thing to be evaluated, and V is the quantitative value of the characteristics of the thing to be evaluated [22–24].

The classical domain R_j is shown in Equation (10):

$$R_j = [N_j, C, V_j] = \begin{bmatrix} N_j & c_1 & V_{1j} \\ & c_2 & V_{2j} \\ & \vdots & \vdots \\ & c_n & V_{nj} \end{bmatrix} = \begin{bmatrix} N_j & c_1 & \left(c_{1j}^-, c_{1j}^+\right) \\ & c_2 & \left(c_{2j}^-, c_{2j}^+\right) \\ & \vdots & \vdots \\ & c_n & \left(c_{nj}^-, c_{nj}^+\right) \end{bmatrix} \tag{10}$$

where N_j is the j-th evaluation level divided; c_i is the i-th evaluation index; $V_{ij} = \left(c_{ij}^-, c_{ij}^+\right)$ is the range of quantities classified by N_j with respect to the index c_i.

The section domain R_p is shown in Equation (11):

$$R_p = [N_p, C, V_p] = \begin{bmatrix} N_p & c_1 & V_{1p} \\ & c_2 & V_{2p} \\ & \vdots & \vdots \\ & c_n & V_{np} \end{bmatrix} = \begin{bmatrix} N_p & c_1 & \left(c_{1p}^-, c_{1p}^+\right) \\ & c_2 & \left(c_{2p}^-, c_{2p}^+\right) \\ & \vdots & \vdots \\ & c_n & \left(c_{np}^-, c_{np}^+\right) \end{bmatrix} \tag{11}$$

where $V_{ij} \subset V_{iP}$; c_{ip}^-, c_{ip}^+ are the minimum and maximum values taken for the evaluation indexes under each level, respectively.

The information of each evaluation index of the object to be evaluated is represented by the matter element, i.e., the matter element to be evaluated, as shown in Equation (12):

$$R_0 = \begin{bmatrix} N_0 & c_1 & v_1 \\ & c_2 & v_2 \\ & \vdots & \vdots \\ & c_n & v_n \end{bmatrix} \tag{12}$$

where N_0 is the object to be evaluated; v_i is the measured sample value corresponding to the evaluation index.

2.2.2. Principle of Coupling Model

The constructed set pair analysis and extension coupling model, based on the set pair identical-discrepancy-contrary principle and the connection between extension sets, constructs the set pair and extension set theoretical domain between the measured sample values of the hydraulic tunnel lining structure durability index and the evaluation level. The correspondence of the coupling model is shown in Figure 1 and the calculation process is as follows [15,25].

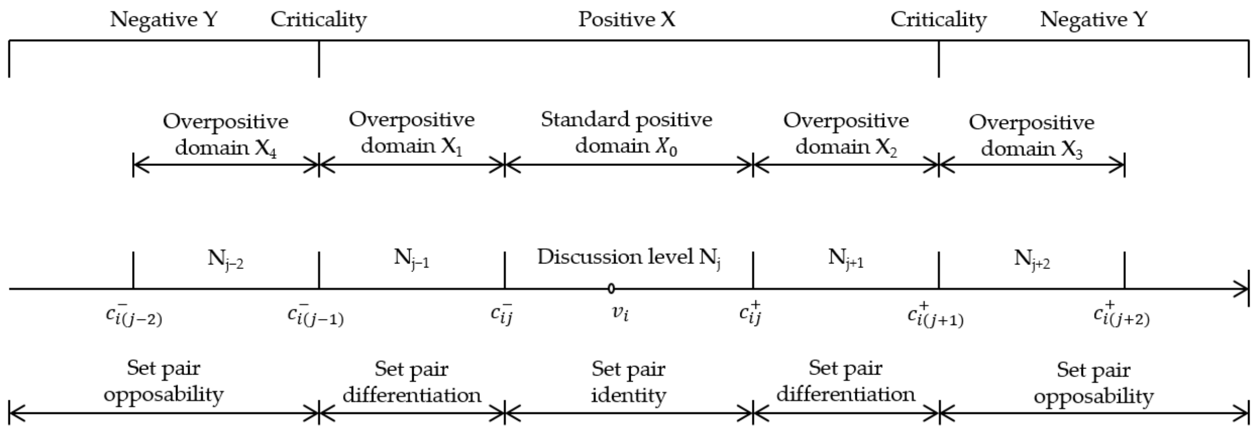

Figure 1. Schematic diagram of the set pair extension relationship.

(1) When the measured sample value of the index v_i belongs to the discussion level j, it means that the value is within the standard positive domain X_0 of the discussion level j and exhibits the identity relation, and the linkage affiliation can be calculated according to Equations (13) and (14):

$$\mu_{N_j}(v_i) = \frac{-\rho(v_i, X_0)}{\left|c^+_{ij} - c^-_{ij}\right|} \tag{13}$$

$$\rho(v_i, X_0) = \left|v_i - \frac{c^+_{ij} + c^-_{ij}}{2}\right| - \frac{c^+_{ij} - c^-_{ij}}{2} \tag{14}$$

(2) When the measured sample value of the index v_i belongs to the discussion level $j-1$ or $j+1$, it means that the value is within the overpositive domain X_1 or X_2 of the discussion level and exhibits a differential relation, which can be calculated according to Equations (15)–(19) to calculate the linkage affiliation:

$$\mu_{N_{j-1}}(v_i) = \frac{\rho(v_i, X_1)}{\rho(v_i, X) - \rho(v_i, X_1)} \tag{15}$$

$$\mu_{N_{j+1}}(v_i) = \frac{\rho(v_i, X_2)}{\rho(v_i, X) - \rho(v_i, X_2)} \tag{16}$$

$$\rho(v_i, X) = \left|v_i - \frac{c^+_{i(j+1)} + c^-_{i(j-1)}}{2}\right| - \frac{c^+_{i(j+1)} - c^-_{i(j-1)}}{2} \tag{17}$$

$$\rho(v_i, X_1) = \left|v_i - \frac{c^-_{ij} + c^-_{i(j-1)}}{2}\right| - \frac{c^-_{ij} - c^-_{i(j-1)}}{2} \tag{18}$$

$$\rho(v_i, X_2) = \left|v_i - \frac{c^+_{i(j+1)} + c^+_{ij}}{2}\right| - \frac{c^+_{i(j+1)} - c^+_{ij}}{2} \tag{19}$$

(3) When the measured sample value of the index v_i belongs to the discussion level $j-2$ or $j+2$, it means that the value is antagonistic to the discussion level j, and the linkage affiliation can be calculated according to Equation (20):

$$\mu_{N_{j-2,j+2}}(v_i) = -1 \tag{20}$$

2.2.3. Calculating the Comprehensive Linkage Affiliation and Set Pair Potential

Based on the linkage affiliation $\mu_{N_j}(v_i)$ and the combination weight ω_i of the evaluation index to be evaluated, the comprehensive linkage affiliation is calculated according to Equation (21), and the durability class of hydraulic tunnel lining structure is determined based on the principle of maximum affiliation.

$$\mu_{N_j} = \sum_{i=1}^{n} \omega_i \mu_{N_j}(v_i) \tag{21}$$

The durability state of hydraulic tunnel lining structure will change with the operational working state of the tunnel, and the expression of the connection degree of the hydraulic tunnel lining structure N_0 can be established according to the weight ω_i of each evaluation index, as shown in Equation (22). Then, the trend and possibility of transformation of the hydraulic tunnel lining structure N_0 to other durability N_j is analyzed according to the set pair potential constructed by Equation (23).

$$\varepsilon = \sum_{v_i \in N_j} \omega_i + \sum_{v_i \in N_{j-1,j+1}} \omega_i p + \sum_{v_i \in N_{j-2,j+2}} \omega_i q \tag{22}$$

$$\text{SHI(H)} = \frac{\sum_{v_i \in N_j} \omega_i}{\sum_{v_i \in N_{j-2,j+2}} \omega_i} \tag{23}$$

where ε is the connection degree; p is the coefficient of difference; q is the coefficient of opposition; SHI(H) is the set pair potential.

3. Durability Evaluation Process of Hydraulic Tunnel Lining Structure Based on Set Pair Analysis and Extension Coupling Model

Durability evaluation of hydraulic tunnel lining structure is the process of analyzing the factors affecting its durability, adopting appropriate methods to organize and evaluate these influencing factors, and determining the durability status of the tunnel. In this paper, we first define the object to be evaluated which is the hydraulic tunnel lining structure, and construct the durability evaluation index system from four aspects: material deterioration, lining cracks, water leakage, lining thickness, and cavity behind; determine the classical and nodal domains of index to be evaluated, construct the set pair and extension set theoretical domain between the measured sample values of hydraulic tunnel lining structure durability index and the evaluation level, establish the corresponding linkage affiliation function, and using the combination weight of the evaluation index obtained from the MIE principle, a comprehensive linkage affiliation of the object to be evaluated is calculated. Finally, the durability class of the hydraulic tunnel lining structure is determined based on the principle of maximum affiliation. The specific evaluation flow chart is shown in Figure 2.

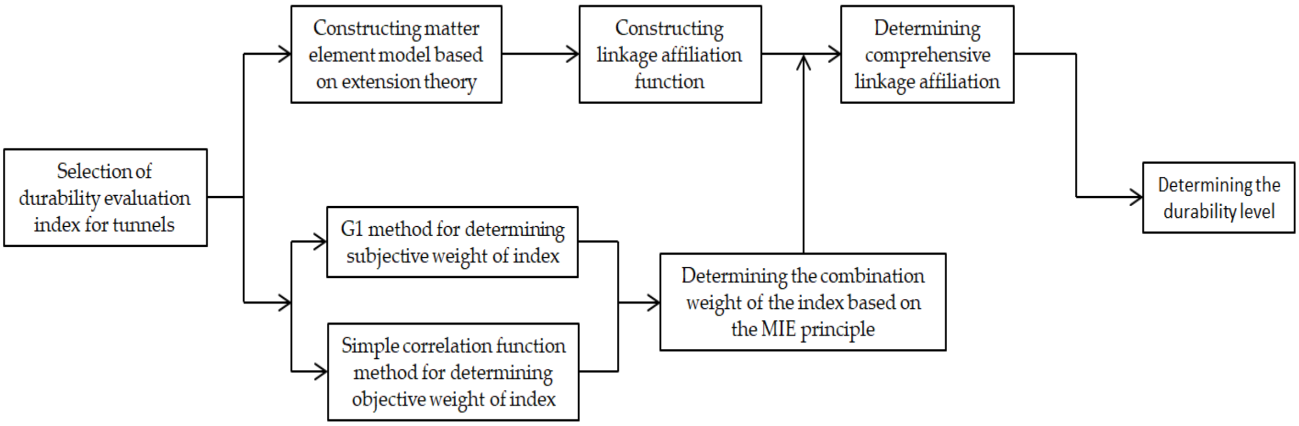

Figure 2. Durability evaluation process of hydraulic tunnel lining structure based on set pair analysis and extension coupling model.

4. Example Application

4.1. Project Example

In order to verify the rationality and scientific validity of this method, an application study is carried out on a section of hydraulic tunnel, located at the junction of Gansu and Qinghai, which is a pressureless hydraulic tunnel, completed and opened to water in 1994, with a length of 2099.25 m. It has been in operation for 29 years, and although several repairs and reinforcements have been carried out, there are still cracks, water leakage, and other diseases due to long operation. The lining structure has been subjected to groundwater scour and corrosion for a long time, resulting in a decrease in the strength of the secondary lining structure. The above unfavorable conditions affect the normal operation and safety of the tunnel. Therefore, it is necessary to evaluate the durability of the tunnel to understand its current condition and to provide a theoretical basis for maintenance and repair work. The structure of the tunnel is shown in Figure 3.

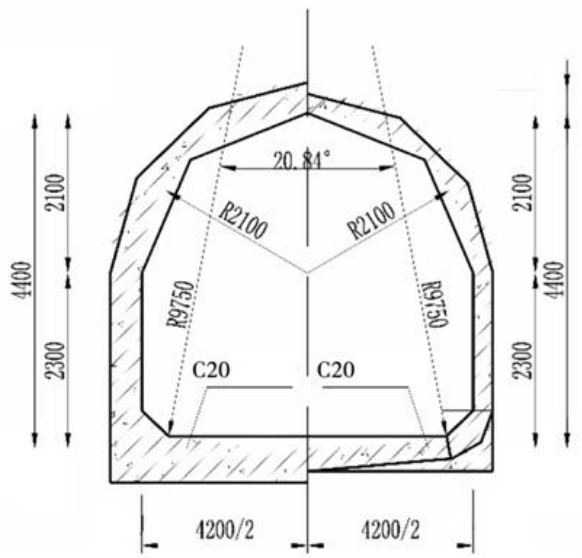

Figure 3. Structure of the tunnel.

4.2. Constructing a Comprehensive Evaluation Index System of Durability

Combined with the actual engineering situation and existing literature, the factors affecting the durability of hydraulic tunnel lining structure are divided into four categories: material deterioration, lining cracks, water leakage, and lining thickness and cavity behind, each of which is composed of several evaluation indexes, and the established three-layer evaluation index system of hydraulic tunnel lining structure durability is shown in Figure 4.

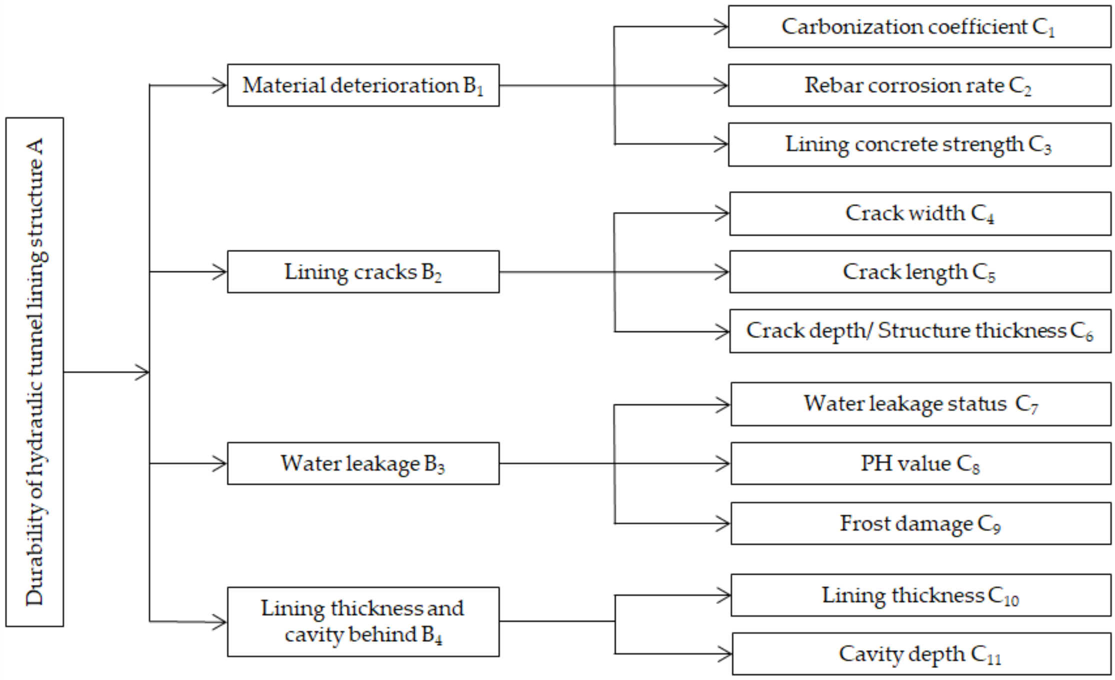

Figure 4. Durability evaluation index system of hydraulic tunnel lining structure.

4.3. Determining the Durability Evaluation Level and Classification Criteria

To reasonably evaluate the durability status of hydraulic tunnel lining structure, the evaluation results need to be appropriately graded. At present, there is no relevant specification for hydraulic tunnel lining in China. In this paper, the durability of tunnel lining structure is divided into four grades by referring to the Code for Durability Design of Concrete Structures in Highway Engineering (JTGT3310-2019) [26] and related literatures [1,27,28], as shown in Table 2.

Table 2. Durability classification of hydraulic tunnel lining structure.

Durability Grade	Durability Status of Hydraulic Tunnel Lining Structure	Performance Status
I	Slightly damaged	No damage or minor damage to the lining structure, no effect on normal operation
II	Generally damaged	Damage to the lining structure exists and has a potential impact on normal operations, requiring appropriate repairs
III	Medium damaged	Moderate damage to the lining structure, affecting operational safety and requiring major repairs
IV	Severely damaged	Serious damage to the lining structure, which seriously affects the safety of operation and requires timely drainage and reinforcement

There is no established division standard for the evaluation level of index, and the evaluation criteria is carried out for a tunnel section from 6 + 240 to 6 + 346.26 with reference to the Technical Code for Detection and Evaluation of Hydraulic Concrete Structure (DL/T5251-2010) [29], the Specification for Design of Hydraulic Tunnel (SL279-2016) [30], and the research results of related scholars' [1,27,28] delineation, as shown in Table 3.

Table 3. Evaluation criteria for durability evaluation index of a hydraulic tunnel lining structure.

Evaluation Index	I	II	III	IV
Carbonization coefficient (k_a)	(0, 0.4]	(0.4, 0.7]	(0.7, 1]	(1, 1.6]
Rebar corrosion rate (%)	(0, 5]	(5, 15]	(15, 25]	(25, 40]
Lining concrete strength (k_b)	[0.8, 1]	(0.8, 0.6]	(0.6, 0.4]	(0.4, 0]
Crack width (mm)	(0, 0.2]	(0.2, 0.4]	(0.4, 0.6]	(0.6, 3]
Crack length (m)	(0, 1]	(1, 2.5]	(2.5, 5]	(5, 6]
Crack depth/Structure thickness	(0, 1/5]	(1/5, 1/3]	(1/3, 1/2]	(1/2, 1]
Water leakage status	Slightly damaged [0, 2)	Generally damaged [2, 4)	Medium damaged [4, 6)	Severely damaged [6, 8)
PH value	(8, 6]	(6,5]	(5, 4]	(4, 0]
Frost damage	Ice present but not affecting water flow [0, 2)	Ice affecting water flow [2, 4)	Ice can greatly affect the water flow [4, 6)	Ice seriously affects water flow [6, 8)
Lining thickness (k_c)	[2/3, 1]	(2/3, 1/2]	(1/2, 1/3]	(1/3, 0]
Cavity depth (mm)	(0, 40]	(40, 100]	(100, 500]	(500, 800]

Note: k_a = carbonization depth/protective layer thickness; k_b = actual strength of lining/design strength of lining; k_c = actual thickness/design thickness.

4.4. Analysis of Identical-Discrepancy-Contrary Linkage Affiliation and Durability Evaluation Index Weight

In order to eliminate the influence of different evaluation indexes of different magnitudes and make them comparable, the extreme value processing method is used to do dimensionless processing of the original evaluation index data. If the effect of the index on durability is positively correlated, that is, the larger the value, the worse the durability grade, it is processed according to Equation (24); for the contrary, it is processed according to Equation (25).

$$X_i' = \frac{X_i - X_{imin}}{X_{imax} - X_{imin}} \quad (24)$$

$$X_i' = \frac{X_{imax} - X_i}{X_{imax} - X_{imin}} \quad (25)$$

The rank intervals of evaluation indexes after the processing of Equations (24) and (25) are shown in Table 4:

Table 4. Normalized results of each evaluation index.

Evaluation Index	I	II	III	IV
Carbonization coefficient (a1)	(0, 0.25]	(0.25, 0.44]	(0.44, 0.63]	(0.63, 1]
Rebar corrosion rate (a2)	(0, 0.13]	(0.13, 0.38]	(0.38, 0.63]	(0.63, 1]
Lining concrete strength (a3)	(0, 0.2]	(0.2, 0.4]	(0.4, 0.6]	(0.6, 1]
Crack width (a4)	(0, 0.07]	(0.07, 0.13]	(0.13, 0.2]	(0.2, 1]
Crack length (a5)	(0, 0.17]	(0.17, 0.42]	(0.42, 0.83]	(0.83, 1]
Crack depth/Structure thickness (a6)	(0, 1/5]	(1/5, 1/3]	(1/3, 1/2]	(1/2, 1]
Water leakage status (a7)	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1)
PH value (a8)	(0, 0.25]	(0.25, 0.38]	(0.38, 0.5]	(0.5, 1]
Frost damage (a9)	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1)
Lining thickness (a10)	(0, 1/3]	(1/3, 1/2]	(1/2, 2/3]	(2/3, 1]
Cavity depth (a11)	(0, 0.05]	(0.05, 0.13]	(0.13, 0.63]	(0.63, 1]

The test information and data of each evaluation index of a hydraulic tunnel are normalized and dimensionless using Equations (24) and (25), and the processing results are shown in Table 5.

Table 5. Normalized results of each evaluation index test data.

Index	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11
Test data	0.9	18	0.81	0.45	3.5	5/12	3.2	6.3	3.1	7/12	350
Normalization	0.563	0.45	0.19	0.15	0.583	5/12	0.4	0.213	0.388	5/12	0.438

From Equations (10)–(12), the classical domain R_j, the section domain R_p and the element to be evaluated R_0 of the durability evaluation index of the hydraulic tunnel lining structure can be obtained as follows:

$$R_j = \begin{bmatrix} N & N_1 & N_2 & N_3 & N_4 \\ C_1 & (0, 0.25) & (0.25, 0.44) & (0.44, 0.63) & (0.63, 1) \\ C_2 & (0, 0.13) & (0.13, 0.38) & (0.38, 0.63) & (0.63, 1) \\ C_3 & (0, 0.2) & (0.2, 0.4) & (0.4, 0.6) & (0.6, 1) \\ C_4 & (0, 0.07) & (0.07, 0.13) & (0.13, 0.2) & (0.2, 1) \\ C_5 & (0, 0.17) & (0.17, 0.42) & (0.42, 0.83) & (0.83, 1) \\ C_6 & (0, 1/5) & (1/5, 1/3) & (1/3, 1/2) & (1/2, 1) \\ C_7 & (0, 0.25) & (0.25, 0.5) & (0.5, 0.75) & (0.75, 1) \\ C_8 & (0, 0.25) & (0.25, 0.38) & (0.38, 0.5) & (0.5, 1) \\ C_9 & (0, 0.25) & (0.25, 0.5) & (0.5, 0.75) & (0.75, 1) \\ C_{10} & (0, 1/3) & (1/3, 1/2) & (1/2, 2/3) & (2, 3/1) \\ C_{11} & (0, 0.05) & (0.05, 0.13) & (0.13, 0.63) & (0.63, 1) \end{bmatrix}$$

$$R_p = \begin{vmatrix} N_p & \\ & c_1 \sim c_2 \\ & (0, 1) \end{vmatrix}$$

$$R_0 = \begin{vmatrix} N_0 & & & & & & & & & & \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} \\ 0.563 & 0.45 & 0.19 & 0.15 & 0.583 & 5/12 & 0.4 & 0.213 & 0.388 & 5/12 & 0.438 \end{vmatrix}$$

4.4.1. Identical-Discrepancy-Contrary Linkage Affiliation Analysis

The identical-discrepancy-contrary linkage affiliation $\mu_{N_j}(v_i)$ of each evaluation index of hydraulic tunnel lining structure relative to durability class is calculated according to Equations (13)–(20). For example, the dimensionless value of carbonization coefficient C1 is $0.563 \in (0.44, 0.63)$, that is, the dimensionless value v_1 of index C1 lies within the standard positive domain $X_0 = (0.44, 0.63)$, then index C1 belongs to the same degree $\mu_{N_3}(v_1)$ of durability class N_3 is calculated as follows:

$$\mu_{N_3}(v_1) = \frac{-\rho(v_1, X_0)}{\left|c_{ij}^+ - c_{ij}^-\right|} = \frac{-\left|0.563 - \frac{0.63+0.44}{2}\right| + \frac{0.63-0.44}{2}}{|0.63 - 0.44|} = 0.353$$

The degree of difference $\mu_{N_2}(v_1)$ and $\mu_{N_4}(v_1)$ of index C1 belonging to durability classes N_2 and N_4 are calculated as follows:

$$\mu_{N_2}(v_1) = \frac{\rho(v_1, X_1)}{\rho(v_1, X) - \rho(v_1, X_1)} = \frac{0.123}{-0.313 - 0.123} = -0.282$$

$$\mu_{N_4}(v_1) = \frac{\rho(v_1, X_2)}{\rho(v_1, X) - \rho(v_1, X_2)} = \frac{0.067}{-0.313 - 0.067} = -0.176$$

$$\rho(v_1, X) = \left|0.563 - \frac{1 + 0.25}{2}\right| - \frac{1 - 0.25}{2} = -0.313$$

$$\rho(v_1, X_1) = \left|0.563 - \frac{0.44 + 0.25}{2}\right| - \frac{0.44 - 0.25}{2} = 0.123$$

$$\rho(v_1, X_2) = \left|0.563 - \frac{1 + 0.63}{2}\right| - \frac{1 - 0.63}{2} = 0.067$$

The opposite degree of index C1 belonging to durability class N_1 is $\mu_{N_4}(v_1) = -1$.

Similarly, the single-index identical-discrepancy-contrary linkage affiliation of each evaluation index of hydraulic tunnel lining structure for the four durability classes can be calculated, and the results are shown in Table 6.

Table 6. Single-index identical-discrepancy-contrary linkage affiliation.

Index	Single-Index Affiliation			
	I	II	III	IV
C_1	−1.000	−0.282	0.353	−0.176
C_2	−1.000	−0.179	0.280	−0.360
C_3	0.050	−0.050	−1.000	−1.000
C_4	−1.000	−0.200	0.286	−0.385
C_5	−1.000	−0.283	0.398	−0.374
C_6	−1.000	−0.278	0.500	−0.278
C_7	−0.300	0.400	−0.222	−1.000
C_8	0.148	−0.181	−1.000	−1.000
C_9	−0.276	0.448	−0.236	−1.000
C_{10}	−0.250	0.500	−0.250	−1.000
C_{11}	−1.000	−0.443	0.384	−0.331

4.4.2. Determining the Weight of Durability Evaluation Index

The subjective weight of evaluation index is calculated according to the G1 method. Combined with expert opinions, the sequence relation of the criteria layer indexes and the importance ratio of adjacent indexes are determined: B2 > B3 > B1 > B4; $r_2 = 1.2$, $r_3 = 1.1$, $r_4 = 1.4$. According to Formula (2), $\alpha_4 = 0.1728$, then according to Formula (3): $\alpha_3 = r_4\alpha_4 = 0.2419$,$\alpha_2 = r_3\alpha_3 = 0.266$, $\alpha_1 = r_2\alpha_2 = 0.3193$. Therefore, the subjective weights of criteria layer indexes B1, B2, B3, and B4 are 0.2419, 0.3193, 0.2660, and 0.1728, respectively. Similarly, the subjective weight of C1, C2, and C3 in the index layer under the criterion layer B1 can be calculated as 0.3216, 0.3860, and 0.2924, respectively.

According to the above results, the subjective weights of indexes C1, C2, and C3 relative to criterion layer B1 are multiplied by the subjective weight of B1 relative to the target layer, and then the subjective weights of C1, C2, and C3 relative to the target layer are 0.0778, 0.0934, and 0.0707, respectively. Similarly, the subjective weights of other evaluation indexes for the target layer can be obtained, and the specific calculation results are shown in Table 6.

The objective weights of evaluation indexes are calculated by using the simple correlation function method, and in this engineering example, the data in Tables 3 and 4 are brought into Equations (4)–(7) to obtain the objective weights of each evaluation index β_i. Finally, the MIE principle is used to eliminate the deviation of the subjective and objective weights, and the final calculation results of combination weight are obtained by substituting α_i and β_i into Equation (9), as shown in Table 7.

Table 7. Calculation results of index weight.

Index	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11
Subjective weight	0.0778	0.0934	0.0707	0.1027	0.0934	0.1232	0.0976	0.061	0.1074	0.0823	0.0905
Objective weight	0.1137	0.1040	0.0245	0.1048	0.1197	0.1334	0.0800	0.0288	0.0843	0.0889	0.1179
Combination weight	0.0954	0.0999	0.0422	0.1052	0.1072	0.1300	0.0896	0.0425	0.0965	0.0867	0.1048

4.5. Durability Evaluation Grade and Variation Trend Analysis of Hydraulic Tunnel Lining Structure

According to the single-index affiliation $\mu_{N_j}(v_i)$ of the hydraulic tunnel lining structure in Table 5 and the combination weight ω_i of each evaluation index in Table 6, according to Equation (21), the comprehensive linkage affiliation of hydraulic tunnel lining structure for the four durability classes $\mu_{N_j} = (\mu_{N_1}, \mu_{N_2}, \mu_{N_3}, \mu_{N_4}) = (-0.7090, -0.0620, 0.0906, -0.5620)$, the durability class of the hydraulic tunnel lining structure of this section can be determined as III according to the maximum affiliation determination criterion.

According to the combination weight ω_i of each evaluation index of the hydraulic tunnel lining structure in Table 6, the expression $\varepsilon = 0.6425 + 0.2728p + 0.0847q$ for the connection degree of this tunnel can be calculated using Equation (22). By $\sum_{v_i \in N_3} \omega_i = 0.6425, \sum_{v_i \in N_{2,4}} \omega_i = 0.2728, \sum_{v_i \in N_1} \omega_i = 0.0847$, which can be obtained as $\sum_{v_i \in N_3} \omega_i = 0.6425 > \sum_{v_i \in N_{2,4}} \omega_i = 0.2728 > \sum_{v_i \in N_1} \omega_i = 0.0847$, and further calculating the set pair potential $\mathrm{SHI(H)} = 7.5856$ for this section of the tunnel according to Formula (23), indicating that the development trend of durability status of hydraulic tunnel lining structure in this section is weak homoeopathy.

From the above results, it can be seen that the durability class of hydraulic tunnel lining structure is III in this section, which means that the section of the tunnel as a whole is medium damaged, affecting operational safety and requiring major repairs. Through the analysis of tunnel damage and field survey results, it can be found that the lining structure is indeed medium damaged, with a certain degree of cracks, water leakage, and rebar corrosion, the evaluation result is consistent with the actual engineering situation of the tunnel, which illustrates the scientific and reasonable nature of the established hydraulic tunnel lining structure durability evaluation index system and evaluation model. According to the results of the set pair potential of the tunnel, it is known that the development trend of its durability condition is weak homoeopathy, which indicates that the tunnel belongs to level III is not strong, and the possibility of changing to level IV condition is very large, so the relevant departments of the tunnel jurisdiction section should pay attention to it and take effective and reasonable solutions to prevent the deterioration of the durability condition of the tunnel.

4.6. Comparative Study

In order to illustrate the scientificity and rationality of this method, this paper uses the AHP-Extenics method to do a comparative study with the above content.

4.6.1. Analytic Hierarchy Process (AHP)

Analytic Hierarchy Process (AHP) is a subjective weighting method, which is a method for ranking the pros and cons of various factors and analyzing the hierarchical weight decision [31]. The process of calculating the evaluation index weight is as follows:

(1) Constructing a judgment matrix

After stratifying the studied problem based on objectives, criteria, and solutions, different indexes at the same layer can be assigned relative importance by the method of two-by-two comparison, so as to judge the relative importance of the indexes at the lower layer to the indexes at the upper layer, and then calculate the weights of each evaluation index. If there are n evaluation indexes in a certain layer, the judgment matrix of Equation (26) can be constructed.

$$A = (a_{ij})_{n \times n} = \begin{bmatrix} a_{11} & L & a_{1n} \\ L & L & L \\ a_{n1} & L & a_{nn} \end{bmatrix} \tag{26}$$

where $i = 1, 2, \cdots, n; j = 1, 2, \cdots, n; a_{ij}$ is the important result of comparing the two evaluation indexes i and j in the criteria A.

In this paper, based on the expert scoring method, the relative importance between two factors is quantified by comparing the factor indexes two by two according to the

1–9 scale method [32] to determine the underlying data, and the 1–9 scale is shown in Table 8.

Table 8. Relationship between numerical value and extent.

Numerical Value	Extent
1	Equal importance
3	Moderate importance of one over another
5	Essential or strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate value between two adjacent judgements

(2) Calculating the weight of evaluation index

Based on the judgment matrix A obtained from pairwise comparison of evaluation indexes, the eigenvector δ corresponding to the maximum eigenvalue λ_{max} is calculated. Then, the problem of calculating the weight of the evaluation indexes is transformed into the problem of solving for the eigenvector δ, with the following Equation (27):

$$A\delta = \lambda_{max}\delta \tag{27}$$

In this paper, the calculation of the weights is based on the square root method, and the calculation process is as follows [1]:

Step 1: The elements in the judgment matrix A are multiplied by rows to obtain u_i, see Equation (28).

$$u_i = \prod_{j=1}^{n} a_{ij}, i = 1, 2, \cdots n \tag{28}$$

Step 2: u_i is squared n times to obtain u_i', see Equation (29).

$$u_i' = \sqrt[n]{u_i} \tag{29}$$

Step 3: Regularize u_i' to solve for the eigenvector δ, see Equation (30).

$$\delta_i = \frac{u_i'}{\sum_{i=1}^{n} u_i'} \tag{30}$$

Step 4: Calculate the maximum eigenvalue λ_{max} of the judgment matrix A, see Equation (31).

$$\lambda_{max} = \sum_{i=1}^{n} \frac{(A\delta)_i}{n\delta_i} \tag{31}$$

(3) Consistency test

In order to judge the reasonableness of the above weight calculation, the consistency of the judgment matrix A needs to be tested, and the specific process is as follows:

Step 1: Calculate the consistency index CI, see Equation (32).

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{32}$$

Step 2: Calculate the consistency ratio CR, see Equation (33).

$$CR = \frac{CI}{RI} \tag{33}$$

where RI is the average random consistency index, which can be found by checking Table 9 [33].

Table 9. Average random consistency index RI.

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

According to the consistency test criteria, when $CR < 0.10$, the judgment matrix A satisfies the consistency test. In contrast, experts are required to re-score and construct a new judgment matrix until the consistency test is satisfied.

4.6.2. Matter–Element Extension

The theory of matter–element has already been described in the previous section and the focus here is on the calculation of the correlation function values for the element class of the hydraulic tunnel lining structures to be evaluated.

(1) Calculation of the correlation of evaluation index

The correlation of the durability class j of the hydraulic tunnel lining structure to be evaluated is shown in Equations (34)–(36):

$$\rho(v_i, V_{ij}) = \left| v_i - \frac{c_{ij}^{+} + c_{ij}^{-}}{2} \right| - \frac{c_{ij}^{+} - c_{ij}^{-}}{2} \tag{34}$$

$$\rho(v_i, V_{ip}) = \left| v_i - \frac{c_{ip}^{+} + c_{ip}^{-}}{2} \right| - \frac{c_{ip}^{+} - c_{ip}^{-}}{2} \tag{35}$$

$$K_j(v_i) = \frac{\rho(v_i, V_{ij})}{\rho(v_i, V_{ip}) - \rho(v_i, V_{ij})} \tag{36}$$

where $K_j(v_i)$ is the correlation degree of the object to be evaluated when the evaluation level is j.

(2) Determine the durability level of hydraulic tunnel lining structure

By combining the weights of each index and the correlation function values, the rank correlation of the evaluation object can be obtained, see Equation (37); if $K_{j_0}(N_0) = maxK_j(N_0)$, then the evaluation object N_0 belongs to j_0 level.

$$K_j(N_0) = \sum_{i=1}^{n} \omega_i K_j(v_i) \tag{37}$$

where ω_i is the weight value of the evaluation index u_i, the larger its value, the greater the degree of influence of the index on the evaluation object.

4.6.3. Calculation of Durability Evaluation Index

According to the durability evaluation index system of hydraulic tunnel lining structure in Figure 3, experts with rich experience in tunnel construction are invited to score and judgment matrices for different layers of indexes are constructed according to the 1–9 scale method, and weights are calculated for the durability evaluation indexes of hydraulic tunnel lining structure. The judgment matrix of target layer A–B and its weight distribution are shown in Table 10.

Table 10. A–B judgment matrix and weight assignment.

A	B_1	B_2	B_3	B_4	W_i	Consistency Check
B_1	1	1/2	1	2	0.2330	$\lambda_{max} = 4.0457$
B_2	2	1	1	3	0.3647	
B_3	1	1	1	2	0.2771	$CR = 0.0171 < 0.1$
B_4	1/2	1/3	1/2	1	0.1252	

The calculation process of the criterion layer B–C is the same as that of the target layer, and the calculation results are shown in Table 11.

Table 11. Calculation results of the index weight of criterion layer.

Index	W_i	Consistency Check
C_1, C_2, C_3	0.3108, 0.4934, 0.1958	$\lambda_{max} = 3.0536$ $CR = 0.0516 < 0.1$
C_4, C_5, C_6	0.3275, 0.2599, 0.4126	$\lambda_{max} = 3.0536$ $CR = 0.0516 < 0.1$
C_7, C_8, C_9	0.4286, 0.1429, 0.4286	$\lambda_{max} = 3.0000$ $CR = 0 < 0.1$
C_{10}, C_{11}	0.3333, 0.6667	$\lambda_{max} = 2.0000$ $CR = 0 < 0.1$

From the above calculation results, it can be seen that in the durability evaluation analysis of hydraulic tunnel lining structure, the index weight of target layer A–B is $W_{A\text{-}B}$ = (0.2330, 0.3647, 0.2771, 0.1252)^T, the index weight of criterion layer B–C is $W_{B\text{-}C}$ = (0.3108, 0.4934, 0.1958, 0.3275, 0.2599, 0.4126, 0.4286, 0.1429, 0.4286, 0.3333, 0.6667)^T, and the judgment matrix of each layer meets the consistency requirements. On this basis, the comprehensive weight of each durability index can be calculated, as shown in Table 12.

Table 12. Durability evaluation index weight of hydraulic tunnel lining structure.

Target Layer	First-Level Index	Primary Weight	Secondary Index	Secondary Weight	Comprehensive Weight
A	B_1	0.2330	C_1	0.3108	0.0724
			C_2	0.4934	0.1150
			C_3	0.1958	0.0456
	B_2	0.3647	C_4	0.3275	0.1194
			C_5	0.2599	0.0948
			C_6	0.4126	0.1505
	B_3	0.2771	C_7	0.4286	0.1188
			C_8	0.1429	0.0396
			C_9	0.4286	0.1188
	B_4	0.1252	C_{10}	0.3333	0.0417
			C_{11}	0.6667	0.0835

4.6.4. Comprehensive Evaluation of AHP-Extenics

The durability grade range value of each evaluation index and the sample measured value are normalized, and the results are shown in Tables 3 and 4.

The correlation degree of durability evaluation indexes for the hydraulic tunnel lining structure to be evaluated can be calculated from Equations (34)–(36), and the calculation results are as follows:

$$K_j(v_i) = \begin{bmatrix} -0.4170 & -0.2200 & 0.1811 & -0.1329 \\ -0.4160 & -0.1350 & 0.1842 & -0.2857 \\ 0.0556 & -0.0500 & -0.5250 & -0.6833 \\ -0.3480 & -0.1180 & 0.1538 & -0.2500 \\ -0.4980 & -0.2810 & 0.6417 & -0.3720 \\ -0.3420 & -0.1670 & 0.2500 & -0.1677 \\ -0.2730 & 0.3333 & -0.2000 & -0.4667 \\ 0.2102 & -0.1480 & -0.4390 & -0.5740 \\ -0.2620 & 0.4058 & -0.2240 & -0.4827 \\ -0.1670 & 0.2500 & -0.1670 & -0.3750 \\ -0.4700 & -0.4130 & 0.7805 & -0.3048 \end{bmatrix}$$

The weight values of each secondary index calculated by the AHP are: $W_{B1\text{-}C}$ = (0.3108, 0.4934, 0.1958)^T, $W_{B2\text{-}C}$ = (0.3275, 0.2599, 0.4126)^T, $W_{B3\text{-}C}$ = (0.4286, 0.1429, 0.4286)^T, $W_{B4\text{-}C}$ = (0.3333, 0.6667)^T. According to Equation (36), the comprehensive correlation degree of the criterion layer for the durability of this hydraulic tunnel lining structure regarding the evaluation level j can be calculated as:

$$K_j(B_1) = (-0.3239, -0.1445, 0.0444, -0.3161)$$

$$K_j(B_2) = (-0.3844, -0.1803, 0.3203, -0.2473)$$

$$K_j(B_3) = (-0.1993, 0.2956, -0.2450, -0.4889)$$

$$K_j(B_4) = (-0.3687, -0.1919, 0.4648, -0.3282)$$

The weight value of the first-level index calculated by the AHP is $W_{A\text{-}B}$ = (0.2330, 0.3647, 0.2771, 0.1252)^{T}. Similarly, according to Equation (36), the comprehensive correlation degree of the target layer for the durability of this hydraulic tunnel lining structure regarding the evaluation level j can be calculated as:

$$K_j(A) = (-0.3170, -0.0415, 0.1176, -0.3404)$$

According to the principle of maximum affiliation, the durability class of the hydraulic tunnel lining structure can be determined as Class III, which is medium damaged.

The method in this paper is a coupling of the set pair analysis and the extension theory, and to verify its practicality, it is used as a comparison with the research results of related scholars and the AHP-Extenics method. The research of related scholars is based on ANP and Cloud-Model-Improved Matter–Element theory to evaluate the durability of hydraulic tunnel lining. This method is an improvement of the traditional hierarchical analysis and material element theory. The evaluation model takes into account the fuzzy and random nature of durability indexes, and the specific evaluation process is shown in the literature [1]. It can be found that the durability evaluation results of these three evaluation methods for the tunnel section 6 + 240 to 6 + 346.26 are consistent, and the durability class is III, which is also consistent with the actual damage state of this hydraulic tunnel lining structure. In the comparative study, the calculation of index weight through the traditional AHP method is difficult to ensure the accuracy and scientificity of the weight due to the uneven level of experts. In contrast, this paper uses the G1 method and the simple correlation function method to calculate the subjective and objective weight of evaluation index, respectively, which avoids the one-sidedness of weight calculation and takes into account the subjective intention of decision makers and the objective attributes of the data itself. Meanwhile, this paper uses the set pair analysis and extension coupling model to calculate the affiliation of each index, and then determines the durability level, and the obtained evaluation results are consistent with the results determined by the AHP-Extenics method, which highlights the scientific and rational nature of the method proposed in this paper.

5. Conclusions

(1) According to the characteristics of durability evaluation indexes of hydraulic tunnel lining structure, the subjective weights calculated by G1 method and the objective weights calculated by simple correlation function method are optimized and synthesized to obtain the combination weight of each evaluation index, eliminating the one-sidedness of single weight calculation method, improving the rationality and reliability of the weights of tunnel durability evaluation indexes, and making the evaluation results more consistent with reality.

(2) The set pair analysis can describe the definite and indefinite connection of things, and the Extenics theory can quantitatively describe the process of quantitative and qualitative changes of things and solve the objective contradiction problem. Combining the two models, a comprehensive evaluation model based on the set pair analysis and extension coupling is proposed. This method not only provides an objective and comprehensive description of the nature of tunnel durability evaluation through the Extenics set, but also organically integrates the theoretical domain division of the Extenics set and "the identical-discrepancy-contrary idea" of set pair analysis in the description of durability evaluation grade, and then constructs the identical-discrepancy-contrary affiliation function, which realizes the comprehensive evaluation of durability level for the hydraulic tunnel lining structure. At the same time, the set pair potential analysis can reasonably describe the trend and possibility of transformation for the tunnel durability condition, and improve the accuracy of tunnel durability evaluation.

(3) Taking a section of hydraulic tunnel as an example, the model constructed in this paper is used to calculate its durability class as III, and the set pair potential SHI(H) = 7.5856. The evaluation results are consistent with the engineering practice, and a comparative study is done in combination with the AHP-Extenics method to verify the applicability and rationality of the model, which can provide a basis for the maintenance and reinforcement of hydraulic tunnel lining structure.

(4) A comprehensive evaluation index system for the durability of hydraulic tunnel lining structure is constructed by selecting 11 factors from 4 aspects: material deterioration, lining cracks, water leakage, and lining thickness and cavity behind. However, the factors that affect its durability have certain characteristics of randomness and fuzziness. In the future research process, the evaluation index system needs to be more perfect and the grading standard needs to be further refined, which can further improve the accuracy of the durability evaluation results.

(5) The set pair analysis and extension coupling model combines the advantages of set pair analysis and extension theory, fully considers the fuzziness and randomness in the process of durability evaluation, and provides a relatively novel method for the durability evaluation of hydraulic tunnel lining structure. In the future, with the help of computer programming software, an evaluation software system based on this model can be developed to form a management network for durability testing and evaluation of hydraulic tunnel lining structures, so as to grasp the durability dynamics of hydraulic tunnel lining structures in a timely manner.

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