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*Article*

# An Integrated Strategy for Rescheduling High-Speed Train Operation under Single-Direction Disruption

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**Abstract:** Comparing to other modes of transportation, high-speed railway has the advantages of energy saving, environment friendly, safety and convenience for passengers, and has been more and more popular. However, unforeseen emergencies may disrupt the normal train operation. In this paper, an integrated dispatch strategy (IDS) is proposed to synergistically reschedule the train timetable and rolling stock circulation plan under single-direction disruptions. A two-objective model is formulated, aiming at minimizing both the delay time of passengers and the operation costs of railway companies, to reschedule the train operation efficiently and economically. An algorithm based on Non-dominated Sorting Genetic Algorithms-II (NSGA-II) is designed to solve the model. To accelerate the solving process, we propose a quick method to generate an assignment plan to serve disrupted passengers, and based on the practical experiences, the algorithm acceleration strategy (AAS) is proposed to improve the quality of initial solutions. The model and algorithm are tested on real-world instances of the Beijing-Shanghai high-speed railway line. The results indicate that the average minimized delay time of passengers is 6,012,386 min and the average minimized additional operation costs (operation mileage of standby rolling stocks) are 1623 km, with a decrease of 28.5% and 18.3%, respectively, indicating the model and algorithm are adaptable to handle single-direction disruptions on the railway line, and AAS can further accelerate the computing speed and improve the solutions quality. Finally, the characteristics of disrupted sections of railway lines are well studied and analyzed.

**Keywords:** high-speed railway; train timetable; rolling stock circulation plan; integrated dispatch strategy (IDS); NSGA-II; algorithm acceleration strategy (AAS)



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## 1. Introduction

In recent years, high-speed railway (HSR) has experienced remarkable growth, emerging as a prominent transportation mode worldwide. It not only has the environmental protection and energy-saving effects, but more importantly, the main source of HSR is electricity rather than oil. Therefore, it greatly reduces the dependence on oil, optimizes the energy consumption structure, reduces carbon emissions, and contributes to the sustainable development of energy.

To ensure the safety and efficiency of HSR operation, the railway companies schedule the train timetable and rolling stock circulation plan before operation. The train timetable specifies the departure time and arrival time of the trains at each station along the journey, and the rolling stock circulation plan specifies the transport tasks of the rolling stocks; that is, trains in the timetable will be performed by which rolling stocks. Timetable ensures the safety and efficiency of train operation, and rolling stock circulation plan is designed for completing the operation of the timetable with as little costs as possible.

Under normal circumstances, the trains operate according to the timetable and rolling stock circulation plan. However, unforeseen emergencies, such as rolling stock equipment failure, track faults, and severe weather, may disrupt train operations. In a disrupted situation, the original timetable cannot be performed as scheduled. As the duration of the disruption increases, more and more trains and passengers will be influenced. From the passengers' point of view, the operators need to arrange adjustment measures to reschedule the timetable and rolling stock circulation plan in a short time, to minimize the negative influences on passenger service. From the railway companies' point of view, some adjustment measures will increase additional operation costs, such as the growth in operating mileage of rolling stocks, which leads to more energy consumption. Thus, it contributes to sustainability to study rescheduling train operation with as low additional operation costs as possible.

On this basis, the study in this paper is conducted to propose a strategy to integrally reschedule the train timetable and rolling stock circulation plan and obtain a Pareto Front, considering both passenger service quality and additional operation costs, so as to provide a set of solutions for operators to handle the disruptions on railway line well. In this paper, we propose an **integrated dispatch strategy (IDS)** to synergistically reschedule the train timetable and rolling stock circulation plan under the circumstance of disruption. IDS includes not only the adjustment measures of rescheduling the train timetable, like cancelling trains, adjusting trains sequence, and coupling short trains, but also the measures of rescheduling rolling stock circulation plan, like adjusting the transport tasks of rolling stocks and operating standby rolling stocks.

1. Cancelling trains

As the duration of the disruption increases, there will be more and more trains disrupted. After the disruption finishes, so many trains need to operate. However, the capacity of the railway line is limited, and it cannot be guaranteed that all trains can be arranged. Thus, in this case, operators should determine which trains will have to be cancelled, to complete the remaining train services in the timetable efficiently and safely.

For passengers of the cancelled trains, they may have to take other trains or cancel their trips, which damage the passenger service quality. For railway companies, canceling trains can reduce the rescheduling complexity.

2. Adjusting trains sequence

As shown in Figure 1a, the gray rectangle indicates a single-direction disruption which ends at $t_{re}$, as shown by the yellow dashed line. The red, green and blue lines represent trains ①, ② and ③, respectively. Letters A−G indicate the stations on the railway line. We suppose that station E only has one platform, which means at most one train is allowed to stop, train ① stops at station E, and trains ②, ③ have to stop at station F. The transport tasks shown by the dashed line cannot be performed, since trains are not allowed to pass through the disrupted section during the disruption. Figure 1b shows the rescheduled timetable that all trains operate as the original sequence after the disruption finishes. We can see that $t_1$ is not utilized, which damages the operation efficiency. In Figure 1c, we adjusting the sequence of trains ① and ② in section E–D. Train ① immediately departs from station E, instead of waiting for train ② passing through. Comparing Figure 1b,c, it can be seen that the timetable with adjusting trains sequence uses less time to complete the train services ($T_2 < T_1$), which can reduce the total delay time of the passengers served by these trains.

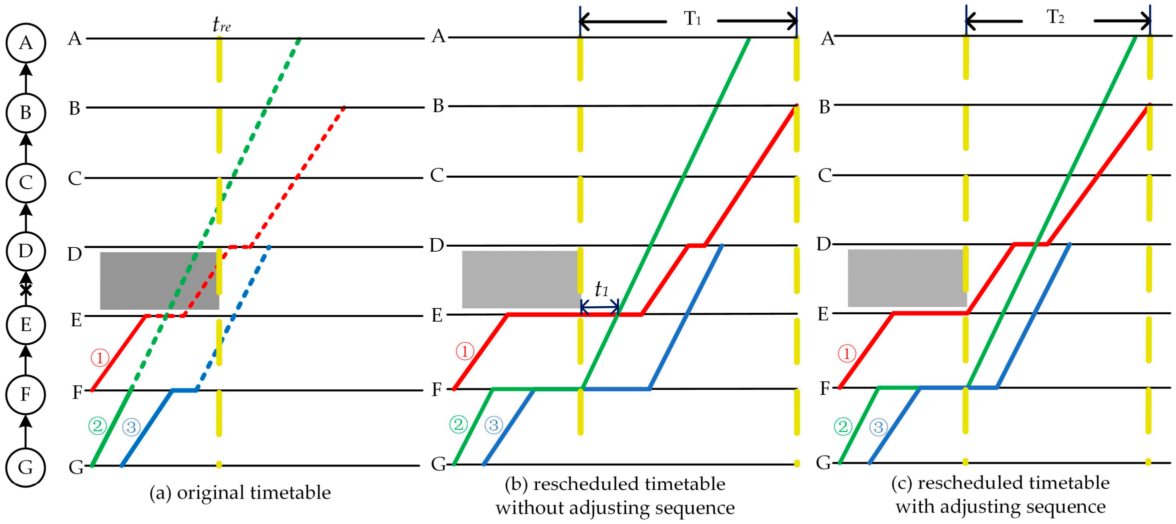


**Figure 1.** Comparison of adjusting trains sequence.

3. Coupling short trains

Coupling and decoupling short trains is a common operation method in many countries. In the rescheduling process, we can use the adjustment measure of coupling short trains. As shown in Figure 2, trains ① and ② are two disrupted short trains, and they will operate as shown by the blue and red dashed line without coupling. If we couple trains ① and ② into train ③, less time will be taken to complete the transport task ($t_2 < t_1$), reducing the delay time of passengers served by the train. Besides, the number of operating trains is reduced, which improves the train operation efficiency. It is essential to determine which trains should be coupled and decoupled according to their journey and stops.

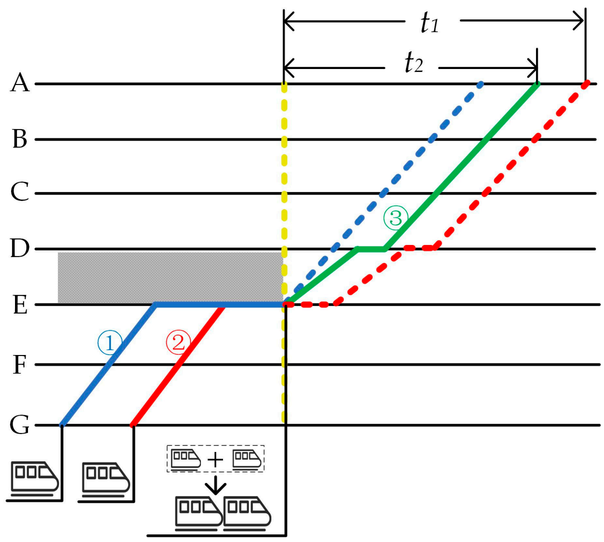


**Figure 2.** Comparison of recoupling short trains.

4. Adjusting the transport tasks of rolling stocks

As shown in Figure 3a, trains ① and ④ are performed by rolling stock *I*, and trains ② and ③ are performed by rolling stock *II*. If rolling stock *II* cannot arrive at station A on time, train ③ may be delayed. If we adjust the transport tasks of rolling stocks like Figure 3b, trains ③ and ④ will operate as scheduled, which reduces the delay time of passengers.

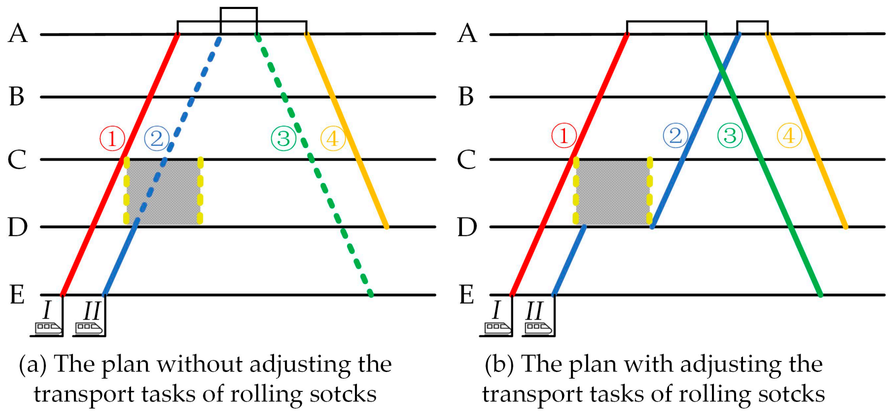


**Figure 3.** Adjusting the transport tasks of rolling stocks.

5. Operating standby rolling stocks

There are depots located near major stations. Depots are responsible for the rolling stocks parking and maintenance. Besides, there are standby rolling stocks in depots, to deal with the temporary transport demands.

As shown in Figure 4a, there are 2 depots near stations A and D, and we call them depots A and D respectively. We suppose that train ① is disrupted and cannot arrive at its destination station on time. Generally, there are 2 standby rolling stocks operation routings we can choose to deal with this situation. The first routing is that we can dispatch standby rolling stock *I* from depot A to perform trains ④ and ⑦ directly, as shown in Figure 4b, and the second routing is that we can dispatch standby rolling stock *II* from depot D to perform trains ④ and ⑦, and part of delayed passengers of train ① can be served by the rolling stock, as shown in Figure 4c. After the transport tasks finish, the standby rolling stocks need to return to their depots. We can see that compared to rolling stock *I*, rolling stock *II* has a longer operation mileage, which means higher operation costs, meanwhile, it serves more delayed passengers, which decreases the delay time of passengers. Thus, it makes great sense to operate standby rolling stocks efficiently and economically, considering there are more depots and delayed trains in real-world operation, which means more standby rolling stocks operation routings to choose.

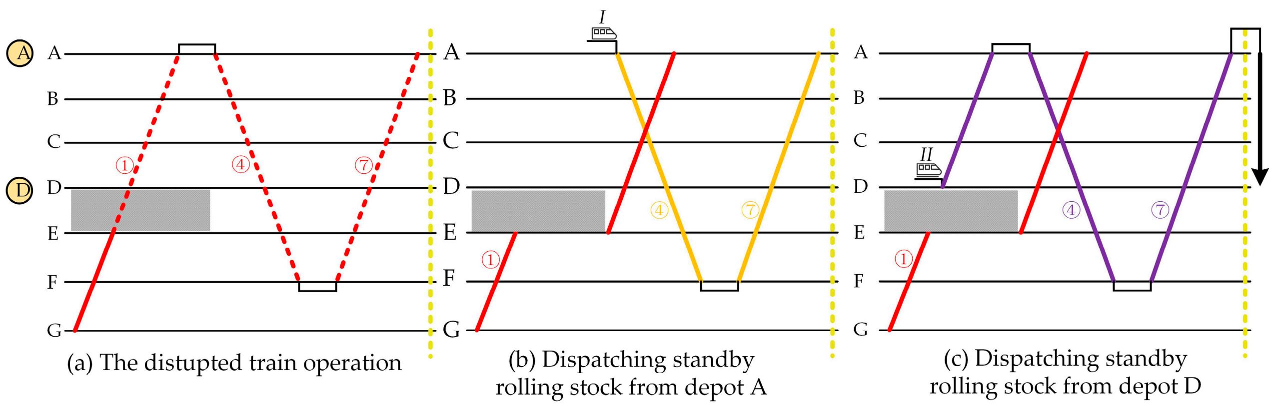


**Figure 4.** Operating standby rolling stocks.

From the analysis of the above measures, we formulate a multi-objective model, including two objectives, that are, from the passengers' point of view, we want to minimize the total delay time of passengers, and from the railway companies' point of view, we want to minimize the additional operation costs. A series of constraints are adopted to ensure the safety and efficiency of train operation. An algorithm based on NSGA-II is designed to solve the model and obtain a Pareto set. To accelerate the solving process, a quick method is devised to generate an assignment plan to serve the delayed passengers and the Algorithm Acceleration Strategy (AAS) is proposed. The model and the algorithm are tested on Beijing-Shanghai HSR line.

## 2. Literature Review

Sustainable development has been a popular research domain in recent years. Yadav [1] conducted a comprehensive review of the Green Lean Six Sigma (GLSS) approach, application status and potential benefits, and provided an avenue for future research work. Yaser [2] reviewed the current work about lean manufacturing and industry 4.0 and proposed a conceptual framework that can guide the implementation of the integration of lean manufacturing and industry 4.0. Malik [3] conducted a systematic literature review, summarized various research areas, and provided insight into Industry 4.0 and environmental sustainability.

HSR is a promising topic on sustainability aspect, and there is more and more research on HSR and sustainability. Azzouz [4] selected and validated social, economic, and environmental factors to evaluate the sustainable performance of HSRs. Chang [5] evaluated the sustainability of high-speed railway (HSR) construction projects in a comprehensive manner.

The train operation rescheduling is one of the main avenues of HSR research, and it has been extensively studied. In terms of the research scope, research on train operation rescheduling can be divided into three aspects, which are rescheduling the train timetable, rescheduling the rolling stock circulation plan, and co-optimization of the train timetable and rolling stock circulation plan.

As for rescheduling the train timetable, D'Ariano [6] viewed the train scheduling problem as a huge job shop scheduling problem with no-store constraints, providing a structured and efficient approach to modeling the scheduling process. Corman [7] studied train rescheduling in the situation of disturbances at a microscopic level and developed a real-time traffic management system. Zhu [8] proposed a timetable rescheduling model where flexible stopping (i.e., skipping stops and adding stops) and flexible short-turning (i.e., full choice of short-turn stations) are innovatively integrated with three dispatching measures: retiming, reordering, and cancelling trains. Louwerse [9] focused on which trains should be operated during the disruption and determining the timetable of these trains, considering both partial and complete railway line blockades. Kumar [10] rescheduled the train timetable with an amalgamated fitness function including minimizing train delay, dwell time, timetable deviation, operational cost service reliability, and the rescheduled timetable's feasibility is checked based on the dictionary-based checking technique. Lamorgese [11] studied the rescheduling problem by decomposing the problem into smaller subproblems associated with the lines and stations. Huo [12] proposed a binary mixed-integer programming model to reschedule the timetable in the emergent incidents based on priority and train order entropy. Reynolds [13] introduced a model for train timetable rescheduling that incorporates statistical methods and historical data to improve accuracy compared to fixed-speed timetable rescheduling models. Wang [14] formulated a mathematical model for timetable rescheduling under train operation time constraints, using reinforcement learning techniques to optimize the departure sequence for trains. Veelenturf [15] developed an integer linear programming model to solve the timetable rescheduling problem, aiming to minimize the number of canceled and delayed train services while adhering to infrastructure and rolling stock capacity constraints.

As for rescheduling rolling stock circulation, Nielsen [16] proposed a rolling horizon approach to reschedule rolling stock and a model was formulated to address real-time situations and applied in the rolling horizon framework. Kroon [17] described a real-time rolling stock rescheduling model that incorporates dynamic passenger flows. An iterative heuristic was presented to solve the rolling stock rescheduling model with consideration of these dynamic passenger flows. Wagenaar [18] introduced dead-heading trips and adjusted passenger demand in the Rolling Stock Rescheduling Problem (RSRP) and formulated a Mixed-Integer Linear Programming model to address the RSRP, taking passenger demand into account more accurately. Budai [19] formulated an integer linear programming model to address the RSRP and presented two heuristics to improve the speed and quality of the solution. Zeng [20] proposed an integer linear programming model based on a multi-commodity flow approach, with the objective of minimizing the difference between the adjusted and original schedule and reducing trip cancellations caused by rolling stock or crew shortages. The model integrated the rolling stock and crew rescheduling processes. van der Hurk [21] studied rolling stock rescheduling in a system with passengers having free route choice. An optimization-based algorithm was presented to minimize passenger inconvenience by providing route advice and coordinating rolling stock rescheduling with passenger advice through a passenger simulation model.

As for the co-optimization of the rescheduling of the timetable and rolling stock circulation plan, there are few research focused on it. Cadarso [22] proposed an integrated model for timetable and rolling stock rescheduling, aiming at minimizing the recovery time, the passenger inconvenience, and the incurred system costs. Hong [23] introduced an integrated approach for the recovery of a timetable by rescheduling train services and rolling stock circulation and proposed a novel integer linear programming model, with the objectives of maximizing the number of disrupted passengers arriving at their pre-planned destinations and minimizing the total delay of all trains and the number of cancelled trains.

In terms of the models and algorithms, a series of models with different objectives are formulated and many algorithms are adopted. Cacchiani [24] provided a comprehensive summary of recovery models and algorithms for real-time railway disturbance and disruption management. The study encompassed various aspects of train operation rescheduling, including real-time timetable rescheduling, rescheduling of rolling stock, and crew duties.

As for objectives of models, some research formulated models with single objective, such as minimizing the delay of passengers [25], minimizing the maximum and average consecutive delay [7,26], and minimizing deviation from original timetable [22,27]. Some research weighted multiple objectives into one objective, such as minimizing the train delays and cancellations [28–30], minimizing the additional cancelled trips the changes to the rolling stock and crew duties [20], and minimizing headway variations, cancellations, and deviation from original timetable [31]. Few research formulated multi-objective models. Fernandez-Rodriguez [32] focused on minimizing the running time and energy consumption of HSR trains.

By summarizing the above models, we find that most of the models mentioned above include one objective or weight the multiple objectives into one according to the importance of the objectives. Few research formulates multi-objective models.

As for the algorithm, many algorithms have been widely applied in the field of the optimization of train operation rescheduling, such as Column generation algorithm [33], Lagrange relaxation algorithm [34], Ant colony algorithm [35], Simulated annealing algorithm [36], etc. Wu [37] reviewed the applications of particle swarm optimization in the railway domain, such as [38–41]. In 2002, Deb [42] proposed NSGA-II, which can deal with multi-objective optimization well. In recent years, NSGA-II has been widely used in mechanical, electrical, transportation, and other fields. Martinez-Salazar [43] used NSGA-II to solve a Transportation Location Routing Problem (TLRP). Meng [44] formulated a multi-objective optimization model based on the fair ramp metering problem and used NSGA-II to solve the model. Parallel multi-objective Optimization techniques are also adopted in recent studies, to accelerate the computing speed. Wu [45,46] presented and discussed a parallel computing scheme and examined the computing performance of parallel multi-objective particle swarm optimization (pMOPSO) and parallel multi-objective genetic algorithm (pMOGA). Recently, artificial intelligence has been used to solve the train rescheduling problem. Kumar [47] used Brownian motion weighted-based salp swarm optimization (BMW-SSO) algorithm and Modified weight-based deep learning neural network (MWDLNN) algorithm to solve the train rescheduling problem.

By summarizing the above algorithms, we find that most of them mainly solve single objective model, such as [33–36,38–40]. For multi-objective model, NSGA-II is an effective algorithm [43,44].

1. Summarizing the literature mentioned above, we sum up the main work in train operation rescheduling domain and conduct the analysis as follows: Most research in train operation rescheduling domain studies the train timetable and rolling stock circulation plan independently or through a two-stage approach. However, the train timetable and rolling stock circulation plan are highly interrelated and have a significant impact on each other's performance. Thus, it will make great sense to reschedule the train timetable and rolling stock circulation plan synergistically.
2. Most of the research on train rescheduling focuses on minimizing the delay of trains, while few research considers the influenced passengers on these trains adequately. When a delay occurs, passengers may take other trains in the original timetable or standby trains. As the duration of disruption increases, more and more passengers may cancel their trip. Thus, a delayed passenger assignment plan needs to be proposed to serve them, to reduce the damage to passenger service quality.
3. The train operation rescheduling problem involves many optimization objectives, such as minimizing the delay time, minimizing the number of cancelled trains, minimizing deviation from the original timetable, etc. Some research formulates a single-objective model, which can obtain the satisfied solution in one aspect, while few of them considering the operation costs in rescheduling process well. Some research weights multiple objectives to one single objective according to their importance in the model formulating process, while there is not an acknowledged method to determine the weighting factors, and the relationship between the objectives cannot be described comprehensively. Thus, the operation costs need to be considered, and the objectives of the model need to be described better.

Based on the analysis mentioned above, we sum up the gap in train operation rescheduling. There are three main points. Firstly, few research focuses on synergistically rescheduling the train timetable and rolling stock circulation plan. Secondly, most research does not consider the influenced passengers adequately. Thirdly, few research takes additional operation costs into account.

Gearing towards addressing the gap mentioned above, this paper has three innovations. Firstly, we propose IDS to synergistically rescheduling the train timetable and rolling stock circulation plan. Secondly, we design quick method to generate an assignment plan to serve influenced passengers in the rescheduling process. Thirdly, we formulate a two-objective model and obtain a Pareto Front, considering both minimizing the delay time of passengers and additional operation costs.

The contributions of this paper are mainly in the following aspects:

1. We propose an integrated dispatch strategy called IDS, including five adjustment measures, which affect not only the operation time and stops of the trains but also the transport tasks of the rolling stocks. In the rescheduling process, we use the measures in IDS integrally, to optimize the train timetable and rolling stock circulation plan synergistically.
2. We propose a quick method to generate an assignment plan for the influenced passengers, which will not only improve passenger service quality but also simplify the solution process, making it more practical and feasible for real-world implementation.
3. We formulate a model including two objectives, i.e., minimizing the delay time of passengers and the additional operation costs, which can reduce the negative influence on passenger service and the energy consumption. The algorithm is designed based on NSGA-II, and a Pareto Front will be outputted. Railway companies could choose appropriate solutions according to the actual rescheduling scenario.
4. By analyzing the influence of using adjustment measures on train operation and passenger service, we propose an algorithm acceleration strategy (AAS) to preprocess the initial solution. The quality of the initial solution and the solving efficiency

are both improved, and the overall effectiveness of the rescheduling approach can also be enhanced.

The rest of this paper is organized as follows. In Section 3, a two-objective model is formulated to model the train operation reschedule problem. In Section 4, we introduce the algorithm, including the passenger assignment plan and AAS. In Section 5, based on the disrupted conditions on Beijing-Shanghai HSR line, we present and analyze our computational results. In Section 6, conclusions and future research are discussed.

## 3. Model Formulation

### *3.1. Problem Description*

The HSR line in this paper has double tracks in each section, with one track for each direction. When a single-direction disruption occurs in one section, trains cannot pass through the section in the direction, until the disruption finishes. We have two objectives which are minimizing the delay time of passengers and the additional operation costs.

Figure 5 gives an example of the problem which we deal with in this paper. There are six stations and two depots. A disruption with the direction of station E to D occurs. Trains running in the direction of the disruption are called **upstream trains**, such as trains ①, ②, ③, and ④, and trains running in the opposite direction are called **downstream trains**, such as trains ⑤, ⑥, and ⑦. Part of the upstream trains in the original timetable are disrupted, such as trains ① and ②, and we call these trains **disrupted trains**. The set of passengers with same origin and destination station in one train is called one **passenger demand** of this train.

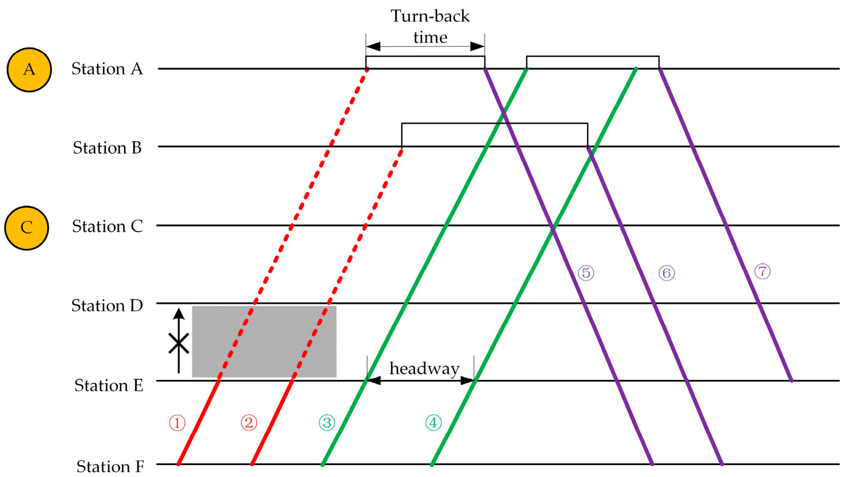


**Figure 5.** Timetable with single-direction disruption.

As mentioned in IDS, the upstream trains (①, ②, ③, and ④) can be cancelled and coupled, and their operation sequence can be adjusted. The downstream trains (⑤, ⑥ and ⑦) can be performed by the rolling stocks which perform the upstream trains in the original timetable, or the standby rolling stocks from depots A and C, and if one downstream train is not performed by any rolling stock, the train and the passenger demand served by it will have to be cancelled. The disrupted passenger demands of upstream trains can be served by standby rolling stocks, and the cancelled passenger demands of upstream trains can be served by other upstream trains which depart later than them.

To ensure the safety of the train operation, some constraints must be met. Firstly, the rolling stocks need a period to be prepared between performing two consecutive trains. The minimum time of this period is called the **minimum turn-back time**. Then, for any two adjacent trains in any section, the departure and arrival time of them, which can be called **headway**, should not be less than the minimum of it, to ensure a safe distance between the two trains. Besides, if one train stops at one station, arriving at or departing from the station will take extra time, and we call them **arrival supplement time** and **departure supplement time**, respectively.

### 3.2. Symbol Definition

The sets, parameters, and decision variables are illustrated in Tables 1–3, respectively.

**Table 1.** Sets.

| Symbols | Definition |
|---|---|
| $T^a$ | Set of disrupted trains in original train timetable, train $t \in T^a$ |
| $T^u$ | Set of upstream trains in original train timetable, train $t \in T^u$, $T^a \in T^u$ |
| $T^d$ | Set of downstream trains in original train timetable, train $t \in T^d$ |
| $T^o$ | Set of trains in the original timetable, train $t \in T^d$, $T^u + T^d = T^o$ |
| $X^t$ | Set of passenger demands of train $t$, demand $x \in X^t$ |
| $I$ | Set of standby rolling stocks, rolling stock $i \in I$ |
| $SEC$ | Set of sections in the HSR line, section $k \in SEC$ |
| $J$ | Set of stations in the HSR line, station $j \in J$ |

**Table 2.** Parameters.

| Symbols | Definition |
|---|---|
| $AO_t^j$ | The arrival time at station $j$ of train $t$ in the original timetable |
| $DO_t^j$ | The departure time at station $j$ of train $t$ in the original timetable |
| $SOR_t^j$ | Whether train $t$ stops at station $j$ in the original timetable. If yes, $SOR_t^j = 1$, else $SOR_t^j = 0$. |
| $OS_t$ | The origin station of train $t$ |
| $DS_t$ | The destination station of train $t$ |
| $OS^k$ | The origin station of section $k$ |
| $DS^k$ | The destination station of section $k$ |
| $BOS_i$ | The origin station of standby rolling stock $i$ |
| $BDS_i$ | The destination station of standby rolling stock $i$ |
| $OS_{t,x}$ | The origin station of passenger demand $x$ served by train $t$ |
| $DS_{t,x}$ | The destination station of passenger demand $x$ served by train $t$ |
| $Q_{t,x}$ | The passenger's number of demand $x$ served by train $t$ |
| $CAP_t$ | The maximum number of passengers train $t$ can serve in one section |
| $CAP_i$ | The maximum number of passengers standby rolling stock $i$ can serve in one section |
| $TRM_k$ | Travel time for trains in section $k$ |
| $MIL_t$ | The operation mileage of train $t$ |
| $DIS_j^{j'}$ | The distance between station $j$ and $j'$ |
| $LEN_t$ | Whether train $t$ is short train, if train $t$ is short train, $LEN_t = 1$, else $LEN_t = 0$ |
| $EXA$ | Arrival supplement time |
| $EXD$ | Departure supplement time |
| $MINST$ | The minimum stop time for one train at one station |
| $MAXST$ | The maximum stop time for one train at one station |
| $HEAD$ | The minimum headway for railway trains |
| $MANT$ | Minimum turn-back time |
| $LST_t$ | The latest time that train $t$ can depart from origin station |
| $SP_i$ | The station which is nearest to the depot of the standby rolling stock $i$ |
| $DISOS$ | The origin station of the disrupted section |
| $DISDS$ | The destination station of the disrupted section |
| $DIT$ | The time that the disruption starts |
| $RET$ | The time that the disruption finishes |
| $M$ | A large enough integer |

**Table 3.** Decision variables.

| Symbols | Definition |
|---|---|
| $can_t$ | Binary variable, if $can_t = 1$, train $t$ is cancelled |
| $b_i$ | Binary variable, if $b_i = 1$, standby rolling stock $i$ is operated |
| $cp_{t,t'}$ | Binary variable, if $cp_{t,t'} = 1$, train $t$ and train $t'$ are coupled |
| $se_k^t$ | The sequence of train $t$ in section $k$ |
| $s_t^j$ | Binary variable, if $s_t^j = 1$, train $t$ stops at station $j$ |
| $bt_i$ | The time that standby rolling stock $i$ is prepared to perform trains |
| $ar_t^j$ | The arrival time of train $t$ at station $j$ after rescheduling |
| $dr_t^j$ | The departure time of train $t$ at station $j$ after rescheduling |
| $e_t^{t'}$ | Binary variable, if $e_t^{t'} = 1$, train $t'$ is performed by the rolling stock which performs train $t$ |
| $be_i^t$ | Binary variable, if $be_i^t = 1$, train $t$ is performed by standby rolling stock $i$ |
| $xe_{t'}^{t,x}$ | Binary variable, if $xe_{t'}^{t,x} = 1$, train $t'$ serve the passenger demand $x$ which is originally served by train $t$ |
| $xb_i^{t,x}$ | Binary variable, if $xb_i^{t,x} = 1$, standby rolling stock $i$ serve the passenger demand $x$ which is originally served by train $t$ |

### *3.3. Objective Function*

1. Additional operation costs

The additional operation costs are represented by the operation mileage of the standby rolling stocks. The minimization of additional operation costs can be expressed as:

$$\min \sum_{i \in I} \sum_{t \in T^d} be_i^t \times \left( DIS_{OS_t}^{SP_i} + MIL_t + DIS_{SP_i}^{DS_t} \right) \tag{1}$$

where, $DIS_{OS_t}^{SP_i}$ is the distance between the origin station of train $t$ and the depot of the standby rolling stock $i$. $MIL^t$ is the operation mileage of train $t$. $DIS_{SP_i}^{DS_t}$ is the distance between the destination station of train $t$ and the depot.

2. The delay time of passengers

The minimization of the delay time of passengers can be expressed as:

$$\begin{gathered} \min \sum_{t \in T^o} \sum_{x \in X^t} Q_{t,x} \times (\Phi_b + \Phi_e + \Phi_c) \\ \Phi_b = \sum_{t' \in T^b} xb_{t'}^{t,x} \times \left( ar_{t'}^{DS_{t,x}} - AO_t^{DS_{t,x}} \right) \\ \Phi_e = \sum_{t' \in T^o} xe_{t'}^{t,x} \times \left( ar_{t'}^{DS_{t,x}} - AO_t^{DS_{t,x}} \right) \\ \Phi_c = \left( 1 - \sum_{t' \in T^b} xb_{t'}^{t,x} - \sum_{t' \in T^o} xe_{t'}^{t,x} \right) \times 1440 \end{gathered} \tag{2}$$

where, only one of $\Phi_b$, $\Phi_e$ and $\Phi_c$ is non-zero. If $\Phi_b$ is non-zero, the passenger demand is served by standby rolling stocks; if $\Phi_e$ is non-zero, the passenger demand is served by trains in the original timetable; and if $\Phi_c$ is non-zero, the passenger demand is not served. For the not served demands, we assign a delay time of 1440 min.

### *3.4. Constraints*

#### 3.4.1. Constraints of Operating Standby Rolling Stocks

If standby rolling stock $i$ is operated, it must perform one train, as shown in constrain (3)

$$\sum_{t \in T^d} be_i^t = b_i, \forall i \in I \tag{3}$$

The origin and destination station of standby rolling stock $i$ should meet the Constraints (4) and (5).

$$BOS_i = SP_i, \forall i \in I_i \tag{4}$$

$$BDS_i = \sum_{t \in T^d} be_i^t \times OS_t, \forall i \in I \tag{5}$$

As for serving passengers, if one passenger demand is served by standby rolling stock $i$, the origin and destination station of them should be same, as shown in Constraints (6) and (7).

$$xb_i^{t,x} \times OS_{t,x} = BOS_i, \forall i \in I, \forall t \in T^a, \forall x \in X^t \tag{6}$$

$$xb_i^{t,x} \times DS_{t,x} = BDS_i, \forall i \in I, \forall t \in T^a, \forall x \in X^t \tag{7}$$

Constraint (8) ensures that passengers served by standby rolling stock $i$ should be less than the passenger capacity of the rolling stock.

$$\sum_{t \in T^a} \sum_{x \in X^t} xb_i^{t,x} \times Q_{t,x} \leq b_i \times CAP_i, \forall i \in I, \tag{8}$$

Constraint (9) ensures that after finishing serving passenger demands, standby rolling stock $i$ must be prepared for at least the minimum turn-back time before performing the downstream train.

$$bt_i \geq xb_i^{t,x} \times AO_t^{DS_{t,x}} + MANT, \forall i \in I, \forall t \in T^a, \forall x \in X^t \tag{9}$$

Since the standby rolling stocks cannot pass through the disrupted section during the disruption, the standby rolling stocks whose depot is before the disrupted section cannot be operated, as shown in Constraint (10).

$$SP_i \times b_i \leq DISDS, \forall i \in I \tag{10}$$

### 3.4.2. Constraint of Coupling Short Trains

The decision variable $cp_{t,t'}$ must satisfy constraint (11)

$$cp_{t,t'} = cp_{t',t}, \forall t, t' \in T^u \tag{11}$$

Constraint (12) ensures that only short trains can be coupled.

$$LEN_t \times cp_{t,t'} = 0, \forall t, t' \in T^u \tag{12}$$

If two trains are coupled, the initial station and the terminal station of them must be same, as shown in Constraints (13) and (14).

$$cp_{t,t'} \times (OS_t - OS_{t'}) = 0, \forall t, t' \in T^u \tag{13}$$

$$cp_{t,t'} \times (DS_t - DS_{t'}) = 0, \forall t, t' \in T^u \tag{14}$$

Constraints (15) and (16) ensure that each short train can be coupled at most one time.

$$\sum_{t' \in T^u} cp_{t,t'} \leq 1, \forall t \in T^u \tag{15}$$

$$\sum_{t' \in T^u} cp_{t',t} \leq 1, \forall t \in T^u \tag{16}$$

Constraint (17) ensures that one train cannot be coupled to itself.

$$cp_{t,t} = 0, \forall t \in T^u \tag{17}$$

If two short trains are coupled, they should have the same departure time, arrival time, and stops, as shown in Constraints (18)–(20).

$$(1 - cp_{t,t'}) \times M \geq \left| dr_t^j - dr_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J \tag{18}$$

$$(1 - cp_{t,t'}) \times M \geq \left| ar_t^j - ar_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J \tag{19}$$

$$1 - cp_{t,t'} \geq \left| s_t^j - s_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J \tag{20}$$

where, $M$ is a large enough integer. We can see that the constraints are nonlinear. The Constraints (18)–(20) are transformed to linear constraints as follows:

$$\begin{aligned}
&(1 - cp_{t,t'}) \times M \geq dr_t^j - dr_{t'}^j, (1 - cp_{t,t'}) \times M \geq dr_{t'}^j - dr_t^j, \\
&(1 - cp_{t,t'}) \times M \geq ar_t^j - ar_{t'}^j, (1 - cp_{t,t'}) \times M \geq ar_{t'}^j - ar_t^j, \\
&1 - cp_{t,t'} \geq s_t^j - s_{t'}^j, 1 - cp_{t,t'} \geq s_{t'}^j - s_t^j, \\
&\forall t, t' \in T^u, \forall j \in J
\end{aligned}$$

If one of the coupled trains stop at one station in the original timetable, the two trains must stop at the station, as shown in Constraints (21)–(22).

$$s_t^j \geq cp_{t,t'} \times \max\left\{ SOR_t^j, SOR_{t'}^j \right\}, \forall t, t' \in T^u, \forall j \in J \tag{21}$$

$$s_{t'}^j \geq cp_{t,t'} \times \max\left\{ SOR_t^j, SOR_{t'}^j \right\}, \forall t, t' \in T^u, \forall j \in J \tag{22}$$

3.4.3. Constrains of Cancelling Trains

Constraint (23) ensures that if one upstream train is cancelled, the rolling stock of the train in the original circulation plan cannot perform downstream trains anymore.

$$\sum_{t' \in T^d} e_t^{t'} \leq 1 - can_t, \forall t \in T^u \tag{23}$$

Constraint (24) ensures that each downstream train will be performed by standby rolling stocks or rolling stocks which perform trains in the original timetable. Otherwise, it will have to be cancelled.

$$\sum\nolimits_{i \in I} be_i^{t'} + \sum\nolimits_{t \in T^u} e_t^{t'} = 1 - can_{t'}, \forall t' \in T^d \tag{24}$$

If the disrupted trains which have already departed from the origin station are cancelled, the passengers on them will have to transfer to other trains, which damages passenger service quality seriously. Thus, these trains cannot be cancelled, as shown in Constraint (25).

$$(DIT - DO_t^{OS_t}) \times can_t \leq 0, \forall t \in T^a \tag{25}$$

3.4.4. Constrains of Train Operation Sequences

Trains operating in the sections should meet the constraints of headway, to ensure the operation safety, as shown in Constraints (26) and (27).

$$\left| dr_t^{OS^k} - dr_{t'}^{OS^k} \right| \geq \left| se_k^t - se_k^{t'} \right| \times HEAD, \forall t, t' \in T^u, \forall k \in SEC \tag{26}$$

$$\left| ar_t^{DS^k} - ar_{t'}^{DS^k} \right| \geq \left| se_k^t - se_k^{t'} \right| \times HEAD, \forall t, t' \in T^u, \forall k \in SEC \tag{27}$$

These two constraints are also expressed as that in any section, the departure and arrival time difference between two adjacent trains should not be less than the minimum headway. Let's introduce a binary variable $setr_k^{t,t'}$. If $setr_k^{t,t'} = 1$, trains $t$ and $t'$ are adjacent and train $t'$ operates after train $t$ in section $k$. Constraints (26) and (27) can be expressed as Constraints (28) and (29), to transform the constraints to linear ones.

$$setr_k^{t,t'} \times (dr_{t'}^{OS^k} - dr_t^{OS^k}) + (1 - setr_k^{t,t'}) \times M \geq HEAD, \forall t, t' \in T^u, \forall k \in SEC \quad (28)$$

$$setr_k^{t,t'} \times (ar_{t'}^{DS^k} - ar_t^{DS^k}) + (1 - setr_k^{t,t'}) \times M \geq HEAD, \forall t, t' \in T^u, \forall k \in SEC \quad (29)$$

$se_k^t$, $se_k^{t'}$ and $setr_k^{t,t'}$ have the relationship as shown in Constraint (30).

$$(1 - setr_k^{t,t'}) \times M \geq se_k^{t'} - se_k^t - 1, \forall t, t' \in T^u, \forall k \in SEC \quad (30)$$

### 3.4.5. Constraints of Performing Downstream Trains

If one downstream train is performed by one standby rolling stock or one rolling stock which performs the upstream train in the original timetable, the rolling stock should get ready before the latest departure time of the downstream train, as shown in Constraint (31).

$$\sum_{i \in I} be_i^{t'} \times bt_i + \sum_{t \in T^u} e_t^{t'} \times (ar_t^{DS_t} + MANT) \leq LST_{t'}, \forall t' \in T^d \quad (31)$$

If one rolling stock performs one upstream train and one downstream train consecutively, the destination station of the upstream train and the origin station of the downstream train must be same, as shown in Constraint (32).

$$(1 - e_t^{t'}) \times M \geq \left| DS^t - OS^{t'} \right|, \forall t \in T^u, \forall t' \in T^d \quad (32)$$

The constraint can be transformed linearly as follows:

$$(1 - e_t^{t'}) \times M \geq DS^t - OS^{t'}, \forall t \in T^u, \forall t' \in T^d,$$
$$(1 - e_t^{t'}) \times M \geq OS^{t'} - DS^t, \forall t \in T^u, \forall t' \in T^d$$

### 3.4.6. Constraints of Trains Serving Passengers

If one train is cancelled, no passenger demands will be served by it, as shown in Constraint (33), where $M$ is a large enough integer.

$$\sum_{t' \in T^o} \sum_{x \in X^{t'}} xe_t^{t',x} \leq (1 - can_t) \times M, \forall t \in T^o \quad (33)$$

For the trains which are not cancelled and disrupted, the passenger demands served by them in the original timetable will not be served by other trains, as shown in Constraint (34).

$$xe_t^{t,x} \leq 1 - can_t, \forall t \in T^o, t \notin T^a \quad (34)$$

In each section, the passengers served by one train cannot exceed its capacity, as shown in Constraint (35).

$$\sum_{t \in T^u} \sum_{x \in X^t} xe_{t'}^{t,x} \times Q_{t,x} \times \alpha_{t,x}^k \leq (1 - can_{t'}) \times CAP_{t'}, \forall t' \in T^u, \forall k \in SEC \quad (35)$$

where, $\alpha_{t,x}^k$ represents whether the passenger demand $x$ of train $t$ includes section $k$.

Constraints (36) and (37) ensure that if one passenger demand is served by train $t'$, the sections which the demand passes through must be included in the operation sections of the train.

$$xe_{t'}^{t,x} \times OS_{t,x} \leq OS_{t'}, \forall t, t' \in T^u, \forall x \in X^t \quad (36)$$

$$xe_{t'}^{t,x} \times DS_{t,x} \geq DS_{t'} - (1 - xe_{t'}^{t,x}) \times M, \forall t, t' \in T^u, \forall x \in X^t \quad (37)$$

Constraint (38) ensures that if one passenger demand is served by one train, the departure time at the origin station of the demand should not be earlier than the scheduled time of the demand in the original timetable,

$$dr_{t'}^{OS_{t,x}} \geq DO_{t}^{OS_{t,x}} \times xe_{t'}^{t,x}, \forall t, t' \in T^u, \forall x \in X^t \quad (38)$$

Constraints (39) and (40) ensure if one train serves one passenger demand, the train must stop at the origin and destination station of the demand.

$$s_{t'}^{OS_{t,x}} - xe_{t'}^{t,x} \geq 0, \forall t, t' \in T^u, \forall x \in X^t \quad (39)$$

$$s_{t'}^{DS_{t,x}} - xe_{t'}^{t,x} \geq 0, \forall t, t' \in T^u, \forall x \in X^t \quad (40)$$

3.4.7. Constraints of Operation Time

For each train, the departure and arrival time in each station should meet the constraint of travel time, as shown in Constraint (41).

$$dr_{t}^{OS^k} + s_{t}^{OS^k} \times EXD + TRM_k + s_{t}^{DS^k} \times EXA \leq ar_{t}^{DS^k}, \forall k \in SEC, \forall t \in T^u \quad (41)$$

If one train stops at a station, the stop time should not be less than the minimum stop time or more than the maximum stop time, and if it passes through a station, the departure time at the station should be equal to the arrival time, as shown in Constraint (42)

$$s_{t}^{j} \times MINST \leq dr_{t}^{j} - ar_{t}^{j} \leq s_{t}^{j} \times MAXST, \forall t \in T^u, \forall j \in J \quad (42)$$

Constraint (43) ensures that no trains are allowed to operate in the disrupted section before the disruption finishes.

$$dr_{t}^{DISOS} \geq RET, \forall t \in T^u \quad (43)$$

## 4. Algorithm

NSGA-II is an effective algorithm for multi-objective optimization. It can output a set of solutions and has such advantages of fast solving speed, low computational complexity, retention of elite individuals, and so on. In recent years, it has wildly used in mechanical, electrical, transportation, and other fields. The algorithm in this paper is designed based on NSGA-II, and in order to accelerate the solving process, based on practical experience, an algorithm acceleration strategy (AAS) is proposed and adopted.

### *4.1. Conceptual Illustration*

1. Pareto dominance

We suppose that $x_a$ and $x_b$ are the two solutions of the multi-objective minimization problem with $k$ objective functions. The solution $x_a$ can be viewed as better than $x_b$ if Condition (44) is satisfied:

$$\begin{cases} f_i(x_a) \leq f_i(x_b), \;\; for\ all\ i = \{1, 2, \ldots, k\} \\ f_i(x_a) < f_i(x_b), \;\; for\ at\ least\ one\ i = \{1, 2, \ldots, k\} \end{cases} \quad (44)$$

where, $f_i(x)$ is the value of the $i$th objective function for decision vector $x$. In this case, we can say that $x_a$ dominates $x_b$, or $x_b$ is dominated by $x_a$.

2. Non-dominated sorting

The process begins with picking up all the non-dominated individuals from the initial population and assigning them the first rank. Then these first-ranked individuals are removed from the original population. After that, non-dominated individuals are picked from all the remaining individuals and assigned the second rank. This procedure continues until the whole population individuals are assigned different ranks. The individuals with the first rank form the Pareto Front, as shown in Figure 6.

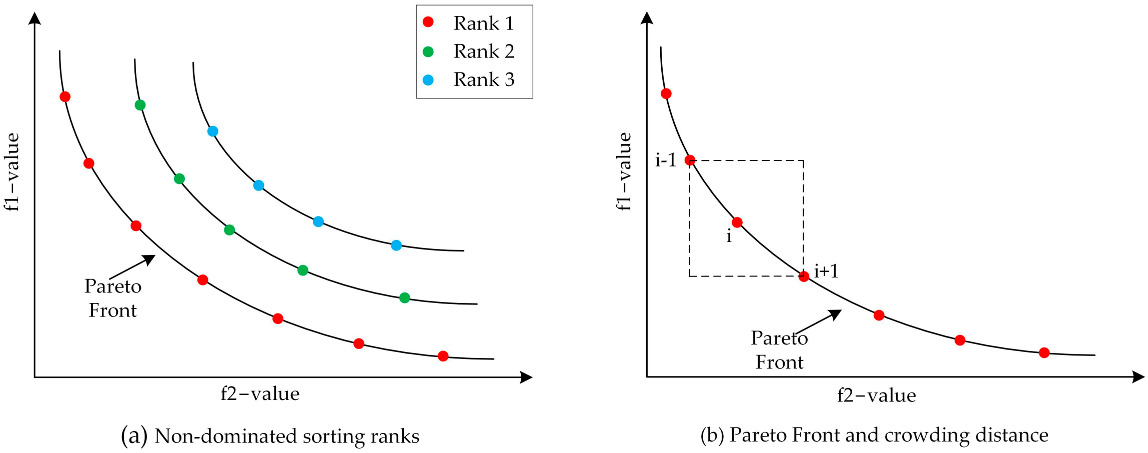


**Figure 6.** Non-dominated sorting and crowding distance.

3. Crowding distance

The density of the solutions in a population can be estimated by the crowding distance. If the crowding distance of a solution is large, it can be considered in a less crowded region, and selecting the solutions with a large crowding distance to the next generation can ensure the population diversity. The crowding distance of $i$th individual $crowd_i$ is defined as follows:

$$crowd_i = \sum_{j=1}^{k} \frac{f_j^{i+1} - f_j^{i-1}}{f_j^{\max} - f_j^{\min}} \tag{45}$$

where, $k$ is the number of objective functions, $f_j^{i+1}$ and $f_j^{i-1}$ are the values of the $j$th objective function for the $(i+1)$th and $(i-1)$th individual, respectively, and $f_j^{\max}$, $f_j^{\min}$ are maximum and minimum values of the $j$th objective function.

## *4.2. Representation Scheme*

In the algorithm, for each individual, part of the decision variable is encoded into one chromosome, including four gene fragments, as shown in Figure 7.

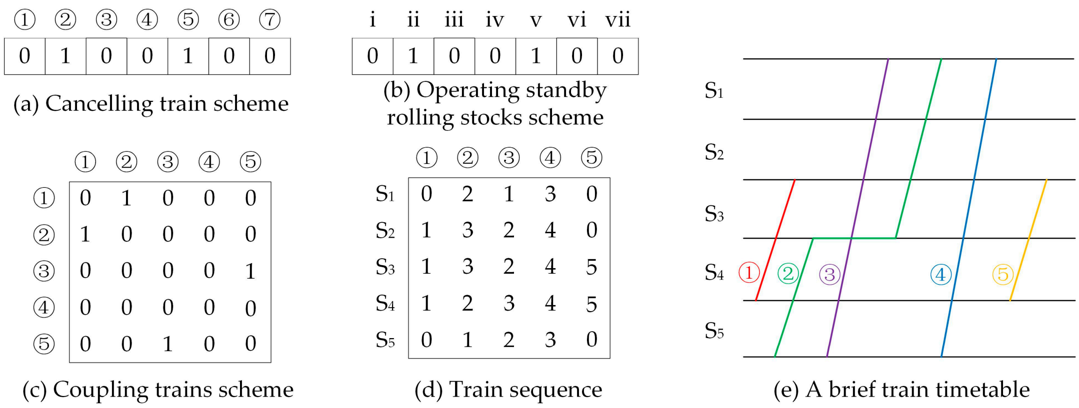


**Figure 7.** Representation scheme for chromosomes.

Gene fragment I represents the cancelling train scheme of the trains in the original timetable. The length of gene fragment I equals the number of the trains. The value of each gene in this fragment is 1 or 0, indicating the train is cancelled or not. As shown in Figure 7a, trains ② and ⑤ are cancelled.

Gene fragment II represents the operating standby rolling stocks scheme. The value of each gene in this fragment is 1 or 0, indicating the standby rolling stock is operated or not. As shown in Figure 7b, rolling stocks *ii* and *v* are operated.

Gene fragment III is a 0–1 matrix to represent the coupling trains scheme. If the value is 1, the two respective trains are coupled. As shown in Figure 7c, trains ① and ②, trains ③ and ⑤ are coupled.

Gene fragment IV is a matrix to represent the train sequence in each section, in which the rows represent the sections, and the columns represent the trains. For example, as shown in Figure 7d, the operation sequence of train ② in section $S_3$ is 3. According to the operation sequence in Figure 7d, a brief timetable can be plotted, as shown in Figure 7e. If one train is cancelled, or does not run in one section, the respective value in the matrix is 0.

### *4.3. Initial Solution Generation and Infeasible Solution Adjustment*

The initial gene values in gene fragments I, II, and III are generated randomly. As for fragments IV, trains run in many sections, and the influence of adjusting train sequence will be propagated. In most cases, generating a train operation sequence randomly cannot obtain a good solution. In the algorithm, the train operation sequence in the original timetable is used to generate the initial train sequence matrix.

The chromosome of some solutions may not satisfy the constraints. Therefore, each solution chromosome needs to be examined, and the infeasible solutions should be adjusted. The adjustment process is as follows:

1. If the value of the *j*th gene in fragment I is 1, which means train *j* is cancelled, all the values in the row *j* and column *j* of the matrix in fragment III should adjust to 0, and values in the column *j* of the matrix in fragment IV should adjust to 0, which means train *j* will not be coupled, and has no operation sequence.
2. If the value of row *i* and column *j* (expressed as (*i*, *j*) hereinafter) in the matrix in fragment III is 1, which means trains *i* and *j* are coupled, the other values in row *i* and column *j* should be adjusted to 0, which means one train can be coupled at most one times, and (*j*, *i*) should be adjusted to 1.
3. If (*i*, *j*) in the matrix in fragment III is 1, in the matrix in fragment IV, the values of column *i* in each row should be adjusted to equal the values in column *j*, which means the coupled trains have the same operation sequence.
4. Non-zero values of each row in the matrix in fragment IV should be adjusted to continuous. Specially, the sequence of coupled trains should also be the same. For example, if values of one row are 0, 1, 4, 2, 0, and 5, they should be adjusted to 0, 1, 3, 2, 0, and 4, which means the operation sequence of trains 2, 3, 4 and 6 in the corresponding section are 1, 3, 2 and 4, respectively.

### *4.4. Decoding*

The chromosomes include four gene fragments which represent cancelling train scheme, operating standby rolling stocks scheme, coupling trains scheme, and train operation sequence. Thus, the chromosomes must be decoded, to derive the train timetable and rolling stock circulation plan, as well as assignment plan for passengers. The decoding process is divided into three parts.

#### 4.4.1. Calculating Train Timetable

The gene values of the four chromosomes can be assigned to four decision variables, which are $can_t$, $b_i$, $cp_{t,t'}$ and $se_k^t$, respectively, as mentioned in Section 3.2. For the stop schemes of trains ($s_t^j$), if one train stops at one station in the original timetable, it must also stop in the rescheduled timetable. On this basis, a rescheduled timetable for upstream trains can be calculated, with the objective of all upstream trains finishing operation as early as possible. We formulate a sub-model to obtain a train timetable, and the objective function and constraints of it can be expressed as shown in (46) and (47):

$$\min \sum_{t \in T^u} ar_t^{DS_t}, \forall t \in T^u \tag{46}$$

$$s.t. \begin{cases} (1 - cp_{t,t'}) \times M \geq \left| dr_t^j - dr_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J, \\ (1 - cp_{t,t'}) \times M \geq \left| ar_t^j - ar_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J, \\ 1 - cp_{t,t'} \geq \left| s_t^j - s_{t'}^j \right|, \forall t, t' \in T^u, \forall j \in J \\ s_t^j \geq SOR_t^j, \forall t \in T^u, \forall j \in J \\ \left| dr_t^{OS^k} - dr_{t'}^{OS^k} \right| \geq \left| se_k^t - se_k^{t'} \right| \times HEAD, \forall t, t' \in T^u, \forall k \in SEC \\ \left| ar_t^{DS^k} - ar_{t'}^{DS^k} \right| \geq \left| se_k^t - se_k^{t'} \right| \times HEAD, \forall t, t' \in T^u, \forall k \in SEC \\ dr_t^{OS^k} + s_t^{OS^k} \times EXD + TRM_k + s_t^{DS^k} \times EXA \leq ar_t^{DS^k}, \forall k \in SEC, \forall t \in T^u \\ s_t^j \times MINST \leq dr_t^j - ar_t^j \leq s_t^j \times MAXST, \forall t \in T^u, \forall j \in J \\ dr_t^{DISOS} \geq RET, \forall t \in T^u \end{cases} \tag{47}$$

The variables and parameters are illustrated in Section 3.2. We use CPLEX to solve the sub-model, so as to obtain a group of values of $s_t^j$, $dr_t^j$ and $ar_t^j$, and generate a new timetable for upstream trains. In this part, the value of $can_t$, $b_i$, $cp_{t,t'}$, $se_k^t$, $s_t^j$, $dr_t^j$ and $ar_t^j$ can be determined.

#### 4.4.2. Rescheduling Rolling Stock Circulation Plan

After calculation in Section 4.4.1, we can get the arrival time of all upstream trains at the destination station, some downstream trains cannot be performed as scheduled due to the delay. In the rescheduling process, these downstream trains can be performed by other rolling stocks in the original circulation plan or standby rolling stocks. The rescheduling process is as follows:

Step 1: Find all the downstream trains which cannot be performed as scheduled and generate the set for these trains (PT).

Step 2: Judging whether the trains in PT can be performed by other rolling stocks in the original circulation plan. We sort the trains according to their departure time at the origin station. Judging from the earliest train, if there is at least one rolling stock ready to perform the train before the departure time of it, make the earliest rolling stock perform this train, then remove it from PT and judge the next train, until all trains in PT have been judged.

Step 3: Dispatching standby rolling stocks to perform the remaining trains in PT. We sort the remaining trains according to the number of passengers served by the trains and make standby rolling stocks perform the trains with more passengers preferentially.

For the remaining trains in PT after the three steps, there is no rolling stock performing them, and the passengers of them have to cancel their trip. In this part, the value of $e_{t'}^t$ and $be_i^t$ can be determined, and the rolling stock circulation plan is rescheduled.

#### 4.4.3. Generating Assignment Plan for Passengers

We design a quick method to generate an assignment plan for passengers, to serve the influenced passengers better and simplify the solution. The assignment plan is divided into two main parts. For the passenger demands of disrupted upstream trains, they can be arranged to standby rolling stocks, as illustrated in steps 2–4; for the passenger demands of cancelled upstream trains, they can be arranged to other trains in the original timetable, as illustrated in steps 5–7. The passenger demands of the trains which are not cancelled or disrupted are still served by the original trains. The steps of the assignment plan are as follows:

Step 1: Generate four sets, which are the set of passenger demands of disrupted upstream trains (DS), the set of passenger demands of cancelled upstream trains (CS), the set of operated standby rolling stocks (BS), and the set of uncancelled upstream trains (TS). Suppose that DS, CS, BS, and TS include $m$, $n$, $p$, and $q$ elements, respectively. Define $i_{ds}$, $i_{cs}$, $i_{bs}$, and $i_{ts}$ as the index of the four sets. For example, passenger demand $i_{ds}$ is the $i_{ds}$th demand in DS. Make $i_{ds}$ = 1, $i_{cs}$ = 1, $i_{bs}$ = 1, and $i_{ts}$ = 1, and go to step 2.

Step 2: Judging whether $i_{bs} = p + 1$. If yes, go to step 5; if no, go to step 3.

Step 3: Judging whether $i_{ds} = m + 1$. If yes, make $i_{ds}$ = 1, $i_{bs} = i_{bs} + 1$, and go to step 2; if no, go to step 4.

Step 4: Judging whether standby rolling stock $i_{bs}$ can serve disrupted passenger demand $i_{ds}$. If standby rolling stock $i_{bs}$ can serve passenger demand $i_{ds}$, five conditions need to be met. (1) The origin and destination station of them need to be the same. (2) The original departure time of demand $i_{ds}$ should be later than the rescheduled departure time of standby rolling stock $i_{bs}$. (3) The arrival time demand $i_{ds}$ should be earlier than the latest arrival time of standby rolling stock $i_{bs}$, so as not to delay the downstream train served by it. (4) The number of passengers served by standby rolling stock $i_{bs}$ should not exceed its capacity of it. (5) Demand $i_{ds}$ has not been served by other standby rolling stocks yet. If yes, assign demand $i_{ds}$ to standby rolling stock $i_{bs}$. Make $i_{ds} = i_{ds} + 1$ and go to step 3.

Step 5: Judging whether $i_{ts} = q + 1$. If yes, go to step 8; if no, go to step 6.

Step 6: Judging whether $i_{cs} = n + 1$. If yes, make $i_{cs}$ = 1, $i_{ts} = i_{ts} + 1$, and go to step 5; if no, go to step 7.

Step 7: Judging whether train $i_{ts}$ can serve passenger demand $i_{cs}$. If train $i_{ts}$ can serve passenger demand $i_{cs}$, four conditions need to be met. (1) Train $i_{ts}$ must stop at the origin and destination station of demand $i_{cs}$. (2) The original departure time of demand $i_{cs}$ should be later than the rescheduled departure time of train $i_{ts}$. (3) The number of passengers served by the train should not exceed the capacity of it in any section. (4) Demand $i_{cs}$ has not been served by trains yet. If yes, assign demand $i_{cs}$ to train $i_{ts}$. Make $i_{cs} = i_{cs} + 1$ and go to step 6.

Step 8: End.

The process of the assignment plan of influenced passengers is shown in Figure 8.

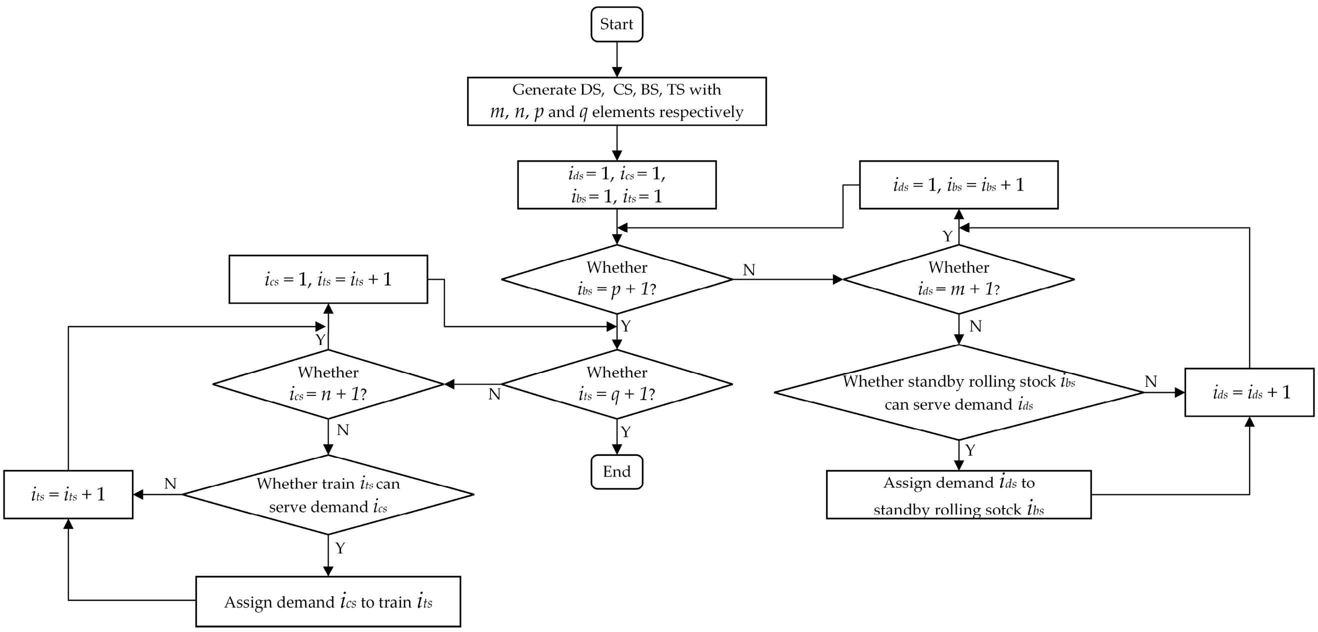


**Figure 8.** A quick method to generate an assignment plan for influenced passengers.

In this part, the value of $xe_{t'}^{t,x}$, $xb_i^{t,x}$ and $bt_i$ can be determined.

After the decoding process of Section 4.4.1, Section 4.4.2, Section 4.4.3, all decision variables mentioned in Section 3.2 are determined.

### *4.5. Fitness Functions*

There are two fitness functions in the algorithm, which are the operation mileage of standby rolling stocks ($Z_1$), as shown in (48), and the delay time of passengers ($Z_2$), as shown in (49).

$$Z_1 = \sum_{i \in I} \sum_{t \in T^d} be_i^t \times \left( DIS_{OS_t}^{SP_i} + MIL_t + DIS_{SP_i}^{DS_t} \right) \tag{48}$$

$$\begin{array}{c} Z_2 = \sum_{t \in T^o \vee T^b} \sum_{x \in X^t} Q_{t,x} \times (\Phi_b + \Phi_e + \Phi_c) \\ \Phi_b = \sum_{t' \in T^b} xb_{t'}^{t,x} \times \left(ar_{t'}^{DS_{t,x}} - AO_t^{DS_{t,x}}\right) \\ \Phi_e = \sum_{t' \in T^o} xe_{t'}^{t,x} \times \left(ar_{t'}^{DS_{t,x}} - AO_t^{DS_{t,x}}\right) \\ \Phi_c = \left(1 - \sum_{t' \in T^b} xb_{t'}^{t,x} - \sum_{t' \in T^o} xe_{t'}^{t,x}\right) \times 1440 \end{array} \tag{49}$$

In each iteration, after calculating the two fitness functions, the Pareto Fronts can be obtained according to the method mentioned in Section 4.1.

### *4.6. Genetic Operator*

There are three genetic operators, which are selection, crossover, and mutation, respectively. The selection operator can select excellent individuals from the current population and generate a new population; the crossover operator combines chromosomes from two chosen individuals to generate a new individual; the mutation operator evolves the chromosomes by changing the value of genes with a certain probability.

#### 4.6.1. Selection

In the selection process, we need to select excellent individuals from the current population and generate a new population. Firstly, non-dominated sort the current individuals and assign different ranks to them. Then, from rank 1, judge whether all the individuals with the rank can be added to the new population. If the number of individuals does not exceed the population size after adding the individuals with the current rank, add these individuals to the new population, and judge the individuals with the next rank. Otherwise, calculate the crowding distance of the individuals with the current rank, and add the individuals with the largest crowding distance to the population, until the number of individuals reaches the population size.

The selection process is shown in Figure 9.

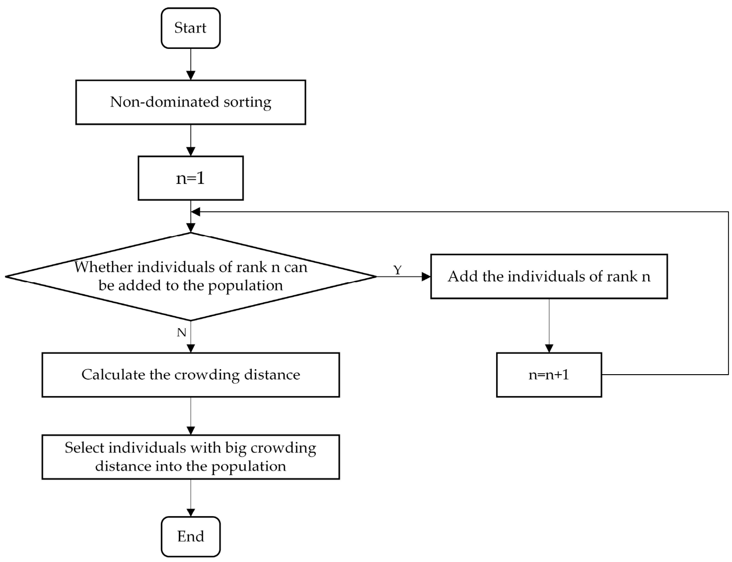


**Figure 9.** Selection process.

4.6.2. Crossover

For each chromosome, we choose another chromosome from the current population to cross with it. Each chromosome is chosen with the probability of $P_c$. The higher the non-dominated sorting rank, the higher $P_c$. We suppose that there are $n$ non-dominated sorting ranks in total. $P_c$ is obtained as follows.

$$P_c = \frac{P_m}{N_m} \tag{50}$$

where $P_m$ is the probability of choosing a chromosome from rank $m$, and $N_m$ is the number of individuals included in rank $m$. $P_m$ can be expressed as follow.

$$\begin{aligned} P_m &= (1-0.6)^{m-1} \times 0.6, \forall m < n \\ P_m &= (1-0.6)^{m-2} \times 0.4, m = n \end{aligned} \tag{51}$$

Specially, if there's only one rank, the chosen probability of every individual is equal, as shown in Equation (52).

$$P_c = \frac{1}{N} \tag{52}$$

where $N$ is the number of individuals in the current population.

Then we use the two chromosomes to generate a new chromosome. The values of the gene in fragments I, II, and III are obtained from one of the two chromosomes at random, and we select one train sequence matrix from the two chromosomes as the gene fragment IV of the new chromosome. After crossover, the new chromosomes should be examined, and the infeasible solutions should be adjusted, as mentioned in Section 4.3.

4.6.3. Mutation

The mutation probability is set to 10% in the algorithm. We respectively apply the mutation process for each fragment.

For gene fragment I and II, we randomly select a gene and change the value of it. Since their genes are 0–1 binary variable, the value of the genes will change from 0 to 1 or 1 to 0.

For gene fragment III, we randomly decouple two coupled trains and randomly select two uncoupled short trains and couple them. The gene value of the decoupled trains will change from 1 to 0, and the gene value of coupled trains will change from 0 to 1.

For gene fragment IV, we randomly select two adjacent trains in one section and make one of the two trains overtake another one. Figure 10 shows an example of mutation. The train sequence matrix before mutation is shown in Figure 10a, and the brief timetable plotted according to the sequence is shown in Figure 10b. Then there is one mutation. Train ④ overtakes train ② at section $S_3$, as shown in Figure 10d. The sequence matrix after the mutation is shown in Figure 10c.

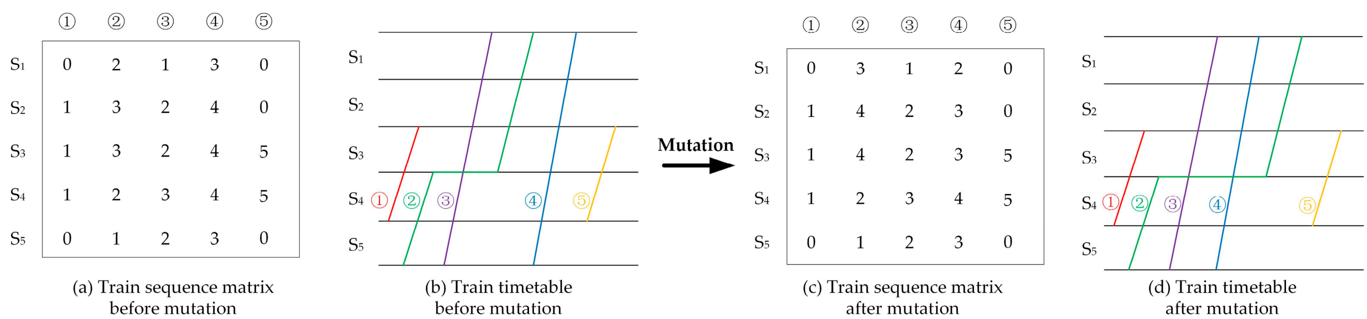


**Figure 10.** An example of a mutation in trains operation sequence.

### 4.7. Termination Conditions

After 50 consecutive generations without new individuals being added to Pareto Front, the algorithm terminates.

### 4.8. Algorithm Procedure

The algorithm process is shown in Figure 11.

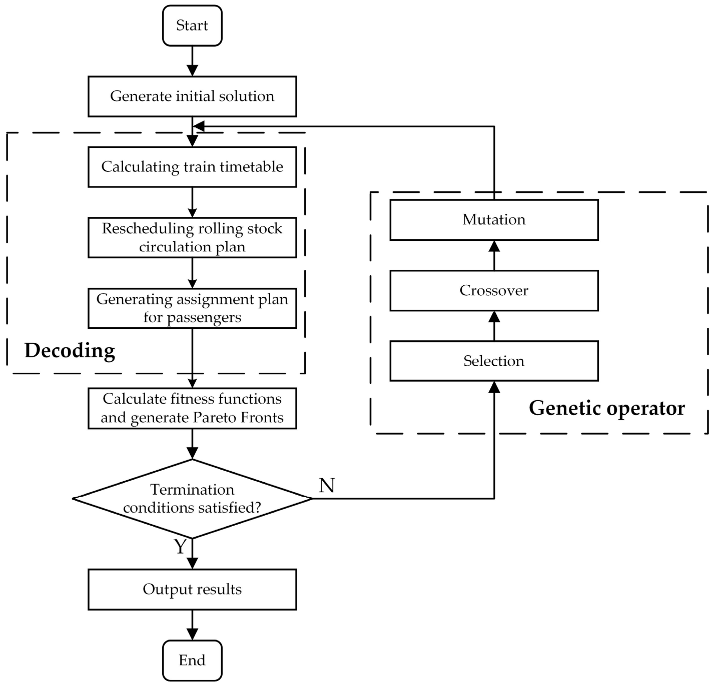


**Figure 11.** Algorithm process.

### 4.9. Algorithm Acceleration Strategy (AAS)

In practice, there are many experiences about using the adjustment measures included in IDS, which are more likely to obtain better results in the rescheduling process. However, in the algorithm, the initial solutions are generated randomly, and the quality of them may be poor. Therefore, we propose the Algorithm Acceleration Strategy (AAS). AAS is a strategy which can improve the quality of initial solutions based on the practical experiences. In AAS, some adjustment measures in initial solutions are replaced by new measures with a certain probability, including cancelling trains, coupling short trains, and operating standby rolling stocks. According to the experiences, the new adjustment measures may be more reasonable than the original ones. The new measures are called AAS measures, and the probability is called AAS probability. The AAS measures for different adjustment measures are illustrated as follows.

As for cancelling trains, according to the experiences, more passengers and more stops may result in a longer delay time. Thus, for each cancelled train in the initial solution, the AAS measure is that selecting another train with more passengers and stops to replace the train to be cancelled. The rule for selecting trains is as follows. Firstly, non-dominated sort all the uncancelled trains according to their passenger quantity and stops and assign them different ranks. Then, randomly select one train from rank 1 to replace the cancelled train.

As for coupling short trains, according to the experiences, the more stop difference may result in longer delay time. Thus, for each two coupled trains in the initial solution, the AAS measure is selecting two trains with the least difference in stop scheme from all uncoupled short trains and coupling them to replace the original coupled trains.

As for operating standby rolling stocks, according to the experiences, the number of standby rolling stocks dispatching from each depot should be proportional to the number of disrupted passengers at the station nearest to the depot. In other words, the more disrupted passengers at the station nearest the depot, the more standby rolling stocks should be dispatched from the depot. Thus, the AAS measure is as follows. Firstly, we calculate the number of disrupted passengers at the stations nearest to the depots. Then, keeping the total number of operated standby rolling stocks constant, change the proportion of standby rolling stocks dispatching from each depot to make it as close as possible to the proportion of the disrupted passengers at the respective station nearest to the depot.

In the following research, we test whether using AAS can accelerate the solving process and improve the quality of solutions.

## 5. Case Study

In this paper, we use the real-world operation data of the Beijing-Shanghai HSR line in 2019 for analysis. Beijing-Shanghai HSR line is one of the longest railway lines in China. There are 23 stations and 6 depots along the line. To describe the case simply, we number each station, as shown in Figure 12. For example, Beijing South Station is called station 1. The 6 depots locate near stations 1, 3, 6, 11, 16, and 23, and we call them depot 1, 3, 6, 11, 16, and 23, respectively.

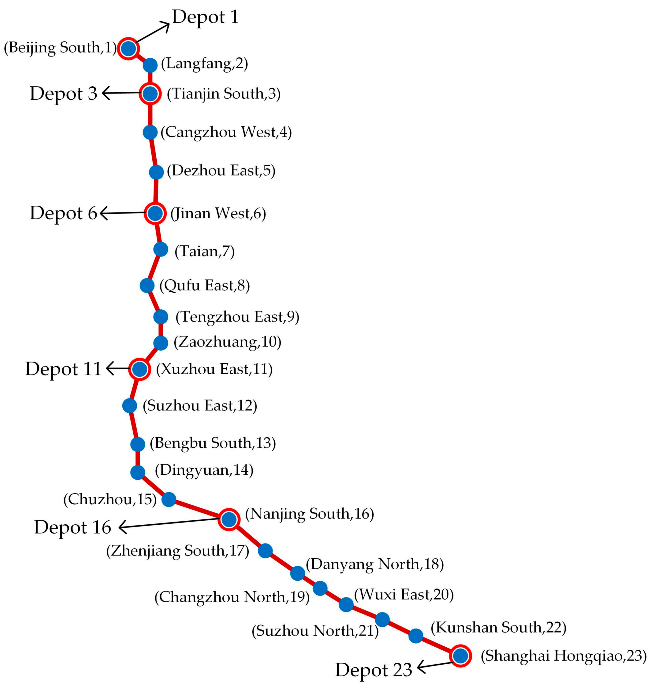


**Figure 12.** Stations on Beijing-Shanghai HSR line.

Table 4 shows the parameters used in the case, and Table 5 shows the number of standby rolling stocks allocated in each depot.

**Table 4.** Operation data and parameters.

| Parameter | Value |
|---|---|
| Upstream trains | 292 |
| Downstream trains | 295 |
| Minimum turn-back time | 20 min |
| Departure supplement time | 2 min |
| Arrival supplement time | 3 min |
| Minimum headway | 4 min |
| Maximum stop time | 10 min |
| Minimum stop time | 2 min |

**Table 5.** Number of standby rolling stocks.

| Depot | Number |
|---|---|
| 1 | 3 |
| 3 | 1 |
| 6 | 3 |
| 11 | 2 |
| 16 | 4 |
| 23 | 3 |

### 5.1. Computational Results

We design a single-direction disruption from station 13 to 12, which starts at 10:00 and lasts for 60 min. During the disruption, trains are not allowed to pass the disrupted section in the disrupted direction.

#### 5.1.1. Solving Model without Using AAS

Firstly, we solve the model by using NSGA-II without AAS. If no new individuals have been added to the Pareto Front for 50 consecutive generations, the iterative process will end. It takes 1263 s to solve the model in total, with 386 iterations. The Pareto Fronts before and after optimization include 7 and 12 individuals respectively, and the delay time of passengers and additional operation costs of them are shown in Table 6.

**Table 6.** Pareto Front before and after optimization.

| Before Optimization | | | After Optimization | | |
|---|---|---|---|---|---|
| No. | The Delay Time of Passengers (min) | Additional Operation Costs (km) | No. | The Delay Time of Passengers (min) | Additional Operation Costs (km) |
| 1 | 10,457,345 | 437 | 1 | 6,635,283 | 0 |
| 2 | 9,423,462 | 796 | 2 | 6,584,721 | 205 |
| 3 | 8,934,562 | 1575 | 3 | 6,348,169 | 506 |
| 4 | 8,245,236 | 1783 | 4 | 6,279,810 | 626 |
| 5 | 7,634,532 | 2365 | 5 | 6,124,290 | 912 |
| 6 | 7,364,132 | 2977 | 6 | 6,103,395 | 1196 |
| 7 | 6,823,785 | 3495 | 7 | 6,020,239 | 1638 |
| - | - | - | 8 | 5,973,819 | 2253 |
| - | - | - | 9 | 5,880,655 | 2450 |
| - | - | - | 10 | 5,506,119 | 2559 |
| - | - | - | 11 | 5,470,830 | 3165 |
| - | - | - | 12 | 5,221,298 | 3967 |
| Average | 8,411,865 | 1918 | Average | 6,012,386 | 1623 |

It can be seen that the solutions have been greatly improved from the initial solution, with an average decrease of 28.5% in the delay time of passengers and 18.3% in additional operation costs. Besides, there are more individuals in Pareto Front after iteration, which means the diversity of the Pareto Front has increased. A set of good solutions can be obtained in a relatively short time, indicating that the model and algorithm are feasible to deal with the rescheduling problem under single-direction disruptions.

#### 5.1.2. Improvement of Initial Solutions by Using AAS

AAS is a strategy to accelerate the solution by improving the initial solutions. Before the iteration process, the original adjustment measures in each initial solution are replaced by AAS measures under a certain probability called AAS probability. We set AAS probabilities to 30%, 60%, and 90%, and call the AAS with these probabilities AAS-30, AAS-60, and AAS-90, respectively. AAS-0 indicates that no AAS measures are adopted, since the probability of taking AAS measures is 0. In order to quantitatively measure the improvement of AAS on enhancing the quality of initial solutions, AAS-30, AAS-60, and AAS-90 are used to deal with the initial solution, and three Pareto Fronts of improved initial solutions can be obtained. The average delay time of passengers and additional operation costs of the initial solutions' Pareto Fronts before and after using AAS are shown in Table 7.

**Table 7.** The objectives' values and improvement of the initial solutions by using AAS.

| | Average Delay Time of Passengers | | Average Additional Operation Costs | |
|---|---|---|---|---|
| | **Value(min)** | **Improvement** | **Value (km)** | **Improvement** |
| AAS-0 (not improved) | 8,411,865 | - | 1918 | - |
| AAS-30 | 7,445,634 | 11.5% | 1617 | 15.7% |
| AAS-60 | 7,193,463 | 14.5% | 1725 | 10.0% |
| AAS-90 | 6,987,856 | 17.0% | 1800 | 6.2% |

We can see that AAS can enhance the quality of the initial solution, with an average improvement of 11.5%, 14.5%, and 17.0% in delay time of passengers and 15.7%, 10.0%, and 6.2% in additional operation costs.

#### 5.1.3. Solving Model with Using AAS

In order to examine the effect of AAS, we solve the model with AAS-30, AAS-60, and AAS-90 respectively. The comparison of solving time and iteration times between AAS-30, AAS-60, AAS-90, and AAS-0 (1263 s, 386 times, as mentioned in Section 5.1.1) are shown in Table 8.

**Table 8.** Comparison of the solving time and iteration times.

| | Solving Time | | Iteration Times | |
|---|---|---|---|---|
| | **Value(s)** | **Decrease Percentage** | **Value** | **Decrease Percentage** |
| AAS-0 (without AAS) | 1263 | - | 386 | - |
| AAS-30 | 808 | 36.0% | 243 | 36.6% |
| AAS-60 | 904 | 28.4% | 273 | 29.4% |
| AAS-90 | 874 | 30.8% | 266 | 31.1% |

We can find that comparing to solving the model without AAS, AAS can greatly accelerate the computing speed, with a decrease of 36.0%, 28.4%, and 30.8% in solving time and a decrease of 36.0%, 28.4%, and 30.8% in iteration times, respectively.

The Pareto Fronts of AAS-30, AAS-60, and AAS-90 include 12, 12, and 11 individuals, respectively, and the two objectives of them, which are delay time of passengers and additional operation costs, are shown in Table 9, and the average value of the two objectives of AAS-0, AAS-30, AAS-60, and AAS-90 are shown in Table 10.

We can see that using AAS can improve the quality of the Pareto Front. Compared to AAS-0, the other three Pareto Fronts have shorter average delay time of passengers, with a decrease of 2.0%, 2.4% and 8.3%, respectively, and lower average additional operation costs, with a decrease of 34.3%, 28.9% and 4.1%, respectively, indicating that the passenger service quality is improved, and the standby rolling stocks are operated more economically.

Besides, we find that in the results of AAS-30, AAS-60, and AAS-90 in Table 10, as the AAS probability increases, average delay time decreases and additional operation costs increase, indicating that when AAS probability is higher, more standby rolling stocks tend to be operated, and less time is delayed.

**Table 9.** Pareto Fronts of using AAS with three probabilities.

| AAS-30 | | | AAS-60 | | | AAS-90 | | |
|---|---|---|---|---|---|---|---|---|
| **No.** | **The Delay Time of Passengers (min)** | **Additional Operation Costs (km)** | **No.** | **The Delay Time of Passengers (min)** | **Additional Operation Costs (km)** | **No.** | **The Delay Time of Passengers (min)** | **Additional Operation Costs (km)** |
| 1 | 6,603,452 | 0 | 1 | 6,594,534 | 0 | 1 | 6,583,452 | 0 |
| 2 | 6,548,756 | 196 | 2 | 6,523,452 | 237 | 2 | 6,295,674 | 516 |
| 3 | 6,347,345 | 498 | 3 | 6,423,423 | 364 | 3 | 6,204,523 | 705 |
| 4 | 6,289,567 | 545 | 4 | 6,282,342 | 607 | 4 | 6,120,234 | 923 |
| 5 | 6,253,474 | 657 | 5 | 6,153,474 | 800 | 5 | 5,934,532 | 1434 |
| 6 | 6,173,453 | 744 | 6 | 6,092,352 | 996 | 6 | 5,693,452 | 1745 |
| 7 | 6,137,457 | 855 | 7 | 6,073,642 | 1103 | 7 | 5,456,345 | 1976 |
| 8 | 6,114,534 | 950 | 8 | 6,000,345 | 1352 | 8 | 4,734,352 | 2364 |
| 9 | 5,745,345 | 1534 | 9 | 5,712,341 | 1601 | 9 | 4,678,567 | 2390 |
| 10 | 5,045,634 | 2088 | 10 | 5,298,078 | 1994 | 10 | 4,537,649 | 2499 |
| 11 | 4,845,634 | 2287 | 11 | 4,945,342 | 2178 | 11 | 4,424,563 | 2574 |
| 12 | 4,594,563 | 2438 | 12 | 4,364,551 | 2617 | - | - | - |

**Table 10.** Comparison of the average delay time of passengers and additional operation costs.

| | **Average Delay Time of Passengers** | | **Average Additional Operation Costs** | |
|---|---|---|---|---|
| | **Value (min)** | **Decrease Percentage** | **Value (km)** | **Decrease Percentage** |
| AAS-0 (without AAS) | 6,012,386 | - | 1623 | - |
| AAS-30 | 5,891,601 | 2.0% | 1066 | 34.3% |
| AAS-60 | 5,870,323 | 2.4% | 1154 | 28.9% |
| AAS-90 | 5,514,849 | 8.3% | 1556 | 4.1% |

It should be noted that the average delay time of passengers and additional operation costs of the individuals in the Pareto Front can only indicate the quality in general, and it cannot show the relationship between individuals. We plot the Pareto Fronts of AAS-0, AAS-30, AAS-60, and AAS-90, to study the individuals in the Fronts, and Figure 13 shows the comparison of the Pareto Front of AAS-0 with the Pareto Fronts of AAS-30, AAS-60, and AAS-90, respectively.

By analyzing Figure 13, we can find that:

1. According to the calculation, there are 9, 8, and 8 individuals in the Pareto Front of AAS-0 dominated by Pareto Fronts of AAS-30, AAS-60, and AAS-90, respectively, indicating that using AAS can improve the quality of the Pareto Front and obtain better solutions.
2. Most of the dominated individuals in the Pareto Front of AAS-0 are distributed at the top of the graph, which means these dominated individuals have high additional operation costs, as shown by the blue diamond in Figure 13. We consider that using AAS can more significantly improve the quality of the individuals with high additional operation costs by operating standby rolling stocks efficiently and economically.
3. We calculate the value of $\Delta$ (a metric to express the nonuniformity of the distribution of individuals in the Pareto Front, and Pareto Front with larger $\Delta$ is less uniform, as introduced in Deb [33]). The $\Delta$ of Pareto Fronts AAS-30, AAS-60, and AAS-90 are 0.837, 1.093, and 1.885 respectively, which means that the distribution of AAS-90 Pareto Front is more nonuniform. As shown in the green point in Figure 13, we find that in the AAS-90 Pareto Front, there are more individuals in the upper left direction, which means they have short delay time and high additional operation costs. So, we think that using AAS with high probability tends to obtain more Pareto individuals with a short delay time and high additional operation costs.

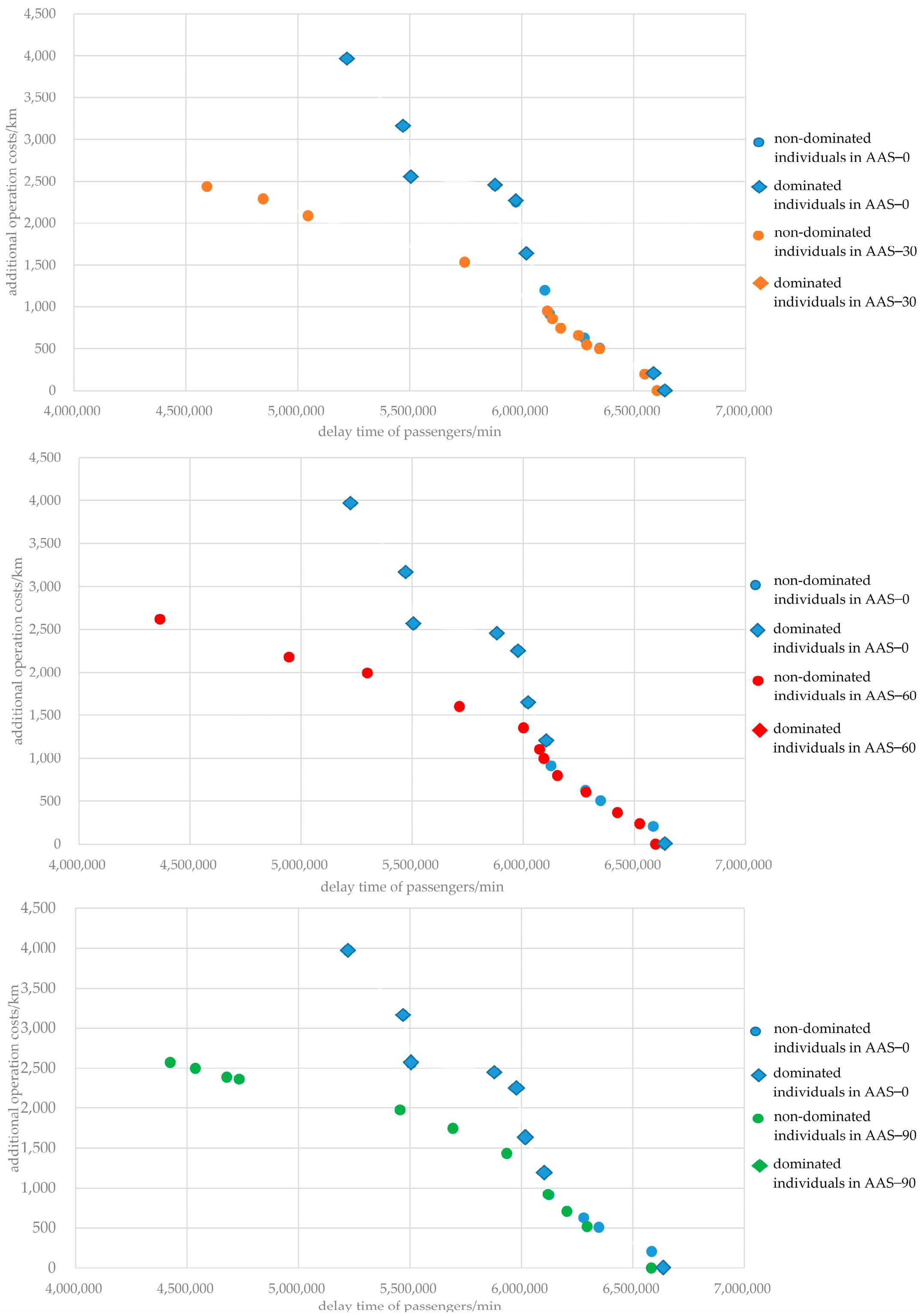


**Figure 13.** Comparison of the Pareto Front of AAS-0 with AAS-30, AAS-60, and AAS-90.

### *5.2. Analyzing Disrupted Sections*

For the HSR railway line, disruptions occurring in different sections lead to different negative influences on passenger service quality, since different trains and passenger demands are disrupted by the disruptions. Thus, it is significant to analyze the negative influence on passengers of the disruptions occurring in different sections.

To analyze the disrupted sections, we set a group of disruptions in every section respectively which have the same start time and duration, then calculate the Pareto Fronts under each disruption. We call the group of disruptions a **disrupted scenario**, and the minimum delay time of passengers in each Pareto Front **critical delay time** under the corresponding disruption. The critical delay time can be seen as the delay time after optimization without limiting operation costs. The longer the critical delay time, the more negative influence on passenger service quality. We call the section with maximum critical delay time **critical section** under the disrupted scenario, where a disruption occurring tends to result in most negative influence on passenger service.

The group of disrupted scenarios are set as follows. We use **{m, n}** to express the disrupted scenario that disruptions start at time **m** and last for **n** min. The critical sections and the critical delay time of them under different disrupted scenarios are shown in Table 11.

**Table 11.** Critical sections under different disrupted scenarios.

| Disrupted Scenarios | Critical Section | Critical Delay Time (min) | Disrupted Scenarios | Critical Section | Critical Delay Time (min) |
|---|---|---|---|---|---|
| {9:00, 60 min} | 16–15 | 5,603,453 | {15:00, 60 min} | 16–15 | 4,496,545 |
| {9:00, 120 min} | 16–15 | 9,898,723 | {15:00, 120 min} | 16–15 | 8,280,964 |
| {9:00, 180 min} | 23–22 | 14,794,582 | {15:00, 180 min} | 23–22 | 11,794,523 |
| {11:00, 60 min} | 16–15 | 5,378,453 | {17:00, 60 min} | 16–15 | 4,094,523 |
| {11:00, 120 min} | 16–15 | 9,983,423 | {17:00, 120 min} | 16–15 | 7,793,423 |
| {11:00, 180 min} | 23–22 | 14,082,342 | {17:00, 180 min} | 23–22 | 9,983,423 |
| {13:00, 60 min} | 16–15 | 4,957,394 | {19:00, 60 min} | 6–5 | 3,157,394 |
| {13:00, 120 min} | 16–15 | 8,885,784 | {19:00, 120 min} | 11–10 | 4,685,784 |
| {13:00, 180 min} | 16–15 | 13,085,739 | {19:00, 180 min} | 16–15 | 5,585,739 |

We can see that in most scenarios, section 16–15 is the critical section, which means that the disruption occurring in section 16–15 tends to damage the passenger service quality mostly. We think the reason is that there are most trains operating in section 16–15 in the original timetable, and if a disruption occurs in the section, more trains and passengers are disrupted, which lead to a longer delay time.

To analyze the influence on passenger service quality of disruptions in different sections, we select the disruptions in {9:00, 60 min} scenario as an example and plot the critical delay time of each section under the scenario, as shown in Figure 14.

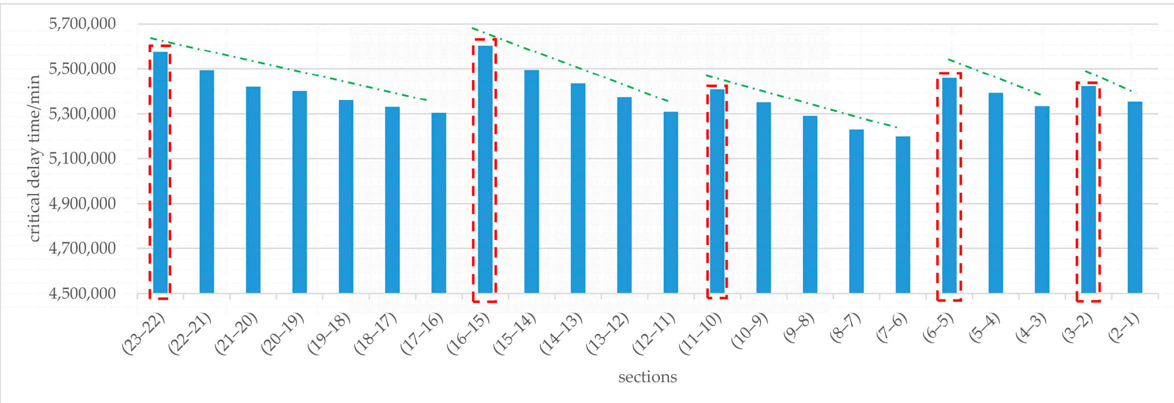


**Figure 14.** The critical delay time of each section.

By analyzing Figure 14, we can find that:

1. Generally speaking, the disruption occurring further away from the destination station influences more passenger demands of disrupted trains, since the trains cannot serve the passenger demands after the disrupted section on time. However, as shown in Figure 14, it is not the case that the further away from the destination station the

disruption occurs, the longer the critical delay time is. We think the reason is that the standby rolling stocks can be dispatched from the depots to serve the influenced passenger demands of disrupted trains, which can reduce the delay time of them. Thus, we think that operating standby rolling stocks plays an important role in reducing the delay time of passengers in the rescheduling process.

2. Figure 14 is divided into five **segments** by the sections framed by the red dashed lines (sections 23–22, 16–15, 11–10, 6–5, and 3–2), and the critical delay time of these sections are obviously longer than the previous section. We think the reason is that there are depots located near station 23, 16, 11, 6, and 3, as mentioned in Section 5.1, and if disruptions occur in the framed sections, fewer standby rolling stocks can be dispatched to serve disrupted passengers compared to the previous sections. For example, if one disruption occurs in section 17–16, the disrupted passengers who get on trains at stations 16–12 can be served by the standby rolling stocks from depot 16, while if the disruption occurs in section 16–15, the standby rolling stocks from depot 16 cannot be dispatched anymore, and no standby rolling stocks can serve these disrupted passengers, which increase the delay time of passengers. Thus, we think that the sections next to the depot in disrupted direction should be paid more attention to, to reduce the potential damage to passenger service quality.
3. In each segment in Figure 14, the critical delay time decrease, as the sections get closer to the destination station, as shown by the green dashed lines which is the trend line for each segment. We think it is because no matter the disruption occurs in which section of the segment, the number of available standby rolling stocks is always the same, and as getting closer to the destination station, less passenger demands of disrupted trains are influenced.

To sum up the above analysis, we consider that operating standby rolling stocks is significant in reducing the delay time of passengers, and the sections next to the depots in disrupted direction should be paid more attention to, in which the disruptions occurring tend to damage the passenger service quality more.

## 6. Conclusions

In this paper, we studied the train rescheduling problem under single-direction disruption. IDS, which includes five adjustment measures in practice, is used to synergistically reschedule the train timetable and rolling stock circulation plan. In the rescheduling process, two objectives are considered, which are minimizing the delay time of passengers, which can reduce the damage to passenger service quality, and minimizing additional operation costs, which can reduce energy consumption and help railway companies reschedule the train operation economically. On this basis, a two-objective model is formulated.

Then, the algorithm based on NSGA-II is designed to solve the model, and a set of Pareto Fronts can be outputted. Compared to weighting different kinds of objectives, the Pareto Front can indicate the relationship between the objectives better, and it can provide more solutions for railway companies to choose according to different situations. To accelerate the solving process, a quick method to generate the assignment plan is proposed to serve passengers, which simplifies the solution, and based on practical experiences, AAS is proposed to improve initial solutions.

A real-world instance of Beijing-Shanghai HSR line is used to test our model and algorithm. According to the computational results, after taking 1263 s to solve the model, the average minimized delay time of passengers and additional operation costs (operation mileage of standby rolling stocks) of the Pareto Front are 6,012,386 min and 1623 km, with a decrease of 28.5% and 18.3%, respectively. Since a set of good solutions can be obtained in a relatively short time, the model and algorithm are feasible to support operators to handle the single-direction disruptions. Besides, AAS can both accelerate the computing speed and improve the quality of solutions. Comparing to the results without AAS, in terms of computing speed, the solving time with AAS is 31.73% lower on average; in terms of the quality of solutions, the average delay time of passengers and additional operation costs decrease 4.2% and 22.43% respectively, indicating that the standby rolling stocks are operated more economically. Then, a group of disrupted scenarios is set to analyze the influence on passenger service quality of different disrupted sections. We find that operating standby rolling stocks is of great significance to reduce the damage to passenger service quality, and the sections next to the depots in disrupted direction tend to be critical sections, which should be paid more attention to.

The research in this paper has the following limitations, and several directions for further research are available. In terms of IDS and the model, this paper does not consider the deployment of the standby rolling stocks, the dynamic disruption duration, and the double-direction disruptions on railway lines and in railway stations, and further research on these aspects will be developed. In terms of the algorithms, other multi-objective optimization algorithms, such as MOPSO, are also widely used in railway domain. In the future, the comparative analysis for different multi-objective optimization algorithms will be studied. What's more, parallel multi-objective optimization techniques are also widely adopted, which can accelerate the computing speed. In the future, we will study the application of parallel multi-objective optimization techniques.

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