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The Role of Dynamic Geometry Software in Teacher–Student Interactions: Stories from Three Chinese Mathematics Teachers

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Abstract: Teacher–student interactions are critical for effective mathematics teaching and learning. Certain technologies, especially dynamic geometry software (DGS), can have a great impact on interactions between teachers and students. In Chinese mathematics educational environments, we selected three mathematics teachers as examples to analyze how DGS impacts their method of teaching and feedback and what role it plays in their interactions with students. Based on the data, we found mathematics to be the main focus for Chinese teachers, even though DGS was widely used in their lessons. DGS was mainly used as a projector to present geometric properties. This implies that the interactivity of the DGS was simply used for pressing buttons on the screen during lessons. Although teachers may have used DGS for many years, they still lack sufficient skills to integrate this software into mathematics teaching.

Keywords: teacher–student interactions; dynamic geometry software; Chinese mathematics lessons; technology

Citation: Zhu, F.; Xu, B. The Role of Dynamic Geometry Software in Teacher–Student Interactions: Stories from Three Chinese Mathematics Teachers. *Sustainability* **2023**, *15*, 7660. https://doi.org/10.3390/su15097660

Academic Editors: Jian-Hong Ye, Yung-Wei Hao, Yu-Feng Wu and Savvas A. Chatzichristofis

Received: 8 March 2023
Revised: 4 May 2023
Accepted: 5 May 2023
Published: 6 May 2023

1. Introduction

Teacher–student interactions form a critical part of classroom teaching through posing questions and giving feedback, affecting students' learning processes [1]. For a teacher, a student's answers to questions indicate their understanding and knowledge surrounding a certain topic [2]. Additionally, it is also important that teachers ask suitable questions in learning environments with the aid of technology to help students learn mathematics [3]. Technology gives students more opportunities to develop their mathematical skills [4] via their interactions with teachers. For example, the use of technology might encourage students to ask worthwhile questions that may not be posed by teachers [5]. Additionally, answers to these questions can be a catalyst for improving students' problem-solving strategies when their original strategies are unsuitable [6].

Dynamic geometry software (DGS) is the most commonly used technology in mathematics teaching and learning. Its main function, the dragging mode [7], is frequently used by teachers to explain geometry. Studies on this software often focus on the design of different tasks for teachers that use DGS [8,9]. However, few studies focus on what kinds of questions teachers pose to students via the use of dynamic geometry software [3]. Within the situation created by DGS, teachers need to think about designing suitable questions in order to allow students to improve their mathematical knowledge. In this study, we focused on both questions and feedback regarding the interactions between teachers and students using DGS in the classroom.

2. Literature Review

2.1. Dynamic Geometry Software in Mathematics Education

Currently, teachers use different kinds of software to support their teaching process. These kinds of software are designed to teach mathematics as clearly as possible and show the results of mathematical processes [10]. They all have these basic features, such as primitive objects (points, lines, circles, etc.), basic construction tools (parallel, perpendicular, etc.), transformations, and measuring and dragging capabilities [3]. DGS has been used for almost three decades, and many studies have demonstrated its contribution to improving learning and teaching mathematics [11–13] via its dragging mode, which is a problem-solving tool with a number of applications, including construction, searching for commonalities, conjecturing, and proving and refuting conjectures [14,15]. When students drag geometric objects with DGS, the geometrical properties of the figure defined in its construction are preserved.

Consequently, teachers must develop new methods for this new learning environment, in which students can have direct interactions with DGS. Previously, teachers had to directly teach students mathematical concepts [16], whereas, with DGS, teachers can let students independently explore and experiment [17]. Despite this, in most lessons, teachers always reveal answers using DGS without letting students explore solutions independently [18]. Students can only observe the outcomes found by teachers using DGS and have no opportunities to question whether the teachers are actually correct [19].

Because of the complex processes of interaction with this software, a teacher may find that students' experiences of interacting with this software can lead to unexpected outcomes [7].

2.2. Framework

2.2.1. Roles of DGS in Mathematics Teaching and Learning: Amplifier and Reorganizer

In this paper, we aimed to explore the roles that DGS plays during the interactions between teachers and students in geometry tasks. Pea [20] found two distinct roles of DGS in mathematics teaching: amplification and reorganization. According to Pea, technologies are always seen as an amplifier that can help users to solve tasks more efficiently. However, this is not sufficient for mathematical learning and teaching. Therefore, users need to make changes when solving tasks with DGS [21]. Over the years, the use of DGS has begun to change from being a visual amplifier to a reorganizer of the conceptual understanding of learners [22]. Based on these two roles, some researchers have proposed four different roles that DGS plays in mathematics tasks [8,9]:

1. DGS is used to facilitate the material aspects of a task without changing it conceptually. For example, a triangle is constructed using the midpoints of its sides and then its medians.
2. Alternatively, DGS is used as a visual amplifier in the task of identifying properties. It is easier to observe that three midlines of one triangle always intersect at one point via dragging with DGS than by using a pencil and paper.

These two roles can be seen as amplifiers in mathematics tasks. DGS also offers new tasks with new and improved standard geometric knowledge that cannot be achieved without its use.

3. DGS can modify the solving strategies of a task because of its powerful functions, which may make the tasks more difficult, similar to creating a parallelogram without using parallel line tools in DGS.
4. The task itself takes its meaning or its 'raison d'être' from Cabri. These two roles show that DGS can be used to generate new tasks in mathematics.

2.2.2. Initial Categories of Focusing Questions

Teachers, students, and mathematics are the foundations of mathematics classes [3,23,24], but in the 21st century, technology is becoming an increasingly important aspect of our daily lives. The emergence of technology compels us to reconsider the relationship between these elements. Tall [10] proposed a tetrahedron model, placing technology as one of the most important elements in mathematical teaching and learning (Figure 1). This model indicates that teachers may either focus on technology and give students a little opportunity for mathematical learning, or they may focus mostly on mathematics and not provide any technological instructions at all [25,26].

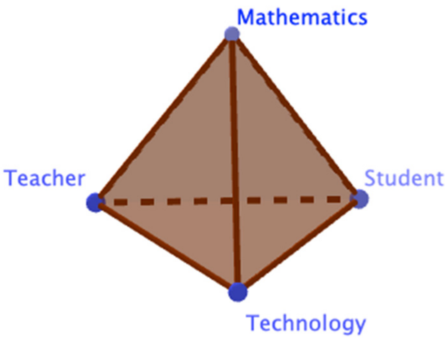
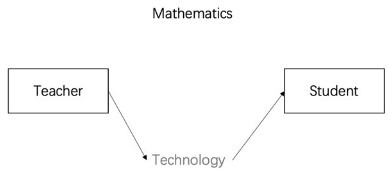
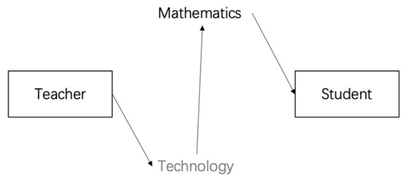
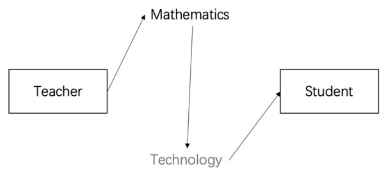
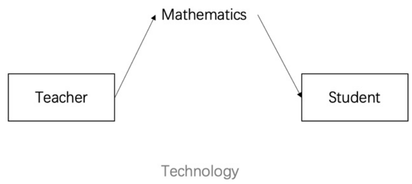


Figure 1. Didactic tetrahedron.

Because of the change in teaching methods caused by DGS, teacher–student interactions can show a different format. We can use Table 1 to describe the interactions between teachers, students, mathematics, and DGS in our lessons.

Table 1. Four initial foci of questions.

Focus	Description	Example Questions	Coding Diagram
Focus on technology.	Teacher's question or feedback is only about the technology.	Try to drag and click the circle button and draw a circle on your screen.	
Focus on technology to notice mathematics.	Teacher's question or feedback is related to a technological action, which is related to a mathematical idea.	If you select the center of the circle you have drawn and move it, what do you see?	
Focus on mathematics with the use of technology.	Teacher's question or feedback is about mathematics, and they assume that students need to use technology to find answers.	Can you make a circle that circumscribes the triangle given on your screen?	
Focus on mathematics.	Teacher's question or feedback are only about mathematics. Additionally, students can answer them without the help of technology.	Try to conclude ways to prove the drawn triangles.	

From this table, four different routes can be identified to show how teachers interact with their students based on mathematics and technology [3]. The route of teacher–mathematics–student means that teachers think that mathematics knowledge is the most important and overlook the technology. In the route of teacher–technology–student, teachers may need to explain the function of the technology they use, and at those times, they do not pay attention to the mathematics. Because of the development of technology, in the routes of teacher–technology–mathematics–students, and teacher–mathematics–technology–students, teachers consider ways to integrate technology to help their students learn mathematics effectively. Sometimes they may want their students to understand how to use technology, so they may pose some mathematics questions as examples to teach. In other cases, teachers may want students to know how to solve mathematical problems with the help of technology.

3. Research Questions

In this study, we addressed the following questions:

1. What technology-specific actions do mathematics teachers focus on when using DGS in mathematics lessons during teacher–student interactions?
2. What roles does DGS play as an amplifier and reorganizer during teacher–student interactions?

4. Methodology

In this section, we describe the criteria for selecting the teachers; their basic information, resources, and tasks; and further explain the data collection and analysis process.

To address RQ1, we paid attention to lesson videos, analyzing the kinds of questions that teachers posed to their students and the feedback given after students answered the questions.

To address RQ2, we focused on how DGS appears as a technological resource to support teachers' interactions with students during lessons.

4.1. Participants

Considering the multiple factors impacting teachers' use of DGS in the classroom, the following criteria were considered: (1) teachers' experience (both experienced and novice teachers); (2) teachers' willingness to use DGS in their lessons; and (3) teachers' experience of using DGS. Based on these criteria and to address our research questions, we decided to choose three Chinese mathematics teachers with different teaching experiences of DGS (two experienced teachers and one novice teacher), who were all willing to use DGS and other technological resources.

Teachers ZH and J were the two most experienced teachers from the same school and the leading mathematics teachers in their respective grades. Tables 2 and 3 present the tasks used by the teachers during this study.

Table 2. The task used by Mr. ZH.

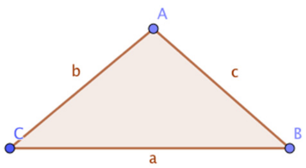
<table>
<tr><th colspan="2">Task</th></tr>
<tr><td>
</td><td>Mr. ZH designed one diagram of a triangle whose side lengths AC and BC were determined (AC = 4, BC = 6). Point A could be moved, while points B and C were stable. From the diagram, the trace of point A is similar to a circle. The main question: please find the relationship between the three sides of the triangle. (Here the lowercase letters a, b, and c mean side BC, side AC, and side AB)</td></tr>
</table>

Table 3. The task used by Madame J.

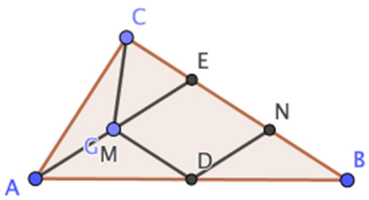
<table>
<tr><th colspan="2">Task</th></tr>
<tr><td>

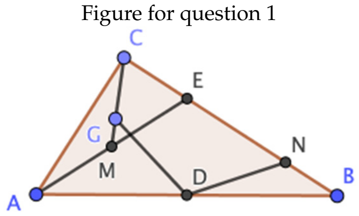
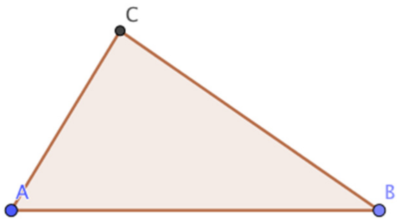
Figure for question 1


Figure for question 2</td><td>In the picture, there is a right triangle ABC, ∠ACB = 90°, point D is the midpoint of AB, point E is on segment BC, AE = BE, point M is the midpoint of AE, point G is on segment CM, and point N is on segment BC, which makes ∠GDN = ∠AEB
Question 1: as in picture 1, if point G coincides with point M, this proves that quadrilateral DMEN is a diamond.
Question 2: as in picture 2, when point G does not coincide with point M and point C, prove GD = DN</td></tr>
</table>

Teacher Y was a novice teacher from another school who needed support from colleagues to help her use DGS in the classroom. The task used in her lesson is shown in Table 4. All the lessons observed in this research were held with grade 7 students.

Table 4. The task used by Mrs. Y.

Task	
	Task 1: DGS is used to construct a new triangle, which is congruent with the given triangle ABC, in which AB = 7, AC = 4, BC = 6. In this triangle, point A and B can be dragged, while point C is stable. When point A is dragged, the whole triangle moves without changing its type and size. When point B is dragged, point A does not move. The trajectory of point B is a circle whose center is point A and radius is the length of AB.

4.2. Data Collection and Analysis

According to the research objective, some details from the pictures and interviews may be ignored. The first author observed all three teachers' lessons with DGS and made lesson videos (40 min for each lesson). The following were the main elements that we focused on: the types of tasks (with DGS); the teacher's interactions with students [27]; the organization and configuration of classes; and potential comments made by both teachers and students [28].

Due to the research objective and for ethical reasons, the videos were focused on the teachers, and only the teachers' faces were shown in order to protect their privacy.

These videos were reanalyzed based on the role of DGS and the focus on teachers' questions and feedback. We first cut each lesson video into short teaching episodes according to the different mathematics tasks used in the lessons.

Then, the interactions between the teachers and students were analyzed to determine the focus of the teachers when using DGS in the classroom. (1) If the question or piece of feedback focused on the technology, then it could be classified as the route "teacher–technology–student". (2) If the question or piece of feedback focused on mathematics, then it could be classified as the "teacher–mathematics–student" route. (3) If the question or feedback focused on how students use technology to solve mathematics tasks, it could be classified as the "teacher–mathematics–technology–student" route. (4) If the question or feedback focused on using mathematics contents to present the function of the technology, it could be classified as the "teacher–technology–mathematics–student" route.

To answer RQ2, we analyzed the video recordings according to the different roles that DGS plays in teacher–student interactions (amplifier or generator), and we examined the interviews with these teachers to determine their uses of and opinions on DGS in teaching mathematics. The audio from all the interviews was recorded. First, we transcribed all the audio recordings and cut them into small sections according to the interview questions. Then, all the teachers' answers were analyzed sentence by sentence. We focused on the keywords that applied to teaching methods involving DGS (one example is presented in Appendix A).

5. Results

In this section, we present the interactions involved in the above task to analyze which teachers' questions and feedback points focused on when DGS was used in the classroom and the roles that DGS played in these interactions.

5.1. Stories from Mr. ZH's Class

Based on the task situation (Table 2), Mr. ZH posed the following questions for discussion:

1. What is the maximum length of side c? What is the relationship between sides a, b, and c? What shape is it now?

2. What is the minimum length of side c? What is the relationship between sides a, b, and c? What shape is it now?
3. Based on the conclusions of questions 1 and 2, what you can find?

The students needed to find the mathematical relationships between the three sides of the triangles. Mr. ZH did not expect students to use the software to find the answers; the students just observed what occurred on the screen when the software was running.

Before the discussion, Mr. ZH let the students watch the screen and use DGS as a projector to present the task and diagram. During the discussion, Mr. ZH did not directly interact with his students but acted as a listener. After students finished their work, Mr. ZH continued the lesson with the following question, as presented in Table 5: "OK, the first one, what is the maximum of side C?"

Table 5. Teacher–student interactions in Mr. ZH's lesson.

Questions and Feedback	Focus
T: "OK, the first one, what is the maximum of side c?"	Only focused on mathematics.
St 1: "The maximum is 10" T: "OK, maximum is 10. I have said in the video, so the maximum is 10"	The teacher's feedback just repeated what the student answered. The answer only focused on mathematics.
T: "Now what is the relation?"	Only focused on mathematics.
St 1:" They are all on the same line." T: "Good, he said on the same line."	The teacher's feedback just repeated what the student answered. The answer only focused on mathematics
T: "Let us see, this is position. OK, now c is 10 and they are all on the same line."	The teacher moved the point on the screen at that time. The feedback focused on technology to notice mathematics.
T: "But what is the mathematical relation between the three sides?"	Only focused on mathematics.
St 1: "b + c = a" T: "b + c = a, good, sit down. b + c = a, we can also say a − b = c."	The teacher's feedback just repeated what the student answered. The answer only focused on mathematics.

One student (St1) was asked to answer the question. From Table 5, both Mr. ZH's questions and the student's answers focused more on mathematics. Sometimes, Mr. ZH tried to let the students observe the mathematical content with the help of DGS, but DGS was only used by Mr. ZH (Figure 2) and not his students. This implies that DGS was more like an amplifier that was used to enlarge the geometry properties and support the teachers' explanations. During this time, the students still did not truly explore knowledge with the help of DGS. When the teachers let the students solve the task together or discuss the questions that they posed, the teachers acted as listeners and often did not participate in the students' discussions. Additionally, although they may have used DGS to teach, it did not play an important role during discussions.

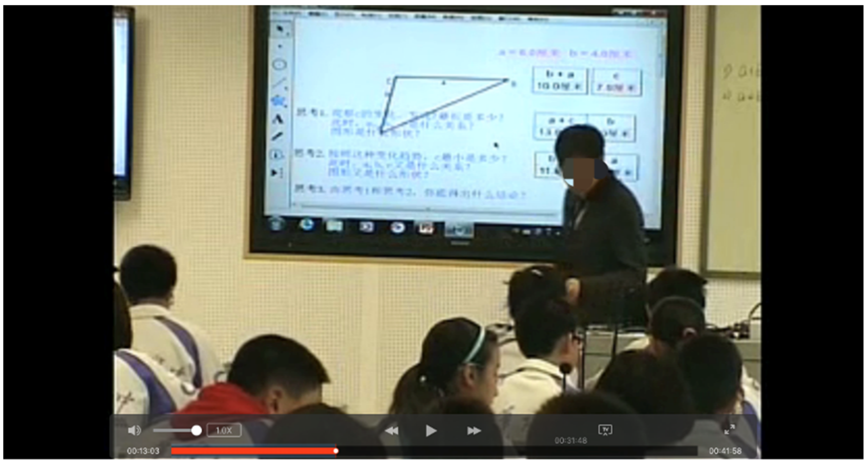


Figure 2. Mr. ZH operated DGS during teacher–student interactions. In this figure, the Chinese terms mean: question 1, please find what is the smallest value of a + b; question 2, please find what is the largest value of a + b; and question 3, what conclusion can you make?

The interaction process was supported by Mr. ZH's preference for using DGS in mathematics education. In the interview, Mr. ZH told us he generally always used DGS to explain complex tasks; for him, it was a personal preference. Because of examinations, Mr. ZH could not allocate the students too much time to use DGS for exploring mathematical concepts. As he stated, in China, students need to pass exams without the help of any software.

5.2. Stories from Madame J's Class

Here, we describe another teacher's classroom interactions through the use of DGS. As mentioned above, Madame J was familiar with DGS in mathematics education. Sometimes, she searched for different tasks on the internet or in some books to learn how to effectively use DGS in the classroom. We chose two examples of teacher–student interactions (Tables 6 and 7) according to the same task (Table 3). One example shows that a student had the incorrect answers and the other one shows that the student had the correct answer.

Table 6. Teacher–student interactions in Madame J's lesson.

Questions and Feedback	Focus
T: "You need to justify what type is the quadrilateral."	Focused on mathematics.
St1: "It is diamond." T: "Diamond. How can you know it?"	The teacher's feedback repeated what this student answered. Additionally, the question let this student justify why they believed that quadrilateral DMEN was a diamond. Therefore, the feedback point and question focused on mathematics.
St1: "First point G and M are coincided." T: "Yes"	The teacher gave positive feedback. They did not focus on mathematics or technology.
St1: "Then, DM, point D is the midpoint of AB, point M is the midpoint of AE, so DM the median line of this triangle." T: "DM is the median line of triangle ABE. You know it is median line, then"	The teacher repeated the conclusion this student made. It still focused on mathematics.
St1:" Then DM is parallel to BE T: DM is parallel to BE"	The teacher repeated the conclusion this student made. It still focused on mathematics.
St1: "So angle MDN is add angle END is 180" T: "It is this angle, angle MDN and angle END, angle MDN plus angle END is 180"	The teacher wrote down some marks through DGS on the screen. The feedback focused on technology to notice mathematics
St1: "Then, it said, angle GEN is equal to angle MDN and AEB" T: "Angle MDN is equal to this angle, angle GEN. It means this angle is equal to angle GEN."	The teacher wrote down some marks through DGS on the screen. The feedback focused on technology to teach mathematics
St1: "So angle END plus angle GEN is 180, then DN is parallel to ME" T: "Why you justify DN is parallel to ME? St1: So NDME is parallelogram"	This question only focused on mathematics.

In her lessons, Madame J always used an iPad as one of her resources, making her lessons a little different than those of other Chinese teachers. As shown in Table 6, Madame J did not pose many questions; instead, she repeated the student's answers or gave feedback such as "ok", and "yes" to let the student continue their explanation. From some critical terms in her questions, such as "diamond" and "parallel", we can see that these questions were more focused on mathematics. Therefore, DGS, similar to the situation in Mr. ZH's lesson, was used to project and display diagrams for the students to observe. They did not need to operate the software to help them find the correct answer (Figure 3).

The next interaction (Table 7) occurred when St1 failed to provide an answer to the question. Madame J asked another question to continue the explanation.

Table 7. Another teacher–student interaction in Madame J's lesson.

Questions and Feedback	Focus
St2: "In the conditions, AE is equal to BE T: AE is equal to BE, what it means?"	The teachers' question focused on mathematics.
St2: "So we know MD is the median line. Then, MD is 1/2 BE" T: "Ok, the median line can help us know not only this one. However, also DM is equal to 1/2 BE."	The teacher explained why this student arrived at their conclusion. The feedback focused on mathematics.
St2: "Because point M and N, M is the midpoint of AE, then, so, it is also useful, it means AM is equal to ME and 1/2AE T: Equal to 1/2AE"	The feedback focused on mathematics
St2: "Then, AE is equal to BE, so MD is equal to ME" T: "MD is equal to " St2: "ME" T: "So now, when AE is equal to BE, this condition is also useful, AE is equal to BE, another one is ME is equal to, we need another condition, it is ME is equal to 1/2 AE. Ok, from these two conditions, we can find ME is equal to DM, then it can be a diamond"	After this student finished their answer, the teacher made a conclusion that focused on mathematics.

Figure 3. Students did not need to operate the software.

Compared with the first interaction, Madame J did not change her teaching strategies, despite the fact that the first students failed to answer her question. She gave both of them enough time to provide their own explanations without any interruptions. However, there were no terms related to DGS mentioned during the interaction. The main focus was only on mathematics. No explanation or method was presented during the interaction. Madame J simply wrote the student's answers on the screen or whiteboard (Figure 4).

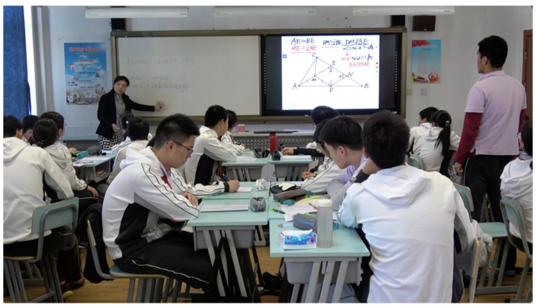


Figure 4. Madame J wrote down students' answers on the screen and whiteboard.

DGS was used to project images since Madame J preferred this technology. However, she believed that using DGS requires teachers' guidance and control in lessons.

5.3. Stories from Mrs. Y's Class

Mrs. Y did not have as much teaching experience with DGS compared to the other teachers in this study. Therefore, it was interesting for us to analyze her interactions with students using DGS. Additionally, it helped us to paint a clearer picture of teachers' practices using DGS in mathematics education. The following interaction (Table 8) shows how the students participated in the classroom when Mrs. Y operated DGS and explained what happened.

Table 8. Teacher–student interaction in Mrs. Y's lesson.

Questions and Feedback	Focus
T:"Let's draw another triangle whose sides are also 4, 7, and 6. Then, we can overlap the two triangles and see whether the two triangles coincided, if they coincided, then the two triangles are, congruent. How to draw it?"	The teacher wanted the students to draw a triangle with the help of DGS. Her question and statement focused on mathematics with the use of technology.
St 1: "First draw one base side" T: "First draw one base side. [teacher operated the computer, choose the tool "segment"] We can find the tool "segments with given length" in the tool "lines". Then, here, we draw a segment, 7, right? Here we tap 7. [teacher clicked the tool, and tapped "7"] Now you can see a segment and its length is 7. Ok, then, how to draw this point on side AC, that is side DF?"	The teacher used DGS to show the students the whole drawing process. Her question focused on mathematics with use of technology.
St 2: "Use compass" T: "Use compass, it means the radius of this circle is?"	The teacher's feedback just repeated what the student answered. It focused on mathematics.
All St: "4" T: "4, which one is its center?"	Focused on mathematics.
St 3: "D" T: "D, ok, now we can find circle. [teacher operated the computer, find the tool "circle"] Then, you can click it, it showed "one center and one point". We need to find the circle and radius, it means the radius is determined, click it. [teacher clicked the tool "circle with center and radius", then looked at the students] Ok, now point D is the center, click it. What is the radius?"	The teacher used DGS to show the students the whole drawing process. Her statements focused on mathematics with use of technology, while one question focused on mathematics.
All St: "4" T: "Then, here tap "4", then you can see a circle. [teacher tapped "4", the screen showed one circle whose center is D and radius is 4] Ok, then is this side, its length is 6. Ok, its center is point E, what is the radius? 6. [teacher clicked "circle with center and radius", tapped 6] Ok, now you can find the intersect point of the two circle?"	The teacher used DGS to show the students the whole drawing process. Her statements focused on mathematics with use of technology, while the question focused on mathematics.
St 4: "Two" T: "Yes, two, it's the same with the two points up or down"	Focused on mathematics.
St 5: "Yes, it is same" T: "Ok, same, here I just draw one, this point, which is the intersect point?"	Focused on mathematics.
St 6:" F" T: "F, it is F here, then, we can draw this triangle. [teacher clicked the intersect point] Link these three points, DEF. [teacher operated the mouse, linked point D, E, F] This triangle, ok, these lines are supportive, we can delete them."	The teacher used DGS to show the students the whole drawing process. Her statements focused on mathematics with the use of technology.

According to the task situation (Table 4), the students were asked to construct a triangle with three given sides using DGS. This kind of task is seldom used in Chinese mathematics lessons because Chinese students do not have that much time to directly operate DGS. Therefore, Mrs. Y made a drastic change to the lesson; she let some of her students operate DGS to solve the task.

As shown in the table, we found that Mrs. Y posed some questions focusing on mathematics: "Which one is the center of the circle? Which is the intersecting point?" DGS was not important in these questions (there were no terms related to the software), and the students did not need to use DGS to find the correct answers.

Unlike the other two teachers in our study, we noticed that Mrs. Y posed some feedback that focused on teaching mathematics using technology by using terminology on how to operate DGS, such as "click it", "delete", and "tap". At this time, Mrs. Y wanted the students to know how to use DGS (Figure 5), For her, DGS was a tool that could help the students to solve mathematics tasks.

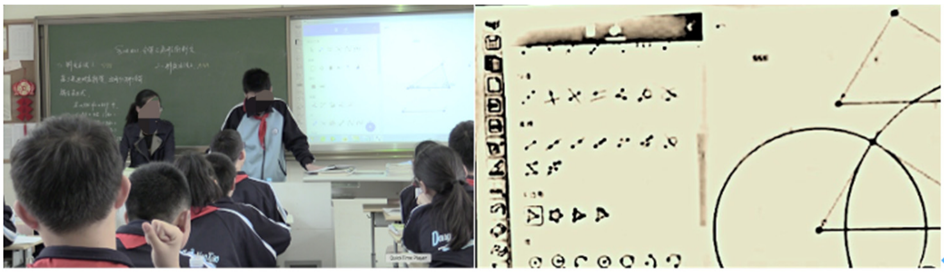


Figure 5. Mrs. Y tried to ask one student to use DGS to construct a triangle.

Based on her teaching experiences, Mrs. Y found that Chinese teachers always told students the answers directly: "I think maybe Chinese teachers will not let the students think by themselves." This may mean that students may not understand the construction method. Therefore, Mrs. Y wanted her students to operate DGS directly and explore the differences between DGS and the use of a pencil and paper. In this way, DGS can help students to develop a clear understanding of geometry.

6. Discussion and Conclusions

This study found that teacher–student interactions are often guided by the initiation–response–evaluation pattern [29]. Additionally, questions and answers may not be derived from students' thought processes during lessons [4]. This implies that teachers may be worried that their students are not familiar with situations in which technology can be used or that they engage in off-task activities, which are discouraged by teachers [30].

Furthermore, although researchers have proposed that technology can create opportunities for students to effectively learn mathematics [4], the teachers in this study seldom linked their questions with DGS. In this study, using DGS did not mean that the teachers changed their interactions with the students. Most of the questions focused only on mathematics, which means that the students were able to answer them without DGS. Therefore, there were not enough opportunities for the students to explore mathematics with the help of DGS in the classroom. This implies that the teachers did not believe that DGS needed to be included in the problem-solving processes of the students. Additionally, they wanted the students to focus on the answers rather than how to explain them using DGS or to consider the concepts of the mathematical tasks [31]. They used DGS as a projector or an amplifier [8,9] without dragging or moving the diagrams. Additionally, the teachers designed many step-by-step tasks for the students to use with technology, with fewer opportunities for the students to engage in mathematical reasoning [4]. This means that the teachers' practices did not present DGS to the students to solve problems [14,15] or explore mathematics [18], but they told the students the mathematical concepts directly [16]. The students were more passive during lessons.

According to research, mathematical interactivity in technology-based classrooms causes students to react to their environment in a mathematical sense. In this research, we assumed that the students must observe these reactions and obtain information on their actions using DGS [6]. This means that the students could use DGS to drag the objects on the screen to explore or check the mathematical content. However, in these lessons, DGS was used more for presenting content. This implies that the dragging mode was controlled by the teachers, not the students; therefore, the meaning of the interactivity of DGS is ambiguous, and DGS was used just for clicking buttons or touching objects on the screen [6]. Using DGS in this way does not impact students' problem-solving processes and does not modify student learning situations and teacher practices in geometry. With DGS, teachers are also required to address new mathematical practices [32,33] and pay more attention to how to design suitable teaching and learning activities [34]. However, the results of this research also imply that the teachers may have lacked sufficient knowledge to help the students via technology [3,35]. Another important factor might be that teachers learned mathematics in a totally different environment to that of modern students; therefore, it was difficult for the teachers to evaluate the students' learning process [3]. In this research, although we could find dynamic elements in each of the tasks used by the teachers, such as moving points, the teachers still used DGS to present the final outcomes and not the entire moving process. When designing some special tasks such as the construction of geometric objects, the teachers chose to interact with the students in a new way [3].

These conclusions concerning the three teachers may not be generalized to other Chinese situations. However, our research seems to present a detailed picture, showing the teachers' strategies regarding the use of DGS in mathematics education. In this regard, further research is required to advance our understanding of how technology resources such as DGS impact the teaching of mathematics. It would be essential to find out whether there are other factors such as a limited class time, heavily didactical tasks, and examinations that may also affect teachers' practices and interactions with students.

Author Contributions: F.Z. conceived, designed, coded, and analyzed the data and drafted the initial manuscript. B.X. supervised and finalized the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding authors.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. An Example of Analyzing Interview Data

Transcription:

Interviewer: what changes do you think DGS take in your lessons and how they can help you during your teaching.

Mrs. J: Of course, it helps me a lot (1) I think technology helps us (2). Now I have used technology for more than 20 years (3). I think I am more and more confident with them (4). I think technology is a supporter (5).

Step 1: We divided this transcription into the following sentences:

Of course, it helps me a lot (1).
I think DGS can help us (2).
Now I have used DGS for more than 20 years (3).
I think I am more and more confident with them (4).
I think technology is a supporter (5).

Step 2: For each sentence, we determined the keywords and their meanings. From the transcript, the first and second sentence had the same meanings. Mrs. J thought that DGS could help her lessons a lot. The third sentence implies that Mrs. J was familiar with DGS. The fourth one shows that Mrs. J was confident when using DGS in the lessons. In addition, the last sentence means that for Mrs. J, DGS was more like a supporter.

Step 3: Afterwards, we discussed each sentence to find out which of them could help us explain how and why the teachers used DGS and interacted with students. For example, in this transcription, because Mrs. J saw DGS as a supporter, she did not always let her students operate DGS, and her questions or feedback always focused on mathematics.

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