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Financing a Capital-Constrained Supply Chain under Risk Regulations: Traditional Finance versus Platform Finance

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Abstract: Small- and medium-sized enterprises (SMEs) frequently face challenges in obtaining financial assistance from traditional banks. Platform Supply Chain Finance (PSCF) has emerged as a promising solution for financing issues among SMEs, with an added focus on integrating sustainability aspects. This study focused on a two-tier supply chain as its primary research topic to find strategies to enhance supplier financial viability and improve the efficiency and profitability of the main manufacturing enterprise. In this study, we establish three distinct hypotheses corresponding to the three models involving supplier and manufacturer participation, encompassing parameters such as production batch size, pricing, and supply chain profit. First, it examined financing decisions through the lens of core enterprise-led platform finance. Second, it applied the Stackelberg game theory to investigate financing decisions in three distinct modes: traditional finance, platform internal finance, and external platform finance. Suppliers, manufacturers, and banks can be seen as participants in a Stackelberg game. In this game, suppliers act as leaders, making production and procurement decisions first, while manufacturers and banks act as followers, adjusting their behavior based on the suppliers' decisions. Finally, it performed a comparative analysis of decisions and supply chain efficiency across these modes. When the risk regulation cost coefficient falls below a certain threshold, suppliers are willing to set up their own PSCF and there is an optimal level of risk regulation effort within the interval (0, 1). We compare platform finance with traditional finance and find that the traditional finance model maximizes profits for suppliers, while the external financing model maximizes profits for manufacturers and the overall supply chain profit. Findings provide insights for platforms, suppliers, manufacturers, and banks to implement optimal financing and channel structures to increase their profits and promote the sustainable development of the financial supply chain. In addition, future research on blockchain platform models would be highly meaningful.

Keywords: platform finance; risk regulations; Stackelberg game; capital constraint; small- and medium-sized enterprises; sustainability

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1. Introduction

Small- and Medium-Sized Manufacturing Enterprises (SMMEs) are defined as a company employing fewer than 250 persons, with a total turnover not exceeding 50 million EUR and with an annual balance sheet total not exceeding 43 million EUR [1]. SMEs have, compared to larger companies, fewer resources and less experience in managing new technologies. Industry 4.0 drives management research toward creating new concepts, models, and methodologies to realize synergy effects across industries that work together on a single business platform and/or system. The challenges facing both macroeconomic and microeconomic spheres, particularly at the operational management level, are set to intensify in the coming era [2]. SMMEs have long been grappling with the paradox of a 'high demand' for financing coupled with 'low trust' from lenders, a situation that significantly impacts their production efficiency [3]. Supply Chain Finance (SCF) emerges as a viable and often preferred financial solution, capable of alleviating the financial strains on supply chain partners. It offers benefits such as lower interest rates, extended payment terms, and increased working capital, which are crucial for addressing the financial constraints of SMMEs [4]. As information and internet technologies continue to evolve at a rapid pace, the significance of online supply chain finance is growing. It is instrumental in enhancing supply chain performance across various dimensions, including capital management and information flow [5]. Supply chain finance leverages the credit and market position of core enterprises to provide financial support and credit value-added services to SMEs up and down the supply chain, thereby solving the financing problems of SMEs and promoting the stability and efficient operation of the industrial chain.

A platform supply chain is defined as a novel format of online agency selling corresponding with the latest development in e-business, where online marketplaces are provided as platforms for manufacturers or sellers to direct access to their customers [6]. Such examples include Amazon (US) and Taobao (China), among others. Recently, an increasing number of industry-dominating core enterprises have ventured into online supply chain finance models. They have established their own supply chain finance platforms as part of the ongoing trend of digital transformation, thereby creating open and interconnected information platforms for logistics, financial flows, and data streams within the industrial chain. With the development of e-commerce, more and more companies are starting to sell their products through E-commerce platforms/retailers, with online sales of agricultural products standing out [7]. For example, in the United States, Amazon extends credit to its qualified vendors, offering loans ranging from USD 1000 to USD 750,000 with terms of up to 12 months and interest rates between 6% and 16% [8]. Alibaba is a leading e-commerce platform with a strong market presence and customer loyalty. Its advanced data capabilities allow it to provide efficient financial support, solidifying its position as a key player in supply chain financing [9]. Ant Group's platform significantly contributed to China's short-term loans, with a substantial impact on small and micro-business financing. Finaxar extends a credit line facility to Lazada sellers, with amounts ranging from USD 3650 to USD 730,000. This initiative is aimed at bolstering the seller base in Southeast Asia, thereby tapping into a market of 650 million potential consumers [10]. Core enterprises, through tailored supply chain finance platforms, can enhance service quality and supply chain capabilities. Despite their competitive advantages, they must navigate the challenges of resource contention and competition, playing a key role in supporting SMMEs' financing. In the digital economy, these platforms are a growing trend, with core enterprises maintaining significant influence. PSCF has risen as an innovative solution to address the financing challenges faced by SMEs, distinctively enhancing their sustainability prospects. By embedding sustainability into its core, PSCF not only alleviates capital constraints but also propels SMEs toward environmentally and socially responsible growth, fostering a transformative approach to business practices that align with the principles of sustainable development. This strategic integration positions PSCF as a conduit for economic progress that is both responsible and resilient, setting a precedent for the financial industry's pivotal role in nurturing a sustainable corporate ecosystem.

In light of the critical role that platform finance plays in the financial landscape, this paper delves into the strategic decisions surrounding supplier financing and the efficacy of platform models. The primary objective is to enhance the financing efficiency of small- and medium-sized enterprises, thereby fostering greater cohesion and resilience within the supply chain. This paper delves into the under-explored area of platform regulatory capabilities within the realm of supply chain finance, where current research predominantly concentrates on financial concepts, financing decisions, and capital constraints. Additionally, it addresses the gap in comparative studies of internal versus external financing modes within platform finance. By employing a comparative analysis method, the paper aims to provide insights into the financing decisions across various modes. Based on the above analysis, the aim of this paper is to address the following four problems. (i) In the context of the three modes, how do suppliers and manufacturers formulate decision-making strategies? (ii) Under the platform financing mode, what impact does wholesale pricing have on supplier production decisions? How do financing interest rates and the level of platform risk regulation efforts influence supplier production decisions? (iii) Under what conditions are suppliers willing to establish their own supply chain finance platforms? (iv) Is the optimal level of platform risk regulation efforts the same between the external financing mode and the internal financing mode? Can manufacturers and banks achieve parity between external financing rates and internal financing rates by negotiating the profit-sharing ratio between banks and enterprises? We consider three modes: (i) a traditional financing supply chain; (ii) a platform's internal financing supply chain; and (iii) a platform's external financing supply chain.

The main contributions of this paper can be summarized as follows. (1) It takes the viewpoint of core enterprises to investigate the financing decisions within supply chain finance platforms. (2) It accounts for the volatility of market demand, delving into the financing decisions within a two-tier supply chain framework that encompasses both a capital-constrained supplier and a core enterprise manufacturer. (3) It introduces platform regulatory capabilities as a distinguishing feature between the platform and traditional supply chain finance modes and uses the level of platform risk regulation efforts as a factor in the core enterprise's platform decisions. (4) It leverages the Stackelberg game theoretical model to dissect financing decisions across three distinct operational modalities and performs a comparative analysis of the efficacy of these decisions in enhancing supply chain efficiency.

The remainder of this paper is organized as follows: Section 2 is mainly about problem description and model construction. Section 3 discusses the external financing model of the platform. The solution and numerical results are discussed in Section 4. Parametric sensitivity analysis is performed in Section 5. The conclusions are outlined in Section 6. Finally, inspiration and prospects are outlined in Section 7.

2. Literature Review

Our paper is closely aligned with three major academic research streams. The first stream focuses on issues related to supply chain finance (SCF). The second stream investigates supply chain finance under capital constraints, while the third stream explores platform finance.

2.1. Research on Supply Chain Finance

SCF represents an effective approach to reducing financing costs, enhancing financing efficiency, and improving the effectiveness of financial operations. Many scholars have engaged in theoretical investigations into the decision-making processes associated with SCF. Yan et al. [11] scrutinized two supply chain financing schemes tailored for capital-constrained retailers and analyzed the retailer's ordering decisions and the supplier's pricing decisions. Qin et al. [12] explored the value of advance payment financing in the context of carbon emission reduction and production within a supply chain. In supply chain financing decision making, scholars have observed that SMEs are often significantly influenced by financial constraints during their decision-making processes. SMEs encounter a multitude of challenges in their financing endeavors, including unfavorable financing terms, information asymmetry, and limitations related to asset collateral. Yan et al. [13] designed a supply chain finance system comprising a capital-constrained retailer, a manufacturer, and a commercial bank, incorporating a partial credit guarantee contract for SCF. Huang et al. [14] examined the impact of transportation fees charged by third-party logistics providers (3PL) on capital-constrained supply chains. Qin et al. [15] found that, when the manufacturer is capital-constrained, funds can be accessed through platform financing, supplier credit financing, and mixed financing modes. Wu et al. [16] examined the joint impact of vertical cross-shareholdings and external financing, including trade credit and bank loans, on the order strategy of a capital-constrained retailer. Li et al. [17]

examine the past and the future of the interface of operation management and finance with Fintech in the supply chain.

2.2. Research on Capital-Constrained Supply Chain Finance

To address financial constraints, scholars have further explored supply chain finance models to find effective solutions for SMEs grappling with financial constraints. Gao et al. [18] explored a Supply Chain Finance (SCF) system wherein a manufacturer, faced with uncertain demand from a retailer in a single period, navigates capital constraints by utilizing an online P2P lending platform, determining loan service rates. Yun et al. [19] investigated the parallels between supply chain finance and options and introduced American call options to supply chain financial products in the context of SMEs' accounts receivable financing. Liu et al. [20] derived applicable conditions for blockchain platform finance by comparing blockchain platform finance with SME-independent finance (SIF). Huang et al. [21] formulated a framework for supply chain finance based on a general supply chain and discussed equilibrium strategies under three distinct financing modes. Cheng et al. [22] investigated the impact of the prisoner's dilemma on manufacturers' emission reduction investments within competitive supply chains, as well as the dual economic and environmental effects of capital constraints faced by retailers.

2.3. Research on Platform Supply Chain Finance

With the increasing availability of huge amounts of data, operations and finance interfaces in a cycle of material, financial, and information flows [23]. Smart supply chain finance, facilitated by network platforms, effectively addresses financing challenges for SMEs. Sun et al. [24] investigated the operational decisions in a dual-channel supply chain with a capital-constrained manufacturer, an e-commerce platform (ECP), and a third-party logistics company (3PL), using two game models to derive equilibrium solutions and comparing strategies under different financing modes. In light of this technology, an increasing number of companies across various industries have considered its use to support supply chain finance [25]. Liu et al. [26] examined a three-tier supply chain comprising a manufacturer, distributor, and capital-constrained retailer. They explored the operational strategies of blockchain platform finance (BPF). Wang et al. [27] conducted an exploratory case study of a blockchain-based platform offering SCF solutions for supply chain partners. Liu et al. [26] discussed platform preferences for different modes and corresponding government supervision measures. Chen et al. [28] identified critical factors influencing the performance of SCF platforms. Fu et al. [29] considered a platform supply chain and investigated supply chain coordination by characterizing the optimal operational decisions of the three firms under any given equity financing strategy. The Hotelling model and Pyramid Spatial model are established to describe the competition between different platforms and the choice of agents and the Stackelberg game is constructed with commission rates as the decision variables [30].

2.4. Literature Summary

In summary, the differences between this paper and the most related studies are shown in Table 1.

Table 1. The main difference between our research and previous research.

Authors	Supply Chain Finance	Decision and Coordination	Platform Finance		Capital Constraint	Risk Regulation
			Internal	External		
Yan et al. [11]	√	√				
Chen et al. [28]	√		√			√
Wang et al. [27]	√	√				
Yan et al. [13]	√				√	
Huang et al. [21]	√				√	
Wu et al. [16]	√				√	
Liu et al. [26]	√		√			
Our paper	√	√	√	√	√	√

Table 1 presents a comparative analysis of the main differences between our research and previous studies within the domain of supply chain finance. The table categorizes various aspects of research, including supply chain finance, decision and coordination, platform finance, capital constraints, and risk regulation. Our review indicates that while some studies, such as those by Yan et al. [11] and Chen et al. [28], have addressed both supply chain finance and decision and coordination, they have not explicitly incorporated platform finance or risk regulation into their analysis. Wang et al. [27] and Yan et al. [13] have included capital constraints in their research but their focus on platform finance and risk regulation remains limited. Huang et al. [21] and Wu et al. [16] have expanded their scope to include risk regulation, yet the integration of platform finance is still not fully explored. Liu et al. [26] have made strides in considering both platform finance and risk regulation but there seems to be a gap in the comprehensive analysis of capital constraints and decision and coordination mechanisms. Our paper aims to bridge these gaps by providing a holistic approach that encompasses all the aforementioned aspects. We have integrated supply chain finance with decision and coordination, platform finance, capital constraints, and risk regulation to offer a more comprehensive understanding of the dynamics within supply chain finance. This multifaceted approach allows for a deeper insight into the complex interplay between these elements and their impact on the overall efficiency and stability of supply chain operations.

In summary, our research contributes to the literature by offering a more integrated perspective on supply chain finance, addressing the limitations of previous studies and providing a foundation for further exploration in this field. Our paper has identified several research gaps in the relevant literature.

Firstly, in the domain of supply chain finance research, the scholarly community has predominantly concentrated on financial conceptual issues, the decision-making processes involved in financing, and the challenges posed by capital constraints. However, the literature has not extensively addressed the regulatory capabilities that platforms possess within the context of supply chain finance. This oversight is significant because the platform's ability to regulate can significantly influence the efficiency and effectiveness of financial transactions within the supply chain. Secondly, regarding the selection of financing modes, there is a noticeable gap in the scholarly discourse when it comes to comparative analyses of internal versus external financing options within platform financing. Most research has not delved deeply into the nuanced differences and implications of choosing between these two financing modes. The selection of financing mode is a critical decision that can have profound effects on the financial health and strategic direction of a firm within a supply chain. Therefore, this paper aims to bridge these gaps by employing a comparative analysis approach. Drawing on the insights and methodologies suggested by previous scholars, we will explore how different financing modes impact the decision-making process. Specifically, we will examine the advantages and disadvantages of internal versus external financing within the platform financing context, considering factors such as cost, accessibility, control, and risk management. By doing so, we hope to provide a more comprehensive understanding of the complexities involved in financing mode selection and contribute to the body of knowledge in supply chain finance.

3. Model

With the advancement of big data, cloud computing, blockchain, and other technologies, the transformation from traditional supply chain finance models to online supply chain finance platforms has accelerated. Supply chain finance platforms can be divided into core enterprise-led supply chain finance platforms, bank-led supply chain finance platforms, and fintech company-led supply chain finance platforms according to the division of the leading body. This paper focuses on the core enterprise-led supply chain finance platform and bank-led supply chain finance platform.

3.1. Model Description

Table 2 delineates the salient features of traditional platforms in the context of financing. In the realm of traditional finance, supply chain participants are contingent upon conventional banking institutions for the provision of financing services, which inevitably entails the payment of service fees. Conversely, under the platform finance paradigm, these participants harness the capabilities of the platform, thereby obviating the need for traditional banking services and, consequently, the associated service fees. In scenarios where sales revenue falls short of fulfilling loan obligations, the traditional financing model leaves the residual amount irretrievably uncollected. In stark contrast, the platform financing model offers a glimmer of hope for the recovery of the residual amount, introducing an element of recuperation that is absent in its traditional counterpart.

Table 2. Features of traditional finance versus platform finance.

	Traditional Supply Chain Financing	**Platform Supply Chain Financing**
Supply chain financing	Relies on banks (or other third parties)	Uses platform directly between the sellers and buyers
Operational cost	No additional cost per transaction but service fees are needed (to be paid to the banks)	Cost of risky regulatory efforts Other costs
Risk management	In instances where the sales revenue proves insufficient to cover the loan obligations, the residual amount remains unrecoverable	In instances where the sales revenue proves insufficient to cover the loan obligations, the residual amount carries a probability of being reclaimed

In essence, the platform's role in facilitating Supply Chain Finance (SCF) is undeniably pivotal and it holds the potential to significantly bolster operational efficiency. The distinction between platform and traditional financing is not merely a matter of preference but a transformative shift that could redefine the financial landscape for supply chains.

The paper mainly studies the following two supply chain finance platform financing modes, including core enterprise platform internal capital financing and core enterprise platform external capital financing, as shown in Table 3. Core enterprise-led platforms are characterized by internal funding sourced directly from the core business, enabling tailored financing solutions that leverage the core enterprise's deep insights into supply chain operations to foster internal synergy. In contrast, bank-led platforms, which operate through a cooperative model with the core business and are externally funded by banks, offer standardized financial services with broader capital access, potentially at the expense of a specialized understanding of the unique supply chain dynamics. The divergence lies in the core enterprise-led model's focus on customized services and supply chain collaboration, versus the bank-led model's emphasis on risk management and credit approval stringency.

Table 3. Financing mode of supply chain finance platform.

Core Enterprise Platform Financing	**Leading Party**	**Source of Funds**
Internal	Core business	Core business
External	Core business (Bank–enterprise cooperation)	Banks

The paper examines a two-echelon supply chain consisting of a newsboy-type supplier and a dominant manufacturer, where the dominant manufacturer is the core firm in the supply chain and the capital-constrained supplier is regarded as an SME in the supply chain. The supplier provides products (raw materials) to the manufacturer's core enterprise, and the core enterprise carries out further production. Under random market demand, for capital-constrained SMEs, production within their own funds often fails to meet the market demand, and their output cannot satisfy the core enterprise for further processing and manufacturing, so the capital-rich core enterprise in the supply chain can provide financing assistance to its upstream suppliers in order to increase the output of the upstream suppliers and further improve its own revenue. In this study, we establish three distinct hypotheses corresponding to the three models involving supplier and manufacturer participation, encompassing parameters such as production batch size, pricing, and supply chain profit. To begin, we hypothesize that the optimal production quantity will be influenced by the interaction between supplier and manufacturer strategies, reflecting a balance between market demand and cost considerations. Furthermore, we postulate that pricing decisions will be significantly determined by the cost structures inherent in each model, with the aim of maximizing supply chain profitability while remaining competitive. Lastly, the suppliers and manufacturers are risk-neutral and both aim to maximize their own profits. (See Figure 1). The hypotheses are as follows:

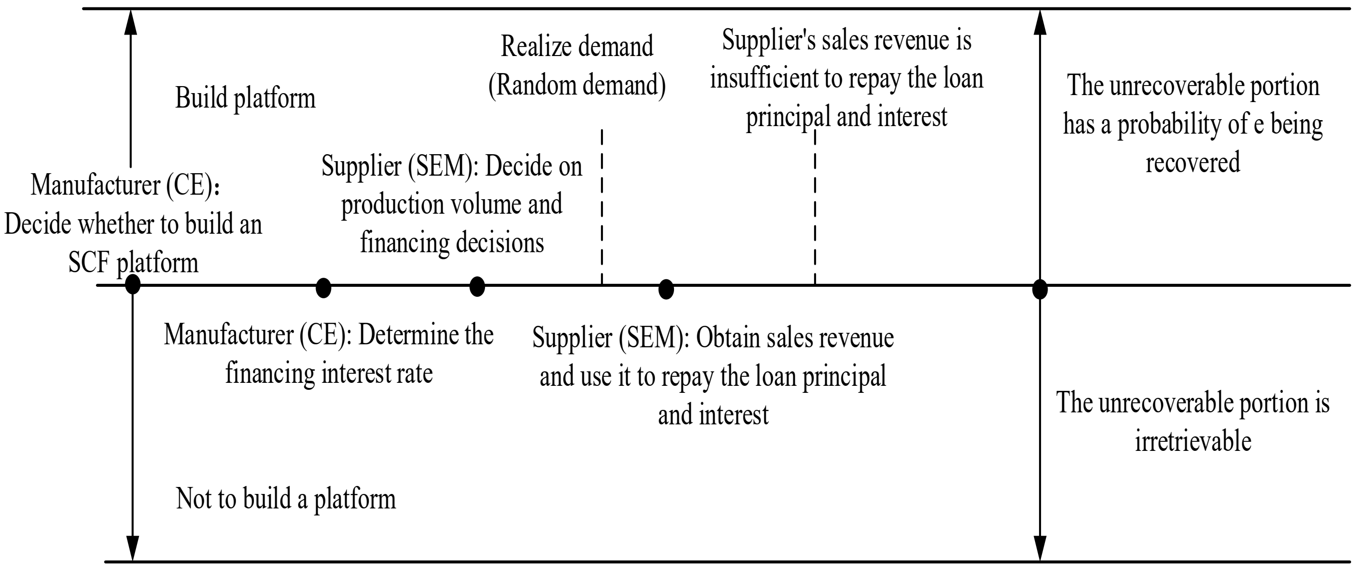

Figure 1. The timeline of supply chain events.

Hypothesis 1: *According to Wang et al. [31], whose probability density function is $f(x)$, the cumulative distribution function is $F(x)$ and $z(x) = f(x)/\overline{F}(x)$ is an increasing function, $\overline{F}(x) = 1 - F(x)$. The supplier produces raw materials that are subsequently sold to the manufacturer for further processing and manufacturing. We assume that the supplier's cost per unit of raw materials is denoted as c, while the cost per unit of raw material production is denoted as w. Let q represent the market price of a product manufactured by the processing of these raw materials. Prior to market demand materializing, the supplier determines the output of raw materials and evaluates the necessity for financing. For the sake of discussion, the wholesale price w is not considered a decision variable in this context. The manufacturer, during this phase, determines the interest rate r for the supplier's loan and the level of risk regulation efforts e associated with the supply chain finance platform. The 'level of risk regulation effort' refers to the measures and resources invested by the platform in the risk management process, including the identification, assessment, mitigation of risks, and communication and coordination with relevant parties. Upon observing the market demand, the manufacturer places orders for raw materials corresponding to the demand. Once the demand is satisfied, both the supplier and the manufacturer receive the sales proceeds. The supplier, in possession of the sales proceeds, repays the principal and interest of the loan.*

Hypothesis 2: *In the framework of the traditional supply chain finance model, the supplier, facing capital constraints, determines the production volume q. The portion of the supplier's own capital insufficient to support the production $(cq - B)$ can be supplemented by a loan obtained from the* *manufacturer at an interest rate r. In this traditional model, any shortfall in sales revenue to cover the loan is irrecoverable by the manufacturer. In contrast to the traditional supply chain finance model, the "platform financing" model introduces enhanced supervision of suppliers by the manufacturer through a supply chain finance platform. Notably, in this model, there is a probability of recovering the shortfall in sales revenue necessary to meet the principal and interest obligations of the loan. This recovery probability is contingent upon the level of risk supervision efforts exerted by the platform, denoted as e, where e represents the probability of recovery.*

Hypothesis 3: *The suppliers and manufacturers are risk-neutral and both aim to maximize their own profits. The supplier aims to maximize expected profit π_S and the manufacturer aims to maximize its expected profit π_M. For simplicity, assume that the bank's risk-free interest rate is 0. Compared with the bank's risk-free interest rate, the manufacturer's cost of funds for providing loans to suppliers is higher than the risk-free interest rate, assuming that the manufacturer's cost of funds corresponds to the interest rate r_f. The lending rate at which the supplier receives a loan from the manufacturer is r, which is greater than or equal to r_f. The decision variables and parameters involved in this paper are listed in Table 4.*

Table 4. Model parameters and decision variables.

Parameters	
X	Stochastic market demand
$f(x)$	The probability density function of the demand X
$F(x)$	The cumulative distribution function of the demand X, $\overline{F}(x) = 1 - F(x)$
$z(x)$	Failure rate. $z(x) = f(x)/\overline{F}(x)$ Incremental
c	Cost of raw materials per unit of production by suppliers
w	Wholesale price at which suppliers sell raw materials to manufacturers
p	Market prices of products produced by manufacturers through the processing of raw materials and manufacturing
B	Initial financing for financially constrained suppliers
r_f	Interest rate corresponding to the manufacturer's cost of capital
$C(e)$	According to Wang et al. [32], the cost of risky regulatory efforts. $C(e) = -a\ln(1 - e)$ where a is the effort cost factor, $a > 0$
π_{Si}	Expected revenue for supplier (S) under the model i
π_{Mi}	Expected revenue for manufacturer (M) under the model i
i	$i = 1$ when it indicates "traditional supply chain finance" mode, and $i = 2$ When it means "platform's own capital financing" mode. $i = 3$ When it means "platform external capital financing" mode.
Decision variables	
q	Supplier-determined raw material production
r	The supplier's lending rate, $r \geq r_f$, $c(1 + r) < w < p$
e	Level of platform risk regulation efforts

3.2. Supplier Decision Making

Under the traditional supply chain finance model, the supplier can apply for a loan from the manufacturer, and the sales revenue will be used to repay the principal and interest of the loan. If the sales revenue is less than the principal and interest of the loan, the insufficient portion of the payment can be excluded from the payment under this model. Under the "platform financing" model, the supplier obtains a loan through the manufacturer-led supply chain finance platform, and the loan funds come from the manufacturer. As the platform has stronger regulatory capabilities, if the sales revenue is less than the loan principal and interest, the platform has a probability e of recovering the remaining loan principal and interest. For more detailed data, please refer to Appendix A.

3.2.1. Traditional Supply Chain Finance

When the supplier's own initial capital B is not sufficient to support its production program, the supplier takes a loan from the manufacturer in the amount of $cq - B$. After the demand is realized, the supplier receives sales revenue $\mathrm{wmin}(q, X)$, which is prioritized for repayment of the loan principal and interest $(cq - B)(1 + r)$. When the sales revenue is greater than the principal and interest on the loan, the difference between the sales revenue and the principal and interest payable $\mathrm{wmin}(q, X) - (cq - B)(1 + r)$ is the supplier's final revenue; when sales revenue is less than the principal and interest on the loan, $\mathrm{wmin}(q, X) - (cq - B)(1 + r) < 0$, it is insufficient to pay the principal and interest on the loan and the supplier goes bankrupt.

Let $k = \frac{(cq-B)(1+r)}{w}$, $E[\mathrm{wmin}(q, X) - (cq - B)(1 + r)] = 0$ when $x = k$. $E[\mathrm{wmin}(q, D) - (cq - B)(1 + r)] < 0$ when $x < k$, which means that at this time, the supplier's sales revenue is less than the principal and interest of the loan, the supplier's sales revenue is unable to repay the full principal and interest of the loan, and the supplier is insolvent. Therefore, k can be defined as the supplier's "bankruptcy threshold".

In this paper, it is assumed that the manufacturer is unable to recover all the loan principal and interest after the supplier's bankruptcy under the traditional supply chain finance model. When $x > k$, the supplier's expected revenue is $E[\mathrm{wmin}(q, X) - (cq - B)(1 + r)]$. When $x < k$, the supplier's expected revenue is 0. The supplier's expected revenue consists of sales revenue, loan principal and interest repayment, and cost of own funds. So, the total expected revenue of the supplier under the traditional finance model is $E[\mathrm{wmin}(q, X) - (cq - B)(1 + r)]^+ - B = w\int_0^q \overline{F}(x)dx - w\int_0^k \overline{F}(x)dx - B$.

Therefore, the decision problem for suppliers in this model is as follows.

$$\max \pi_{S1}(q) = w\int_0^q \overline{F}(x)dx - w\int_0^k \overline{F}(x)dx - B \tag{1}$$

$$\text{s.t.} \quad q > \frac{B}{c} \tag{2}$$

Proposition 1. *In the traditional supply chain finance model, the optimal output of the supplier is* $q^* = q_1$*, where* q_1 *satisfies* $\overline{F}(q_1) = \frac{c(1+r)}{w}\overline{F}(k_1)$.

3.2.2. Platform Supply Chain Finance

Different from the traditional supply chain finance model, under the platform financing model, when the supplier's sales revenue is insufficient to repay the loan principal and interest, there is still e probability of paying the entire loan principal and interest.

When $x > k$, the supplier expects a revenue of $E[\mathrm{wmin}(q, X) - (cq - B)(1 + r)]$. At this time, the expected revenue is $wq\int_q^{+\infty} f(x)dx + w\int_k^q xf(x)dx - \int_k^{+\infty}(cq - B)(1 + r)f(x)dx$.

When $x < k$, under the platform financing model, the supplier has e probability of having to pay the remaining unpaid principal and interest on the loan. Therefore, when $x < k$, the supplier expects a revenue of $e * E[\mathrm{wmin}(q, X) - (cq - B)(1 + r)]$. At this time, the expected revenue is $e\int_0^k [wx - (cq - B)(1 + r)]f(x)dx$. The two parts are added and simplified to give $w\int_0^q \overline{F}(x)dx - (1 - e)w\int_0^k \overline{F}(x)dx - e(cq - B)(1 + r) - B$.

In summary, the issues of supplier decision making under the platform financing model are as follows:

$$\max \pi_{S2}(q) = w\int_0^q \overline{F}(x)dx - (1 - e)w\int_0^k \overline{F}(x)dx - e(cq - B)(1 + r) - B \tag{3}$$

$$\text{s.t.} \quad q > \frac{B}{c} \tag{4}$$

Proposition 2. *The optimal output of the supplier under the platform financing model is* $q^* = q_2$*, where* q_2 *satisfies* $\overline{F}(q_2) = \frac{c(1+r)}{w}\overline{F}(k_2) + \frac{c(1+r)}{w}eF(k_2)$.

Proposition 3. $q_2 < q_1$; $\pi_{S2}{}^* < \pi_{S1}{}^*$.

Proposition 3 establishes that, in the comparison of the three models, suppliers experiencing capital inadequacy achieve maximum yield and revenue under the traditional supply chain finance financing model. Furthermore, it indicates that the optimal yield under the platform financing model q_2 is less than the optimal yield under the traditional model q_1. This disparity is attributed to the fact that the risk regulatory measures implemented by the platform act as constraints on suppliers' tendencies toward uninformed loans and production. Specifically, the reason for $\pi_{S2}{}^* < \pi_{S1}{}^*$ is grounded in the risk regulation mechanisms enforced by the platform. These measures serve to curtail instances of suppliers engaging in blind loans and production practices. Even in situations where market demand is low and the supplier's sales revenue proves insufficient to cover the loan's principal and interest under the platform mode, the supplier remains exposed to a certain probability of risk. This probability imposes a necessary constraint, fostering a more methodical and scientifically informed approach to production by suppliers while discouraging blind loans within the supply chain.

Proposition 4. *Within the platform financing model and while keeping other parameters constant, several relationships can be observed. Firstly, as the wholesale price w increases, the supplier's optimal output q also rises. Secondly, an increase in the level of risky regulatory effort e results in a decrease in the supplier's optimal output q. Lastly, an elevation in the financing interest rate r is associated with a decrease in the supplier's optimal output q.*

Proposition 4 shows that, when wholesale prices increase, the optimal production level for suppliers rises because the elevated prices incentivize them to allocate more resources to attain greater profits. As the level of risk regulation intensifies, the optimal production level for suppliers may decrease due to heightened regulations, which augment uncertainties and compliance costs in the production process. An increase in financing interest rates may result in financial constraints for suppliers, leading to a reduction in the optimal production level to accommodate the elevated costs of financing.

3.3. Manufacturer Decision Making

In the study of this paper, the manufacturer is the dominant core firm in the supply chain, and after the market demand is observed, the manufacturer then orders the product from the supplier to fulfill the market demand. Generally, in studies similar to this paper, some scholars set the wholesale price w as the decision variable of the manufacturer and the selling price of the manufacturer p as the decision variable. Because the problem explored in this paper focuses on financing decisions rather than production decisions, in order to facilitate the analysis of the financing decisions of suppliers and manufacturers and combined with the fact that the wholesale price and retail price are greatly influenced by the market price, this paper does not discuss the wholesale price w as the manufacturer's decision-making variable.

The financing decisions of both the supplier and the manufacturer, in which the manufacturer primarily places the financing interest rate r and the level of risk supervision efforts of supply chain finance platforms e as decision variables to be analyzed, should be considered. Among them, the manufacturer makes an interest rate decision in the traditional supply chain finance model and applies that interest rate in the platform financing model. This paper assumes that the financing interest rates in the traditional supply chain finance model and the core enterprise's own supply chain finance platform own funds financing are equal.

3.3.1. Manufacturer Interest Rate Decisions

As can be seen above, the manufacturer's interest rate decision is sought under the traditional supply chain finance model. The manufacturer's expected revenue under this model can be categorized into three parts: the sales proceeds, some or all of the principal and interest recovered on the loan, and the cost of capital for providing the loan.

In the supplier non-financing model, the manufacturer's revenue is equal to the sales revenue, as follows:

$$\pi_M = E[(p-w)\min(q,X)] = (p-w)\int_0^q \overline{F}(x)dx \quad (5)$$

Under the traditional supply chain finance model, the manufacturer's sales proceeds are $E[(p-w)\min(q,X)]$. The recovered loan amount is $E[\min[w\min(q,X), (cq-B)(1+r)]]$. The cost of capital is $(cq-B)(1+r_f)$. Therefore, the manufacturer's total revenue is

$$E[(p-w)\min(q,X) + \min[w\min(q,X), (cq-B)(1+r)]] - (cq-B)(1+r_f)$$

$$= (p-w)\int_0^q \overline{F}(x)dx + w\int_0^k \overline{F}(x)dx - (cq-B)(1+r_f)$$

Thus, the supplier interest rate decision problem in the traditional supply chain finance model is obtained as follows:

$$\pi_{M1}(r) = (p-w)\int_0^q \overline{F}(x)dx + w\int_0^k \overline{F}(x)dx - (cq-B)(1+r_f) \quad (6)$$

$$\text{s.t.} \begin{cases} r_f \le r < \frac{w-c}{c} \\ \overline{F}(q) = \frac{c(1+r)}{w}\overline{F}(k) \end{cases} \quad (7)$$

Proposition 5. *The optimal decision of the manufacturer under the constraints is* $r^* = r_f$.

Proposition 5 shows that the larger the financing rate in the traditional supply chain finance model, the smaller the total revenue to the manufacturer instead, so the optimal interest rate decision for the manufacturer is the smallest interest rate within the constraints, i.e., the interest rate that is equal to the opportunity cost of capital r_f.

3.3.2. Manufacturer Platform Risk Regulation Level of Effort Decisions

Under the platform financing model, the manufacturer's expected revenue can be divided into four parts: first, sales revenue, second, part or all of the principal and interest recovered, third, the capital cost of providing the loan, and fourth, the cost of building the platform. Among them, the recovered principal and interest of the loan differs from that under the traditional supply chain finance model and additionally increases the platform construction cost.

By making the platform construction cost $C(e) = a\ln(1-e)$, the cost of platform risk regulatory effort, the higher the level of effort, the higher the probability that the supplier repays the full principal and interest of the loan. According to Xu et al. [33], the a in the platform building cost function represents the cost of effort coefficient. $C(0) = 0$ can particularly represent the cost when the manufacturer does not build a supply chain finance platform, which translates into the manufacturer's problem under the traditional supply chain finance model. $C(1) = +\infty$ represents the infinite effort cost when the manufacturer pursues the level of effort to recover 100% of the loan principal and interest.

Under the platform financing model, the manufacturer's expected revenue on sales is $E[(p-w)\min(q,X)]$; the manufacturer's cost of capital to provide the loan is $(cq-B)(1+r_f)$; and the manufacturer's platform construction cost is $a\ln(1-e)$. Some or all of the principal and interest recovered by the manufacturer is discussed below.

When $x > k$, the principal and interest on the loan recovered by the manufacturer is $(cq-B)(1+r)$. When $x < k$, the supplier has e probability of repaying the entire loan principal and interest, i.e., $e(cq-B)(1+r)$. The supplier has $1-e$ probability of repaying only part of the loan principal and interest, i.e., $(1-e)E[w\min(q,X) - (cq-B)(1+r)]$. Therefore, the recovered loan principal and interest under the platform model is found to be $ewk + (1-e)w\int_0^k \overline{F}(x)dx$.

Thus, the manufacturer's total revenue under the platform financing model is $\pi_{M2} = (p - w)\int_0^q \overline{F}(x)dx + ewk + (1-e)w\int_0^k \overline{F}(x)dx - (cq - B)(1 + r_f) + a\ln(1-e)$.

Proposition 6. *Discuss the feasibility of a manufacturer building its own supply chain finance platform. When the cost coefficient of risk monitoring effort $a < w\int_0^k F(x)dx$, manufacturers are willing to build their own supply chain finance platforms.*

Based on this, under the $a < w\int_0^k F(x)dx$ conditions, the manufacturer's decision problem on the level of risky regulatory effort in the platform financing model can be written as

$$\pi_{M2} = (p - w)\int_0^q \overline{F}(x)dx - (1-e)w\int_0^k F(x)dx + a\ln(1-e) \tag{8}$$

$$\text{s.t.}\begin{cases} r = r_f \\ \overline{F}(q) = \frac{c(1+r)}{w}\overline{F}(k) + \frac{c(1+r)}{w}eF(k) \\ a < w\int_0^k F(x)dx \end{cases} \tag{9}$$

Proposition 7. *Manufacturer's optimal level of risk regulatory effort e^* Satisfies $\left[(1-e)k'^2 f(k) - f(q)\right]q' = k'F(k)$, where q' satisfies $\left[p\overline{F}(q) - c\left(1 + r_f\right)\right]q' + w\int_0^k F(x)dx = \frac{a}{1-e}$.*

Proposition 8. *The overall supply chain revenue π_{SM2} under the traditional supply chain finance model is greater than the overall supply chain revenue π_{SM3} under the platform finance model.*

Proposition 8 shows that the overall supply chain yield is higher under the traditional supply chain finance model when comparing the overall supply chain yield. This is because the suppliers do not bear the financing risk at this time and the yield is relatively higher. Manufacturers, on the other hand, are more willing to build a supply chain finance platform in order to protect their capital security, reduce capital risk, and gain greater benefits for themselves.

3.4. Platform External Financing Decisions

Under the core enterprise-led supply chain finance platform external fund financing mode, the bank obtains the profit and loss generated by the loan through the platform. The core enterprise generates the cost that exists due to the construction of the platform and extracts a part of the revenue from the principal and interest of the loan recovered by the bank as the platform service fee. However, for suppliers financing through the core enterprise-led supply chain finance platform, there is no difference between the different sources of funds except for the possible difference in the financing interest rate, so this chapter does not specifically discuss the decision-making problem of suppliers under this model.

Suppliers obtain loans for production through the platform. Market demand is observed and then sold to the manufacturer (the core enterprise), and sales revenue is used to repay loan principal and interest. The platform loan funds come from banks outside the chain, and the banks obtain platform services through the manufacturer's own supply chain finance platform, provide loans to suppliers, and share the recovered loan principal and interest to the platform as a platform service fee at a ratio of θ. The platform determines the level of risk supervision efforts e. The higher the level e, the higher the probability of recovering the loan principal and interest when the supplier's sales revenue fails to cover the loan principal and interest. The behavioral relationship among suppliers, manufacturers, and banks in this model is shown in Figure 2.

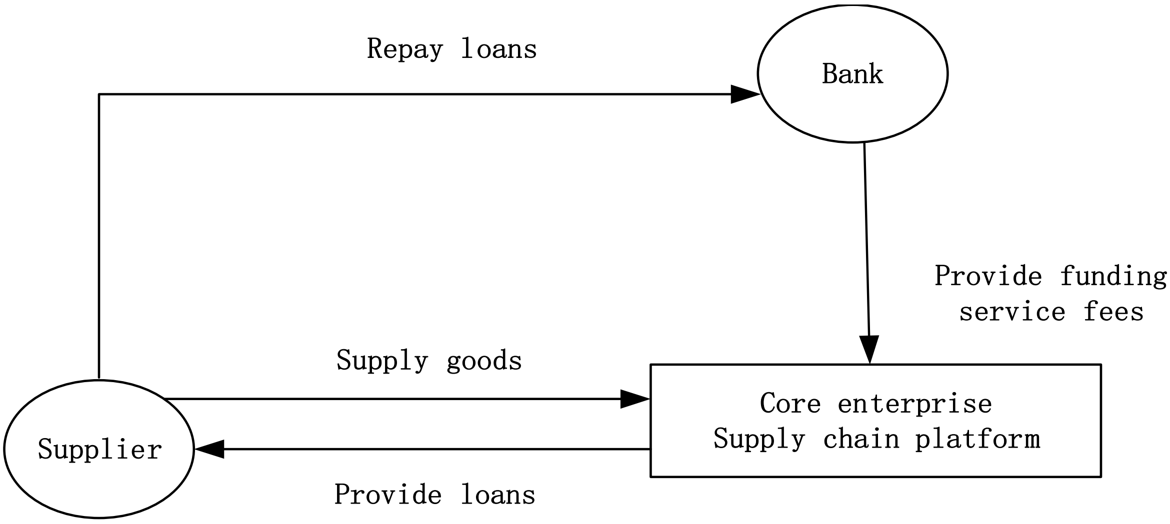

Figure 2. The behavior relationship between suppliers, manufacturers, and banks.

The decision sequence of the external fund financing of the core firm-led supply chain finance platform is as follows. First, the manufacturer's own platform decides the level of the platform's risk regulatory effort, then the bank provides a loan contract to the manufacturer's own platform (i.e., the interest rate of the platform's external capital financing loan), and finally, the supplier decides the amount of financing through the platform's loan (i.e., the production output). In supply chain finance, suppliers, manufacturers, and banks can be seen as participants in a Stackelberg game. In this game, suppliers act as leaders, making production and procurement decisions first, while manufacturers and banks act as followers, adjusting their behavior based on the suppliers' decisions. This structure reflects the hierarchical relationships and information asymmetry present in the supply chain, where suppliers must make capital-constrained decisions under market demand uncertainty, similar to the newsvendor problem. Therefore, the Stackelberg game theory provides us with a powerful analytical framework to study how suppliers optimize their strategies taking into account the reactions of manufacturers and banks. Thus, suppliers, manufacturers, and banks can be viewed as being in a Stackelberg game, and suppliers can be viewed as capital-constrained newsboys under stochastic market demand.

Combining the behavioral relationship and decision sequence of suppliers, manufacturers, and banks, the supplier's financing decision is analyzed first in the Stackelberg game, while the bank's interest rate decision is analyzed later. And finally, the level of risk regulation efforts on the manufacturer's own platform is analyzed.

As can be seen from the above, compared with the core enterprise-led supply chain finance platform's own capital financing decision, the suppliers are the same as in the platform's internal financing mode except for the loan interest rate. Therefore, let the loan interest rate under the external platform financing mode be $\widetilde{r}$. Therefore, the supplier's expected revenue under the external platform financing mode is

$$\max\pi_{S3}(q) = \int_0^q \overline{F}(x)dx - (1-e)w\int_0^k \overline{F}(x)dx - e(cq - B)(1+\widetilde{r}) - B \tag{10}$$

$$\text{s.t.} \quad q > \frac{B}{c} \tag{11}$$

The optimal output of the supplier is $q^* = q_3$, where q_3 satisfies $\overline{F}(q_3) = \frac{c(1+\widetilde{r})}{w}\overline{F}(k) + \frac{c(1+\widetilde{r})}{w}eF(k)$.

Since the manufacturer provides platform services, it can receive a percentage θ of the recovered loan principal and interest as a platform service fee of the funds. The actual funds received by the bank out of the principal and interest recovered on the loan are $(1-\theta)\left[\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}\right]$. Considering a perfectly competitive market, the bank's expected revenue on financing should be equal to the bank's risk-free interest rate. This paper assumes that the bank's risk-free interest rate is; therefore, letting $(1-\theta)\left[\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}\right]=\mathrm{cq}-\mathrm{B}$, the $\widetilde{\mathrm{r}}=\frac{(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\mathrm{F}(\mathrm{x})\mathrm{dx}}{(\mathrm{cq}-\mathrm{B})}+\frac{\theta}{(1-\theta)}$.

Proposition 9. *The bank's interest rate decision $\widetilde{r}$ in this model satisfies $(1-\theta)\left[ewk+(1-e)w\int_0^k\overline{F}(x)dx\right]=cq-B$. i.e., $\widetilde{r}=\frac{(1-e)w\int_0^k F(x)dx}{(cq-B)}+\frac{\theta}{(1-\theta)}$.*

From Proposition 9, the bank's interest rate decision is affected by the platform's level of risky regulatory effort in addition to the bank–enterprise revenue-sharing ratio, which is that the higher θ, the higher the bank's interest rate decision.

Manufacturer's expected revenues under the external financing model of the platform can be categorized into three parts: sales proceeds, the share of principal and interest recovered on the loan, and the cost of building the platform. The sales proceeds and platform construction costs are the same as in the platform internal capital financing model. The recovered loan principal and interest under the platform model is $\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}$. The manufacturer's share of the loan principal and interest is $\theta\left[\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}\right]$.

Thus, the manufacturer's expected revenue is as follows:

$$\pi_{\mathrm{M3}}=(\mathrm{p}-\mathrm{w})\int_0^{\mathrm{q}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}+\theta\left[\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}\right]+\mathrm{aln}(1-\mathrm{e})$$

By substituting for the bank's interest rate decision $\widetilde{\mathrm{r}}=\frac{(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\mathrm{F}(\mathrm{x})\mathrm{dx}}{(\mathrm{cq}-\mathrm{B})}+\frac{\theta}{(1-\theta)}$, i.e., $(1-\theta)\left[\mathrm{ewk}+(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}\right]=\mathrm{cq}-\mathrm{B}$, the manufacturer risk regulatory effort level decision problem can be written as

$$\max\pi_{\mathrm{M3}}(\mathrm{e})=(\mathrm{p}-\mathrm{w})\int_0^{\mathrm{q}}\overline{\mathrm{F}}(\mathrm{x})\mathrm{dx}-(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\mathrm{F}(\mathrm{x})\mathrm{dx}+\mathrm{aln}(1-\mathrm{e})+(\mathrm{cq}-\mathrm{B})\widetilde{\mathrm{r}} \quad (12)$$

Among other things, $(1-\mathrm{e})\mathrm{w}\int_0^{\mathrm{k}}\mathrm{F}(\mathrm{x})\mathrm{dx}$ denotes the part of loan principal and interest repaid by the supply chain financial platform after recovering part of the loan principal and interest repaid by the suppliers, which can also be regarded as the financing risk of the supply chain financial platform.

Proposition 10. *Manufacturer's optimal level of risk regulatory effort e^* satisfies*

$$\left[(p-w)\overline{F}(q)+\frac{\theta c}{1-\theta}\right]q'-\frac{a}{1-e}=0$$

Among them, the q_3 satisfies $\overline{F}(q_3)=\frac{c(1+\widetilde{r})}{w}\overline{F}(k)+\frac{c(1+\widetilde{r})}{w}eF(k)$ and $\widetilde{r}$ satisfies $(1-\theta)\left[ewk+(1-e)w\int_0^k\overline{F}(x)dx\right]=cq-B$. $q'=\frac{F(k)k'}{(1-e)f(k)k'k'-f(q)}$.

The relationship between the manufacturer's revenue and the platform's regulatory effort function is shown in Figure 3.

Firstly, when the platform risk regulatory efforts are at a low level (within the range of 0 to 0.2), an increase in regulatory intensity correlates with an augmentation of the total revenue for manufacturers. This may be attributed to the fact that lower levels of regulatory efforts can provide a certain degree of standardization and order, thereby aiding in reducing market risks and fostering economic activities among manufacturers. However, when the platform risk regulatory efforts exceed 0.2, the total revenue for manufacturers gradually diminishes. This may suggest that beyond a certain threshold of regulatory intensity, the costs associated with regulation may start to outweigh its benefits. Excessive regulation might lead to higher compliance costs and restrictions for manufacturers, thereby suppressing their economic activities and influencing the overall performance of total revenue. This interpretation contributes to an understanding of the complexity of platform risk regulation and highlights the challenges involved in finding an optimal balance, where regulatory efforts are effective in maintaining market order without imposing undue burdens on industries, thus achieving optimal economic benefits.

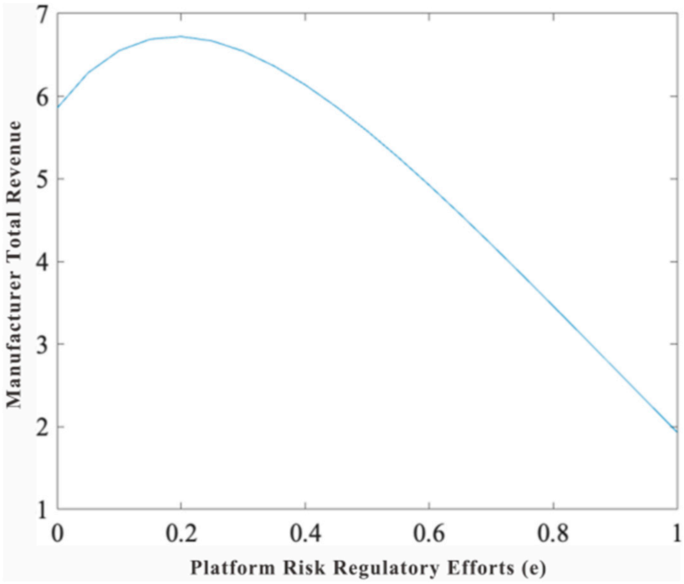

Figure 3. The relationship between manufacturer revenue and platform regulatory effort.

4. Comparative Analysis of the Platform Internal and External Financing

From Proposition 9, it follows that since the manufacturer provides platform services and can collect a percentage of the repaid principal and interest as platform service fees, the actual amount of funds that the bank receives from the repaid principal and interest is $(1-\theta)\left[ewk+(1-e)w\int_0^k \overline{F}(x)dx\right]$. Considering a perfectly competitive market, the expected return on financing for banks should be equal to the risk-free interest rate of banks. This paper assumes the risk-free interest rate for banks to be $\widetilde{r}$; hence, let $(1-\theta)\left[ewk+(1-e)w\int_0^k \overline{F}(x)dx\right] = cq-B$. From the above, it can be seen that due to the platform's external financing model, the bank's interest rate decision is $\widetilde{r}=\frac{(1-e)w\int_0^k F(x)dx}{(cq-B)}+\frac{\theta}{(1-\theta)}$. Suppliers obtain loans through the platform to finance production. After observing market demand, they sell to the manufacturer (core enterprise) and the sales revenue is used to repay the principal and interest of the loan. The platform loan funds come from banks outside the supply chain. Banks, by joining the manufacturer's own supply chain finance platform, provide platform services, extend loans to suppliers, and share a proportion θ of the repaid principal and interest with the platform as a platform service fee. The platform determines the level of risk regulation effort e; the higher the level, the greater the probability of recovering the principal and interest when the supplier's sales revenue is insufficient to cover the loan principal and interest.

From Proposition 9, its interest rate decision is not only affected by the level of the platform's risk supervision efforts but also by the revenue sharing ratio between the bank and the enterprise. When $\widetilde{r} = r_f$ and $\frac{(1-e)w\int_0^k F(x)dx}{(cq-B)}+\frac{\theta}{(1-\theta)} = r_f$, $\theta\left[(1+r_f)(cq-B)-(1-e)w\int_0^k F(x)dx\right] = r_f(cq-B)-(1-e)w\int_0^k F(x)dx$. Therefore, the bank–enterprise revenue sharing ratio is required to satisfy $\theta = 1-\frac{(cq-B)}{(1+r_f)(cq-B)-(1-e)w\int_0^k F(x)dx}$. This section compares and analyzes the platform's risk supervision effort level, manufacturer's revenue, and mode choice under the platform's internal and external capital financing mode when the interest rate is the same.

Proposition 11. $\pi_{M3}{}^{*} > \pi_{M2}{}^{*}$. *Manufacturers' profits under the external platform financing model exceed those under the internal platform financing model.*

Proposition 12. $e_3^* < e_2^*$. *The level of regulatory effort in the platform under the external financing model is lower than the level of regulatory effort under the internal financing model.*

Proposition 11 shows that manufacturers' profits under the external platform financing model exceed those under the internal platform financing model. Manufacturers typically achieve higher profits with external platform financing due to greater access to capital, risk diversification, specialized financial services, operational flexibility, and potentially better credit terms, compared to the more limited scope and resources of internal platform financing.

Proposition 12 shows that the level of regulatory effort in the platform under the external financing model is lower than the level of regulatory effort under the internal financing model. It highlights that platforms typically invest less in regulatory efforts under external financing models due to shared responsibilities with external financiers, distributed risks, reliance on external expertise, cost reduction, and adherence to simpler regulatory practices, reflecting a strategic approach to balancing oversight with operational efficiency.

When a manufacturer builds its own supply chain platform, it can reduce the level of platform risk supervision efforts e, reduce platform inputs, and increase its own revenue when it introduces external financing from banks and ensures that the financing rate of external financing is the same as r_f. The reduction in the platform's risk supervision effort level e can also appropriately expand the output of suppliers and better promote the healthy and scientific development of the supply chain.

Under the platform's external financing model, from $\widetilde{r} = \frac{(1-e)w\int_0^k F(x)dx}{(cq-B)} + \frac{\theta}{(1-\theta)}$, we obtain $(cq-B)(1+r) - (1-e)w\int_0^k F(x)dx = \frac{1}{(1-\theta)}(cq-B)$. Due to $\frac{1}{(1-\theta)}(cq-B) > (cq-B)$, we obtain $(cq-B)r > (1-e)w\int_0^k F(x)dx$. Therefore, the financing interest rate under the external financing mode of the platform can compensate for the financing risk of the supply chain finance platform.

5. Numerical Analysis

In this section, further numerical analysis is conducted based on the previous studies. To analyze the trend of changes in the decision variables q, r, and e, we will assign values to the remaining parameters. According to Wang C et al. [32], and and Dan et al. [34] in conjunction with the assumptions of this paper, $p = 10$, $w = 8$, $c = 5$, $r_f = 0.06$, $B = 0.8$, and $a = 0.005$. Uniform distribution is widely used in statistical analysis due to its mathematical properties, which include simplicity, linearity in expectation and variance, equal probability distribution, ease of calculation and simulation, unbiasedness, and easy comparison with other distributions. It offers an intuitive method for describing data and is an appropriate choice when data are uniformly distributed within a certain range without a clear concentration trend. Therefore, we assume that the stochastic demand obeys a standard uniform distribution, i.e., $D \sim U(0,1)$.

5.1. Impact of Different Factors on Supplier Decision Making

In light of Proposition 4, this concise paper delves into the pivotal role that wholesale pricing, the intensity of platform risk regulatory efforts, and the financing interest rate play in shaping suppliers' dual decisions regarding financing and output within the framework of a platform financing model. The interplay of these three factors in supplier decision making is vividly depicted in Figure 4. As illustrated in Figure 4, the supplier's decision making is negatively correlated with the platform risk regulatory effort and the financing rate, while it positively correlates with the wholesale price. In this context, an escalation in the platform risk regulation level (e) can impose additional constraints and limitations on suppliers. This is due to the platform potentially enforcing more stringent regulations and oversight, which could, in turn, escalate the suppliers' operational costs and risks.

Consequently, suppliers might opt to curtail production volumes (q) to navigate these risks and align with the heightened regulatory demands. Similarly, an uptick in the financing rate (r) could compel suppliers to shoulder higher interest expenses or increased financial burdens. This financial strain might detract from their capital allocation toward production, resulting in a reduction in production volumes. Conversely, an increase in the wholesale price (w) presents suppliers with the opportunity to reap greater profits. This financial incentive could encourage them to ramp up production volumes in pursuit of enhanced revenue streams. This analysis underscores the dynamic and interdependent nature of supplier decisions within the platform financing ecosystem, highlighting the need for a balanced approach to risk management, regulatory compliance, and financial strategy to optimize operational outcomes.

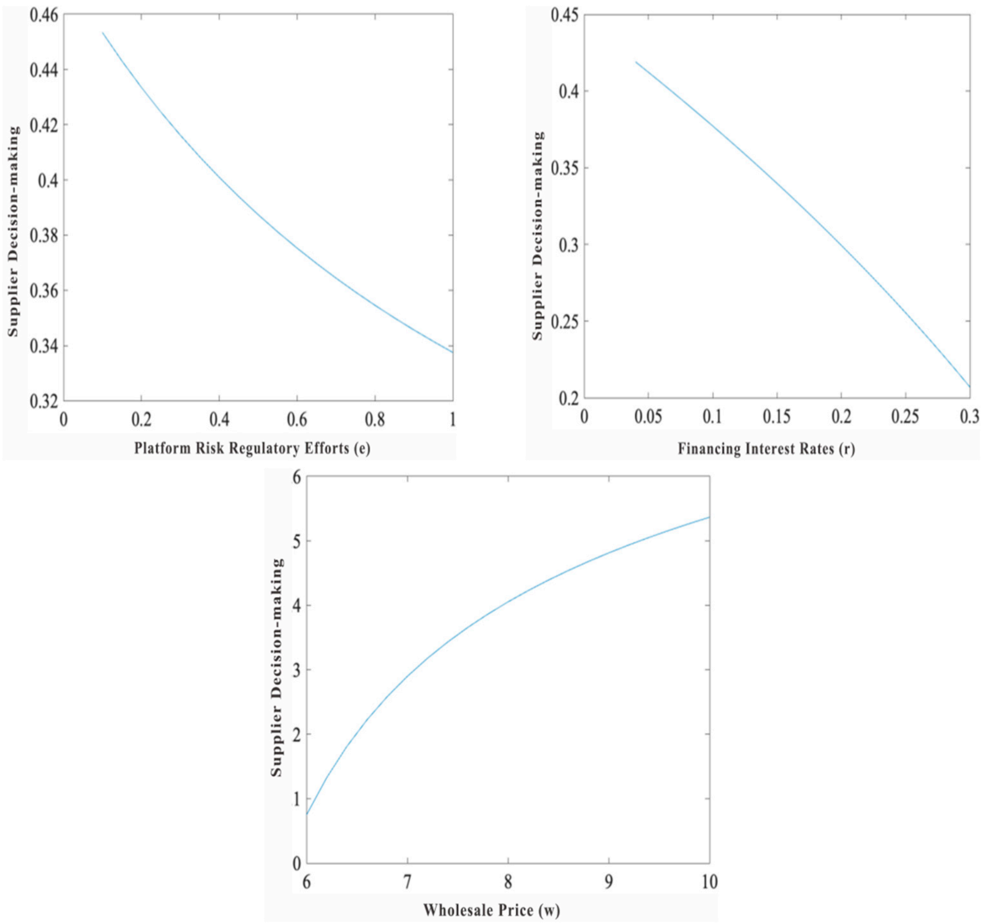

Figure 4. The impact of different influencing factors on supplier decision making.

5.2. Comparison of Decision Making in Different Models

Under the demand obeying the standard uniform distribution, the optimal decisions and maximum revenues of suppliers and manufacturers under the three modes of traditional supply chain finance mode, the platform's own fund mode, and the platform's external fund mode are calculated as shown in Figure 5.

From Figure 5, comparing the maximum revenue of suppliers under different models, $\pi_{S1} > \pi_{S3} > \pi_{S2}$, the manufacturer's maximum gain is $\pi_{M3} > \pi_{M2} > \pi_{M1}$.

In our analysis of the three distinct supply chain finance modes, suppliers realize their highest profits under the traditional finance model, amounting to 0.6022. This outcome underscores the potential benefits of established financial frameworks that are deeply integrated into current supply chain practices. Manufacturers find their peak profitability in the platform's external financing mode, with profits reaching 0.6672. The external financing model, facilitated by the platform, appears to offer manufacturers a more advantageous position in terms of profit margins, possibly due to better access to capital or more favorable financing terms. The overall supply chain sees its maximum profit in the platform's external financing mode, with a combined profit of 1.239. This suggests that while individual stakeholders may have varying levels of profit across different models, the external financing model by the platform optimizes the collective financial health of the entire supply chain. It suggests a nuanced relationship between the structure of supply chain finance and the distribution of profits among stakeholders. The traditional finance mode, while beneficial for suppliers, may not necessarily maximize the profits for manufacturers or the supply chain as a whole. Conversely, the platform's external financing mode seems to strike a balance that not only benefits manufacturers but also enhances the overall profitability of the supply chain. This disparity in profit distribution raises important questions about the design of supply chain finance models. In conclusion, while the traditional finance mode offers suppliers the highest individual profits, the platform's external financing mode emerges as a more balanced and potentially more beneficial approach for both manufacturers and the supply chain as a whole. Future research should continue to explore the dynamics of these different finance modes and their implications for supply chain management and strategy.

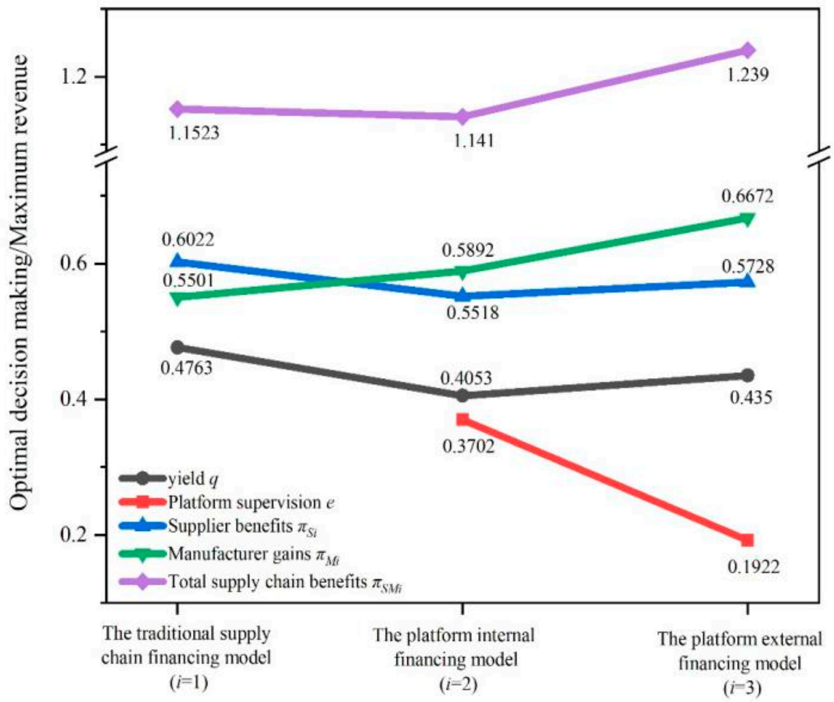

Figure 5. Optimal decision and maximum revenue.

In the realm of supply chain finance, the platform's external financing model confers the most substantial benefits upon manufacturers, surpassing even the platform's own financing model. The rationale behind this phenomenon can be attributed to two principal factors. Firstly, manufacturers experience a reduction in the cost of capital under the external financing model, as external financiers may provide more favorable rates or terms that diminish the overall borrowing expenses. Secondly, the level of supervision from the platform is diminished, alleviating the administrative burden and costs for manufacturers, who are no longer subject to the stringent monitoring and reporting obligations inherent in the platform's own financing model. Furthermore, the total revenue of the supply chain is maximized under the platform's external financing model. This outcome can be ascribed to several interrelated factors. Efficiency gains within the supply chain, stemming from more streamlined capital flows, contribute to enhanced operational performance and, consequently, increased revenue. The diversification of risk through the involvement of external financiers fosters more stable and potentially superior revenue generation over time. Additionally, improved access to capital for manufacturers enables investment in growth opportunities, which can further augment the overall revenue of the supply chain. In essence, the platform's external financing model offers manufacturers significant advantages by reducing both the cost of capital and the level of supervision, culminating in a higher total revenue for the supply chain. This model exemplifies a strategic approach to financing that not only benefits individual manufacturers but also bolsters the financial health and performance of the entire supply chain ecosystem.

The paper distinguishes itself by offering a unique perspective on supply chain finance, focusing on the strategic decisions made by core enterprises within these platforms. Firstly, it delves into how these enterprises approach financing within the supply chain, acknowledging the inherent variability in market demand. Secondly, it explores the dynamics of a two-tier supply chain, highlighting the interplay between a capital-constrained supplier and a core enterprise manufacturer. A key differentiator is the introduction of platform regulatory capabilities, which sets it apart from traditional finance models. The paper emphasizes the role of the platform's risk regulation efforts in influencing the core enterprise's strategic choices. Finally, employing a Stackelberg game model provides a comprehensive analysis of financing decisions across three distinct modes, offering insights into how these decisions impact supply chain efficiency, thereby setting a new benchmark in the field of supply chain finance research.

6. Conclusions

The paper investigates the financing decisions in a two-tier supply chain involving suppliers and manufacturers with stochastic demand under three financing modes: traditional supply chain finance, platform's internal financing, and platform's external financing.

Through solving the optimization problems, we determine the optimal production levels for suppliers in all three modes, as well as the optimal financing amounts for suppliers in the traditional supply chain finance and platform's internal financing modes. Simultaneously, we obtain the manufacturers' optimal interest rates and risk regulation effort levels. This study proposes three hypotheses, each corresponding to different models of supplier and manufacturer participation, focusing on key parameters such as production batch size, pricing, and supply chain profit. Initially, it examines financing decisions from the perspective of core enterprise-led platform finance. Subsequently, it applies Stackelberg game theory to analyze financing decisions in three distinct modes, traditional finance, platform internal finance, and external platform finance, where suppliers act as leaders by making production and procurement decisions first, followed by manufacturers and banks who adjust their strategies based on the suppliers' decisions. Finally, a comparative numerical analysis of decisions and supply chain efficiency across these modes is conducted to draw conclusions.

These models in our paper advance supply chain finance by differentiating between internal and external platform financing and introducing the risk regulation effort parameter. It offers a deeper insight into the operational and strategic impacts of various financing mechanisms. Internal financing benefits from lower costs and quicker access but is contingent on the platform's financial stability. External financing provides broader capital at potentially lower costs yet entails compliance and interest expenses. The model's emphasis on proactive risk management through assessment tools, contingency planning, and credit practices enhances supply chain stability. It equips managers with strategies for capital and risk decisions and assists policymakers in fostering a resilient supply chain finance ecosystem.

The research findings indicate the following. (1) A supplier's own financing level influences their choice of financing mode. Under the platform financing mode, wholesale prices positively impact supplier production decisions, while financing interest rates and the level of platform risk regulation efforts negatively influence their production decisions. The financing interest rate offered by manufacturers should meet the minimum value of the supplier's financing opportunity cost, encouraging increased production and thereby enhancing the manufacturer's sales revenue. (2) Concerning the optimal platform risk regulation effort level, suppliers are willing to establish their own supply chain finance platforms when the risk regulation cost coefficient is below a certain threshold. An optimal risk regulation effort level e exists within the interval (0, 1). (3) Among the three modes, suppliers achieve maximum profits in the traditional supply chain finance mode, manufacturers attain maximum profits in the platform's external financing mode, and the overall supply chain profit is maximized in the platform's external financing mode. (4) Manufacturers and banks can achieve parity between external financing rates and internal financing rates by negotiating the profit-sharing ratio between banks and enterprises. Under equal interest rates, the optimal platform risk regulation effort level in the platform's external financing mode is lower than that in the internal financing mode but the profits are higher than in the internal financing mode. The paper holds profound significance in both theory and practice.

(1) The paper makes significant theoretical contributions by innovatively incorporating the concept of platform regulatory capabilities, which not only enriches the theoretical framework of supply chain finance but also deepens the understanding of supply chain risk management. Through in-depth analysis using the Stackelberg game model, it extends the application of relevant theories and provides new theoretical insights for improving supply chain efficiency. Furthermore, by integrating theories from multiple disciplines, this paper promotes innovation in research methodology and facilitates in-depth academic discussions in the field of supply chain finance, thus making important contributions to the academic development of this area;
(2) The paper holds significant practical implications by offering a unique perspective on strategic decisions made by core enterprises within supply chain finance platforms. It not only enhances enterprises' adaptability to market demand fluctuations and optimizes supply chain structures but also innovates in risk management and provides scientific support for strategic decision making. Furthermore, by deeply analyzing the impact of various financing models on supply chain efficiency, it provides practical guidance for improving operational performance. Ultimately, setting a new benchmark in the field of supply chain finance research aids in the formation of industry standards and offers valuable references for practitioners and theorists alike.

7. Inspiration and Prospects

The paper investigates the financing decisions in a two-echelon supply chain involving suppliers and manufacturers with stochastic demand under three financing modes. Based on our analysis, we offer the following managerial recommendations to optimize supply chain finance model selection and management strategies:

(1) Within the platform financing model, managers are advised to focus on the control of platform oversight. Over-regulation could potentially deter SMEs from pursuing financing opportunities and may diminish the profits for all parties involved. Hence, it is essential for managers to strike a balance in oversight levels, ensuring effective risk management without undermining the viability of supply chain finance;
(2) Core enterprises are encouraged to consider the establishment of proprietary supply chain finance platforms. Such an initiative can draw external funding into the platform, alleviating their financial strain and reducing the cost of capital. Additionally, owning a supply chain finance platform allows for more customized financing terms that can better accommodate the diverse needs of supply chain participants;
(3) Manufacturers, when extending credit to suppliers, should set reasonable interest rates that reflect the suppliers' opportunity costs of capital. This approach can incentivize suppliers to boost production and deliveries, thereby enhancing the overall efficiency of the supply chain;
(4) In core enterprise-led external financing models, manufacturers and banks can negotiate a profit-sharing ratio that aligns external financing rates with internal rates. This cooperative strategy can attract more external investment to the platform and increase overall revenue;

(5) Risk management and monitoring are paramount, regardless of the financing model adopted. It is imperative for managers to develop a robust risk monitoring system that can swiftly identify and mitigate potential risks, safeguarding the stable functioning of supply chain finance;
(6) To promote the sustainable development of financial supply chains, platforms should adopt comprehensive measures, including the implementation of data-driven risk management to enhance decision-making transparency, integration of blockchain technology to improve supply chain visibility, incorporation of ESG standards to foster environmental and social responsibility, innovation of financial products to meet diverse financing needs, and establishment of partnerships to jointly advance industry development, thereby creating a more favorable financial environment.

Several limitations exist in this work, which can be possible research directions in the future. The following prospects are presented below:

Although blockchain technology has a potentially significant impact on supply chain finance, our study did not fully consider this aspect. Blockchain can offer transparency, security, and decentralization, which are particularly important for platform finance models. Future research should explore how blockchain affects the structure, processes, and outcomes of supply chain finance. Our analysis has primarily focused on the independent optimal decisions of manufacturers and suppliers under three distinct modes, without addressing decision-making issues in collaborative scenarios. In actual business environments, collaboration among supply chain participants is crucial for improving overall efficiency and responding to market changes. Future research should consider how cooperation can optimize the decision-making process. We did not delve into the development of rational contractual agreements tailored to different modes. Contractual agreements are key tools for coordinating the interests of all supply chain parties and reducing transaction costs. Future research should analyze how to design contracts to fit various supply chain finance models and promote cooperation among all parties. For computational simplicity, this study assumes that the financing interest rates for traditional supply chain finance models and self-funded financing by core enterprises are the same. However, in the real world, different financing channels and conditions can lead to varying financing costs. Exploring the impact of diverse financing interest rates on supply chain finance decisions and efficiency will provide important insights for understanding and optimizing financing strategies. Although our research provides theoretical models and numerical analysis, it does not include actual case studies to validate and enrich our findings. Real case studies can demonstrate the application of theoretical models in the real world and offer more specific insights and recommendations. Future studies could examine platform finance on platforms like Alibaba and JD.com to draw more meaningful conclusions.

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Appendix A. Proofs

Proof of Proposition 1. The derivative of $\pi_{S1}(q)$ with respect to q gives $\pi'_{S1}(q) = w\overline{F}(q) - c(1+r)\overline{F}(k)$ that $\pi'_{S1} = 0$ i.e., $\frac{\overline{F}(q)}{\overline{F}(k)} = \frac{c(1+r)}{w}$ and taking the logarithm on both sides gives

$\ln \overline{F}(q) - \ln \overline{F}(k) = \ln \frac{c(1+r)}{w}$. Deriving from the left side of the equation yields $-z(q) + \frac{c(1+r)}{w} z(k)$. Since $k < q$ and $z(k) < z(q)$ and because $\frac{c(1+r)}{w} < 1$, $-z(q) + \frac{c(1+r)}{w} z(k) \le 0$, so π'_{s1} is first positive and then negative. The maximum value of π_{S1} is obtained at $\pi'_{S1} = 0$, such that q_1 satisfies $\overline{F}(q_1) = \frac{c(1+r)}{w} \overline{F}(k)$ and π_{S1} obtains the maximum value when $q = q_1$.

As can be seen from the previous content, $q_1 > \frac{B}{c}$, when the supplier does not have sufficient funds, participating in supply chain finance to finance from the manufacturer can effectively increase its output and revenue. □

Proof of Proposition 2. The derivative of $\pi_{S2}(q)$ with respect to q gives $\pi'_{S2}(q) = w\overline{F}(q) - (1-e)c(1+r)\overline{F}(k) - ec(1+r)$. $\frac{\overline{F}(q)}{(1-e)\overline{F}(k)+e} = \frac{c(1+r)}{w}$, when $\pi'_{S2}(q) = 0$. Taking logarithms of the left and right sides of this equation yields $\ln \overline{F}(q) - \ln\{(1-e)\overline{F}(k) + e\} = \ln \frac{c(1+r)}{w}$. Taking the derivative on the left side of this equation yields $-\frac{f(q)}{\overline{F}(q)} + \frac{(1-e)f(k)k'}{(1-e)\overline{F}(k)+e} = -z(q) + z(k)\frac{(1-e)k'}{(1-e)+e/\overline{F}(k)}$. It is known that $z(k) < z(q)$ and $(1-e)\frac{c(1+r)}{w} < 1$. It is also known that $\frac{1}{\overline{F}(k)} - 1 > 0$ and $\frac{e}{\overline{F}(k)} - e > 0$. Therefore, $\frac{(1-e)k'}{(1-e)+e/\overline{F}(k)} < 1$ and the right-hand side of the above equation $-z(q) + z(k)\frac{(1-e)k'}{(1-e)+\frac{e}{\overline{F}(k)}} < 0$. The maximum value of π_{s3} is obtained at $\pi'_{s2} = 0$. Such that q_2 satisfies $\overline{F}(q_2) = \frac{c(1+r)}{w}\overline{F}(k) + \frac{c(1+r)}{w} eF(k)$, i.e., $q = q_2$. π_{S2} obtains the maximum value when $\pi'_{S2}(q_2) = 0$. □

Proof of Proposition 3. Such that q_1 satisfies $\overline{F}(q_1) = \frac{c(1+r)}{w}\overline{F}(k_1)$ and q_2 satisfies $\overline{F}(q_2) = \frac{c(1+r)}{w}\overline{F}(k_2) + \frac{c(1+r)}{w} eF(k_2)$. The following proves that $q_2 < q_1$:

$\overline{F}(q_1) - \overline{F}(q_2) = \frac{c(1+r)}{w}\overline{F}(k_1) - \frac{c(1+r)}{w}\overline{F}(k_2) - \frac{c(1+r)}{w} eF(k_2)$. That is $\left[\overline{F}(q_1) - \frac{c(1+r)}{w}\overline{F}(k_1)\right] - \left[\overline{F}(q_2) - \frac{c(1+r)}{w}\overline{F}(k_2)\right] = -\frac{c(1+r)}{w} eF(k_2)$, $-\frac{c(1+r)}{w} eF(k_2) < 0$. Let the function $G(x) = \overline{F}(x) - \frac{c(1+r)}{w}\overline{F}\left(\frac{(cq-B)(1+r)}{w}\right)$; from the proof of Proposition 2, we know that $w\overline{F}(x) - c(1+r)\overline{F}\left(\frac{(cq-B)(1+r)}{w}\right)$ monotonically decreases. It is easy to know that the function $G(x)$ is monotonically decreasing. $\left[\overline{F}(q_1) - \frac{c(1+r)}{w}\overline{F}(k_1)\right] - \left[\overline{F}(q_2) - \frac{c(1+r)}{w}\overline{F}(k_2)\right]$ can be written as $G(q_1) - G(q_2) < 0$, i.e., $q_2 > q_3$.

$\pi_{S1}(q_2) - \pi_{S2}{}^*(q_2) = ew\left(\int_0^k \overline{F}(x)dx + k\right) > 0$. That is $\pi_{S1}(q_2) > \pi_{S2}{}^*(q_2)$. It has been proved before that $\pi_{S1}(q)$ obtains the maximum value when $q = q_1$. Therefore, $\pi_{S1}(q_1) > \pi_{s1}(q_2), \pi_{S1}(q_1) > \pi_{S2}{}^*(q_2)$, i.e., $\pi_{S1}{}^* > \pi_{S2}{}^*$. □

Proof Proposition 4. By $\pi'_{S2}(q) = w\overline{F}(q) - (1-e)c(1+r)\overline{F}(k) - ec(1+r), k = \frac{(cq-B)(1+r)}{w}$. Such that $a = \frac{B(1+r)}{w}$. $\pi'_{S2}(q)$ can be written as $\pi'_{S2}(q) = \frac{w}{q}\left[q\overline{F}(q) - (1-e)(k+a)\overline{F}(k) - e(k+a)\right]$. $\pi'_{S2}(q) = 0$, we have $q\overline{F}(q) = (1-e)(k+a)\overline{F}(k) + e(k+a)$. The $(1-e)(k+a)\overline{F}(k) + e(k+a) = (k+a)\overline{F}(k) + e(k+a)F(k) > (k+a)\overline{F}(k)$. Therefore, $q\overline{F}(q) > (k+a)\overline{F}(k)$.

Hence, the function $H(k) = (k+a)\overline{F}(k)$; it is easy to know that the function $H(k)$ increases first and then decreases. Next, prove that k is on the increasing interval of the function $H(k)$.

Assuming that the k is not on the increasing interval of the function $H(k)$, k should be on the monotonically decreasing interval of $H(k)$. $q > k$, q is also on the monotonically decreasing interval of $H(k)$. $(k+a)\overline{F}(k) > (q+a)\overline{F}(q) > q\overline{F}(q)$. In contradiction to the above $q\overline{F}(q) > (k+a)\overline{F}(k)$, so k is on the increasing interval of the function $H(k)$.

Based on this, the next section demonstrates the effect of each parameter on the optimal decision q.

As w increases, the variable k experiences a decrease, leading to a decrease in the function $H(k)$. By $\pi'_{S2}(q) = \frac{w}{q}\left[q\overline{F}(q) - (k+a)\overline{F}(k) - e(k+a)F(k)\right]$, at this time $\pi'_{S2}(q) > 0$. $\pi'_{S2}(q) = 0$. Indeed, as q increases, the optimal production decision experiences an increase. Likewise, it can be demonstrated that as the level of risky regulatory effort e or the financing interest rate r increases, the optimal production output q decreases. □

Proof of Proposition 5. $\pi_{M1}(r) = (p-w)\int_0^q \overline{F}(x)dx + w\int_0^k \overline{F}(x)dx - (cq-B)(1+r_f)$. Consider q as a function of r. The derivative of $\pi_{M1}(r)$ with respect to r gives $\pi'_{M1}(r) = \left[(p-w)\overline{F}(q) + c(1+r)\overline{F}(k) - c(1+r_f)\right]q'(r) = \left[p\overline{F}(q) - c(1+r_f)\right]q'(r)$. $p\overline{F}(q) - c(1+r_f)$ is monotonically decreasing. From the above analysis, q decreases with increasing r. Combined with the constraint, $\pi_{M1}(r)$ monotonically decreases when $r_f \le r < \frac{w-c}{c}$. The manufacturer's gain is maximized when $r = r_f$. □

Proof of Proposition 6. The difference between the manufacturer's revenues under the platform financing model and the traditional supply chain finance model for the same output is $\pi_{M2} - \pi_{M1} = ew\int_0^k F(x)dx + a\ln(1-e)$. Let $J(e) = ew\int_0^k F(x)dx + a\ln(1-e)$. The derivative of J(e) with respect to e gives $J'(e) = w\int_0^k F(x)dx - \frac{a}{1-e}$. Since $J''(e) < 0$, the maximum value of J(e) is found at $J'(e) = 0$, i.e., $e^* = 1 - \frac{a}{w\int_0^k F(x)dx}$. When $a < w\int_0^k F(x)dx$, $0 < e^* < 1$, the assumptions are satisfied. When $a > w\int_0^k F(x)dx$, J(e) is monotonically decreasing on the interval $[0, 1]$ and obtains the maximum value when $e = 0$, which indicates that the manufacturer is not willing to build its own supply chain finance platform. □

Proof of Proposition 7. The derivative of the supplier profit formula with respect to e gives $\pi_{M2}{}' = \left[-wk' + p\overline{F}(q)\right]q' + w\int_0^k F(x)dx - \frac{a}{1-e}$. According to the previous analys, q' is less than zero and $\pi_{M2}{}'$ decreases as e increases. Hence, the maximum value of π_{M2} is obtained at $\pi_{M2} = 0$. The optimal level of risky regulatory effort e^* satisfies $\left[(1-e^*)k'^2 f(k) - f(q)\right]q' = k'F(k)$. Combining $\overline{F}(q) = \frac{c(1+r)}{w}\overline{F}(k) + \frac{c(1+r)}{w}eF(k)$, q' satisfies $\left[p\overline{F}(q) - c(1+r_f)\right]q' + w\int_0^k F(x)dx = \frac{a}{1-e}$. □

Proof of Proposition 8. The overall revenue to the supply chain under the traditional unified supply chain finance model is $\pi_{SM1} = p\int_0^q \overline{F}(x)dx - (cq-B)(1+r_f) - B$. The total revenue of the supply chain under the platform financing model is $\pi_{SM2} = p\int_0^q \overline{F}(x)dx - (cq-B)(1+r_f) - B + a\ln(1-e)$. When the yields are equal, it is clear that $\pi_{SM2} - \pi_{SM1} = a\ln(1-e) < 0$ and the optimal yield under the two models $q_1 > q_2$. At this time there is $\pi_{SM1} > \pi_{SM2}$. □

Proof of Proposition 11. From the above, when $\widetilde{r} = r_f$,

$$\pi_{M2} = (p-w)\int_0^q \overline{F}(x)dx - (1-e)w\int_0^k F(x)dx + a\ln(1-e)$$

The e_2 is the manufacturer's optimal decision and $\pi_{M2}(e_2)$ is the manufacturer's maximum revenue in the model.

$$\pi_{M3} = (p-w)\int_0^q \overline{F}(x)dx - (1-e)w\int_0^k F(x)dx + a\ln(1-e) + (cq-B)r_f$$

The e_3 is the manufacturer's optimal decision and $\pi_{M3}(e_3)$ is the manufacturer's maximum revenue in the model.

Then, $\pi_{M3}(e_3) > \pi_{M3}(e_2) > \pi_{M2}(e_2)$; hence, the evidence is obtained. □

Proof of Proposition 12. The derivative of π_{M2} with respect to e gives $\pi'_{M2} = \left[p\overline{F}(q) - c(1+r_f)\right]q' + w\int_0^k F(x)dx - \frac{a}{1-e}$, which is a subtractive function with respect to e. The derivative of π_{M4} with respect to e gives $\pi'_{M3} = \left[p\overline{F}(q) - c(1+r_f) + cr_f\right]q' + w\int_0^k F(x)dx - \frac{a}{1-e}$, which is also a subtractive function of e. Therefore, the e_2^* satisfies

$[p\bar{F}(q) - c(1 + r_f)]q' + w\int_0^k F(x)dx - \frac{a}{1-e_3} = 0$ and e_3^* satisfies $[p\bar{F}(q) - c(1 + r_f) + cr_f]q' + w\int_0^k F(x)dx - \frac{a}{1-e_3} = 0$. $H(e) = [p\bar{F}(q) - c(1 + r_f)]q' + w\int_0^k F(x)dx - \frac{a}{1-e}$ (a)$H(e_2^*) = 0$; and $H(e_3^*) > 0$. $H(e)$ is the subtractive function of e. From $H(e_3^*) > H(e_2^*)$, we obtain $e_3^* < e_2^*$.

Under the platform's external financing model, from $\tilde{r} = \frac{(1-e)w\int_0^k F(x)dx}{(cq-B)} + \frac{\theta}{(1-\theta)}$, we obtain $(cq - B)(1 + r) - (1 - e)w\int_0^k F(x)dx = \frac{1}{(1-\theta)}(cq - B)$. Due to $\frac{1}{(1-\theta)}(cq - B) > (cq - B)$, we obtain $(cq - B)r > (1 - e)w\int_0^k F(x)dx$. Therefore, the financing interest rate under the external financing mode of the platform can compensate for the financing risk of the supply chain finance platform. □

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