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Robust Financing Decisions of Green Supply Chain under Market Risk

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Abstract: In the face of global climate change and the collision of consumer preferences towards green and low-carbon, businesses need to accelerate the transition to sustainable development to achieve long-term growth. Companies must raise significant funds to support this transition and manage high market risk. The existing research on green innovation within supply chains often overlooks market risks, particularly those associated with incomplete information. Hence, this paper considers a two-echelon supply chain system composed of a manufacturer and a retailer. Manufacturers are willing to carry out green innovation and make a single product for sale in the consumer market with green preferences. However, innovation is risky due to the uncertainty in the sales volume of green products. In addition, the manufacturer may lack internal capital to invest in the innovation activities and may seek external financial resources, e.g., bank loans or retail prepayment financing. Hence, the manufacturer and retailer must decide which financial option to adopt. The results show that when the market risk is high, the supply chain members tend to make conservative decisions, no matter which financial modes they choose. However, with the robust optimization approach, the manufacturer and the retailer may earn a higher profit when the market risk is high. When the prepayment rate and bank loan interest rate are equal, regardless of the market risk, the manufacturer's optimal decision is to choose prepayment financing from the retailer. However, when the prepayment rate is higher than the bank loan interest rate, there is no dominant strategy for the manufacturer to choose.

Keywords: green innovation; financial constraints; market risk; robust optimization

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1. Introduction

In recent years, climate change, environmental protection, and sustainable development have emerged as central concerns on the global political agenda (Perera et al., 2020; Bai et al., 2023) [1,2]. The Paris Agreement, which was reached at the 2015 United Nations Climate Change Conference, mandates all the participating parties to intensify efforts to address the global threat of climate change and to strive to limit the increase in the global average temperature to 1.5 degrees Celsius. The product's greenness refers to the friendliness of the product to people and nature, and the product's greenness significantly affects consumers' purchase choices (Song and Gao, 2018) [3]. Under environmental regulations and consumer awareness pressure, supply chain companies need to undergo a green transformation. However, implementing green transformation in the supply chain requires substantial capital investment, which poses a significant challenge to upstream manufacturers in the supply chain. Companies in developing countries are the most vulnerable to climate change, with relatively weak coping skills and insufficient finance, technology, and capacity-building support. The funding gap is a significant challenge for companies undertaking a green transformation. Currently, the issue of inadequate funding for supply chain companies is being addressed primarily through financing.

Prepayment financing and bank loans are extensively employed in green supply chain innovation practices. First, prepayment financing is used by well-capitalized retailers to incentivize upstream manufacturers to undertake green innovation. This approach ensures relatively low wholesale prices, improves product sustainability to meet consumer demand, and creates a mutually beneficial scenario. The apparel retailer PUMA, for example, has implemented a prepayment financing strategy to help financially constrained manufacturers solve financing issues and improve the environmental sustainability of their products. Second, due to the introduction of green credit policies, many banks are willing to offer loans and financing services to companies engaged in green innovation. For example, Shanghai Pudong Development Bank (SPDB) provides loans and financing services to companies facing financial constraints while implementing carbon emission reduction projects. Academic scholars have also conducted relevant research on green supply chain financing issues. For example, Wu et al. (2019) [4] and Zhang et al. (2021) [5] constructed a two-echelon supply chain model consisting of a manufacturer and a financially constrained retailer. They examined the impact of bank and trade credit financing on emissions reduction innovation within the supply chain. Tang et al. (2020) [6] formulated a low-carbon supply chain model involving a manufacturer with limited capital and a well-funded retailer. They investigated the optimal level of emission reduction under both bank loan financing and prepayment financing.

In addition, companies often face significant uncertainties and diminished returns in green innovation efforts (Agrawal et al., 2023) [7]. In the sale of innovative products, a lack of consumer understanding of product quality; technical security provided by supply chain companies; the high uncertainty of market demand for green, low-carbon products; and uncertified market demand can introduce a range of unintended risks into a financially constrained supply chain, thereby jeopardizing the interests of the supply chain companies (Gupta and Kanthi Herath, 2005) [8]. Therefore, it is crucial to consider the risks of market uncertainty when examining the financing constraints on supply chain innovation. Yang et al. (2021) [9] investigated market risk in supply chains involving suppliers and retailers with limited financing resources. They analyzed optimal supply chain decision making under bank loan and trade credit financing models. The traditional analysis of market demand uncertainty often requires obtaining information such as probability distribution (Kouvelis and Zhao, 2016; Bi et al., 2022) [10,11]. However, obtaining the exact distribution function of market demand is often tricky in practice. Nevertheless, relatively accurate partial information about the distribution function of market demand can be obtained from historical data, including upper and lower bounds and expected variance. Consequently, some researchers have used robust optimization in supply chain management when only partial market demand information is available (Du et al., 2014; Qiu et al., 2022) [12,13]. Zhao et al. (2022) [14] investigated the secondary supply chain of suppliers and retailers with financial constraints, possessing only the knowledge of market demand below a certain threshold, and formulated a resumption strategy for supply chains under conditions of uncertainty.

The existing literature focuses mainly on financial decisions in green supply chains and explores financial options (Wu et al., 2019; Tang et al., 2020) [4,6]. In comparison, other researchers have explored financial options in supply chains with market demand risk when retailers are financially constrained (Yang et al., 2021; Zhao et al., 2022) [9,14]. To the best of our knowledge, no research paper has addressed the issue of manufacturers' green innovation decisions with the interacting effects of financial constraints and uncertain market demand. Building on the previous discussion, this research addresses the following questions:

(1) How can manufacturers use robust optimization for financing and green innovation decisions at the risk of incomplete market demand information?
(2) How does the risk of incomplete market demand information affect optimal supply chain decision making under prepayment and bank loan financing?
(3) Given the risks of incomplete market demand information, what is the optimal financing strategy for manufacturers dealing with prepayment and bank loan financing?

To address these questions, we consider the incomplete risk associated with market demand information and use the robust optimization method to investigate optimal production decisions and financing strategies within green supply chains in prepayment financing and bank loan financing models, referred to as "robust financing decisions".

The main contribution of this paper is mainly reflected in the following three aspects: Firstly, it enriches the theoretical green supply chain theory research; so far, there is no literature dedicated to the green innovation decision making of capital-constrained manufacturers under the risk of incomplete information about market demand. The study explores the decision-making behavior of supply chain firms in this specific context. Secondly, this paper compares the impact of two financing modes, advance payment and bank loan, on the optimal decision making of supply chains under the risk of incomplete information about market demand. This comparative analysis helps understand supply chain parties' behaviors and decisions under different financing modes. It helps enterprises choose suitable financing methods in practice. Thirdly, this paper considers the risk of incomplete information on market demand, a factor of great practical significance in supply chain management. Analyzing the impact of this risk on supply chain financing and decision making provides theoretical support and practical guidance for risk management when enterprises face an uncertain market environment.

The following sections are structured as follows: Section 2 presents a summary of the relevant literature. Section 3 introduces a green supply chain financing model, followed by Section 4, which examines two financing strategies involving prepayment financing and bank loans under the risk of incomplete market demand information. Section 5 provides an in-depth discussion and analysis of optimal financing strategies. Section 6 provides both a numerical analysis and extended studies. Finally, Section 7 provides concluding remarks.

2. Reviews of Literature and Motivations

This paper investigates financing strategy choices for green supply chains amid market demand uncertainty, exploring two facets: green supply chain financing and supply chain management considering uncertain market needs.

2.1. Green Supply Chain Financing Research

In situations of constrained supply chain funding, two primary financing channels emerge. The first involves internal financing within the supply chain, encompassing late payments from upstream enterprises, prepayment financing from downstream companies, investments, and sell-offs between trading members (Zia and Taleizadeh, 2015; Tiwari, etc., 2018) [15,16]. The second channel pertains to external financing, specifically loans from third-party institutions, predominantly in bank loans (Kouvelis and Zhao, 2011; Yan et al., 2016) [17,18]. Both domestically and internationally, scholars have extensively researched the financing behavior of green supply chains, primarily categorized into internal and external financing. Wu et al. (2022) [19] investigated the green supply chain involving suppliers with limited funds and reputable retailers. They analyzed the impact of retailers' procurement order financing and prepayment internal financing methods on decision making within the green supply chain. Additionally, scholars have delved into external supply chain financing. Huang et al. (2020) [20] examined the impact of bank green borrowing on decision making within supply chains comprising individually funded green manufacturers and retailers. Cong et al. (2020) [21] investigated the impact of bank green credit on carbon reduction within supply chains facing financial constraints. Fan et al. (2023) [22] established a secondary supply chain involving funded upstream companies engaged in green research and development. They explored the impact of external financing on green research and development within supply chains. In contrast to the singular focus on internal and external financing, some scholars have started comparing distinct financing strategies within green supply chains. Fang and Xu (2020) [23] compared two credit strategies: bank green credit financing, retailer partial prepayment, and a hybrid approach involving banking green credit. Luo et al. (2020) [24] investigated bank financing and supplier financing strategies for secondary green supply chains facing financial constraints.

In comparing financing decisions for fund-constrained low-carbon supply chains, most studies have concluded that internal financing strategies outperform bank loan financing. Under the Carbon Trading Mechanism, Cao et al. (2019) [25] analyzed whether supply chain enterprises factored in the impact of carbon reduction investment on financing strategies. They concluded that irrespective of considering carbon reduction investment, supplier internal financing represents a distinctive financing equilibrium for the manufacturer. Wu et al. (2019) [4] investigated carbon emissions in low-carbon supply chains under bank and internal financing models. They discovered that internal finance was more effective in enhancing supply chain emission reduction. Zhang et al. (2021) [5] examined the decision making in green supply chain financing for manufacturers and retailers with limited funding. They compared the impact of retailers choosing internal financing or bank loan financing strategies on the performance of the green supplier chain. They found that internal finance was more effective in enhancing performance. Considering the circumstances of manufacturers under carbon limits and transaction mechanisms, Cao et al. (2019) [25] found that internal financing was preferable to obtaining direct loans from banks. Most of the existing studies have been conducted on green supply chain financing in the context of market demand identification. However, based on the market's current situation, the demand is generally random and uncertain, and the problem of financial constraint usually occurs when improving the greenness of products or carrying out the green transformation, which needs to be further analyzed.

2.2. Research on Supply Chain Management at Market Risk

The influence of market demand risk on supply chain enterprise decision making is significant, prompting numerous domestic and foreign scholars to delve into supply chain research grounded in market demand uncertainty. Lai et al. (2009) [26] analyzed inventory risk in response to demand uncertainty. They assessed the efficiency of supply chain operations using pre-purchase, post-sales, and combined models, considering only the average and standard differences in market demand. Martí et al. (2015) [27] considered demand uncertainty by assuming the knowledge of the distribution function of market demand. They explored supply chain response decisions under various carbon policies, including ceilings for supply chains' carbon footprints, markets' carbon footprints, and carbon taxes. Nouri et al. (2018) [28] investigated the market randomly, considering supply chain coordination involving manufacturers' innovation and retailers' promotional efforts. Sarkar et al. (2023) [29] treated demand parameters as interval values, considered uncertain environments, studied optimal decision making for participants, and determined the profit intervals of supply chain stakeholders. The measurement of market demand uncertainty has primarily focused on two aspects. The first involves situations where information uncertainty has a probability distribution or known demand functions (Wu et al., 2022; Lai et al., 2009) [19,26]. In such cases, historical data, market trends, or relevant models can be employed to establish probability distributions or demand functions. The second scenario pertains to the random distribution of market need information, with only the knowledge of the upper boundary of market demand, making it challenging to accurately determine the demand function (Sarkar et al., 2023) [29]. For such situations, solutions can involve robust optimization, vague reasoning, decision trees, and other methods (Sainidis, 2004) [30].

Robust optimization has achieved remarkable results in all the areas of risk management applications. In the field of supply chain management, a random variable is usually used to represent a specific type of risk, assuming that the random variables submit to a particular distribution. However, obtaining an accurate distribution of a random variant is often difficult. As some scholars applied the idea of the optimization method of Rubber to the processing of random variable distribution functions that only knew part of the information; Scarf (1958) [31] first used peer-to-peer technology to give the distribution of information shortcomings (knowing only demand averages and differences) to Rubber's ordering decision making. Gallego and Moon (1993) [32] reaffirmed and analyzed Scarf's results using the method of probability inequality. However, also criticized for always seeking to make decisions based on the worst-case situation, resulting in conservative results, Du and Chen et al. (2014) [12] analyzed the efficiency of supply chains based on wholesale price contracts when only knowing the upper and lower boundaries of demand distribution and found that the supply chain system can achieve higher efficiency when the upper boundary ratio satisfies certain conditions, and the results are not necessarily conservative. Leung et al. (2007) [33] used uncertainty data to minimize the total cost of production costs, manpower costs, inventory costs, and labor change costs. They proposed a robust optimization model for multi-site production plans.

2.3. Literature Summary

To summarize, at present, green supply chains subject to financial constraints are mainly financed by external financing (bank loans, green credits, etc.) and internal financing (early payment, delayed payment, etc.), and there are fewer studies on financing under risky scenarios, especially the lack of studies on financing under market risk. In the research of supply chain management under the risk of market demand uncertainty, most scholars assume that the market demand has a distribution function method for modeling. However, the market demand distribution function is often complex and difficult to obtain accurately. The robust optimization method is an effective method for solving incomplete information decision making.

In the field of supply chain management, there is a scarcity of literature addressing supply chain management while considering financial constraints and market risks. Yang et al. (2021) [9] incorporate market risk into supply chains with pre-existing capital constraints, assuming market demand as a random variable with known averages and differences. They analyze optimal supply chain decision making under both the financing models of bank loans and trade credit. However, accurately capturing the distribution function of market demand information is challenging. Qiu et al. (2022) [13] investigated multi-term inventory problems in supply chains with capital constraints and demand uncertainty. They optimized the model to mitigate demand uncertainties using buffer stock but did not consider the choice of supply chain financing strategy. Zhao et al. (2022) [14] explored the secondary supply chain of suppliers and retailers with limited funding, considering only the knowledge of market demand. They utilized the robust optimization method and obtained regrettable order quantities for retailers, both maximum and minimum values. Nevertheless, they did not address the issue of green innovation in the supply chain, and it is assumed that retailers face financial constraints. Currently, no literature addresses the risk of incomplete market demand information and financial constraints while studying the financing strategy of green innovation in the supply chain through robust optimization.

No literature considers the risk of incomplete information about market demand and financial constraints when studying financing strategies for green innovation in supply chains through robust optimization methods. Therefore, to investigate the optimal financing strategy and the optimal production decision of the supply chain under the risk of incomplete market demand information when the financially constrained manufacturer faces two financing methods, i.e., prepayment and bank loan, the study finds the equilibrium solutions of the financially constrained manufacturer and retailer with the help of robust optimization method, and comparatively analyze the optimal financing strategy of the manufacturer under the risk of incomplete market demand information.

3. Problem Description and Hypothesis

This paper explores a two-echelon green supply chain comprising a solitary manufacturer and a lone retailer. To address the uncertainty of market demand parameters in the supply chain model, robust optimization is employed to enhance the robustness and stability of decision making, establishing a manufacturer-led Stackelberg game. Within this model, the manufacturer assumes the role of a leader, optimizing its objective function to determine the wholesale price and product greenness. Simultaneously, the retailer, acting as a follower, sets the retail price based on the manufacturer's decision. Throughout this process, the manufacturer encounters financial constraints and contemplates two financing methods: prepayment financing and bank loans. The key assumptions include the following:

1. The manufacturer incurs not only the unit production cost c in the production process but also invests in green innovation to enhance the product's sustainability. Let us assume the manufacturer's green input cost is denoted as $\frac{1}{2}\theta g^2$.
2. Assume that the market demand for green products is influenced not only by the retail price p but also by the product's greenness g, denoted as $D = a - \beta p + \lambda g$.
3. Assume that the actual market demand size is denoted as a. Due to the absence of relevant data, the manufacturer and retailer can only perceive the market demand size as a random variable and are aware only of $a_x \in [a_l, a_h]$, where $a_l > 0$.
4. Confronting the risk of incomplete information, we introduce a robust risk decision-making mechanism. Both the manufacturer and retailer, being risk-averse, adopt this mechanism while making decisions. In essence, the manufacturer aims to make decisions on product greenness and wholesale price to maximize profit under the worst market demand scale. The retailer pursues the retail price decision that maximizes its profit or utility under the worst market demand scale, assuming symmetrical supply chain information.
5. To ensure that the decision has an equilibrium solution and is positive. Thus, assume $\beta\theta(1 + r) - \frac{\lambda^2}{4} > 0$.
6. Additionally, the subscripts RF and BF denote the financing modes of prepayment financing and bank loans, respectively, while the superscript * signifies the situation of complete information. Specific symbols are provided in Table 1.
7. Under prepayment financing, the manufacturer encounters challenges related to inadequate production and insufficient funds for green innovation research and development. To mitigate capital chain risks, the manufacturer offers a prepayment rate, enticing the retailer to engage in financing. It is assumed that the prepayment financing rate is exogenous. At this juncture, the manufacturer's initial capital satisfies the following: $B < c(a_x - \beta p + \lambda g) + \frac{1}{2}\theta g^2$;

Table 1. Symbols and meanings.

Symbol	Meaning
π_M	Manufacturer's profit under incomplete information
π_R	Retailer profit under incomplete information
π	Supply chain profit under incomplete information
D	Market demand for green products
a	The actual scale of market demand
a_x	The manufacturer and retailer grasp the size of the market demand
c	Manufacturer's unit cost of production
w	Wholesale price
p	Retail Price
β	Consumer price sensitivity coefficient
λ	Consumer green preference level
g	Product greenness
θ	Cost coefficient of green innovation
r	Prepayment rate
r_b	Bank lending rate
B	Manufacturer's initial capital

When the supplier faces initial fund constraints, the retailer, equipped with ample funds, is incentivized to offer prepayment financing to ensure product availability for sale. The financing amount is as follows: $L = c(a_x - \beta p + \lambda g) + \frac{1}{2}\theta g^2 - B$;

After the completion of production activities, the manufacturer delivers the green product of the unit $\frac{(1+r)L}{w}$ to the retailer, and the retailer purchases the green product of the unit $D - \frac{(1+r)L}{w}$ at the unit price to meet the market demand D. At this point, the profit function of the manufacturer and retailer is as follows:

$$\pi_M^{RF} = (w-c)(a_x - \beta p + \lambda g) - \frac{1}{2}\theta g^2 - r\left[c(a_x - \beta p + \lambda g) + \frac{1}{2}\theta g^2 - B\right]$$

$$\pi_R^{RF} = (p-w)(a_x - \beta p + \lambda g) + r\left[c(a_x - \beta p + \lambda g) + \frac{1}{2}\theta g^2 - B\right]$$

Under the bank loan financing model, the manufacturer obtains loans from the bank at a certain interest rate, and the profit function of the manufacturer and the retailer is as follows:

$$\pi_M^{BF} = (w-c)(a_x - \beta p + \lambda g) - \frac{1}{2}\theta g^2 - r_b\left[c(a_x - \beta p + \lambda g) + \frac{1}{2}\theta g^2 - B\right]$$

$$\pi_R^{BF} = (p-w)(a_x - \beta p + \lambda g)$$

Building upon the assumptions above, Figure 1 encapsulates the decision-making process involving the risk of incomplete market demand information. The manufacturer is positioned at the forefront of the supply chain and grapples with financial constraints while engaging in green innovation. In contrast, in a subordinate role, the retailer contends with incomplete market demand information, possessing solely the upper and lower bounds of market demand details—ensuring symmetry within the supply chain system. Initially, the manufacturer opts for prepayment or bank loan financing for product production and green innovation. Firstly, the manufacturer decides the greenness (g) of the product, wholesale price (w). The prepayment financing rate and bank interest rate are exogenous variables. Following this, the retailer determines the product's retail price (p), ultimately culminating in the product's introduction to the market.

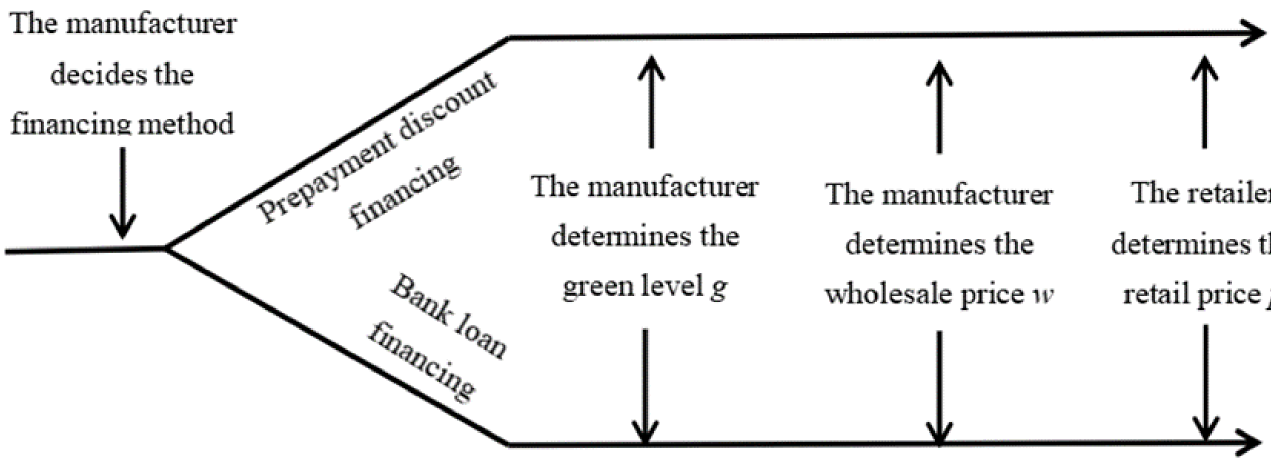

Figure 1. The decision diagram under the risk of incomplete market demand information.

4. Analysis of Prepayment and Bank Loan Model under Market Risk

4.1. Prepayment Financing

Confronted with incomplete information and inherent risks, the manufacturer and retailer exhibit risk aversion and act as rational decision-makers, aiming to maximize profits. The manufacturer initiates decisions regarding product greenness and wholesale prices through a robust risk decision-making mechanism, followed by the retailer determining retail prices employing a similar mechanism. In this context, the manufacturer's decision-making model can be delineated as (1) and the retailer's decision model as (2).

$$\max_{w,g} \min_{a_x} \pi_M^{RF} = (w-c)(a_x - \beta p + \lambda g) - \tfrac{1}{2}\theta g^2 - r[c(a_x - \beta p + \lambda g) + \tfrac{1}{2}\theta g^2 - B] \quad s.t. a_x \in [a_l, a_h] \tag{1}$$

$$\max_{p} \min_{a_x} \pi_R^{RF} = (p-w)(a_x - \beta p + \lambda g) + r[c(a_x - \beta p + \lambda g) + \tfrac{1}{2}\theta g^2 - B] \quad s.t. a_x \in [a_l, a_h] \tag{2}$$

By analyzing model (2), it can be seen that when the market demand is uncertain, under the premise of $a_x \in [a_l, a_h]$, for any p, the target value of this formula will increase with the increase in a_x. Therefore, model (2) can be transformed into the following:

$$\max_p \pi_R^{RF} = (p-w)(a_l - \beta p + \lambda g) + r[c(a_l - \beta p + \lambda g) + \frac{1}{2}\theta g^2 - B] \tag{3}$$

First, take the first derivative of concerning, giving $\frac{\partial \pi_R^{RF}}{\partial p} = 0$.

$$p = \frac{a_l + \beta w - rc\beta + \lambda g}{2\beta} \tag{4}$$

On this basis, the second derivative can be obtained as follows: $\frac{\partial^2 \pi_R^{RF}}{\partial p^2} = -2\beta < 0$, which determines that the objective function has a maximum value.

By bringing (4) into (1), the manufacturer's decision model is transformed into the following:

$$\max_{w,g} \min_{a_x} \pi_M^{RF} \begin{aligned} &= (w-c)(a_l - \beta \tfrac{a_l + \beta w - rc\beta + \lambda g}{2\beta} + \lambda g) - \tfrac{1}{2}\theta g^2 \\ &-r[c(a_x - \beta \tfrac{a_l + \beta w - rc\beta + \lambda g}{2\beta} + \lambda g) + \tfrac{1}{2}\theta g^2 - B] \end{aligned} \tag{5}$$

Take the first derivative of (5) concerning, g, let $\frac{\partial \pi_M^{RF}}{\partial w} = 0$, $\frac{\partial \pi_M^{RF}}{\partial g} = 0$, then:

$$w = \frac{a_l + \beta c + 2rc\beta + \lambda g}{2\beta} \tag{6}$$

$$g = -\frac{(c - w + rc)\lambda}{2(1+r)\theta} \tag{7}$$

On this basis, the second derivative of the wholesale price and greenness g is obtained, and the second order Hessian matrix, as shown in the equation below, is obtained as follows:

$$\begin{vmatrix} -\beta & \frac{\lambda}{2} \\ \frac{\lambda}{2} & -(1+r)\theta \end{vmatrix} = \beta\theta(1+r) - \frac{\lambda^2}{4} > 0 \tag{8}$$

An optimal solution exists if the negative definite condition is met, indicating a negative definite Hessian matrix. The supply chain's optimal decision can be derived using the inverse induction solution method, as illustrated in Theorem 1.

Theorem 1. *In the prepayment rate financing mode, when market risk is present, the optimal product greenness, wholesale price, and retail price of the supply chain are as follows:*

$$g^{RF} = \frac{\lambda(a_l - c\beta)}{4\beta\theta(1+r) - \lambda^2} \tag{9}$$

$$w^{RF} = \frac{(1+r)(2a_l\theta + 2c\beta\theta + 4rc\beta\theta - c\lambda^2)}{4\beta\theta(1+r) - \lambda^2} \tag{10}$$

$$p^{RF} = \frac{3a_l\theta(1+r) + c\beta\theta(1+r) - c\lambda^2}{4\beta\theta(1+r) - \lambda^2} \tag{11}$$

Bring (8)–(10) into (1) and (2) to obtain:

Theorem 2. *When the prepayment financing method is adopted, the profits of the manufacturer and the retailer under the risk of incomplete information are as follows:*

$$\begin{aligned}\pi_M^{RF} &= \frac{1}{8\beta\theta(1+r)-2\lambda^2}[c^2\beta^2\theta + 8B\beta r\theta + rc^2\beta^2\theta + 8B\beta r^2\theta - 3a_l^2(1+r)\theta \\ &-4ac\beta\theta(1+r) + 2a_l\theta(2a+c\beta)(1+r) - 2Br\lambda^2]\end{aligned} \tag{12}$$

$$\begin{aligned}\pi_R^{RF} &= -Br + \frac{r\theta\lambda^2(a_l-c\beta)^2}{2(-4\beta\theta(1+r)+\lambda^2)^2} - \frac{1}{(-4\beta\theta(1+r)+\lambda^2)^2}[\theta(a_l - c\beta)(1 \\ &+r)(3a_l\beta\theta(1+r) - 4a\beta\theta(1+r) + c\beta^2\theta(1+r) - a_l\lambda^2 + a\lambda^2)]\end{aligned} \tag{13}$$

Inference 1. *Greenness, consumer green sensitivity coefficient, green innovation cost coefficient, and manufacturer's initial capital for a given supply chain product. The conclusion is established as follows:* $\frac{\partial\pi_M^{RF}}{\partial\lambda} > 0$*; that is, as consumer green preferences increase, manufacturer profits enhance. When* $a_l < \frac{2a+c\beta}{3}$*, there is* $\frac{\partial\pi_M^{RF}}{\partial a_l} > 0$*; that is, with the increase in* a_l*, the profit of the manufacturer increases; otherwise, with the increase in* a_l*, the manufacturer's profit decreases.*

Proof. Please see Appendix A. □

Given the market demand scale a, a higher value indicates a less accurate understanding of the market demand, and the magnitude of market risk can be denoted by a_l; the smaller the a_l, the greater the market risk. Conversely, a larger a_l implies a more negligible market risk. Incomplete information implies that decision-makers in the supply chain lack a comprehensive understanding of the market, competitors, demand, and other crucial factors. In situations with limited, incomplete information, the manufacturer can employ robust optimization models for decision making, leading to increased profits. As the risk of incomplete information rises, decision-makers encounter heightened uncertainty and risk, potentially prompting the adoption of conservative strategies, costly decisions, or overlooking market opportunities. Hence, the manufacturer's and retailer's profits are intricately linked to the completeness and accuracy of information. In situations involving incomplete information, decision-makers in the supply chain should explore enhanced methods for information acquisition and analysis to minimize uncertainty and improve decision-making accuracy and effectiveness, ultimately maximizing profits.

In the case of $a_l = a = a_h$, there is no risk of incomplete information. In this scenario, the risk of incomplete information is mitigated, transforming it into a risk-free environment with complete information. Consequently, we employ the inverse method to solve the prepayment model. The results are presented in Table 2.

Table 2. The results of incomplete information risk and complete information without risk.

	Incomplete Information Risk	Complete Information without Risk
g	$\frac{\lambda(a_l-c\beta)}{4\beta\theta(1+r)-\lambda^2}$	$\frac{\lambda(a-c\beta)}{4\beta\theta(1+r)-\lambda^2}$
w	$\frac{(1+r)(2a_l\theta+2c\beta\theta+4rc\beta\theta-c\lambda^2)}{4\beta\theta(1+r)-\lambda^2}$	$\frac{(1+r)(2a\theta+2c\beta\theta+4rc\beta\theta-c\lambda^2)}{4\beta\theta(1+r)-\lambda^2}$
p	$\frac{3a_l\theta(1+r)+c\beta\theta(1+r)-c\lambda^2}{4\beta\theta(1+r)-\lambda^2}$	$\frac{3a\theta(1+r)+c\beta\theta(1+r)-c\lambda^2}{4\beta\theta(1+r)-\lambda^2}$

If g^{RF*}, w^{RF*}, and p^{RF*} under complete information are brought into the profit function of the manufacturer and retailer, and prepayment financing is adopted for financing under the condition of complete information and no risk, the profit of the manufacturer and retailer is as follows:

$$\begin{aligned}\pi_M^{BF*} = \tfrac{1}{8\beta\theta(1+r)-2\lambda^2}[a^2\theta(1+r) - 2ac\beta\theta(1+r) \\ +c^2\beta^2\theta(1+r) + 8B\beta r\theta(1+r) - 2Br\lambda^2]\end{aligned} \quad (14)$$

$$\begin{aligned}\pi_R^{RF*} \quad &= -Br + \tfrac{1}{2(-4\beta\theta(1+r)+\lambda^2)^2}[a^2\theta(2\beta\theta(1+r)^2 + r\lambda^2) \\ &-2ac\beta\theta(2\beta\theta(1+r)^2 + r\lambda^2) + c^2\beta^2\theta(2\beta\theta(1+r)^2 + r\lambda^2)]\end{aligned} \quad (15)$$

Proposition 1. *In the context of prepayment financing, the outcomes under incomplete information risk and the decision variables in the absence of risk with the manufacturer's profit resultsas follows:*

1. $g^{RF} < g^{RF*}$, $w^{RF} < w^{RF*}$, $p^{RF} < p^{RF*}$;
2. *When* $a_l > \frac{a+2c\beta}{3}$, $\pi_M^{RF} - \pi_M^{RF*} > 0$; *otherwise,* $\pi_M^{RF} - \pi_M^{RF*} < 0$.

Proof. Please see Appendix B. □

Faced with market risks, supply chain members tend to employ conservative decision-making strategies, leading to the adjustment of both the wholesale and selling prices and a compromise on product greenness when dealing with incomplete information risks. This conservative approach reflects their inclination to mitigate uncertainty and minimize potential risks. In situations with incomplete information, making accurate predictions about market demand and competitive landscapes becomes challenging, prompting the adoption of conservative strategies to mitigate potential losses. Dealing with incomplete information, the manufacturer may reduce wholesale prices to encourage more retailer collaborations, trimming product greenness to manage costs and risks. At the same time, they may establish comparatively lower selling prices to enhance sales volume and minimize inventory risks amid market fluctuations. Conversely, situations without market risks contribute to stable demand for green innovative products or services, simplifying supply chain planning and management and creating a more favorable environment for green innovation.

Nevertheless, it is crucial to acknowledge that the impact of market risks on manufacturer profits is not consistently negative, particularly under high levels of incomplete information risks. When facing substantial incomplete information risks, the manufacturer might experience reduced profits. Intriguingly, as the level of incomplete information risks diminishes, the adoption of a robust risk decision mechanism empowers the manufacturer to achieve higher profits, surpassing those in scenarios without risk but with complete information. This counterintuitive outcome emphasizes the importance and effectiveness of a robust risk decision mechanism. To summarize, the interaction between market risks and incomplete information substantially impacts supply chain decision making. Although market risks frequently lead to conservative strategies, robust risk decision mechanisms can paradoxically result in higher profits in scenarios with incomplete information and reduced risk levels.

In scenarios with high levels of incomplete information risk, the effect of market risk on the manufacturer's profit is not consistently negative. The manufacturer can only attain reduced profits in situations with substantial incomplete information risk. Intriguingly, in scenarios with low incomplete information risk, the adoption of the robust risk decision mechanism enables the manufacturer and retailer to achieve higher profits under incomplete information risk level $a_l > \frac{a+2c\beta}{3}$ compared to situations without risk but with complete information. This is because in situations with low market risk, the manufacturer and retailer possess complete information about the market demand without any risk. However, the competitive pricing of both parties prevents them from obtaining greater profits. Conversely, in the scenario of incomplete information risk $a_l < \frac{a+2c\beta}{3}$, the manufacturer possesses only partial demand information. The robust decision-making mechanism allows both parties to obtain more profits. However, this decision-making mechanism becomes less effective when the market risk is too large.

4.2. Bank Loan Financing

In the bank loan financing model, the manufacturer is required to cover the interest expenses incurred by the bank loan, with the interest rate denoted as r_b. The decision sequence in the interaction between the manufacturer and the retailer aligns with that observed in the prepayment rate financing scenario. In the context of incomplete information risk, the decision models for both the manufacturer and the retailer are outlined as follows:

$$\begin{gathered}\max_{w,g}\min_{a_x}\pi_M^{BF}=(w-c)(a_x-\beta p+\lambda g)-\tfrac{1}{2}\theta g^2- \\ r_b[c(a_x-\beta p+\lambda g)+\tfrac{1}{2}\theta g^2-B] \\ s.t.\ a_x\in[a_l,a_h]\end{gathered} \tag{16}$$

$$\begin{gathered}\max_{p}\min_{a_x}\pi_R^{F}=(p-w)(a_x-\beta p+\lambda g) \\ s.t.\ a_x\in[a_l,a_h]\end{gathered} \tag{17}$$

When the demand is uncertain, under the premise of $a_x \in [a_l, a_h]$, the analysis model (16) shows that for a given p, the target value of the formula will increase with the increase in a_x. Therefore, model (16) can be transformed into the following:

$$\max_{p}\pi_R^{BF}=(p-w)(a_l-\beta p+\lambda g) \tag{18}$$

Take the first derivative of concerning and give $\frac{\partial \pi_R^{BF}}{\partial p}=0$:

$$p=\frac{a_l+\beta w+\lambda g}{2\beta} \tag{19}$$

On this basis, a second derivative $\frac{\partial^2 \pi_R^{BF}}{\partial p^2}=-2\beta<0$ is obtained, and the objective function has a maximum value.

Transpose (18) into (15) manufacturer's decision model into the following:

$$\begin{aligned}\max_{w,g}\min_{a_x}\pi_M^{BF}&=(w-c)(a_l-\beta\tfrac{a_l+\beta w+\lambda g}{2\beta}+\lambda g)-\tfrac{1}{2}\theta g^2 \\ &-r_b(c(a_x-\beta\tfrac{a_l+\beta w+\lambda g}{2\beta}+\lambda g)+\tfrac{1}{2}\theta g^2-B)\end{aligned} \tag{20}$$

Take the first derivative of π_M^{BF} with respect to w and g, and let $\frac{\partial \pi_M^{BF}}{\partial w}=0$ and $\frac{\partial \pi_M^{BF}}{\partial g}=0$ obtain the following:

$$w=\frac{a_l+\beta c+r_b c\beta+\lambda g}{2\beta} \tag{21}$$

$$g=-\frac{(c-w+r_b c)\lambda}{2(1+r)\theta} \tag{22}$$

On this basis, the second derivative of wholesale price w and greenness g is obtained as follows:

$$\begin{vmatrix}-\beta & \frac{\lambda}{2} \\ \frac{\lambda}{2} & -(1+r_b)\theta\end{vmatrix}=\beta\theta(1+r_b)-\frac{\lambda^2}{4}>0 \tag{23}$$

Suppose the adverse definite condition is met, indicating a negative definite Hessian matrix and the existence of an optimal solution. In that case, the optimal decision for the supply chain can be derived by solving it. This is illustrated in Theorem 4 below.

Theorem 3. *In the bank loan model, when market risk is present, the optimal product greenness, wholesale price, and retail price for the supply chain are as follows:*

$$g^{BF} = \frac{\lambda(a_l - c\beta(1+r_b))}{4\beta\theta(1+r_b) - \lambda^2} \tag{24}$$

$$w^{BF} = \frac{(1+r_b)(2a_l\theta + 2c\beta\theta + 2r_bc\beta\theta - c\lambda^2)}{4\beta\theta(1+r_b) - \lambda^2} \tag{25}$$

$$p^{BF} = \frac{(1+r_b)(3a_l\theta + c\beta\theta(1+r_b) - c\lambda^2)}{4\beta\theta(1+r) - \lambda^2} \tag{26}$$

Bring (22)–(24) into (15) and (16) to obtain the following:

Theorem 4. *When bank loan financing is adopted, the manufacturer's and retailer's profits under the risk of incomplete information are as follows:*

$$\begin{aligned}\pi_M^{BF} = \tfrac{1}{8\beta\theta(1+r_b)-2\lambda^2}[c^2\beta^2\theta + 8B\beta r_b\theta + 3r_bc^2\beta^2\theta + 8B\beta {r_b}^2\theta + 3\theta c^2\beta^2 r_b^2 + \theta c^2\beta^2 r_b^3 \\ -3a_l^2(1+r_b)\theta - 4ac\beta\theta(1+r_b)^2 + 2a_l\theta(1+r_b)(2a + c\beta(1+r_b)) - 2Br_b\lambda^2)]\end{aligned} \tag{27}$$

$$\begin{aligned}\pi_R^{BF} &= \tfrac{1}{(-4\beta\theta(1+r_b)+\lambda^2)^2}[(1+r_b)(-a_l + c\beta(1+r_b))\theta(3a_l\beta\theta(1+r_b) \\ &-4a\beta\theta(1+r_b) + c\theta\beta^2(1+r_b)^2 - a_l\lambda^2 + a\lambda^2]\end{aligned} \tag{28}$$

As in the previous section, Inference 1, under bank loans, the conclusion is valid if there is a similar prepayment pattern for a given variable. When $a_l < \frac{2a+c\beta}{3}$, there is $\frac{\partial \pi_M^{BF}}{\partial a_l} > 0$; that is, with the increase in a_l, the manufacturer's and retailer's profit increases; when $a_l > \frac{2a+c\beta}{3}$, there is $\frac{\partial \pi_M^{BF}}{\partial a_l} < 0$; that is, with the increase in a_l, the manufacturer's and retailer's profit decreases; $\frac{\partial \pi_R^{BF}}{\partial a_l}$ cannot compare sizes. It can be seen that whether it is prepayment financing or bank loans, when the manufacturer and retailer play the Stackelberg game, the impact of market risk on the manufacturer's profits presents an inverted "U" shape.

As in the previous section, when $a_l = a = a_h$, the incomplete information risk situation is transformed into the complete information risk-free situation, and the complete information risk-free situation under the bank loan mode is solved, and the results are shown in Table 3.

Table 3. Results of incomplete information risk and complete information without risk.

	Incomplete Information Risk	**Complete Information without Risk**
g	$\frac{\lambda(a_l-c\beta(1+r_b))}{4\beta\theta(1+r_b)-\lambda^2}$	$\frac{\lambda(a-c\beta(1+r_b))}{4\beta\theta(1+r_b)-\lambda^2}$
w	$\frac{(1+r_b)(2a_l\theta+2c\beta\theta+2r_bc\beta\theta-c\lambda^2)}{4\beta\theta(1+r_b)-\lambda^2}$	$\frac{(1+r_b)(2a\theta+2c\beta\theta+2r_bc\beta\theta-c\lambda^2)}{4\beta\theta(1+r_b)-\lambda^2}$
p	$\frac{(1+r_b)(3a_l+c\beta\theta(1+r_b)-c\lambda^2)}{4\beta\theta(1+r)-\lambda^2}$	$\frac{(1+r_b)(3a+c\beta\theta(1+r_b)-c\lambda^2)}{4\beta\theta(1+r)-\lambda^2}$

By bringing g^{BF*}, w^{BF*}, and p^{BF*} under complete information into the profit function of the manufacturer and retailer, the profit of the manufacturer and retailer can be obtained as follows:

$$\begin{aligned}\pi_M^{BF*} &= \tfrac{1}{8\beta\theta(1+r_b)-2\lambda^2}[a^2\theta(1+r_b) - 2ac\beta\theta(1+r_b)^2 + c^2\beta^2\theta(1+r_b)^3 \\ &+8B\beta r_b\theta(1+r_b) - 2Br_b\lambda^2]\end{aligned} \tag{29}$$

$$\pi_R^{BF*} = \frac{\beta(1+r_b)^2(a-c\beta(1+r_b))^2\theta^2}{(-4\beta\theta(1+r_b)+\lambda^2)^2} \tag{30}$$

Proposition 2. *Under the bank loan financing model, the results under incomplete information risk and the results of decision variables under complete information risk-free are compared as follows:*

1. $g^{BF} < g^{BF*}$, $w^{BF} < w^{BF*}$, $p^{BF} < p^{BF*}$;
2. *When* $a_l > \frac{a+2c\beta}{3}$, $\pi_M^{BF} - \pi_M^{BF*} > 0$; *otherwise,* $\pi_M^{BF} - \pi_M^{BF*} < 0$.

Proof. Please see Appendix C. □

Proposition 2 demonstrates that supply chain members employing a robust risk decision-making mechanism under the bank loan model are inclined to adopt conservative strategies when confronted with market risks, mirroring the behavior observed in the prepayment model. Based on the comprehensive analysis in Proposition 1, whether in the prepayment financing or bank loan model, the product greenness, wholesale price, and retail price reach their peaks under the conditions of complete information without risk. In scenarios with high incomplete information risk, the manufacturer can only secure lower profits under these conditions. However, in situations where incomplete information risk is low, aided by the robust risk decision mechanism, the manufacturer can achieve higher profits under incomplete information risk compared to scenarios without any risk. The decisions made under incomplete information risk are lower than those made in situations without complete information risk. This challenges the intuition that conservative strategies inevitably result in lower profit.

5. Comparative Analysis of Prepayment Financing and Bank Loan Financing under Market Risk

When the manufacturer is faced with financial constraints, the two financing options prepayment financing and bank loan are compared and analyzed to explore the optimal financing strategy of the supply chain. In order to compare which financing strategy is better between the two types of financing, prepayment financing and bank loans, we study them to obtain Proposition 3 and Proposition 4.

Proposition 3. *When the rate of prepayment financing is equal to the interest rate of a bank loan, compared with the decision level of the supply chain under prepayment financing and bank loan, the greenness of the product* $g^{RF} > g^{BF}$*; the product wholesale price* $w^{RF} > w^{BF}$.

Proof. Please see Appendix D. □

According to Proposition 3, when comparing product greenness, wholesale price, and retail price under uncertain risk, the difference in the financing decision between the advance payment financing method and the bank loan financing method remains unaffected by market risk at the same interest rate. This holds regardless of the presence of market uncertainty. Opting for advance payment financing is more favorable for enhancing product greenness and wholesale prices. Specifically, when the retailer extends advance payment financing to a financially constrained manufacturer, the manufacturer receives rate compensation through elevated wholesale prices, resulting in higher wholesale prices under the advance payment method. The retail prices of products are not directly comparable. The retail price of products cannot be compared. When the green sensitivity coefficient of consumer products is $\beta\theta(1+r) < \lambda^2 < 4\beta\theta(1+r)$, $p^{RF} > p^{BF}$, but when the green sensitivity coefficient of consumer products is $0 < \lambda^2 < \beta\theta(1+r)$, $p^{RF} < p^{BF}$; it can be seen that for products with high green sensitivity, under the risk of incomplete information, the retail price decision of products with prepayment financing is higher than that of products with the bank loan. Conversely, when the green sensitivity coefficient of consumers is low, products with high green sensitivity exhibit higher retail prices compared to products financed by bank loans. Retail price decisions are lower when using prepayment compared to using bank loans. This phenomenon may arise because consumers place greater importance on the product's green attributes in product categories with high green sensitivity. The retailer, in turn, faces higher prepayment financing costs under the advance payment financing model, potentially leading to increased capital costs. This distinction can be partially attributed to the varying degree of financial constraints imposed on the retailer by the two financing methods—prepayment financing and bank loans—as well as the distinct sensitivity of consumers to green attributes. Supply chain members select the appropriate financing model based on market demand and consumer preferences, carefully considering costs, profits, and market competition in their pricing decisions.

Proposition 4. *When the prepayment financing rate equals the bank loan interest rate,* $\pi_M^{RF} > \pi_M^{BF}$.

Proof. Please see Appendix E. □

When the prepayment financing rate matches a bank loan's interest rate, comparing the manufacturer's and retailer's profit levels between prepayment financing and bank loan financing under market risk consistently demonstrates a trend. Regardless of market risk fluctuations, the manufacturer consistently achieves higher profits through prepayment financing than bank loans. This pattern stems from prepayment financing, which usually does not require collateral or security, leading to lower fees and interest expenses than bank loans. When market risks suddenly escalate, the manufacturer may need immediate access to capital to address emerging challenges. In contrast, bank loans frequently entail prolonged approval and processing times, potentially causing the manufacturer to miss critical opportunities for adapting to market changes. Prepayment financing, conversely, usually undergoes a faster approval process due to its collaborative nature with the retailer, circumventing the cumbersome procedures associated with banks. Furthermore, the retailer is often willing to share risks with the manufacturer in prepayment financing. This collaborative risk-sharing approach mitigates the manufacturer's risk exposure, ensuring a more favorable profit margin. Mutual risk sharing fosters a collaborative environment and shields the manufacturer from excessive risk exposure, contributing to their ability to secure higher profits.

6. Analysis of Numerical Examples

6.1. Numerical Simulation Analysis

Aiming at the above model, under the condition of incomplete information risk, this section analyzes the impact of incomplete information risk on the profits of all the parties in the supply chain under the two financing modes of prepayment financing and bank loan. Incomplete market risk a_l is taken as an experimental factor to study the effects of different market risks on the manufacturer's profits π_M, the retailer's profits π_R, and the overall profits π of the supply chain. We select the fundamental parameter values $a = 200$, $c = 20$, $\beta = 1$, $\lambda = 3$, $\theta = 5$, $B = 10$, and $r = r_b = 0.2$. The parameters' meanings are as follows: The actual market demand scale of the product is 200, the manufacturer's unit production cost is 20, the consumer price sensitivity coefficient is 1, the green sensitivity coefficient is 3, the green in-novation cost coefficient of the product is 5, the available capital of the manufacturing enterprise is 10, and the interest rate for prepayment financing and the bank loan interest rate are both set at 0.2. The results are presented in Figures 2–7.

Figures 2–4 illustrate that irrespective of market uncertainty, opting for prepayment financing significantly enhances product greenness and reduces wholesale prices. Retail price comparisons depend on factors like consumer sensitivity coefficients, green consumer awareness, and the cost coefficient of green innovation. In the prepayment financing model, the retailer allocates a portion of the upfront payment to the manufacturer, resulting in a lower wholesale price. This financial arrangement provides the manufacturer flexibility for green innovation investment. These funds can improve production processes, adopt eco-friendly materials and technologies, and support research and development for green innovation. Conversely, bank loan financing may involve higher capital costs, constraining the manufacturer's budget for green innovation. Thus, prepayment financing eases financial burdens, allowing substantial investments in green innovation and enhancing product ecological standards. This validates Proposition 3. Figure 5 demonstrates that robust optimization yields higher profits under low market risks than risk-free scenarios, emphasizing the value of robust optimization. Additionally, comparing prepayment financing and bank loan options under market risk reveals a consistent trend. Regardless of market fluctuations, the optimal financing choice for the manufacturer remains prepayment financing, establishing Proposition 4.

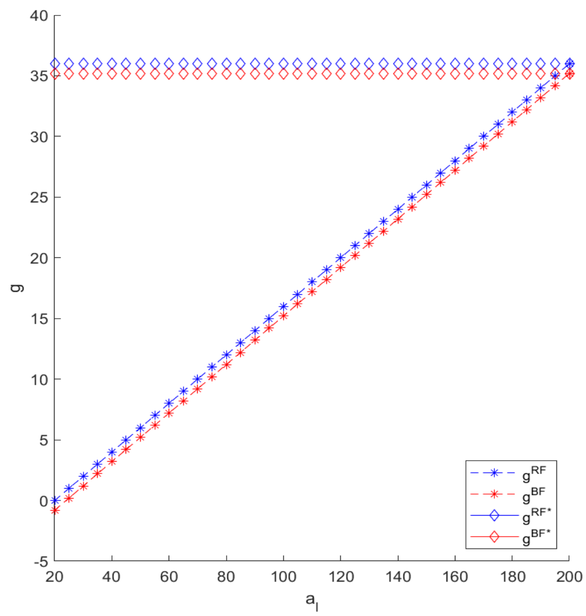

Figure 2. Product greenness level under market risk.

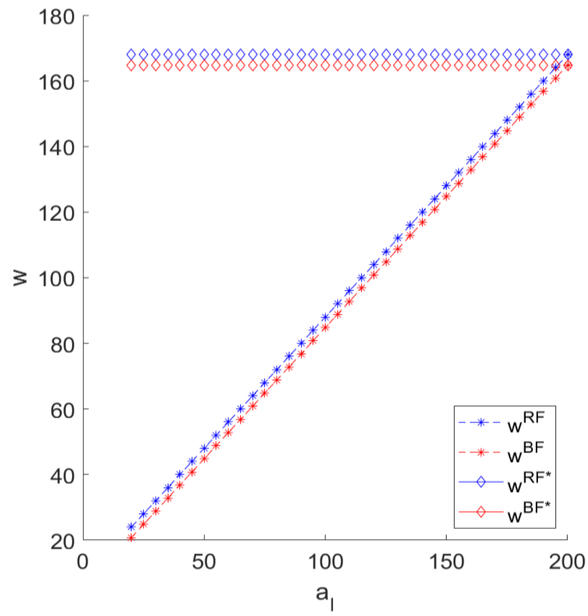

Figure 3. Retail price under market risk.

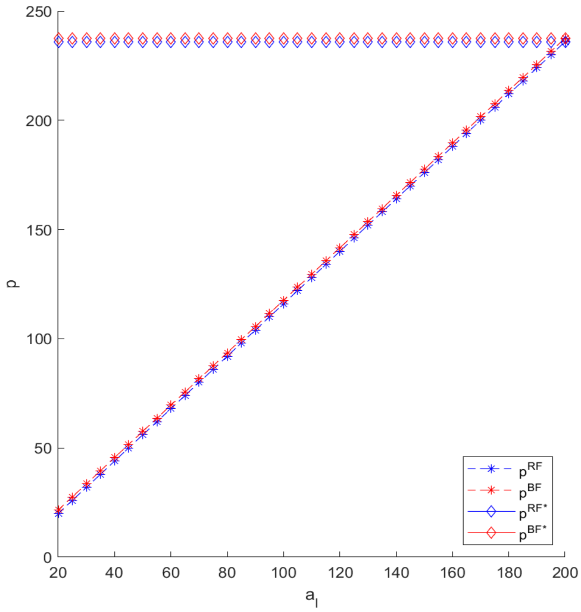

Figure 4. Wholesale price under market risk.

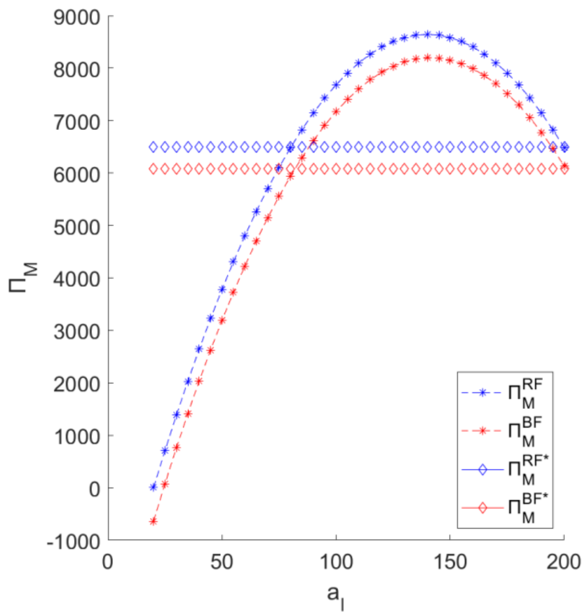

Figure 5. Manufacturer's profit under market risk.

Figures 6 and 7 further show that not only the manufacturer profits but also retailers and supply chain systems may be more profitable with the help of robust optimization in the face of incomplete information risk under different market risks, which is a deviation from intuition and emphasizes the importance of considering risk and incomplete information when studying supply chain decisions. From a profit function standpoint, despite the reduction in wholesale price, greenness, and selling price, there is an increase in sales volume under the robust risk decision-making mechanism. Under specific parameter conditions, the rise in sales volume will substantially impact the retailer's (manufacturer's) profit more than the reduction in selling price (wholesale price). In scenarios with low incomplete information risk, there is no need for both parties to invest time and cost in obtaining complete information. Profits can be secured by adopting a robust risk decision-making mechanism. In situations with substantial incomplete information risk, the manufacturer and retailer should proactively comprehend the market demand for green products to safeguard their profits. This can be achieved by, for instance, intensifying market research and employing other methods to diminish the risk of incomplete information regarding market demand. In the context of the prepayment financing model, the analysis of incomplete information risk and a risk-free situation with complete information reveals that in scenarios with high market risk, robust risk optimization can enhance the overall profit of the supply chain, highlighting the value of the robust optimization model.

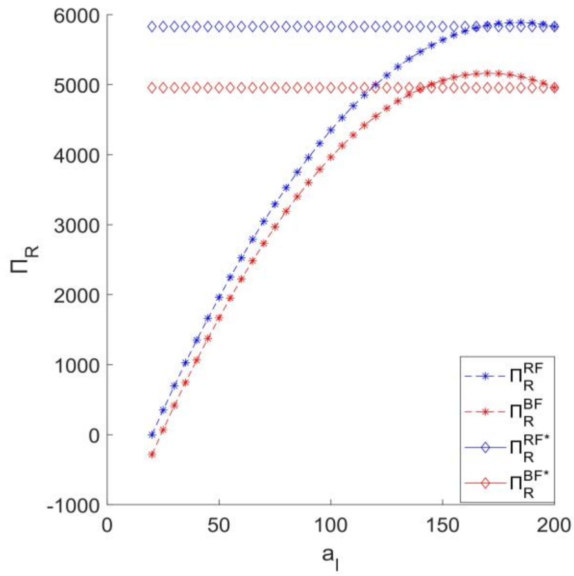

Figure 6. Retailer's profit under market risk.

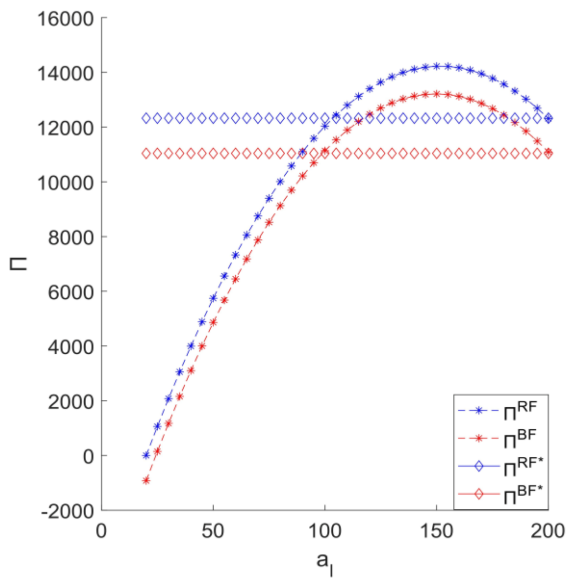

Figure 7. Profitability of supply chain under market risk.

Figures 6 and 7 further show that not only the manufacturer profits but also retailers and supply chain systems may be more profitable with the help of robust optimization in the face of incomplete information risk under different market risks, which is a deviation from intuition and emphasizes the importance of considering risk and incomplete information when studying supply chain decisions. From a profit function standpoint, despite the reduction in wholesale price, greenness, and selling price, there is an increase in sales volume under the robust risk decision-making mechanism. Under specific parameter conditions, the rise in sales volume will substantially impact the retailer's (manufacturer's) profit more than the reduction in selling price (wholesale price). In scenarios with low incomplete information risk, there is no need for both parties to invest time and cost in obtaining complete information. Profits can be secured by adopting a robust risk decision-making mechanism. In situations with substantial incomplete information risk, the manufacturer and retailer should proactively comprehend the market demand for green products to safeguard their profits. This can be achieved by, for instance, intensifying market research and employing other methods to diminish the risk of incomplete information regarding market demand. In the context of the prepayment financing model, the analysis of incomplete information risk and a risk-free situation with complete information reveals that in scenarios with high market risk, robust risk optimization can enhance the overall profit of the supply chain, highlighting the value of the robust optimization model.

6.2. Extended Analysis

When the prepayment is not equal to the bank loan interest rate, the loan interest rate of order is studied, where x is the relationship coefficient between the bank loan interest rate and the prepayment rate r. To analyze the manufacturer's decision choice when the prepayment rate is not equal to the bank loan interest rate in the presence of market risk, this paper takes $r = 0.2$, and $x = 1.5, 1.25, 1, 0.75$, and 0.5. Meanwhile, the fundamental parameter values $a = 200$, $c = 20$, $\beta = 1$, $\lambda = 3$, $\theta = 5$, and $B = 10$ are selected according to the above. Then, we explore the influence of different market risks on the manufacturer's profits, the retailer's profits, and the overall profits of the supply chain. The results are shown in Table 4 below in the context provided, where "x" represents the multiplier for the bank loan interest rate and prepayment rate, and "a_l" signifies the risk level, corresponding to the manufacturer's profits across different periods.

Table 4. The magnitude of π_M^{BF} when the interest rates are not equal.

	$x = 1.5$		$x = 1.25$		$x = 1$		$x = 0.75$		$x = 0.5$	
	π_M^{RF}	π_M^{BF}	π_M^{RF}	π_M^{BF}	π_M^{RF}	π_M^{BF}	π_M^{RF}	π_M^{BF}	π_M^{RF}	π_M^{BF}
$a_1 = 40$	2642.0	1694.9	2642.0	1859.2	2642.0	2034.8	2642.0	2224.0	2642.0	2429.9
$a_1 = 60$	4802.0	3805.4	4802.0	4007.7	4802.0	4226.8	4802.0	4466.5	4802.0	4731.4
$a_1 = 80$	6482.0	5457.2	6482.0	5687.4	6482.0	5938.8	6482.0	6216.2	6482.0	6525.3
$a_1 = 100$	7682.0	6650.2	7682.0	6898.3	7682.0	7170.8	7682.0	7473.0	7682.0	7811.4
$a_1 = 120$	8402.0	7384.3	8402.0	7640.5	8402.0	7922.8	8402.0	8236.9	8402.0	8589.9
$a_1 = 140$	8642.0	7659.6	8642.0	7913.9	8642.0	8194.8	8642.0	8508.0	8642.0	8860.7

When the prepayment rate differs from the bank loan rate, and market risk is present, the manufacturer lacks a universally superior decision choice. As evident from the table, in the realm of market risk, opting for prepayment financing is not restricted to the cases where the bank loan interest rate exceeds the prepayment rate. Even when the bank loan interest rate is less than the prepayment rate, prepayment remains a viable choice under certain circumstances. For instance, when the bank loan interest rate is 0.8 times the prepayment financing rate, irrespective of market risk fluctuations, the manufacturer consistently attains higher profits with prepayment financing than with bank loan financing. In other words, the manufacturer's optimal financing strategy is prepayment financing. Yet, when the bank loan interest rate falls below the prepayment rate, as observed at rates such as 0.25 times and 0.5 times the prepayment rate, the manufacturer's optimal financing strategy is not exclusively dictated by market risk. Specifically, in scenarios with low market risk (i.e., large a_l), choosing bank loan financing is optimal for the manufacturer. Conversely, in high-market risk scenarios (i.e., small a_l), the manufacturer's optimal financing strategy shifts to prepayment financing. This nuanced decision making arises because the manufacturer must consider various factors comprehensively regarding market risk, non-equal prepayment financing rates, and bank loan interest rates. This includes forecasting market interest rates, evaluating the reliability of supply chain partners, assessing capital requirements, gauging risk tolerance, and considering financial status. Each financing option presents its own merits and drawbacks, suited to different situations and objectives. Hence, manufacturers must conduct thorough assessments and make decisions based on specific circumstances, acknowledging that there is no universally superior decision choice.

7. Conclusions

The findings are outlined as follows: (1) Compared with bank loan financing, the prepayment financing method proves more conducive to enterprise product innovation, enhancing product greenness, and expediting product green transformation and development when the manufacturer faces financial constraints irrespective of market risk. Simultaneously, it is observed that the retailer can secure higher wholesale prices when making advance payments to the financially constrained manufacturer. (2) Comparing the effects of the presence or absence of market risk on the optimal production decisions of the supply chain, it is found that the level of green innovation and the wholesale price of the product is higher without market risk, which indicates that manufacturers and retailers are more conservative in their decisions when faced with market risk. (3) In market risk scenarios, robust optimization allows risk-averse manufacturers to attain higher profits than those without market risk. This counterintuitive result underscores the value and applicability of robust optimization. (4) When the manufacturer encounters financial constraints, the prepayment financing method is advisable under market risk-free conditions, aligning with the previous conclusions. In situations with market risk and when the prepayment rate equals the bank loan interest rate, regardless of market risk fluctuations, the optimal financing choice for the manufacturer remains prepayment. However, when the prepayment rate deviates from the bank loan interest rate, the manufacturer lacks a universally superior decision choice due to the impact of market risk.

The study suggests that (1) when manufacturers face financial constraints, the use of prepayment financing is more conducive to product innovation and green transformation than bank loan financing. This finding provides clear guidance for managers in choosing financing methods, especially when companies wish to upgrade the green level of their products; they can prioritize prepayment financing, which can accelerate the innovation and transformation of green products, as well as respond to the global trend in sustainable development and promote the industry's green transformation. (2) Manufacturers and retailers make more conservative decisions when faced with market risk. However, by employing robust optimization, risk-averse manufacturers can earn higher profits with market risk than in the no-market-risk scenario. This finding highlights the importance of robust optimization in risk management and suggests that managers adopt robust optimization strategies to achieve higher profits in uncertain market environments. (3) Advance payment financing can alleviate the financial constraints of manufacturers and ensure the continuity and stability of production, which in turn improves the stability and efficiency of the entire supply chain. Firms can establish closer and more mutually beneficial supply chain relationships through this approach. The study results provide specific references for enterprises in choosing financing methods, help enterprises make the best financing decisions in different market environments, and enhance the efficiency of capital utilization and financial management level of enterprises.

This paper explores the optimal financing level and strategy for enterprises facing financial constraints, specifically under the condition of incomplete information risk. This refers to situations where only market demand's upper and lower bounds are known and distributed randomly. Subsequent research could investigate strategies when the supply chain system contends with both market demand information risk and information asymmetry risk. Market demand affects the relevant decisions of enterprises in the green supply chain. Enterprises should vigorously publicize their green concept and the advantages of their products to consumers to increase consumers' attention to green products. Future research can also take retailers' green publicity into account. Additionally, future studies might delve into comparing and analyzing supply chains with three or more levels, along with examining multiple financing methods to understand the financing decisions within green supply chains under the market risk posed by capital constraints.

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Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of Inference 1

$\frac{\partial \pi_M^{RF}}{\partial \lambda} = \frac{(a_l - c\beta)(4a - 3a_l - c\beta)(1+r)\theta\lambda}{(4\beta\theta(1+r) - \lambda^2)^2} > 0$, The derivation of π_R^{RF} with respect to λ yields the following: $\frac{\partial \pi_R^{RF}}{\partial \lambda} = \frac{\theta\lambda(a_l - c\beta)(2a(1+r)(4\beta(1+r)\theta - \lambda^2))}{(4\beta\theta(1+r)-\lambda^2)^3} + \frac{\theta\lambda(a_l - c\beta)c\beta(4\beta(1+3r+2r^2)\theta + r\lambda^2)}{(4\beta\theta(1+r)-\lambda^2)^3} + \frac{\theta\lambda(a_l - c\beta)a_l(4\beta(1+r)\theta - (2+3r)\lambda^2)}{(4\beta\theta(1+r)-\lambda^2)^3}$.

Because of the uncertainty of the parameters and the complexity of the analytic formula, it is impossible to judge the size.

$\frac{\partial \pi_M^{RF}}{\partial a_l} = \frac{(2a - 3a_l + c\beta)(1+r)}{4\beta\theta(1+r) - \lambda^2}$, when $a_l < \frac{2a + c\beta}{3}$, $2a - 3a_l + c\beta > 0$, $\frac{\partial \pi_M^{RF}}{\partial a_l} > 0$, and when $a_l > \frac{2a + c\beta}{3}$, $2a - 3a_l + c\beta < 0$, $\frac{\partial \pi_M^{RF}}{\partial a_l} < 0$;

The $\frac{\partial \pi_R^{RF}}{\partial a_l} = \frac{\theta(a(1+r)(4\beta\theta(1+r)-\lambda^2) + c\beta\theta(-2\beta(1+r)^2 - (1+2r)\lambda^2) + a((2+3r)\lambda^2 - 6\beta\theta(1+r)^2))}{(4\beta\theta(1+r)-\lambda^2)^2}$ analytic expression is too complex to judge the size.

$\frac{\partial \pi_M^{BF}}{\partial a_l} = \frac{\theta(1+r_b)(2a - 3a_l + c\beta(1+r_b))}{4\beta\theta(1+r) - \lambda^2}$, when $a_l > \frac{2a + c\beta}{3}$, $2a - 3a_l + c\beta > 0$, $\frac{\partial \pi_M^{BF}}{\partial a_l} > 0$, and when $a_l > \frac{2a + c\beta}{3}$, $2a - 3a_l + c\beta < 0$, $\frac{\partial \pi_M^{BF}}{\partial a_l} < 0$;

$\frac{\partial \pi_R^{BF}}{\partial a_l} = \frac{[\theta(1+r_b)(c\beta(1+r_b)(2\beta\theta(1+r_b)-\lambda^2) + a(4\beta\theta(1+r_b)-\lambda^2) + a(-6\beta\theta(1+r)^2 + 2\lambda^2)]}{(4\beta\theta(1+r)-\lambda^2)^2}$ Because of the uncertainty of the parameters and the complexity of the analytic formula, it is impossible to judge the size.

Appendix B. Proof of Proposition 1

$\pi_M^{RF} - \pi_M^{RF*} = \frac{\theta(1+r)(a - a_l)(3a_l - a - 2c\beta)}{8\beta\theta(1+r) - 2\lambda^2}$, when $a_l > \frac{a + 2c\beta}{3}$, $3a_l - a - 2c\beta > 0$, $\pi_M^{RF} - \pi_M^{RF*} > 0$, and when $a_l < \frac{(a+2c\beta)}{3}$, $3a_l - a - 2c\beta < 0$, $\pi_M^{RF} - \pi_M^{RF*} < 0$;

$$\pi_R^{RF} - \pi_R^{RF*} = \frac{\begin{array}{c}\theta(a - a_l)(-a(2\beta\theta(1+r)^2 + r\lambda^2) + 2c\beta(-2\beta\theta(1+r)^2 \\ +(1+2r)\lambda^2) + a_l(6\beta\theta(1+r)^2 - (2+3r)\lambda^2))\end{array}}{2(-4\beta\theta(1+r) - \lambda^2)^2}$$

$$\pi^{RF} - \pi^{RF*} = \frac{\begin{array}{c}\theta(a-a_l)((18a_l\beta\theta(1+r)^2 - 12c\beta^2\theta(1+r)^2 + 2c\beta(2+3r)\lambda^2 \\ -a_l(5+6r)\lambda^2 + a(-6\beta\theta(1+r)^2 + \lambda^2))\end{array}}{2(-4\beta\theta(1+r)-\lambda^2)^2}$$

The analytical formula is too complex, and the final sign positive and negative are difficult to determine, and the size cannot be compared.

Appendix C. Proof of Proposition 2

$\pi_M^{BF} - \pi_M^{BF*} = \frac{\theta(a-a_l)(1+r_b)(3a_l-a-c\beta(1+r_b))}{8\beta\theta(1+r_b)-2\lambda^2}$, when $a_l > \frac{a+2c\beta}{3}$, $\pi_M^{BF} - \pi_M^{BF*} > 0$, and when $a_l < \frac{a+2c\beta}{3}$, $\pi_M^{BF} - \pi_M^{BF*} < 0$.

$$\pi_R^{BF} - \pi_R^{BF*} = \frac{\begin{array}{c}\theta(a-a_l)(1+r_b)(a_l(3\beta\theta(1+r_b)-\lambda^2) \\ -\beta(1+r_b)(a\theta + 2c\beta\theta(1+r_b) - c\lambda^2))\end{array}}{(-4\beta\theta(1+r_b)+\lambda^2)^2}$$

$$\pi^{BF} - \pi^{BF*} = \frac{\begin{array}{c}\theta(a-a_l)(1+r_b)(a_l(18\beta\theta(1+r_b)-5\lambda^2) - 4c\beta(1+r_b)(3\beta\theta(1+r_b)-\lambda^2) \\ +a(-6\beta\theta(1+r_b)+\lambda^2))\end{array}}{2(-4\beta\theta(1+r_b)+\lambda^2)^2}$$

Because the analytical formula is too complex, and the final signs positive and negative are difficult to determine, and the size cannot be compared.

Appendix D. Proof of Proposition 3

Product greenness $g^{RF} - g^{BF} = \frac{c\lambda\beta r}{4\beta\theta(1+r)-\lambda^2} > 0$, that is $g^{RF} > g^{BF}$; product wholesale price $w^{RF} - w^{BF} = \frac{2c\beta r\theta(1+r)}{4\beta\theta(1+r)-\lambda^2} > 0$, that is $w^{RF} > w^{BF}$; product wholesale price under the mode of prepayment financing is lower; and product retail price $p^{RF} - p^{BF} = \frac{cr(-\beta\theta(1+r)+\lambda^2)}{4\beta\theta(1+r)-\lambda^2}$, the positive or negative value of $-\beta\theta(1+r)+\lambda^2$ cannot be judged and cannot be compared.

Appendix E. Proof of Proposition 4

Proof: $\pi_M^{RF} - \pi_M^{BF} = \frac{c\beta\theta(1+r)(4a-2a_l-c\beta(2+r))}{8\beta\theta(1+r)-2\lambda^2}$; because $a - c\beta(1+r) > 0$, $a - c\beta(1+\frac{r}{2}) > 0$, that is $4a - 2a_l - c\beta(2+r) > 0$, so $\pi_M^{RF} > \pi_M^{BF}$;

$$\begin{array}{c}\pi_R^{RF} - \pi_R^{BF} = -Br + \frac{r\theta}{2(-4\beta\theta(1+r)+\lambda^2)^2}[(a_l^2\lambda^2 - 2a_lc\beta(2\beta\theta(1+r)^2 - r\lambda^2) \\ +c\beta(2a(1+r)(4\beta\theta(1+r)-\lambda^2) - c\beta(2\beta\theta(1+r)^2(2+r)-\lambda^2))]\end{array}$$

Because the analytical formula is too complex, and the final signs positive and negative are difficult to determine, and the size cannot be compared.

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