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An Integrated SIMUS–Game Theory Approach for Sustainable Decision Making—An Application for Route and Transport Operator Selection

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Abstract: The choice of management strategy for companies operating in different sectors of the economy is of great importance for their sustainable development. In many cases, companies are in competition within the scope of the same activities, meaning that the profit of one company is at the expense of the other. The choice of strategies for each of the firms in this case can be optimized using game theory for a non-cooperative game case where the two players have antagonistic interests. The aim of this research is to develop a methodology which, in non-cooperative games, accounts for the benefits of different criteria for each of the strategies of the two participants. In this research a new integrated sequential interactive model for urban systems (SIMUS)–game theory technique for decision making in the case of non-cooperative games is proposed. The methodology includes three steps. The first step consists of a determination of the strategies of both players and the selection of criteria for their assessment. In the second step the SIMUS method for multi-criteria analysis is applied to identify the benefits of the strategies for both players according to the criteria. The model formation in game theory is drawn up in the third step. The payoff matrix of the game is formed based on the benefits obtained from the SIMUS method. The strategies of both players are solved by dual linear programming. Finally, to verify the results of the new approach we apply four criteria to make a decision—Laplace's criterion, the minimax and maximin criteria, Savage's criterion and Hurwitz's criterion. The new integrated SIMUS–game theory approach is applied to a real example in the transport sector. The Bulgarian transport network is investigated regarding route and transport type selection for a carriage of containers between a starting point, Sofia, and a destination, Varna, in the case of competition between railway and road operators. Two strategies for a railway operator and three strategies for a road operator are examined. The benefits of the strategies for both operators are determined using the SIMUS method, based on seven criteria representing environmental, technological, infrastructural, economic, security and safety factors. The optimal strategies for both operators are determined using the game model and dual linear programming. It is discovered that the railway operator will apply their first strategy and that the road operator will also apply their first strategy. Both players will obtain a profit if they implement their optimal strategies. The new integrated SIMUS–game theory approach can be used in different areas of research, when the strategies for both players in non-cooperatives games need to be established.

Keywords: game theory; SIMUS method; non-cooperative game; multi-criteria analysis; benefits; sustainable decision making; route selection; container carriage

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1. Introduction

Sustainable decision making is of great importance when selecting strategies for the operation and development of systems in various fields of research. The decision-making process involves the consideration of various factors that are related to solving a given problem. Game theory is applied in various fields when there are conflict situations and strategic interactions. Game theory involves two or more players, each of which has strategies. A payoff matrix is constructed considering the strategies of both players. The beginning of game theory was established by Neumann and Morgenstern in [1]. Later, John Hash influenced the development of game theory as a tool for strategic decisions [2,3]. There are two main groups of games—cooperative and non-cooperative games. The former deal with how coalitions interact; the payoffs are known. The latter include strategic games with competition between the players. These can be zero-sum-games and non-zero-sum games. This research studies non-cooperative games. In zero-sum games, the gain of one player is equal to the loss of the other player. These games are solved mathematically using the dual problem of linear programming by determining the strategies of both players. Such a game is Pareto optimal. The payoff matrix is the basis on which the dual mathematical model is built. The payoff matrix can only consider one factor. Usually, the payoff matrix takes into account the profit of one player. As the players have conflicting interests, the matrix shows the other player's losses accordingly. The non-zero-sum games permit all participants to win or lose at the same time. There is no competition between the players.

The hypothesis of this research is that the payoff matrix of non-cooperative games could account for the influence of many criteria expressed as benefits for each player. The benefits can be determined using the methods of multi-criteria analysis. A suitable method for determining the benefits is the sequential interactive model for urban systems (SIMUS) method [4]. The main advantages of the SIMUS method, which make it preferable to other multi-criteria methods are the following:

- It uses linear optimization.
- It is not a subjective method; it is objective because it is based on optimization.
- It does not use experts' assessment.
- It allows both quantitative and qualitative decision criteria to be explored.
- It does not use weights of criteria.
- After performing the optimizations for each of the criteria, the weights of the objectives can be defined, which serve only for analysis and not for follow-up actions.
- It allows one to analyze the sensitivity of the input data, i.e., to determine the upper and lower values for each of the criteria under which the obtained ranking of the alternatives is preserved. The values of the upper and lower limits of the criteria are determined as results of the linear optimization models.
- It permits one to undergo a preliminary analysis of results. The preliminary analysis permits one to compare the values for each objective and the optimum values obtained by using the SIMUS method. This makes it possible to analyze the real state of the studied system and opportunities for its improvement.

The determination of the payoff matrix based on the scores of the SIMUS method permit us to assess the influence of predetermined quantitative and qualitative criteria on the choice of strategy and of obtaining common benefits. The integration of the SIMUS and game theory methods enable us to use their advantages and opportunities to make decisions.

The goal of this study is to expand scientific knowledge regarding sustainable decision making based on the advantages of the SIMUS method and game theory for choosing strategies in non-cooperative games.

The research questions for this paper consist of the following problems:

- Assessment of the benefits of the strategies of both competing players.
- Establishment the impact of different criteria on the behavior of the players.
- Formation of the payoff matrix so that it considers the benefits of all the quantitative and qualitative criteria that influence the players' choice of strategies.
- Making decisions about the most suitable strategies of both competing players considering the benefits of different criteria.

The advantages of the integrated SIMUS–game theory approach are as follows:

(1) It can assess the common benefits of different quantitative and qualitative criteria for each strategy and interactions between the players.

(2) The determination of the benefits of the criteria is carried out by linear optimization, which is an objective method of decision making.
(3) The strategies for both players are formed while taking into account a complex of quantitative and qualitative criteria.

This paper presents a unique study which integrated the power of game theory with the advantages of the SIMUS method. Here, this approach is presented for the first time.

Game theory models can be used to solve problems in various fields such as economics, business, finance, politics, military affairs, logistics, transport, systems science, information technology.

In this study the transport sector is chosen to present the new integrated approach. In transport, game theory can be used to find a solution in the cases of competition between modes of transport that utilize carriages on parallel lines for highway or city transport, competition between carriers of the same mode of transport, selection of transport equipment for servicing logistics centers, management of congestion in cities, route selection, pricing, and determining transport company management strategy, when allocating transport services among carriers. In all cases, the main objective is to obtain maximum utility or maximization of possible profit and to make a decision about a management strategy.

Both cooperative and non-cooperative games can be applied in the transportation sector. In the first case, for example, transport carriers from one mode of transport can coalesce to achieve a common goal. In the second case, in non-cooperative games, transport firms have antagonistic interests, with one's gain being the other's loss. The goal here is for both players to gain utility.

In this paper, a new approach, which makes use of SIMUS game theory, is presented through a real example about route and transport type selection for the carriage of containers between a starting point and a final point in the case of competing transport operators. This is an important problem because it is part of sustainable transport development. The choice of transport strategy is important for the sustainability of transport companies in the transport services market and depends on a complex of indicators that need to be considered.

In this research a case of the Bulgarian transport network is studied.

At present, the interests of the railway and road carriers on the Bulgarian transport network are in conflict in the field of freight transportation.

(1) The motorways, first- and second-class roads in Bulgaria are overloaded by trucks of different types—from 3.5 to over 40 tons. The emitted harmful emissions and fine dust particles from trucks pollute the air. At the same time, a small number of freight trains run on the railway highways, although this type of transport is environmentally friendly.
(2) Transportation by trucks is tolerated by the state. For example, according to data from the National Statistical Institute of Bulgaria, in 2023 for inland carriages, 11.26 million tons of cargo were transported by freight rail transport, while 42.70 million tons were transported by road with trucks over 25 t. The total length of running track is 4029 km, and the total length of motorway and category I roads is 3719 km. Furthermore, the state invests in more road repair projects.
(3) There is the possibility of transportation on parallel lines, i.e., by rail and road. In these cases, both modes of transport are in a state of competition and have antagonistic interests, i.e., the gain of one is at the expense of the other. The purpose of both types of transport is to make a profit.

This paper investigates the selection of route and transport type for the carriage of containers between a starting point, Sofia, and destination, Varna, on the Bulgarian transport network in the case of competing railway and road operators. Two route selection strategies for a railway operator and three route strategies for a road operator were studied.

This paper expands the literature as follows:

(1) The game relations between rail and road transport are considered.

(2) An approach is proposed to evaluate the total utility of quantitative and qualitative criteria for evaluating strategies for both players.
(3) The strategies for route selection for both players are determined based on the total utility, which is applied in the game non-cooperative model.

This paper is organized as follows: Section 2 presents the literature review. Section 3 introduces materials and methods where the novel integrated SIMUS–game theory approach is explained. Section 4 presents the results obtained by applying the new approach for route and transport operator selection. Section 5 discusses the obtained results and the advantages of the model. Finally, the conclusions are given.

2. Literature Review

Game theory has been successfully applied in different areas of research. A detailed analysis of the game models in business areas is given in [5]. In transport, game theory has been applied to make decisions about transport problems [6,7], route choices in transport network [8,9], transport policy [10–12], travel demand management [13,14], high-speed rail and air transport [15–17], railway companies [18–20], transport companies [21–27], urban public transport [28–32], logistics and supply chains [33,34], and intermodal transport [35].

In [6], the authors describe the applications of non-cooperative games theory models for solving transport problems. Four groups of games were proposed, according to the relevant players. The first group includes games against a demon. In these games there is one player that wishes to maximize the objective function and the other player wants to minimize it. The second group consists of games between travelers. The third group covers games between authorities. In this group at least one player is an authority. The fourth group includes travelers and authorities as players.

An analysis of game theory applications in transport was conducted in [7]. The authors classified these applications into two groups: macro-policy analysis and micro-behavior simulation. The authors discussed the game theory applications in both cases.

Route choices on the international transport network were studied by applying game theory and a congestion game [8]. The model was experimented with regard to three ports and an intermodal terminal in Budapest. The players were drivers and logistics operators who are not in full conflict. As a criterion for choosing a route in the game, the authors used minimum transportation costs. Game theory non-cooperative approach in the assessment of network reliability is presented in [9]. The players are the network user and the state of the network. The criterion of optimization is the link cost. An example of a network with six paths is presented. The problem was defined as a linear programming model.

In [10], a concept, based on cooperative game theory, for EU policy to improve the international cooperation of countries along rail freight corridors is elaborated. The case of a transferable utility game with characteristic function was studied. The author defined as players the governments along the corridors, each of which have a common goal and have to act together. The research proposes a new concept of allocation rules for transport networks based on the Shapley value and cooperative game theory.

In [11], the authors present and discuss the game models and their application to transport markets. Four groups of models were analyzed, including common oligopoly models, traffic assignment models, spatial models, and auctioning related to transport. The authors analyzed the road, rail, aviation, port and multi-modal literature to describe the real-word application with a game theory component.

A game theory model was elaborated in [12] to optimize strategy for the transport sector in Iraq. The players are public transport and private transport. Both forms of transport include different types of carriage, such as sea, air, land, rail and port transport. The minimax–maximin principle and linear programming model were applied to determine the strategies of both operators.

In [13], game theory was applied for the travel demand management of urban transport systems. The model investigated three districts, three passenger flows, and public transport, meaning that the game model includes seven players. Passengers' travel time, environmental conditions, residents' travel time, and an optimal proportion between using a private car and public transport were studied in the game model.

In [14], a gaming model about the mode choice behavior of travelers is presented. The model includes two players, the parking operator and transit agency. The application of the model is presented for Isfahan, Iran.

A game model to analyze the competition between high-speed rail and air transport was elaborated on in [15]. The model includes low cost and network carriers. A generalized profit function for the different operators was determined and the utility function in terms of total trip time, fares and frequencies was applied in the model. The methodology was applied to the analysis of four trans-European networks.

In [16], a game model was applied to study the competition between high-speed rail and airlines in India. The fare and frequency were chosen as the parameters of the game. The speed was investigated at three levels as an additional parameter.

The competition between a high-speed rail operator and aviation operator was studied using game theory [17]. The game was presented in two stages. In the first stage, based on a linear programming model, the game gave results regarding the costs. The second stage included a game between each operator and the goal was to maximize the profit.

In [18], the authors presented a concept for applying game theory in some sectors of the railway systems, such as railway privatization and maintenance of railway infrastructure and rolling stock. The game model includes three players (track operator, rolling stock operator and track maintenance contractor). The railway privatization processes in Sweden and the UK were studied.

In [19], the authors present a two-stage model for evaluating the strategic decisions made by rail operators. The first stage includes a game model with which to solve whether the railway operators decide to participate in the market. In the second stage, the game model includes the railway operators and the new operators. The model was studied for the Madrid–Barcelona European high-speed rail corridor.

In [20], a game model for a railroad and a shipper was studied. The game model investigated two strategies for the railroad and three strategies for the shipper. The strategies for the railroad refer to the pricing to maximize the profits and to avoid litigation. The strategies of the shipper are related to the prices offered by the railway line.

Game theory was used to determine an alternative strategy for two players—online transportation companies and drivers [21]. Six strategies were studied. The values of strategies were determined through a survey. Both players have antagonistic interests, i.e., the online transportation companies aim to minimize the maximum loss, while the drivers want to maximize the minimum profit. Linear optimization was applied to solve the problem under investigation.

A game theory model that takes into account the competition between two oil-product transportation systems was elaborated in [22]. The main variables of the model that were investigated were the transportation prices and tanker truck fleet. The Nash and Stackelberg equilibrium models were applied to formulate and solve the problem.

A coalition game theory model was elaborated in [23] for route planning in a shared transportation system. The authors studied the problem of automated vehicle collection.

The subsidy problem in railways was investigated by applying a game model [24]. The players were railway operators, shippers and local governments. The game model was implemented for a railway express line in China.

Game theory was used to study inter-city transportation pricing [25]. The authors applied the game model with the aim to maximize profit. The authors studied a rail and bus company.

A Stackelberg game model was used to study railway transportation companies and customers regarding costs and prices [26].

A game theory model to investigate an urban public transport integration policy is given in [27]. The problem is staged for a two-player game and multi-player game, providing services in separate and the same districts.

A review of game theory that was applied to investigate urban traffic congestion management systems is undertaken in [28]. Here, the authors discuss different applications of game models.

In [29], a game theory approach was adopted to optimize public transport traffic. The two players are the passenger and authorities. Strategies were determined for each of the players. To provide the Nash equilibrium, a set of conditions had to be satisfied: the frequency of transport and the payoff functions of players.

Game theory and the Golden Template algorithm were integrated with the purpose to investigate urban mobility [30]. The relevant intersections were chosen as players. Two game models were studied, one for public transport operators, and the other on the number of passengers. The model permits us to plan multimodal transportation routes in urban conditions. The methodology was tested for public transport in Bucharest.

In [31], game theory was used to establish the best line in urban transport. The payoff matrix was formed by means of a survey conducted among residents for route service preference. The function of risk of expected losses was chosen as the element of the payoff matrix. The linear programming method was applied to make a decision.

The sharing of transportation between a distribution center and service outlets in a city was investigated based on game theory [32]. The model aimed to reduce the costs for transportation. The multiagent game model was elaborated on to realize transportation sharing between multiple enterprises.

Game theory was applied in the area of supply chains [33]. The game model was formed based on Nash competition and Stackelberg competition. The Nash competition was described between the retailers of the supply chains and between the manufacturers of the supply chains. The Stackelberg competition was presented between the manufacturer and the retailer.

A cooperative game theory approach was used to make a decision about the selection of a transportation network in a multi-level supply chain [34]. Linear transportation programming was used to make models for multiple supply chains. The Shapley value was applied to determine the total value of the coalition and establish the optimal mode in terms of transportation.

Intermodal rail–road transport was studied using Stackelberg game theory in order to reduce both the subsidy and carbon emissions [35].

It can be concluded that previous literature has elaborated on different game theory models and has thus contributed to the development of transport planning. The criteria used for making a decision based on the game model were as follows: transportation costs [8,32], link costs [9], total costs (profit) [10,16,17,19,25], subsidy [24,35], utility function [15], number of passengers [30], function of risk of expected losses [31], and pricing [20].

The authors investigated cooperative and non-cooperative games. In cooperative games the players make coalitions and have a common interest. The non-cooperative games present the competition between the players. The linear programming method is applied to make a decision in these cases [9,12,17,20,21,31,33].

In [36], the authors investigated the transport time, costs, transport quality, the service in transport, transport tools, and the social benefits as criteria by which to assess different routes for multimodal transportation. A fuzzy analytic hierarchy process (FAHP) and an artificial neural network (ANN) were applied for route selection. The transport time, transport price, congestion time, accident risk and noise, and CO_2 emissions were chosen for route selection in [37]. The authors used the AHP method to establish the weights of criteria, and the PROMETHEE method for ranking routes. The criteria—travel time, transportation cost, and CO_2 emissions generated by all transportation modes—were applied in a multi-objective optimization model for transport mode selection [38].

The criteria of transportation costs, transportation time, security risk, operational risk, infrastructure risk, macro risk, policy, legality risks, and environmental risk were used in [39] for route selection in multimodal transport network. The author proposed a hybrid multi-criteria approach integrating the analytic hierarchy process (AHP), data envelopment analysis (DEA), and the technique for order preference by similarity to ideal solution (TOPSIS). The case of route selection between Thailand and Vietnam was studied.

Five main groups of criteria, eco-sociological, urban planning, constructional, technical, and transport and economic were introduced in [40,41] with purpose of evaluating a road route. Sub-criteria were set for each main group of criteria. The PROMETHEE method was used to rank the alternative routes. The weights of the criteria were calculated through experts' assessment. The case of optimal railroad route between Rijeka and Zagreb was studied.

In [42], the authors developed a multi-objective optimization model for optimizing transportation routes. The criteria that were chosen were minimizing the total transportation time, transportation costs and container usage costs. The example of transportation in Panzhihua City, Sichuan Province was studied.

In [43], four main groups of criteria—technology, economy, society and environment—were studied for railway route selection. TOPSIS, the entropy weight method and Grey correlation analysis was applied to make a decision. The case study was investigated for China's railway.

On the basis of this research, it can be concluded that the most important factors for route selection are transport price [36–43], transport time [36–39,42], and CO_2 emissions [38,39,42].

Some authors have focused efforts to integrate game theory and multi-criteria decision making (MCDM) methods to support decision making in different areas of research.

A hybrid methodology comprising stepwise weight assessment ratio analysis (SWARA), weighted aggregated sum product assessment (WASPAS) and game theory is elaborated in [44]. The authors applied the SWARA method to assess the criteria weights, and the WASPAS method to evaluate and select the best Nash equilibriums to make a decision. The methodology was tested for supply chain management.

In [45], the authors elaborate an integrated model comprising the analytic network process (ANP) method, entropy weight, game theory, the decision-making trial and evaluation laboratory (DEMATEL) method and evidence theory. Supplier management of an uncertain case was studied. The weights of criteria were determined using a combination of game theory and the DEMATEL method.

The analytical hierarchy process (AHP) method and game theory were integrated to select the most suitable network, [46]. Non-cooperative game theory was applied. Both of the players in the game (for network and for user) participated without any form of collaboration. The payoff matrix of the game was formed by taking into account the weight estimation of the networks given by AHP method.

The concept of a combination of fuzzy AHP and game theory is presented in [47], in which the risk factors associated with a tunnel project were assessed. A case study for the Resalat tunnel in Iran is presented. The fuzzy AHP was used to analyze qualitative decisions into quantitative scores. Both non-cooperative and cooperative games were investigated.

An integration between the TOPSIS method and cooperative games is presented in [48]. The authors investigated airlines. The authors evaluated the impacts of alliances and mergers on both airlines and passengers using Shapley values and the Nash equilibrium. The TOPSIS method was applied to assess the factors affecting airline mergers.

In [49], a multi-agent MCDM is represented as an evolutionary game. The criteria and alternatives were considered as strategies of the agents (players). A case study for the Indian tea industry was considered.

A hybrid game theory technique and multi-criteria decision-making methods are discussed in [50]. A total of 56 papers from the year 2008 to 2020 have been analyzed to provide different applications studied from previous research. The authors found that the most used multi-criteria methods which have been integrated with game theory are AHP and TOPSIS. The authors concluded that the integration between MCDM and game theory has been increasingly used to support decision-making.

In summary, multi-criteria methods can be said to be a powerful tool when integrated with game theory to support the making of sustainable decisions. The authors applied AHP, ANP, TOPSIS, SWARA, WASPAS, DEMATEL methods for multi-criteria analysis. However, all of these methods use expert evaluation of the criteria, which makes them subjective.

This paper proposes the integration of the SIMUS method of multi-criteria analysis with game theory for the first time. This is a non-subjective MCDM method.

The difference between this study and the literature described above is the development of a new integrated approach. The combination of multi-criteria analysis based on the SIMUS method and game theory has not yet been presented in the literature. This paper deals with a new approach which permits us to assess the common benefits of quantitative and qualitative criteria when choosing strategies for two players with conflicting interests in non-cooperative games. These benefits are considered as the total utility, which is used to construct the payoff matrix in the game model.

3. Materials and Methods

The methodology of the new integrated SIMUS–game theory approach for decision making consists of the following steps, as in Figure 1:

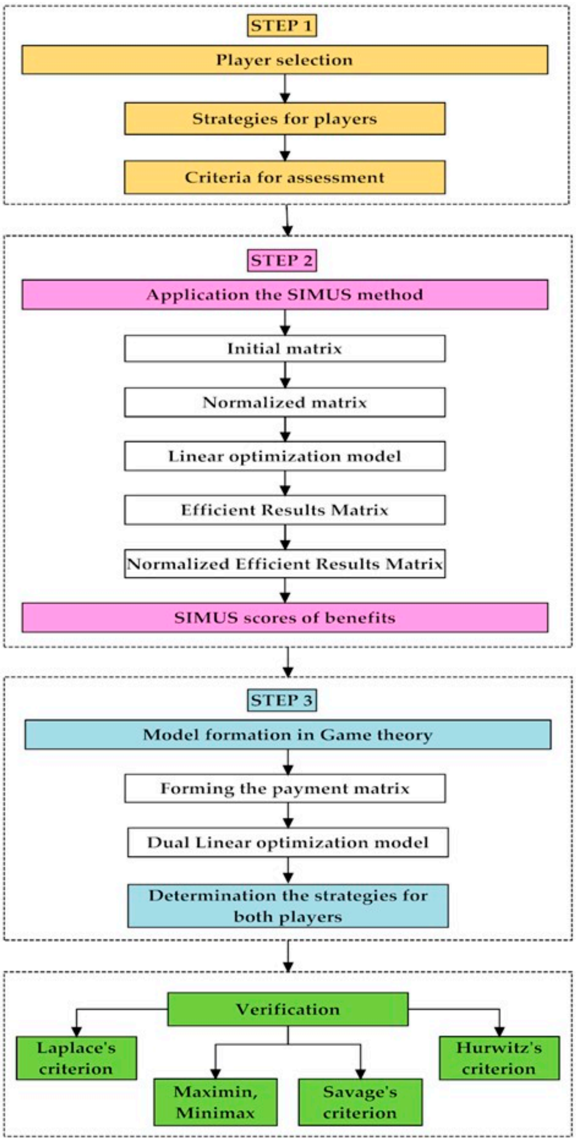

Figure 1. Scheme of the methodology.

Step 1: Determination of the strategies for players A and B and determination of the criteria to evaluate these strategies.

Step 2. Application of the SIMUS method for determining the benefits of the strategies, taking into account the criteria.

Step 3: Model formation in game theory. Determination of the strategies for both players.

Finally, to verify the results of the new approach we propose the following four criteria for decision making: Laplace's criterion, minimax and maximin criteria, Savage's criterion and Hurwitz's criterion [51].

3.1. Step 1: Determination of the Strategies for Players A and B—Determination of the Criteria for Assessment of the Strategies

The players have antagonistic interests, i.e., one player's gain is another player's loss. The objective is to determine the strategies of both players, so that each of them has a profit.

Initial decision matrices A_{mxn1} and B_{mxn2} are constructed for each of the players. Matrix A and matrix B contain the criteria values for each strategy of both players.

$$A_{mxn1} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1i} & \dots & a_{1n1} \\ a_{21} & a_{22} & \dots & a_{2i} & \dots & a_{2n1} \\ \dots & \dots & \dots & \dots & \dots & .. \\ a_{k1} & a_{k2} & \dots & a_{ki} & \dots & a_{kn1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mi} & \dots & a_{mn1} \end{pmatrix}$$
$$B_{mxn2} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1i} & \dots & b_{1n2} \\ b_{21} & b_{22} & \dots & b_{2i} & \dots & b_{2n2} \\ \dots & \dots & \dots & \dots & \dots & .. \\ b_{k1} & b_{k2} & \dots & b_{ki} & \dots & b_{kn2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mi} & \dots & b_{mn2} \end{pmatrix}, \quad (1)$$

where $k = 1, \dots, m$ is the number of criteria, $i = 1, \dots, n_1$ is the number of strategies for player A, and $j = 1, \dots, n_2$ is the number of strategies for player B.

3.2. Step 2: Application of the SIMUS Method for Solving the Benefits of the Strategies

SIMUS is a multi-criteria method that uses linear programming, weighted sum, and outranking [4]. Its main advantage is that it does not use criteria weights, i.e., it does not use a subjective approach.

The SIMUS method includes the following operations:

- Determination of the initial decision matrix.
- Calculation of the normalized decision matrix.
- Formulation and solving of the linear optimization models for each criterion as an objective function.
- Determination of the efficient results matrix.
- Calculation of the normalized efficient results matrix.
- Calculation of the scores of alternatives.

The initial decision matrix determined by means of the SIMUS method is formed with alternatives in columns and criteria in rows using the matrices A_{mxn1} and B_{mxn2} of players A and B, respectively, that were composed in the previous step.

The initial decision matrix is normalized. For this purpose, different methods of normalization can be used. The ranking of the alternatives is not affected by the normalization method. This research proposes the use of the total sum in row method.

The thresholds for each row are determined. This determination depends on the type of optimization. The threshold is equal to the maximum normalized value of the row when the optimization is of the maximum of the criterion. The threshold is equal to the minimum normalized value of the row when the optimization is of the minimum of the criterion. The thresholds are necessary for the formulation of the restrictive conditions of linear optimization models.

The simplex algorithm is applied to solve the linear optimization models. The unknowns are the scores of each alternative. The number of criteria determine the number of optimization models. The first linear optimization model is formed by taking into account the first criterion as the objective function. In this case the restrictive conditions are formed by the other rows. This procedure is applied consistently to all criteria considered as an objective function. When the optimization is of a maximum, the operator of the restrictive condition is "≤";when the optimization is of a minimum the operator of the restrictive condition is "≥". The results of the optimizations are recorded in the efficient results matrix (ERM). The elements of the ERM are the optimal values of the scores of each alternative for each optimization model. This matrix is a new decision matrix.

The normalization procedure is applied again to ERM using the sum of the row method. The SIMUS procedure uses normalized efficient results matrix (NERM) to determine the ranking of the alternatives. For this purpose, the sum of all elements in each column (SC) and the Participation Factor (PF) are consistently determined.

The sum of all elements in each column ($SCA_i; SCB_j$) is determined for each player as follows:

$$SCA_i = \sum_{k=1}^{m} \frac{x_{mi}}{\sum_{i=1}^{n1} x_{mi} + \sum_{j=1}^{n2} y_{mj}},\ i = 1, \ldots,\ n_1;\ j = 1, \ldots,\ n_2 \quad (2)$$

$$SCB_j = \sum_{k=1}^{m} \frac{x_{mj}}{\sum_{i=1}^{n1} x_{mi} + \sum_{j=1}^{n2} y_{mj}},\ i = 1, \ldots,\ n_1;\ j = 1, \ldots,\ n_2; \quad (3)$$

where x_{mi} represents the scores of each alternative for player A; y_{mj} represents the scores of each alternative for player B; SCA_i represents the sum of all elements in each strategy for player A; SCB_j represents the sum of all elements in each strategy for player B; and where $i = 1, \ldots,\ n_1; j = 1, \ldots,\ n_2; k = 1, \ldots, m$.

The number of participations of each alternative in each column of NERM represents the participation factor (PF). The number of participations of each strategy i of player A are PFA_i and the number of participations of each strategy j of player B are PFB_j.

The normalized values of the participation factor (NPF) are calculated dividing each value of PF $(PFA_i; PFB_j)$ by the total number of criteria m.

The scores of the strategies are determined as follows:

$$a_i = \frac{PFA_i}{m} \cdot SCA_i \quad (4)$$

$$b_j = \frac{PFB_j}{m} \cdot SCB_j \quad (5)$$

where a_i is the score of strategy i of player A; $i = 1, \ldots, n_1$ is the number of strategies for player A; b_j is the score of strategy j of player B; and $j = 1, \ldots, n_2$ is the number of strategies for player B.

The maximal values of all scores shows the best alternative.

3.3. Step 3: Model Formation in Game Theory—Determination of the Strategies for Both Players

In this step, a payoff matrix is formed and a mathematical representation of the game theory problem for both players is undertaken. The probabilities when applying the strategies for each player are determined using the dual simplex method.

The payoff matrix $(p_{ij})_{n1xn2}$ represents the game, as in Table 1.

The strategies for player A ($i = 1, \ldots, n_1$) are given by rows and the strategies for player B ($j = 1, \ldots, n_2$) are given by the columns. For example, p_{i1} is the profit of player A for his ith strategy and the first strategy for player B.

Table 1. Payoff matrix for game theory.

Player	**Probability**	B_1	B_2		B_j		B_{n2}
		y_1	y_2		y_j		y_{n2}
A_1	x_1	$p_{11} = a_1 - b_1$	$p_{12} = a_1 - b_2$	...	$p_{1j} = a_1 - b_j$	...	$p_{1n2} = a_1 - b_{n2}$
A_2	x_2	$p_{21} = a_2 - b_1$	$p_{22} = a_2 - b_2$	...	$p_{2j} = a_2 - b_j$	...	$p_{2n2} = a_2 - b_{n_2}$
...		...	...	...	...	...	...
A_i	x_i	$p_{i1} = a_i - b_1$	$p_{i2} = a_i - b_2$	...	$p_{ij} = a_i - b_2$		$p_{in2} = a_{n1} - b_{n_2}$
...		...	...	...	...	...	...
A_{n1}	x_{n1}	$p_{n11} = a_{n1} - b_1$	$p_{n12} = a_{n1} - b_2$	...	$p_{n1j} = a_2 - b_j$	...	$p_{n1n2} = a_{n1} - b_{n_2}$

The elements of the payoff matrix are formed using the results of the scores of the alternatives of both players received by the SIMUS method. The payoff matrix is made up of rows expressing the payoff of player A. As the SIMUS scores express the benefits of each strategy for both players, and as they have antagonistic interests, the profit of player A is reduced by the profit of player B. For example, p_{21} is the profit of player A for his second strategy and the first strategy for player B, i.e., the value of p_{21}, is calculated by reducing the benefit of the second strategy, a_2, for player A by the benefit of the first strategy, b_1, for B. The reduction of the benefits of all strategies of player A when player B uses his first strategy is found by subtracting the benefits of this strategy by the benefits of all of the strategies of player A. This is determined in order to account for the competition between the players. It is possible that some of the elements in the matrix are negative. This means that, in this case, there are losses. If all values in the payoff matrix are negative, then the problem is solved against player B. In this case the payoff matrix is determined by reducing the benefits of all of the strategies of player B by subtracting the benefits of player A.

Game theory indicates a two-person zero-sum game wherein the gain of one player is equal to the loss for the other. The game is of a mixed strategy when both players have several optimal strategies. In this case, dual linear optimization is applied to determine the strategies of both players. This makes it possible to determine the optimal strategies for both players simultaneously. The linear optimization model for player A involves maximizing their benefits, just as the linear optimization model for player B involves minimizing their losses. The value of the game is the same for both optimization problems.

For player A, the optimal mixed strategies are as follows:

$$\max_{x_i}\left\{\min\left(\sum_{i=1}^{n1} p_{i1}x_i,\ \sum_{i=1}^{n1} p_{i2}x_i, \ldots, \sum_{i=1}^{n1} p_{in2}x_i\right)\right\}, \quad (6)$$

$$x_1 + x_2 + \ldots + x_{n1} = 1, \quad (7)$$

$$0 \le x_i \le 1, i = 1, 2, \ldots, n_1 \quad (8)$$

where x_i represents the probability of player A using strategy A_i.

For player B, the optimal mixed strategies are as follows:

$$\min_{y_j}\left\{\max\left(\sum_{j=1}^{n2} p_{1j}y_j,\ \sum_{j=1}^{n2} a_{2j}y_j, \ldots, \sum_{j=1}^{n2} a_{mj}y_j,\right)\right\}, \quad (9)$$

$$y_1 + y_2 + \ldots + y_{n2} = 1, \quad (10)$$

$$0 \le y_j \le 1, j = 1, 2, \ldots, n_2 \quad (11)$$

where y_j represents the respective probabilities for strategy B_j for player B.

To be able to solve the indicated mathematical models, transformations are made in order to apply a dual simplex method.

For the model presented by Equation (6) the following transformation is made:

$$\zeta = \min\left(\sum_{i=1}^{n1} p_{i1}x_i,\ \sum_{i=1}^{n1} p_{i2}x_i, \ldots, \sum_{i=1}^{n1} p_{in2}x_i\right) \quad (12)$$

For the model presented by Equation (9) the following transformation is made:

$$\omega = \max\left(\sum_{j=1}^{n2} p_{1j}y_j,\ \sum_{j=1}^{n2} p_{2j}y_j, \ldots, \sum_{j=1}^{n2} y_j,\right) \tag{13}$$

Based on Equations (12) and (13), the problem for both players can be written as follows:

$$\text{Maximize } \zeta = \vartheta, \tag{14}$$

where ϑ represents the value of the game.

Subject to the following:

$$\left| \begin{array}{c} p_{11}x_1 + p_{12}x_2 + \ldots + p_{n11}x_{n1} \geq \vartheta \\ p_{12}x_1 + p_{22}x_2 + \ldots + p_{n12}x_{n1} \geq \vartheta \\ \ldots \\ p_{1n2}x_1 + p_{2n2}x_2 + \ldots + p_{n1n2}x_{n1} \geq \vartheta \end{array} \right. \tag{15}$$

and Equations (7) and (8).

$$\text{Minimize } \omega = \vartheta, \tag{16}$$

Subject to the following:

$$\begin{array}{c} p_{11}y_1 + p_{12}y_2 + \ldots + p_{1n}y_{n2} \leq \vartheta \\ p_{21}y_1 + p_{22}y_2 + \ldots + p_{2n}y_{n2} \leq \vartheta \\ \ldots \\ p_{n1}y_1 + p_{n12}y_2 + \ldots + p_{n1n2}y_{n2} \leq \vartheta \end{array} \tag{17}$$

and Equations (10) and (11).

Using additional transformations models (14)–(17) are represented by dual linear optimization.

The problem of player A is transformed as follows:

$$\text{Minimize } \zeta = X_1 + X_2 + \ldots + X_{n1} \tag{18}$$

Subject to the following:

$$\begin{array}{c} p_{11}X_1 + p_{21}X_2 + \ldots + p_{n11}X_{n1} \geq 1 \\ p_{12}xX_1 + p_{22}X_2 + \ldots + p_{n12}X_{n1} \geq 1 \\ \ldots \\ p_{1n2}X_1 + p_{2n2}X_2 + \ldots + p_{n1n2}X_{n1} \geq 1 \end{array} \tag{19}$$

$$X_i \geq 0, i = 1, 2, \ldots, n_2 \tag{20}$$

$$X_i = \frac{x_i}{\vartheta}; \vartheta = \frac{1}{\zeta} \tag{21}$$

The problem of player B is transformed as follows:

$$\text{Maximize } \omega = (Y_1 + Y_2 + \ldots + Y_n) \tag{22}$$

Subject to the following:

$$\begin{array}{c} p_{11}Y_1 + p_{12}Y_2 + \ldots + p_{1n}y_{n2} \leq 1 \\ p_{21}Y_1 + p_{22}Y_2 + \ldots + p_{2n}y_{n2} \leq 1 \\ \ldots \\ p_{n1}Y_1 + p_{n12}Y_2 + \ldots + p_{n1n2}y_{n2} \leq 1 \end{array} \tag{23}$$

$$Y_j \geq 0, j = 1, 2, \ldots, n_2 \tag{24}$$

$$Y_j = \frac{y_i}{\vartheta};\ \vartheta = \frac{1}{\omega} \tag{25}$$

The results for the game model are given by the solution of the transformed models.

4. Results

The elaborated new integrated SIMUS–game theory approach was applied to the decision making of strategies of railway and road operators for the carriage of containers. Both transport operators serve the in the direction from Sofia to Varna and have antagonistic interests, i.e., the rail operator's gain is the road operator's loss.

The limitations of this research are as follows:

- The players are competitors.
- The players have antagonistic interests.
- The transport is in 40-foot containers.
- The player benefits are determined by preset criteria.
- The strategies of both players are about the choice of the route between two points.

4.1. Step 1: Determination of the Strategies for Railway and Road Operator—Determination of the Criteria for Assessment of the Strategies

Let us assume that the railway operator is player A, and that the road operator is player B.

Player A has two strategies for the carriage of container block trains in the Sofia–Varna direction, which differ from each other on the route between Sofia and Varna. The first strategy (A1) is carriage along the Sofia–Gorna Oryahovitsa–Varna route; the second strategy (A2) is carriage along the Sofia–Karlovo–Karnobat–Varna route. A1 is served by a fully double-tracked railway line. A2 is, for the most part, a single-track railway, with some double-track sections. Figure 2 illustrates the strategies for both operators.

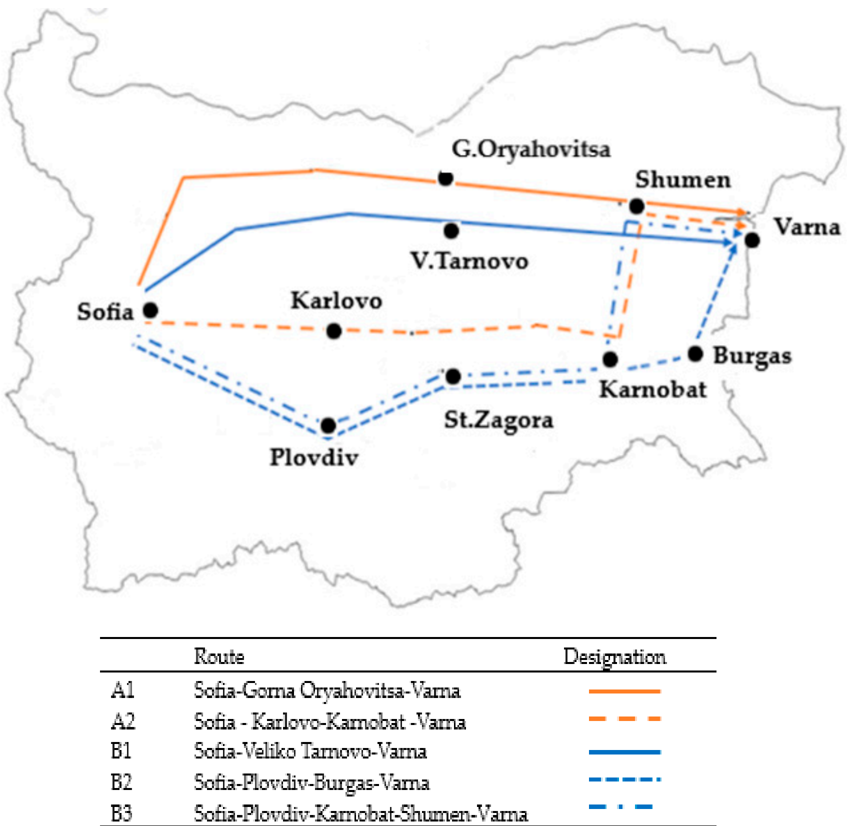

Figure 2. Scheme of the strategies for the railway and road operator.

It is assumed that the container block trains have 20 wagons with 40-foot containers and a gross mass of 20 t. The gross train weight is 1086 t. The container trucks each have load capacity of 24 tons and a gross weight of 36 tons.

Player B has three strategies for carriage by container trucks, called road trains in the Sofia–Varna direction which differ by route. The first strategy (B1) is carriage by road trains along the Sofia–Veliko Tarnovo–Varna route; the second strategy (B2) is carriage by road train along the Sofia–Plovdiv–Burgas–Varna route; the third strategy (B3) is carriage by road trains along the Sofia–Plovdiv–Karnobat–Shumen–Varna route.

Strategy B1 has about 280 km of secondary road. This road crosses northern Bulgaria and is well maintained throughout the year. The road passes through many populated areas. The speed of movement is limited, which makes maneuvering difficult.

Strategy B2 mainly involves motorway. There is 125 km of secondary road which is located in the eastern part of the country.

Strategy B3, compared with road B2, has approximately the same length of secondary road. This road passes through a mountainous section and has many sharp turns.

This study uses the criteria for the assessment of container carriages determined in [52], as follows:

C1—carbon dioxide emissions during transport, g/UTI;
C2—operational costs and fees for loading and unloading operations, EUR/UTI;
C3—charges for the use of railway and road infrastructure, EUR/UTI;
C4—duration of transportation, h;
C5—light on the route, km;
C6—infrastructure factor, coef. This describes the transport infrastructure and can have values between 1 and 2. C6 = 2 in the case of a two-track railway line or a motorway, while C6 = 1 in the case of a single-track railway line or a first-class/second-class road. When the infrastructure consists of mixed sections, the infrastructure factor is determined by taking into account its percentage, i.e., C6 = 1 + k, where k is a parameter that indicates the relative share of double-track sections or motorway for the route.
C7—security and safety, coef. This can have values of 0 or 1. When C7 = 1 this indicates a more secure and safe transport.

The criteria included in the study cover ecology (criterion C1—carbon dioxide emissions during transport), economic criteria related to operating costs and infrastructure fees (C2 and C3), technology (C4), infrastructure (C5 and C6), and security and safety (C7). It can be summarized that various aspects of achieving transport sustainability are covered—ecology, technology, infrastructure, costs and fees for transport, security and safety.

Based on the literature review it can be concluded that the criteria that refer to costs, transport time and carbon dioxide emissions during transport were used by most authors when they explored route selection. This study also applies criteria related to the type of infrastructure and security and safety. Considering the type of infrastructure is important, as it affects the speed and capacity of the transport.

In the first step of the methodology, initial decision matrices $A_{7\times2}$ and $B_{7\times3}$ are constructed for each player. Matrix A and matrix B contain the criteria values for each strategy of both players.

In order to have commensurability between the container truck and the container train composition, the data regarding the railway operator in the matrix represent one transport unit, i.e., one wagon (one container).

$$A_{7\times2} = \begin{pmatrix} 238,257.00 & 248,250.00 \\ 152.00 & 147.00 \\ 96.00 & 94.00 \\ 9.05 & 8.57 \\ 543.00 & 514.00 \\ 2.00 & 1.00 \\ 2.00 & 2.00 \end{pmatrix}; \; B_{7\times3} = \begin{pmatrix} 442,887.00 & 519,180.00 & 545,931.00 \\ 190.00 & 227.00 & 189.00 \\ 51.00 & 61.00 & 65.00 \\ 7.14 & 8.12 & 8.62 \\ 447.00 & 524.00 & 551.00 \\ 1.53 & 1.75 & 1.82 \\ 1.00 & 1.00 & 1.00 \end{pmatrix} \quad (26)$$

where $k = 1, \ldots, 7$ represents the number of criteria, $i = 1,\ 2$ represents the number of strategies for player A, and $j = 1,\ 2,\ 3$ represents the number of strategies for player B.

In this study were used data obtained from [52].

The values of carbon dioxide emissions during transport (C1) were determined while taking into account the consumption of electricity for the movement of the trains and the consumption of fuel for the movement of the trucks. The consumption of electricity for train movement was determined according to data provided by the Bulgarian State Railways Holding. The amount of carbon dioxide (CO_2) during train movement was determined while taking into account the amount of CO_2 t/MWh released by the power plants during the production of electricity. This research used a value of 0.460 t/MWh amount of CO_2, according to data from the National Statistics Institute of Bulgaria.

The data from the real test of the European Truck Challenge 2014 were used to calculate the amount of carbon dioxide released during the movement of a truck on the considered routes. The value of 49.54 g CO_2/ton.km were used. It is assumed that the trucks are Euro 6 standard.

The operational costs (C2) for a freight train were determined cost rates per locomotive kilometer and wagon kilometer. These rates include the costs for movement, depreciation costs, salary and social security costs. The study used a rate of 0.6 EUR/loc.km and rate of 0.1 EUR/wag.km for a freight wagon, according to Bulgarian State Railways Holding. The operating costs included the costs for electricity, depreciation costs, salary and social security costs, costs for maintenance and repair of rolling stock. The operating costs for a freight train with a composition of 20 wagons and 2 locomotives are 3.2 EUR/train.km.

The operational costs (C2) for a truck include fuel consumption, driver wages, taxes and insurance, the costs of tires, lubricants, maintenance, and other types of costs. Fuel consumption is close to 60% of a transport company's operating costs. The data from the European Truck Challenge 2014 were used to determine the value of operational costs based on average rate for variable costs of 0.467 EUR/km, and average rate for fixed costs of 0.223 EUR/km, for a total of 0.69 EUR/km. In this situation, fuel consumption is 60% of the operating costs, and the other groups of operating costs make up the remaining 40%.

The charges (C3) for the use of railway infrastructure were determined according to the tariff for infrastructure fees, determined by the National Railway Infrastructure Company of Bulgaria. The charges (C3) for the use of road infrastructure were determined by tariff for toll road charges.

The duration of the transportation by rail and by road (C4) were determined while taking into account the timetable of the trains and the permitted speed for trucks on the road infrastructure.

The light on the route by rail and by road (C5) was determined according to data on the sections in the transport infrastructure.

The infrastructure factor (C6) was determined regarding the type of transport infrastructure by sections of the transport network.

The value of the criterion security and safety (C7) is either 0 or 1. These values are determined based on analysis of freight train and truck accidents for the studied sections. In recent years, there have been no incidents with container trains on both railway routes. There are accidents with trucks on highways and road infrastructure. In general, rail transport is the safer mode of transport because it runs on a separate infrastructure. The trucks move on motorways and on road networks on which private cars also move.

4.2. Step 2: Application of the SIMUS Method for Determining the Benefits of the Strategies for Both Transport Operators

The next step of the methodology includes determination of the scores of both players by applying the SIMUS method.

Table 2 represents the initial decision matrix. The left-hand side (LHS) of the initial decision matrix presents the values of the criteria for the alternatives of both players. The last column shows the type of optimization for each criterion. Table 3 represents the normalized decision matrix. The sum of the rows is used.

Table 2. Initial decision matrix.

Criteria		Player A		Player B			Type
		Alternatives		Alternatives			
Symbol	Dimension	A1	A2	B 1	B2	B3	
C1	CO_2, g/UTI	238,257.00	248,250.00	442,887.00	519,180.00	545,931.00	min
C2	EUR/UTI	152.00	147.00	190.00	227.00	189.00	min
C3	EUR/UTI	96.00	94.00	51.00	61.00	65.00	min
C4	h	9.05	8.57	7.14	8.12	8.62	min
C5	km	543.00	514.00	447.00	524.00	551.00	min
C6	-	2.00	1.00	1.53	1.75	1.82	max
C7	-	2.00	2.00	1.00	1.00	1.00	max

Table 3. Normalized decision matrix.

Criteria		Player A		Player B			Type	Operator	RHS Values
		Alternatives		Alternatives					
Symbol	Dimension	A1	A2	B1	B2	B3			
C1	CO_2, g/UTI	0.12	0.12	0.22	0.26	0.27	Min	≥	0.12
C2	EUR/UTI	0.17	0.16	0.21	0.25	0.21	Min	≥	0.16
C3	EUR/UTI	0.26	0.26	0.14	0.17	0.18	Min	≥	0.14
C4	h	0.22	0.21	0.17	0.20	0.21	Min	≥	0.17
C5	km	0.21	0.20	0.17	0.20	0.21	Min	≥	0.17
C6	-	0.25	0.12	0.19	0.22	0.22	Max	≤	0.25
C7	-	0.29	0.29	0.14	0.14	0.14	Max	≤	0.29

The elements of the ERM represent the scores of each alternative.

The procedure includes successively constructing and solving linear optimizations targeting each of the criteria. The first optimization is drawn up with the first criterion considered as an optimization function. It is removed from the decision matrix. The restrictive conditions are considered by the other rows of the normalized matrix. This procedure is repeated successively with the other criteria. The results of all optimization models are structured in an efficient results matrix (ERM), as in Table 4.

Table 4. Efficient results matrix (ERM). Preliminary analysis.

Efficient Results Matrix						Preliminary Analysis				
Objective	Player A		Player B			Objective Function Values	RHS Values	Type	Objective Satisfied?	Comment
	Alternatives		Alternatives							
	A1	A2	B1	B2	B3					
Z1	0.97	0.00	0.00	0.00	0.00	0.12	0.12	Min	Yes	Satisfied 100%
Z2	0.68	0.00	0.00	0.00	0.14	0.14	0.16	Min	No	The objective function value is lower than the RHS value
Z3	0.00	0.00	1.00	0.00	0.00	0.14	0.14	Min	Yes	Satisfied 100%
Z4	0.00	0.00	0.00	0.85	0.00	0.17	0.17	Min	Yes	Satisfied 100%
Z5	0.00	0.53	0.36	0.00	0.00	0.17	0.17	Min	Yes	Satisfied 100%
Z6	0.00	0.00	0.00	0.00	2.00	0.45	0.25	Max	No	The objective function is greater than the RHS value
Z7	0.00	2.00	0.00	0.00	0.00	0.57	0.29	Max	No	The objective function is greater than the RHS value

Z1–Z7 are the objectives equivalent to criteria C1–C7.

The second part of Table 4 represents the values of the objective function for each of the optimization models, the right-hand side (RHS) values and the preliminary analysis of optimization. The preliminary analysis makes a comparison between the RHS values, and the results for objective functions obtained through the SIMUS method. The RHS presents the defined values for each objective for the criteria. The objective is satisfied 100% in the case of equality of the objective function values and RHS. Objective Z2, operating costs, which have to be minimized, shows that they are a little less (0.14) than the defined value (0.16). This could be due to the small amount of freight for railways, or the high frequencies of carriage of container trucks. Objectives Z6 and Z7 refer respectively to the state of infrastructure, and to security and safety, which must be maximum. The computed values of the objective function (0.45) and (0.57) are greater than the RHS values (0.25) and (0.29) in the original table. It can be concluded that the actual state of the infrastructure and security are higher than expected. This is positive for freight transport, but for transport company finances it indicates a strain on operating costs.

Table 5 shows the normalized efficient results matrix (NERM), and the determination of the scores of alternatives.

Table 5. Normalized efficient results matrix (NERM).

Objective	Player A		Player B		
	Alternatives		Alternatives		
	A1	A2	B1	B2	B3
Z1	1.00	0.00	0.00	0.00	0.00
Z2	0.83	0.00	0.00	0.00	0.17
Z3	0.00	0.00	1.00	0.00	0.00
Z4	0.00	0.00	0.00	1.00	0.00
Z5	0.00	0.59	0.41	0.00	0.00
Z6	0.00	0.00	0.00	0.00	1.00
Z7	0.00	1.00	0.00	0.00	0.00
SC (SCA_i; SCB_j)	1.83	1.59	1.41	1.00	1.17
PF (PFA_i; PFB_j)	2	2	2	1	2
NPF	0.29	0.29	0.29	0.14	0.29
SC × NPF	a_1	a_2	b_1	b_2	b_3
	0.52	0.46	0.40	0.14	0.33

SC—sum of values in the column; PF—participation factor; NPF—norm. participation factor; SC × NPF—final result. Number of targets = 7; ranking: A1–A2–B1–B5–B4.

4.3. Step 3: Model Formation in Game Theory

The next part of the methodology consists of the formation of the game theory model. The elements of the payoff matrix are structured based on the results of the scores of alternatives for both players, determined using the SIMUS method.

The payoff matrix is made up of rows for player A. The values of the matrix are calculated by reducing the profit of player A with the profit of player B for each row of payoff matrix. Table 6 shows the payoff matrix.

Table 6. Payoff matrix of SIMUS–Game Theory Approach.

Player	Probability	B_1 y_1	B_2 y_2	B_3 y_3
A_1	x_1	$p_{11} = a_1 - b_1 =$ $0.52 - 0.40 = 0.12$	$p_{12} = a_1 - b_2 =$ $0.52 - 0.14 = 0.38$	$p_{12} = a_1 - b_{n_3} =$ $0.52 - 0.33 = 0.19$
A_2	x_2	$p_{21} = a_2 - b_1 =$ $0.46 - 0.40 = 0.06$	$p_{12} = a_2 - b_2 =$ $0.46 - 0.14 = 0.32$	$p_{13} = a_2 - b_{n_2} =$ $0.46 - 0.33 = 0.13$

The mathematical representation of the game model is as follows:

For player A can be written as follows:

$$\text{Maximize } \zeta = \vartheta, \quad (27)$$

Subject to the following:

$$\begin{array}{c} 0.12 \cdot x_1 + 0.06 \cdot x_2 \geq \vartheta \\ 0.38 \cdot x_1 + 0.32 \cdot x_2 \geq \vartheta \\ 0.19 \cdot x_1 + 0.13 \cdot x_2 \geq \vartheta \\ x_1 + x_2 = 1 \end{array} \quad (28)$$

For player A can be written as follows:

$$\text{Minimize } \zeta = X_1 + X_2, \quad (29)$$

Subject to the following:

$$\begin{array}{c} 0.12 \cdot X_1 + 0.06 \cdot X_2 \geq 1 \\ 0.38 \cdot X_1 + 0.32 \cdot X_2 \geq 1 \\ 0.19 \cdot X_1 + 0.13 \cdot X_2 \geq 1 \\ X_1 + X_2 = 1 \\ X_i \geq 0, i = 1,2 \\ X_1 = \frac{x_1}{\vartheta};\ X_2 = \frac{x_2}{\vartheta};\ \vartheta = \frac{1}{\zeta} \end{array} \quad (30)$$

The problem of player B can be written as follows:

$$\text{Minimize } \omega = \vartheta, \quad (31)$$

Subject to the following:

$$\begin{array}{c} 0.12y_1 + 0.38y_2 + 0.19y_3 \leq \vartheta \\ 0.06y_1 + 0.32y_2 + 0.13y_3 \leq \vartheta \\ y_1 + y_2 + y_3 = 1, \\ 0 \leq y_j \leq 1, j = 1,2,3 \end{array} \quad (32)$$

The Game model for player B is as follows:

$$\text{Maximize } \omega = (Y_1 + Y_2 + Y_3) \quad (33)$$

Subject to the following:

$$\begin{array}{c} 0.12Y_1 + 0.38Y_2 + 0.19Y_3 \leq 1 \\ 0.06Y_1 + 0.32Y_2 + 0.13Y_3 \leq 1 \\ Y_j \geq 0, j = 1,2,3. \\ Y_1 = \frac{y_1}{\vartheta};\ Y_2 = \frac{y_2}{\vartheta}; Y_3 = \frac{y_3}{\vartheta}; \vartheta = \frac{1}{\omega} \end{array} \quad (34)$$

The game models were solved by means of dual linear programming. The results are given in Table 7. For player A, the optimal strategy is A1, while for player B, the optimal strategy is B1. Player A will apply their first strategy with probability 1, player B will also apply their first strategy with probability 1. The value of the game is 0.12. This means that both players with antagonistic interests will have a profit if they implement their optimal strategies.

Table 7. Results of the game models of SIMUS–Game Theory Approach.

Player A			Player B			
Probabilities		**Value of the Game**	**Probabilities**			**Value of the Game**
x_1 8.33	x_2 0.00	ζ 8.33	y_1 8.33	y_2 0.00	y_3 0.00	ω 8.33
$X_1 = \frac{x_1}{\vartheta}$ 1.00	$X_2 = \frac{x_2}{\vartheta}$ 0.00	$\vartheta = \frac{1}{\zeta}$ 0.12	$Y_1 = \frac{y_1}{\vartheta}$ 1.00	$Y_2 = \frac{y_2}{\vartheta}$ 0.00	$Y_3 = \frac{y_3}{\vartheta}$ 0.00	$\vartheta = \frac{1}{\omega}$ 0.12

5. Discussion

5.1. Verification of the Results

The new integrated SIMUS–game theory approach for decision making elaborated herein permits us to determine the strategies of both players when they have antagonistic interests.

The results were verified by applying the following four criteria for decision making: Laplace's criterion, minimax and maximin criteria, Savage's criterion and Hurwitz's criterion [51]. These criteria serve to determine the optimal strategies for each of the players. The payoff matrix given in Table 6 is used to calculate the values of the criteria.

The goal of player A is to choose a strategy with maximum utility. The value of Laplace's criterion is determined as follows:

$$L_i = \frac{\sum_{j=1}^{n2} p_{ij}}{n_2},\ i = 1, \ldots, n_1, \quad (35)$$

where $i = 1, \ldots, n_1$ is the number of strategies for player A and p_{ij} represents the elements of the payoff matrix.

The maximum value of Laplace's criterion determines the optimal alternative for player A, as follows:

$$L_{opt} = \max_i L_i. \quad (36)$$

The goal of player B is to reduce their losses, as both players have antagonistic interests. In this case, the value of Laplace's criterion is determined as follows:

$$L_j = \frac{\sum_{i=1}^{n1} p_{ij}}{n_1},\ i = 1, \ldots, n_1, \quad (37)$$

where $j = 1, \ldots, n_2$ is the number of strategies for player B and p_{ij} represents the elements of the payoff matrix.

The maximin criterion expresses the choice of the worst option among the best options. The optimal alternative for player A is determined by the maximin criterion, as follows:

$$L_{opt} = \min_j L_j,\ j = 1, \ldots, n_2. \quad (38)$$

The maximin criterion expresses the choice of the worst option among the best options. The optimal alternative for player A is determined by the maximin criterion, as follows:

$$\max_i \left\{ \min_j (p_{ij}) \right\},\ i = 1, \ldots, n_1; \mathrm{j} = 1, \ldots . n_2 \quad (39)$$

The minimax criterion expresses the choice of the best option among the worst options. The optimal alternative for player B is determined by the minimax criterion, as follows:

$$\min_j \left\{ \max_i (p_{ij}) \right\},\ i = 1, \ldots, n_1; \mathrm{j} = 1, \ldots . n_2. \quad (40)$$

Savage's criterion is determined based on the newly formed matrix with the following elements:

$$r_{ij} = r_{ij} - \min_j (p_{ij}),\ \text{when } p_{ij} \text{ represents the costs} \quad (41)$$

$$r_{ij} = \max_i (p_{ij}) - p_{ij}, \text{when } p_{ij} \text{ represents the profits} \quad (42)$$

The newly formed matrix represents a loss. The minimax criterion is applied to make a decision. For this purpose, the maximum costs by rows for each strategy of the new-formed matrix are determined. The optimal strategy is that for which the maximum cost is the smallest. The minimax criterion for both players is determined as follows:

$$\min_i \left\{ \max_j (r_{ij}) \right\} \text{for player A} \tag{43}$$

$$\min_j \left\{ \max_i (r_{ij}) \right\}, \text{for player B} \tag{44}$$

Hurwitz's criterion uses the minimum and the maximum values of the rows to make a decision. The optimal strategy is determined as follows:

$$H_{opt} = \max_i \left\{ \alpha \max_j p_{ij} + (1 - \alpha) \min_j p_{ij} \right\}, \text{when } p_{ij} \text{ represents the profits} \tag{45}$$

$$H_{opt} = \min_j \left\{ \alpha \min_j p_{ij} + (1 - \alpha) \max_j p_{ij} \right\}, \text{when } p_{ij} \text{ represents the costs} \tag{46}$$

where α is a coefficient of optimism, $0 \leq \alpha \leq 1$. Generally, $\alpha = 0.5$.

The strategy with the highest profit is the aim for player A. Table 8 represents the values of the criteria for verification for player A.

Table 8. Parameters of criteria for verification of player A.

Player A	Decision Matrix			Laplace's Criterion	Maximin Criterion	r_{ij}			Savage's Criterion	Max	Min	Hurwitz's Criterion
	Player B											
	B1	B2	B3			B1	B2	B3				
A1	0.12	0.23	0.38	0.19	0.12	0	0	0	0	0.38	0.12	0.25
A2	0.06	0.17	0.32	0.13	0.06	0.06	0.06	0.06	0.06	0.32	0.06	0.19

Player B aims to implement the strategy with the least loss. The decision matrix for player B is structured as a transposed matrix of player A, because the profit of player A is a loss for player B. Table 9 represents the values of the criteria for verification for player B. Both operators have opposite interests.

Table 9. Parameters of criteria for verification for player B.

Player B	Decision Matrix		Laplace's Criterion	Minimax Criterion	r_{ij}		Savage's Criterion	Max	Min	Hurwitz's Criterion
	Player A									
	A1	A2			A1	A2				
B1	0.12	0.09	0.06	0.12	0	0	0	0.12	0.06	0.09
B2	0.38	0.35	0.32	0.06	0.26	0.26	0.06	0.38	0.32	0.35
B3	0.19	0.16	0.13	0.12	0.07	0.07	0.07	0.19	0.13	0.16

The results show that the optimal strategy for player A is A1 since it has a maximum value for Laplace's criterion, maximin criterion and Hurwitz's criterion, and minimum value for Savage's criterion. The optimal strategy for player B is strategy B1 since it has a minimum value for Laplace's criterion, minimax criterion, Savage's criterion and Hurwitz's criterion.

Figures 3 and 4 illustrate the results for all criteria for verification.

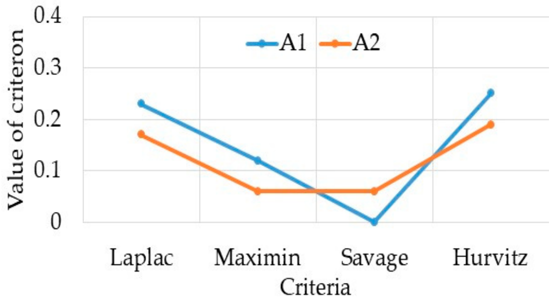

Figure 3. Criteria for verification. Player A.

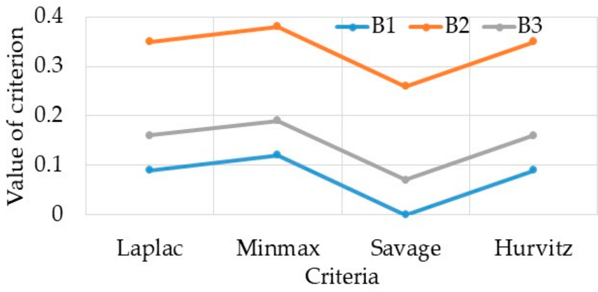

Figure 4. Criteria for verification. Player B.

It can be concluded that the results obtained by using the criteria of verification give the same choice of strategies for both players as those are obtained through game theory.

The verification of the results confirms the hypothesis of this research about the possibility of decision making by means of the new integrated SIMUS–game theory approach elaborated on herein.

5.2. Calculation of the Weights of Criteria

The SIMUS method allows one to determine the weights of criteria based on the results in the normalized ERM matrix. It can be used only for analysis. Further procedures with the weights are not carried out. The number of columns in the NERM matrix is calculated as a sum of strategies for player A and player B $(n_1 + n_2)$ and the number of rows correspond to the number of criteria. The maximum value of each row $\max_q NERM_{kq}$ is determined.

The weights are determined as follows:

$$w_k = \frac{\max_q NERM_{kq}}{\sum_{k=1}^{m} NERM_{kq}}, \tag{47}$$

$$0 \leq w_k \leq 1, \tag{48}$$

$$\sum_{k=1}^{m} w_k = 1, \tag{49}$$

where $k = 1, \ldots, m$ is the number of criteria, $q = 1, \ldots, n_1 + n_2$ is the total number of strategies for player A and player B, n_1 is the number of strategies for player A, and n_2 is the number of strategies for player B.

Table 10 presents the NERM matrix and the weights of the criteria. The criteria with the greatest impact are those of carbon dioxide emissions (C1), infrastructure charges for the use of railway and road infrastructure (C3), duration of transportation (C4), infrastructure factor (C6), and security and safety (C7), as in Figure 5.

Table 10. Weights of objectives obtained using the SIMUS method.

Objective	**Player A Alternatives**		**Player B Alternatives**			$\max_q NERM_{kq}$	w_k
	A1	**A2**	**B1**	**B2**	**B3**		
Z1	1.00	0.00	0.00	0.00	0.00	1.00	0.16
Z2	0.83	0.00	0.00	0.00	0.17	0.83	0.13
Z3	0.00	0.00	1.00	0.00	0.00	1.00	0.16
Z4	0.00	0.00	0.00	1.00	0.00	1.00	0.16
Z5	0.00	0.59	0.41	0.00	0.00	0.59	0.09
Z6	0.00	0.00	0.00	0.00	1.00	1.00	0.16
Z7	0.00	1.00	0.00	0.00	0.00	1.00	0.16
Total						6.42	1.00

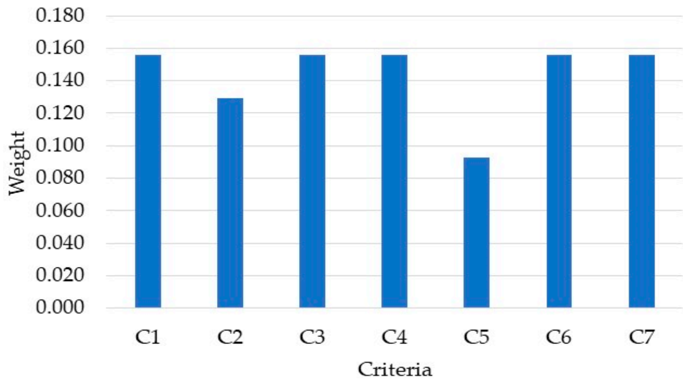

Figure 5. Weights of criteria determined using the SIMUS method.

The determined values of weights could serve a decision maker only for analysis and not for consequent actions.

5.3. Comparison with the Game Cost-Based Model

In order to demonstrate the advantage of the new integrated SIMUS–game theory methodology, a comparison of the obtained results is made with an approach for applying a game theory model based only on costs. For this purpose, the data for operational costs and fees for loading and unloading operations (C2) presented in Table 2 were used. As the costs of player B regarding all of his strategies are greater than those of player A, the payoff matrix was formed for player B. This is shown in Table 11. The results for the game cost-based model is represented in Table 12.

Table 11. Payoff matrix of game model by costs.

Player	Costs EUR/UTI	Probability	A_1	A_2
			152	**147**
			x_1	x_2
B_1	190	y_1	$190 - 152 = 38$	$170 - 147 = 43$
B_2	227	y_2	$227 - 152 = 75$	$227 - 147 = 80$
B_3	189	y_3	$189 - 152 = 37$	$189 - 147 = 42$

Table 12. Results of game models by costs.

Player A			Player B			
Probabilities		**Value of the Game**	**Probabilities**			**Value of the Game**
x_1 0.00	x_2 0.024	ζ 0.024	y_1 8.33	y_2 0.00	y_3 0.024	ω 0.024
$X_1 = \frac{x_1}{\vartheta}$ 0.00	$X_2 = \frac{x_2}{\vartheta}$ 1.00	$\vartheta = \frac{1}{\zeta}$ 42	$Y_1 = \frac{y_1}{\vartheta}$ 0.00	$Y_2 = \frac{y_2}{\vartheta}$ 0.00	$Y_3 = \frac{y_3}{\vartheta}$ 1.00	$\vartheta = \frac{1}{\omega}$ 42

It can be seen that the solution in this case shows that the optimal strategy for player A is the second one, while for player B the optimal strategy is the third one. The results are different compared with those of the integrated SIMUS–game theory approach. This is due to the fact that only costs are used, and not a set of criteria with their total benefit.

When the game theory is used by itself, the strategies of both players are determined only in terms of the revenue or cost criteria. The ranking of the alternatives of both players is performed, and their strategies are considered jointly. The best alternative among all of the strategies of both players is thus determined.

5.4. Sensitivity Analysis Using SIMUS

The SIMUS method utilizes linear programming. This makes it possible to define, for each optimization, the limits of the values of the criteria at which the solution is preserved.

For each objective we performed a sensitivity analysis. The allowable range of variation of criteria were thus determined. Table 13 shows the upper limit (U) and the lower limit (L) for each of the criteria determined by SIMUS.

Table 13. Sensitivity analysis using SIMUS.

Objectives		**Player A**		**Player B**		
		A1	**A2**	**B1**	**B2**	**B3**
C1	U	∞	∞	∞	∞	∞
		238,257.00	248,250.00	442,887.00	519,180.00	545,931.00
	L	0	230,420	297,821	355,818	296,254
C2	U	152	∞	∞	∞	195
		152.00	147.00	190.00	227.00	189.00
	L	82	146	153	180	154
C3	U	∞	∞	52	∞	∞
		96.00	94.00	51.00	61.00	65.00
	L	62	59	0	60	63
C4	U	∞	∞	7.26	∞	∞
		9.05	8.57	7.14	8.12	8.62
	L	5	5	1	8	8
C5	U	∞	516	466	∞	∞
		543.00	514.00	447.00	524.00	551.00
	L	541	420	428	511	528
C6	U	2	2	2	2	2
		2.00	1.00	1.53	1.75	1.82
	L	1	1	1	1	1
C7	U	2	2	2	2	2
		2.00	2.00	1.00	1.00	1.00
	L	1	1	1	1	1

It can be seen in Table 13 that some values are unlimited or equal to zero. These values are only theoretical.

The following conditions have to be taken into account:

- The limits for the criterion "carbon dioxide emissions" (C1) depends on the electricity consumption for train movement and fuel consumption for truck movement. It is also related to the permissible speed for movement in the sections, the type of cargo transported, the weight of the cargo, and the restrictions in the infrastructure.
- The limits of the criterion "operational costs" (C2) depend on the cost rates for fixed and variable costs, depreciation of the rolling stock, operating personnel, and the characteristics of the type of transport.
- The limits of the criterion "infrastructure charges" (C3) depend on the state's policy and the state of the transport infrastructure.
- The limits of the criterion "duration of transportation" (C4) depend on the restrictions on the speed of movement in the sections of the transport infrastructure, the state of the infrastructure, and the traffic load.
- The value of the criterion "light on the route" (C5) cannot be changed. It cannot be reduced or increased as no changes are planned in the studied routes. The upper and the lower limits are only theoretical.
- The upper limit for both of the criteria C6 and C7 is 2. The lower limit is 1.

The obtained values for the criteria limits could also be used for analysis in cases when the criteria values are not precisely set. If some or all of the criteria are changed within the established limits, the obtained decision for the scores of the alternatives does not change.

5.5. Comparison with the SIMUS Ranking Model

The results of the newly integrated approach can be compared with the results obtained using the SIMUS method. For this purpose, Tables 5 and 7 are considered. The last row in Table 5 shows the ranking of the strategies of the players when all strategies are considered together.

It should be noted that the SIMUS method aims to rank the alternatives and not to determine the optimal strategies for each of the players. It can be seen that the most appropriate route selection strategy is A1, i.e., travel by carriage via the Sofia–Gorna Oryahovitsa–Varna railway route, as in Table 5. This strategy is also optimal for the railway operator according to the results obtained through the new approach. If the ranking obtained by SIMUS is analyzed, it can be seen that, out of all strategies of the road operator, strategy B1 has the highest rating, i.e., carriage by container trucks along the Sofia–Veliko Tarnovo–Varna route. When analyzed separately, the rail and road operator rankings show which strategies are most appropriate for each of them. This procedure can also be used to verify the results obtained through the new integrated approach. The independent application of the SIMUS method is not appropriate in case of antagonistic interests between two transport operators. Although the ranking is based on the overall utility of the quantitative and qualitative criteria used, this method does not show what the cost of the game is and what is the probability that each of the players will use each of their strategies under antagonistic interests. This is achieved with the newly integrated SIMUS–game theory approach proposed in this research.

5.6. Comparison of the Results

A comparison of the three procedures—individual application of the SIMUS method, game theory using a costs-based model and the integrated SIMUS–game theory approach—is shown in Figure 6. The most suitable alternative among all investigated alternatives of both players is determined.

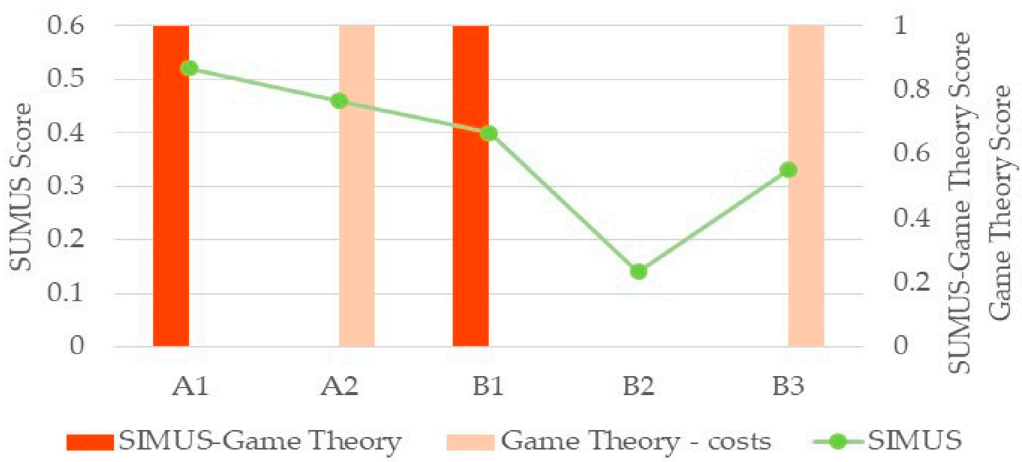

Figure 6. Comparison of the results using the SIMUS method, SIMUS–game theory and game theory by costs.

The results obtained through the game theory cost-based model show that the best alternative for the railway operator is the use of container block trains on the Sofia–Karlovo–Karnobat–Varna route (A2). This transport service offers minimal operating costs and transport time, but the carbon dioxide emissions are larger compared with railway route A1. The proposed strategy for the road operator is carriage by container trucks on the

Sofia–Plovdiv–Karnobat–Shumen–Varna route (B3). This alternative offers comparable costs with route B1, but time travel and carbon dioxide emissions are greater.

It can be seen in Figure 6 that the SIMUS ranking is A1 > A2 > B1 > B3 > B2. The most appropriate strategy in this case is A1 (container block trains on the Sofia–Gorna Oryahovitsa–Varna route).

The newly integrated SIMUS–game theory approach determines the strategies of each of the players, in this case A1 for player A and B1 for player B; each of these will implement their strategy with a probability equal to 1. This means that, for the railway operator, the optimal strategy is carriage by container block trains on the Sofia–Gorna Oryahovitsa–Varna route, while, for the road operator, the optimal strategy is carriage by container trucks on the Sofia–Veliko Tarnovo–Varna route. For the railway operator this result shows minimal carbon dioxide emissions, comparable operating costs and travel times with the alternative rail transport strategy (Sofia–Karlovo–Karnobat–Varna). For the road operator the result shows carriage with minimal carbon dioxide emissions, operating costs close to the minimum and the shortest transport time. These results demonstrate the overall benefits of the investigated criteria. It can be said that the results obtained through the new integrated SIMUS–game theory approach are better when compared with those obtained though the game theory cost-based model regarding operating costs, time during and carbon dioxide emissions.

5.7. Concept of Application in the Case of Games with Imperfect Information: Fuzzy SIMUS–Game Theory Approach

There are many cases when the information regarding the criteria values of the two players cannot be precisely determined. In this situation, the studied system is in a state of uncertainty. In these cases, the proposed new integrated approach SIMUS–game theory can be extended by applying a fuzzy SIMUS method. The application is also for non-cooperative games. The fuzzy SIMUS method is elaborated in [53]. The steps of the integrated fuzzy SIMUS–game theory approach are as follows:

- For each player, three initial decision matrices with the values of the criteria for each alternative are set—lower (L), medium (M), and upper (U).
- An average decision matrix is formed. Its elements are calculated as an average value by using the lower, medium, and upper values.
- Determination of the normalized average, upper and lower matrices.
- Determination of the objective function per criterion for both lower and upper matrices.
- Fuzzy linear optimization with linear membership function. For each objective, the optimization is formed and calculated sequentially based on the average decision matrix.
- The results of fuzzy linear models are recorded in fuzzy efficient results matrix.
- The SIMUS procedure ranking is then applied.
- The payoff matrix for game model is formed as in the SIMUS–game theory approach.

The applicability of the new fuzzy SIMUS–game theory approach is shown below for the case study from point 4 under uncertainty. Tables 14 and 15 represent the upper and lower matrices. It can be assumed that the matrix shown in Table 2 is a medium matrix. Criteria C5, C6 and C7 are set with constant values in the matrices. The criterion C5 presents the light of the route, which is constant for each route, criterion C6 presents the infrastructure factor, and criterion C7 presents the security. The changes in carbon dioxide values can be explained by changes in load weight, electricity consumption, and changes in speed of movement. The changes in operating costs and tariffs can be explained by changes in prices and government policy.

The SIMUS method was calculated separately for each of the given matrices. Table 15 shows the results for objective functions for both upper and lower matrices $(Z_{i,U},\ Z_{i,L})$ and the threshold values $(RHS_{j,U},\ RHS_{j,L})$. These results were used in fuzzy linear models for membership functions. For objectives Z5, Z6 and Z7 we applied the SIMUS linear procedure by using the average matrix. This is because only one value is set for criteria C5, C6 and C7.

Table 14. Upper decision matrix.

Criteria		Player A		Player B			Type
		Alternatives		Alternatives			
Symbol	Dimension	A1	A2	B1	B2	B3	
C1	CO_2, g/UTI	262,083	273,075	487,176	571,098	600,524	min
C2	EUR/UTI	167	162	209	250	208	min
C3	EUR/UTI	101	99	54	64	68	min
C4	h	9.2	9.15	7.25	8.3	9	min
C5	km	543	514	447	524	551	min
C6	-	2	1	1.53	1.75	1.82	max
C7	-	2	2	1	1	1	max

Table 15. Lower decision matrix.

Criteria		Player A		Player B			Type
		Alternatives		Alternatives			
Symbol	Dimension	A1	A2	B1	B2	B3	
C1	CO_2, g/UTI	214,431	223,425	398,598	467,262	491,338	min
C2	EUR/UTI	137	132	171	204	170	min
C3	EUR/UTI	91	89	48	58	62	min
C4	h	9	8.55	7.1	8.1	8.55	min
C5	km	543	514	447	524	551	min
C6	-	2	1	1.53	1.75	1.82	max
C7	-	2	2	1	1	1	max

Fuzzy linear models were formed for each criterion. For this purpose, the average decision matrix was used. The results of the SIMUS method for normalized lower and upper matrices are applied to build the optimization models. The fuzzy linear models are solved by linear membership function.

The efficient results fuzzy matrix (ERFM) presents the results of fuzzy linear models. The first part of Table 16 shows the results of FERM and the values of objectives. The rows represent the scores of the alternatives obtained by optimization models. The second part of Table 17 represents the results of normalized ERFM and the SIMUS scores.

Table 16. Results of the SIMUS method for normalized lower and upper matrices.

Objective	$RHS_{j,U}$	$Z_{i,U}$	$RHS_{j,L}$	$Z_{i,L}$	$\frac{Z_{i,U}}{Z_{i,U}-Z_{i,L}}$	$\frac{Z_{i,L}}{Z_{i,U}-Z_{i,L}}$	$\frac{RHS_{j,U}}{RHS_{j,U}-RHS_{j,L}}$	$\frac{RHS_{j,L}}{RHS_{j,U}-RHS_{j,L}}$
Z1	0.133	0.133	0.126	0.127	147.916	146.92	490,849.91	490,848.91
Z2	0.102	0.095	0.105	0.097	367.193	368.19	333.00	332.00
Z3	0.116	0.116	0.117	0.119	71.182	70.18	71.18	70.18
Z4	0.112	0.119	0.113	0.121	0.059	1.06	0.07	1.07
Z5	0.104	0.099	0.104	0.097	-	-	-	-
Z6	0.167	0.169	0.167	0.169	-	-	-	-
Z7	0.120	0.120	0.124	0.126	-	-	-	-

The results of fuzzy SIMUS are applied to form the payoff matrix, which is made up of rows for player A. The values of the matrix are calculated by reducing the profit of player A with the profit of player B for each row of payoff matrix. Table 18 shows the payoff matrix. The game models were formed using the payoff matrix. Table 19 shows the results.

Table 17. Normalized efficient results fuzzy matrix (NERFM). Results of fuzzy SIMUS score.

Objective	Efficient Results Fuzzy Matrix					Normalized Efficient Results Matrix				
	Player A		Player B			Player A		Player B		
	Alternatives		Alternatives			Alternatives		Alternatives		
	A1	**A2**	**B1**	**B2**	**B3**	**A1**	**A2**	**B1**	**B2**	**B3**
Z1	0.000	0.870	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
Z2	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
Z3	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
Z4	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
Z5	0	0.53	0.36	0	0	0.000	0.596	0.404	0.000	0.000
Z6	0	0	0	0	2	0.000	0.000	0.000	0.000	1.000
Z7	0	2	0	0	0	0.000	1.000	0.000	0.000	0.000
SC (SCA_i; SCB_j)	-	-	-	-	-	3.000	2.596	0.404	0.000	1.000
PF (PFA_i; PFB_j)	-	-	-	-	-	3	4	1	0	1
NPF	-	-	-	-	-	0.429	0.571	0.143	0.000	0.143
SC × NPF	-	-	-	-	-	a_1	a_2	b_1	b_2	b_3
						1.286	1.483	0.058	0.000	0.143

Table 18. Payoff matrix of Fuzzy SIMUS–Game Theory Approach.

Player	Probability	B_1 y_1	B_2 y_2	B_3 y_3
A_1	x_1	$p_{11} = a_1 - b_1 = 1.228$	$p_{12} = a_1 - b_2 = 1.286$	$p_{12} = a_1 - b_{n_3} = 1.143$
A_2	x_2	$p_{21} = a_2 - b_1 = 1.054$	$p_{12} = a_2 - b_2 = 1.112$	$p_{13} = a_2 - b_{n_2} = 0.969$

Table 19. Results of the game models of Fuzzy SIMUS–Game Theory Approach.

Player A			Player B			
Probabilities		Value of the Game	Probabilities			Value of the Game
x_1 0.875	x_2 0.000	ζ 0.875	y_1 0.00	y_2 0.00	y_3 0.875	ω 0.746
$X_1 = \frac{x_1}{\vartheta}$ 1.00	$X_2 = \frac{x_2}{\vartheta}$ 0.00	$\vartheta = \frac{1}{\zeta}$ 1.14	$Y_1 = \frac{y_1}{\vartheta}$ 0.00	$Y_2 = \frac{y_2}{\vartheta}$ 0.00	$Y_3 = \frac{y_3}{\vartheta}$ 1.00	$\vartheta = \frac{1}{\omega}$ 1.14

It can be seen that the optimal strategy for player A is A1, the optimal strategy for player B is B3. The value of the game is 1.14. This means that both players with antagonistic interests will have a profit if they implement their optimal strategies. It can be seen that the results are different from those in the case of certainty obtained by the SIMUS–game theory approach.

The change in results can be explained by the changes in the values of the criteria. Some upper limits of criteria values determined by the linear optimization models using the SIMUS method are exceeded in the upper decision matrix, Table 12.

6. Conclusions

The contributions of the conducted research can be summarized in the following aspects:

(1) Regarding decision theory—A new integrated approach to determine the optimal strategies of two players in non-cooperative games is developed, in which the payoff matrix of the game model consists of the benefits of quantitative and qualitative criteria for evaluating the strategies determined using the SIMUS multicriteria analysis method. This approach can be applied in various fields of research. A concept has

been developed for the case of uncertainty, where the fuzzy SIMUS–game theory approach is applied.

(2) For society—The proposed approach can be used in various fields of research. The article examines a case for the Bulgarian transport network for choosing a route for the transport of containers between Sofia and Varna by competing railway and road operators. Applying the proposed approach allows the two transport operators to choose their optimal route strategies so that they can both benefit. This includes not only operating costs, but also environmental, technological, infrastructural criteria, as well as security and safety of transportation.

(3) Regarding the transport technique—The study examines two types of transport, rail and road, which are in competition with antagonistic interests, and the corresponding transport technique, container trains and container trucks. The application of the elaborated methodology allows both types of transport to be distributed along efficient routes, taking into account the benefit of a set of criteria applied in the study. This approach allows both competing operators to carry out transportation along a route that will bring them equal benefit. The use of different types of transport on parallel lines between the starting point and the final point allows for adequate use of the transport infrastructure and transport technique.

(4) Regarding the researched example of the Bulgarian transport network—The strategies for railway and road operator for the carriage of containers between Sofia and Varna were obtained based on the new integrated SIMUS–game theory approach. It was determined that the proposed strategies for both operators determined by the new integrated approach are better when compared with those obtained by the game theory cost-based model concerning operating costs, time travel and carbon dioxide emissions. For the railway operator, the proposed route offers minimal carbon dioxide emissions and operating costs and travel times that are close to the minimum values. For the road operator the result shows that carriage has minimal carbon dioxide emissions, operating costs that are close to minimum, and the shortest transport time.

This paper creates new scientific knowledge regarding decision making using the SIMUS method and non-cooperative games in a new approach decision making.

The advantages of the new integrated SIMUS–game theory approach can be summarized as follows:

- It allows for strategies of competing players to be determined taking into account the benefits of criteria affecting the transport process.
- The integration of the game theory method with the SIMUS method permits us to take into account different criteria that influence decision making about the optimal strategies for each of the players. In this case, the payment matrix for game theory is compiled taking into account the benefits of all criteria determined through the SIMUS method.
- In this research, the SIMUS method is chosen among numerous MCDM to be integrated with game theory because it uses linear optimization, the ranking is based on benefits, criteria weights are not used, expert evaluations are not used, and for this reason there is no subjectivity in decision making.
- The combination of the SIMUS method and game theory allow one to assess the weights of criteria in the formation of the common benefit. Thus, it can assess which criteria have the greatest influence on the total benefit of all criteria in determining the strategies of the competing players. The calculation of the weights of the criteria can be used only for analysis by the decision maker after performing the optimization procedures. The criteria weights are not used in subsequent operations.
- The individual application of the SIMUS method does not allow one to determine what the probability is for each of the players to apply their most appropriate strategy in the case of antagonistic interests. It ranks only alternatives based on benefits. The combination of the SIMUS method and game theory makes it possible to determine

the probabilities of the application of the most suitable strategies for both players with antagonistic interests, taking into account the benefits of a set of criteria.

The new methodology was applied for sustainable decision making for a railway and a road operator, with antagonistic interests, for the carriage of containers on the Bulgarian transport network. Seven criteria were applied to assess the benefits for both players. The strategies for both operators were investigated. The results were verified using four criteria for decision making.

The new integrated SIMUS–game theory methodology can be applied in different areas of research, when the strategies for two players in non-cooperatives games have to be established. The procedure helps the decision maker to establish those strategies for the players that will bring them benefits. The benefits are established through the overall impact of various criteria.

In future work the new integrated approach will be extended to the case of cooperative games. This is of interest because many non-antagonistic conflicts allow participants to cooperate to maximize their profits.

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References

1. Neumann, J.; Morgenstern, O. *Theory of Games and Economic Behavior*; 60th Anniversary Commemorative Edition (Princeton Classic Editions); Princeton University Press: Princeton, NJ, USA, 2004; 776p.
2. Nash, J. Non-Cooperative Games. *Ann. Math.* **1951**, *54*, 286–295. [CrossRef]
3. Crawford, V.P. John Nash and the analysis of strategic behavior. *Econ. Lett.* **2002**, *75*, 377–382. [CrossRef]
4. Munier, N. *A Strategy for Using Multi-Criteria Analysis in Decision-Making*; Springer: Berlin/Heidelberg, Germany, 2011.
5. Chatterjee, K.; Samuelson, W. (Eds.) Game Theory and Business Applications. In *International Series in Operations Research & Management Science*, 2nd ed.; Springer: New York, NY, USA, 2014; p. 406. [CrossRef]
6. Hollander, Y.; Prashker, J.N. The applicability of non-cooperative game theory in transport analysis. *Transportation* **2006**, *33*, 481–496. [CrossRef]
7. Zhang, H.; Su, Y.; Peng, L.; Yao, D. A review of game theory applications in transportation analysis. In Proceedings of the 2010 International Conference on Computer and Information Application, Tianjin, China, 3–5 December 2010; pp. 152–157. [CrossRef]
8. Bukvic, L.; Škrinjar, J.P.; Abramovic, B.; Zitrický, V. Route Selection Decision-Making in an Intermodal Transport. Network Using Game Theory. *Sustainability* **2021**, *13*, 4443. [CrossRef]
9. Bell, M.G.H. A game theory approach to measuring the performance reliability of transport networks. *Transp. Res. Part B* **2000**, *34*, 533–545. [CrossRef]
10. Prokić, M. The applicability of cooperative game theory in rail freight corridors framework. In Proceedings of the 3rd International Interdepartmental Conference "Shaping Post-COVID World—Challenges for Economic Theory and Policy", Belgrade, Serbia, 14–15 September 2022; 2023, ISBN 978-86-403-1780-1. Available online: https://www.ekof.bg.ac.rs/wp-content/uploads/2023/01/Shaping-Post-COVID-World.pdf (accessed on 1 July 2024).
11. Adler, N.; Brudner, A.; Proost, S. A review of transport market modeling using game-theoretic principles. *Eur. J. Oper. Res.* **2021**, *291*, 808–829. [CrossRef]
12. Abboud, S.F. Using Game Theory to Determine the Optimal Strategy for the Transportation Sector in Iraq. *J. Econ. Adm. Sci.* **2022**, *28*, 157–173.
13. Koryagin, M. Urban Planning: A Game Theory Application for the Travel Demand Management. *Period. Polytech. Transp. Eng.* **2018**, *46*, 171–178. [CrossRef]
14. Emami, M.; Haghshenas, H.; Talebian, A.; Kermanshahi, S. A game theoretic approach to study the impact of transportation policies on the competition between transit and private car in the urban context. *Transp. Res. Part A* **2022**, *163*, 320–337. [CrossRef]
15. Adler, N.; Pels, E.; Nash, C. High-speed rail and air transport competition: Game engineering as tool for cost-benefit analysis. *Transp. Res. Part B Methodol.* **2020**, *44*, 812–833. [CrossRef]

16. Raturi, V.; Verma, A. A game-theoretic approach to analyse inter-modal competition between high-speed rail and airlines in the Indian context. *Transp. Plan. Technol.* **2019**, *43*, 20–47. [CrossRef]
17. Jiang, X.; Zhang, X.; Lu, W.; Zhang, L.; Chen, X. Competition between High-Speed Rail and Airline Based on Game Theory. *Math. Probl. Eng.* **2017**, 1748691. [CrossRef]
18. Jack, N.; Murthy, D.N.P.; Kumar, U. Application of Game Theory to Railway Decision Making. *Curr. Trends Reliab. Availab. Maintainab. Saf.* **2015**, 395–408. [CrossRef]
19. Ruiz-Rúa, A.; Palacín, R. Towards a liberalised European high speed railway sector: Analysis and modelling of competition using Game Theory. *Eur. Transp. Res. Rev.* **2013**, *5*, 53–63. [CrossRef]
20. Huneke, W. A Refined Game Theory Approach to Railroad- Shipper Negotiations. In In Proceedings of the 46th Annual Transportation Research Forum, Washington, DC, USA, 6–8 March 2005; p. 208202. [CrossRef]
21. Sinaga, M.S.; Molik, A.Y.; Febrian, D. Game theory application on online transportation company and driver income levels during the pandemic. *Barekeng J. Math. Its Appl.* **2022**, *16*, 713–720. [CrossRef]
22. Foomani-Dana, C.; Tamannaei, M. A Game-Theoretic Approach for Transportation of Oil Products in a Duopolistic Supply Chain. A. *AUT J. Civ. Eng.* **2021**, *5*, 115–128. [CrossRef]
23. Hadded, M.; Minet, P.; Lasgouttes, J.M. A game theory-based route planning approach for automated vehicle collection. *Concurr. Comput. Pract. Exp.* **2021**, *33*, 16. [CrossRef]
24. Feng, F.; Zhang, T.; Liu, C.; Fan, L. China Railway Express Subsidy Model Based on Game Theory under "the Belt and Road" Initiative. *Sustainability* **2020**, *12*, 2083. [CrossRef]
25. Wu, L.; Zhang, L.; Wang, Y. Inter-city Public Transportation Pricing Based on Game Theory Analysis. *Appl. Mech. Mater.* **2012**, *209–211*, 902–906. [CrossRef]
26. Zhang, J.; Yang, T.; Guo, J. Study on Discrete Railway Bulk Transportation Option Based on Game Theory, International Conference on Internet of Things and Smart City (IoTSC 2021). *J. Phys. Conf. Ser.* **2021**, *1972*, 012127. [CrossRef]
27. Roumboutsos, A.; Kapros, S. A game theory approach to urban public transport integration policy. *Transp. Policy* **2008**, *15*, 209–215. [CrossRef]
28. Ahmad, F.; Almarri, O.; Shah, Z.; Al-Fagih, L. Game theory applications in traffic management: A review of authority-based travel modelling. *Travel Behav. Soc.* **2023**, *32*, 100585. [CrossRef]
29. Koryagin, M. Game theory approach to optimizing of public transport traffic under conditions of travel mode choice by passengers. *Transp. Probl.* **2014**, *9*, 117–124.
30. Radu, V.; Dumitrescu, C.; Vasile, E.; Tabîrca, A.I.; Stefan, M.C.; Manea, L.; Rad, F. Modeling and Prediction of Sustainable Urban Mobility Using Game Theory Multiagent and the Golden Template Algorithm. *Electronics* **2023**, *12*, 1288. [CrossRef]
31. Škrinjar, J.P.; Abramović, B.; Brnjac, N. The use of game theory in urban transport planning. *Tech. Gaz.* **2015**, *22*, 1617–1621. [CrossRef]
32. Pan, G.; Jiang, H.; Jin, Q.; Zhao, T.; Wang, J.; Wang, L. Study on the Sharing Transportation Based on Game Theory. *Sustainability* **2021**, *13*, 9347. [CrossRef]
33. Jamali, M.B.; Rasti-Barzoki, M. A game theoretic approach for green and non-green product pricing in chain-to-chain competitive sustainable and regular dual-channel supply chains. *J. Clean. Prod.* **2018**, *170*, 1029–1043. [CrossRef]
34. Darvishsefat, M.; Rezaeian, J.; Pourpasha, M.M. Designing an Optimal Selection Model of Transportation Network in A Multi-level Supply Chain Using Game Theory. *J. Syst. Manag.* **2023**, *9*, 77–87. [CrossRef]
35. Shi, Y.; Wang, H.; Di, S.; Chen, L. Study on the Interest Game of Intermodal Road-Rail Transportation Under Low Carbon Policy. *Tech. Gaz.* **2022**, *29*, 2038–2047. [CrossRef]
36. Qu, L.; Chen, Y. A Hybrid MCDM Method for Route Selection of Multimodal TransportationNetwork. In *Advances in Neural Networks*; Sun, F., Zhang, J., Tan, Y., Cao, J., Yu, W., Eds.; Lecture Notes in Computer Science; Springer: Berlin/Heidelberg, Germany, 2008; Volume 5263. [CrossRef]
37. Macharis, C.; Meers, D.; Lier, T.V. Modal choice in freight transport: Combining multi-criteriadecision analysis and geographic information systems. *Int. J. Multicriteria Decis. Mak.* **2015**, *5*, 355. [CrossRef]
38. Oudani, M. A combined multi-objective multi criteria approach for blockchain-based synchromodaltransportation. *Comput. Ind. Eng.* **2023**, *176*, 108996. [CrossRef]
39. Koohathongsumrit, N.; Meethom, W. Route selection in multimodal transportation networks: A hybrid multiple criteria decision-making approach. *J. Ind. Prod. Eng.* **2021**, *38*, 171–185. [CrossRef]
40. Vilke, S.; Krpan, L.I.; Milković, M. Application of the Multi-Criteria Analysis in the Process of Road Route Evaluation. *Teh. Vjesn.* **2018**, *25*, 1851–1859. [CrossRef]
41. Vilke, S.; Petrovic, I.; Tadic, F. Evaluation and Selection of the Railroad Route between Rijeka and Zagreb. *Appl. Sci.* **2022**, *12*, 1306. [CrossRef]
42. Chen, D.; Zhang, Y.; Gao, L.; Thompson, R.G. Optimizing Multimodal Transportation Routes Considering Container Use. *Sustainability* **2019**, *11*, 5320. [CrossRef]
43. Zhou, Y.; Liu, X.; Li, F.; Jiang, W. Entropy weight method-gray correlation improvement TOPSIS. *IOP Conf. Ser. Earth Environ. Sci.* **2019**, *304*, 032112. [CrossRef]
44. Hashemkhani Zolfani, S.; Maknoon, R.; Zavadskas, E.K. Multiple Nash equilibriums and evaluation of strategies. New application of MCDM methods. *J. Bus. Econ. Manag.* **2014**, *16*, 290–306. [CrossRef]

45. Liu, T.; Deng, Y.; Chan, F. Evidential Supplier Selection Based on DEMATEL and Game Theory. *Int. J. Fuzzy Syst.* **2018**, *20*, 1321–1333. [CrossRef]
46. Chowdhury, M.Z.; Rahman, M.T.; Jang, Y.M. An Analytical Hierarchy Process Combined with Game Theory for Interface Selection in 5G Heterogeneous Networks. *KSII Trans. Internet Inf. Syst.* **2020**, *14*, 1817–1836. [CrossRef]
47. Aliahmadi, A.; Sadjadi, S.J.; Jafari-Eskandari, M. Design a new intelligence expert decision making using game theory and fuzzy AHP to risk management in design, construction, and operation of tunnel projects (case studies: Resalat tunnel). *Int. J. Adv. Manuf. Technol.* **2011**, *53*, 789–798. [CrossRef]
48. Shyr Oliver, F.; Kuo, Y.-P. Applying TOPSIS and cooperative game theory in airline merging and coalition decisions. *J. Mar. Sci. Technol.* **2008**, *16*, 2. [CrossRef]
49. Debnath, A.; Bandyopadhyay, A.; Roy, J.; Kar, S. Game theory based multi criteria decision making problem under uncertainty: A case study on Indian Tea Industry. *J. Bus. Econ. Manag.* **2018**, *19*, 154–175. [CrossRef]
50. Ibrahim, M.A.R.; Jaini, N.I.; Khalif, K.M.N.K. A comprehensive review of hybrid game theory techniques and multi-criteria decision-making Methods. *J. Phys. Conf. Ser.* **2021**, *1988*, 012056. [CrossRef]
51. Srivastava, T.N.; Rego, S. *Statistics for Management*; Tata McCraw-Hill Publishing Company: New York, NY, USA, 2008.
52. Stoilova, S. Multi-criteria approaches for choosing intermodal technology for container transportation. *Eng. Rural. Dev.* **2023**, *22*, 943–950. [CrossRef]
53. Stoilova, S.; Munier, N. A novel fuzzy simus multicriteria decision-making method. An application in railway passenger transport planning. *Symmetry* **2021**, *13*, 483. [CrossRef]