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Green Tollways: Strategizing Carbon-Emissions-Based Government-Owned Public Toll Road Operations in China

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Abstract: When build–operate–transfer (BOT) roads are transferred back to the government upon the expiry of their contract, they are typically considered to be public roads and are no longer subject to tolls. However, in China, BOT roads, after being transferred to the government, remain tolled by the government in order to maintain efficiency. Therefore, such roads are termed public toll roads (PTRs). During the operational phase of PTRs, ongoing operating costs become a significant financial burden compared to the initial investment made for their construction. Against the backdrop of global carbon emission efforts, this paper studies the operational strategy of PTRs in terms of car emission costs, which constitute a portion of PTRs' operation costs. This paper explores the operational strategy of PTRs, including whether the government should operate the road independently or outsource their operation to a competent private firm. Our analysis concludes that the operator should manage PTRs for the entire duration of their operation by maintaining self-financing while also accounting for operation costs. In this study, governmental regulations for the cost of carbon emissions are also studied.

Keywords: build–operate–transfer; public toll road; operation cost; carbon emission cost

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1. Introduction

To improve the construction of infrastructure in the country, the Chinese government decided to commence the building of various infrastructure projects using the capital of private firms that could profit from operating these projects. In recent decades, the build–operate–transfer (BOT) contract model has been applied worldwide to build infrastructure, especially for the construction of highways [1]. Investment companies have operated BOT roads for decades. When the concession period ends, the ownership of managing BOT roads is transferred back to the government. Based on traditional knowledge, this kind of road then becomes a public road used for free. The government operates and maintains said roads within this budget. Although the road operation cost is much less than the initial building investment, the government must invest more funds to cover operation costs to maintain road operation standards, especially in the context of reducing global carbon emissions. Transportation emissions make a substantial contribution to greenhouse gas issues. Statistics show that total transportation-related carbon emissions worldwide account for around 20% of the total 53.8 gigatons and 3.71 gigatons emitted by vehicles in 2022 [2].

In order to achieve sustainable development, reductions in carbon emissions must be considered. Continuing to toll these roads is an adequate traffic operation strategy to provide quality transportation service, decrease financial burden, and reduce car emissions. To some degree, continuing tolling may become a universal traffic operation strategy for this kind of public road. Internationally, BOT road concessions frequently span extensive durations, often up to 99 years, with private firms typically overseeing their management [3]. Conversely, the normative concession period for BOT roads is approximately 30 years in China.

Consequently, the Chinese government has witnessed multiple BOT road contract expiration instances. In response to these expirations and as a strategy for carbon emission reduction, the Chinese transportation agency has opted to continue charging road users [4]. Based on the above description, when BOT roads are transferred from private firms to the government, and users continue to be tolled for using them, they are then referred to as PTRs [5–7]. When BOT roads become PTRs, whether they are repaired or not, the traffic operation strategies of PTRs then become new modes that are different from those of BOT roads. The road operation period starts a new period, defined as the PTR period. Table 1 outlines comparisons between BOT roads and PTRs.

Table 1. Comparison between BOT roads and PTRs.

	BOT	PTR
Project Model	For city infrastructure with a tight budget.	Variety of models, including government or PPP operation.
Operator	Private firm.	Government, private firm, or joint venture.
Operation Period	Fixed concession period.	No fixed period.
Costs	Construction-focused.	Maintenance- and operation-focused.
Tolling Scheme	Set based on profitability.	Flexible and based on city needs and traffic conditions.
Traffic Rules	Based on individual road conditions.	Adjusted to city-wide traffic conditions.

Under the BOT model, private firms typically undertake city infrastructure projects when government budgets are constrained, focusing on construction costs and setting tolls based on profitability. The operational period is fixed, with traffic rules being tailored to individual road conditions. Conversely, the PTR model offers a more flexible approach, accommodating various operating models such as government-run, private, or public–private partnerships. Unlike BOT roads, PTRs do not have a set operational period and emphasize maintenance and operational costs. Tolling schemes in PTRs are adaptable, designed to meet the specific needs of a city and its traffic conditions, which also inform the adjustment of traffic rules. This flexibility in the PTR model allows for a more dynamic response to urban transportation demands and evolving infrastructural needs.

The expiration of BOT road contracts is an eventuality that will affect such roads worldwide. An increasing number of BOT roads are anticipated to transition to PTRs. Consequently, identifying the most effective strategies for operating PTRs, especially in the context of reducing vehicle carbon emissions, is emerging as a critical challenge. Successfully answering this question will significantly aid in developing robust PTR operational strategies while also contributing to the broader goal of reducing carbon emissions. Thus, with this paper, we attempt to investigate the following research questions:

(1) In the context of carbon emission reduction, what criteria should the government use to select an appropriate PTR strategy upon the expiration of BOT contracts?
(2) How does the imperative of the costs of carbon emissions and operations impact the government's choice of PTR strategies?
(3) To reduce carbon emissions, what regulations should the government implement to guide the operation of PTRs?

Table 2 summarizes the most related studies on the infrastructure construction problem regarding the operational mode, operator, period, objectives, and variables. We found that the majority of previous studies focused on BOT roads. Shang et al. (2022) studied PTRs with heterogeneous road users [7]. The BOT or PTR model being based simultaneously on homogeneous users, and the cost of carbon emissions in the model is rarely observed.

Table 2. Comparison of studies closely related to this paper.

Operational Mode	Objective(s)	Operator	Road Users	Period	Considering Road Operation Cost/Car Emissions Cost	Publications
BOT	Operation strategy: price and concession periods.	Private firm	Homogeneous	Concession period of BOT	No/No	[1,3,8–10]
PTR	Determining the operator of PTRs	Government or private firm	Homogeneous	Operation period of PTRs	Yes/No	[6]
	Determining the effect of critical indicators of PTRs		Heterogeneous			[7]
	Analyzing the operation strategy of PTRs based on carbon emissions		Homogeneous	Operation period of PTRs	Yes/Yes	This paper

This paper studies the operation strategies for PTRs considering carbon emissions in order to fill in the previously outlined research gaps. This study makes the following contributions to the literature and provides traffic operation strategies for the government.

(1) This paper aims to thoroughly study the impacts of operation costs on PTRs with homogeneous users under the condition that operation cost is a function of travel demand and road capacity.
(2) This paper studies the carbon emission costs of vehicles in the model, which is a part of the operation cost function.
(3) This paper studies the application of the self-financing theory on PTRs. The results show that the total revenue of PTRs can cover the operation costs of PTRs under certain conditions.

Operation cost, factoring in the cost of carbon emissions, is an essential definition in our paper and significantly affects road operation during the PTR period. In our model, by using the operation cost, we analyze the operational situation of the government and private firms, and add a constraint of operation cost in optimizing social welfare and the private profit model.

However, in China, the franchise of BOT roads has to be transferred from private firms to the government when contracts end. This paper assumes that the PTR period begins when BOT road franchises are transferred to the government. The relationship between the BOT concession and PTR periods is illustrated in Figure 1.

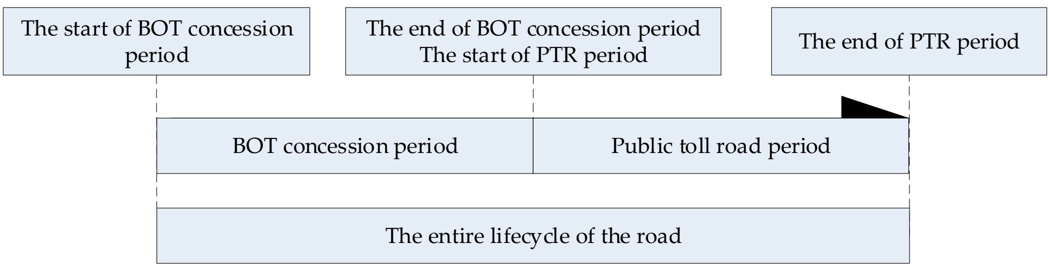

Figure 1. Relationship between the BOT concession period and PTR period [4,6].

The road life after the BOT concession period is defined as the PTR period. During the PTR period, the transportation agencies decide on traffic operation strategies, such as whether to charge PTRs alone or authorize a private firm to undertake this task. With permission from the government, a private firm considers the whole lifecycle of the BOT road as a concession period in the franchise contract [3]. The results of Tan et al.'s study (2010) could apply to this paper if PTRs are operated alone [3]. Private firms select whether to operate PTRs during the whole PTR period; meanwhile, if the government decides to operate PTRs on its own, it must manage them during the entire PTR period.

The remainder of this paper is structured as follows. Section 2 reviews the literature. We introduce some notations and assumptions in Section 3. Section 4 presents the properties of the operation cost and volume/capacity ratio. In Section 5, first-best and second-best contracts are introduced, along with the constrained profit problem of private firms and the self-financing problem. In Section 6, we analyze the government's regulations to control car emissions. Section 7 details our discussion. In Section 8, conclusions and implications are drawn.

2. Literature Review

The body of research on BOT roads has traditionally focused on economic and operational dynamics, such as the toll charge, concession period, and road capacity. Most researchers have constructed analytical frameworks to guide the development of traffic operation strategies to balance social welfare with private sector profitability [1,11–14]. Recent investigations have extended those discussions by providing analytical frameworks to guide the development of traffic operation strategies that consider the complex interplay between social welfare and private profitability [3,15–17].

The adoption of second-best congestion pricing and variable traffic condition analyses by Verhoef and Rouwendal (2004) and Tsai and Chu (2003) has been central to understanding financial mechanisms in BOT projects [18,19]. This body of work was complemented by the application of the mathematical Pareto model by Niu and Zhang (2013) to scrutinize BOT investments amidst uncertain demand and by Yan and Chong's (2019) use of inequity aversion theory to probe the distribution of risks and benefits within such projects [8,9]. Furthermore, Feng et al. (2018) and Shi et al. (2020) have contributed models that allow for the negotiation of tolls and road capacities to maximize social welfare while ensuring the economic viability of these projects [10,16]. Lu and Meng (2023) research the principal-agent model to design BOT project contracts that can regulate private firms, providing a report with their real costs based on the condition of asymmetric investment cost information. Then, the government can maximize social welfare, and the firms can earn a reserved level of profit [20]. Later, Hoang-Tung et al. (2021) extend this stream of research by building a ceiling-price model based on a simple two-route network to protect government social welfare and allow private firms to achieve their profit. This model analyzes the negotiating possibility between government and private firms by considering travelers' benefits and risk identification [21].

As the realm of transportation research expands, the environmental impacts of road transportation, specifically carbon emission reductions, have garnered increasing attention. Zhu et al. (2012) provided an early foray into the effectiveness of road dust emission control measures [22]. Studies have paved the way for more comprehensive reviews, such as those conducted by Ferrer and Thomé (2023), which underscores the need for transportation services to adapt to climate change imperatives [23].

Further bridging the gap between environmental consequences and transportation, Aminzadegan et al. (2022) conducted a thorough investigation into the greenhouse gases emitted by the transportation industry, suggesting a series of practical solutions for policy development [24]. These environmental considerations are critical for the sustainable management of PTRs, and are becoming integral to the strategies that govern them.

Additionally, the work of Xia et al. (2023), introducing the Standardized Driver Aggressiveness Index, highlighted the potential of behavioral changes to contribute significantly to emission reductions [25]. This perspective is enriched by Lin (2013), who integrated traffic flow and emission models, and Qi et al. (2023), who provided a detailed examination of the management of air pollutants and CO_2 emissions from vehicles [26,27]. Using nonlinear model predictive control, Bakibillah et al. (2024) evaluated a novel optimal ecological driving scheme to evaluate reducing consumption and carbon emissions [28]. These studies advocate for comprehensive strategies that address immediate operational concerns and the broader environmental challenges of climate change and air quality.

Roman (2022) synthesized these diverse strands of research, offering a holistic review that connects sustainable transport's economic and operational aspects with the pressing global objectives of environmental policy [29]. This body of literature stresses the importance of using a multifaceted approach toward transportation research that seamlessly integrates social, economic, and environmental sustainability considerations within the context of PTR management.

Despite the extensive research on BOT roads, there remains a significant gap in the literature regarding the integration of carbon emissions strategies into the operational and financial models of government-owned public toll roads. While previous studies have laid the groundwork for understanding the financial mechanisms and environmental impacts of BOT projects, they have not sufficiently addressed how these factors can be cohesively aligned with carbon emissions reduction targets. Particularly, there is a lack of comprehensive frameworks that incorporate the cost of carbon emissions into the financial and operational decision-making processes of public toll roads. Moreover, the existing literature has yet to fully explore the role of government regulations and policies in steering the transition towards green tollways that prioritize both environmental sustainability and economic viability. This study seeks to bridge this gap by proposing an integrated approach that considers the carbon emissions footprint within the lifecycle of toll road operations, post-BOT contract expiry, and evaluates the efficacy of potential government interventions in the context of China's evolving environmental policy landscape.

3. Notation and Assumptions

It is assumed that private firms and the government encompass all the information about operation costs and the travel demand of PTRs in the future. This paper also considers that the technical characteristics of PTRs are exogenous. In the following section of this paper, some variables may have the subscript g for government operation, and some may have the subscript s for private firm operation. If the variable does not have subscripts g and s, it will be given subscripts g and s when using them in the analysis of the following section of this paper. The notations used in these equations are listed in Table 3.

Table 3. Definition of the notations.

Notation	Definition	Notation	Definition
D	The potential PTR travel demand	$C_s(D) = c_{s1}D$	The demand-related operation cost of a private firm
y	The road capacity	$C_s(y) = c_{s2}y$	The capacity-related operation cost of a private firm
$t(D, y)$	The trip duration time function of D and y	$C_g(D) = c_{g1}D$	The demand-related operation cost of the government
$B(D)$	The travelling cost of road users	$C_g(y) = c_{g2}y$	The capacity-related operation cost of the government
p	The toll rate for each road user	$C_{eg}(D) = c_{eg}D$	The carbon emission cost of the private firm
β	The value of time	$C_{es}(D) = c_{es}D$	The carbon emission cost of the private firm
$S(D, y)$	The unit time social surplus	$P(D, y)$	Profit
$R(D, y)$	The unit time revenue of private firms	$W_g(D, y)$	Social welfare under government operation
k	The constant of construction cost	$W_s(D, y)$	Social welfare under private firm operation

Let $D \geq 0$ be the potential PTR travel demand, $y \geq 0$ be the road capacity during the PTR period, and $t(D, y)$ be the trip duration time function of D and y. $B(D)$ represents the inverse function of the demand [5–7]. Note that D is measured by the number of vehicles per unit of time.

The supply–demand balance holds as follows:

$$B(D) = p + \beta t(D, y) \tag{1}$$

where p is the toll rate for each road user and β is the value of time, where time can be converted into an equivalent monetary cost (only considering homogenous users). Condition (1) means that PTR trip demand D is highly associated with total trip cost.

From Formula (1), p can be expressed as a function of D and y, mathematically, as follows:

$$p(D, y) = B(D) - \beta t(D, y) \tag{2}$$

p can be determined by D for a given y. Therefore, selecting (p, y) as decision variables is equivalent to choosing (D, y). Hereafter, D can be substituted by p. The assumptions about $B(D)$ and $t(D, y)$ are valid for this paper, which is the same finding as that of Shang et al. (2016) [5].

Assumption 1. *For any given $D \geq 0$, $B(D)$ is a strictly continuously decreasing and differentiable function of D, and the function $D \cdot B(D)$ is strictly concave in D. $t(D, y)$ is a continuously differentiable function of both $D(D \geq 0)$ and $y(y \geq 0)$; it is a convex and increasing function in D for any given $y \geq 0$ and a decreasing function in y for any given $D \geq 0$. $t(D, y)$ is homogeneous of degree zero in D and y, i.e., $t(\alpha D, \alpha y) = t(D, y)$ for any given $\alpha > 0$.*

It is shown that vehicle travel time is only relevant to the volume/capacity ratio $r = D/y$. The Bureau of Public Roads travel time function satisfies this widely used assumption in transportation research.

We introduce $S(D, y)$, the unit time social surplus, and $R(D, y)$, the unit time revenue, of private firms. Next, $S(D, y)$, the PTR period, can be determined as follows:

$$S(D, y) = \int_0^D B(w)dw - \beta \cdot Dt(D, y) \tag{3}$$

Under Assumption 1, we can see that $S(D, y)$ is strictly concave in D.

And $R(D, y)$ during the PTR period can be calculated as follows:

$$R(D, y) = D \cdot p(D, y) = D \cdot B(D) - \beta \cdot Dt(D, y) \tag{4}$$

where D is determined by Equation (1).

In line with Shi et al.'s study (2016), we use $C_s(D) = c_{s1}D$ and $C_s(y) = c_{s2}y$ as the demand-related operation cost and the capacity-related operation cost of private firms for PTRs, respectively [30]; let $C_g(D) = c_{g1}D$ and $C_g(y) = c_{g2}y$ be the demand-related operation cost and the capacity-related operation cost of the government for PTRs, respectively. We use $C_{eg}(D) = c_{eg}D$ and $C_{es}(D) = c_{es}D$ as the carbon emission cost of the government and private firms, and $I(y)$ is the road construction cost. Then,

$$C_s(D, y) = C_{es}(D) + C_s(D) + C_s(y) \tag{5}$$

$$C_g(D, y) = C_{eg}(D) + C_g(D) + C_g(y) \tag{6}$$

$$I(y) = ky \tag{7}$$

where c_{g1}, c_{s1} are the demand-related marginal operation cost of private firms and the government, respectively; c_{g2}, c_{s2} are the capacity-related marginal operation cost of private firms and the government, respectively; and c_{es}, c_{eg} are the carbon emission marginal cost of private firms and the government, respectively. According to Chinese policies [4,31,32], in this paper, we assume that PTR operators buy unit carbon emissions with the same price, namely, $c_e = c_{es} = c_{eg}$ and $C_e(D) = C_{es}(D) = C_{eg}(D)$; k is the constant of the construction cost. From the above content, governmental and private operation costs are constant returns to scale.

Assumption 2. *Functions $C_g(y)$, $C_s(y)$, $I(y)$ are continuously increasing and differentiable with y for given $y > 0$. For any given $y > 0$, $R(D,y)$ is a strictly concave function of D for given $D \geq 0$, i.e., $\partial R^2/\partial D^2 < 0$.*

Let the PTR period L_{con} ($L_{con} > 0$) be the optimal life of the road with a reasonable level of service. Firstly, the private firm's problem is taken into consideration. A combination (D,y) is selected to maximize its profit $P_s(D,y)$:

$$P_s(D,y) = L_{con} \cdot [R(D,y) - C_s(D,y)] - I(y) \tag{8}$$

The first term of Equation (8) is the total profit earned by the private firm; the second to fourth terms are the private operation cost and road construction cost. With Equation (4), Equation (8) can be written as follows:

$$P_s(D,y) = L_{con}[DB(D) - \beta \cdot Dt(D,y) - C_s(D,y)] - I(y) \tag{9}$$

According to Assumption 1, $t(D,y)$ is the convex function in D; for given $y > 0$, the term $D \cdot t(D,y)$ in Equation (9) is also convex in D. With the strict concavity of $D \cdot B(D)$, $P_s(D,y)$ is strictly concave in D for granted $y > 0$ during the PTR period.

When operating PTRs, the government usually prefers to select combinations (D,y) to optimize social welfare. When operation cost is considered, two objective social welfare functions exist. The first one is that when the government itself operates PTRs, we calculate social welfare with $C_g(D)$ using the following $M_g(y)$:

$$W_g(D,y) = L_{con}[S(D,y) - C_g(D,y)] - I(y) \tag{10}$$

where $W_g(D,y)$ is social welfare under government operation.

The other is when private firms operate PTRs. Social welfare is calculated with the following formula $C_s(D)$ and $C_s(y)$:

$$W_s(D,y) = L_{con}[S(D,y) - C_s(D,y)] - I(y) \tag{11}$$

where $W_s(D,y)$ is social welfare under private firm operation.

4. Properties of Operation Cost

The operation cost properties are studied in a PTR problem based on both private firms and the government sharing the same degree of knowledge, such as travel demand D, construction cost $I(y)$, and operation cost, including carbon emission operation cost, demand-related operation cost, and capacity-related operation cost.

In reality, a private firm usually desires to operate the road for as long as possible in order to generate more profit by selecting road capacity, an appropriate price, and saved operation cost.

With BOT road franchise rights expiring in China, almost all private firms want to prolong their operation period, such as Huayu Company in Shenzhen, which operates the Shui-guan highway in Shenzhen. Governmental highway companies (such as highway companies in Shandong province) would also like to extend the PTR period. Niu and Zhang (2013) found a similar result within the context of uncertain travel demand, but operation cost was not studied thoroughly in their work [8].

Based on the above content, private firm social welfare $C_s(D, y)$ and private firm profit $P_s(D, y)$ can be written as follows:

$$W_s(D, y) = L_{con} \cdot [\int_0^D B(w)dw - \beta Dt(D, y) - C_s(D, y)] - I(y) \tag{12}$$

$$P_s(D, y) = L_{con} \cdot [DB(D) - \beta Dt(D, y) - C_s(D, y)] - I(y) \tag{13}$$

4.1. Toll and Capacity of Social Welfare with Governmental Operation

If $C_g(D,\ y) < C_s(D,\ y)$, the government would operate PTRs by itself in order to achieve maximal social welfare by selecting optimal price and capacity. Maximizing social welfare is the most important goal of the government. Then, this problem can be expressed by Equation (10).

We assume that $(\widetilde{D}_g, \widetilde{y}_g)$ is the social welfare optimal (SO) solution that can maximize $W_g(D, y)$. Then, this would meet the first-order optimal conditions as follows:

$$\frac{\partial W_g}{\partial D} = B(\widetilde{D}_g) - \beta t(\widetilde{r}_g) - \beta \widetilde{r} t'(\widetilde{r}_g) - c_e - c_{g1} = 0 \tag{14}$$

$$\frac{\partial W_g}{\partial y} = L_{con} \cdot \beta \widetilde{r}_g^2 t'(\widetilde{r}_g) - (L_{con} \cdot c_{g2} + k) = 0 \tag{15}$$

where $\widetilde{r}_g$ denotes the SO volume/capacity ratio with government operation, namely $\widetilde{r}_g = \widetilde{D}_g / \widetilde{y}_g$. $\widetilde{r}_g$ can be calculated by Condition (15),

$$L_{con} \cdot \beta \widetilde{r}_g^2 t(\widetilde{r}_g) = (L_{con} \cdot c_{g2} + k) \tag{16}$$

Toll charges under optimal social welfare can be calculated by (2) and (14).

$$\widetilde{p}_g = B(\widetilde{D}_g) - \beta t(\widetilde{r}_g) = \beta \widetilde{r}_g t'(\widetilde{r}_g) + c_e + c_{g1} \tag{17}$$

The government can implement this capacity and toll value to obtain optimal social welfare. From the left side of the formula, we can see that price includes the marginal cost of carbon emissions. The government can regulate the transportation policy based on carbon emission factors.

In China, many highways are operated by the government. Therefore, it is essential to study how social welfare can be maximized under the condition where the government obtains zero profit; that is, $R(D, y)$, which is equivalent to the total investment, including operation cost and construction cost.

This problem can be defined as

$$\max_{y \geq 0, L_{con} \geq 0} W_g(D, y) = L_{con} \cdot [S(D, y) - C_g(D, y)] - I(y) \tag{18}$$

subject to

$$P_g(D, y) = L_{con} \cdot [R_g(D, y) - C_g(D, y)] - I(y) \geq 0 \tag{19}$$

where $P_g(D, y)$ is the profit earned by the governmental company. $R_g(D, y)$ is the revenue of the governmental company. Assume that (D_g^*, y_g^*) is an optimal solution to the above model; then, (D_g^*, y_g^*) is the solution for the following Lagrange problem:

$$\begin{aligned} L_g(D, y, \eta) &= L_{con} \cdot [\textstyle\int_0^D B(w)dw - \beta q t(\frac{D}{y}) - C_g(D, y)] - I(y) \\ &+ \eta \left\{ L_{con} \cdot [D \cdot B(D) - \beta D t(\frac{D}{y}) - C_g(D, y)] - I(y) - P_g^* \right\} \end{aligned} \tag{20}$$

where $\eta \geq 0$ is the Lagrange multiplier and P_g^* is the value of $P_g(D_g^*, y_g^*)$. Then, the following first-order conditions can be obtained:

$$\frac{\partial L_g}{\partial D} = (1+\eta)L_{con}B(D_g^*) - (1+\eta)L_{con}(\beta t(\frac{D_g^*}{y_g^*}) + \beta \frac{D_g^*}{y_g^*}\frac{\partial t(D_g^*/y_g^*)}{\partial D}) - (1+\eta)L_{con}(c_e + c_{g1}) + \eta L_{con}D_g^* B'(D_g^*) \quad (21)$$

$$\frac{\partial L_g}{\partial y} = (1+\eta)\cdot\{L_{con}\cdot[\beta(\frac{D_g^*}{y_g^*})^2 \frac{\partial t(D_g^*/y_g^*)}{\partial y} - c_{g2}] - k\} = 0 \quad (22)$$

and

$$\frac{\partial L_g}{\partial \eta} = L_{con}\cdot[R_g(D_g^*, y_g^*) - C_e(D_g^*) - C_g(D_g^*) - C_g(y_g^*)] - I(y_g^*) = 0 \quad (23)$$

Denote $r_g^* = \frac{D_g^*}{y_g^*}$ as the y ratio. Since $\eta \geq 0$, Equation (22) can be simplified as

$$L_{con}\cdot\beta(r_g^*)^2 t'(r_g^*) = L_{con}\cdot c_{g2} + k \quad (24)$$

Equation (21) is a unique solution for $t(D, y)$, which is a strictly convex function, and solution r_g^* can be obtained from Equation (22). Furthermore, we can determine the toll charge and capacity from Equations (1) and (21). The government operation cost is usually higher than that of a private firm because a private firm has more flexibility and vigorous execution. However, in China, government companies usually adopt higher toll charges and operation costs with a higher road capacity. If the government manages the road by itself, there will be no contract.

In this section, one simple numerical example of the PTR operation is presented to demonstrate the above results that PTRs will be operated by the government. We make an assumption that the capacity of PTRs, $y = 6000(vehicels/h)$), is not changed, which means that the government will not invest any construction costs. The travel time of the free-flow on PTRs is 1.6 (h). We set the other factor values as follows: $\beta = 30(\$/h), c_e = 1, c_{g1} = 2, c_{g2} = 2$. We only compute one year of social welfare and profit, and the period is 365*24 = 8750 h. The following equations link the travel time function and travel cost function, which are linear functions, as follows:

$$t(D, y) = 1.6\cdot[1 + (D/y)]$$

$$B(D) = 20 + 30t(D, y)$$

Based on Equations (18) and (19), the PTR problem under the cost of carbon emissions is

$$\max_{y\geq 0, L_{con}\geq 0} W_g(D, y) = 8760\cdot[\int_0^D B(w)dw - 30\cdot Dt(D, y) - (1\cdot D + 2\cdot D + 2\cdot y)]$$

subject to

$$P_g(D, y) = 8760\cdot[D\cdot p - (1\cdot D + 2\cdot D + 2\cdot y)] \geq 0$$

Figure 2 shows that social welfare decreases with the increase in travelling based on operation cost, including demand-related cost and carbon emission cost. When demand equals 706 vehicles/h, profit is equal to 0. This means that if the government wants to obtain social welfare under the condition of zero profit, the travel flow must be larger than 706. From the government's perspective, the government must incentive more vehicles to use PTRs, but with the increase in traffic, social welfare decreases. When traffic flow is larger than 5000 vehicles/h, social welfare starts to approach zero. When traffic flow starts to approach 6000, causing traffic congestion, social welfare becomes negative. Then, operation cost becomes higher, along with the carbon emission cost.

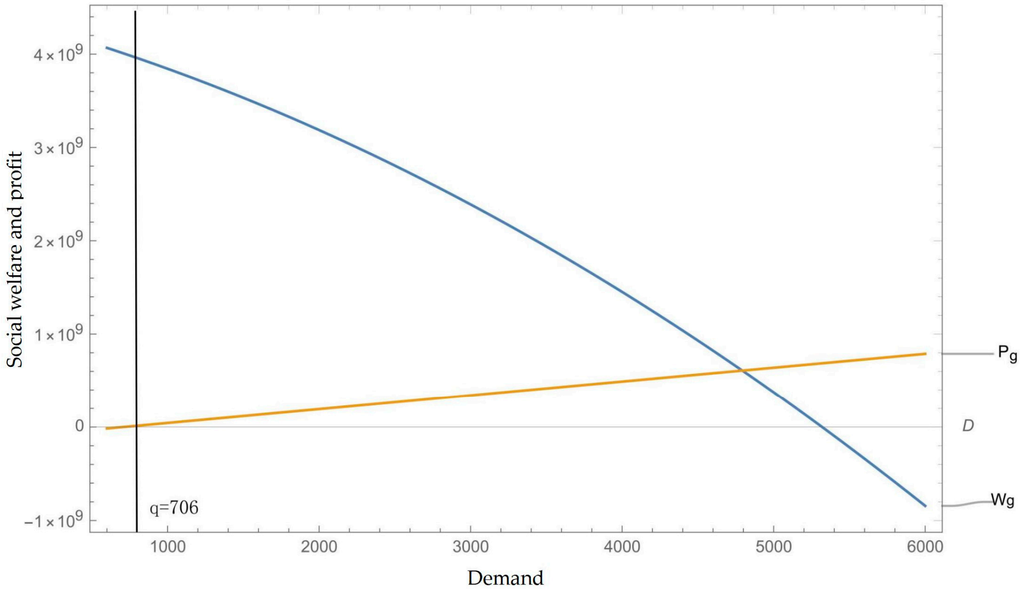

Figure 2. Social welfare and profit of PTRs.

4.2. The Ratio of Operation Cost in PTR Contracts

Let $(\widetilde{D}_s, \widetilde{y}_s)$ be the social optimal solution that the road is operated by a private firm, which optimizes social welfare $C_s(D, y)$; let $(\overline{D}_s, \overline{y}_s)$ be the monopoly optimal solution which maximizes private profit $P_s(D, y)$. Thus, the first-order conditions of D and y with Equations (12) and (13), respectively, can be illustrated as follows:

$$\frac{\partial W_s}{\partial D} = B(\widetilde{D}_s) - \beta t(\widetilde{r}_s) - \beta \widetilde{r} t'(\widetilde{r}_s) - c_e - c_{s1} = 0 \tag{25}$$

$$\frac{\partial W_s}{\partial y} = L_{con} \cdot \beta \widetilde{r}_s^2 t(\widetilde{r}_s) - (L_{con} \cdot c_{s2} + k) = 0 \tag{26}$$

and

$$\frac{\partial P_s}{\partial D} = B(\overline{D}_s) + \overline{D}_s B'(\overline{D}_s) - \beta t(\overline{r}_s) - \beta \overline{r} t'(\overline{r}_s) - c_e - c_{s1} = 0 \tag{27}$$

$$\frac{\partial P_s}{\partial y} = L_{con} \cdot \beta \overline{r}_s^2 t(\overline{r}_s) - (L_{con} \cdot c_{s2} + k) = 0 \tag{28}$$

where $\widetilde{r}$ and $\overline{r}$ are the social optimal volume/capacity ratios, respectively. Comparing Equations (26) and (28), we have $\widetilde{r} = \overline{r}$, with the fact that $r^2 t'(r)$ is an increasing function of r. We know that if $C_g(D, y) \geq C_s(D, y)$, social welfare and private profit can be expressed as Equations (12) and (13) because the government will let private firms undertake the operation; then, the following proposition with the D/y ratio for PTRs contract is achieved.

Proposition 1. *For a given unit-capacity-related operation cost c_{s2}, trip time function $t(D, y)$, unit capital cost k, D/y, and ratio r of road have the solution $L_{con} \cdot \beta r^2 t'(r) = L_{con} \cdot c_{s2} + k$, and are constant and equal to the social optimal and private profit D/y ratio $\widetilde{r}$ and $\overline{r}$.*

Proof. Assume that (D_s^*, y_s^*) is an optimal combination. Then, (D_s^*, y_s^*) solves the following Lagrange problem.

$$\begin{aligned} L(D, y, \eta) = {} & L_{con} \cdot [\int_0^D B(w) dw - \beta D t(\tfrac{D}{y}) - C_s(D, y)] - I(y) \\ & + \eta \left\{ L_{con} \cdot [D B(D) - \beta D t(\tfrac{D}{y}) - C_s(D, y)] - I(y) - P_s^* \right\} \end{aligned} \tag{29}$$

where $\eta \geq 0$ is the Lagrange multiplier and P_s^* is the value of $P_s(D_s^*, y_s^*)$. Then, the first-order conditions can be obtained as follows:

$$\frac{\partial L}{\partial D} = (1+\eta)L_{con}B(D_s^*) - (1+\eta)L_{con}(\beta t(\frac{D_s^*}{y_s^*}) + \beta\frac{D_s^*}{y_s^*}\frac{\partial t(D_s^*/y_s^*)}{\partial D}) \\ -(1+\eta)L_{con}(c_e + c_{s1}) + \eta L_{con}D_s^* B'(D_s^*) \tag{30}$$

and

$$\frac{\partial L}{\partial y} = (1+\eta)\left(L_{con}\beta(\frac{D_s^*}{y_s^*})^2 \frac{\partial t(D_s^*/y_s^*)}{\partial y} - L_{con}c_{s2} - k\right) = 0 \tag{31}$$

$r_s^* = \frac{D_s^*}{y_s^*}$ is denoted as the ratio. Since $\eta \geq 0$, then Equation (31) can be reduced to

$$L_{con}\cdot\beta(r_s^*)^2 t'(r_s^*) = L_{con}\cdot c_{s2} + k \tag{32}$$

Equation (32) has a unique solution in that t is strictly a convex function. And we have $r_s^* = \widetilde{r} = \bar{r}$. This completes the proof. □

The volume/capacity ratio r affects users' travel time. Tan et al. (2010) proved that the volume/capacity ratio of the Pareto efficient is also constant. Based on the above literature, Wu et al. (2011) showed that, in a general traffic network, the r of a private toll road is independent of another private firm's toll and road capacity choices [3,33]. All the above studies from the literature show that r on a private toll road is constant, assuming that road flow is continuously differentiable concerning toll charge and road capacity. Furthermore, Wang et al. (2013) examined the fundamental properties of the volume/capacity ratio of toll roads in a general network through relaxing assumptions [34]. The situation identical to proposition 1 is that private firms tend to provide a lower road capacity and higher toll charge, resulting in more inadequate traffic flow. In contrast, the government offers a higher road capacity and lower toll charges with an increased traffic flow. Therefore, the volume/capacity values of both parties are identical.

5. The First-Best and Second-Best Contract

5.1. The Profit Constraint Problem

The above sections construct the unconstrained maximal profit model and unconstrained maximal social welfare model. The constrained maximal profit model is built under the assumption of $C_g(D, y) > C_s(D, y)$. Generally, a private firm will optimize its profit by selecting y and $p(y)$. Therefore, the constrained profit-maximizing model can be expressed through total revenue minus total construction cost and private operation cost as follows:

$$\max_{y\geq 0, L_{con}\geq 0} (W_s(D,y), P_s(D,y)) \tag{33}$$

subject to

$$L_{con}\cdot[Dp(D, y) - C_s(D, y)] - I(y) \geq \widetilde{P} \tag{34}$$

$$L_{con}\cdot[Dp(D, y) - C_s(D, y)] - I(y) \leq L_{con}\cdot[C_g(D, y) - C_s(D, y)] \tag{35}$$

$$c_g \geq c_s, D \geq 0, y \geq 0, L_{con} \geq 0 \tag{36}$$

where $\widetilde{P} \geq 0$ is the minimum profit that a private firm can accept. So, if the government would prefer the private firm to operate the road, it meets condition $c_g \geq c_s$ and Condition (34). Simultaneously, if the government wants to attract the right private firm to operate the road, the profit must be bigger than marginal profit $\widetilde{P}$. Constrained Condition (35) shows that private profit will not exceed the difference between governmental and private operation costs; if the payoff is more significant than the difference, the government will manage the road itself.

Condition (35) can be reduced as

$$L_{con}[D \cdot p(D,y) - C_g(D,y)] - I(y) \leq 0 \tag{37}$$

Equation (37) shows that the government would like to select a private firm to operate PTRs when its profit is negative, which means that $C_g(D,\ y) > C_s(D,\ y)$. With Condition (37), our model is different from the traditional BOT model, which neglects operation cost constraints in the model [3,8,31,32].

Based on the above problem, the definition of first-best PTRs and second-best PTRs contracts is introduced.

Definition of first-best and second-best contracts. Let (D,y) be the optimal solution to Models (33)–(36), where (D,y) is a first-best contract if (D,y) is the optimal solution to Problem (12); otherwise, (D,y) is a second-best contract.

The first-best and second-best contracts are two critical concepts that differentiate the problem. In reality, a private firm usually obtains the second-best contract with the government since the first-best contract cannot easily be achieved.

Proposition 2. *If selecting the second-best contract, private firms hold out for the second-best solution* (D_s, y_s)*, which is* $\overline{D} \leq D_s \leq \widetilde{D}$*.*

Proof. Assume that (D_s, y_s) is the second-best optimal solution. Firstly, the partial derivatives of $W_s(D,y)$ and $P_s(D,y)$ in D and y is taken

$$\frac{\partial W_s(D,y)}{\partial D} = L_{con} \cdot [B(D) - \beta t(D,y) - \beta D \frac{\partial t(D,y)}{\partial D} - c_e - c_{s1}] \tag{38}$$

$$\frac{\partial W_s(D,y)}{\partial y} = -\left(L_{con} \cdot \beta D \frac{\partial t(D,y)}{\partial y} + c_{s2} + I'(y)\right) \tag{39}$$

and

$$\frac{\partial P_s(D,y)}{\partial D} = L_{con} \cdot \left(B(D) + DB'(D) - \beta t(D,y) - \beta D \frac{\partial t(D,y)}{\partial D} - c_e - c_{s1}\right) \tag{40}$$

$$\frac{\partial P_s(D,y)}{\partial y} = -\left(L_{con} \cdot [\beta \cdot D \frac{\partial t(D,y)}{\partial y} + c_{s2}] + I'(y)\right) \tag{41}$$

Assume that $\widetilde{D}$ can maximize $W_s(D,y)$, and the term on the right side of Equation (38) is zero; i.e.,

$$B(\widetilde{D}) - \beta t(\widetilde{D}, y_s) - \beta \widetilde{D} \frac{\partial t(\widetilde{D}, y_s)}{\partial D} - c_{s1} = 0 \tag{42}$$

Assume that $\overline{D}$ can maximize $P_s(D,y)$, and the term on the right side of Equation (40) is also zero; i.e.,

$$B(\overline{D}) + \overline{D}B'(\overline{D}) - \beta t(\overline{D}, y_s) - \beta \overline{D} \frac{\partial t(\overline{D}, y_s)}{\partial D} - c_{s1} = 0 \tag{43}$$

Under Assumption 1, $\widetilde{D}$ and $\overline{D}$ exist, which could meet Equations (38) and (40), respectively, and $\overline{D} < \widetilde{D}$.

By comparing Equations (38) and (40), for any feasible solution (D,y), $\partial W/\partial D > \partial P/\partial D$ can be obtained with the condition of $B'(D) < 0$. If (D_s, y_s) is a second-best solution, $\partial W(D_s, y_s)/\partial D$ and $\partial P(D_s, y_s)/\partial D$ cannot be positive and negative simultaneously, and increasing or decreasing D will alter social welfare and profit simultaneously, which shows that (D_s, y_s) is the second-best solution. Thus, $\partial W(D_s, y_s)/\partial D \geq 0$ and $\partial q(D_s, y_s)/\partial D \leq 0$ are obtained. This completes the proof. □

5.2. The Classic Self-Financing Theory with PTRs

The famous classic self-financing theory was deduced from the first-best environment based on a single road with homogeneous users [35,36]. It stated that the revenue collected from toll road users equals its investment under certain conditions. Now, we will use this theory to examine PTRs with homogeneous users. In our model, we assume that $D > 0$ and $y > 0$, because if $y = 0$, this means that there is no road to be constructed, and $D = 0$ means that there are no cars on the road. Otherwise, it is unnecessary to build a new road.

If $(D > 0, y > 0)$ at the unconstrained social optimum, the first-order optimal condition for the unconstrained social Problem (12) is given by

$$B(D) - \beta\left(t(D,y) + D\frac{\partial t(D,y)}{\partial D}\right) - c_e - c_s = 0 \quad (44)$$

$$-L_{con}\cdot\beta D\frac{\partial t(D,y)}{\partial y} - L_{con}\cdot C_s'(y) - I'(y) = 0 \quad (45)$$

By combining Equation (44) with (2), a new formula is obtained, as follows:

$$p(D,y) = \beta D\frac{\partial t(D,y)}{\partial D} + c_e + c_s \quad (46)$$

Looking at Equation (46), an unconstrained social optimum toll can be obtained, which equals congestion externality.

In the self-financing problem, there is an important assumption that $t(D,y)$ is homogeneous of degree zero in D and y, which means that $t(\theta D, \theta y) = t(D,y)$ for any given $\theta > 0$. Then, from the Euler equation, taking a partial derivative in terms of θ on both sides, we can obtain the equation for any given D and y, as follows:

$$y\frac{\partial t(D,y)}{\partial y} = -D\frac{\partial t(D,y)}{\partial D} \quad (47)$$

By applying (47) to solve the unconstrained social optimum problem, the first-order Condition (45) equals

$$L_{con}\cdot D\cdot\beta D\frac{\partial t(D,y)}{\partial D} = L_{con}\cdot C_s(y)E_{Cs}^{y} + I(y)E_I^{y} \quad (48)$$

where

$$E_{Cs}^{y} = \frac{dC_s(y)/C_s(y)}{dy/y} = C_s'(y)\frac{y}{C_s(y)}$$

$$E_I^{y} = \frac{dI(y)/I(y)}{dy/y} = I'(y)\frac{y}{I(y)}$$

E_{Cs}^{y} is the capacity-related operation cost elasticity in terms of y, and E_I^{y} is the construction cost elasticity $I(y)$ under y. Furthermore, by substituting the unconstrained social optimum toll from Equation (46) into (45), the following equation can be obtained:

$$L_{con}\cdot D\cdot p(D,y) = L_{con}\cdot[C_e(D)E_e^{D} + C_s(D)E_{Cs}^{D} + C_s(y)E_{Cs}^{y}] + I(y)E_I^{y} \quad (49)$$

where

$$E_e^{D} = \frac{dC_e(D)/C_e(D)}{dD/D} = C_e'(D)\frac{D}{C_e(D)}$$

$$E_{Cs}^{D} = \frac{dM_s(D)/M_s(D)}{dD/D} = M_s'(D)\frac{D}{M_s(D)}$$

E_e^{D} is the emission operation cost elasticity in terms of D; E_{Cs}^{D} is the demand-related operation cost elasticity in terms of D. The left-hand term in Equation (49) is the total revenue of the private firm during the PTR period for the unconstrained social optimum toll and road capacity. The right-hand term of the equation has a close relationship with operation cost and road construction cost.

If the return to scale in both the construction and operation of PTRs is constant, namely, $E_e^D = 1$ or $C_e(D) = c_e D$, $E_{Cs}^D = 1$ or $C_s(D) = c_{s1} D$, $E_{Cs}^y = 1$ or $C_s(y) = c_{s2} y$, and $E_I^y = 1$ or $I(y) = ky$ simultaneously, then Equation (48) can be expressed as $L_{con} \cdot D \cdot p(D, y) = L_{con} \cdot C_s(D, y) + ky$, which means that the revenue just covers road operation cost and construction cost. The classic self-financing theory holds when considering operation cost under the common assumption.

Next, constant return is relaxed to examine the effects of non-constant returns to scale, including decreasing and increasing returns, in the PTR period under the following specific construction and operation cost functions.

$$I(y) = ky^\alpha \tag{50}$$

$$C_s(y) = c_{s2} y^\alpha \tag{51}$$

By taking the derivative of the function $L_{con} D \cdot p(D, y) = L_{con}[C_s(D, y)] + I(y)$ with y, the following first-order term yields:

$$L_{con} D \cdot \beta \frac{\partial t}{\partial y} = \alpha L_{con} c_{s2} y^{\alpha-1} + \alpha k y^{\alpha-1} \tag{52}$$

Substituting Equations (2), (46), and (47) into Equation (52) gives rise to

$$\begin{aligned} &L_{con} D p(D, y) = L_{con}[c_e D + c_{s1} D + \alpha c_{s2} y^\alpha] + \alpha k y^\alpha \\ &= L_{con}[C_e(D) + C_s(D) + \alpha C_s(y)] + \alpha I(y) \end{aligned} \tag{53}$$

where α is the elasticity of the operation cost $C_s(y)$ and construction cost $I(y)$. The left-hand term of Equation (53) is the total revenue for the unconstrained social optimum toll and capacity under non-constant returns; it scales to road construction and operation. The right-hand term of the equation also has a close relationship with operation cost and construction cost. When $\alpha = 1$, total revenue just covers the construction cost and operation cost. The following self-financing theory is obtained.

Proposition 3. *Assume that all of the assumptions are met when road operation and construction cost show non-constant returns to scale, and travel time is homogeneous of degree zero with D and y. Total revenue with the goal of social optimum collected during the PTR period is as follows:*

$$L_{con} \cdot D \cdot p(D, y) = L_{con} \cdot C_s(D, y) + I(y) \textit{ if } \alpha = 1.$$

$$L_{con} \cdot D \cdot p(D, y) > L_{con} \cdot C_s(D, y) + I(y) \textit{ if } \alpha > 1.$$

$$L_{con} \cdot D \cdot p(D, y) < L_{con} \cdot C_s(D, y) + I(y) \textit{ if } 0 < \alpha < 1.$$

Most of the existing self-financing research focuses on fundamental analysis without paying more attention to operation cost. However, the above self-financing proposition is established with the assumption that the operation cost is the function of D and y. If $0 < \alpha < 10 < \alpha < 1$, with non-constant returns to scale in road construction and maintenance, it is shown from proposition 3 that there is not enough revenue to cover all the operation costs. Then, when $\alpha > 1$, the total revenue will be more significant than all the operation costs, and the government or private sector will earn a surplus from the road; when $\alpha = 1$, total revenue will just cover operation costs. Conditions (50) and (51) or constant return to scale are not generally satisfied. Still, we can also conclude that total revenue will exceed capital cost if $\alpha \geq 1$, and we consider operation cost in PTR operations.

If the government operates the road and tolls it by itself, we can obtain the same results by substituting c_g for c_s. The self-financing theory also holds.

5.3. The Zero Profit of Private Firms

From self-financing problem analysis, it is known that private firms might have minus profits under a socially optimal PTR combination $(\widetilde{D}, \widetilde{y})$ if non-constant returns to scale $(\alpha < 1)$ are available. It is essential to examine PTR contracts with the zero-profit constraint, where the subscript 'zp' stands for 'zero profit' of the private firm profit. Its existential conditions should be found before looking for the zero-profit combination (D^*_{zp}, y^*_{zp}).

In this section, zero-profit private firms with both constant return to scale and non-constant returns to scale will be examined. When $\alpha = 1$, it is also possible that $\alpha < 1$. First, we must point out that private firm profit should be positive under the second-best contract.

Then, we look into zero profit under the constant return to scale condition. By combining Equations (2) and (32), the unconstrained profit maximal model can be expressed as follows:

$$P_s(\overline{D}, \overline{y}) = L_{con} \cdot \overline{D}\left(B(\overline{D}) - \beta t(\overline{D}, \overline{y}) + \beta \overline{r} t'(\overline{r}) - c_e - c_{s1}\right) \tag{54}$$

Combining Equations (2) and (52), the unconstrained profit maximizing problem under the non-constant returns to scale can be expressed as follows:

$$P_s(\overline{D}, \overline{y}) = L_{con} \cdot \overline{D}\left(B(\overline{D}) - \beta t(\overline{D}, \overline{y}) + \frac{1}{\alpha} \beta \overline{r} t'(\overline{r}) - c_e - c_{s1}\right) \tag{55}$$

If $D \to 0$, average link travel time is close to the free-flow travel time, $t(0)$, for any given y, and congestion externality will be close to zero. Therefore, private profits $P_s(\overline{D}, \overline{y}) \geq 0$, including Equations (54) and (55), are guaranteed by the following condition.

$$B(0) - \beta t(0) > 0 \tag{56}$$

From the above analysis, we can see that zero profit has the same condition no matter whether constant return to scale or non-constant returns to scale is used, namely, $B(0) > \beta t(0)$. However, Condition (56) is too ideal in reality, since traffic demand should be buoyant. Otherwise, it is unnecessary to build a new road.

When Condition (56) is satisfied, a zero-profit contract will exist, and then (D^*_{zp}, y^*_{zp}) can be determined by Equations (32) and (52). The zero-profit state is as follows:

$$P_s(D^*_{zp}, y^*_{zp}) = L_{con} \cdot [D^*_{zp} B(D^*_{zp}) - \beta D^*_{zp} t(D^*_{zp}, y^*_{zp}) - C_s(D, y)] - I(y^*_{zp}) = 0 \tag{57}$$

Toll charges under zero profit can be determined by

$$p^*_{zp} = B(D^*_{zp}) - \beta t(D^*_{zp}, y^*_{zp}) = c_e + c_{s1} - \beta r^*_{zp} t'(r^*_{zp});\ if\ \alpha = 1 \tag{58}$$

$$p^*_{zp} = B(D^*_{zp}) - \beta t(D^*_{zp}, y^*_{zp}) = c_e + c_{s1} - \frac{\beta}{\alpha} r^*_{zp} t'(r^*_{zp});\ if\ 0 < \alpha < 1 \tag{59}$$

With both constant and non-constant return to scale, an optimal combination (D^*, y^*) would yield a positive profit if $0 < \alpha < 1$; otherwise, the payoff will be negative. From the left side of the equations, the toll on PTRs will change with the trade regulation of carbon emissions.

6. Government Regulation

So far, we have looked into the properties of operation cost and first-best and second-best contracts. Usually, the private firm is a financial schemer who is assumed to chase the maximum amount of profit out of the investment. Once the government sets the limitations of the regulation variables, the private firm will modify other variables in order to achieve the maximum amount of profit. Averch and Johnson (1962) and Takayama (1969) studied a firm's behavior under governmental regulation [37,38]. Tan and Yang (2010) examined the behavior of private firms operating BOT roads under governmental regulation [3]. In this section, we now explore various regulatory mechanisms, including the rate-of-return regulations, demand, and price-cap, which may affect the private firm's choice of the combination of toll and capacity. Section 1 shows that the government will allow private firms to operate PTRs if $C_g(D,y) \geq C_s(D,y)$. The following regulation analysis is all based on this condition.

6.1. Rate-of-Return Regulation

A rate-of-return (ROR) regulatory mechanism is investigated, and private profit is restricted to not exceed a "reasonable" rate-of-return. Private firms can freely select the combination of PTR variables, including toll and capacity, if the project profits do not exceed a reasonable rate.

s is denoted as the ROR of a firm's management and construction investment and s_1^*, $s_1^* \geq 0$ is the fair ROR determined by the government for a given acceptable contract (p, D) when a private firm is responsible for building and operating roads. Under this case, the government will limit private firms as follows:

$$s = \frac{L_{con} D \cdot p - C_s(D,y) - I(y)}{C_s(D,y) + I(y)} \leq s_1^* \tag{60}$$

Under Equation (60), the profit-maximizing problem for private firms can be illustrated as

$$\max_{p \geq 0, y \geq 0} L_{con} pD - C_s(D,y) - I(y) \tag{61}$$

subject to Condition (60).

The following proposition is obtained to explain private firm behavior under the ROR regulations.

Proposition 4. *Let (D^*, y^*) be a non-monopoly optimal solution under the ROR regulation s_1^*. Then, the private firm selects a PTR contract $D < D^*, y \geq y^*$.*

Proof. s^* is the biggest rate-of-return based on a predetermined optimal contract (D^*, y^*). Therefore, it can be expressed as

$$L_{con} pD - C_s(D,y) - I(y) = s^*[C_s(D,y) + I(y)] \tag{62}$$

Therefore, the profit objective function $P(D,y)$ can be expressed as $\max_{p \geq 0, y \geq 0} s^*[C_s(D,y) + I(y)]$; this is equivalent to maximizing the total investment of operation cost C_s and construction cost $I(y)$, or maximal capacity y since $C_s(D,y)$ and $I(y)$ are an increasing function of y. Viewing y and p as functions of D from Equations (62) and (2), we differentiate y from D, as follows:

$$\begin{aligned} &\Big((s^*+1)[C_s(D,y) + I(y)] - \beta L_{con} \tfrac{D^2}{y^2} t'(\tfrac{D}{y})\Big) \tfrac{\partial y}{\partial D} \\ &= L_{con} \cdot \Big(B(D) + qB'(D) - \beta t(\tfrac{D}{y}) - \beta \tfrac{D}{y} t'(\tfrac{D}{y})\Big) \end{aligned} \tag{63}$$

If (D^*, y^*) is a solution of Equation (61), $\partial y / \partial D$ is equal to zero, and hence the right-hand side of Equation (63) is also equal to zero. Because $L_{con} = 0$, the government will operate the road by itself; we have $L_{con} \neq 0$, then the term in the bracket on the right-hand side of Equation (63) equals zero, which is the same as the necessary condition of social optima, as seen in Equation (40). This observation tells us (D^*, y^*), the social optimal solution to the problem, contradicts the optimal solution $\partial y / \partial D \neq 0$. Next, we prove that $\partial y / \partial D < 0$. By subtracting profit $P(D,y)$ from social welfare, we obtain

$$C_s(D,y) - P(D,y) = L_{con} \cdot (\int_0^D B(w)dw - DB(D)). \tag{64}$$

Note that the right-hand side term of Equation (64) represents total consumer surplus, which increases as D.

The derivative $\frac{\partial y}{\partial D} > 0$ at (D^*, y^*) means that capacity at y can increase with demand D increasing under Condition (62). Then, the private firm profit $P(D, y) = s^*(C_s(y) + I(y))$ will increase, since $P(D, y)$ is increasing similar to y from the above analysis. From Equation (64), social welfare $W(D, y)$ strictly increases from the value of $W(D^*, y^*)$ as both profit and demand increase. This result contradicts the optimal assumption. Therefore, under government regulation $s \leq s^*$, the private firm will select a PTR contract with $D < D^*$ and $y > y^*$. Thus, the proposition is proved. □

The above proposition shows that rate-of-return regulation is ineffective, since private firms will select a higher toll charge and road capacity. Travel demand will decrease under the condition of higher toll charges. Private firms will increase the road capacity. So, this kind of overinvestment is wasted.

6.2. Demand Regulation

If (D^*, y^*) is an optimal PTR combination and the government expects travel demand $D \geq D^*$, the private firm will select $D = D^*$; namely, the regulation condition $D \geq D^*$ is equal to $D = D^*$. Under the demand regulation, the government will allow the private firm to make a choice of price p and capacity y subject to the bottom level of traffic volume $D \geq D^*$, where D^* is set by the government. In the following proposition, we show that private firms will select road capacity to obtain acceptable profit under regulation $D \geq D^*$ considering operation cost in the model.

Proposition 5. *Under this assumption, if the government allows the private firm to choose $D \geq D^*$, an optimal PTR contract is obtained.*

Proof. Assume that $P(D^*, y^*)$ is optimal. At the beginning of this paper, we showed that the government will let private firms choose the whole life course of their management of PTRs. Under the travel demand condition $D = D^*$, the private firm will determine capacity by solving the following problem.

$$\max_{y \geq 0} P(D^*, y) = L_{con} \cdot D^* \cdot p(D^*, y) - C_s(D^*, y) - I(y) \quad (65)$$

In view of the toll function $p^* = B(D^*) - \beta t(D^*, y^*)$, where D^* is determined by the government, Problem (65) is equivalent to

$$\min_{y \geq 0} TO_s = L_{con}[\beta D^* t(D^*, y) + C_s(D^*, y)] + I(y) \quad (66)$$

where TO_s is total cost, including the time cost and operation cost of the road user. We assume that capacity y_s is the private firm's choice, and that y_s is the optimal solution to (66). The private firm will choose to put forward the PTR contract (D^*, y_s). Then, we need to prove that (D^*, y_s) is optimal for the following problem.

$$\begin{aligned} \max_{y \geq 0} W(D^*, y) &= L_{con}[\textstyle\int_0^{D^*} B(w)dw - \beta D^* t(D^*, y) - C_s(D^*, y)] - I(y) \\ &= L_{con} \textstyle\int_0^{D^*} B(w)dw - \{L_{con}[\beta D^* t(D^*, y) + C_s(D^*, y)] + I(y)\} \end{aligned} . \quad (67)$$

It is known that the second term of (67) is equal to (66), and that the optimal capacity y^* is the solution to social welfare (67), which was given at the beginning of this section. Comparing capacities y_s and y^*, we know that $W(D^*, y^*) = W(D^*, y_s)$ and (D^*, y_s) is an optimal PTR combination adopted by a private firm. This completes the proof. □

Proposition 5 shows that demand regulation is an effective regulation. Under the minimum value of travel demand set by the government, the private firm will freely choose a preferable combination of toll charge and capacity to maximize its profit. And a private firm will provide a better service for road users and reduce carbon emissions according to government policy.

6.3. Price-Cap Regulation

The price-cap regulation permits private firms to set the toll charge below or equal to a price ceiling set by the government. We suppose that the optimal PTR contract is (D^*, y^*) or (p^*, y^*), and corresponding to the upper limit of the toll is $p^* = B(D^*) - \beta t(D^*, y^*)$. Therefore, under the price-cap regulation constraint that the government sets $p \leq p^*$, a private firm will maximize their profit by regulating (D, y) at the toll value p^*. The model is given as appears below:

$$\max_{q \geq 0, L_{con} \geq 0} P = L_{con}[p^* D - C_e(D) - C_s(D) - C_s(y)] - I(y) \tag{68}$$

subject to Condition (37) and

$$p(D, y) = p^* \tag{69}$$

We first analyze the variable demand D. Taking the derivative of P in D $D = D^*$, we obtain the following equation

$$\left.\frac{dP}{dD}\right|_{D=D^*} = B(D^*) + D^* B'(D^*) - \beta t(D^*, y^*) - \beta D^* t'(D^*, y^*) - c_e - c_{s1} \tag{70}$$

If the targeted optimal solution (D^*, y^*) is a social optimal solution, the private firm will use (D^*, y^*) to implement it. If (D^*, y^*) is not a social optimal solution, then $\frac{dP}{dD} < 0$ at $D = D^*$. Therefore, the private firm will select (D, y) $D < D^*$ under the price-cap regulation $p \leq p^*$ in order to obtain more profit.

Next, we consider capacity y under the price-cap regulation. Assuming D as a function of y, we rewrite objective function Equation (68) for the given Equation (69), namely, $p = p^*$ as

$$P_s(y) = L_{con}[p^* D(y) - C_e(D) - C_s(D) - C_s(y)] - I(y) \tag{71}$$

where $P_s(y)$ is private firm profit under the condition of $p = p^*$. Taking the derivative of Equation (69) in y,

$$\frac{dq(y)}{dy} = -\frac{\partial p(D, y)/\partial y}{\partial p(D, y)/\partial D} \tag{72}$$

Then, we take the derivative of Equation (71) in y at (D^*, y^*) and apply (72) to it.

$$\frac{dP_s(y^*)}{dy} = L_{con} \cdot [\frac{p^*}{-p'(D^*)} \frac{\partial p(D^*, y^*)}{\partial y} - C_s'(y^*)] - I'(y^*), \tag{73}$$

where $p'(D^*) = \partial p(D^*, y^*)/\partial D$. From Equation (1), we have

$$\frac{\partial p}{\partial D} = B'(D) - \beta t'(D, y). \tag{74}$$

From Assumption 1, $B(D)$ decreases with D, and $t(D, y)$ increases with D $\frac{\partial p}{\partial D} < 0$. Equation (1) is transformed to $p(D, y) = B(D) - \beta \cdot t(D, y)$, which is differentiated at y with $D = D^*$, and we obtain $\frac{\partial p}{\partial y} = -\beta t'(D^*, y)|_y$. Based on Assumption 1, $t(D, y)$ decreases with y, then $\frac{\partial p}{\partial y} > 0$. From Assumption 2, both $C_s(y)$ $I(y)$ are increasing y functions $\frac{dP_s(y^*)}{dy} < 0$, which infers that $P_s(y)$ will grow with reducing y. Therefore, private firms will select a road capacity $y < y^*$ under the pricing-cap regulation $p \leq p^*$, and contracts will not be socially optimal.

The price-cap regulation is inefficient unless the targeted contract is the socially optimal solution. The private firm would not be incentivized to offer better service quality for the user and adopt lower road capacity, and the private firm would not increase the road capacity either.

7. Discussion

Traditional research on BOT roads does not include the study of PTRs [3,11,13,14]. From traditional academic perspectives, it is regarded that BOT roads transferred to the government will become public roads for free use. However, continuing tolling these roads will be an adequate traffic operation strategy to provide quality transportation services and efficiency, decrease financial burden, and reduce car emissions. However, there is a notable gap in the operation strategy for PTRs in the literature regarding reducing carbon emissions in the transportation industry. Our study utilizes an optimal model addressing this gap, indicating that the operational strategy for PTRs positively contributes to lowering carbon emissions for the global environment. Our research method aligns with mainstream academic methods [11,13,34,35]. We present our primary results and discussions as follows.

First, our research findings of the ratio r of the road are constant and equal to the social optimal and private profit volume/capacity ratio. This conclusion aligns with mainstream academic results [3,29,30], and it illustrates that private firms tend to provide a lower road capacity and higher toll charge, resulting in more inadequate traffic flow. In contrast, the government offers higher road capacity and lower toll charges with increased traffic flow. Therefore, the volume/capacity values of both parties are identical.

Secondly, the analysis concludes that the operator should manage PTRs for the entire duration by maintaining self-financing while accounting for operation costs. We found that if the government selects a private firm to operate PTRs, the private firm will choose the second-best PTR contract. In the second-best contract, the demand is smaller than the demand that maximizes social welfare and bigger than the demand that maximizes private profit. This means that the government must set the necessary regulations to urge private firms operating roads to consider carbon emissions and social welfare.

Thirdly, we found that the self-financing theory still holds when operation cost is considered, which is the same as that found in mainstream research [36]. Total revenue can or cannot cover operation cost, which is decided by the value return to scale α. If constant return is kept to scale, the total revenue will just cover the operation costs; if returns to scale are increased, the total revenue will be more significant than all the operation costs, and the government or private firms will earn more from the road; and if returns to scale are decreased, there is not enough revenue to cover all the operation costs. This could greatly ease the financial burden on the government. And this finding tells us that if the government or a private firm wish to maximize their social welfare or private profit, they must provide a good service and satisfying road capacity. At the same time, zero profit is analyzed, and the corresponding charging strategy is given, which provides recommendations on setting the charging value for PTRs in the future.

Finally, governmental regulations for carbon emission costs are also studied. The results show that a private firm operating using a PTR strategy will be affected by demand and rate-of-return regulations. Under the regulation of rate-of-return, private firms tend to increase road capacity in order to obtain more profit; under the demand regulation, private firms tend to provide a better service for road users; and under the price-cap regulation, private firms operate PTRs at capacity, meaning that private firms will not improve the transportation service. Therefore, if the government sets this regulation for private firms, the government then has another incentive to urge private firms to provide high-quality transportation services, such as with subsidies.

8. Conclusions and Implications

8.1. Research Conclusions

This paper studied traffic operation strategies for PTRs to reduce carbon emissions based on operation costs. With either the government or a private firm operating PTRs, operation costs may significantly affect carbon emissions due to different capacities and tolls. In the analysis, social welfare, profit, zero profit, and self-financing problems were further complicated, since private firms tend to obtain more profit from operating PTRs, and the government wishes to obtain increased social welfare in the background of considering decreasing carbon emissions globally. The main conclusions of this study are as follows: (1) the ratio r of road is constant and equal to the social optimal and private profit volume/capacity ratio $\widetilde{r}$ and $\bar{r}$. This implies that a private firm and the government will choose the same volume/capacity ratio. However, private firms tend to provide a lower road capacity and higher toll charge, whereas the government offers higher road capacity and lower toll charges with increased traffic flow. (2) When operation cost is considered, the self-financing theory still holds. Total revenue can cover operation cost. (3) The results of the regulation analysis show that private firms tend to increase road capacity under the rate-of-return regulation; under the demand regulation, private firms tend to provide a better service for road users and reduce carbon emissions; and under the price-cap regulation, private firms operate PTRs at capacity when BOT roads are transferred to the government from private firms.

8.2. Policy Recommendations

Based on the above conclusions, some recommendations are proposed.

Firstly, the contracts of BOT roads will confront expiry in the future, so how to operate this kind of road is a wide-scale problem. The construction of PTR operating policies is widely accepted. Our research encourages national or regional governments to develop policies before BOT roads are transferred to the government, ensuring that roads can continue to operate efficiently, whilst also reducing carbon emissions in the background.

Secondly, the government has a positive attitude toward controlling carbon emissions, so the government could develop a travel strategy based on reducing carbon emissions, such as providing incentive subsidies.

Thirdly, our research provides important implications for the government to make policies based on PTR traffic operation strategies, such as selecting a private firm to operate PTRs continuously, signing the operation contract to obtain reasonable social welfare, setting regulations to control traffic flow, or using the public–private partnership (PPP) operation strategy on PTRs, amongst others.

8.3. Research Limitations and Further Research

Our research has some limitations.

Firstly, this study assumes that the government and private firms have the same knowledge of operating PTRs. This is an asymmetric information problem; the government and private firms will obtain different information about operating PTRs.

Secondly, we assume that all road users are homogeneous. However, vehicles are different from each other. Additionally, individual drivers have different driving behaviors, and contribute to global carbon emissions based on their daily driving activities [25]. Different vehicles also have different levels of emissions, and have other impacts on the environment.

Since the operation cost of PTRs raises a series of concerns for further research, this work is just the beginning. The new factor of operation cost in PTRs requires extensive study. Future research should delve into PTR operation strategies based on asymmetric information in order to respond effectively to operational issues. The government and private firms can play a game through contracts based on asymmetric information. Additionally, based on different kinds of vehicles, researchers have the opportunity to build a new model to analyze carbon emissions and provide a new operation strategy. Furthermore, based on a deeper understanding of drivers' behaviors when interacting with each other, other researchers could conduct comprehensive and detailed research on carbon emissions based on drivers' behaviors. These avenues of study could provide a broad perspective on research into PTR operation strategies in terms of national policy and operational regulations.

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