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Remanufacturing Operations in Different Financial Ownership Structures with Consideration of the Upwards Supplier

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Abstract: Under the increasing environmental pressure, remanufacturing has increasingly become a new mode of recycling economy and upgrading and transforming the equipment manufacturing industry. Some enterprises include remanufacturing businesses in the original production system by holding or controlling shares in other remanufacturing enterprises. This paper builds a two-echelon supply chain model composed of a supplier, a manufacturer, and a remanufacturer, considering the different ownership structures (i.e., shareholding and share-controlling) between them, in which the supplier sells non-remanufacturable parts to the manufacturer and the remanufacturer. At the same time, the optimal decisions of each firm are considered. The results show that for the manufacturer, a higher shareholding ratio means that it can obtain more profits. For the supplier, the impact of the shareholding ratio depends on the manufacturing cost. When the manufacturing cost is relatively low, the stock sharing relationship between the manufacturer and the remanufacturer will decrease the supplier's profit. When the manufacturing cost is relatively high, it will depend on the shareholding ratio. In the case of shareholding between the manufacturer and the remanufacturer, a higher shareholding ratio will decrease the supplier's profit. From the perspective of the supply chain, when the production cost is high enough, the supply chain's profit decreases first and then increases with the shareholding ratio. Furthermore, the increase in the manufacturer's shareholding in remanufacturing does not always improve the remanufacturing proportion of products.

Keywords: financial ownership structure; closed-loop supply chain; remanufacturing; supply chain management

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1. Introduction

Remanufacturing is a technical measure or engineering activity that takes products that have reached their service life and makes their quality or performance reach that of the original products through remanufacturing technology and processes. Consistent with the overall growth in manufacturing, during 2012–2017, an annual growth rate of 1.5% was observed in aircraft maintenance, repair, and overhaul and 2% in auto parts remanufacturing (Vlaanderen 2018) [1]. Compared with new products, remanufacturing significantly lower carbon emissions by 26.75–65.13% and saves 36–73% of materials, 28–63% of energy, and 34–88% of water, with significant economic and environmental benefits (Liu et al., 2023) [2]. Remanufacturing is an advanced form of recycling in the circular economy and a new mode of upgrading and transforming the equipment manufacturing industry. It is also an important way to achieve the goal of carbon peak and carbon neutrality. The governments of many countries have issued relevant policies to encourage or support remanufacturing enterprises.

From the perspective of enterprises, manufacturers are paying more and more attention to remanufacturing, and remanufacturing has become an important operation strategy for more manufacturing enterprises. At the same time, in the related operations of

remanufacturing, there is a competitive relationship between manufacturers and remanufacturers. Some manufacturing enterprises, especially some large-scale manufacturers, will build or control their remanufacturing departments when they have sufficient funds or meet the technical standards, such as Caterpillar, Apple, and Boeing. They build up their own remanufacturing companies or departments. Some manufacturers, especially some middle- and small-sized manufacturers, do not establish remanufacturers or remanufacturing departments because of many reasons, including cost or technology factors. In some industries, third-party remanufacturers have dominated the remanufacturing sector (Zou et al. 2016) [3]. Besides the above two cases, some manufacturing enterprises will take part in remanufacturing in an intermediate way, i.e., hold part of the shares of the third-party remanufacturing. For example, Shandong Energy Heavy Equipment Manufacturing Co., Ltd., controls Shandong Energy Heavy Equipment Group Han's Remanufacturing Co., Ltd. (79.8%), and Shaanxi Blower Co., Ltd., partially holds Xi'an Intelligent Remanufacturing Co., Ltd. (10%). When the share ratio is less than 50%, the manufacturer and the remanufacturer have a shareholding relationship. However, the manufacturer and the remanufacturer have a controlling relationship when the share ratio is higher than 50%. Based on the competitive relationship between manufacturers and remanufacturers, the operations of the supply chain become more complex.

In the process of remanufacturing, only high-value and high-durability parts are generally selected for remanufacturing, and low-value and easily worn parts are directly replaced (Fleischmann et al., 2003) [4]. As far as automobile remanufacturing is concerned, the main remanufacturable components include the engine, steering gear, transmission, front and rear axles, and frame, while other parts are new parts in the remanufacturing process. For such non-remanufacturable components, remanufacturers also have to purchase from suppliers. This study takes the upwards procurement of non-remanufacturable components in remanufacturing into consideration. For non-remanufacturable components, such suppliers, as joint partners of manufacturers and remanufacturers, need to adopt different pricing strategies according to different manufacturers' and remanufacturers' conditions and financial ownership structures. Therefore, how the members in the supplier chain deal with the financial ownership structure between them is a new and important question.

Based on the above research background, this research aims to answer the following questions: Given the remanufacturer's financial ownership structure, what is the optimal remanufacturing strategy for the remanufacturer? From the perspective of the supplier, facing manufacturers and remanufacturers with different ownership structures, how should the supplier decide the wholesale prices? What kind of financial ownership structure is most beneficial to the supply chain? To answer these questions, this paper builds a two-level supply chain model composed of a supplier, a manufacturer, and a remanufacturer, considering the different ownership structures (shareholding and share-controlling) between the manufacturer and the remanufacturer, in which the supplier provides non-remanufacturable components to the manufacturer and the remanufacturer at the same time. Further, the optimal decision-making of each enterprise is considered to provide operation suggestions for relevant enterprises.

From the perspective of extant research, the competition and cooperation between manufacturers and remanufacturers are not new. Much research on remanufacturing operations has been carried out by scholars. However, limited research has been devoted to the financial ownership relationships between manufacturers and remanufacturers, especially with the consideration of the upwards supplier. In this paper, we seek to provide insights into how different financial ownership structures influence the remanufacturing operations of the supply chain.

Our results reveal that, for the manufacturer, a higher shareholding ratio means that it can obtain more profits. For the supplier, the impact of the shareholding ratio is dependent on the manufacturing cost. When the manufacturing cost is relatively low, the stock sharing relationship between the manufacturer and the remanufacturer decreases the supplier's profit. When the manufacturing cost is relatively high, it will depend on the

shareholding ratio. In this case, a higher shareholding ratio will decrease the supplier's profit. From the perspective of the supply chain, when the production cost is high enough, the supply chain's profit decreases first and then increases with the shareholding ratio. Furthermore, the increase in the manufacturer's shareholding in remanufacturing does not always improve the remanufacturing proportion of products.

2. Literature Review

There are mainly three research areas related to this research: remanufacturing channel management, supplier decision-making under remanufacturing, and supply chain management with partial ownership.

2.1. Remanufacturing Channel Management

Firstly, regarding the remanufacturing channel management, in the existing research, remanufacturing modes are mainly divided into manufacturer remanufacturing, third-party remanufacturing, retailer remanufacturing, etc. Firstly, for manufacturer remanufacturing, to improve the environmental and economic benefits, OEMs need to coordinate the production cost structure, recovery rate, product life cycle, component durability, etc., in remanufacturing. (Reimann et al. 2019, Ferguson and Toktay 2009, Majumder and Groenevelt 2001, Debo et al. 2005, Huang et al. 2019, Tian et al. 2019) [5–10]. Secondly, some scholars have explored the situation of retailers undertaking remanufacturing. Reimann et al. (2019) analyzed the pricing decisions under a supply chain composed of a manufacturer and a retailer. In this model, remanufacturing is carried out by the manufacturer or the retailer. At the same time, the manufacturer can reduce remanufacturing costs through process innovation and explore pricing decisions and manufacturers' process innovation efforts [5]. Timoumi et al. (2021) explored whether manufacturers should remanufacture by themselves or let their retailers remanufacture [11]. Thirdly, the emergence of the third-party remanufacturer has become the focus of research on the cooperation model choice between OEMs and stakeholders. Through authorization or collaborative cooperation, enterprises can reduce the price of remanufactured products and improve market competitiveness (Agrawal et al. 2015, Oraiopoulos et al. 2012, Atasu et al. 2012, He 2015, Vedantam & Iyer, 2021, Chen & Chen 2019, Esenduran et al. 2019, Sun et al. 2024) [12–19]. He (2015) compared centralized remanufacturing and decentralized remanufacturing channels [15]. Atasu et al. (2012) explored and compared the environmental protection design, profit, and consumer surplus between the two recycling modes: Collaborative Producer Recycling (CPR) and Individual Producer Recycling (IPR) [14]. For this part of the literature, the contribution of this study is to consider the ownership relationship structure between the manufacturer and the third-party remanufacturing and consider the remanufacturing-related decisions on this basis.

2.2. Supplier Decision-Making under Remanufacturing

Secondly, we consider supplier decisions under remanufacturing. Some scholars have conducted in-depth research on remanufacturing-related issues from the perspective of the upstream of the supply chain (i.e., suppliers) (Xiong et al. 2013, Xiong et al. 2016, Wu and Zhou 2019, Jin et al. 2017, Huang et al. 2017, Qian 2020, Duan 2023) [20–26]. Xiong et al. (2013) analyzed a closed-loop supply chain composed of a remanufactured components supplier and manufacturer and analyzed the interaction between participants. In this model, manufacturers can purchase new parts from suppliers to produce new products and also remanufacture waste parts to produce remanufactured products [20]. Xiong et al. (2016) analyzed the two modes of manufacturer remanufacturing and supplier remanufacturing in a decentralized closed-loop supply chain and analyzed the selection of the two modes in different situations from the perspective of different stakeholders [21]. The study of Wu and Zhou (2019), which is similar to this paper, explored a model of a manufacturer and a third-party remanufacturer facing the same supplier and compared the two wholesale price pricing strategies of suppliers to the two enterprises (unified pricing

and differential pricing) and the impact of the two pricing strategies on each enterprise [22]. The difference between this study and our paper is that the ownership structure between the manufacturer and the remanufacturing is considered, and the purpose is to explore the impact of different ownership structures on the relevant strategies of suppliers.

2.3. Supply Chain Management with Partial Ownership

Thirdly, some scholars have also studied the related issues of supply chain management with partial ownership. Aviv and Shamir (2021) studied the impact of cross-shareholdings (retailers hold the shares of their competitors) on two key operational decisions in the supply chain, that is, competitive retailers purchase information and products from a single supplier [27]. The closest research to our paper is that of Shi et al. (2020) [28], which studied a company that is composed of two departments, one responsible for designing and manufacturing new products and the other responsible for remanufacturing business. These two departments have independent accounting and overall accounting. At the same time, the company sells new products and remanufactured products directly to consumers (direct sales) or through independent retailers (indirect sales). The conclusions of this study show that the organizational structure of an enterprise will affect its marketing decisions. The differences between this study and our paper are as follows: firstly, our paper considers the intermediate state (shareholding and share-controlling) between complete independence and complete integration between manufacturers and remanufacturers and studies remanufacturing-related decisions under this research scenario; secondly, their research explored the downstream marketing channel, while our paper studies the upstream procurement channel selection.

2.4. Literature Summary

For relevant research on remanufacturing operations management, most scholars are concerned about competition and cooperation between manufacturers and remanufacturers and the channel choices, while limited research has been devoted to the financial ownership relationships between them, especially with the consideration of the upwards supplier. In Table 1, we list the studies that are closely related to our work and position our research in a literature context.

Table 1. Comparison between this study and closely related research.

	He (2015) [20]	Xiong et al. (2013) [20]	Wu and Zhou (2019) [22]	Shi et al. (2020) [28]	This Study
Upwards supplier	×	√	√	×	√
Financial ownership	×	×	×	√	√
Cooperation between the manufacturers and remanufacturers	√	×	×	√	√

3. The Model and Financial Ownership Structure

We consider a manufacturer and a remanufacturer where the manufacturer produces new products and undertakes remanufacturing business by holding part shareholding of the remanufacturer who collects and remanufactures the returned product. The shareholding ratio of the manufacturer is ρ_i ($i = H,\ C$). We use ρ_H ($0 \le \rho_H \le \frac{1}{2}$) to denote the shareholding ratio in the shareholding mode. In the shareholding mode, the remanufacturer has the independent decision right, and the manufacturer will receive a ratio ρ_H of the profits of the remanufacturer. In this case, if $\rho_H = 0$, it is a special case of an independent remanufacturer, where there is no equity relationship between the manufacturer and the remanufacturer (Xiong et al. 2016, Wu and Zhou 2019) [21,22]. We use ρ_C ($\frac{1}{2} < \rho_C \le 1$,) to denote the shareholding ratio in the share-controlling mode. In the share-controlling mode, the manufacturer has dominant decision-making power over the remanufacturer and it can control the remanufacturer's decision-making. Meanwhile, the manufacturer shares the profits according to the shareholding ratio. In this case, if $\rho_C = 1$, it is a special case of a

centralized manufacturer and remanufacturer, in which the remanufacturer is a division of the manufacturer (Shi et al. 2020) [28].

Both the manufacturer and the remanufacturer need to purchase a component that cannot be remanufactured either independently from the supplier. All the enterprises are profit-maximizing firms, and we assume that the manufacturer and the remanufacturer are in a Cournot competition.

The event sequence is as follows. Firstly, the suppliers decide the wholesale prices w_n and w_r to the manufacturer and the remanufacturer. Then, the manufacturer and the remanufacturer decide the production quantity of new products and remanufacturing products, q_n and q_r. We assume that the manufacturer has enough capacity to fulfill the demand for new products. The remanufacturer cannot remanufacture more than the past sales of new products. For simplicity, we assume that all used products are returned and remanufactured. The production cost of new products and remanufactured products is c_n and c_r, respectively, and $1 > c_n > c_r \geq 0$. For simplicity, we assume that $c_r = 0$. Thus, the manufacturing cost c_n can be regarded as the cost advantage of remanufacturing. The higher the c_n, the more cost advantage the remanufacturing has and the stronger the incentive for remanufacturers to remanufacture returned products.

The willingness to pay (WTP) for the new product of consumers is uniformly distributed in $(0, 1)$, and the market size is normalized to 1. For the same kind of products, the consumer's perceived values of new and remanufactured products have a certain level of difference. In particular, consumers have a lower perceived value of remanufactured products. Consumers are more willing to buy new products than remanufactured ones at the same price because they believe that the quality of remanufactured products is lower than that of new ones. Thus, the difference is reflected by the discount coefficient δ. Similar to Ondemir and Gupta (2014) and Esenduran et al. (2016) [17,29], we assume that if a consumer is willing to pay V for a new product, then their WTP for a remanufactured product is δV. Each customer purchases at most one unit of product, either new or remanufactured. Thus, the prices of new and remanufactured products are as follows:

$$p_n = 1 - q_n - \delta q_r \text{ and } p_r = \delta(1 - q_n - q_r).$$

In this paper, the subscript $i \in \{M, R, S\}$ denotes the manufacturer, the remanufacturer, and the supplier, respectively. We consider two financial ownership structures: shareholding mode and share-controlling mode. The superscript $j \in \{H, I\}$ denotes the shareholding mode and share-controlling mode, respectively. Figure 1 provides a visual illustration of the four models.

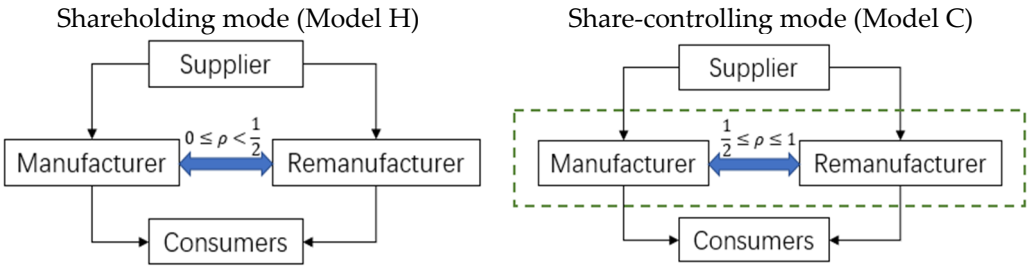

Figure 1. Two mode structures.

4. Main Model

In this section, we consider two models in the shareholding mode (Model H) and the share-controlling mode (Model C).

4.1. Shareholding Mode (Model H)

We first consider the shareholding mode, in which the remanufacturer has the independent decision right, and the manufacturer receives a ratio ρ_H ($0 \leq \rho_H \leq \frac{1}{2}$) of the profits of the remanufacturer. In the shareholding mode, the remanufacturer has the right to make

its decisions independently, and both the manufacturer and the remanufacturer procure components that cannot be remanufactured from the same supplier. The manufacturer's profits come from two parts, one from the production and sale of new products and the other from the shareholding to the remanufacturer. The remanufacturer's profit is the total profit minus the manufacturer's shareholding. The supplier's profit comes from two parts, supplying components to the manufacturer and to the remanufacturer.

The decision sequence is as follows. Firstly, the supplier decides the wholesale prices w_n^H and w_r^H to the manufacturer and the remanufacturer. Given w_n^H and w_r^H, they decide the optimal quantity decisions to maximize their profits. Then, the manufacturer and the remanufacturer decide the production quantity of new products and remanufacturing products, q_n^H and q_r^H, respectively. Thus, we proceed by analyzing the quantity competition between the manufacturer and the remanufacturer. Then, we derive the supplier's optimal wholesale prices.

The profit functions of the manufacturer, the remanufacturer, and the supplier are as follows:

$$\pi_M\left(q_n^H\right) = \left(p_n - w_n^H - c_n\right)q_n^H + \rho_H\left(p_r - w_r^H\right)q_r^H,$$

$$\pi_R\left(q_r^H\right) = (1-\rho_H)\left(p_r - w_r^H\right)q_r^H,$$

$$\pi_S\left(w_n^H, w_r^H\right) = w_n^H q_n^H + w_r^H q_r^H,$$

$$s.t.\ 0 \le q_r^H \le q_n^H.$$

We solve the decision problems of Model H by backward induction and obtain the optimal quantities given the wholesale prices as the following lemma.

Lemma 1. *In Model H, given w_n^H and w_r^H, there exist two thresholds, t_1^H and t_2^H, such that the optimal quantities of new and remanufactured products $\left(q_n^H,\ q_r^H\right)$ are as follows:*

$$\begin{cases} \left(\frac{1-c_n-w_n^H}{2+\delta(1+\rho_H)}, \frac{1-c_n-w_n^H}{2+\delta(1+\rho_H)}\right), if\ w_r^H \le\ t_1^H, \\ \left(\frac{\delta+\delta\rho_H+2c_n+2w_n^H-2-(1+\rho_H)w_r^H}{-4+\delta+\delta\rho_H}, \frac{\delta\left(1+c_n+w_n^H\right)-2w_r^H}{\delta(4-\delta-\delta\rho_H)}\right), if t_1^H < w_r^H \le t_2^H, \\ \left(\frac{1-c_n-w_n^H}{2}, 0\right), if\ w_r^H > t_2^H. \end{cases}$$

where $t_1^H = \frac{\delta(1+(-2+\delta+\delta\rho_H)+3c_n+3w_n)}{2+\delta(1+\rho_H)}$ *and* $t_2^H = \frac{\delta(1+c_n+w_n)}{2}$.

Lemma 1 shows that the supplier's wholesale price to the remanufacturer will correspond to three different scenarios of the remanufacturing strategy. When the wholesale price to the remanufacturer (w_r^H) is low enough, the remanufacturer will have enough profit margin to remanufacture. Therefore, it will adopt the full remanufacturing (FR) strategy and remanufacture the collected products as much as possible. As the w_r^H increases, the profit margin of remanufacturing gradually reduces and so does the remanufacturing scale. In this case, the partial remanufacturing (PR) strategy is adopted by the remanufacturer. If the wholesale price to the remanufacturer (w_r^H) is very high, the remanufacturer becomes unprofitable and will not remanufacture any used products; thus, it adopts a no-remanufacturing (NR) strategy.

To obtain the supplier's optimal wholesale prices, we first derive the supplier's constrained optimal wholesale price and the corresponding profit for each of the three strategies (FR, PR, NR) identified in Lemma 1, and then we explore the supplier's optimal wholesale prices that maximize its profit.

Proposition 1. *In Model H, the optimal wholesale prices of new and remanufactured products are as follows:*

$$\left(w_n^H, w_r^H\right) = \begin{cases} \left(\frac{8-\delta(2+3\rho_H)-(8-\delta(2+\rho_H))c_n}{16-\delta(2+\rho_H)^2}, \frac{\delta(\delta(1+\rho_H)(2+\rho_H)-2(4+\rho_H)+2\rho_H c_n)}{-16+\delta(2+\rho_H)^2}\right), if c_n \le c_1 \\ \left(\frac{2+\delta(5+\rho_H-\delta-\delta\rho_H)-(2+\delta(\rho_H+7))c_n}{4+2\delta(\rho_H+4)}, \frac{\delta(1+\delta(2\rho_H+5))+3c_n)}{4+2\delta(\rho_H+4)}\right), if c_n > c_1 \end{cases}$$

Furthermore, the remanufacturer adopts the PR strategy when $c_n \le c_1$*, and otherwise the remanufacturer adopts the FR strategy, where* $c_1 = \frac{(1-\delta)(2+\rho_H)}{\rho_H+6}$.

According to Proposition 1, in Model H, the supplier sells components to both the manufacturer and the remanufacturer; thus, it needs to make a trade-off between profits from the manufacturer and the remanufacturer when setting wholesale prices. If a higher wholesale price is set for the remanufacturer, it will increase the costs of the remanufacturer and benefit the manufacturer. The specific principle is that when the production cost of the new product is low, the competitive advantage of the new product is obvious, and the supplier can obtain greater profits from selling the new product. Therefore, the wholesale price for the remanufacturer will be appropriately raised to prevent the remanufacturer from producing too many remanufactured products. On the contrary, when the production cost of the new product is high, the remanufactured product has a greater competitive advantage, and the supplier hopes to gain more profits from selling the remanufactured product. Therefore, the wholesale price of the remanufacturer will be appropriately reduced to encourage it to carry out remanufacturing as much as possible.

Following the two situations of Theorem 1, we obtain the optimal remanufacturing strategy in different situations under various production costs, remanufacturing product discount coefficients, and shareholding ratios, as shown in Table 2.

Table 2. Optimal remanufacturing strategies in Model H.

c_n	δ	$\rho_H \le \frac{2-2\delta-6c_n}{-1+\delta+c_n}$	$\frac{2-2\delta-6c_n}{-1+\delta+c_n} < \rho_H < \frac{1}{2}$
$0 < c_n \le \frac{1}{3}$	$0 < \delta < 1 - 3c_n$		PR
	$1 - 3c_n < \delta < \frac{5-13c_n}{5}$	FR	PR
	$\frac{5-13c_n}{5} < \delta < 1$		FR
$\frac{1}{3} < c_n \le \frac{5}{13}$	$0 < \delta < \frac{5-13c_n}{5}$	FR	PR
	$\frac{5-13c_n}{5} < \delta < 1$		FR
$\frac{5}{13} < c_n \le 1$	$0 < \delta < 1$		FR

Table 1 illustrates that the optimal remanufacturing strategy selection of remanufacturers in the HC model is influenced by the production cost of new products, the discount coefficient of remanufactured products, and the shareholding ratio. When the production cost of the new product is high, the remanufacturer will only choose the full remanufacturing strategy (FR). The reason for this is that the cost advantage of remanufacturing is very large, and the remanufacturer will carry out full remanufacturing regardless of the discount coefficient and shareholding ratio of the remanufactured product. With the reduction in the production cost of new products, the remanufacturer will choose the partial remanufacturing strategy (PR) when the discount coefficient of remanufactured products is small and the shareholding ratio is large.

When the production cost of new products is lower than a certain level, if the discount coefficient of remanufactured products is small, the remanufacturer always chooses the partial remanufacturing strategy (PR) regardless of the shareholding ratio. If the discount coefficient of the remanufactured product is moderate and the shareholding ratio is large, the remanufacturer will choose the partial remanufacturing strategy. The reason for this is that when considering the shareholding relationship between the manufacturer and the remanufacturer, if the shareholding ratio is small, the manufacturer and the remanu-

facturer are relatively independent and the competition relationship is more obvious, so the remanufacturer should set a lower wholesale price and encourage them to carry out remanufacturing as much as possible.

When the shareholding ratio is large, the relative advantage of the manufacturer is more obvious. The supplier is more inclined to guarantee more profits from the manufacturer and set a higher wholesale price for the remanufacturer to restrain its remanufacturing. When the production cost is large enough, the cost advantage of the remanufacturer is quite obvious, and the equity ratio will not affect the remanufacturing strategy of the remanufacturer. Figure 2 shows the optimal remanufacturing strategy selection for remanufacturers under different production costs. With the increase in production cost, the area of FR strategy becomes larger and larger, that is to say, remanufacturers are more and more inclined to choose the FR strategy. In addition, to some extent, a larger equity ratio might hinder the remanufacturer from carrying out full remanufacturing. Therefore, to promote remanufacturing, manufacturers should control the equity ratio to some extent.

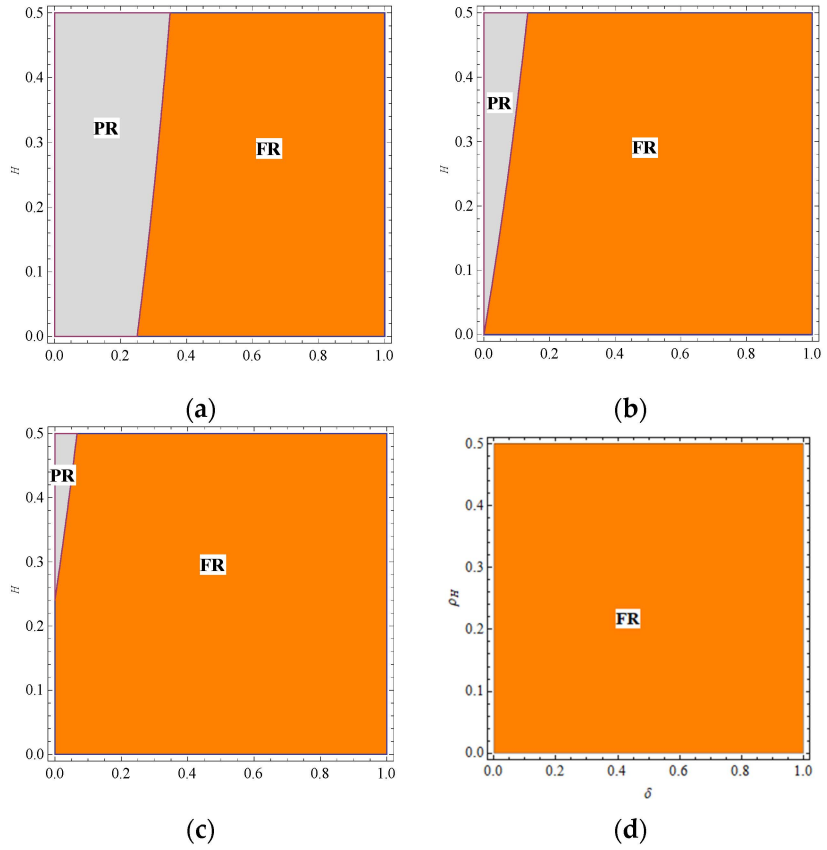

Figure 2. The optimal strategy of the remanufacturer in Model H. (**a**) $c_n = 1/4$; (**b**) $c_n = 1/3$; (**c**) $c_n = 14/39$; (**d**) $c_n = 1/2$.

Next, we further analyze the impact of the shareholding ratio on the wholesale price in Model H.

Corollary 1. *In the HC mode, (1) the wholesale price to the manufacturer decreases with the shareholding ratio; (2) when* $c_n > c_1$ *and* $\frac{\sqrt{1+4c_n}-1}{2} < \delta < 1$, *the wholesale price to the remanufacturer decreases with the shareholding ratio; otherwise, the wholesale price to the remanufacturer increases with the shareholding ratio*.

According to Corollary 1, there are two cases for the impacts of the shareholding ratio on the two kinds of wholesale prices: low production cost ($c_n \leq c_1$) and high production cost ($c_1 < c_n \leq 1$). In the first case, the remanufacturer will adopt a partial remanufacturing

(PR) strategy. At this time, with the increase in the manufacturer's shareholding in the remanufacturer, the sales quantity of remanufactured products has been increased to a certain extent. However, the supplier hopes to obtain more profits from the main source of profit—new products—so as to reduce the wholesale price to the manufacturer. In the second case, the remanufacturer will adopt the full remanufacturing (FR) strategy. At this time, all products will be remanufactured and sold, and the profit of the supplier will be constrained by the sale quantity of the new product. Therefore, the supplier will reduce the wholesale price to the manufacturer to further increase the quantity of products sold, so as to obtain greater profits.

4.2. Share-Controlling Mode (Model C)

In the share-controlling mode (Model C), the manufacturer has dominant decision-making power over the remanufacturer, and it can control the remanufacturer's decision-making. And the manufacturer shares the profits according to the shareholding ratio ρ_C ($0.5 < \rho_C \leq 1$). In the share-controlling mode, the manufacturer has dominant decision-making power over the remanufacturer, and the remanufacturer procures components that cannot be remanufactured from the supplier. In Model C, firstly, the supplier decides the wholesale prices w_n^C and w_r^C to the manufacturer and the remanufacturer. Given w_n^C and w_r^C, they decide their optimal quantity decisions to maximize their profits. Then, the manufacturer decides the production quantity of new products and remanufacturing products, q_n^C and q_r^C. Thus, we proceed by analyzing the quantity competition between the manufacturer and the remanufacturer. Then, we derive the supplier's optimal wholesale prices.

Because the remanufacturer has no decision rights because of the shareholding, we do not list the profit function of the remanufacturer. The profit functions of the manufacturer and the supplier are as follows:

$$\pi_M\left(q_n^C, q_r^C\right) = \left(p_n - w^C - c_n\right)q_n^C + \rho_c\left(p_r - w^C\right)q_r^C \text{ and } \pi_S\left(w^C\right) = w^C\left(q_n^C + q_r^C\right),$$
$$s.t.\ 0 \leq q_r^C \leq q_n^C.$$

We solve the decision problems of Model C by backward induction and obtain the optimal quantities given the wholesale prices as the following lemma.

Lemma 2. *In Model C, given w^C, there exist two thresholds, t_1^C and t_2^C, $t_1^C = \delta + \frac{\delta(1+3\rho_C)(-1+c_n+w_n)}{\rho_C(2+\delta(1+\rho_C))}$ and $t_2^C = \frac{\delta(-1+\rho_C+(1+\rho_C)c_n+(1+\rho_C)w_n)}{2\rho_C}$, such that the optimal quantities of new and remanufactured products $\left(q_n^C,\ q_r^C\right)$ are as follows:*

$$\begin{cases} \left(\frac{1-c_n-w_n^C}{2+\delta(1+\rho_C)}, \frac{1-c_n-w_n^C}{2+\delta(1+\rho_C)}\right), if w_r^C \leq t_1^C \\ \left(\frac{\rho_C\left(-2+\delta+\delta\rho_C+2c_n+2w_n^C-(1+\rho_C)w_r^C\right)}{-4\rho_C+\delta(1+\rho_C)^2}, \frac{\delta-\delta\rho_C-\delta(1+\rho_C)\left(c_n+w_n^C\right)+2\rho_C w_r^C}{\delta\left(-4\rho_C+\delta(1+\rho_C)^2\right)}\right), if t_1^C < w_r^C \leq t_2^C \\ \left(\frac{1}{2}\left(1-c_n-w_n^C\right), 0\right), if w_r^C > t_2^C \end{cases}$$

According to Lemma 2, the wholesale price to the remanufacturer will correspond to three different remanufacturing strategy scenarios. Similar to Lemma 1, when the wholesale price to the remanufacturer is low, the remanufacturer will have enough profit space to carry out remanufacturing, so it will adopt the full remanufacturing strategy (FR). When the supplier's wholesale price to the remanufacturer is moderate, the remanufacturer will gradually shrink the remanufacturing scale and adopt the partial remanufacturing strategy (PR). When the supplier's wholesale price to the remanufacturer is high, the remanufacturer will be unprofitable and will not manufacture, that is, the no-manufacturing strategy (NR). According to the optimal sales quantity in different situations obtained above, the optimal decisions and optimal profits in the case of share-controlling can be obtained, as shown in Proposition 2.

Proposition 2. *In Model C, the optimal wholesale prices of new and remanufactured products are as follows:*

$$\begin{cases} (w_n^c, w_r^c) = \\ \left(\frac{1}{2}(1-c_n), \frac{\delta(-1+3\rho_c+(1+\rho_c)c_n)}{4\rho_c}\right), if c_n \le c_3 \\ \left(\frac{-8\rho_c^2+\delta(1+\rho_c)\left(1+3\rho_c^2\right)-\left(-8\rho_c^2+\delta(1+\rho_c)^3\right)c_n}{-16\rho_c^2+\delta(1+\rho_c)^4}, \frac{\delta\rho_c\left(\delta(1+\rho_c)^3-2(-1+\rho_c(4+\rho_c))+2\left(-1+\rho_c^2\right)c_n\right)}{-16\rho_c^2+\delta(1+\rho_c)^4}\right), \\ if c_3 < c_n \le c_4 \\ \left(\frac{2\rho_c+\delta(2+\rho_c(5+\rho_c-\delta(1+\rho_c)))-(2\rho_c+\delta(2+\rho_c(7+\rho_c)))c_n}{2(2\rho_c+\delta(1+\rho_c(4+\rho_c)))}, \frac{\delta(-1+\rho_c+\delta(1+\rho_c(5+2\rho_c))+(1+3\rho_c)c_n)}{2(2\rho_c+\delta(1+\rho_c(4+\rho_c)))}\right), \\ if c_n > c_4. \end{cases}$$

When $c_n \le c_3$, the remanufacturer adopts the NR strategy; when $c_3 < c_n \le c_4$, the remanufacturer adopts the PR strategy; otherwise, the remanufacturer adopts the FR strategy, where $c_3 = \frac{(1-\rho_C)^2}{(1+\rho_C)^2}$ and $c_4 = \frac{(1-\delta)(1+\rho_C)^2}{1+\rho_C(6+\rho_C)}$.

According to Proposition 3 in Model C, when the production cost of new products is low, the new products will have greater competitive advantages, and the supplier will also appropriately raise the wholesale price to the remanufacturer, to obtain greater profits from the manufacturer. With the increase in the production cost of new products, the competitive advantage of remanufactured products gradually increases, and suppliers prefer to obtain greater profits from the remanufacturer. Therefore, they will appropriately reduce the wholesale price to the remanufacturer to encourage them to carry out remanufacturing as much as possible. Different from Proposition 1, in the share-controlling mode, when the production cost of new products is low, the wholesale price set by the supplier to the remanufacturer will make it impossible for the remanufacturer to carry out remanufacturing, so the NR strategy will be adopted.

Following the three situations of Theorem 3, we obtain the optimal remanufacturing strategy in different situations under various production costs, remanufacturing product discount coefficients, and shareholding ratios, as shown in Table 3.

According to Table 3, in Model C, similar to in Model H, the remanufacturer's optimal remanufacturing strategy selection is also affected by the production cost of new products, the discount coefficient of remanufactured products, and the shareholding ratio. The greater the production cost and the greater the discount factor of remanufactured products, the more inclined the remanufacturer is to adopt the strategy of full remanufacturing (FR) and remanufacture as much as possible.

Table 3. Optimal remanufacturing strategies in Model C.

c_n	δ	ρ	
$0 < c_n < \frac{1}{9}$	$0 < \delta < 1$	$\frac{1}{2} < \rho \le \rho_0$	$\rho_0 < \rho \le 1$
		NR	PR
$\frac{1}{9} < c_n \le \frac{1}{2}$	$0 < \delta < 1-2c_n$	PR	
	$1-2c_n < \delta < \frac{9-17c_n}{9}$	$\frac{1}{2} < \rho \le \rho_1$	$\rho_1 < \rho \le 1$
		PR	FR
	$\frac{9-17c_n}{9} < \delta < 1$	FR	
$\frac{1}{2} < c_n \le \frac{9}{17}$	$0 < \delta < \frac{9-17c_n}{9}$	$\frac{1}{2} < \rho \le \rho_1$	$\rho_1 < \rho \le 1$
		PR	FR
	$\frac{9-17c_n}{9} < \delta < 1$	FR	
$\frac{9}{17} < c_n \le 1$	$0 < \delta < 1$	FR	

Where $\rho_0 = \frac{(1-\sqrt{c_n})^2}{1-c_n}$ and $\rho_1 = \frac{1-\delta-3c_n}{-1+\delta+c_n} - 2\sqrt{\frac{-c_n+\delta c_n+2c_n^2}{(-1+\delta+c_n)^2}}$.

By comparing Tables 2 and 3, we can find that share-controlling can harm remanufacturing in certain circumstances, as shown in Corollary 4.

Corollary 4. *When the new product production cost is low* ($0 < c_n < \frac{1}{9}$)*, share-controlling to a third-party remanufacturer can deter remanufacturing when the share-controlling ratio is relatively low, i.e.,* $\frac{1}{2} < \rho \leq \frac{(1-\sqrt{c_n})^2}{1-c_n}$.

We find that when the new product production cost is low and the share-controlling ratio is relatively low, the new product will have a great competitive advantage, and the upstream supplier will set a higher wholesale price for the remanufacturer, so the manufacturer can only make the remanufacturer stop remanufacturing. Therefore, from the perspective of the environment, in some certain circumstances, the acquisition of a third-party remanufacturer might be harmful to the environment. Therefore, from the perspective of the environmental effect, the manufacturer should think it over when considering acquiring a third-party remanufacturer.

Similarly, under the condition of moderate production cost and relatively low discount factor of remanufactured products, with the increase in the shareholding ratio, the supplier will reduce the wholesale price to the remanufacturer. Under the control of the manufacturer, the remanufacturer will gradually change from partial remanufacturing (PR) to full remanufacturing (FR). Figure 3 shows the optimal remanufacturing strategy selection of the remanufacturer under different production costs in the case of share-controlling. With the increase in production cost, the area of the no-manufacturing strategy gradually decreases, and the area of the FR strategy becomes larger and larger, that is to say, remanufacturers are more and more inclined to choose the FR strategy.

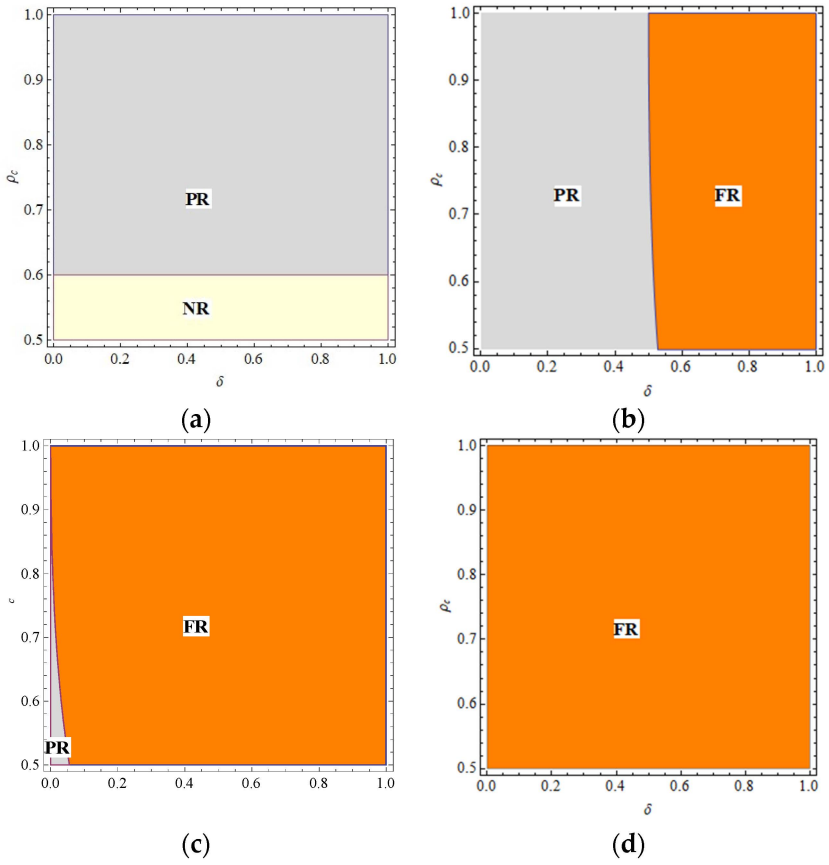

Figure 3. The optimal strategy of the remanufacturer in Model C. (**a**) $c_n = 1/16$; (**b**) $c_n = 1/4$; (**c**) $c_n = 1/2$; (**d**) $c_n = 3/4$.

5. Model Analysis and Comparison

Based on the model results in the above two cases, we carry out some model analysis and comparison in this section. First, the monotonicity of the shareholding rate on the manufacturer's profit is illustrated in Corollary 2.

Corollary 2 (impact on the manufacturer). *The manufacturer's profit increases with the shareholding ratio, and* $\pi_M^H\Big|_{\rho=\frac{1}{2}} < \pi_M^C\Big|_{\rho=\frac{1}{2}}$.

For the manufacturer, a higher shareholding ratio means that it can obtain more profits from the remanufacturing. It is easy to understand that the manufacturer obtains more profit with a higher shareholding ratio. And for the critical value of sharing ratio between the shareholding mode and the share-controlling mode ($\rho = \frac{1}{2}$), we find that the manufacturer's profit in the share-controlling mode is always higher than that in the shareholding mode. When the shareholding ratio is slightly larger than $\frac{1}{2}$ in the share-controlling mode, even though the shareholding ratio changes a little bit, the manufacturer can obtain the decision rights of the remanufacturer and it will be able to balance the new product and remanufactured product quantities. When the production cost of new products is low, the manufacturer can choose to deter remanufacturing to assure that it can obtain enough profit from selling new products. When the production cost of new products is high, which means that the cost advantage of remanufacturing is not obvious, the manufacturer will increase the selling prices of both the new and remanufactured products, which increases the margin profit but decreases the quantities of these two kinds of products. It means that for the aim of economic profits, the manufacturer should increase the shareholding ratio of the remanufacturer as much as possible to acquire more economic profits.

We also explore the impact of share ownership on the supplier. We obtain the following results as Corollary 3.

Corollary 3 (impact on the supplier). *When* $c_n < c_3$*, the profit of the supplier decreases first and then stays unchanged with the shareholding ratio. When* $c_3 < c_n \leq min\{\frac{4\rho_c - \delta(1+\rho_c)^2}{4\rho_c}, c_3\}$*, the profit of the supplier decreases with the shareholding ratio. Otherwise, the profit of the supplier decreases first and then increases with the shareholding ratio, and the supplier obtains the highest profit when* $\rho = 0$.

Specifically, according to Corollary 3 above, when the production cost of the new product is low and the sharing ratio is higher than 1/2, the remanufacturer will not carry out remanufacturing, and the profit of the supplier is all derived from the production business of the manufacturer's new product, so the profit of the supplier has nothing to do with the shareholding ratio. In other cases, the increase in the manufacturer's shareholding ratio to the remanufacturer will have two effects on the supplier's profit. On the one hand, the increase in the manufacturer's shareholding ratio to the remanufacturer will promote the remanufacturing production to a certain extent, to increase the procurement quantity of the remanufacturing end and thus improve the profit of the supplier. On the other hand, the increase in the manufacturer's shareholding ratio to the remanufacturer will make the wholesale price to the remanufacturer decrease, which may damage the profit of the supplier. These two effects are regulated by the production cost. When the production cost is low, with the increase in shareholding ratio, the supplier will reduce the wholesale price to the manufacturer and increase the wholesale price to the remanufacturer, but at the same time, the sales quantity of both new products and remanufactured products will decrease. Comprehensively, the profit of the supplier will decrease. Thus, the larger the shareholding ratio of the manufacturer to the remanufacturer, the stronger the cooperative relationship between the two enterprises, and the smaller the profit space for the supplier. When the

production cost is high, the cost advantage of remanufactured products is greater, and the former plays a major role, and the supplier's profit will increase.

And when manufacturer does not hold stock in the third-party remanufacturer, the supplier will obtain more profit compared to when the manufacturer and the remanufacturer are in a stock relationship. In other words, a supplier which sells non-remanufacturable components will always prefer to cooperate with independent manufacturers and remanufacturers. According to Corollary 2, the supplier's profit is in an approximate U-shape in the stock sharing ratio. So, we need to compare the two boundary cases ($\rho = 0$ and $\rho = 1$). When $\rho = 0$, the supplier obtains a higher profit. The reason for this is that when the manufacturer and the remanufacturer decide independently, they are in a complete competing relationship. And the selling price of the new and remanufactured products will be lower, but the quantities can be higher. Compared to the centralized case, the selling prices of the new and remanufactured products will be higher, but the quantities are lower, and the wholesale prices will be higher. For the supplier, with the combinational effects of increased quantities and lowered wholesale, their profit improves. Therefore, the supplier should choose to collaborate with the independent manufacturer and remanufacturer, in which case the supplier can obtain high profits.

Corollary 4 (impact on the supply chain). *When* $max\{c_1, c_4\} < c_n \leq 1$, *the profit of the supply chain decreases first and then increases with the shareholding ratio, and the independent structure is optimal to the supply chain's profit.*

According to Corollary 4, we can obtain that, from the perspective of the supply chain, when the production cost is high enough, in the shareholding mode, the supply chain's profit decreases with the shareholding ratio; in the share-controlling mode, the supply chain's profit decreases with the shareholding ratio. And the supply chain obtains the most profit when there is no sharing relationship between the manufacturer and the remanufacturer. Intuitively, we may speculate that a centralized structure is beneficial to the whole supply chain. However, in this model, we find that share ownership can harm the efficiency of the supply chain. Therefore, when the manufacturer decides to invest in a third-party remanufacturer in commercial practice, it means that the supply chain's total profit will be decreased to some extent.

The following will analyze the impact of the shareholding ratio on the remanufacturing rate.

Corollary 5 (impact on the remanufacturing rate).

(1) *In Model H, the product remanufacturing rate* θ *decreases with the shareholding ratio when* $c_n \leq 1 - \sqrt{\delta}$; *however, the product remanufacturing rate* θ *increases with the shareholding ratio when* $1 - \sqrt{\delta} < c_n \leq c_1$.

(2) *In Model C, when* $c_n \leq c_3$, *there is no product remanufacturing; when* $c_3 < c_n \leq c_4$, *the product remanufacturing rate* θ *increases with the shareholding ratio.*

The specific principle of Corollary 5 is that, in Model H, when the production cost of new products is low, the new product has a great competitive advantage in the market, and the supplier can obtain greater profits from the manufacturer. With the increase in the shareholding ratio, the manufacturer will reduce the competition with the remanufacturer due to the economic profit so that the sales quantity of remanufactured products will increase. But on the other hand, to obtain more profits, the supplier will increase the wholesale price to the remanufacturer, which will reduce the sales quantity of the remanufactured products. In this case, the latter effect is more obvious, so the product remanufacturing rate in the case of low cost decreases with the shareholding ratio. With the increase in the production cost of new products, the competitive advantage of remanufactured products will gradually increase, and the willingness of suppliers to inhibit remanufacturers will gradually decrease. Remanufacturers will increase the production quantity of remanufactured products, and

the remanufacturing rate will increase with the shareholding ratio, to gradually achieve full remanufacturing.

In Model C, when the production cost of the new product is low, the new product will have a great competitive advantage in the market, and the remanufactured product will have no market. Therefore, the remanufactured product will not exist, and the manufacturer will refuse the remanufacturer under its holding to carry out remanufacturing. With the increase in the production cost of new products, the competition between new products and remanufactured products becomes increasingly fierce. With the increase in the shareholding ratio, although remanufactured products will have a strong competitive effect on new products, manufacturers will obtain a greater profit due to the shareholding relationship. The profit increases with the increase in the holding ratio. To maximize their profit, more and more manufacturers will allow remanufacturers to produce again, to make the product remanufacturing ratio θ with the percentage of shareholding improved; until the new product production cost is high, the remanufacturing product competitive advantage is very obvious.

Therefore, from the perspective of the environment, the government can choose some measures and regulation tools to lead the development of the remanufacturing industry. When most manufacturers and remanufacturers are in a shareholding relationship, if the production cost is low, the government should not encourage manufacturers to collaborate with remanufacturers. On the other hand, if the production cost is high, the government should encourage manufacturers to acquire more third-party remanufacturers. When most manufacturers and remanufacturers are in a share-controlling relationship, the government should take some measures to encourage manufacturers to acquire the remanufacturer completely or establish its own remanufacturer.

6. Extension: Competitive Suppliers Model

In this section, we analyze the models in the competitive supplier environment, in which the manufacturer and the remanufacturer procure components from two different suppliers. And both the shareholding mode and share-controlling mode are analyzed.

6.1. Shareholding Mode (Model HC)

In the HC mode, the manufacturer and the remanufacturer procure components from two different suppliers, the manufacturer receives a ratio ρ_H ($0 \leq \rho_H \leq \frac{1}{2}$) of the profits of the remanufacturer, and the remanufacturer has the independent decision right. Firstly, two suppliers decide the wholesale prices w_n^{HC} and w_r^{HC} to the manufacturer and the remanufacturer, respectively. Given w_n^{HC} and w_r^{HC}, they decide their optimal quantity decisions to maximize their profits. Then, the manufacturer and the remanufacturer decide the production quantity of new products and remanufacturing products, q_n^{HC} and q_r^{HC}. Thus, we proceed by analyzing the quantity competition between the manufacturer and the remanufacturer. Then, we derive the two suppliers' optimal wholesale prices.

The profit functions of the manufacturer, the remanufacturer, and the supplier are as follows:

$$\pi_M\left(q_n^{HC}\right) = \left(p_n - w_n^{HC} - c_n\right)q_n^{HC},$$

$$\pi_R\left(q_r^{HC}\right) = \left(p_r - w_r^{HC}\right)q_r^{HC},$$

$$\pi_{SM}\left(w_n^{HC}\right) = w_n^{HC}q_n^{HC} \text{ and } \pi_{SR}\left(w_r^{HC}\right) = w_r^{HC}q_r^{HC},$$

$$s.t.\ 0 \leq q_r^{HC} \leq q_n^{HC}.$$

We solve the decision problems of Model HC by backward induction and obtain the optimal quantities given the wholesale prices. Since the profit functions of the manufacturer in the HC and H modes are the same, the expression for the sales quantities of new and remanufactured products based on wholesale prices are the same as Lemma 1. According to the optimal sales quantity in different situations obtained by Lemma 1 and the profit

function of two suppliers in the HC mode, two remanufacturing strategies, their optimal decisions, and optimal profits in the H model can be obtained, as shown in Proposition 3.

Proposition 3. *In the HC model, the optimal wholesale prices of new and remanufactured products are follows:*

$$\left(w_n^{HC}, w_r^{HC}\right) = \begin{cases} \left(\frac{-8+3\delta(1+\rho_H)-(-8+\delta+\delta\rho_H)c_n}{-16+\delta+\delta\rho_H}, \frac{\delta(-6+\delta+\delta\rho_H-2c_n)}{-16+\delta+\delta\rho_H}\right), if c_n \le c_2 \\ \left(\begin{array}{c} 8-\delta(10+\rho_H)(\delta\rho_H-2+\delta) \\ \frac{+\left(\delta^2(1+\rho_H)(4+\rho_H)-8-2\delta(16+\rho_H)\right)c_n}{16+\delta^2(-2+\rho_H)(1+\rho_H)+10\delta(4+\rho_H)}, \end{array} \quad \frac{\delta\left(4-\delta^2(2-\rho_H)(1+\rho_H)+4\delta(4+\rho_H)+6(2+\delta+\delta\rho_H)c_n\right)}{16+\delta^2(-2+\rho_H)(1+\rho_H)+10\delta(4+\rho_H)}\right), \\ if c_n > c_2 \end{cases}$$

When $c_n \le c_2$, the remanufacturer adopts the PR strategy; otherwise, the remanufacturer adopts the FR strategy, where $c_2 = 2 + \frac{18}{-10+\delta+\delta\rho_H}$.

In the HC model, the manufacturer and the remanufacturer choose different suppliers, so suppliers do not need to consider the trade-off between the manufacturer and remanufacturer and only make wholesale price decisions according to their corresponding enterprises (manufacturer or remanufacturer). Similar to in Proposition 1, when the production cost of the new product is low, the new product has a greater competitive advantage. In the face of fierce competition from the new product, the remanufacturer will not carry out full remanufacturing but will implement the partial remanufacturing strategy. In this case, the supplier of the remanufacturer will appropriately raise its wholesale price. On the contrary, when the production cost of the new product is high, the remanufactured product will have a greater competitive advantage, and remanufactured production will be carried out as much as possible. In this case, the supplier of the remanufacturer will reduce the wholesale price appropriately.

Following the two situations of Proposition 3, we obtain the optimal remanufacturing strategy in different situations under various production costs, remanufacturing product discount coefficients, and shareholding ratios, as shown in Table 4.

Table 4. Optimal remanufacturing strategies in the HC model.

c_n	δ	$0<\rho_H\le\frac{-2+2\delta+10c_n-\delta c_n}{-2\delta+\delta c_n}$	$\frac{-2+2\delta+10c_n-\delta c_n}{-2\delta+\delta c_n}<\rho_H\le\frac{1}{2}$
$0 < c_n \le \frac{1}{5}$	$0 < \delta \le \frac{-4+20c_n}{-6+3c_n}$	PR	
	$\frac{-4+20c_n}{-6+3c_n} < \delta < \frac{-2+10c_n}{-2+c_n}$	PR	FR
	$\frac{-2+10c_n}{-2+c_n} < \delta < 1$	FR	
$\frac{1}{5} < c_n \le 1$	$0 < \delta < 1$	FR	

According to Table 4, in the HC mode, the remanufacturer's optimal remanufacturing strategy selection is also affected by the production cost of new products, the discount coefficient of remanufactured products, and the shareholding ratio. When the production cost of new products is high, the cost advantage of remanufacturing is great. No matter what the discount factor and shareholding ratio of remanufactured products are, the remanufacturer will always choose FR. However, with the decrease in the production cost of new products, if the discount coefficient of remanufactured products is small, the remanufacturer always chooses PR regardless of the shareholding ratio. If the discount factor of remanufactured products is moderate and the shareholding ratio is large, the remanufacturer will choose a partial remanufacturing strategy. In other cases, the remanufacturer chooses to remanufacture all the products.

Different from the shareholding mode (Model H), when the production cost of new products is low and the discount coefficient of remanufactured products is moderate, a higher shareholding ratio will motivate the remanufacturer to carry out the FR strategy. However, a higher shareholding ratio will lead to the PR strategy being adopted by the

remanufacturer. The reason for this phenomenon is that it is mainly regulated by the pricing of the shared supplier. If the shareholding ratio is small, the manufacturer and the remanufacturer are relatively independent, and the competition relationship is more obvious, so the remanufacturer should set a lower wholesale price to encourage it to carry out remanufacturing as much as possible. When the shareholding ratio is large, the manufacturer's relative advantage is more obvious, and the wholesaler will be more inclined to ensure more profits from the manufacturer, so the wholesaler will set a higher wholesale price for the remanufacturer to inhibit its remanufacturing.

In this case, the supplier does not need to consider the trade-off between manufacturers and remanufacturers again, only according to its corresponding enterprise, a manufacturer or remanufacturer, for wholesale price decision; therefore, if the stake is small, due to intense competition caused by production and the manufacturing chamber of commerce manufacturers caused by the pressure of competition, manufacturers have to reduce the remanufacturing product again, and then select the partial remanufacturing strategy. When the shareholding ratio is large, the manufacturer will relax the intensity of production competition for the remanufacturer due to the consideration of profit, and the remanufacturer will have more market space to adopt FR.

Figure 4 shows the remanufacturer's optimal remanufacturing strategy selection under different production costs. With the increase in the production cost, the area of complete remanufacturing strategy is larger and larger, that is to say, remanufacturers are more and more inclined to choose the FR strategy.

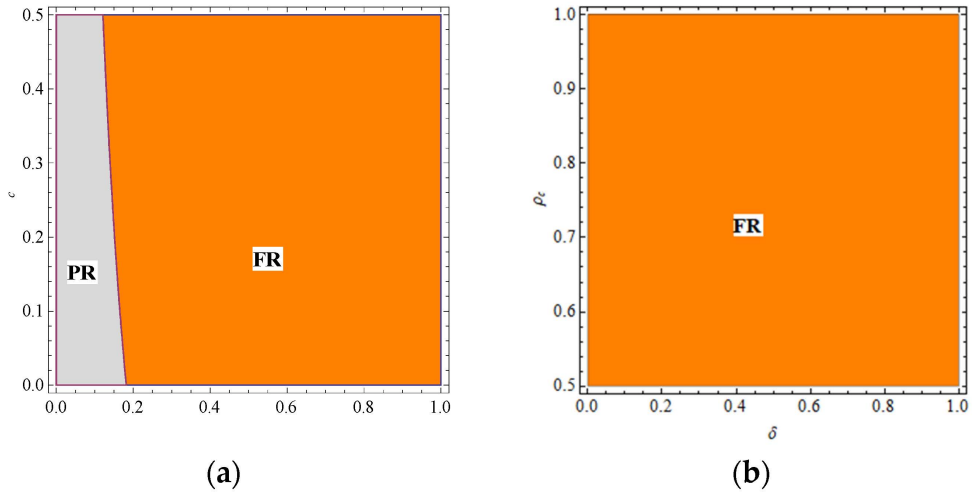

Figure 4. The optimal strategy of the remanufacturer in HC mode. (**a**) $c_n = 1/6$; (**b**) $c_n = 1/4$.

In the following, we will analyze the impact of the shareholding ratio on the remanufacturing rate, the wholesale price, and supplier profit in the HC mode.

Corollary 7. *In the HC mode, when $c_n \leq c_2$, the product remanufacturing rate θ increases with the shareholding ratio; when $c_n > c_2$, the product remanufacturing rate θ is 100%.*

According to Corollary 7, in the HC mode, when the cost of new product production is lower, the cost difference between the new product and the remanufacturing product becomes more obvious; with the increase in the shareholding ratio to the remanufacturer, the manufacturer will relax the production competition strength. The remanufacturer will have more market space, thus gradually increasing the remanufacturing rate. However, when the production cost of the new product is high, the cost difference between the new product and the remanufactured product is large, and the remanufacturer will adopt the FR strategy.

Corollary 8. *In the HC mode, when the remanufacturer takes a PR strategy, the wholesale prices to the manufacturer and remanufacturer decrease with the shareholding ratio to the remanufacturer.*

Corollary 8 shows that, in the HC mode, when manufacturers take the PR strategy, the cost competition between the new products and remanufactured products is more fierce; with the increase in shareholding to the remanufacturer, the manufacturers will reduce the new product production manufacturing, so that the remanufacturer has more space to increase the output of its remanufactured products, and the competition between the manufacturer and the remanufacturer is eased, leaving lower profit space for the upstream supplier. Therefore, the two suppliers will reduce the wholesale price to the manufacturer and the remanufacturer at the same time, to obtain optimal profits.

6.2. Share-Controlling Mode (Model CC)

In Model CC, the manufacturer has dominant decision-making power over the remanufacturer, and the remanufacturer procures components that cannot be remanufactured from two different suppliers. Firstly, two suppliers decide the wholesale prices w_n^{CC} and w_r^{CC} to the manufacturer and the remanufacturer. Given w_n^{CC} and w_r^{CC}, they decide their optimal quantity decisions to maximize their profits. Then, the manufacturer decides the production quantity of both new products and remanufacturing products, q_n^{CC} and q_r^{CC}. Thus, we proceed by analyzing the quantity competition between the manufacturer and the remanufacturer. Then, we derive the two suppliers' optimal wholesale prices.

The profit functions of the manufacturer, the remanufacturer, and the supplier are as follows:

$$\pi_M\left(q_n^{CC}, q_r^{CC}\right) = \left(p_n - w_n^{CC} - c_n\right)q_n^{CC} + \rho_c\left(p_r - \ w_r^{CC}\right)q_r^{CC},$$

$$\pi_{SM}\left(w_n^{CC}\right) = w_n^{CC}q_n^{CC} \text{ and } \pi_{SR}\left(w_r^{CC}\right) = w_r^{CC}q_r^{CC},$$

$$s.t.\ 0 \le q_r^{CC} \le q_n^{CC}.$$

We solve the decision problems of Model CC by backward induction and obtain the optimal quantities given the wholesale prices. Since the profit functions of the manufacturer in the CC and C modes are the same, the expression for the sales quantities of new and remanufactured products based on wholesale prices are the same as Lemma 2. According to the optimal sales quantity in different situations obtained by Lemma 2 and the profit function of two suppliers in the CC mode, two remanufacturing strategies, their optimal decisions, and optimal profits in the CC model can be obtained, as shown in Proposition 4.

Proposition 4. *In the CC mode, the optimal wholesale prices of new and remanufactured products are as follows:*

$$\left(w_n^{cC}, w_r^{cC}\right) =$$

$$\begin{cases} \begin{cases} \left(\frac{-4\rho^2+\delta\left(-1+\rho_c+3\rho_c^2+\rho_c^3\right)+\left(4\rho_c^2-\delta(-1+\rho_c)(1+\rho_c)^2\right)c_n}{-8\rho^2+\delta(-1+\rho_c)(1+\rho_c)^2}, \frac{\delta\rho_c\left(2-6\rho_c+\delta(1+\rho_c)^2-2(1+\rho_c)c_n\right)}{-8\rho_c^2+\delta(-1+\rho_c)(1+\rho_c)^2}\right), \\ if c_n \le c_5 \\ \left(\frac{-8\rho_c+\delta(1+\rho_c)(1+3\rho_c)-\left(-8\rho_c+\delta(1+\rho_c)^2\right)c_n}{-16\rho_c+\delta(1+\rho_c)^2}, \frac{\delta\left(2-6\rho_c+\delta(1+\rho_c)^2-2(1+\rho_c)c_n\right)}{-16\rho_c+\delta(1+\rho_c)^2}\right), \\ if c_5 < c_n \le c_6 \\ \begin{pmatrix} 8\rho_c^2+2\delta(1+\rho_c(5+\rho_c(9+\rho_c))) - \delta^2(1+\rho_c)(1+\rho_c(7+\rho_c(11+\rho_c))) \\ \frac{+\left(-8\rho_c^2+\delta^2(1+\rho_c)^2(1+\rho_c(4+\rho_c))-2\delta(1+\rho_c(7+\rho_c(15+\rho_c)))\right)c_n}{16\rho_c^2+\delta^2(1+\rho_c)^2(-1+(-4+\rho_c)\rho_c)+2\delta(1+\rho_c(7+\rho_c(19+5\rho_c)))}, \\ \frac{\delta\left(4(-1+\rho_c)\rho_c+4\delta\rho_c^2(3+\rho_c)+\delta^2(1+\rho_c)^2(-1+(-4+\rho_c)\rho_c)+2\rho_c(1+3\rho)(2+\delta+\delta\rho)c_n\right)}{16\rho^2+\delta^2(1+\rho)^2(-1+(-4+\rho)\rho)+2\delta(1+\rho(7+\rho(19+5\rho)))} \end{pmatrix}, \\ if c_n > c_6 \end{cases} \end{cases}$$

When $c_n \leq c_5$, *the remanufacturer adopts the NR strategy; when*$c_5 < c_n \leq c_6$, *the remanufacturer adopts the PR strategy; otherwise, the remanufacturer adopts the FR strategy, where* $c_5 = \frac{2-6\rho_c+\delta(1+\rho_c)^2}{2(1+\rho_c)}$ *and* $c_6 = \frac{2(1+\rho_c)(-1+\delta\rho_c)}{\delta(1+\rho_c)^2-2(1+5\rho_c)}$.

In the CC mode, the manufacturer and the remanufacturer choose different suppliers, so suppliers do not need to consider the trade-off between the manufacturer and remanufacturer and only make wholesale price decisions according to their corresponding enterprises (manufacturer or remanufacturer). Similar to Proposition 2, when the production cost of new products is low, the remanufacturer will adopt the FR strategy. When the supplier's wholesale price to the remanufacturer is moderate, the remanufacturer will adopt the PR strategy. When the supplier's wholesale price to the remanufacturer is high, the remanufacturer will be unprofitable and will not remanufacture, that is, the NR strategy is adopted. Different from Proposition 2, in the competitive supplier mode, when the production cost of new products is low, the manufacturer will refuse the remanufacturer to carry out remanufacturing, so the NR strategy is adopted.

Following the three situations of Proposition 4, we obtain the optimal remanufacturing strategy in different situations under various production costs, remanufacturing product discount coefficients, and shareholding ratios as Proposition 5.

Proposition 5. *In the CC mode, the remanufacturer takes the NR strategy when* $0 < c_n < \frac{1}{3}$ *and* $\frac{1}{2} < \rho \leq \frac{3-\delta+c_n-\sqrt{9-8\delta+6c_n+c_n^2}}{\delta}$, *the PR strategy when* $\frac{1-\delta-5c_n+\delta c_n\sqrt{1+2\delta+\delta^2-10c_n+6\delta c_n+25c_n^2-8\delta c_n^2}}{\delta(2-c_n)} \leq \rho < 1$ *and* $\frac{1-\delta}{3-\delta} < c_n < \frac{12-6\delta}{28-9\delta}$ *or* $\frac{12-6\delta}{28-9\delta} < c_n < 1$, *or the FR strategy otherwise.*

According to Proposition 5, in the CC mode, when the production cost of new products is low and the holding ratio is slightly higher than 1/2, the manufacturer will not allow the remanufacturing of the remanufacturer. The reason for this is that when the production cost of a new product is low, the cost difference between the new product and the remanufactured product is small, and the remanufactured products will pose a greater competitive threat to the new products. Therefore, when the shareholding ratio is slightly greater than 1/2, the manufacturer will not allow the remanufacturer to introduce the remanufactured product. With the increase in the shareholding ratio, for the sake of profit, the manufacturer will allow the remanufacturer to gradually start the remanufacturing business, to obtain more profits. When the production cost is large, the manufacturer will arrange for the remanufacturer to carry out remanufacturing as much as possible to obtain the profits of both the new products and remanufactured products together. In addition, there is a situation in which the manufacturer will allow full remanufacturing, that is, the production cost is moderate and the holding proportion is large (close to full holding). In this case, because the manufacturer has a large proportion of shares, the profit brought by remanufacturing products has exceeded their competitive effect. That is to say, when the proportion of shares is large, as long as the production cost of new products is not lower than a certain level, the manufacturer will always make the remanufacturer adopt the FR strategy.

Figure 5 shows the optimal remanufacturing strategy selection of the remanufacturer under different production costs in the case of a competitive supplier and share-controlling. With the increase in the production cost, the area of the no-manufacturing strategy gradually decreases, and the area of the FR strategy becomes larger and larger, that is to say, remanufacturers are more and more inclined to choose the FR strategy.

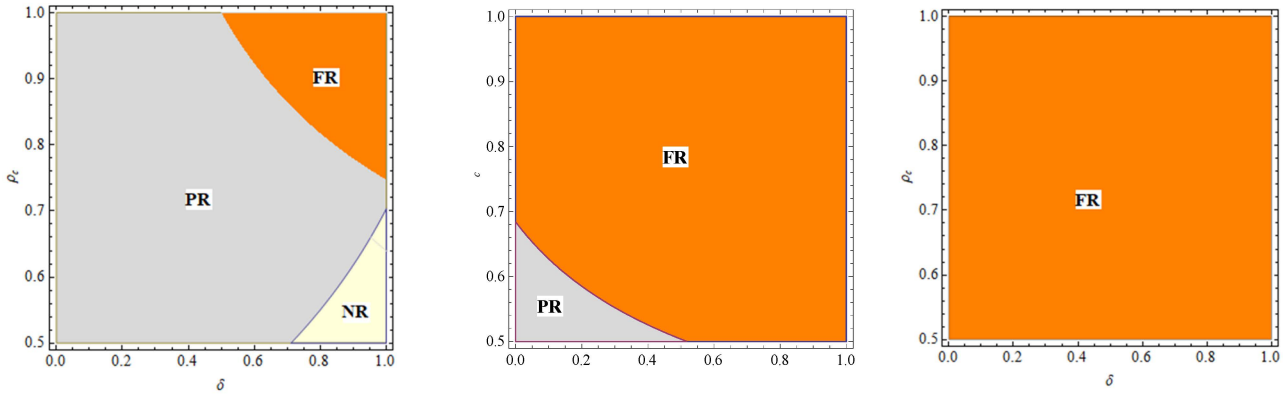

Figure 5. The optimal strategy of the remanufacturer in CC Mode. (**a**) $c_n = 1/5$; (**b**) $c_n = 8/21$; (**c**) $c_n = 1/2$.

7. Conclusions

This paper constructs a closed-loop supply chain model composed of a supplier, a manufacturer, and a remanufacturer. Considering the different ownership structures (shareholding mode and share-controlling mode) between the manufacturer and the remanufacturer, suppliers provide non-reusable components to the manufacturer and the remanufacturer at the same time. Furthermore, the optimal decisions of each enterprise under the two modes are considered. In this study, we first solve and analyze the optimal decision-making, optimal pricing, and profit of each related enterprise under two conditions: shareholding mode and share-controlling mode. On this basis, we further analyze the relevant results. The main conclusions of this study are as follows:

For the manufacturer, a higher shareholding ratio means that it can obtain more profits from the remanufacturing. We find that the manufacturer's profit in the share-controlling mode is always higher than that in the shareholding mode, even though the remanufacturing cost is very high. In the case of high remanufacturing cost (low manufacturing cost), investing in third-party remanufacturing can help the manufacturer balance the new and manufactured products in the market. Therefore, the manufacturer should always invest in the remanufacturer as much as possible, with consideration of the financial capacity.

For the supplier, the impact of the shareholding ratio on the supplier depends on the manufacturing cost. When the manufacturing cost is relatively low, the stock sharing relationship between the manufacturer and the remanufacturer will decrease the supplier's profit. Therefore, in this case, a supplier will prefer to cooperate with a manufacturer and an independent remanufacturer as a priority. A higher shareholding ratio will squeeze the profit margins of the supplier. When the manufacturing cost is relatively high, it will depend on the shareholding. In the case of shareholding between the manufacturer and the remanufacturer, a higher shareholding ratio will decrease the supplier's profit. In the case of share-controlling, a higher shareholding ratio will increase the supplier's profit.

From the perspective of the supply chain, when the production cost is high enough, the supply chain's profit decreases first and then increases with the shareholding ratio. And the supply chain obtains the most profit in a completely decentralized structure. In other words, sharing ownership between the manufacturer and the remanufacturer can harm the efficiency of the supply chain. Therefore, when the manufacturer decides to invest in a third-party remanufacturer in commercial practice, it means that the supply chain's total profit will be decreased to some extent. But when the shareholding ratio becomes higher than 50%, the supply chain's total profit begins to improve.

From the perspective of the environment, the increase in the shareholding ratio does not always improve the remanufacturing ratio of products and enhance the environmental effect. Under certain circumstances, the manufacturer's shareholding in remanufacturing

will reduce the proportion of remanufacturing products. When the production cost of new products is low, the new products have great competitive advantages in the market, and suppliers can obtain greater profits from manufacturers. With the increase in the shareholding ratio, to obtain more profits, suppliers will increase the wholesale price to the remanufacturer, which will reduce the sales quantity of remanufactured products, thus reducing the remanufacturing proportion.

In future research, we can consider some more complex relationships between the manufacturer and the remanufacturer. For example, the shareholding rate is closely related to remanufacturing technology licensing and cooperation. Future studies could further explore some more contracts or non-contract relationships in remanufacturing operations.

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