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Sustainable Water Quality Evaluation Based on Cohesive Mamdani and Sugeno Fuzzy Inference System in Tivoli (Italy)

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Abstract: Clean water is vital for a sustainable environment, human wellness, and welfare, supporting life and contributing to a healthier environment. Fuzzy-logic-based techniques are quite effective at dealing with uncertainty about environmental issues. This study proposes two methodologies for assessing water quality based on Mamdani and Sugeno fuzzy systems, focusing on water's physiochemical attributes, as these provide essential indicators of water's chemical composition and potential health impacts. The goal is to evaluate water quality using a single numerical value which indicates total water quality at a specific location and time. This study utilizes data from the Acea Group and employs the Mamdani fuzzy inference system combined with various defuzzification techniques as well as the Sugeno fuzzy system with the weighted average defuzzification technique. The suggested model comprises three fuzzy middle models along with one ultimate fuzzy model. Each model has three input variables and 27 fuzzy rules, using a dataset of nine key factors to rate water quality for drinking purposes. This methodology is a suitable and alternative tool for effective water-management plans. Results show a final water quality score of 85.4% with Mamdani (centroid defuzzification) and 83.5% with Sugeno (weighted average defuzzification), indicating excellent drinking water quality in Tivoli, Italy. Water quality evaluation is vital for sustainability, ensuring clean resources, protecting biodiversity, and promoting long-term environmental health. Intermediate model evaluations for the Mamdani approach with centroid defuzzification showed amounts of 72.4%, 83.4%, and 92.5% for the first, second, and third fuzzy models, respectively. For the Sugeno method, the corresponding amounts were 76.2%, 83.5%, and 92.5%. These results show the precision of both fuzzy systems in capturing nuanced water quality variations. This study aims to develop fuzzy logic methodologies for evaluating drinking water quality using a single numerical index, ensuring a comprehensive and scalable tool for water management.

Keywords: water quality index; fuzzy systems; water pollution; sustainable water management; sustainable environment; pollution control

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1. Introduction

Water is a limited and vital resource for human well-being, socio-economic development, and ecosystem health. However, over a billion people around the world currently do not have access to safe drinking water, and two-thirds of the countries are experiencing water-related stress [1]. Inequitable distribution leads to wastefulness, contamination, and unsustainable management practices [2], while its uses in sports and entertainment events have long-term implications for its availability [3]. Human activities like increasing population, industrialization, and the use of fertilizers lead to water contamination [4]. Various evaluation techniques have been developed for monitoring water quality, which plays an important role in ecosystem survival and human health. Contaminated water may increase mortality and cause serious health problems [5,6]. Water pollution is a major issue in environmental monitoring, as toxins are released into water bodies, and traditional water quality-evaluation methods are labor-intensive, complex, and inadequate for future needs [7]. Converting complex environmental data into actionable information for the public and policymakers is a major challenge [8–10]. An effective technique is needed to resolve interpretation inconsistencies and clarify findings [11]. Drinking water quality (DWQ) management has gained attention due to groundwater contamination [12–15], with developing countries struggling to preserve water quality while ensuring supply [16–18] and developed nations focusing on improving water quality amid population growth and public health concerns [19,20]. Monitoring water quality in the 21st century is challenging due to the several chemicals used in daily life and commerce. A fuzzy-based prediction approach is proposed to assess water quality more accurately, using various physical, biological, and chemical parameters to address this. Several scholars have applied Water Quality Index (WQI) models for water quality evaluation. Sahu et al. [21] used the ANFIS (Adaptive Neuro-Fuzzy Inference System) to analyze highly contaminated groundwater near mines, applying PCA (Principal Component Analysis) to convert correlated data into uncorrelated data, improving accuracy, though requiring extensive training. Liu et al. [22] applied a support vector machine (SVM) to predict aquaculture water conditions and addressed parameter-selection issues by developing a real-value genetic algorithm support vector regression (RGA-SVR), a genetic algorithm that effectively handles nonlinear time series problems. Sedeño-Díaz and López [23] presented a simple and cost-effective method for calculating water quality in reservoirs using tools like fuzzy logic (FL), resulting in the highest accuracy findings. Khan and See used an Artificial Neural Network (ANN) with a Nonlinear Autoregressive (NAR) time series and Scaled Conjugate Gradient (SCG) to train on (chlorophyll, dissolved oxygen (DO), turbidity, and conductance), achieving improved accuracy at slightly higher implementation costs [24]. Huang et al. proposed a Fuzzy Wavelet Neural Network (FWNN) model, using genetic and gradient descent algorithms to assess river water quality with improved accuracy, performance, and durability for varying and non-seasonal data [25]. Zhou et al. developed two prediction models, including a Long short-term memory (LSTM) neural network with enhanced Gray Relational Analysis (IGRA), to assess water quality. However, they found the models required more data and longer training time [26]. Ahmed et al. used SVM, Group Method of Data Handling (GMDH), and Artificial Neural Network (ANN) for water quality prediction, finding SVM and GMDH more reliable, with WDT-ANFIS recommended for WQI from historical data [27]. Li et al. propose a multimodal water quality prediction model combining ensemble empirical mode decomposition (EEMD) and support vector regression (SVR), demonstrating superior performance in predicting dissolved oxygen [28]. The promising results of the Samantaray et al. study show that SVR-FFAPSO is a viable method for evaluating GWL uncertainty because it produces targeted and financially feasible outcomes without sacrificing any dependability [29]. Binney et al. proposed a Sustainability Assessment of Groundwater in Southeast Ghana's western region. The study evaluated groundwater's biological and physico-chemical characteristics, using Hazard Quotient (HQ) and WQI to assess health risks. It found that 16.7% of samples exceeded WHO limits for total dissolved solids (TDS) and turbidity, while 83.3% had a mildly acidic pH [30]. Barzegar et al. proposed the FIS approach to evaluate drinking water quality in Rome, Italy, using various defuzzification methods [31]. Kimothi et al. developed an IoT and machine learning (ML)-based framework to assess water quality in 13 Uttarakhand locations, finding all samples safe to drink, with pH levels averaging 7.19 and a mode of 0.25 [32]. Hassan et al. classified water quality across different geographical regions in India using different machine-learning techniques. This study identified nitrate, pH, and conductivity as significant variables in the classification of water quality [33]. Bui Quoc Lap et al. used machine learning and feature selection to identify key water quality parameters, with Random Forest achieving 0.94 accuracy for efficient WQI calculation in Vietnam's An Kim Hai system [34]. Salari et al. used an ANN with a backpropagation MLP and the Adam algorithm to estimate water volume from Capacitive Deionization, achieving RMSE values of 0.008 (testing) and 0.003 (training) [35]. Derdour et al. used classification techniques including decision tree (DT), K-Nearest Neighbor (KNN), Data Assimilation (DA), SVM, and Extra Trees (ET) to predict WQI in Algeria's Wilaya of Naâma, with SVM achieving the highest accuracy (95.4% for normalized data, 88.9% for test samples) [36]. Uddin et al. quantified uncertainty in the WQI model using Monte Carlo simulation (MCS) and Gaussian Process Regression (GPR), finding that the number of input indicators significantly impacted model uncertainties [37]. Dhruba et al. developed a real-time water quality evaluation system using IoT and a mobile app to monitor key metrics like pH, TDS, turbidity, and temperature, benefiting environmental and public health. Wu et al. developed a framework combining ANN, discrete wavelet transforms, and LSTM to estimate Jinjiang River's water quality accurately, outperforming earlier models [38]. Mdee et al.'s study in Dodoma, Tanzania, measured water quality using shallow wells and deep boreholes with WQI, Inverse Distance Weighting (IDW), and GIS analysis. They found that 42.5% of samples were good and 57.5% outstanding, while the IDW analysis indicated an eastward improvement in water quality [39]. Mishra et al. [40] analyzed water quality data at seven Doon Valley, India, locations using ArcGIS 10.7 and the IDW method. They determined that pH had the greatest impact on WQI, followed by BOD, DO, and TDS, while noting the limitations of conventional machine learning for large data processing [41]. Dilmi et al. integrated deep learning with feature extraction, achieving 99.72% accuracy in water quality classification at the Tilesdit dam using LSTM RNNs and dimensionality reduction methods like LDA and ICA [42]. Barzegar et al. used a fuzzy inference system to evaluate the severity of privacy problems in a healthcare case study [43]. In the presented research, a cohesive fuzzy model has been applied to water quality assessment in Tivoli, Italy, aiming to address the challenges of handling imprecise, distorted, or noisy data in water quality monitoring. The cohesive fuzzy inference systems are compact and require fewer rules, thus being user-friendly and reliable while treating vague and uncertain information. The Mamdani fuzzy inference system is intuitive and human-oriented, thus most suitable for representing expertise. On the other hand, the Sugeno method is computationally efficient and easy to combine with optimization and adaptive techniques, thus very suitable for dynamic nonlinear systems. The objective of the work is to compare the computational efficiency and performance of Mamdani and Sugeno fuzzy inference systems, establish the applicative efficiency of the cohesive fuzzy model in handling uncertainties within water quality assessment, and deliver appropriate information for policymakers and managers to enable efficient monitoring and management of water quality.

2. Materials and Methods

2.1. Geospatial Representation of the Study Area

This research was conducted in Tivoli, Italy, according to Figure 1. Water sample analysis was carried out in the laboratory of Acea Group. The current study used data from the Acea Elabori group's accessible database [44]. The data comprise some key physical and chemical variables of the drinking water delivered in Tivoli, Italy. The Acea Elabori group, the biggest Italian supplier of integrated water services in terms of population served, works in different Italian areas. They manage the water cycle, from spring to wastewater treatment, for nine million individuals in different Italian areas [44]. Key parameters analyzed included pH, hardness, alkalinity, bicarbonates, sulfates, nitrates, sodium, potassium, and magnesium, according to the D. Lgs 18/2023 guidelines. The pH was determined by a pH meter, while hardness and alkalinity by EDTA titration. Sulfates were analyzed by ion chromatography, nitrates by UV spectrophotometry, and sodium, potassium, and magnesium were determined by flame photometry and atomic absorption spectrometry.

Figure 1. Geospatial representation of the study area.

2.2. Fuzzy Logic Modeling

2.2.1. Fuzzy Logic Overview

Fuzzy logic is a mathematical approach utilized to model ambiguity and uncertainty within decision-making processes. It enabled complex expressions to be translated from natural language discourse to mathematical modeling [45]. L. Zadeh [20] invented fuzzy logic, which has since become one of the most well-known ways to construct environmental indicators. These systems can easily include human thought and expertise in the indices [46]. Fuzzy logic can translate uncertain or unclear data into fuzzy sets for processing by the fuzzy inference system (FIS). Under this study, important parameters of water quality, like pH, turbidity, and dissolved oxygen, have been considered as fuzzy variables. This enabled the developed system to handle imprecise values, such as those from missing data and sensor errors, and therefore produced more reliable and valid results [47–55].

2.2.2. Fuzzy Inference Systems

A fuzzy inference system (FIS) transforms input elements into output through logical reasoning, addressing uncertainty in terms like "significant impact" or "level of concern" [56,57]. This imprecision stems from human judgment and is often expressed with linguistic variables, especially in fields like environmental management and water quality [22]. The two main inference methods are Mamdani (1974) and Takagi–Sugeno (1985). The Sugeno approach differs from Mamdani by using constant functions instead of fuzzy sets in the consequent part of the rule base [58]. Both methods use triangular, Gaussian, or trapezoidal membership functions for inputs, with Mamdani also using these functions for outputs, while Sugeno uses singleton functions [59]. An FIS consists of three components: fuzzification, inference rules, and defuzzification. Fuzzification converts numerical inputs into fuzzy-set membership grades, indicating ranges like low or high [56]. The importance of each variable is integrated into the rules. Once fuzzified, inputs are aggregated using fuzzy operators to determine rule strength. These are combined with output membership functions, and the results are aggregated to produce a fuzzy output. Defuzzification converts the fuzzy output into a crisp value, using methods like centroid, area bisector, or the largest of the maxima (LOM) in Mamdani FIS and weighted sum (wtsum) or weighted average in Sugeno FIS [59]. Finally, the fuzzy output is transformed into a crisp value [60]. The FIS structure is shown in Figure 2.

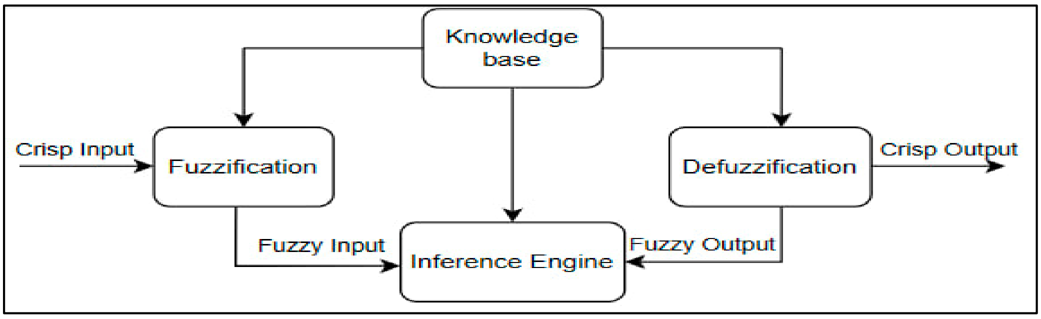


Figure 2. Fuzzy inference system structure.

Figure 3 presents the flowchart of both Mamdani and Sugeno techniques, illustrating all three steps involved in each method. The primary distinction between the Mamdani and Sugeno approaches lies in the defuzzification process. Sugeno FIS utilizes weighted sum (wtsum) and weighted average (wtaver) methods for defuzzification [61].

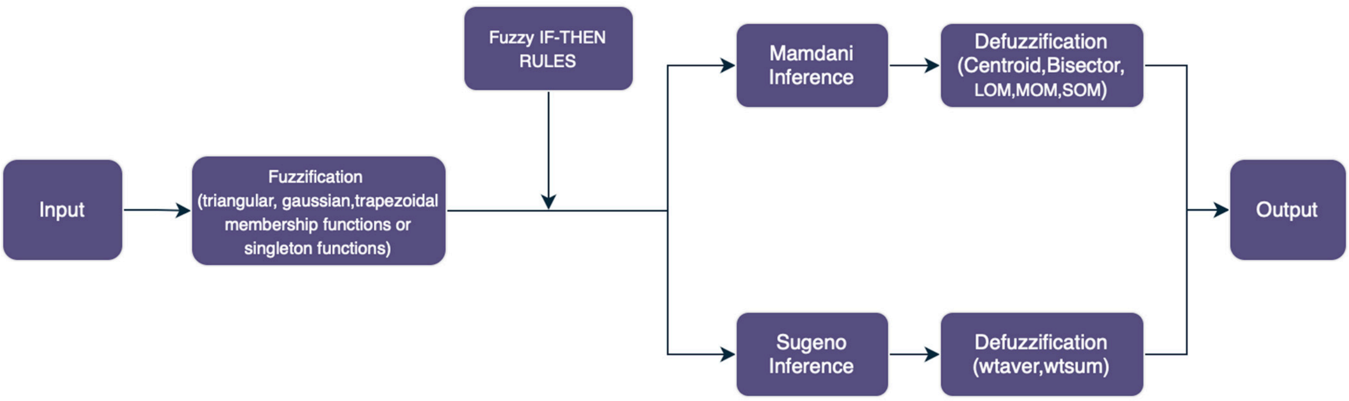


Figure 3. Flowchart of Mamdani and Sugeno techniques.

2.2.3. Defuzzification Techniques

Defuzzification refers to converting fuzzy outputs into precise values. To summarize the results of a water quality assessment, fuzzy results need to be translated into a single crisp value. Various defuzzification techniques can be used, including centroid, height, and area methods. The centroid method calculates the center of gravity (centroid) of the fuzzy-set distribution. This is done by integrating the products of the membership function and variable values across the entire fuzzy-set range and then dividing by the total area. The height method selects the highest membership grade in the fuzzy set and assigns the corresponding crisp value, indicating the dominant classification of water quality. The area-based method evaluates the fuzzy set's area to estimate the numeric score. Techniques such as the center of area or the mean of maximum (MOM) can be applied in this method. Different defuzzification methods include Center of Sums (COS), center of gravity (COG)/Centroid of Area (COA), center of area (BOA), weighted average, First of Maxima (FOM), Last of Maxima (LOM), and Mean of Maxima (MOM).

The COG/COA method calculates the precise value by determining the center of gravity of the fuzzy set. The fuzzy set's total area is divided into sub-areas, and the centroids and areas of these sub-areas are calculated to determine the defuzzified value. These are then summed up to obtain the final defined value [62]. For a discrete membership function, this value, denoted as x* and calculated using the center of gravity (COG) method, is defined as follows:

$$x^* = \frac{\sum_{i=1}^{n} x_i.\mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)} \tag{1}$$

In Equation (1), xi denotes a sample element, $\mu(x_i)$ represents the membership function, and n signifies the total number of elements in the sample.

For a continuous membership function, x^* is defined as in Equation (2):

$$x^* = \frac{\int x\mu_A(x)dx}{\mu_A(x)dx} \tag{2}$$

The Weighted Average: The technique works well with fuzzy sets that resemble the COA technique and have symmetrical-outcome membership functions. This approach requires less computing power. Based on the greatest membership score for every member function, a weighting system is employed.

$$x^* = \frac{\sum \mu(x).x}{\sum \mu(x)} \tag{3}$$

In Equation (3), Σ denotes the algebraic summation, and x is the element with maximum membership function.

Center of Area/The Bisector of Area Method (BOA): This method identifies the point beneath the curve where the areas on both sides of the curvature are equal, as outlined in Equation (4). Two zones with the same area are created by the procedure that the BOA creates.

$$\int_{\alpha}^{x^*} \mu_A(x)dx = \int_{x^*}^{\beta} \mu_A(x)dx, \text{where } \alpha = \min\{x|x\epsilon X\} \text{ and } \beta = \max\{x|x\epsilon X\} \tag{4}$$

The Last of Maxima (LOM): This defuzzification approach utilizes the area with the highest membership score, which has the largest number [63]. The last maximum defuzzification is shown in Equation (5). In Equation (5), the parameter A represents a fuzzy set, while the number one denotes a membership function.

$$y_{LOM} = \max\{y|\mu_A(y) = \max(\mu_A(y))\} \tag{5}$$

The Smallest of the Maxima Method (SOM): This technique ascertains the area with the highest membership score that has the lowest number [64]. The smallest of maximum defuzzification is shown in Equation (6). In Equation (6), the parameter A indicates a fuzzy set, and one indicates a membership function.

$$y_{SOM} = \min\{y|\mu_A(y) = \max(\mu_A(y))\} \quad (6)$$

The Mean of Maxima Method (MOM): The component with the greatest membership scores is determined by taking the defined value. This technique is useful when there is more than one component that has the maximum membership score. The means of the maxima defuzzification method is shown in Equation (7).

$$y_{MOM} = \frac{y_{LOM} + y_{SOM}}{2} \quad (7)$$

2.3. Development of an FIS Framework to Forecast the Quality of the Drinking Water Distributed in Tivoli

The present research effort aims to develop a cohesive fuzzy model to forecast water quality. This water quality-forecasting framework was developed using the MATLAB R2022b program (MathWorks, Natick, MA, USA). We selected a cohesive fuzzy model since the main benefit is that it reduces the overall number of fuzzy rules in comparison to a conventional fuzzy model. The major issue with conventional fuzzy models is that when more inputs are added to the model, it requires creating a huge number of fuzzy if-then rules, making the model complex and time-intensive to develop. The number of fuzzy if-then rules depends on the number of input variables and the number of fuzzy sets for each variable. The conventional model in this study consists of 9 input variables, each with 3 fuzzy sets (low, medium, high). In contrast, the cohesive fuzzy model is structured with 3 fuzzy models in the middle layer, each containing 3 input variables with 3 fuzzy sets per variable. Traditional and proposed cohesive fuzzy models can be seen in Figures 4 and 5, respectively.

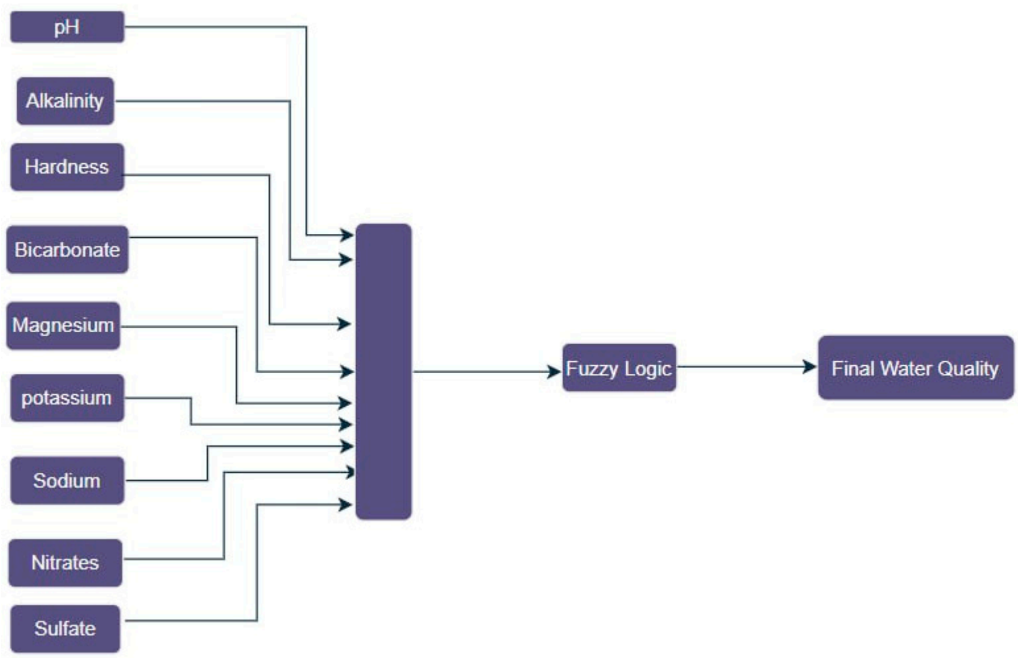


Figure 4. Traditional fuzzy system.

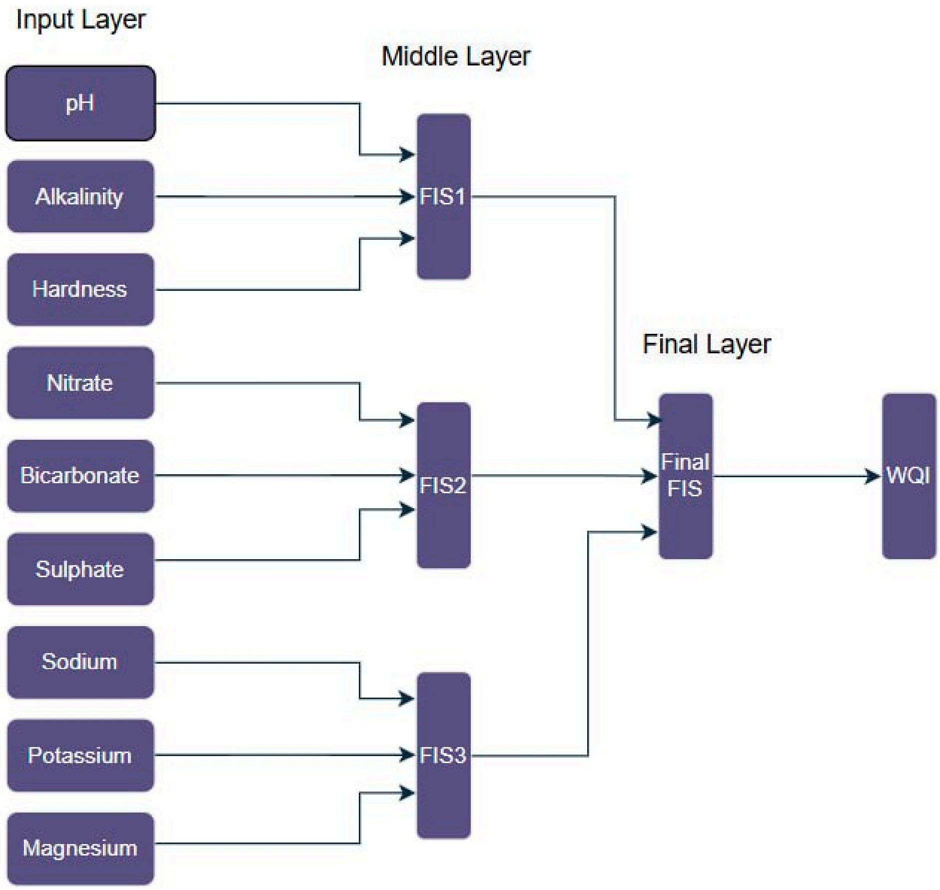
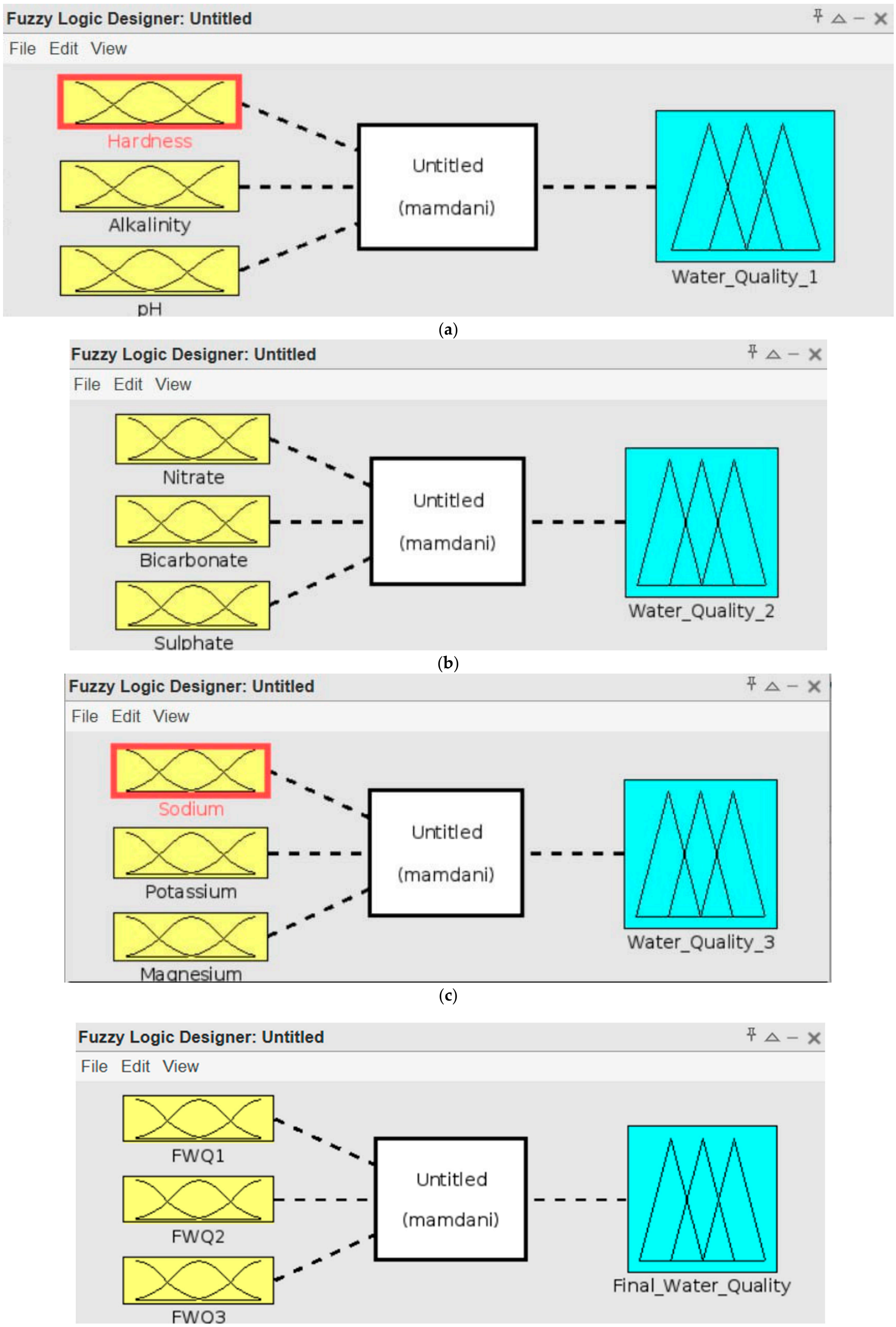


Figure 5. Cohesive fuzzy system.

2.3.1. Hierarchical Model Structure

A hierarchical, cohesive fuzzy inference model for estimating the quality of drinking water is shown in Figure 6a–d. The model in this study consists of two levels. In the first level, we define three intermediate models with three inputs; in the second level, we define the final model. The outputs generated from the first level serve as inputs for the final model in the second level. There are multiple stages throughout the process of creating the model to forecast the quality of the water. The system's input and output parameters must be identified in the first stage. Alkalinity, pH, and hardness are the three variables of the first fuzzy system, according to the fuzzy model's framework. The second fuzzy model similarly has three inputs: nitrate, bicarbonate, and sulfate. Similarly, the third fuzzy model has three inputs: sodium, potassium, and magnesium. The results of all three models in the middle layer will be used as inputs for the final model to evaluate the quality.

Assigning ranges to each input and output comes after the inputs and outputs of the model have been defined. The criteria for drinking water quality (IS 10500) [65] are displayed in Table 1 and are used to determine the range for each variable. To protect human health, regulatory organizations specify that the level and quality of physicochemical and biological factors in water for drinking must be within allowable (or desirable) boundaries. All the model's numerical input parameters are categorized into three groups, high, medium, and low, and the output of the proposed model is divided into seven categories between 0–100. Table 1 outlines the various input variables used in the analysis, specifying their associated linguistic terms and the ranges for each term.



(**d**)

Figure 6. (**a**) Structure of first fuzzy model. (**b**) Structure of second fuzzy model. (**c**) Structure of third fuzzy model. (**d**) Structure of final model.

Table 1. Fuzzy value of all parameters.

Parameters	Low Range	Medium Range	High Range	Data Set of Tivoli	Standard Value
pH	0–7 (Poor)	5.5–9.5 (Good)	7–14 (Moderate)	7.5	6.5–8.5
Alkalinity	0–400 (Good)	100–700 (Moderate)	400–800 (Poor)	318 (mg/L)	200 (mg/L)
Hardness	0–500 (Good)	100–900 (Moderate)	500–1200(Poor)	317(mg/L)	300 (mg/L)
Bicarbonate	0–400 (Good)	100–700 (Moderate)	400–800 (Poor)	388 (mg/L)	100 (mg/L)
Magnesium	0–60 (Good)	20–100 (Moderate)	60–120 (Poor)	21.80 (mg/L)	50 (mg/L)
Potassium	0–15 (Good)	6–25 (Moderate)	15–30 (Poor)	0.65 (mg/L)	12 (mg/L)
Sodium	0–400(Good)	100–700 (Moderate)	400–800 (Poor)	2.9 (mg/L)	200 (mg/L)
Nitrates	0–80 (Good)	20–140 (Moderate)	80–180 (Poor)	2.01 mg/L	45 (mg/L)
Sulfate	0–400 (Good)	150–650 (Moderate)	400–800 (Poor)	3.67 (mg/L)	200 (mg/L)

Figure 7 shows the triangular membership function for bicarbonate and output which is based on the quality of the water. The simplest membership functions formed using straight lines are triangular membership functions. These straight-line membership functions have the advantage of simplicity [66].

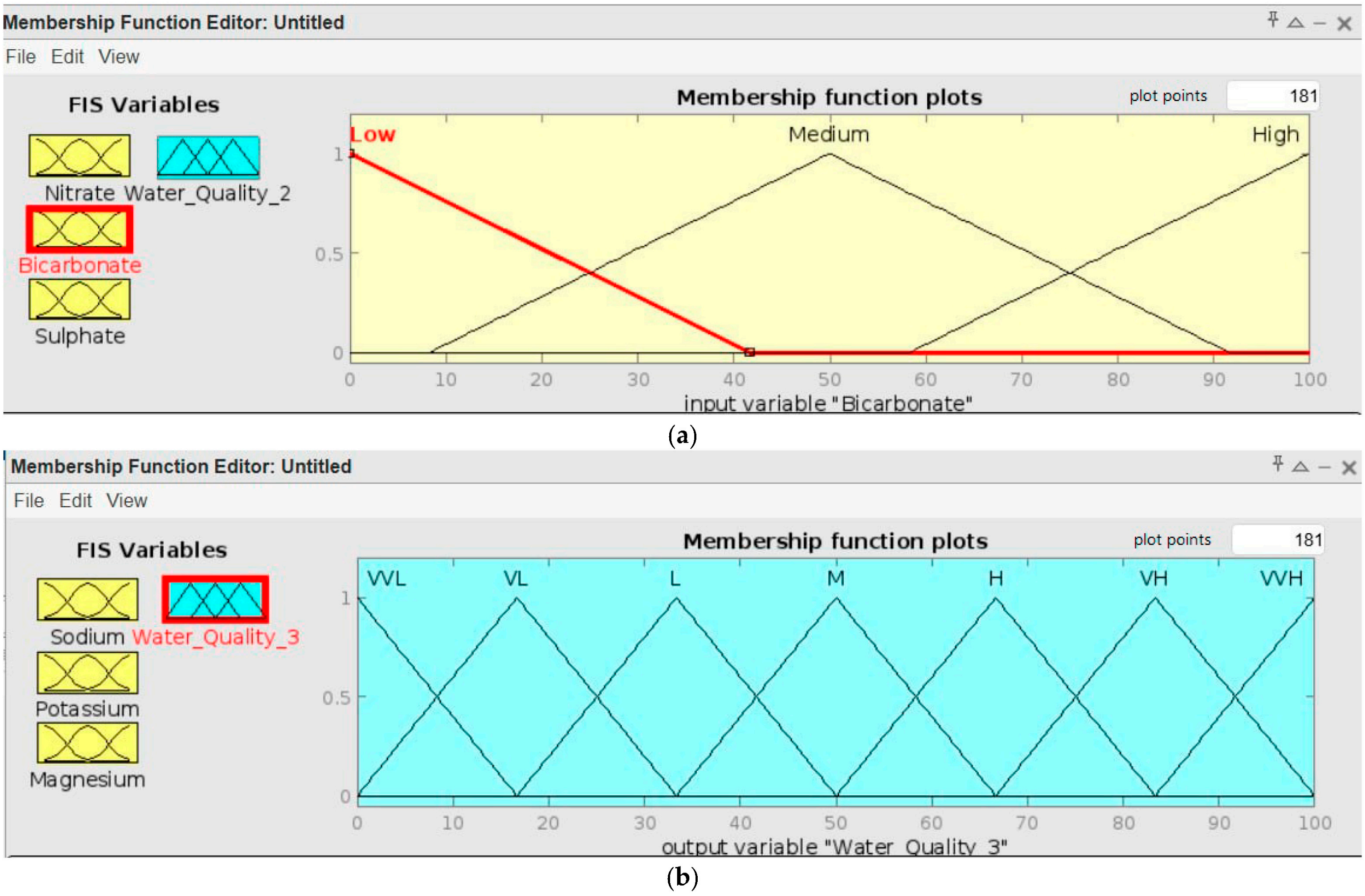


Figure 7. (**a**) Membership function for hardness variable. (**b**) Membership function for output of the fuzzy model.

The FIS is used to measure language phrases represented by t. The fuzzy input can appear in up to three situations, which are defined by the single restriction permitted: μ_c^t (center), μ_l^t (low), and μ_u^t (high). The first scenario is when an upper (C_u^t) and lower limit (C_l^t) have been established to specify the permitted ranges of t. Next, the fuzzy function μ_c^t is ascertained in this manner:

$$\mu_c^t = \max\{\min[\frac{C^t - C_l^t}{m^t - C_l^t}, 1, \frac{C_u^t - C^t}{C_u^t - m^t}], 0\} \quad (8)$$

- C^t: Measured parameter's concentration at time t; C_l^t: Lower limit of the acceptable range at time t; C_u^t: Upper limit of the acceptable range at time t. The lowest acceptable value is l; u is the maximum permissible value; and m^t is the center value.

In the Equation (9), the fuzzy function is specified just by its minimum threshold (c_l^t), and it can be computed as:

$$\mu_U^t = \max\{\min\,[\frac{C^t - C_l^t}{m^t - C_l^t}, 1], 0\} \quad (9)$$

Therefore, when an acceptable range is specified by a single possible upper unit, the fuzzy function could be derived as follows:

$$\mu_L^t = \max\{\min[\frac{C^t - C_l^t}{m^t - C_l^t}, \frac{C_u^t - C^t}{C_u^t - m^t}], 0\} \quad (10)$$

2.3.2. Fuzzy Rule

The fourth step, including "if-then" logical statements, is used for combining antecedents and consequences with the "and" operator [67]. Table 2 illustrates some of these rules for the first fuzzy model. After defining the rules, the inference engine evaluates and combines the rules into one rule. Defuzzification transforms fuzzy results into numerical values. The proposed model has three fuzzy models with three input variables and three fuzzy sets: low, medium, and high. This will result in 3 × 3 × 3 = 27 possible if-then rule combinations per model.

Table 2. Fuzzy rules.

Rule	Hardness	Alkalinity	pH	Water Quality Output
1	Low	Low	Medium	Very High (VH)
2	Low	Low	High	Very High (VH)
3	Low	Low	Low	High (H)
4	Low	Medium	Low	Medium
5	Low	Medium	High	Low (L)
6	Medium	Medium	Low	Medium (M)
7	Medium	Medium	High	Medium (M)
8	Medium	Low	Medium	Medium (M)
9	Medium	High	Low	Medium (M)
10	Medium	High	Medium	Low (L)
11	Medium	High	High	Medium (M)

2.3.3. Fuzzy Operator

After fuzzifying the inputs, we can determine the degree of antecedent satisfaction for every rule. If a rule's antecedent comprises many parts, the fuzzy operator generates a single integer that reflects the rule's outcome. The value is subsequently sent into the outcome function. The fuzzy operator receives multiple membership scores from fuzzified inputs and generates a whole amount of output. This study relied on the logic operator.

2.3.4. Implication Method

The antecedent function reshapes the consequent by mapping an integer input to a fuzzy-set output. Each rule applies an implication, calculating the minimum membership degree (activation degree) among variables to limit the outcome function's membership. These fuzzy sets are then aggregated using the highest membership value, with rule order irrelevant if the operation is commutative. Computing a fuzzy rule involves the following: 1. Combining fuzzy inputs to determine rule strength. 2. Clipping the output membership function at this strength. Figure 8 shows the implication method in the fuzzy inference system.

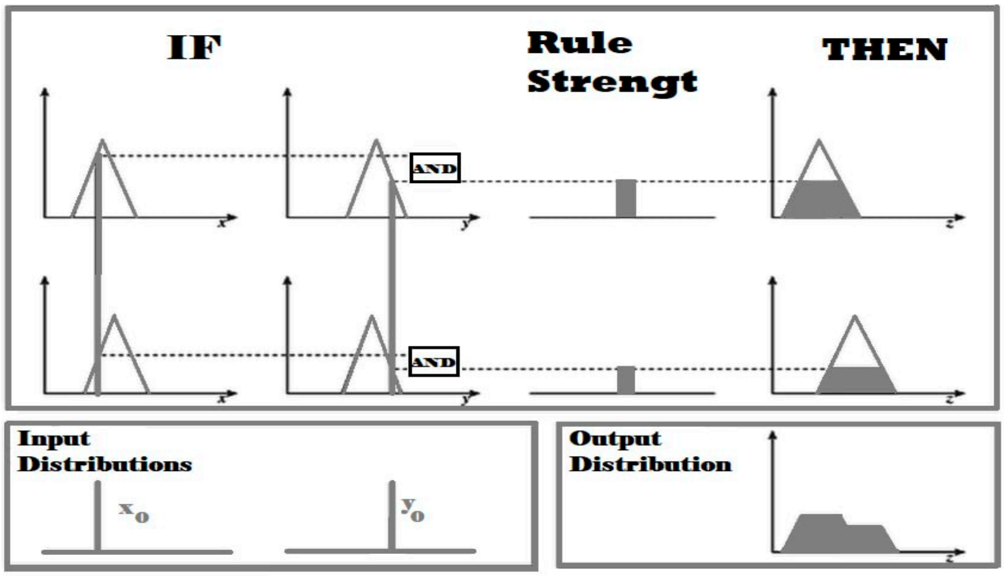


Figure 8. Implication method [68].

The aggregation process involves combining the fuzzy sets representing the outputs of each rule into a single fuzzy set, occurring only once for each output variable before the final defuzzification step, with the input being the list of truncated output functions from the implication process for each rule and resulting in one fuzzy set for each output variable.

3. Results and Discussion

The evaluation rules and surface views of the three intermediate models, FWQ1, FWQ2, FWQ3, and the final FWQ, have been derived in this study using two distinct fuzzy inference engines: the Mamdani engine and the Sugeno fuzzy engine.

3.1. Intermediate Mamdani Fuzzy Model

3.1.1. First Mamdani Fuzzy Model

Figure 9 illustrates the rule viewer for the first model's water quality evaluation, which has three input variables: hardness, alkalinity, and pH. According to the rule-based illustration in Figure 9, the score for water quality in the first fuzzy model is 72.4%, which was obtained using the centroid defuzzification technique for their corresponding average concentrations of alkalinity, pH, and hardness, which are 318 mg/L, 7.5, and 317 mg/L, accordingly.

The surface representation of the first fuzzy model in the middle layer using the centroid defuzzification approach is illustrated in Figure 10. This illustration demonstrates how alkalinity and hardness impact water quality in the first fuzzy model in the middle layer. It indicates that lower concentrations of hardness and alkalinity correspond to better water quality, and vice versa.

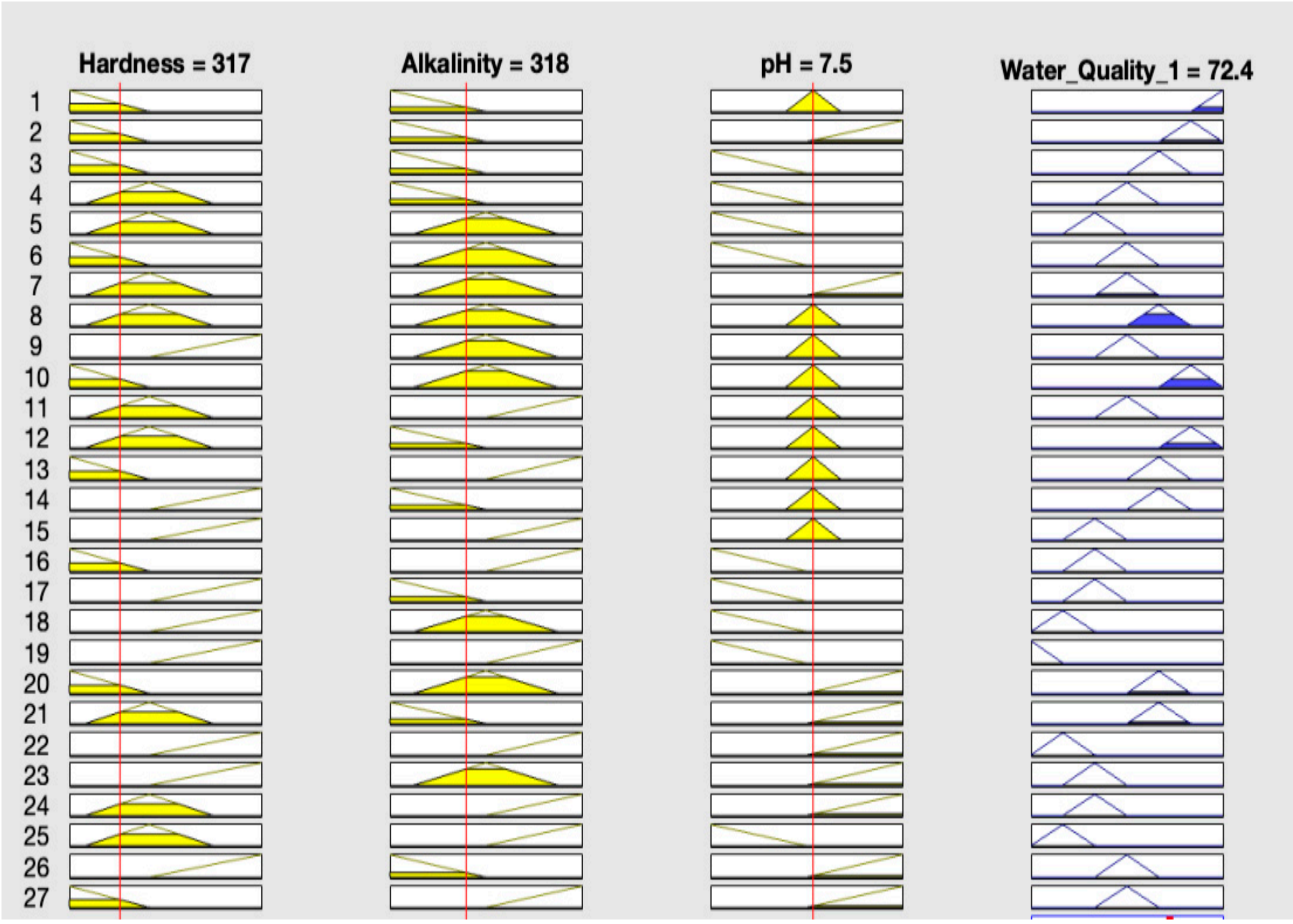


Figure 9. Rule representation in the first fuzzy model.

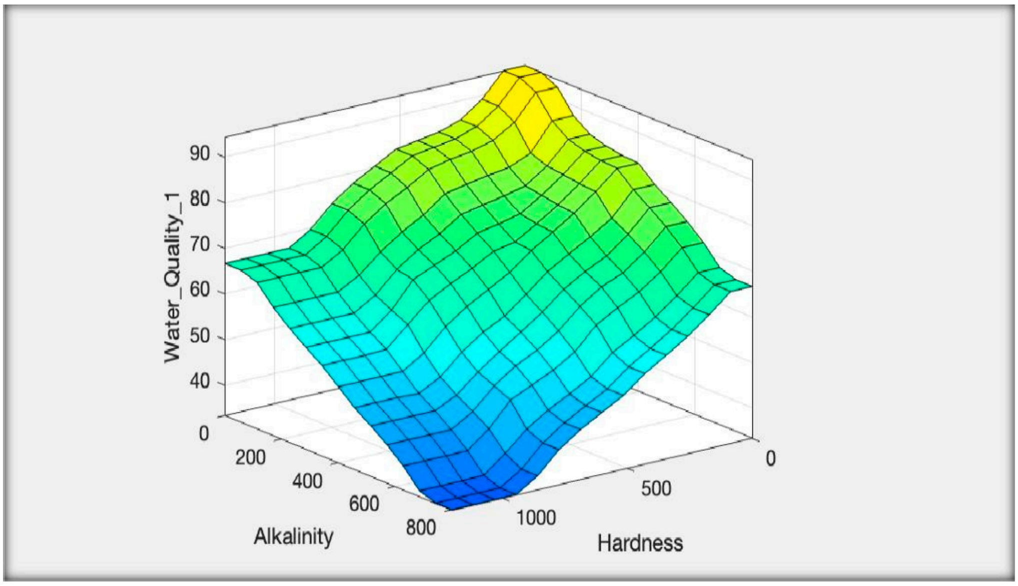


Figure 10. Surface representation of the first Mamdani fuzzy model by centroid defuzzification.

3.1.2. Second Mamdani Fuzzy Model

Figure 11 shows the rule representation of the second model using bicarbonate, sulfate, and nitrate as input factors. The second model has a value of 83.4% using centroid defuzzification for the sulfate, nitrate, and bicarbonate concentrations of 3.67, 2.01, and 388 mg/L, respectively.

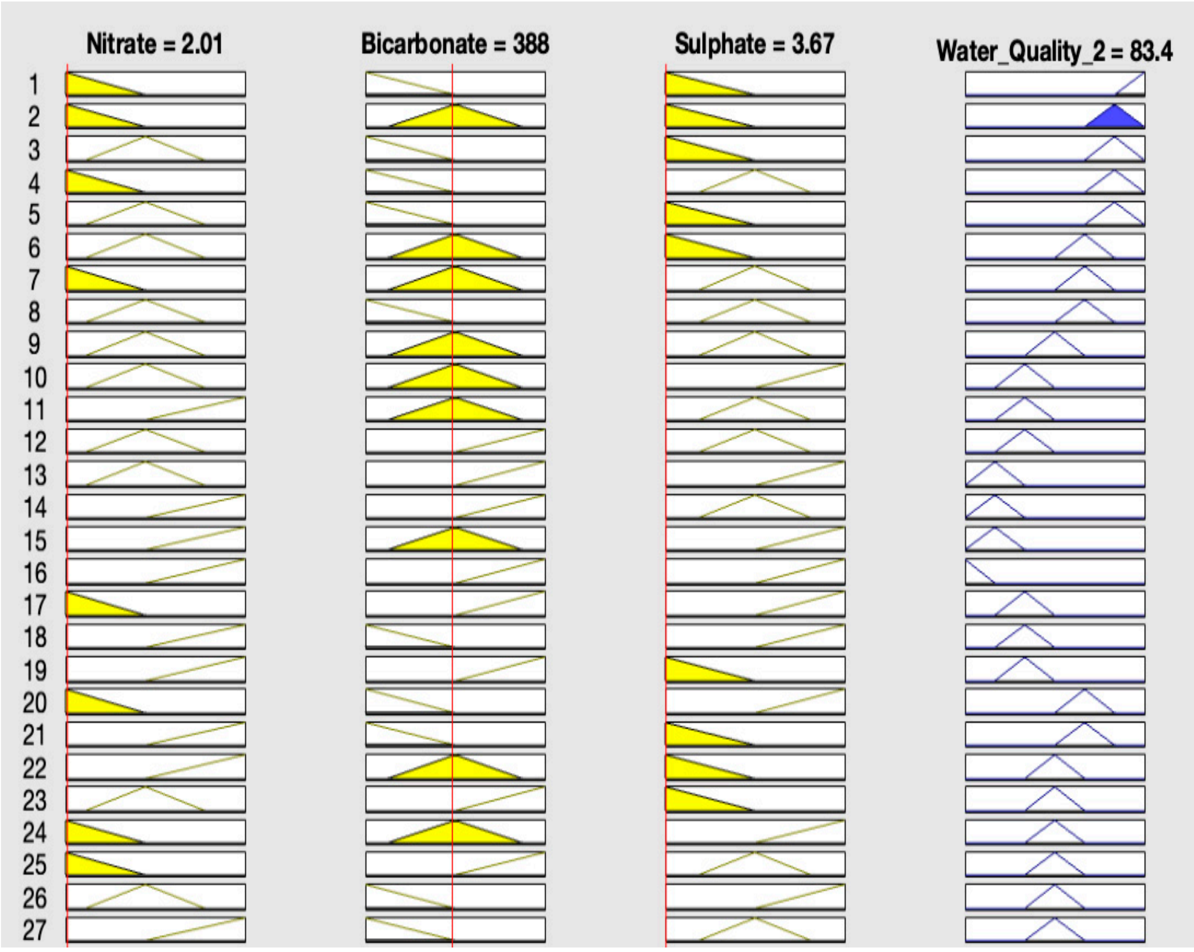


Figure 11. Rule viewer of the second fuzzy system.

Figure 12a illustrates the impact of bicarbonate and nitrates on quality using MOM defuzzification, whereas Figure 12b illustrates the centroid defuzzification approach. Better water quality is correlated with a smaller amount of bicarbonate, fluoride, and nitrate, and conversely.

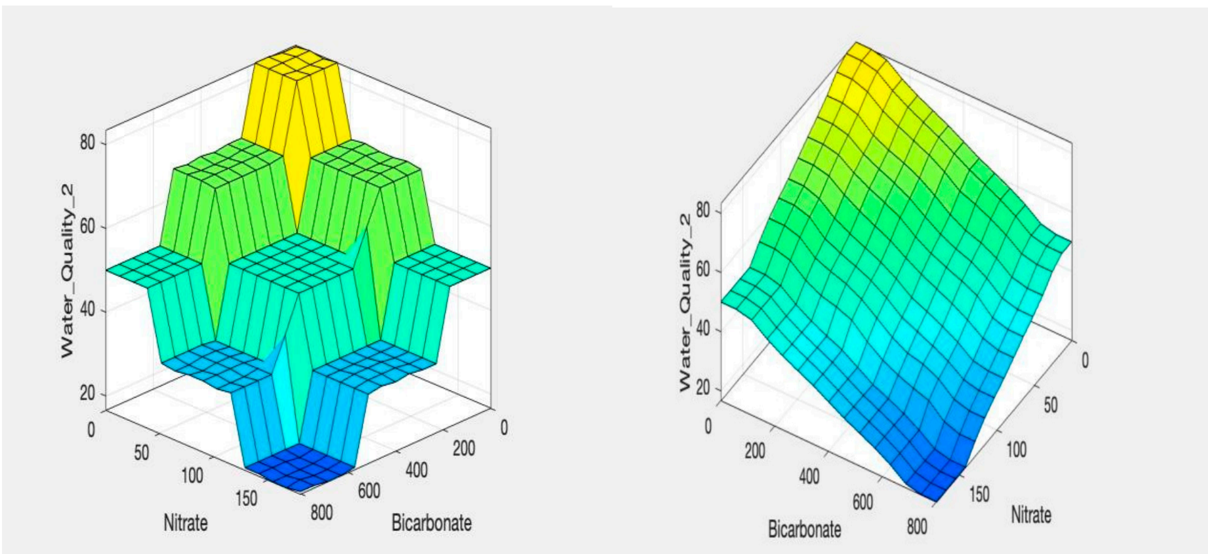


Figure 12. Surface representation of the second fuzzy model. (**a**) Centroid defuzzification. (**b**) MOM defuzzification.

3.1.3. Third Mamdani Fuzzy Model

The rule adopted for the third model in the middle layer, which has three input variables—sodium, potassium, and magnesium—is illustrated in Figure 13. Using the centroid defuzzification approach, the water quality in the third model was determined to be 92.5%, as shown by the rule-based representation in Figure 13. The average concentrations of magnesium, potassium, and sodium were found to be 21.8, 0.65, and 2.9 mg/L, accordingly.

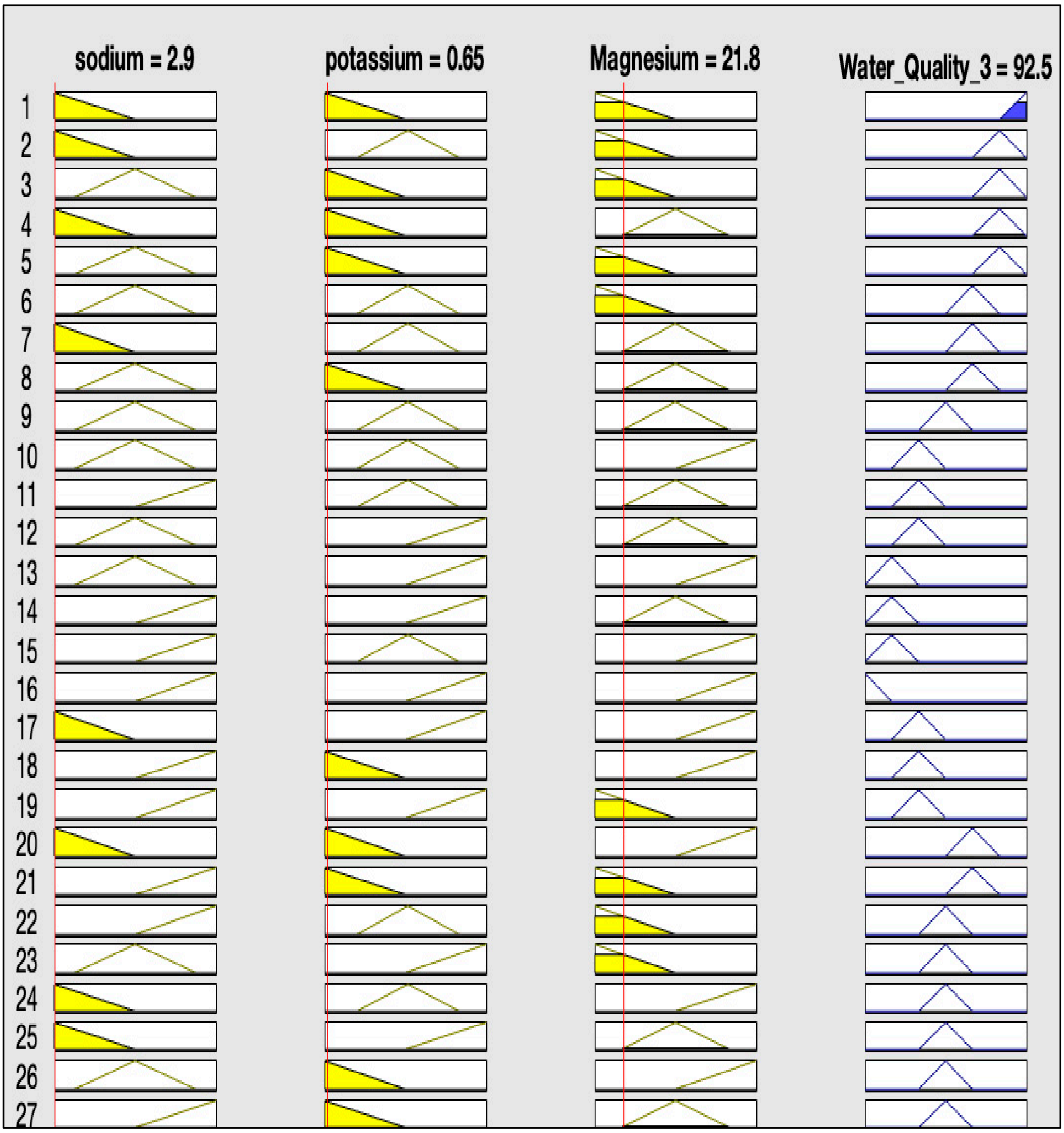
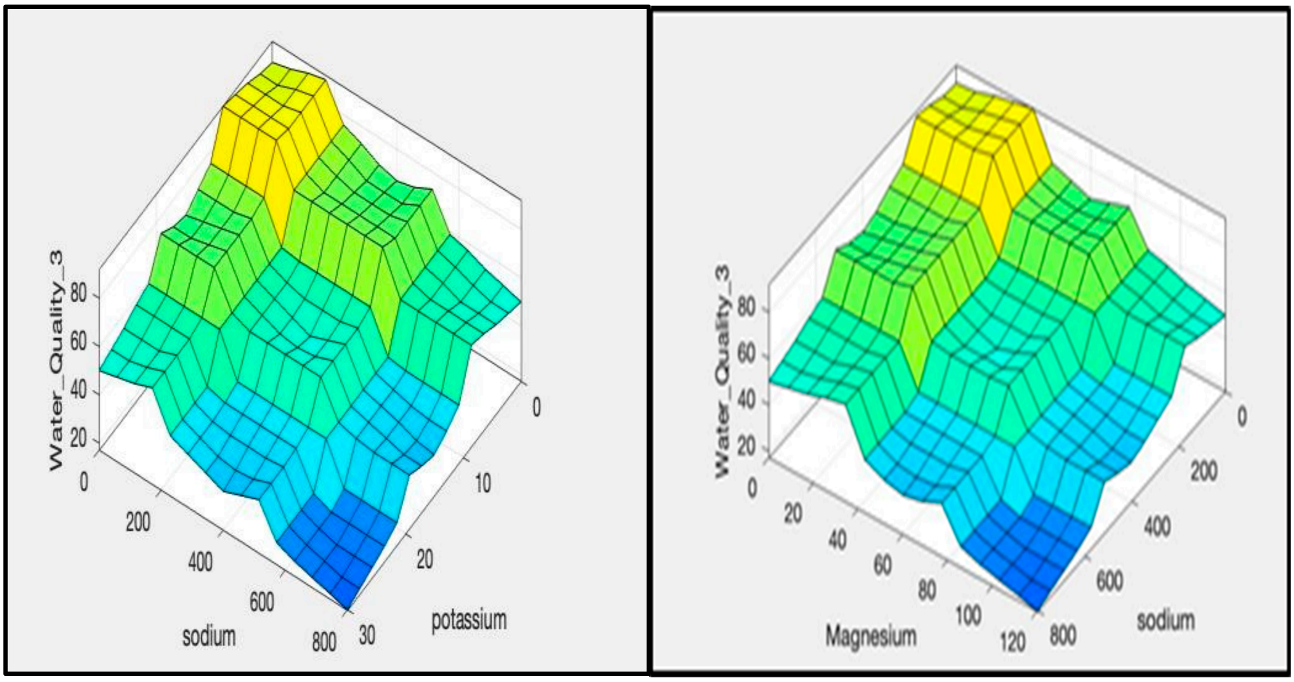


Figure 13. Rule representation of the third model.

3.2. *Final Mamdani Fuzzy Model (MFWQ)*

Figure 14a illustrates the impact of sodium and potassium on the quality of water in the third model with the MOM defuzzification approach. Figure 14b illustrates the impact of magnesium and sodium on water quality. They indicate that the smaller the sodium, potassium, and magnesium levels, the better the water quality and vice versa. Furthermore, it shows that the amount of sodium, potassium, and magnesium directly impacts the water's quality.



(**a**) (**b**)

Figure 14. Surface viewer. (**a**) Sodium and potassium surface viewer. (**b**) The surface viewer for magnesium and sodium.

In Figure 15, the rule-based illustration reveals a Final FWQ value of 85.4%, indicating a very high-quality class between 80–100. The equivalent average values in the first, second, and third fuzzy models are 72.4%, 83.4%, and 92.5%, respectively, determined using the centroid defuzzification technique. Figure 15 provides a visual representation of the rule viewer for evaluating fuzzy water quality in the final model, considering the outputs of the first, second, and third fuzzy models.

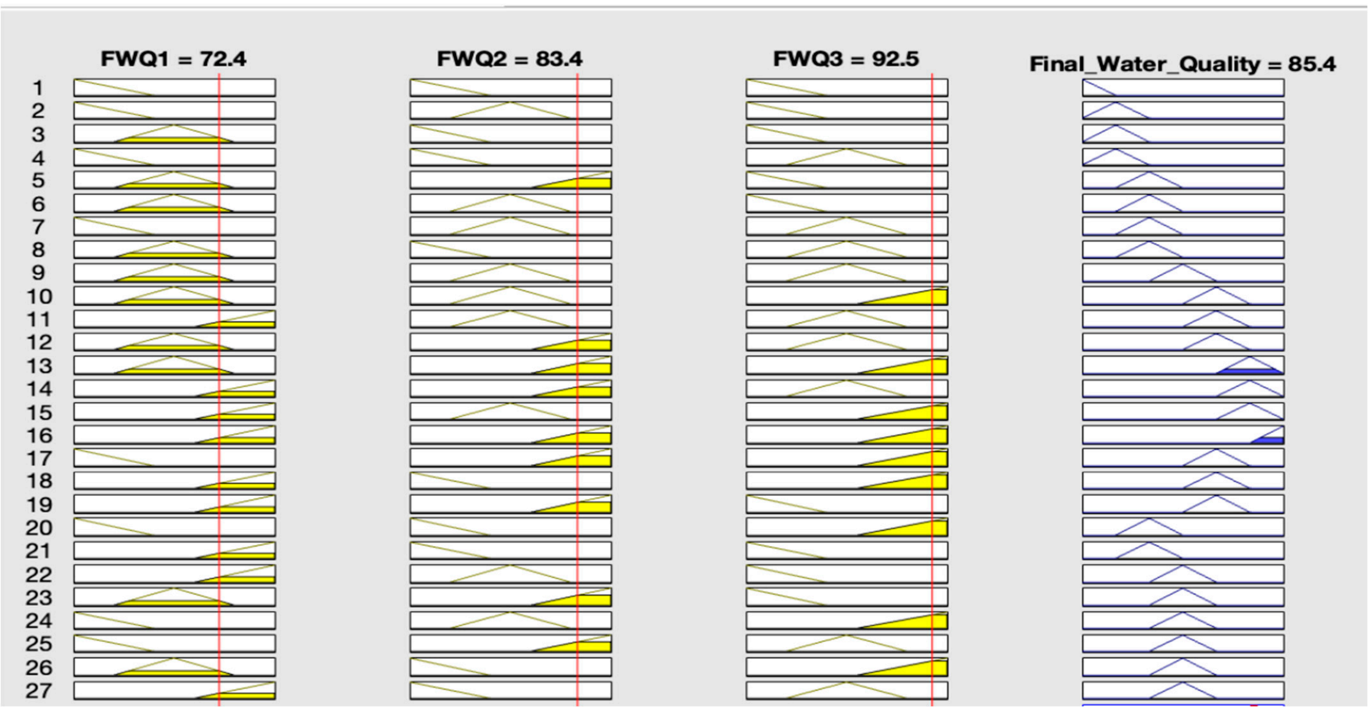


Figure 15. Rule viewer of the final fuzzy model.

Figure 16 represents the surface viewer of the ultimate fuzzy water-quality model by centroid defuzzification technique. Figure 16 illustrates how the second and third levels influence the overall quality of water. Greater water quality of second- and third-model levels correlates with greater overall water quality. Increased levels of fuzzy output in the second and third models resulted in greater ultimate water quality values, and vice versa.

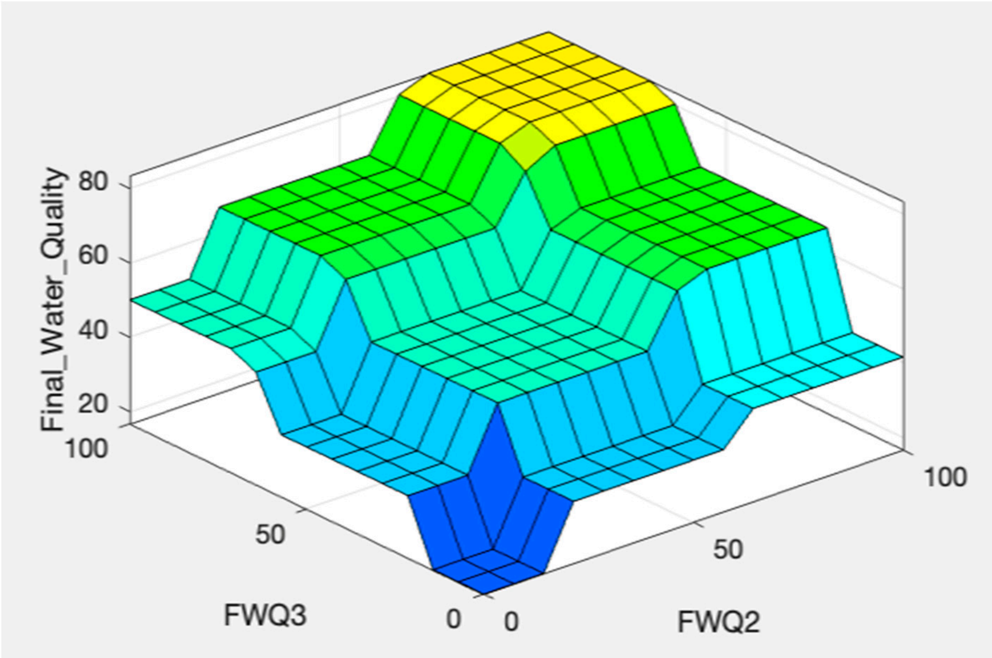


Figure 16. Surface viewer of the final fuzzy model.

3.3. Intermediate Sugeno

The results of water quality with the Sugeno fuzzy inference method with three intermediate models and the final model are presented in Figures 17–20.

3.3.1. First Sugeno Fuzzy System

Figure 17 illustrates the outcomes of the initial Fuzzy system at the intermediate level utilizing the Sugeno method. The water quality result from this first model, employing the weighted average defuzzification technique with three input parameters—hardness at 317 mg/L, alkalinity at 318 mg/L, and pH at 7.5—yields a percentage of 76.2.

3.3.2. Second Sugeno Fuzzy

Figure 18 illustrates the outcomes of the second fuzzy model at the intermediate level utilizing the Sugeno method. The water quality result from this first model, employing the weighted average defuzzification technique with three input parameters—nitrate at 2.01 mg/L, bicarbonate at 388 mg/L, and sulfate at 3.67—yields a percentage of 83.5.

3.3.3. Third Sugeno Fuzzy

Figure 19 illustrates the outcomes of the third fuzzy model at the intermediate level utilizing the Sugeno method. The water quality result from this model, employing the weighted average defuzzification technique with three input parameters—sodium at 2.9 mg/L, potassium at 0.65 mg/L, and magnesium at 3.67—yields a percentage of 92.5.

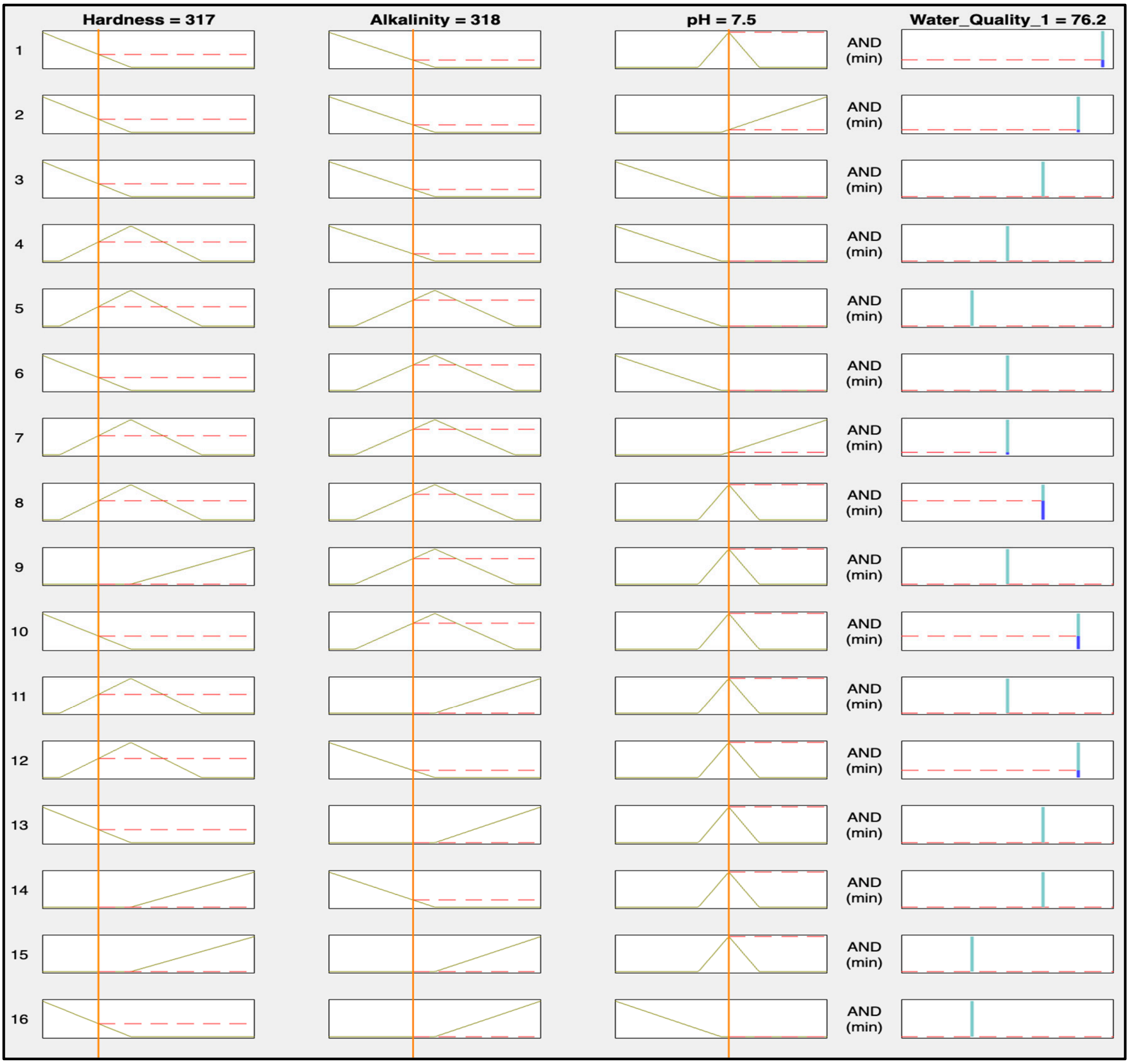


Figure 17. First fuzzy model with Sugeno technique.

3.4. Final Sugeno Fuzzy Inference Engine

Figure 20 illustrates the results of the ultimate fuzzy system at the second level, which employs the Sugeno method. Assessment of the quality derived from this model utilizes the weighted average defuzzification approach. The inputs for this model are derived from the outputs of the intermediate-level models. The final model yields a result of 83.5 percent based on the weighted average defuzzification approach.

Table 3 shows water quality scores for three intermediate models (FWQ1, FWQ2, FWQ3) and the final model (Final FWQ) using Mamdani and Sugeno systems. Mamdani's centroid method scored 72.4%, 83.4%, 92.5%, and 85.4%, while the LOM method achieved the highest scores of 100.0% for FWQ3 and the Final FWQ. Sugeno's weighted average method provided consistent scores of 76.2%, 83.5%, 92.5%, and 83.5%, highlighting its efficiency and stability for dynamic systems. The results demonstrate Mamdani's flexibility and Sugeno's reliability for water quality evaluation.

Figure 18. Second fuzzy model with the Sugeno technique.

Table 3. Water quality scores for intermediate and final fuzzy models using Mamdani and Sugeno systems.

Model	Defuzzification Method	FWQ1 Score (%)	FWQ2 Score (%)	FWQ3 Score (%)	Final FWQ Score (%)
Mamdani	Centroid	72.4	83.4	92.5	85.4
Mamdani	Bisector	72.0	83.0	94.0	86.0
Mamdani	LOM	74.0	83.0	100.0	100.0
Mamdani	MOM	67.0	83.0	97.0	94.5
Mamdani	SOM	60.0	83.0	94.0	89.0
Sugeno	Weighted Average	76.2	83.5	92.5	83.5

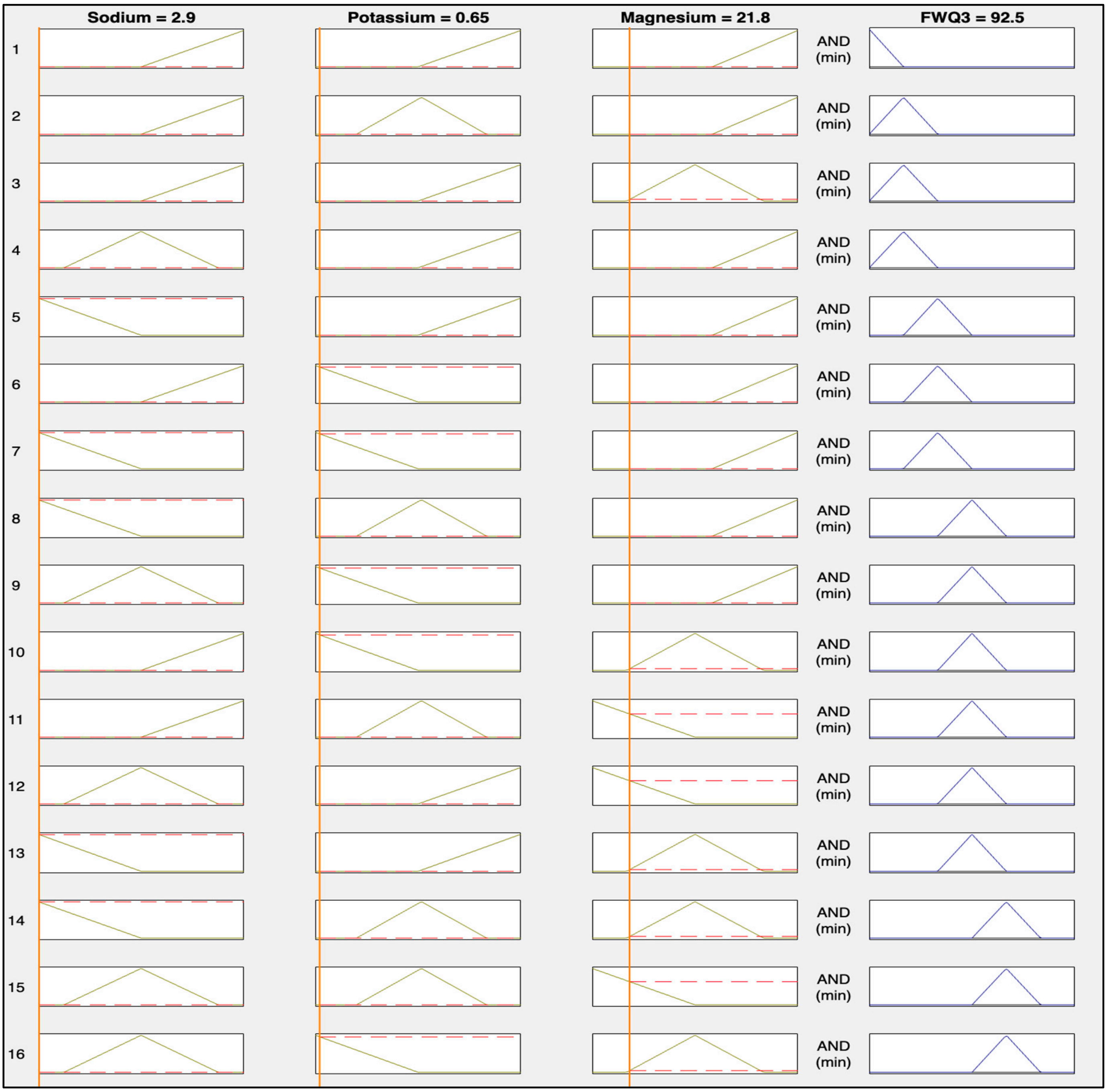


Figure 19. Third fuzzy model with Sugeno technique.

The system proposed in this study provides several advantages over other artificial intelligence methods discussed in the literature:

1. Integrating fuzzy rules within an FIS presents an innovative and efficient way to enhance communication between humans and advanced AI systems.
2. The use of fuzzy rules as a model-specific explanatory method shows varying degrees of interpretability across different prediction models.
3. Fuzzy inference systems are well-suited for embedding linear controllers, offering ease in mathematical analysis, straightforward rule formulation, and simple interpretation and implementation.
4. Fuzzy inference systems are capable of extracting and managing fuzzy and uncertain data features.

5. The Sugeno method demonstrates high computational efficiency; it seamlessly incorporates optimization and adaptive techniques.

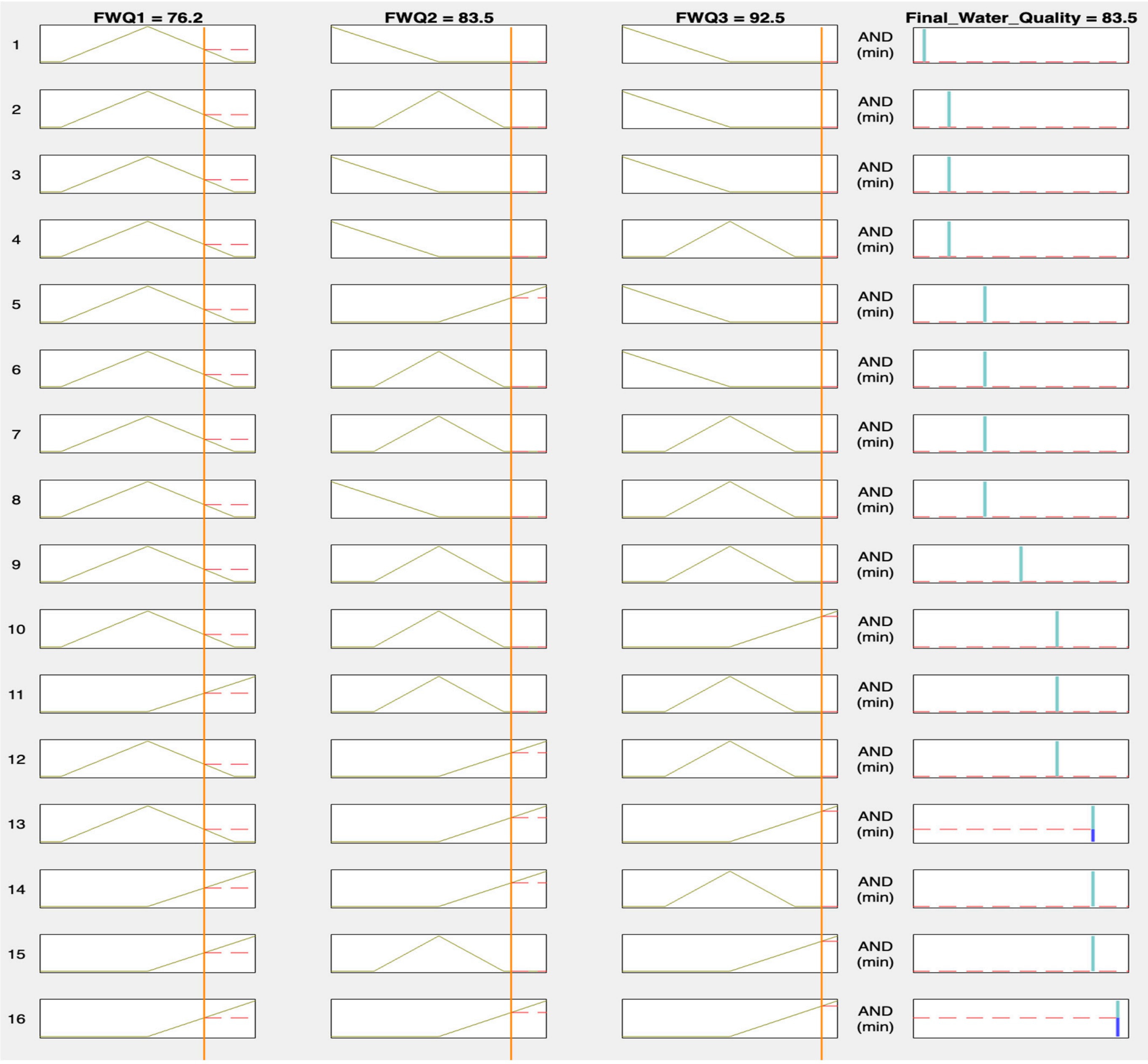


Figure 20. Final fuzzy model with Sugeno technique.

3.5. Comparison and Interpretation of Results

The results of the water quality assessment, conducted using the Mamdani inference engine with various defuzzification methods and the Sugeno inference engine, consistently indicate water quality levels within the range of 80–100 percent. The results from both methods fall within the 'Very Very High' (VVH) category, meaning the water quality is excellent. Both the Mamdani and Sugeno approaches show the same outcome, consistently indicating that the water quality is at a very very high level, confirming its superior quality. Table 4 shows the final result with the Mamdani and Sugeno approaches, indicating that all the results belong to the same water quality category.

Table 4. Comparison result.

Mamdani WQ Centroid	Mamdani WQ Bisector	Mamdani WQ LOM	Mamdani WQ MOM	Mamdani WQ SOM	Sugeno WQ
85.4%	86%	100%	94.5%	89%	83.5%

4. Conclusions

The assessment of drinking water quality in Tivoli, Italy, was carried out by two fuzzy inference systems with different defuzzification techniques. This cohesive fuzzy model proposed various advantages compared to the existing fuzzy systems; it is less complex and has fewer rules. Therefore, it turned out to be reliable and interpretable, integrating nine critical water quality parameters. The results showed that the water quality in Tivoli was always in the category of "very high", with Mamdani and Sugeno systems giving water quality scores of 85.4% and 83.5%, respectively. These results illustrated the capability of fuzzy-logic-based approaches in handling imprecise, uncertain, or noisy data; thus, it is an efficient tool for sustainable water management. Compared to other methods, traditional statistical approaches, or even modern machine learning models such as neural networks and support vector machines, there are advantages to the cohesive fuzzy model. Fuzzy logic works effectively with imprecise, uncertain, or ambiguous information, for which statistical or deterministic models do not work well because of noisy or incomplete data conditions. While machine learning methods are highly accurate, they mainly act as black-box systems, hence limiting interpretability by policymakers and water managers. Other advanced techniques, like the Adaptive Neuro-Fuzzy Inference Systems, combine neural networks with fuzzy logic for high precision at the expense of increased computational complexity and higher data requirements. In contrast, the cohesive fuzzy model strikes an optimum balance between simplicity, efficiency, and accuracy, and thus provides actionable insight understandable by experts and non-experts alike. The proposed methodology not only simplifies the evaluation process but also delivers insights accessible to policymakers and non-experts in pollution control, ecosystem conservation, and sustainable resource utilization. Further research may extend the model by considering extra input parameters, other types of membership functions, or more advanced techniques like ANFIS using neural networks together with fuzzy logic. The results of this study help provide more insight into the assessment of water quality for informed decisions to attain sustainable and healthful longevity.

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Conflicts of Interest: Author Patrizio Pisani was employed by the company Unidata S.p.A. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

1. Heinemann, A.B.; Hoogenboom, G.; De Faria, R.T. Determination of Spatial Water Requirements at County and Regional Levels Using Crop Models and GIS: An Example for the State of Paraná, Brazil. *Agric. Water Manag.* **2002**, *52*, 177–196. [CrossRef]
2. Liang, Y.; Cai, Y.; Wang, X.; Li, C.; Liu, Q. Water Security Assessment with the Improvement of Modifying the Boundary Consistency Between Footprint and Provision. *Sci. Total Environ.* **2021**, *801*, 149639. [CrossRef] [PubMed]
3. Jennings, G. *Water-Based Tourism, Sport, Leisure, and Recreation Experiences*; Routledge: London, UK, 2007; pp. 1–20.
4. Cabral Pinto, M.M.; Ordens, C.M.; Condesso de Melo, M.T.; Inácio, M.; Almeida, A.; Pinto, E.; Ferreira da Silva, E.A. An Interdisciplinary Approach to Evaluate Human Health Risks Due to Long-Term Exposure to Contaminated Groundwater Near a Chemical Complex. *Expo. Health* **2020**, *12*, 199–214. [CrossRef]
5. Zeilhofer, P.; Zeilhofer, L.V.A.C.; Hardoim, E.L.; Lima, Z.M.D.; Oliveira, C.S. GIS Applications for Mapping and Spatial Modeling of Urban-Use Water Quality: A Case Study in District of Cuiabá, Mato Grosso, Brazil. *Cad. Saúde Pública* **2007**, *23*, 875–884. [CrossRef]
6. Kahlown, M.A.; Tahir, M.A.; Rasheed, H. *National Water Quality Monitoring Programme*; Fifth Monitoring Report (2005–2006); Pakistan Council of Research in Water Resources: Islamabad, Pakistan, 2007.
7. Mahapatra, S.S.; Nanda, S.K.; Panigrahy, B.K. A Cascaded Fuzzy Inference System for Indian River Water Quality Prediction. *Adv. Eng. Softw.* **2011**, *42*, 787–796. [CrossRef]
8. European Parliament and Council of the European Union. Water Framework Directive of the European Parliament and the Council, 23 October 2000 No. 60. *Off. J. Eur. Communities* **2000**, *327*, 1–72.
9. Nagels, J.W.; Davies-Colley, R.J.; Smith, D.G. A Water Quality Index for Contact Recreation in New Zealand. *Water Sci. Technol.* **2001**, *43*, 285–292. [CrossRef] [PubMed]
10. McOne, T.; Ashok, W.D. Can Fuzzy Logic Bring Complex Problems into Focus? *J. Environ. Sci. Technol.* **2005**, *39*, 42A–47A. [CrossRef] [PubMed]
11. Silvert, W. Fuzzy Indices of Environmental Conditions. *Ecol. Model.* **2000**, *130*, 111–119. [CrossRef]
12. Guo, X.; Zhang, X.; Yue, H. Evaluation of Hierarchically Weighted Principal Component Analysis for Water Quality Management at Jiaozuo Mine. *Ecol. Indic.* **2018**, *91*, 442–450. [CrossRef]
13. Ighalo, J.O.; Adeniyi, A.G. A Comprehensive Review of Water Quality Monitoring and Assessment in Nigeria. *Chemosphere* **2020**, *260*, 127569. [CrossRef] [PubMed]
14. Motlagh, A.M.; Yang, Z.; Saba, H. Groundwater Quality. *Water Environ. Res.* **2020**, *92*, 1649–1658. [CrossRef]
15. Uddin, M.G.; Nash, S.; Olbert, A.I. A Review of Water Quality Index Models and Their Use for Assessing Surface Water Quality. *Ecol. Indic.* **2021**, *122*, 107218. [CrossRef]
16. Egbueri, J.C.; Ameh, P.D.; Unigwe, C.O. Integrating Entropy-Weighted Water Quality Index and Multiple Pollution Indices Towards a Better Understanding of Drinking Water Quality in Ojoto Area, SE Nigeria. *Sci. Afr.* **2020**, *10*, e00644. [CrossRef]
17. Ortega, D.J.; Pérez, D.A.; Américo, J.H.; De Carvalho, S.L.; Segovia, J.A. Development of an Index of Resilience for Surface Water in Watersheds. *J. Urban Environ. Eng.* **2016**, *10*, 72–82. [CrossRef]
18. Alcamo, J. Water Quality and Its Interlinkages with the Sustainable Development Goals. *Curr. Opin. Environ. Sustain.* **2019**, *36*, 126–140. [CrossRef]
19. Li, P.; Wu, J. Drinking Water Quality and Public Health. *Expo. Health* **2019**, *11*, 73–79. [CrossRef]
20. Zadeh, L.A. Fuzzy Sets. *Inf. Control J.* **1965**, *8*, 338–353. [CrossRef]
21. Sahu, M.; Mahapatra, S.S.; Sahu, H.B.; Patel, R.K. Prediction of Water Quality Index Using Neuro-Fuzzy Inference System. *Water Qual. Expo. Health* **2011**, *3*, 175–191. [CrossRef]
22. Liu, S.; Tai, H.; Ding, Q.; Li, D.; Xu, L.; Wei, Y. A Hybrid Approach of Support Vector Regression with Genetic Algorithm Optimization for Aquaculture Water Quality Prediction. *Math. Comput. Model.* **2013**, *58*, 458–465. [CrossRef]
23. Sedeño-Díaz, J.E.; López-López, E. Fuzzy Logic as a Tool for the Assessment of Water Quality for Reservoirs: A Regional Perspective (Lerma River Basin, Mexico). In *Lake Sciences and Climate Change*; IntechOpen: London, UK, 2016; Volume 5, pp. 155–174. [CrossRef]

24. Khan, Y.; See, C.S. Predicting and Analyzing Water Quality Using Machine Learning: A Comprehensive Model. In Proceedings of the 2016 IEEE Long Island Systems, Applications and Technology Conference (LISAT), Farmingdale, NY, USA, 29 April 2016; IEEE: Piscataway, NJ, USA, 2016; pp. 1–6. [CrossRef]
25. Huang, M.; Tian, D.; Liu, H.; Zhang, C.; Yi, X.; Cai, J.; Ying, G. A Hybrid Fuzzy Wavelet Neural Network Model with Self-Adaptive Fuzzy c-Means Clustering and Genetic Algorithm for Water Quality Prediction in Rivers. *Complexity* **2018**, *2018*, 8241342. [CrossRef]
26. Zhou, J.; Wang, Y.; Xiao, F.; Wang, Y.; Sun, L. Water Quality Prediction Method Based on IGRA and LSTM. *Water* **2018**, *10*, 1148. [CrossRef]
27. Ahmed, A.N.A.; Othman, F.B.; Afan, H.A.A.; Ibrahim, R.K.; Chow, F.C.; Hossain, M.S.; Elshafie, A.E. Machine Learning Methods for Better Water Quality Prediction. *J. Hydrol.* **2019**, *578*, 124084. [CrossRef]
28. Li, X.; Cheng, Z.; Yu, Q.; Bai, Y.; Li, C. Water-Quality Prediction Using Multimodal Support Vector Regression: Case Study of Jialing River, China. *J. Environ. Eng.* **2017**, *143*, 04017070. [CrossRef]
29. Samantaray, S.; Sahoo, A.; Baliarsingh, F. Groundwater Level Prediction Using an Improved SVR Model Integrated with Hybrid Particle Swarm Optimization and Firefly Algorithm. *Clean. Water* **2024**, *1*, 100003. [CrossRef]
30. Biney, E.A.; Ankomah, E.A.; Agbenorhevi, A.E.; Yankey, D.B.; Annan, E. Sustainability Assessment of Groundwater in South-Eastern Parts of the Western Region of Ghana for Water Supply. *Sustainability* **2024**, *16*, 1234. [CrossRef]
31. Barzegar, Y.; Gorelova, I.; Bellini, F.; D'Ascenzo, F. Drinking Water Quality Assessment Using a Fuzzy Inference System Method: A Case Study of Rome (Italy). *Int. J. Environ. Res. Public Health* **2023**, *20*, 6522. [CrossRef] [PubMed]
32. Kimothi, S.; Thapliyal, A.; Akram, S.V.; Singh, R.; Gehlot, A.; Mohamed, H.G.; Noya, I.D. Big Data Analysis Framework for Water Quality Indicators with Assimilation of IoT and ML. *Electronics* **2022**, *11*, 1927. [CrossRef]
33. Hassan, M.M.; Hassan, M.M.; Akter, L.; Akhter, S.; Sarker, S. Efficient Prediction of Water Quality Index (WQI) Using Machine Learning Algorithms. *Hum.-Cent. Intell. Syst.* **2021**, *1*, 86–97. [CrossRef]
34. Lap, B.Q.; Phan, T.-T.-H.; Nguyen, H.D.; Quang, L.X.; Hang, P.T.; Phi, N.Q.; Hoang, V.T.; Linh, P.G.; Hang, B.T.T. Predicting Water Quality Index (WQI) by Feature Selection and Machine Learning: A Case Study of An Kim Hai Irrigation System. *Ecol. Inform.* **2023**, *74*, 101991. [CrossRef]
35. Salari, K.; Zarafshan, P.; Khashehchi, M.; Pipelzadeh, E.; Chegini, G. Modeling and Predicting Water Production by Capacitive Deionization Method Using Artificial Neural Networks. *Desalination* **2022**, *540*, 115992. [CrossRef]
36. Derdour, A.; Jodar-Abellan, A.; Pardo, M.Á.; Ghoneim, S.S.; Hussein, E.E. Designing Efficient and Sustainable Predictions of WQIs at the Regional Scale Using Machine Learning Algorithms. *Water* **2022**, *14*, 2801. [CrossRef]
37. Uddin, M.G.; Nash, S.; Rahman, A.; Olbert, A.I. A Novel Approach for Estimating and Predicting Uncertainty in Water Quality Index Model Using Machine Learning Approaches. *Water Res.* **2023**, *229*, 119422. [CrossRef]
38. Wu, J.; Wang, Z. A Hybrid Model for Water Quality Prediction Based on an Artificial Neural Network, Wavelet Transform, and Long Short-Term Memory. *Water* **2022**, *14*, 610. [CrossRef]
39. Mdee, O.J.; Mndolwa, B.; Sadiki, N. Water Quality Assessment and Spatial Distribution of Water Quality Parameters of Dodoma Urban, Tanzania. *Afr. J. Aquat. Sci.* **2024**, *49*, 95–105. [CrossRef]
40. Mishra, A.P.; Singh, S.; Jani, M.; Singh, K.A.; Pande, C.B.; Varade, A.M. Assessment of Water Quality Index Using Analytic Hierarchy Process (AHP) and GIS: A Case Study of a Struggling Asan River. *Int. J. Environ. Anal. Chem.* **2024**, *104*, 1159–1171. [CrossRef]
41. Cuong, B.C.; Pham, V.H. Some Fuzzy Logic Operators for Picture Fuzzy Sets. In Proceedings of the 2015 Seventh International Conference on Knowledge and Systems Engineering (KSE), Ho Chi Minh City, Vietnam, 8–10 October 2015; IEEE: Piscataway, NJ, USA, 2015; pp. 132–137.
42. Dilmi, S.; Ladjal, M. A Novel Approach for Water Quality Classification Based on the Integration of Deep Learning and Feature Extraction Techniques. *Chemom. Intell. Lab. Syst.* **2021**, *214*, 104329. [CrossRef]
43. Barzegar, A.; Campanile, L.; Marrone, S.; Marulli, F.; Verde, L.; Mastroianni, M. Fuzzy-Based Severity Evaluation in Privacy Problems: An Application to Healthcare. In Proceedings of the 2024 19th European Dependable Computing Conference (EDCC), Munich, Germany, 22–25 April 2024; IEEE: Piscataway, NJ, USA, 2024; pp. 147–154.
44. ACEEA. 2018 Sustainability Report. Available online: http://sostenibilita2018.gruppo.acea.it/en/relations-environment/water-segment/water-quality (accessed on 10 March 2023).
45. Chen, H.W.; Chang, N.B. Identification of River Water Quality Using the Fuzzy Synthetic Evaluation Approach. *J. Environ. Manag.* **2001**, *63*, 293–305. [CrossRef]
46. Icaga, Y. Fuzzy Evaluation of Water Quality Classification. *Ecol. Indic.* **2007**, *7*, 710–718. [CrossRef]
47. Haider, H.; Ghumman, A.R.; Al-Salamah, I.S.; Thabit, H. Assessment Framework for Natural Groundwater Contamination in Arid Regions: Development of Indices and Wells Ranking System Using Fuzzy VIKOR Method. *Water* **2020**, *12*, 423. [CrossRef]
48. Muhammetoglu, A.; Yardimci, A. A Fuzzy Logic Approach to Assess Groundwater Pollution Levels Below Agricultural Fields. *Environ. Monit. Assess.* **2006**, *118*, 337–354. [CrossRef]

49. Mirabbasi, R.; Mazloumzadeh, S.M.; Rahnama, M.B. Evaluation of Irrigation Water Quality Using Fuzzy Logic. *Res. J. Environ. Sci.* **2008**, *2*, 340–352.
50. Gharibi, H.; Mahvi, A.H.; Nabizadeh, R.; Arabalibeik, H.; Yunesian, M.; Sowlat, M.H. A Novel Approach in Water Quality Assessment Based on Fuzzy Logic. *J. Environ. Manag.* **2012**, *112*, 87–95. [CrossRef]
51. Venkatramanan, S.; Chung, S.Y.; Rajesh, R.; Lee, S.Y.; Ramkumar, T.; Prasanna, M.V. Comprehensive Studies of Hydrogeochemical Processes and Quality Status of Groundwater with Tools of Cluster, Grouping Analysis, and Fuzzy Set Method Using GIS Platform: A Case Study of Dalcheon in Ulsan City, Korea. *Environ. Sci. Pollut. Res.* **2015**, *22*, 11209–11223. [CrossRef]
52. Agoubi, B.; Souid, F.; Kharroubi, A.; Abdallaoui, A. Assessment of Hot Groundwater in an Arid Area in Tunisia Using Geochemical and Fuzzy Logic Approaches. *Environ. Earth Sci.* **2016**, *75*, 1497. [CrossRef]
53. Vadiati, M.; Asghari-Moghaddam, A.; Nakhaei, M.; Adamowski, J.; Akbarzadeh, A.H. A Fuzzy Logic-Based Decision-Making Approach for Identification of Groundwater Quality Based on Groundwater Quality Indices. *J. Environ. Manag.* **2016**, *184*, 255–270. [CrossRef]
54. Tiri, A.; Belkhiri, L.; Mouni, L. Evaluation of Surface Water Quality for Drinking Purposes Using Fuzzy Inference System. *Groundw. Sustain. Dev.* **2018**, *6*, 235–244. [CrossRef]
55. Qiu, H.; Gui, H.; Cui, L.; Pan, Z.; Lu, B. Hydrogeochemical Characteristics and Water Quality Assessment of Shallow Groundwater: A Case Study from Linhuan Coal-Mining District in Northern Anhui Province, China. *Water Supply* **2019**, *19*, 1572–1578. [CrossRef]
56. McNeil, F.M.; Thro, E. *Fuzzy Logic: A Practical Approach*; Academic Press: Boston, MA, USA, 1994; p. 294.
57. Barzegar, Y.; Barzegar, A.; Bellini, F.; Marrone, S.; Verde, L. Fuzzy Inference System for Risk Assessment of Wheat Flour Product Manufacturing Systems. *Procedia Comput. Sci.* **2024**, *246*, 4431–4440. [CrossRef]
58. Raj, R.; Mohan, B.M. Modeling and Analysis of the Simplest Fuzzy PI/PD Controllers of the Takagi-Sugeno Type. *IFAC-Pap.* **2016**, *49*, 537–542.
59. Amaral, B.C.; Bahuti, M.; Junior, T.Y.; de Lima, R.R.; Campos, A.T.; Fassani, É.J. Proficiencies of Different Fuzzy Inference Systems in Predicting the Production Performance of Broiler Chickens. *Comput. Electron. Agric.* **2023**, *209*, 107860. [CrossRef]
60. Jayetileke, H.R.; De Mel, W.R.; Mukhopadhyay, S.C. Real-time Metaheuristic Algorithm for Dynamic Fuzzification, Defuzzification, and Fuzzy Reasoning Processes. *Appl. Sci.* **2022**, *12*, 8242. [CrossRef]
61. Jain, K.; Soni, A. Comparison of Mamdani and Sugeno Fuzzy Inference System for Deciding the Set Point for a Hydro Power Plant Dam Reservoir Based on Power Generation Requirement. *Int. J. Eng. Manag. Sci.* **2015**, *2*, 43–47.
62. Sabounchi, N.; Triantis, K.; Sarangi, S.; Liu, S. Fuzzy Modeling of Linguistic Variables in a System Dynamics Context. In Proceedings of the 29th International Conference of the System Dynamics Society, Washington, DC, USA, 24–28 July 2011.
63. Gilda, K.; Satarkar, S. Defuzzification: Maxima Methods with Improved Efficiency and Performance Evaluation. In Proceedings of the 2022 IEEE Delhi Section Conference (DELCON), New Delhi, India, 11–13 February 2022; IEEE: Piscataway, NJ, USA, 2022; pp. 1–6.
64. Koçak, S.; Tóth-Laufer, E.; Pokorádi, L. Comparison of the Defuzzification Methods in Risk Assessment Applications. In Proceedings of the 2018 IEEE 18th International Symposium on Computational Intelligence and Informatics (CINTI), Budapest, Hungary, 21–22 November 2018; IEEE: Piscataway, NJ, USA, 2018; pp. 229–234.
65. *IS 10500:2012*; Indian Standard Specifications for Drinking Water. Bureau of Indian Standards: New Delhi, India, 2012. Available online: https://cpcb.nic.in/wqm/BIS_Drinking_Water_Specification.pdf (accessed on 15 November 2024).
66. Sadollah, A. Which Membership Function Is Appropriate in Fuzzy System? In *Fuzzy Logic-Based in Optimization Methods and Control Systems and Its Applications*; Sadollah, A., Ed.; IntechOpen: London, UK, 2018; pp. 1–10.
67. Adriaenssens, V.; De Baets, B.; Goethals, P.L.; De Pauw, N. Fuzzy Rule-Based Models for Decision Support in Ecosystem Management. *Sci. Total Environ.* **2004**, *319*, 1–12. [CrossRef] [PubMed]
68. Knapp, G. Fuzzy Logic: A Tutorial. Available online: https://www.cs.princeton.edu/courses/archive/fall07/cos436/HIDDEN/Knapp/fuzzy004.htm (accessed on 31 December 2024).