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Article

Workspace Representation and Optimization of a Novel Parallel Mechanism with Three-Degrees-of-Freedom

Zhen Gao and Dan Zhang *

Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Oshawa ON L1H 7K4, Canada; E-Mail: Zhen.Gao@uoit.ca

* Author to whom correspondence should be addressed; E-Mail: Dan. Zhang@uoit.ca; Tel.: +1-905-721-8668 ext. 5721; Fax: +1-905-721-3370.

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Abstract: The development of a new parallel mechanism based on simulation driven design is a rapid approach to discover the unique features or advantages of a conceptual model. In this research, one novel parallel mechanism which can generate three degrees-of-freedom translations is proposed. The kinematic model and Jacobian matrix is derived. The workspace generation and mapping is investigated based on simplified boundary searching method. The particle swarm algorithm is applied to search for the optimal volume of workspace.

Keywords: parallel mechanism; kinematic model; workspace; particle swarm algorithm

1. Introduction

The traditional parallel robots have proved their advantages in aspects of stiffness, rigidness, dexterity, reconfigurability, with the extensive applications in machine tools [1-3], motion simulators [4], picking and placing, sensors [5,6] and so on [7,8].

However, compared with the serial one, the drawbacks of the parallel robot is obvious [9,10]. Take its application in manufacturing as an example. The parallel robotic machines are usually composed with the active prismatic joint, the passive revolute joint, the passive universal joint or the passive spherical joint. According to the experience of real experiments, if the control system is broken or the vibration of the power supply, the unexpected motion will happen in these joints which may induce the permanent damage of the joints.

For the scenario of medical micromanipulation, the requirement of precision and safety is high. The compliant mechanism is fabricated with very flexible hinge or other flexure joint. The integration of compliant mechanism and parallel mechanism can provide an effective solution especially in precision and dexterity [11-14]. Besides, simulation driven design (SDD) as an effective method has been widely applied for the rapid prototype development. Based on SDD, the unique features or advantages of the conceptual model can be discovered in a high efficiency.

Many restriction factors of the parallel robot, including degree-of-freedom and configuration constraint of various joints, mechanical collision of different components, actuators stroke and singular limitations, affect the performances of workspace. Many scholars have developed different approaches and algorithms to investigate the features of workspace, especially its volume [15-27]. In [15], the joint workspace of a parallel kinematic machine (PKM) was calculated using the forward kinematic model. Since many PKMs are developed and fabricated based on decoupled kinematic structure, the single actuated joint usually cannot move dependently. Knowing the workspace of the active joints is important for motor selection and path planning. In [16], the relationship between task workspace and joint workspace was explained. In [18], the analogous symmetry characteristics of the reachable workspace for the symmetry group of the parallel mechanism were investigated. In [19], the reachable workspace was mapped without considering the joints limits and interferences between links. In [20], the path planning for singularity free route in a reachable workspace which was generated with a generic numerical algorithm was developed. In [21], the configuration optimization of a Delta-type parallel kinematic mechanism called Orthoglide was conducted based on a Cartesian workspace with prescribed kinetostatic performances. In [26], the modularity property of the parallel mechanism was utilized to calculate the volume of the workspace. In [27], the segmentation on the boundary curve of cross-section of a Stewart-Gough parallel manipulator was implemented to compute the size of the orientation workspace. In this research, the simplified workspace representation and optimization approaches are developed for a novel parallel mechanism. The proposed methods are generic and suitable for visual analysis, modeling and optimization of workspace for the different types of parallel manipulators.

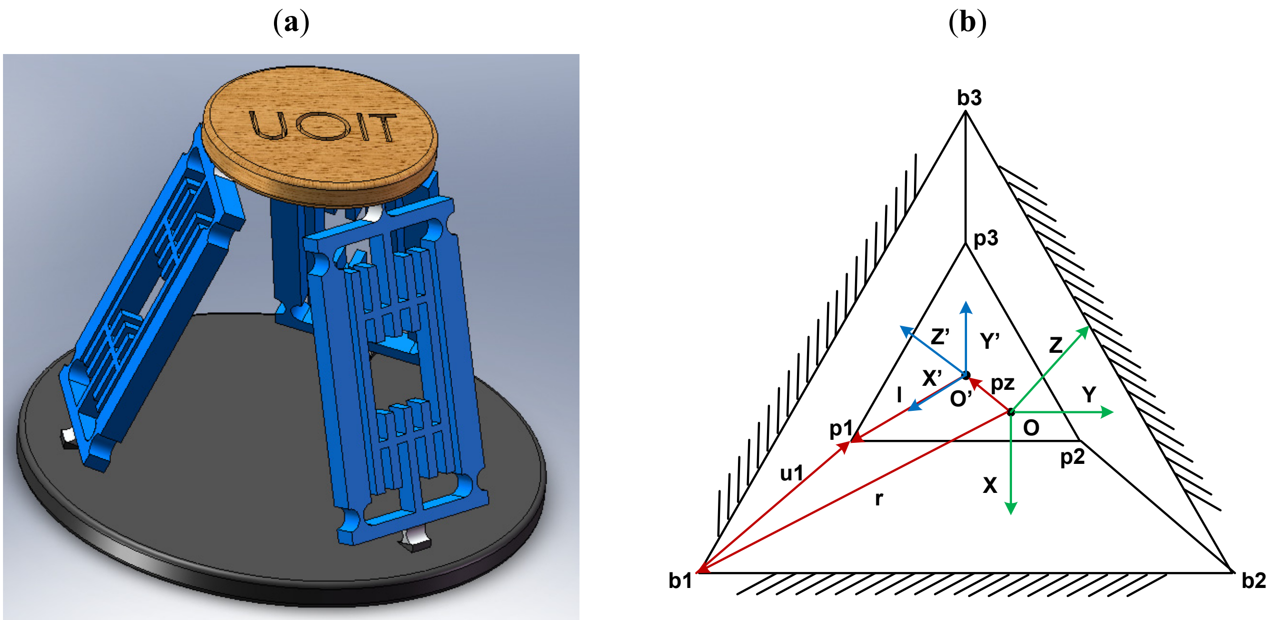
In what follows, a new parallel mechanism that can generate three degrees-of-freedom translations is developed. Its kinematic model and Jacobian matrix is derived in Section 2. The workspace as the one of the most important indices of parallel mechanism is calculated and mapped in Section 3. The particle swarm algorithm based performance optimization is conducted to maximize the volume of workspace in Section 4. Section 5 gives the conclusions.

2. Conceptual Design and Kinematics Analysis

2.1. CAD Model

The novel parallel mechanism with three degrees-of-freedom translations is composed of a base structure, a moving platform and three legs connecting the base and platform. In each leg, a compliant revolute joint is attached to the moving platform. A four bar mechanism is hereafter connected to the revolute joint. The linear driven mechanism is embedded in the four bar mechanism which is actuated by a PZT. Another revolute joint which is perpendicular to the above-mentioned one is connected to the four bar mechanism and the base. The CAD model of the proposed 3-DOF parallel mechanism is shown in Figure 1.

Figure 1. The proposed parallel mechanism; (**a**) CAD model, (**b**) kinematic model.



2.2. Kinematics Analysis and Jacobian Matrix

Because of the nature of the compliant mechanism, generally speaking, the analysis of the parallel mechanism will be different to the conventional parallel mechanism. However, the detailed analysis will depend on the given case. Just like the traditional parallel manipulator, the proposed parallel mechanism has many performance indices, *i.e.*, stiffness, dexterity, workspace, manipulability and so on. If the research topic is related to the stiffness of the parallel mechanism, the compliance of the flexure joint and link must be considered. Otherwise, the model would be inaccurate. Since in this case we would apply the PZTs as the actuators, the displacement of the moving platform will be mainly generated by the PZTs. Besides, in the research, the motion amplification effect produced by the PZTs is ignored. Based on these assumptions, the analysis of the proposed parallel mechanism can be simplified.

A kinematics model of the manipulator is shown in Figure 1(b). The vertices of the moving platform are p_i (i = 1, 2, 3), and the vertices of the base are b_i (i = 1, 2, 3). A global reference system O-XYZ is located at the center of the base. Another reference system O'-X'Y'Z', called the moving frame, is located at the center of the moving platform. Note that $Ob_1 = Ob_2 = Ob_3 = r$, and $O'p_1 = O'p_2 = O'p_3 = l$.

The position vector of b_i (i = 1, 2, 3) with respect to the global reference system is expressed as follows:

$$\boldsymbol{b}_1 = \begin{bmatrix} -\frac{1}{2}r & -\frac{\sqrt{3}}{2}r & 0 \end{bmatrix}^T ; \boldsymbol{b}_2 = \begin{bmatrix} r & 0 & 0 \end{bmatrix}^T ; \boldsymbol{b}_3 = \begin{bmatrix} -\frac{1}{2}r & \frac{\sqrt{3}}{2}r & 0 \end{bmatrix}^T \quad (1)$$

The position vector of p_i (i = 1, 2, 3) with respect to the moving frame is expressed as below:

$$\boldsymbol{p}_{1(P)} = \begin{bmatrix} -\frac{1}{2}l & -\frac{\sqrt{3}}{2}l & 0 \end{bmatrix}^T ; \boldsymbol{p}_{2(P)} = \begin{bmatrix} l & 0 & 0 \end{bmatrix}^T ; \boldsymbol{p}_{3(P)} = \begin{bmatrix} -\frac{1}{2}l & \frac{\sqrt{3}}{2}l & 0 \end{bmatrix}^T \quad (2)$$

Thus, the position vector of p_i (i = 1, 2, 3) with respect to the global reference system is derived as:

$$\boldsymbol{p}_{1(O)} = \boldsymbol{Q} \cdot \boldsymbol{p}_{1(P)} + \boldsymbol{p} = \begin{bmatrix} -\frac{1}{2}l + p_x & -\frac{\sqrt{3}}{2}l + p_y & p_z \end{bmatrix}^T \quad (3)$$

$$\boldsymbol{p}_{2(O)} = \boldsymbol{Q} \cdot \boldsymbol{p}_{2(P)} + \boldsymbol{p} = \begin{bmatrix} l + p_x & p_y & p_z \end{bmatrix}^T \quad (4)$$

$$\boldsymbol{p}_{3(O)} = \boldsymbol{Q} \cdot \boldsymbol{p}_{3(P)} + \boldsymbol{p} = \begin{bmatrix} -\frac{1}{2}l + p_x & \frac{\sqrt{3}}{2}l + p_y & p_z \end{bmatrix}^T \quad (5)$$

where, $\boldsymbol{Q}$ is the rotation matrix and $\boldsymbol{p}$ is the position vector is point O' with respect to the global reference system.

Thus, the inverse kinematics of the proposed parallel mechanism can be derived as:

$$d_i^2 = [\boldsymbol{p}_{i(O)} - \boldsymbol{b}_i]^T \cdot [\boldsymbol{p}_{i(O)} - \boldsymbol{b}_i] \quad (6)$$

where

$$d_1^2 = \begin{bmatrix} -\frac{1}{2}l + p_x + \frac{1}{2}r & -\frac{\sqrt{3}}{2}l + p_y + \frac{\sqrt{3}}{2}r & p_z \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2}l + p_x + \frac{1}{2}r & -\frac{\sqrt{3}}{2}l + p_y + \frac{\sqrt{3}}{2}r & p_z \end{bmatrix} \quad (7)$$

$$d_2^2 = \begin{bmatrix} l + p_x - r & p_y & p_z \end{bmatrix}^T \cdot \begin{bmatrix} l + p_x - r & p_y & p_z \end{bmatrix} \quad (8)$$

$$d_3^2 = \begin{bmatrix} -\frac{1}{2}l + p_x + \frac{1}{2}r & \frac{\sqrt{3}}{2}l + p_y - \frac{\sqrt{3}}{2}r & p_z \end{bmatrix}^T \cdot \begin{bmatrix} -\frac{1}{2}l + p_x + \frac{1}{2}r & \frac{\sqrt{3}}{2}l + p_y - \frac{\sqrt{3}}{2}r & p_z \end{bmatrix} \quad (9)$$

Since the relationship of the differential for the input joints and the output displacements can be expressed as:

$$\begin{bmatrix} \delta d_1 \\ \delta d_2 \\ \delta d_3 \end{bmatrix} = \boldsymbol{J} \cdot \begin{bmatrix} \delta p_x \\ \delta p_y \\ \delta p_z \end{bmatrix} \quad (10)$$

Thus, the Jacobian matrix of the proposed parallel mechanism is obtained as:

$$\boldsymbol{J} = \begin{bmatrix} \dfrac{p_x + \frac{1}{2}(r-l)}{d_1} & \dfrac{p_y + \frac{\sqrt{3}}{2}(r-l)}{d_1} & \dfrac{p_z}{d_1} \\ \dfrac{p_x + (l-r)}{d_2} & \dfrac{p_y}{d_2} & \dfrac{p_z}{d_2} \\ \dfrac{p_x + \frac{1}{2}(r-l)}{d_3} & \dfrac{p_y + \frac{\sqrt{3}}{2}(l-r)}{d_3} & \dfrac{p_z}{d_3} \end{bmatrix} \tag{11}$$

3. Workspace Mapping

As was defined in [15], generally, the workspace of a parallel mechanism can be roughly divided into task workspace and the joint workspace. The task workspace refers the motion scopes of the moving platform in two or three dimensions. The total area was calculated to describe the performance of a 2D task workspace. The total volume was calculated to describe the performance of a 3D task workspace.

The generation of workspace for the parallel mechanisms includes geometrical approach, numerical method and discretization method. With the integration of geometrical approach, discretization method and inverse kinematics model, a simplified boundary searching method (SBS) is developed to acquire the task workspace.

The calculating process of SBS method is described as follows:

s1: Define the input parameters of the proposed parallel mechanism. These parameters include l, r, $d_{\min}$, $d_{\max}$ and h. Here, l and r are the radii of the moving platform and base, respectively. $d_{\min}$ and $d_{\max}$ are the motion scopes of the PZT. h is the height of the moving platform.

s2: Confirm the bounds of input parameters and other boundary conditions. Initially, the moving platform is located at the home position.

s3: Let counter = 0.

s4: Set the step-size for the movement of mobile platform in directions of x, y and z from initial pose. The step-size should be as small as possible to improve the calculation accuracy.

s5: Calculate the analytical solution of inverse kinematics model.

s6: The mobile platform transfers from the original/former position to the given position with the step-size.

s7: According to the bounds of input parameters and other boundary conditions, determining that whether the boundary conditions are exceeded when the mobile platform is located at a certain pose.

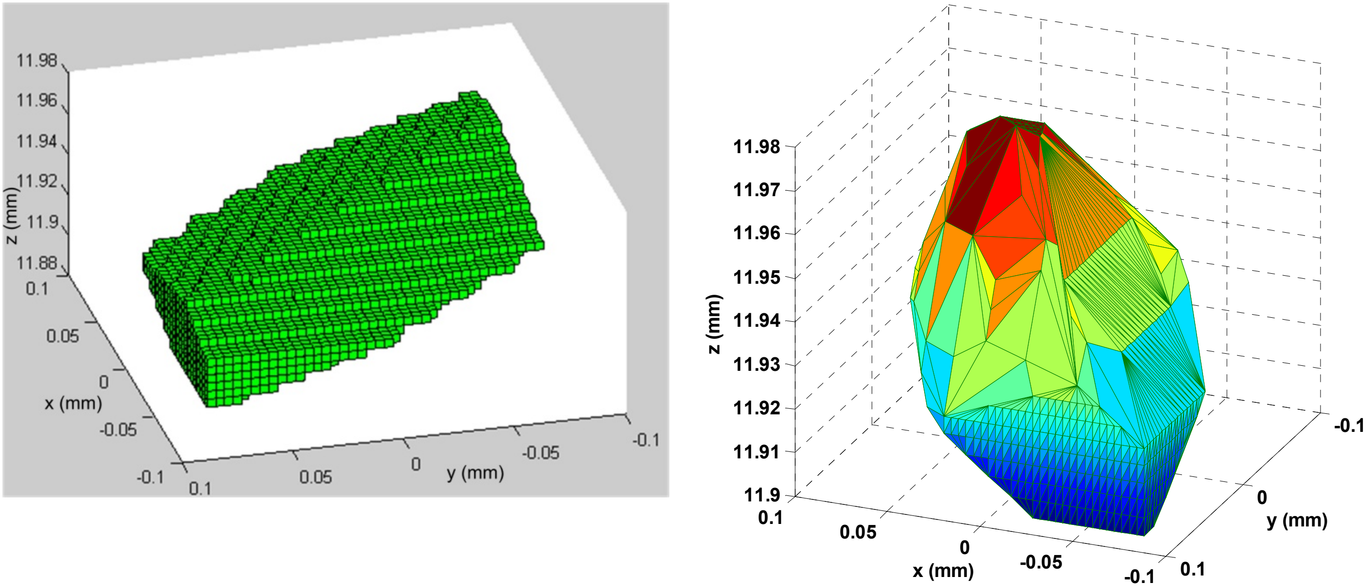
s8: If the answer to s7 is yes, then go to s9. Otherwise, counter = counter + 1, and jump back to s6.

s9: In this step, since the moving platform is reaching the bound to the workspace, the algorithm terminates. Thus, generate and draw the workspace with the cubes (0.004 mm × 0.004 mm × 0.004 mm) in Cartesian coordinate system as shown in Figure 2(a).

s10: Plot the envelope of the workspace generated in s9, as shown in Figure 2(b).

Figure 2. The reachable workspace; (**a**) with cube, (**b**) with colored envelope.

(**a**) (**b**)



The features of workspace including shape and volume are related with the dimensions of the proposed parallel mechanism. Figure 3 shows the different mapping of workspace when the two representative dimensions l and r are defined with the optional values.

Figure 3. The mapping of workspace under different input parameters; (**a**) when l = 4.4 *mm* and r = 8.0 *mm*, (**b**) when l = 4.3 *mm* and r = 8.0 *mm*, (**c**) when l = 4.0 *mm* and r = 8.04 *mm*, (**d**) when l = 4.2 *mm* and r = 7.8 *mm*.

(**a**) (**b**)

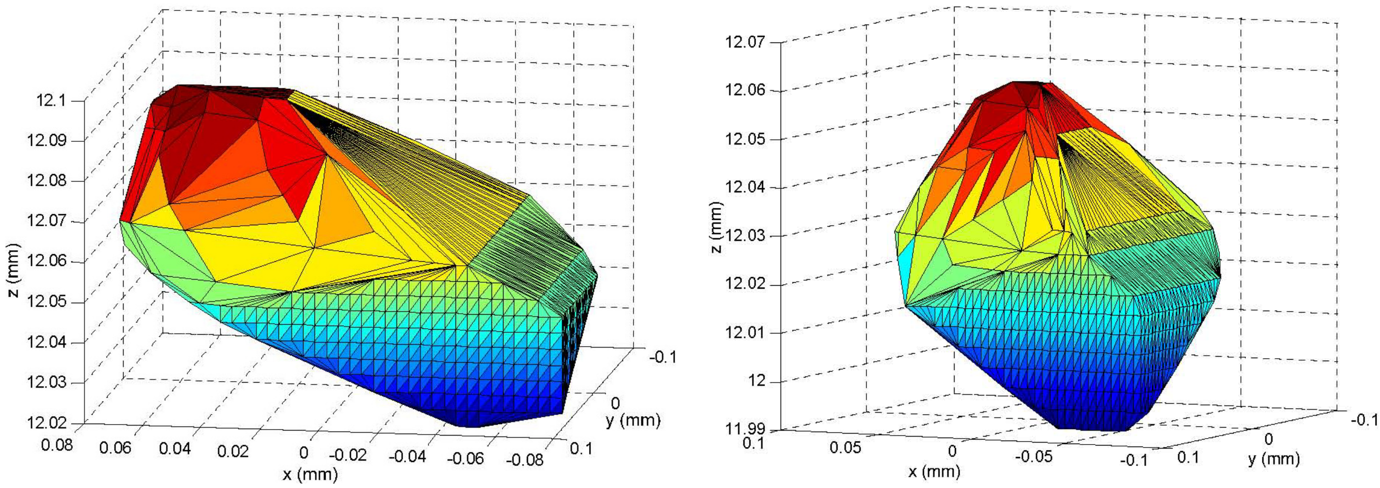
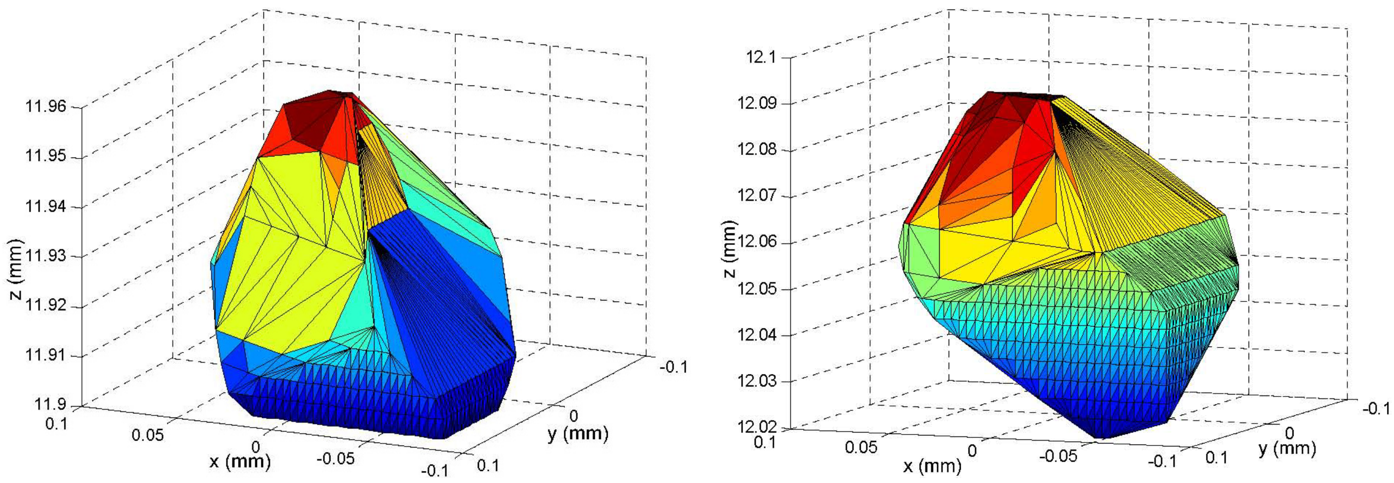


Figure 3. *Cont.*

(c) **(d)**



4. Workspace Optimization

The traditional optimization methods usually adopt the local search by a convergent stepwise procedure which possibly falls into local optimal solution. If the complex function to be optimized does not possess convexity characteristics that essentially satisfy that the local extreme point is a global optimum, a global optimization algorithm is required. As an advanced computational intelligence method, particle swarm optimization (PSO) is inspired by simulating the swarm behavior such as bird flocking. Without the traditional evolution operators including crossover and mutation, PSO can be viewed as the extension and improvement of the working principle of genetic algorithm. Thus, the particle swarm algorithms will be used to search the overall optimal performance.

The general PSO algorithm is constituted with the following velocity and position [28]:

$$v_i(n+1) = v_i(n) + \gamma_{1i}(bestP_i - x_i(n)) + \gamma_{2i}(bestG - x_i(n)) \tag{12a}$$

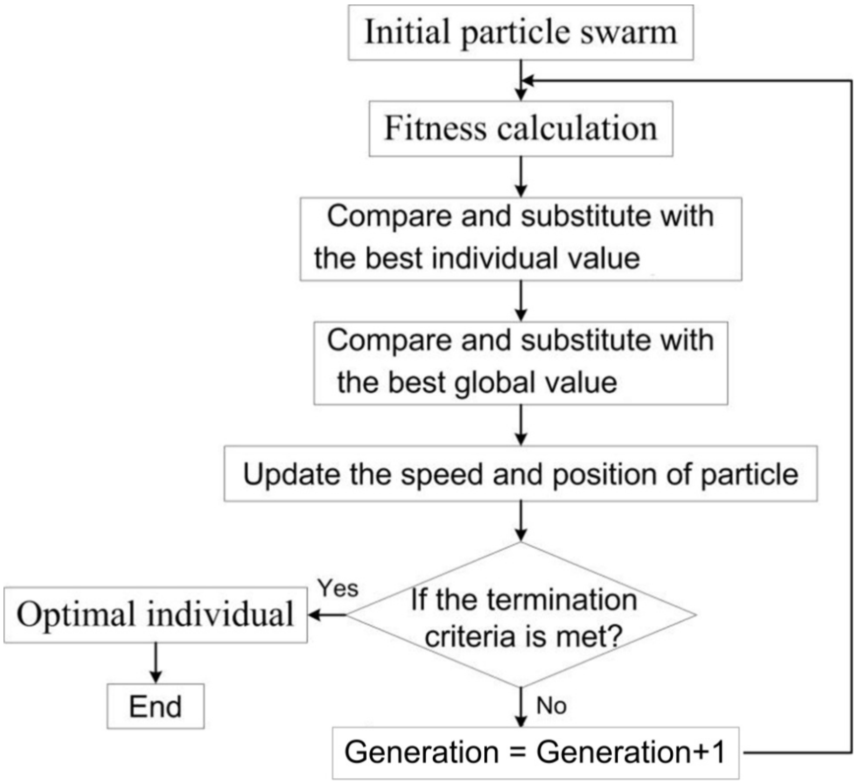
$$x_i(n+1) = x_i(n) + v_i(n+1) \tag{12b}$$

where, i denotes the particle index. n is the discrete time index. v_i is the velocity of ith particle. x_i denotes the position of ith particle. $bestP_i$ means the best local position found by ith particle. $bestP$ is the global best position found by swarm. Figure 4 explains the working principle of PSO.

The mechanism parameters chosen for optimization are l, r, $d_{\min}$, $d_{\max}$ and h. After considering the initial values and their possible changes, the ranges are set as

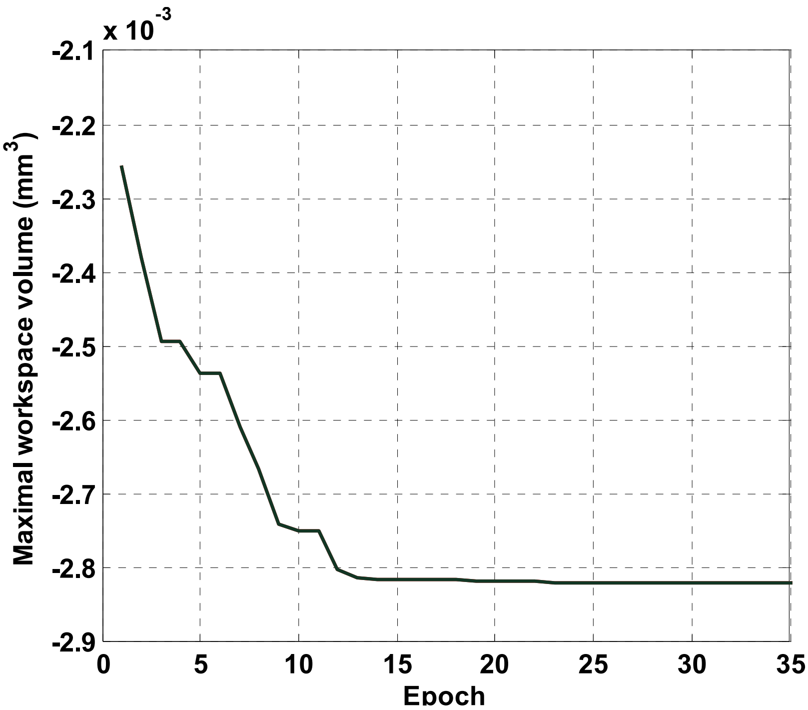
$l \in [3.9, 4.5]mm$,
$r \in [7.7, 8.1]mm$,
$d_{\min} \in [12.5, 12.58]mm$,
$d_{\max} \in [12.58, 12.62]mm$,
$h \in [11.98, 12.02]mm$.

Figure 4. The working principle of PSO.



In this scenario, the common PSO with inertia is utilized to perform the optimization process. The maximal velocity divisor is 2, the particles number is 100. Figure 5 illustrates the evolutionary process with PSO.

Figure 5. The evolution process of the maximal workspace volume per epoch.



Before optimization, the value of objective function is equal to 5.42×10^{-4} mm^3, with the initial parameters as follows:

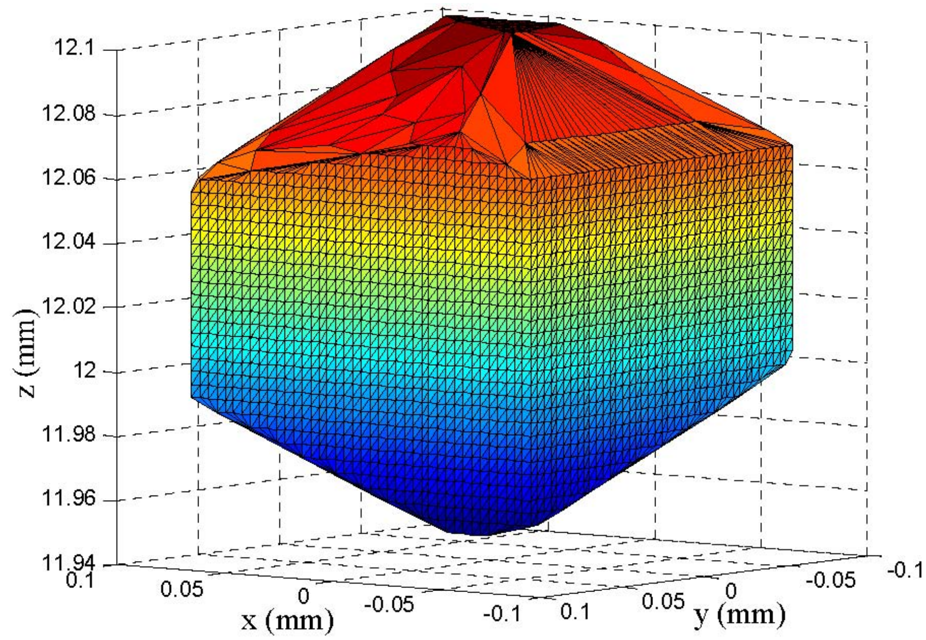
$l = 4$ mm,
$r = 8$ mm,
$d_{\min} = 12.56$ mm,
$d_{\max} = 12.6$ mm,
$h = 12$ mm.

After optimization with 35 epochs, the volume of workspace is 2.82×10^{-3} mm^3, improved by a factor of 5.2. The best individuals of the five parameters are:

$L' = 4.2854$ mm,
$R' = 7.8943$ mm,
$d'_{\min} = 12.5$ mm,
$d'_{\max} = 12.62$ mm,
$h' = 11.996$ mm.

Figure 6 describes the envelope of the workspace with the maximal volume after optimization based on PSO.

Figure 6. The envelope of the optimal workspace.



5. Conclusions

The main contributions of this paper are summarized as follows:

(1) A new parallel mechanism which can generate three degrees-of-freedom translations is designed. The related kinematics modeling is investigated.
(2) A general approach called simplified boundary searching method is developed to generate the reachable workspace of the proposed parallel mechanism.

(3) Optimization of the dimensional parameters is conducted to obtain the maximal volume of workspace based on particle swarm algorithm.

This research focuses on developing a generic and simplified method for the modeling, mapping, calculation and optimization of workspace for a parallel mechanism. Currently, the modeling and optimization of parallel mechanisms is still one open issue for the scholars and engineers in the world who are interested in or already investigating this topic. The newly developed method about workspace generation and improvement is presented in a clear and operational way. Through case study, it can be proved that this approach is very efficient in modeling and computing time. The proposed method is feasible as one efficient solution for the open issue of workspace.

This study has endeavored in design optimization of the workspace for a novel moving stage which can generate three degrees-of-freedom translations. The kinematic model and Jacobian matrix is derived. The simplified boundary searching method to generate the workspace of the proposed parallel mechanism is generic. The particle swarm algorithm is applied to improve the volume of workspace. For the future work, the prototype will be fabricated based on the proposed modeling and optimization approach.

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Conflict of Interest

The authors declare no conflict of interest.

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