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Article

Robustness of Acoustic Scattering Cancellation to Parameter Variations

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Abstract: This contribution aims at investigating the possibility to cloak a spherical object from an acoustic wave by applying the scattering cancellation approach. In electromagnetism, the scattering problem is treated using the Mie expansion technique, through which the scattered field by a spherical object can be represented as a superposition of TE and TM spherical harmonics. It is possible to extend this concept to the acoustic field by defining an analogous approach; the pressure field, generated by an elastic wave impinging on a spherical object, can be expressed applying the Mie expansion technique, as well. In acoustics, to achieve scattering suppression at a given frequency, the constitutive parameters to control are density and compressibility. By varying these parameter values, it is possible to define an engineered material with anomalous properties, which cannot be found in nature, able to reduce the scattering cross-section (SCS) from a spherical object. We propose a study about the effectiveness of the SCS reduction from an elastic sphere coated with a properly-designed acoustic metamaterial. The sensitivity of the SCS to parameter variations is analyzed for different coating thicknesses and sphere dimensions. Our analysis is supported by both the analytical modelling of the structure and numerical simulations.

Keywords: acoustic metamaterial; scattering cancellation; Mie scattering; acoustic cloaking

1. Introduction

Metamaterials are defined as artificial engineered materials exhibiting unique or unusual properties that cannot be found in natural materials at the frequencies of interest, and thus, they allow going beyond some of the limitations encountered when using natural materials [1]. Employing properly-designed metamaterials for microwave and optical components, it is possible to achieve anomalous properties and useful operation that can overcome several limitations connected to conventional and natural materials, opening the door to an innovative technology able to strongly improve the performances of existing devices. Metamaterial macroscopic response and physical properties depend on dimensions and geometrical characteristics of their components and also on constitutive parameter variations [2]. Such degrees of freedom for realizing an engineered material allow employing them for many different applications over a desired range of frequencies. In electromagnetism, recent developments in metamaterials and metasurfaces have led to innovative designs of radiating and transmitting components [3–13].

In the last few years, advances in metamaterial design have led to invisibility, transparency and cloaking applications by employing different techniques based on the interaction between waves and metamaterials [14–17]. Currently, one of the most diffused cloaking techniques is the so-called transformation-based one, which is based on the electromagnetic properties of an inhomogeneous cover guiding an impinging wave around an object. The difficulties in manufacturing this specific kind of cover led to a very limited applicability [18]. Another viable and effective technique, able to cloak an arbitrary object, is the scattering cancellation method. In this case, the cover is characterized by a homogeneous layer properly designed to achieve the scattering suppression in the frequency range of interest. Recently, several findings in the literature have shown that it is possible to reach the scattering suppression, not only through bulk metamaterials, but also by employing thin metasurfaces, an approach known as mantle cloak [19–22].

Several metamaterial concepts have been extended and applied also to acoustics [23,24], and this opens the door to a strong innovation in the field of sound control. Properly-designed acoustic metamaterials provide a negative effective dynamic mass density, thus allowing one to obtain increased transmission loss without increasing the thickness of the acoustic panel. This is usually realized by means of membrane-type acoustic metamaterials, also known as locally-resonant sonic materials, essentially based on an array of elastic resonators composed of a heavy core surrounded by a soft coating layer [25–27]. Transformation-based technique can also be applied for sound insulation, but this technique is of limited practical appeal, as it requires highly inhomogeneous materials [28].

In this paper, we focus on the sound control issue consisting in cloaking an object from an acoustic wave using the scattering cancellation approach. In particular, we investigate the effectiveness of the scattering cross-section (SCS) reduction from an elastic sphere in air and coated with a properly-designed acoustic metamaterial, by analyzing the sensitivity of the SCS to some parameters. Two different sets of constitutive parameter values have been analyzed, the plasmonic and the anti-resonant region. We show that the reduction of the SCS is possible only in the plasmonic region, where, in addition, the effectiveness of scattering cancellation is shown to be quite robust to coating parameter variations.

2. Theoretical Background

The scattering cancellation technique is one of the most popular approaches to cloak an object from an electromagnetic wave. It is based on the employment of a single homogeneous thin layer of properly-designed characteristics to cover an object in order to obtain the suppression of the scattered field in a given frequency band. Differently from transformation-based cloaking, this method allows using an isotropic homogeneous material to achieve the cancellation of the field around the object. Recently, this technique has been extended and applied to acoustic waves and to the cloaking of elastic objects [23–27].

Acoustic Scattering Cancellation

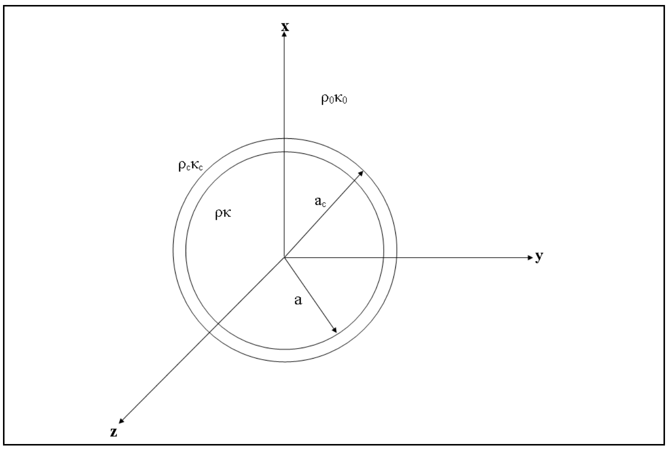
Similarly to what happens for electromagnetic waves [29], also in acoustics, cloaking can be obtained by means of scattering cancellation based on the Mie expansion technique. In this analytical approach, that has been developed in [23] and is reported here for convenience, an acoustic plane wave impinging on a spherical object of radius *a* produces a scattered field that can be expressed as a sum of spherical harmonics:

$$p_{\mathrm{sc}} = \sum_{n=0}^{\infty} i^n (2n+1) A_n h_n^{(1)}(k_0 r) P_n(\cos\vartheta) \tag{1}$$

where A_n is the scattering coefficient for the n-th mode, $h_n^{(1)}$ the spherical Hankel function of the first kind, P_n the Legendre polynomials, $k_0 = \omega\sqrt{\rho_0\kappa_0}$, ρ_0 and κ_0 wavenumber, density and bulk modulus in the surrounding host fluid, respectively, and ω the angular frequency of the impinging harmonic plane wave.

In order to suppress this scattered pressure field, a homogeneous coating layer with outer radius a_c can be applied to the sphere (see Figure 1).

Figure 1. Geometry of the problem: the coated sphere.



Pressure and radial velocity continuity at the interfaces have to be applied, giving rise to a 4 × 4 linear system to be solved as a function of the scattering coefficients, which, in turn, depend on the cover elastic properties, ρ_c and κ_c .

The scattering coefficients can be calculated by:

$$A_n = -\frac{U_n}{U_n + iV_n} \tag{2}$$

with:

$$U_n = \begin{vmatrix} -j_n(ka) & j_n(k_c a) & y_n(k_c a) & 0 \\ -\frac{1}{\rho} ka j_n'(ka) & \frac{1}{\rho_c} k_c a j_n'(k_c a) & \frac{1}{\rho_c} k_c a y_n'(k_c a) & 0 \\ 0 & j_n(k_c a_c) & y_n(k_c a_c) & j_n(k_0 a_c) \\ 0 & \frac{1}{\rho_c} k_c a_c j_n'(k_c a_c) & \frac{1}{\rho_c} k_c a_c y_n'(k_c a_c) & \frac{1}{\rho_0} k_0 a_c j_n'(k_0 a_c) \end{vmatrix} \tag{3}$$

$$V_n = \begin{vmatrix} -j_n(ka) & j_n(k_c a) & y_n(k_c a) & 0 \\ -\frac{1}{\rho} ka j_n'(ka) & \frac{1}{\rho_c} k_c a j_n'(k_c a) & \frac{1}{\rho_c} k_c a y_n'(k_c a) & 0 \\ 0 & j_n(k_c a_c) & y_n(k_c a_c) & y_n(k_0 a_c) \\ 0 & \frac{1}{\rho_c} k_c a_c j_n'(k_c a_c) & \frac{1}{\rho_c} k_c a_c y_n'(k_c a_c) & \frac{1}{\rho_0} k_0 a_c y_n'(k_0 a_c) \end{vmatrix} \tag{4}$$

where $j_n(.)$ and $y_n(.)$ indicate the n-th order spherical Bessel functions of the first kind and of the second kind, respectively, and ' denotes derivative with respect to the argument.

The resulting total SCS is given by:

$$\sigma_{tot} = \frac{4\pi}{k_0^2} \sum_{n=0}^{\infty} (2n+1) \left| A_n \right|^2 \tag{5}$$

Reducing identically to zero the A_n coefficients for all the n scattering orders, it is possible to obtain the total suppression of the scattered pressure field. In particular, if the acoustical size, $k_0 a$, of the coated sphere is smaller than the wavelength of the incoming plane wave, only the modes $n = 0$ and $n = 1$ in Equation (5) give a significant contribution, and the scattering cancellation problem can be solved by looking for a cover with density ρ_c and bulk modulus κ_c that minimize the SCS.

3. Results and Discussion

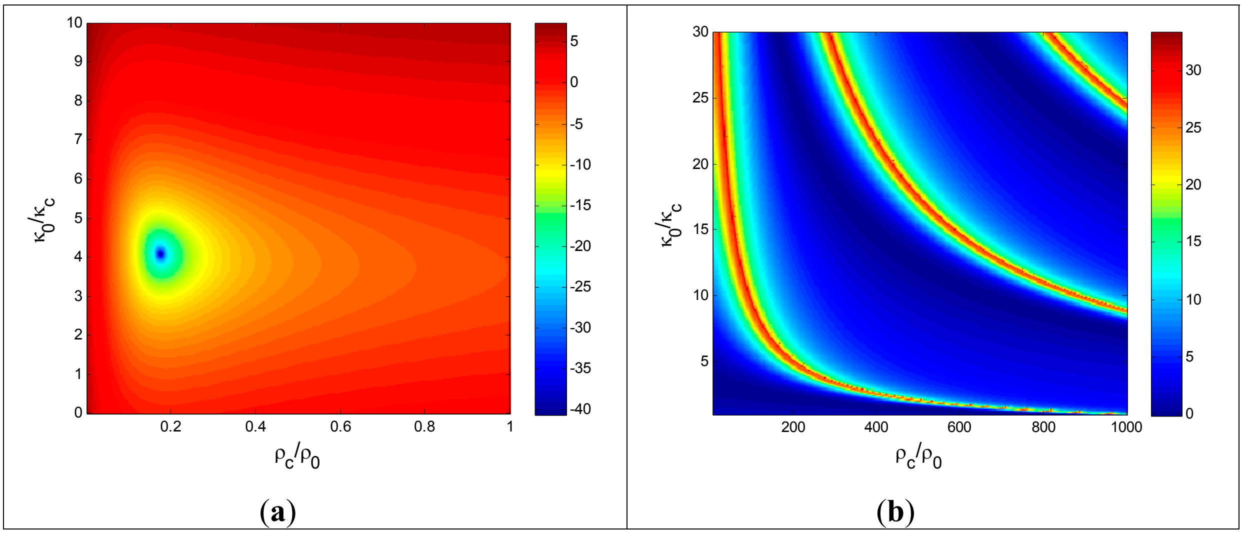
The minimization of the SCS returns the values of density and the bulk modulus of the cover material needed to cloak the object. It makes sense then to analyze to what extent the effectiveness of scattering cancellation is preserved when material parameters present deviations from the required designed values (e.g., incomplete knowledge, construction tolerances or temperature dependence).

The sensitivity of the SCS cancellation to parameter variations is analyzed for different coating thicknesses and sphere dimensions, exploring two different sets of constitutive parameter values, the plasmonic and the anti-resonant region.

As our starting configuration, we consider the situation in which an acoustic plane wave, at a frequency of 1,000 Hz, impinges in air on an elastic rubber sphere with acoustical dimensions $k_0a = 0.5$ and with a cover with radius $a_c = 1.1\ a$. In the plasmonic cloaking region the parameters that minimize the SCS are $\rho_c = 0.174\ \rho_0$, $\kappa_c = \kappa_0/4.102$ (see Figure 2a), leading to an SCS reduction of 41.7 dB compared to the uncloaked sphere.

Differently from what was observed in [23] for applications in water, in air, the anti-resonant regions, where the mismatch between coating and air density and compressibility is larger, cannot be exploited to reduce the SCS, which is only negligibly decreased in our example (see Figure 2b).

Figure 2. Scattering cross-section (SCS) variation (dB) obtained by varying the density and bulk modulus of the coating. The plasmonic scattering region is shown in (**a**) and resonant/antiresonant regions are displayed in (**b**).



We therefore focus on the plasmonic scattering region and analyze the sensitivity of the SCS to parameter variations for different coating thicknesses ($a_c = 1.1a$ and $1.5a$) and sphere dimensions ($k_0a = 0.1$, 0.5 and 1.0). All of these cases satisfy the condition of object smaller than the wavelength of the impinging wave.

Figure 3 reports the results of this analysis, highlighting, with the contour plots in the right panels, the regions where the SCS is reduced below a prescribed level (the reference being in all cases the uncloaked sphere). It is possible to select, e.g., a −20 dB threshold to identify the limiting values of cover constitutive parameters that allow a SCS reduction below that level. Such values are reported in Table 1 for the cases illustrated in Figure 3. Regions outside the curves labeled with “0” in Figure 3 indicate, instead, that a cover with those parameter values gives rise to an increase of SCS. It can be deduced from Table 1 that thinner covers should be used to obtain a design that is more robust against parameter variations.

Figure 3. SCS variation (dB) obtained by varying the density and bulk modulus of the coating in the plasmonic scattering region. k_0a and a_c values are indicated in each panel.

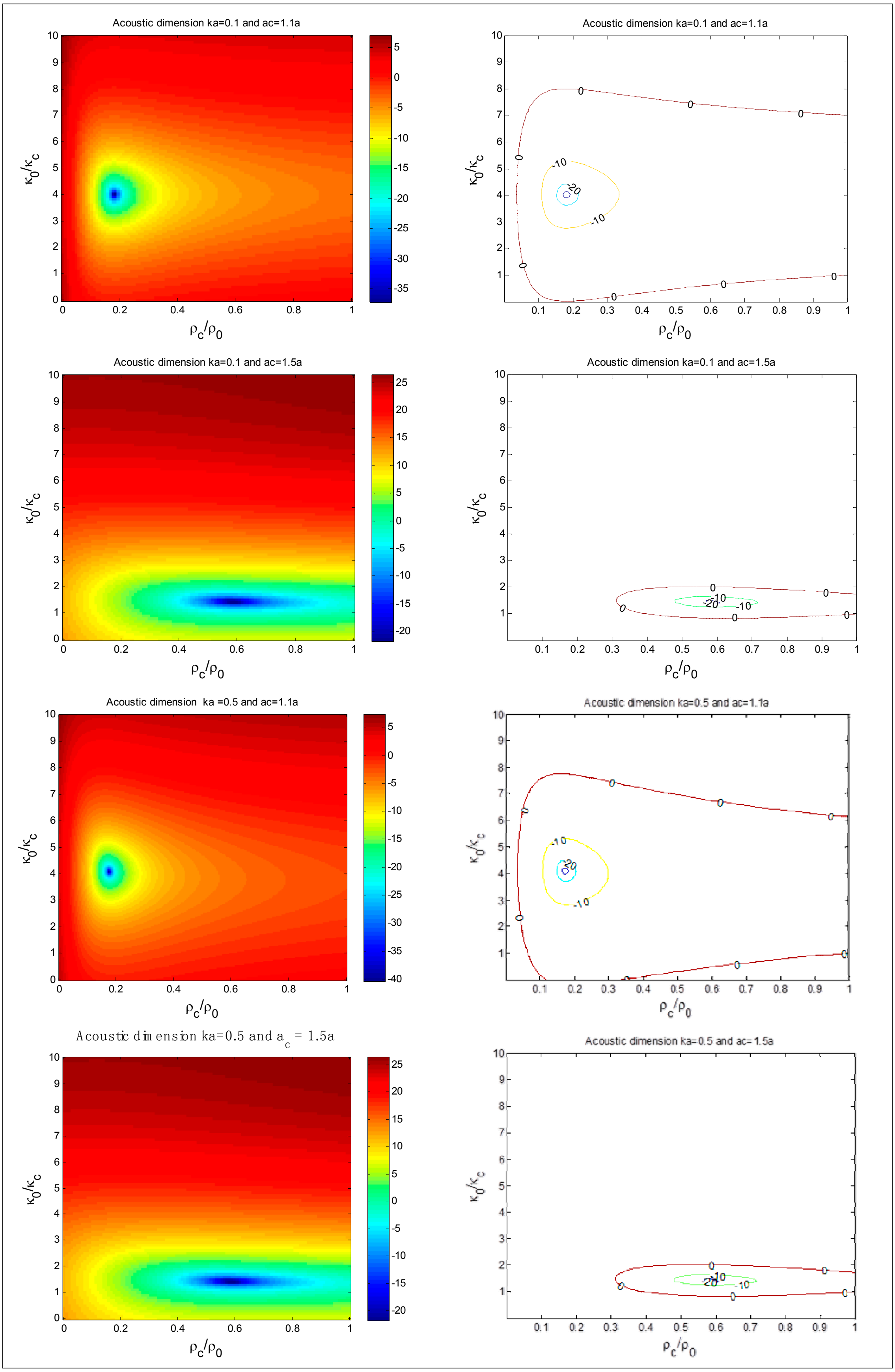


Figure 3. *Cont.*

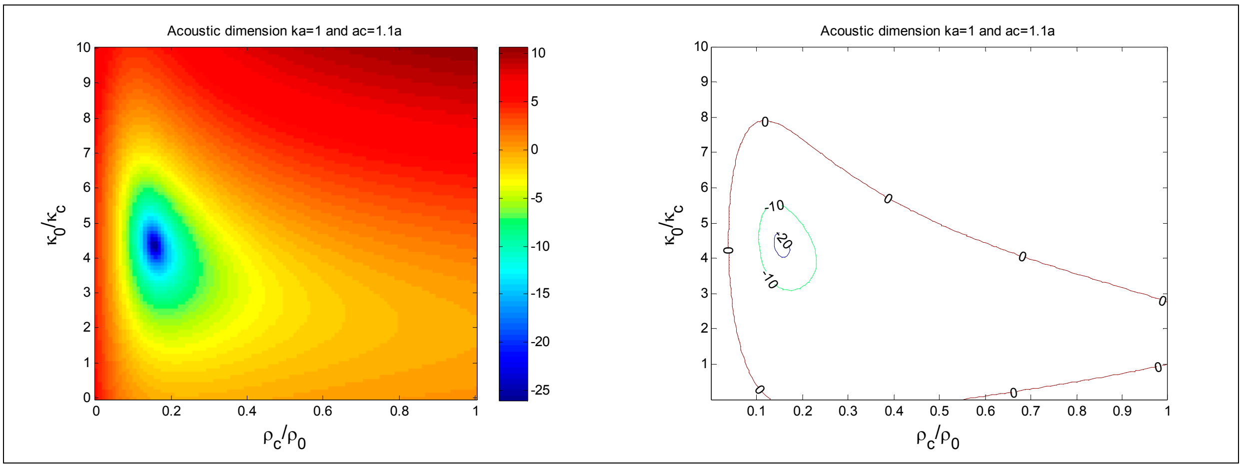


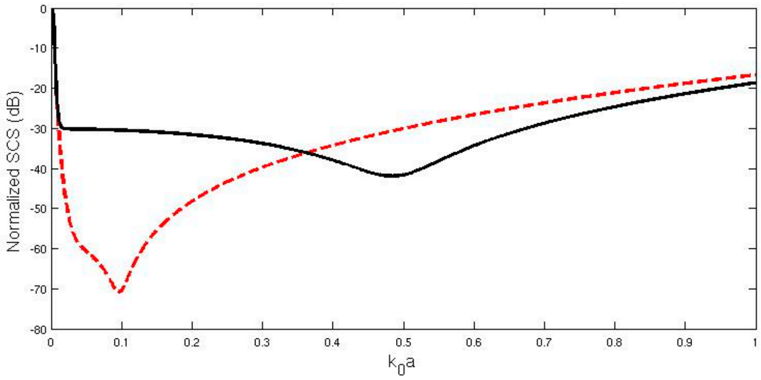
Table 1. The maximum percent parameter variations allowed in order to keep an SCS reduction of at least 20 dB. Compressibility values are reported instead of bulk moduli.

k0a = 0.1 ac = 1.1a	**Lower limit (%)**	**Upper limit (%)**	**k_0a = 0.1 a_c = 1.5a**	**Lower limit (%)**	**Upper limit (%)**
density	−15.3	19.5		−6.6	7.2
compressibility	−9.9	9.9		−3.9	3.9
k_0a = 0.5 a_c = 1.1a			**k_0a = 0.5 a_c = 1.5a**		
density	−14.3	17.3		−6.0	6.4
compressibility	−9.7	9.5		−3.9	3.9
k_0a = 1.0 a_c = 1.1a			**k_0a = 1.0 a_c = 1.5a**		
density	−10.3	11.8		–	–
compressibility	−8.0	7.8		–	–

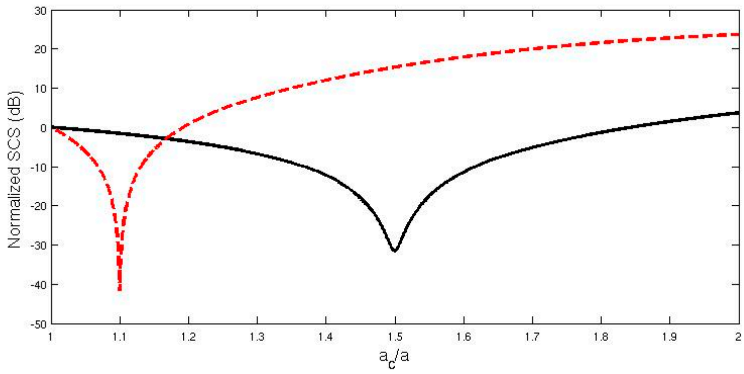
Further indications on the effectiveness of the SCS reduction can be obtained by reporting its values as a function of the normalized frequency k_0a or of the cover radius a_c. The first analysis (reported in Figure 4a) allows appreciating the frequency bandwidth over which it is possible to keep the SCS reduction below a desired level, showing that a satisfactory performance can be obtained in an extended frequency range.

The sensitivity to inaccurate realizations of the cover thickness is finally displayed in Figure 4b: a tolerance of ±0.9% is allowed for k_0a = 0.1 and of ±2.7% for k_0a = 0.5.

Figure 4. SCS variation (dB) obtained by varying the frequency of the incoming plane wave (**a**) or cover radius (**b**). In both cases, the design of the cover is done for $a_c = 1.1a$, while red dashed lines refer to a cover whose constitutive parameters minimize the SCS for a sphere with $k_0a = 0.1$ and black solid lines refer to the design for a sphere with $k_0a = 0.5$.



(**a**)



(**b**)

4. Conclusions

The design of an acoustic metamaterial able to suppress the scattered pressure field of an elastic object in air has been explored. We have provided a sensitivity analysis of scattering cross-section reduction *versus* variations of coating constitutive parameters and thicknesses, sphere dimensions and frequency. We have shown the effectiveness of working in the plasmonic regime and related parameter variation robustness, compared to the anti-resonant mode.

In particular, the range of parameter variations in which it is possible to obtain an SCS reduction of at least 20 dB has been defined. It has been shown that there is a tolerance varying between ±5% and ±15%, depending on coating thickness and scatterer acoustic size.

The material to realize the coating layer in the proposed design has a density and compressibility lower than the corresponding values in air. This requirement can be met by means of a homogeneous material, like an aerogel, or, rather, by resorting to an acoustic metamaterial, in which the resulting homogenized properties can be tailored by building inclusions with proper shape, size and spacing of a selected material in a host medium.

Sustainable acoustic design could greatly benefit from the possibilities offered by acoustic metamaterials. In particular, they promise a good versatility in the control of sound for acoustic comfort and noise pollution limitation, by the use of materials that are lighter and/or less expensive than conventional ones.

The results of this study can be extended to analyze the behavior of an array of elastic spheres under the same conditions and different geometries of the object or multilayer cover.

Heat transfer is also a very promising application field where the scattering cancellation-based cloaking could be employed. It is currently being explored in our research group for the design of innovative thermal devices.

Author Contributions

Filiberto Bilotti, Alessandro Toscano, Paola Gori and Roberto de Lieto Vollaro conceived the research; Claudia Guattari, Luca Evangelisti, Gabriele Battista and Carmine Basilicata performed the research and analyzed the data; Claudia Guattari and Paola Gori wrote the paper.

Conflicts of Interest

The authors declare no conflict of interest.

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