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Distributed Optimisation Algorithm for Demand Side Management in a Grid-Connected Smart Microgrid

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Abstract: The contributions of Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) for Demand Side Management (DSM) purposes in a smart macrogrid or microgrid cannot be over-emphasised. However, standalone DEG and DES can lead to under-utilisation of energy generation by consumers and financial investments; in grid-connection mode, though, DEG and DES can offer arbitrage opportunities for consumers and utility provider(s). A grid-connected smart microgrid comprising heterogeneous (active and passive) smart consumers, electric vehicles and a large-scale centralised energy storage is considered in this paper. Efficient energy management by each smart entity is carried out by the proposed Microgrid Energy Management Distributed Optimisation Algorithm (MEM-DOA) installed distributively within the network according to consumer type. Each smart consumer optimises its energy consumption and trading for comfort (demand satisfaction) and profit. The proposed model was observed to yield better consumer satisfaction, higher financial savings, and reduced Peak-to-Average-Ratio (PAR) demand on the utility grid. Other associated benefits of the model include reduced investment on peaker plants, grid reliability and environmental benefits. The MEM-DOA also offered participating smart consumers energy and tariff incentives so that passive smart consumers do not benefit more than active smart consumers, as was the case with some previous energy management algorithms.

Keywords: distributed energy generation (DEG); distributed energy storage (DES); demand side management (DSM); Microgrid Energy Management-Distributed Optimisation Algorithm (MEM-DOA); smart microgrid

1. Introduction

Smart grid provides an enabling environment for the integration of Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) for Demand Side Management (DSM) and Demand Response (DR) purposes, with mutual benefits to electricity utility providers and consumers. The incorporation of DEGs and DES devices into the supply mix of the smart grid is expected to help in balancing energy demand and supply curves in (near) real time. These energy pockets maybe distributed within a smart grid in consumer premises or as microgrids. A microgrid can be a regional or communal energy system comprised of distributed energy sources (renewable and/or non-renewable) often in order to optimise power quality, reliability, efficiency and sustainability with accompanying economic benefits (cheaper cost of energy, local employment generation and economic development) and environmental benefits (if renewable energy sources are used). Microgrids would be a common feature in the smart grid, either in standalone [1–3] or grid-connected [4,5] mode, but the latter form is explored in this paper, in order to access its arbitrage opportunities.

Some literature has shown the contributions of DEG [6–8] and DES [7,9–13] in the smart grid with possible locations in consumer premises [7,9,11,12] and centralised locations (owned by third parties) [10,13]. However, the former is easier and faster to deploy than the latter. Also, DEG and DES devices can be installed in on-grid (grid-connected) or off-grid modes. Grid-connected DEG and DES can offer certain advantages, such as reduced investment on additional generation and transmission infrastructures by utility providers to meet growing demand, provision of spinning reserve, power factor correction, reduction/elimination of occurrence of load shedding and blackouts, Peak-to-Average demand reduction, and reduction in CO_2 emissions, etc.

The energy management algorithm could be centralised (sequential) [14,15] or decentralised (distributed) [6,16] depending on the aim of the algorithm. While centralised algorithms enable utilities to control aggregate load with high dissatisfaction to consumers, decentralised algorithms are characterised by lower implementation costs, limited security and privacy challenges and better consumer satisfaction. Therefore, decentralised algorithms have found more applications in DSM and DR algorithms, including the Microgrid Energy Management Distributed Optimisation Algorithm (MEM-DOA).

Efficient energy management for profitable energy trading in a smart grid-connected microgrid is one of the motivations for this paper. Therefore, this work focuses on a possible heterogeneous community of smart consumers with local DEG and DES in a grid-connected smart microgrid with a centralised large-scale Microgrid Energy Storage (MES) device for arbitrage opportunities. Each smart entity (smart consumers, MES device and utility) optimises its benefits in the energy market through the proposed Microgrid Energy Management - Distributed Optimisation Algorithm (MEM-DOA). The MEM-DOA is made up of energy consumption scheduling, storage and generation optimisation algorithms. The MEM-DOA approach is proposed in order to enhance the scalability of deployment, privacy and security in the smart microgrid. The proposed algorithm can be installed in the smart meters of consumers, and Energy Management Controller (EMC) of the MES device and Plug-in Hybrid Electric Vehicles (PHEVs) and the utility grid. Another contribution of this work, which goes beyond existing literature on energy management [6,12,16], is that it guides against a situation where passive smart consumers can benefit more than active smart consumers in a smart (micro)grid. Also, it encourages the penetration of DEG and DES devices in the future energy web. Furthermore, the proposed MEM-DOA enhances consumer satisfaction by factoring tolerance into dissatisfaction problems in order to ensure that every appliance scheduling is within what the consumer is willing to tolerate, unlike most algorithms in the literature. This type of architecture can additionally offer grid reliability and stability, financial benefits to all its smart entities, consumers' social welfare, reduction in Peak-to-Average-Ratio (PAR) demand, and CO_2 emissions. Grid-connected DEG [6–8] and DES [7,9,10,17] can offer grid resilience and stability as it mitigates energy imbalance and emergencies, and the same benefits would be experienced with the deployment of MEM-DOA proposed in this paper.

The rest of the paper is organised as follows: the smart microgrid model is described in Section 2, while its mathematical formulation is presented in Section 3. The MEM-DOA problems and the simulation results are presented in Sections 4 and 5 respectively, and conclusions are presented in Section 6.

2. Description of a Smart Microgrid Energy Management Model

This work proposes a novel model for a Smart Microgrid Energy Management (SMEM) system, comprised of heterogeneous consumers who are connected to the utility grid and a large scale Microgrid Energy Storage (MES) device. A sketch of the proposed grid-connected smart microgrid energy management architecture is presented in Figure 1. This type of architecture is envisaged as a possibility in the future, even among residential consumers, with increasing penetration of DEG and DES in the smart grid.

The smart microgrid has a large scale DES installed as a grid-connected-and-consumer-connected energy storage device providing an alternative centralised source of power to consumers in the smart microgrid. This centralised MES device can be charged from the grid or by any active consumer in the microgrid with their excess energy generation or storage at low price periods, and sell back the stored energy to the consumers and grid as the need arises at a higher price; thereby, enhancing profitability in its energy trading.

Passive consumers are connected uni-directionally for energy flow with the grid and MES device, but bi-directionally for information and communication flow because they neither sell energy to the grid nor the MES device. However, active consumers and PHEVs are connected bi-directionally with both MES device and the utility grid for energy, information and communication flows. Hence, consumers are said to be passive in the SMEM network if they always buy all their energy consumption, or active if they have the ability to both buy and sell energy in the network. A household is regarded as a consumer unit in this work; while it may consist of more than one person, as mostly is the case, it will simply be referred to as consumer throughout this paper. The data used for analyses is the aggregate consumption per household, irrespective of how many individuals it is comprised of.

Therefore, a consumer in the smart microgrid can either be passive (Type-A) consumer $a \in \mathcal{Z}_A, \mathcal{Z}_A \subset \mathbb{A}$ or an active consumer belonging to $\mathcal{W}_V$. The active consumers are further sub-divided into: Type-B consumer $a \in \mathcal{Z}_B$, $\mathcal{Z}_B \subset \mathbb{A}$ with DES, e.g., in-Home Energy Storage (iHES) device only; Type-C consumer $a \in \mathcal{Z}_C$, $\mathcal{Z}_C \subset \mathbb{A}$ with DEG only; Type-D consumer $a \in \mathcal{Z}_D$, $\mathcal{Z}_D \subset \mathbb{A}$ with iHES device and DEG; and PHEVs $v \in \mathcal{V}$, where $\mathcal{W}_V = \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D \cup \mathcal{V}$.

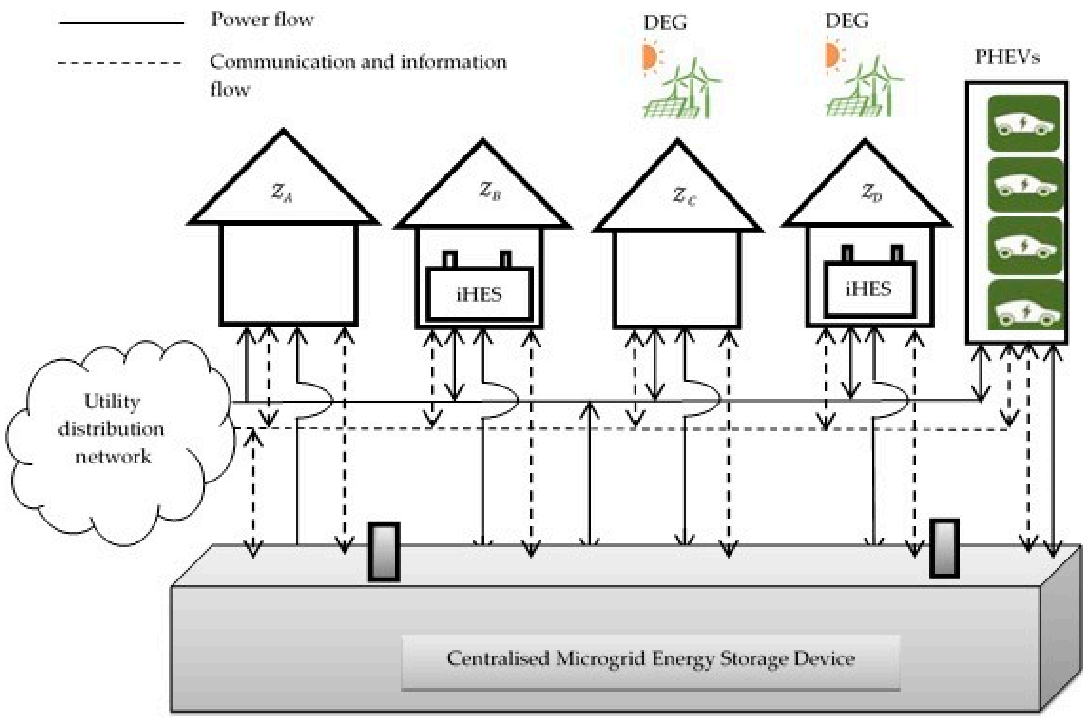

Figure 1. Proposed grid-connected smart microgrid architecture.

These consumers' categories are chosen to reflect the possible different types of consumers that can exist in a smart grid or smart microgrid. An active consumer meets its local demand at every time $t \in \mathbb{T}$ from the energy generated by its local DEG and/or DES, utility grid or MES device depending on energy prices from these sources at the time that the energy is needed. If the active consumer demand is greater than the amount of energy available locally from its DEG and/or DES, then it purchases the difference from the cheaper seller between the grid and MES device. This can, in a way, reduce the need for investment on peaker plants by the utilities. Type-B consumers buy energy for storage and consumption from the cheaper seller between the utility and MES at the optimal buying time, and sells its excess storage (i.e., energy stored beyond daily household consumption) to the highest bidder between utility and MES in order to maximise profit. Similarly, every Type-C consumer optimises its energy consumption, generation, and trading for maximised profit, by selling excess generation to the highest bidder between the utility and MES during the day, and buys from the cheaper seller at night to meet its demand. Also, a Type-D consumer optimises its energy consumption, generation, storage and trading for maximised profit by selling excess generation and/or storage to the highest bidder between the utility and MES and buys from the cheaper seller between the utility and MES, whenever its demand exceeds its local generation (from solar) and storage (iHES) capacities. The PHEVs are also modelled as distributed load, but scheduled in order to avoid over-loading of the grid at any particular time [18–20]. The MEM-DOA architecture can also offer better power quality and reliability, while mitigating intermittency of renewable energy resources, e.g., solar, wind.

3. MEM-DOA Problem Formulation

The proposed model will be made up of appliance consumption scheduling and dissatisfaction models for all residential consumers belonging to set $\mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D$; energy storage models for active consumers $\mathcal{Z}_B \cup \mathcal{Z}_D$, PHEVs v and the microgrid μ; and energy production models for active consumers $\mathcal{Z}_C \cup \mathcal{Z}_D$ and the grid r. Each model is mathematically formulated and presented in this section. A distributed optimisation approach is observed in this work so that each smart consumer can autonomously optimise its energy consumption and expenditure.

3.1. Appliance Energy Consumption Scheduling Model

The consumer's load is categorised into non-shiftable, flexible, interruptible deferrable and uninterruptible deferrable smart appliances. Let every smart consumer $a \in \mathbb{A}$, where $\mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D$ in the smart microgrid, have non-shiftable appliances $i \in \mathbb{I}$, flexible appliances $j \in \mathbb{J}$, uninterruptible deferrable appliances (e.g., dish washer) $f \in \mathbb{F}$ and interruptible deferrable appliances $l \in \mathbb{L}$. The flexible and deferrable appliances would have their consumption shifted in power and time respectively. Therefore, all the smart appliances in a consumer premise belong to the set, $\mathbb{G} = \mathbb{I} \cup \mathbb{J} \cup \mathbb{F} \cup \mathbb{L} = \mathbb{I} \cup \mathbb{H}$, where $\mathbb{H} = \mathbb{J} \cup \mathbb{F} \cup \mathbb{L}$. The total appliance load $x_{a,t}$ of consumer a at any time $t \in \mathbb{T}$, where $\mathbb{T} = [1,\ 2,\ \ldots,\ t]$ is given by [11]:

$$x_{a,t} = \sum_{i \in \mathbb{I}} x_{a,i,t} + \sum_{j \in \mathbb{J}} x_{a,j,t} + \sum_{f \in \mathbb{F}} x_{a,f,t} + \sum_{l \in \mathbb{L}} x_{a,l,t}. \tag{1}$$

The daily load vector for each consumer $a \in \mathbb{A}$ is $x_a = [x_{a,1}, x_{a,2}, \ldots, x_{a,t}]'$, while its total daily load x_a is $x_a = \sum_{t \in \mathbb{T}} x_{a,t}$. If the feasible period of operation $\mathcal{T}_{a,g}$ of any appliance g in the household has a start time $t^s_{a,g}$ and an end time $t^e_{a,g}$, where $\mathcal{T}_{a,g} = \{t \big| t^s_{a,g} \le t \le t^e_{a,g}\}$ and $g = \{i, j, f, l\}$, $\forall g \in \mathbb{G}$, then, total energy $e_{a,g}$ consumed by any appliance g in the smart home is given by:

$$e_{a,g} = \begin{cases} \sum\limits_{t^s_{a,g}}^{t^e_{a,g}} x_{a,g,t},\ \forall t \in \mathbb{T},\ x^{min}_{a,g} \le x_{a,g,t} \le x^{max}_{a,g},\ g = \{i,j,f,l\},\ \forall g \in \mathbb{G} \\ 0,\ \forall t \in \mathbb{T} \backslash \mathcal{T}_{a,g},\ g = \{i,j,f,l\},\ \forall g \in \mathbb{G} \end{cases}. \tag{2}$$

where $x^{min}_{a,g} \le x_{a,g,t} \le x^{max}_{a,g}$ describes the power level constraint for each appliance such that $x^{min}_{a,g} \ge 0$, $x^{min}_{a,g}$ and $x^{max}_{a,g}$ are the minimum power level (OFF or standby mode) and maximum power level of each smart appliance respectively. The total energy x_t consumed by all smart appliances owned by all of the consumers in the smart microgrid at a time t is given by:

$$x_t = \sum_{a \in \mathbb{A}} \sum_{g \in \mathbb{G}} e_{a,g},\ g = \{i,j,f,l\},\ \forall t \in \mathbb{T}. \tag{3}$$

The appliances considered are categorised as non-schedulable (lighting, television, DVD player, electric stove, computer, and refrigerator, etc.) and schedulable appliances (washing machine, dish washer, rice cooker, pool pump, clothes dryer, air-conditioner, room heater, and water heater, etc.).

3.2. Appliance Scheduling Dissatisfaction Model

The dissatisfaction associated with appliance scheduling is modeled for the schedulable appliances only. The schedulable appliances are classified as power-shiftable (flexible) and time-shiftable (deferrable) appliances, which is further divided as uninterruptible deferrable and interruptible deferrable appliances.

The total dissatisfaction cost $\overline{d_a}$ in a consumer's premises, can be defined as the summation of the load dissatisfaction costs for all shiftable/schedulable appliances with respect to energy tariff and is given as [11]:

$$\overline{d_a} = \sum_{j\in\mathbb{J},\, t\in\mathbb{T}} \overline{d}^t_{a,j} + \sum_{f\in\mathbb{F}, t\in\mathbb{T}} \overline{d}^t_{a,f} + \sum_{l\in\mathbb{L}, t\in\mathbb{T}} \overline{d}^t_{a,l}, \forall a \in \mathbb{A}. \quad (4)$$

Such that:

$$\overline{d}^t_{a,j} = \alpha_{a,j}\left(u_{a,j,t}\theta_t\left[1-\left(\tfrac{x_{a,j,t}}{u_{a,j,t}}\right)^{\gamma_t}\right]\right),\ 0 \le \alpha_{a,j} \le 1,\ \gamma_t < 1,\ \gamma_t\theta_t < 0,\ \gamma_t, \theta_t \in \mathbb{R},\ \forall j \in \mathbb{J},\ \forall a \in \mathbb{A},\ \forall t \in \mathbb{T}, \quad (5)$$

$$\overline{d}^t_{a,f} = \alpha_{a,f}\left|t^{s,s}_{a,f} - t^s_{a,f}\right|,\ 0 \le \alpha_{a,f} \le 1,\ t^{e,s}_{a,f} \ge t^{s,s}_{a,f} + \eta_{a,f},\ \forall f \in \mathbb{F},\ \forall t \in \mathbb{T}, \forall a \in \mathbb{A}, \quad (6)$$

$$\overline{d}^t_{a,l} = \alpha_{a,l}\left|\eta_{a,l} - \eta^s_{a,l}\right|,\ 0 \le \alpha_{a,l} \le 1,\ \forall l \in \mathbb{L},\ \eta_{a,l} = \left|t^e_{a,l} - t^s_{a,l}\right|, \eta^s_{a,l} = \left|t^{e,s}_{a,l} - t^{s,s}_{a,l}\right|,\ \forall t \in \mathbb{T}, \forall a \in \mathbb{A}. \quad (7)$$

where $\overline{d}^t_{a,j}$ is the dissatisfaction cost due to scheduling flexible smart appliances from a nominal load $u_{a,j,t}$ to an actual load $x_{a,j,t}$; $\overline{d}^t_{a,f}$ is dissatisfaction cost for scheduling start time of operation for uninterruptible deferrable load f to an actual start time $t^{s,s}_{a,f}$ from a nominal start time $t^s_{a,f}$; $\overline{d}^t_{a,l}$ is the dissatisfaction cost due to scheduling an interruptible deferrable appliance l from its nominal duration $\eta_{a,l}$ to an actual duration $\eta^s_{a,l}$ for the entire task to be completed; $\alpha_{a,j}$ is flexibility tolerance; $\alpha_{a,f}$ is uninterruptibility tolerance; $\alpha_{a,l}$ is interruptibility tolerance; provided $\gamma_t < 1$, $\gamma_t\theta_t < 0$, $\gamma_t, \theta_t \in \mathbb{R}$, $t^{e,s}_{a,f} \ge t^{s,s}_{a,f} + \eta_{a,f}$, $\forall f \in \mathbb{F}$, $\forall t \in \mathbb{T}, \forall a \in \mathbb{A}$. The values of $\alpha_{a,j}$, $\alpha_{a,f}$, $\alpha_{a,l}$, γ_t and θ_t are varied to model different levels of consumer dissatisfaction. The solutions to (4)–(7) constrains the dissatisfaction of the consumer to its choice level of tolerance thereby offering scheduling with minimised dissatisfaction. Although this paper does not go into detail, thermal analysis of heating/cooling appliances [21,22], consumer satisfaction is still ensured using (5) whose second derivative is less than zero for $x_{a,j,t} < u_{a,j,t}$ and $x_{a,j,t} < u_{a,j,t}$. This, therefore, ensures that the consumer is not infinitely satisfied by increasing appliance power consumption.

3.3. Local Distributed Energy Storage Model

The local DES (the iHES device, e.g., battery) model applies only to Type-B active consumer $a \in \mathcal{Z}_B, \mathcal{Z}_B \subset \mathbb{A}$ and Type-D active consumer $a \in \mathcal{Z}_D,\ \mathcal{Z}_D \subset \mathbb{A}$ in this smart microgrid model. If $b_{a,t}$ is the energy stored in the battery at time $t \in \mathbb{T}$ for consumer $a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}$, then, the battery daily energy storage scheduling vector is $\boldsymbol{b}_a = [b_{a,1}, b_{a,2}, \ldots,\ b_{a,t}, \ldots,\ b_{a,t}]'$. Therefore, $b_{a,t}$ can be expressed in terms of the energy charging profile $b^+_{a,t}$ and energy discharging profile $b^-_{a,t}$ as $b_{a,t} = b^+_{a,t} - b^-_{a,t}$, where $b^+_{a,t},\ b^-_{a,t} \ge 0$. The charging efficiency β^+_a and discharging efficiency β^-_a fulfil conditions $0 < \beta^+_a \le 1$ and $\beta^-_a \ge 1$ respectively. Therefore, the battery is only effectively charged and discharged with $\beta^+_a b^+_{a,t}$ and $\beta^-_a b^-_{a,t}$ amounts of energy, respectively. The charging and discharging efficiency vector $\boldsymbol{\beta}_a = [\beta^+_a, -\beta^-_a]'$ and per-timeslot storage scheduling vector is $\boldsymbol{b}_{a,t} = [b^+_{a,t}, b^-_{a,t}]'$. This implies that $\boldsymbol{\beta}'_a\boldsymbol{b}_{a,t}$

is the energy charged/discharged at time $t \in \mathbb{T}$. Since the maximum charging/discharging rate b_a^{max} of the battery cannot be exceeded at any charging/discharging time then, the constraint (8) is introduced:

$$\boldsymbol{\beta}_a' \boldsymbol{b}_{a,t} \leq b_a^{max}, \ a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}, \ \forall t \in \mathbb{T}. \tag{8}$$

The energy leakage rate λ_a of the battery is constrained as $0 < \lambda_a \leq 1$. If $q_{a,t-1}$ is the charge level of the battery at time $t-1$, which was reduced at λ_a leakage rate, then, the present time t charge level can be expressed as: $q_{a,t} = q_{a,t-1}(1-\lambda_a) + \boldsymbol{\beta}_a' \boldsymbol{b}_{a,t}, \ a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}, \ \forall t \in \mathbb{T}$. Also, the charge level $q_{a,t}$ of the battery is bounded as $0 \leq q_{a,t} \leq b_{a,cap}$, where $b_{a,cap}$ is the battery capacity. Therefore, for every smart consumer $a \in \{\mathcal{Z}_B, \mathcal{Z}_D\},$:

$$-q_{a,t-1}(1-\lambda_a) \leq \boldsymbol{\beta}_a' \boldsymbol{b}_{a,t} \leq b_{a,cap} - q_{a,t-1}(1-\lambda_a), \ \forall a \in \{\mathcal{Z}_B, \mathcal{Z}_D\},. \tag{9}$$

Also, $q_{a,t}$ and the initial charge level q_{a,t_0} are related by:

$$q_{a,t} = q_{a,t_0}(1-\lambda_{a,t}) + \sum_{t=t_0}^{t} \lambda_{a,t-t_0} \boldsymbol{\beta}_a' \boldsymbol{b}_{a,t}, a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}. \tag{10}$$

The storage device can go through integer number of charging and discharging cycles, which oppose fluctuations in the daily energy demand of the consumer. Therefore, q_{a,t_0} and the daily final charge level q_{a,t_t} can be related by:

$$|q_{a,t_t} - q_{a,t_0}| \leq \mho_a, \ \mho_a \in \mathbb{R}^+, \ a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}, \ \forall t \in \mathbb{T}, \tag{11}$$

where $\mho_a$ is a sufficiently small positive constant. Each battery is assumed to be sufficiently small compared to the aggregate load, so as not to influence tariffs during charging and discharging periods. Examples of possible local DES devices include lithium-ion batteries, lead-acid batteries, etc.

3.4. Consumer Distributed Energy Generation Model

A consumer's DEG can be either a dispatchable or non-dispatchable energy generator. Dispatchable generators include micro-turbines, internal combustion engines, etc., while non-dispatchable generators include solar panels, wind turbines, etc. Only non-dispatchable generators are considered in this work, due to their associated environmental friendliness and ease of deployment.

For a non-dispatchable generator owned by consumer $a \in \{\mathcal{Z}_C, \mathcal{Z}_D\}$, the DEG production at time t is $\mathfrak{g}_{a,t}$. The non-dispatchable generators produce energy based on available intermittent resources, e.g., solar radiation. A consumer can sell its excess local generation to the grid or MES device and buy it back later again at periods when these resources are naturally not available or are less than the quantity required to meet the consumer's demand.

3.5. Microgrid Energy Storage Model

The MES device is modelled similarly to consumers' DES devices and applies the same explanations and formulations. Therefore, if the daily energy storage scheduling vector is $\boldsymbol{b}_\mu = [b_{\mu,1}, b_{\mu,2}, \ldots, b_{\mu,t}, \ldots, b_{\mu,t}]$ for the MES device, then (8)–(10) can be adopted and re-written for the MES device as follows:

$$\boldsymbol{\beta}_\mu' \boldsymbol{b}_{\mu,t} \leq b_\mu^{max}, \ 0 < \beta_\mu^+ \leq 1, \beta_\mu^- \geq 1, b_{\mu,t}^+, b_{\mu,t}^- \geq 0, \forall t, \tag{12}$$

$$-q_{\mu,t-1}(1-\lambda_\mu) \leq \boldsymbol{\beta}_\mu' \boldsymbol{b}_{\mu,t} \leq b_{\mu,cap} - q_{\mu,t-1}(1-\lambda_\mu), \tag{13}$$

$$q_{\mu,t} = q_{\mu,t_0}(1-\lambda_{\mu,t}) + \sum_{t=t_0}^{t} \lambda_{\mu,t-t_0} \boldsymbol{\beta}_\mu' \boldsymbol{b}_{\mu,t}, \ \forall t \in \mathbb{T}, \tag{14}$$

The energy charged/discharged by the MES device $\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t}$ at time t is further simplified as:

$$\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t} = (\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t})^{r} + (\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t})^{\mathcal{W}_V} + ({\boldsymbol{\beta}'_{\mu}}^{-}{\boldsymbol{b}_{\mu,t}}^{-})^{\mathcal{Z}_A},\ \mathcal{W}_V = \{\mathcal{Z}_B, \mathcal{Z}_C, \mathcal{Z}_D, \mathcal{V}\}, \tag{15}$$

where $(\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t})^{r}$ and $(\boldsymbol{\beta}'_{\mu}\boldsymbol{b}_{\mu,t})^{\mathcal{W}_V}$ are the charged/discharged energy by the grid and active consumers respectively, and $({\boldsymbol{\beta}'_{\mu}}^{-}{\boldsymbol{b}_{\mu,t}}^{-})^{\mathcal{Z}_A}$ is the MES discharging profile towards consumer $\mathcal{Z}_A$, $({\boldsymbol{b}_{\mu,t}}^{+})^{\mathcal{Z}_A} = 0$ since consumer $a \in \mathcal{Z}_A$ does not sell energy to the MES device. The quantity of charge $q_{\mu,t}$ in the MES device at any time t is the aggregate of the charges stored in it by the grid and active consumers and is given as:

$$q_{\mu,t} = q^{r}_{\mu,t} + q^{\mathcal{Z}_B}_{\mu,t} + q^{\mathcal{Z}_C}_{\mu,t} + q^{\mathcal{Z}_D}_{\mu,t} + q^{v}_{\mu,t}, \tag{16}$$

where $q^{r}_{\mu,t}$, $q^{\mathcal{Z}_B}_{\mu,t}$, $q^{\mathcal{Z}_C}_{\mu,t}$, $q^{\mathcal{Z}_D}_{\mu,t}$ and $q^{v}_{\mu,t}$ are the quantities of charge stored in the MES device by the grid, consumers $a \in \mathcal{Z}_B$, $a \in \mathcal{Z}_C$ and $a \in \mathcal{Z}_D$, and PHEVs respectively.

Some storage devices that can serve as MES devices include Compressed-Air Energy Storage (CAES), Pumped-Storage Hydroelectric (PSH), etc. The MES device is a form of large-scale energy storage that can be owned by a private operator or utility provider.

3.6. Plug-In Hybrid Electric Vehicle Battery Model

The PHEVs in the smart microgrid shall be modeled with respect to its battery characteristics only, and not driving pattern. Let $b_{v,t}$ be a PHEV charging/discharging profile at time t; then, the daily storage vector for every PHEV battery $v \in \mathcal{V}$ can be denoted as $\boldsymbol{b}_v = [b_{v,1}, b_{v,2}, \ldots, b_{v,t}, \ldots, b_{v,t}]$. Then, the storage profile for the PHEV can be modeled as follows:

$$\boldsymbol{\beta}'_{v}\boldsymbol{b}_{v,t} \le b^{max}_{v},\ v \in \mathcal{V},\ \forall t \in \mathbb{T}, \tag{17}$$

$$q_{v,t} = q_{v,t-1}(1 - \lambda_v) + \boldsymbol{\beta}'_{v}\boldsymbol{b}_{v,t},\ v \in \mathcal{V},\ \forall t \in \mathbb{T}, \tag{18}$$

$$-q_{v,t-1}(1 - \lambda_v) \le \boldsymbol{\beta}'_{v}\boldsymbol{b}_{v,t} \le b_{v,cap} - q_{v,t-1}(1 - \lambda_v),\ \forall v, \tag{19}$$

$$q_{v,t} = q_{v,t_0}(1 - \lambda_{v,t}) + \sum_{t=t_0}^{t} \lambda_{v,t-t_0}\boldsymbol{\beta}'_{v}\boldsymbol{b}_{v,t},\ v \in \mathcal{V},\ \forall t \in \mathbb{T}, \tag{20}$$

and

$$|q_{v,t_t} - q_{v,t_0}| \le \mho_v,\ v \in \mathcal{V},\ \forall t \in \mathbb{T}, \tag{21}$$

In order to prevent the PHEVs from increasing peak demand beyond grid and MES capacities, their charging/discharging profiles $b_{v,t}$ and, hence, the load $\mathcal{R}_{v,t} = b_{v,t}$ are centrally scheduled within the microgrid and are constrained by:

$$0 \le \sum_{v \in \mathcal{V}} \mathcal{R}_{v,t} \le \overline{b}_{v,t},\ \forall v \in \mathcal{V},\ \forall t \in \mathbb{T}. \tag{22}$$

where $\overline{b}_{v,t} = (\overline{b}_{v,t})^{r} + (\overline{b}_{v,t})^{\mu}$ is the maximum energy the PHEVs can draw from the utility grid and MES device at any timeslot respectively. The PHEVs are modeled as separate aggregate loads in the microgrid without attachment to any particular consumer, although they could also play similar roles as iHES device in consumer premises depending on their configurations. The PHEVs considered here are not directly attached to any consumer in the grid as shown in Figure 1, but could also be owned by any of the consumers considered. Also, the charging location could either be the consumers' premises or the public charging station.

4. MEM-DOA Optimisation Problems

4.1. Microgrid Energy Storage Cost Model

The MES device buys energy from the grid and active consumers during low price periods and sells energy back to them at a higher price than the purchasing price, in order to maximise its profit. If the charging/discharging load of the MES device is $\mathcal{R}_{\mu,t} = b_{\mu,t}, \ \forall t \in \mathbb{T}$, then, the MES daily cost function $C_\mu(\boldsymbol{\mathcal{R}}_\mu)$ is given as:

$$C_\mu(\boldsymbol{\mathcal{R}}_\mu) = \sum_{t \in \mathbb{T}} (P^{SP}_{\mu \to \mathscr{Y},t} \boldsymbol{b}^-_{\mu,t} - P^{BP}_{\mu,t} \boldsymbol{b}^+_{\mu,t} - P^{O}_{\mu,t} b_{\mu,t}), \ \mathscr{Y} = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D\}, \forall t \in \mathbb{T}, \tag{23}$$

where $P^{SP}_{\mu,t}$ and $P^{BP}_{\mu,t} = min(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_B,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t})$ are the respective selling and buying prices of the MES and $P^{O}_{\mu,t} b_{\mu,t}$ is its charging/discharging operating cost. Type-A consumers are passive consumers in the microgrid and hence, would always buy energy from the MES device with a penalty price. For instance, the selling price of energy from the MES device to any buyer $P^{SP}_{\mu \to \mathscr{Y},t}$ is given by:

$$P^{SP}_{\mu \to \mathscr{Y},t} = \begin{cases} \omega_{\mu,t} P^{BP}_{\mu,t}, \text{if } q^{\mathscr{Y}}_{\mu,t} \geq b^{\mu-}_{\mu \to \mathscr{Y},t}, \ \forall t \in \mathbb{T} \\ \omega_{\mu,t} P^{BP}_{\mu,t} \mathcal{P}_{\mathscr{Y},t}, \text{if } q^{\mathscr{Y}}_{\mu,t} < b^{\mu-}_{\mu \to \mathscr{Y},t}, \ \forall t \in \mathbb{T} \end{cases}, \tag{24}$$

where $\mathscr{Y} = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D, v\}, \omega_{\mu,t}$ is a preset MES provider coefficient of profit in order to maximise reasonable profit for the MES device provider, $\mathcal{P}_{\mathscr{Y},t}$ is the buyer's price penalty for requesting more energy than contributed to the MES present charge level, $q^{\mathscr{Y}}_{\mu,t}$ is the energy contribution by a buyer $\mathscr{Y}$ to the MES charge level and $b^{\mu-}_{\mu \to \mathscr{Y},t}$ is the amount of energy to be discharged from the MES device to buyer $\mathscr{Y}$ at time t. The MES selling price (SP) to the passive consumers would be the highest every time $t \in \mathbb{T}$ since they do not make a contribution to the energy stored in the MES device. Also, $P^{BP}_{\mu,t} = min(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_B,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t})$, where $P^{SP}_{r,t}$, $P^{SP}_{a,\mathcal{Z}_B,t} P^{SP}_{a,\mathcal{Z}_C,t}$ and $P^{SP}_{a,\mathcal{Z}_D,t}$ are selling prices for grid and active consumers $\mathcal{Z}_B$, $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The value of $\omega_{\mu,t}$ is constrained as $\omega_{\mu,t} > 1$ to ensure compliance with the rate-of-return on investment regulations. This would help the MES device provider to set a SP or tariff that is high enough to attract further capital investment and also low enough so as not to negatively affect customers' welfare. In this work, a buyer's price penalty $\mathcal{P}_{\mathscr{Y},t}$ is given by:

$$\mathcal{P}_{\mathscr{Y},t} = \frac{max(P^{SP}_{-\mathscr{Y},t})}{min(P^{SP}_{-\mathscr{Y},t})}, \ q^{\mathscr{Y}}_{\mu,t} < b^{\mu-}_{\mu \to \mathscr{Y},t}, \ \mathscr{Y} = \{r, \mathcal{Z}_A, \mathcal{Z}_B, \mathcal{Z}_C, \mathcal{Z}_D\}. \tag{25}$$

where $P^{SP}_{-\mathscr{Y},t}$ is the SP of other buyers excluding $\mathscr{Y}$ at time t. The MEM-DOA for the MES device is formulated as a linear program and solved using simplex method [23,24]:

$$\min_{\boldsymbol{\mathcal{R}}_\mu \in \mathbb{R}} C_\mu(\boldsymbol{\mathcal{R}}_\mu) \ s.t. \ (12) - (16), \ (23), \ (24), \ (25), \ P^{BP}_{\mu,t} = min(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t}), \ \forall t \in \mathbb{T}. \tag{26}$$

4.2. Utility Cost Model

Let $\mathfrak{g}_{r,t}$ be the energy generation by the electricity utility provider at time t and bounded by the utility grid maximum energy production capacity $\mathfrak{g}^{max}_r$ be given as:

$$0 \leq \mathfrak{g}_{r,t} \leq \mathfrak{g}^{max}_r, \ \forall t \in \mathbb{T}. \tag{27}$$

The constrain in (27) ensures that all the load from all devices connected to the grid does not exceed grid capacity at any given time. Also, the load balance on the grid at any time t can be given as:

$$\mathfrak{g}_{r,t} = \mathcal{R}_{a,\mathcal{Z}_A,t} + \mathcal{R}_{a,\mathcal{Z}_B,t} + \mathcal{R}_{a,\mathcal{Z}_C,t} + \mathcal{R}_{a,\mathcal{Z}_D,t} + \mathcal{R}_{\mu,t}. \tag{28}$$

where $\mathcal{R}_{a,\mathcal{Z}_A,t}$, $\mathcal{R}_{a,\mathcal{Z}_C,t}$ and $\mathcal{R}_{a,\mathcal{Z}_D,t}$ are the total grid loads from consumers $\mathcal{Z}_A$, $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The utility cost function, $C_t(\mathfrak{g}_{r,t})$ is the cost to the utility for providing $\mathfrak{g}_{r,t}$ supply and can be modeled as a non-decreasing convex function using the energy cost function for thermal generators [6,7,14]:

$$C_t(\mathfrak{g}_{r,t}) = c_1^t(\mathfrak{g}_{r,t})^2 + c_2^t\mathfrak{g}_{r,t} + c_3^t,\ \forall t \in \mathbb{T}, \tag{29}$$

where $c_1^t > 0$ and $c_2^t, c_3^t \geq 0$. Also, in accordance with the rate-of-return on investment regulations, $P_{r,t}^{SP}$ and the utility buying price $P_{r,t}^{BP}$ are modified from [6] and given as:

$$P_{r,t}^{SP} = \omega_{r,t}\frac{C_t(\mathfrak{g}_{r,t})}{\mathfrak{g}_{r,t}} = \omega_{r,t}P_{r,t}^{BP},\ \forall t \in \mathbb{T}, \tag{30}$$

where $\omega_{r,t} > 1$ is a preset utility profit coefficient. This ensures mutual financial benefits between utility, consumers and MES provider. The total daily cost of electricity vector to the utility $\boldsymbol{C}_r$ is the total cost of generation to meet its load and cost of energy purchases from the active consumers and MES device, and it is given as:

$$\begin{aligned}\boldsymbol{C}_r = \sum_{t\in\mathbb{T}}(C_t(\mathfrak{g}_{r,t}) \quad & + P_{r,t}^{BP}(b_{a,\mathcal{Z}_B,t} - x_{a,\mathcal{Z}_B,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_B})^{+} \\ & + P_{r,t}^{BP}(\mathfrak{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_C})^{+} \\ & + P_{r,t}^{BP}(\mathfrak{g}_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_D})^{+} \\ & + P_{r,t}^{BP}\sum_{v\in\mathcal{V}}({b_{\mu,t}}^{-})^{r} + P_{r,t}^{BP}({b_{\mu,t}}^{-})^{r}\Big),\ \forall t \in \mathbb{T}.\end{aligned} \tag{31}$$

where $({b_{\mu,t}}^{+})^{\mathcal{Z}_B}$, $({b_{\mu,t}}^{+})^{\mathcal{Z}_C}$ and $({b_{\mu,t}}^{+})^{\mathcal{Z}_D}$ are energy sold to the MES device by consumers $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively and $({b_{\mu,t}}^{-})^{r}$ is energy bought from the MES device by the grid. The MEM-DOA for the utility grid is formulated as a convex programming problem [25] and solved using interior-point method [26], as follows:

$$\min_{\mathfrak{g}_{r,t}\in\mathbb{R}} \boldsymbol{C}_r s.t.\ (27)-(31), P_{r,t}^{BP} = min(P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP},\ P_{\mu\rightarrow r,t}^{SP}). \tag{32}$$

4.3. Type-A Consumer Cost Model

Since the Type-A consumer is a passive consumer, its cost function is basically the cost of meeting its local demand from the grid or MES device depending on their energy selling prices. Therefore, the daily cost $C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A})$ of Type-A smart consumer $a \in \mathcal{Z}_A$, $\mathcal{Z}_A \subset \mathbb{A}$, is given as:

$$C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A}) = \boldsymbol{P}_{a,\mathcal{Z}_A}^{BP}\mathbf{x}_{a,\mathcal{Z}_A} + \overline{\mathbf{d}}_{a,\mathcal{Z}_A}^{t},\ a \in \mathcal{Z}_A, \tag{33}$$

where $\boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\rightarrow\mathcal{Z}_A}^{SP}) = min([P_{r,1}^{SP}, P_{r,2}^{SP}, \ldots, P_{r,t}^{SP}], [P_{\mu\rightarrow\mathcal{Z}_A,1}^{SP}, P_{\mu\rightarrow\mathcal{Z}_A,2}^{SP}, \ldots, P_{\mu\rightarrow\mathcal{Z}_A,t}^{SP}]) = [P_{a,\mathcal{Z}_A,1}^{BP}, P_{a,\mathcal{Z}_A,2}^{BP}, \ldots, P_{a,\mathcal{Z}_A,t}^{BP}]$ is consumer $a \in \mathcal{Z}_A$ purchasing or buying price and $\mathbf{x}_{a,\mathcal{Z}_A} = [x_{a,\mathcal{Z}_A,1}, x_{a,\mathcal{Z}_A,2}, \ldots, x_{a,\mathcal{Z}_A,t}]$ is the total appliance load for consumer $a \in \mathcal{Z}_A$ at time t. The MEM-DOA for Type-A passive consumer shall be formulated as a convex programming problem [25] solved using interior-point method [26] and is given as:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_A}, \overline{d}_a^t\in\mathbb{R}} C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A})\ s.t.\ (1)-(7),\ \boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\rightarrow\mathcal{Z}_A}^{SP}),\ a \in \mathcal{Z}_A,\ \forall t \in \mathbb{T}. \tag{34}$$

4.4. Type-B Consumer Cost Model

The daily cost function $C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B})$ for each Type-B consumer is given as:

$$C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B}) = \sum_{t\in\mathbb{T}} P^{BP}_{a,\mathcal{Z}_B,t}(x_{a,\mathcal{Z}_B,t} - b_{a,\mathcal{Z}_B,t})^+ - \sum_{t\in\mathbb{T}} P^{SP}_{a,\mathcal{Z}_B,t}(b_{a,\mathcal{Z}_B,t} - x_{a,\mathcal{Z}_B,t})^+ + \sum_{t\in\mathbb{T}} \vec{d}^{\,t}_{a,\mathcal{Z}_B},\ a \in \mathcal{Z}_B, \quad (35)$$

The MEM-DOA for Type-B active smart consumer is formulated as a convex programming problem [25] as follows:

$$\begin{aligned} \min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B}, \vec{d}^{\,t}_a \in \mathbb{R}} \quad & C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B}) \text{ s.t. } (1)-(7),\ (35), P^{BP}_{a,\mathcal{Z}_B,t} = min(P^{SP}_{r,t}, P^{SP}_{\mu,t}),\ P^{SP}_{a,\mathcal{Z}_B,t} = max(P^{BP}_{r,t}, P^{BP}_{\mu,t}), \\ & \mathcal{P}_{\mathcal{Z}_B,t} = \frac{max(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t}, P^{SP}_{v,t})}{min(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t}, P^{SP}_{v,t})}, \text{ if } q^{\mathcal{Z}_B}_{\mu,t} < b^{\mu-}_{\mu\to\mathcal{Z}_B,t},\ a \in \mathcal{Z}_B,\ \forall t \in \mathbb{T}, \forall t. \end{aligned} \quad (36)$$

Solving (36) for each Type-B consumer ensures minimised energy consumption and expenditure from the utility grid at peak times with an accompanying maximised consumer satisfaction.

4.5. Type-C Consumer Cost Model

A Type-C smart consumer $a \in \mathcal{Z}_C$ possesses non-dispatchable DEG locally. Since the consumer does not have a storage device, it would have to sell out its excess generation during the day to the grid or MES device. Therefore, the per timeslot load $\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C,t}$ and daily cost function $C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C})$ for Type-C consumer are given by (37) and (38) respectively:

$$\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C,t} = x_{a,\mathcal{Z}_C,t} - \mathfrak{g}_{a,\mathcal{Z}_C,t},\ a \in \mathcal{Z}_C,\ \mathcal{Z}_C \subset \mathbb{A}, \quad (37)$$

$$C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}) = \sum_{t\in\mathbb{T}} P^{BP}_{a,\mathcal{Z}_C,t}(x_{a,\mathcal{Z}_C,t} - \mathfrak{g}_{a,\mathcal{Z}_C,t})^+ - \sum_{t\in\mathbb{T}} P^{SP}_{a,\mathcal{Z}_C,t}(\mathfrak{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t})^+ + \sum_{t\in\mathbb{T}} \vec{d}^{\,t}_{a,\mathcal{Z}_C},\ a \in \mathcal{Z}_C, \quad (38)$$

where $P^{BP}_{a,\mathcal{Z}_C,t} = min(P^{SP}_{r,t}, P^{SP}_{\mu,t})$ and $P^{SP}_{a,\mathcal{Z}_C,t} = max(P^{BP}_{r,t}, P^{BP}_{\mu,t})$ are buying and selling prices respectively, $x_{a,\mathcal{Z}_C,t}$ is total appliances demand and $\mathfrak{g}_{a,\mathcal{Z}_C,t}$ is generation by consumer $a \in \mathcal{Z}_C$ at time t. Each Type-C smart consumer also has its MEM-DOA formulated as a convex programming problem [25] and solved using interior-point method [26] is given as:

$$\begin{aligned} \min_{\mathcal{R}_{a,\mathcal{Z}_C}, \vec{d}^{\,t}_a \in \mathbb{R}} \quad & C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}) \text{ s.t. } (1)-(7),\ (38),\ P^{BP}_{a,\mathcal{Z}_C,t} = min(P^{SP}_{r,t}, P^{SP}_{\mu\to\mathcal{Z}_C,t}), \\ & P^{SP}_{a,\mathcal{Z}_C,t} = max(P^{BP}_{r,t}, P^{BP}_{\mu,t}),\ a \in \mathcal{Z}_C,\ \forall t \in \mathbb{T}. \end{aligned} \quad (39)$$

4.6. Type-D Consumer Cost Model

The Type-D consumer $a \in \mathcal{Z}_D$ is the active consumer that possesses both non-dispatchable DEG and DES device in its premise. Its total load $\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D,t}$ at any time t is given by:

$$\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D,t} = x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathfrak{g}_{a,\mathcal{Z}_D,t},\ a \in \mathcal{Z}_D,\ \mathcal{Z}_D \subset \mathbb{A}, \quad (40)$$

where $x_{a,\mathcal{Z}_D,t}$ is the consumer's total appliances demand, $b_{a,\mathcal{Z}_D,t}$ is the energy charging/discharging profile for its DES device and $\mathfrak{g}_{a,\mathcal{Z}_D,t}$ is the generation from its DEG at time t. Therefore, the daily cost function $C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D})$ for each Type-D consumer is given as:

$$\begin{aligned} C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}) &= \sum_{t\in\mathbb{T}} P^{BP}_{a,\mathcal{Z}_D,t}(x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathfrak{g}_{a,\mathcal{Z}_D,t})^+ \\ &- \sum_{t\in\mathbb{T}} P^{SP}_{a,\mathcal{Z}_D,t}(\mathfrak{g}_{a,\mathcal{Z}_D,t} - b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t})^+ + \sum_{t\in\mathbb{T}} \vec{d}^{\,t}_{a,\mathcal{Z}_D}. \end{aligned} \quad (41)$$

where $P^{BP}_{a,\mathcal{Z}_D,t} = min(P^{SP}_{r,t}, P^{SP}_{\mu,t})$ and $P^{SP}_{a,\mathcal{Z}_D,t} = max(P^{BP}_{r,t}, P^{BP}_{\mu,t})$ are consumer $a \in \mathcal{Z}_D$ buying and selling prices respectively at time t. Finally, the MEM-DOA for Type-D active smart consumer is formulated as a convex programming problem [25] and solved using interior-point method [26] as follows:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_D}, \vec{d}^t_a \in \mathbb{R}} C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}) \text{ s.t. } (1)-(7),\ (41), P^{BP}_{a,\mathcal{Z}_D,t} = min(P^{SP}_{r,t}, P^{SP}_{\mu\to\mathcal{Z}_D,t}),$$
$$P^{SP}_{a,\mathcal{Z}_D,t} = max(P^{BP}_{r,t}, P^{BP}_{\mu,t}),\ a \in \mathcal{Z}_D,\ \forall t \in \mathbb{T}. \tag{42}$$

The solutions to (34), (36), (39) and (42) offer the smart consumers optimised satisfaction, energy consumption and expenditure with financial savings.

4.7. Plug-In Hybrid Electric Vehicle Battery Storage Cost Model

The MEM-DOA for the PHEVs is centralised within the PHEVs community network, but distributed in relation to other smart entities in the smart microgrid and is formulated as a linear programming problem which can be solved using simplex method [23,24]:

$$\min_{\boldsymbol{\mathcal{R}}_v \in \mathbb{R}} C_v(\boldsymbol{\mathcal{R}}_v) \text{ s.t. } (17)-(22),\ \mathcal{R}_{v,t} = b_{v,t},\ \forall t \in \mathbb{T},$$
$$\mathcal{P}_{v,t} = \frac{max(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_B,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t})}{min(P^{SP}_{r,t}, P^{SP}_{a,\mathcal{Z}_B,t}, P^{SP}_{a,\mathcal{Z}_C,t}, P^{SP}_{a,\mathcal{Z}_D,t})},\ \text{if } q^v_{\mu,t} < b^{\mu-}_{\mu\to v,t}, \tag{43}$$
$$b_{v,t} \leq \bar{b}_{v,t},\ \bar{b}_{v,t} = (\bar{b}_{v,t})^r + (\bar{b}_{v,t})^\mu,\ 0 \leq \sum_{v\in\mathcal{V}} \mathcal{R}_{v,t} \leq \sum_{v\in\mathcal{V}} \bar{b}_{v,t},\ \forall t.$$

Peak-to-Average-Ratio (PAR) demand from the grid can be found using (44) and solved using simplex method [23,24]:

$$PAR = \frac{Peak\ demand}{Average\ demand} = \frac{\max_{t\in\mathbb{T}} \sum_{a\in\mathbb{A}} Y^t_{a,r}}{\frac{1}{t}\sum_{a\in\mathbb{A},t\in\mathbb{T}} Y^t_{a,r}}. \tag{44}$$

5. Numerical Results and Discussions

The MEM-DOA simulation was considered for four hundred consumers (one hundred households in each category of consumer) with residential data obtained from [27] and Time-of-Use (TOU) pricing tariffs in South Africa adopted [28]. The obtained data was used as the nominal consumption for all consumers. The results of the simulations after rigorous analyses are presented in this section. The demand from the utility grid is presented in Figure 2 (for Type-A and Type-B smart consumers) and Figure 3 (for Type-C and Type-D smart consumers).

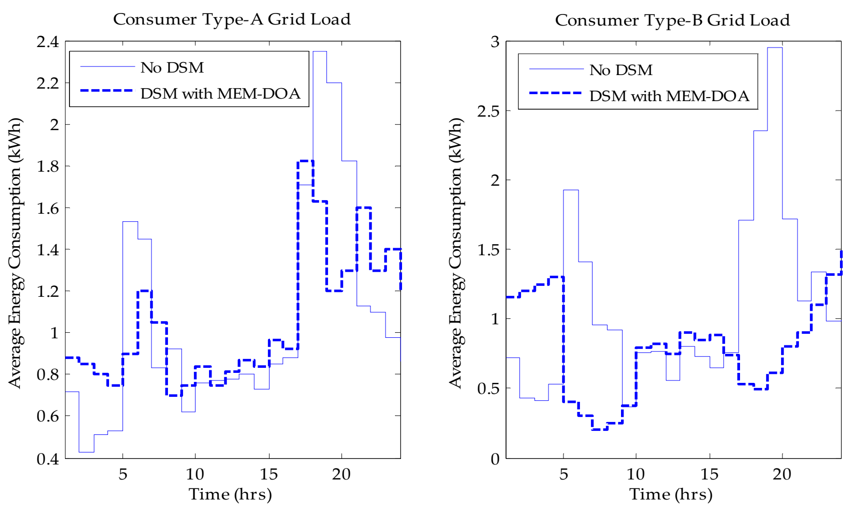

Figure 2. Average grid consumption for Type-A passive and Type-B active smart consumers.

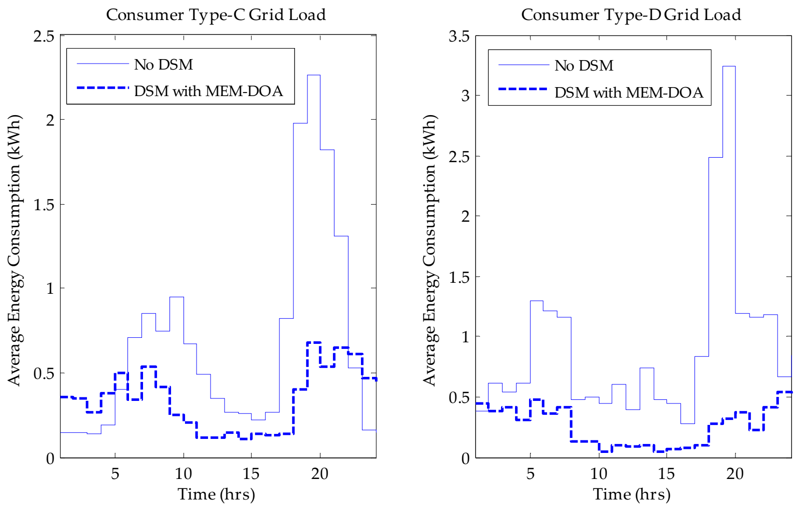

Figure 3. Average grid consumption for Type-C and Type-D active smart consumers.

Since Type-A smart consumers are passive smart consumers, they optimise their energy consumption, source of energy supply in their premises, and energy expenditure using the MEM-DOA. They are constrained to always having to buy energy from the either the utility or MES and do not benefit from any incentive from the MES provider. These factors, therefore, place them at the lowest level of satisfaction (highest dissatisfaction) and financial savings among consumers in the microgrid. The consumers also shift schedulable loads to periods of low tariff (mostly in the nights) in order to obtain a lower energy expenditure at the end of the month.

For Type-B smart consumers with an iHES device (or local DES device), the consumer load, battery charging/discharging and energy expenditure are optimised for maximised financial savings and demand satisfaction. Its MEM-DOA ensures that the battery is only charged from grid or MES at low price/off-peak periods, but discharged primarily to meet consumer demand at peak/high price periods. This type of consumer prefers to buy energy from the MES because of the price incentives it receives, but sometimes has to buy from the grid when its demand from the MES exceeds its allowed incentivised energy (i.e., quantity of energy initially sold to the MES). At such times, the MES treats an active consumer as if it were a passive consumer according to (24).

Active Type-C consumers have their local generation from roof-top solar panel prioritised for meeting demand in consumers' premises. However, since solar resources are only available during the day and generation from the 3 kW solar panel (assumed in this study) mostly exceeding consumer instantaneous demands during the day, the excess generation is sold mostly to the MES device, due to the price incentive available to it from the MES provider within the constrain of (24). These consumers therefore purchase more energy from the MES than the grid, as is evident in Figure 3.

Type-D smart consumers can store their excess energy generation in their iHES and use the stored charge at peak times and only make energy request from the MES or utility when their demand exceeds their total local generation and storage levels. The iHES device could be charged from either the solar panel locally or externally from the grid or MES device and hence offers the consumers highest demand satisfaction and financial savings in this study. The MEM-DOA model has shown to offer reduction in grid peak demand in all considered scenarios with increasing penetration of DES and DEG in consumer premises as shown in Figures 2 and 3. There is a lower reduction in grid peak demand for Type-B than for Type-C smart consumers because Type-C smart consumers do not have local storage for their excess generation during the day and would have to purchase from the grid or MES device at peak periods.

Additionally, the centralised MEM-DOA for the PHEVs ensures that only limited PHEVs are scheduled to be charged from the grid and MES device at peak periods, while most of the PHEV loads are scheduled for charging at night (low price period). The aggregate battery charging/discharging load profile for the hundred PHEVs considered is presented in Figure 4. The load profile showed more consumption at non-peak periods than peak periods as compared to the nominal (No DSM) scenario where individual PHEVs owners could decide to charge their PHEVs in the evenings especially, upon arrival at home. The morning peak observed between 7:00 and11:00 a.m. was observed to be due to some PHEV owners working in this environment and those coming to charge their batteries at public charging stations located within the microgrid. This MEM-DOA load profile offers the PHEV owners an average of 18% savings on energy expenditure.

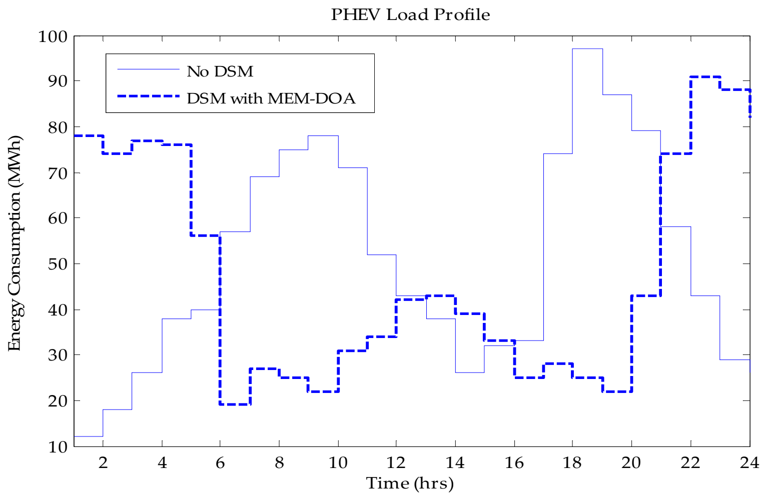

Figure 4. Plug-in Hybrid Electric Vehicle (PHEV) aggregate load profile.

Figures 2 and 3 show a reduction in peak demand from the utility grid by the consumers because the higher the demand met by local energy generation and storage (DEG and DES devices), the less the demand met by the utility or a third party (e.g., MES) in active consumers' premises, thereby, reducing a utility's investment in peaker plants (designed to meet peak demand) and making grid supply more reliable for those totally dependent on it such as Type-A consumers. The increased penetration of renewable resources (as DEG) and DES, as shown in this paper, would also lead to reduced carbon emissions and a safer environment for all, unlike most traditional generators using fossil fuels.

Additionally, box plots per 100 households are presented in Figure 5 for smart residential consumers Types A to D and aggregate energy consumption (including PHEV demand) respectively in the smart microgrid, showing the relationship in the consumption distribution between its average and peak values. The MEM-DOA plots are seen to be better that the initial unscheduled consumption.

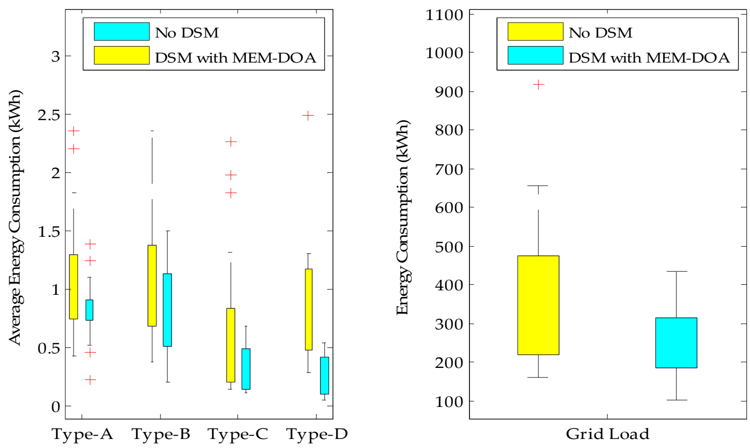

Figure 5. Comparison of consumer type consumption and aggregate grid load.

The aggregate grid load is presented in Figure 6 and it can be seen how the MEM-DOA had greatly reduced the peak demand from the grid resulting in a near-table load profile. It can also be noticed that the aggregate peak demand in the MEM-DOA reduced by 68% compared to the traditional peak demand with an aggregate PAR demand reduction by 46% from 2.9 to 1.56. Therefore, instead of the utility acquiring a peaker plant to meet approximately 1.1 MWh, it would only need to purchase one to meet 0.4 MWh demand, which would result in lower tariffs, lower carbon emissions and reduced investment cost in peaker plants. For instance, in South Africa, saving 0.916 kWh of energy reduces CO_2 emissions by 1 kg [29,30].

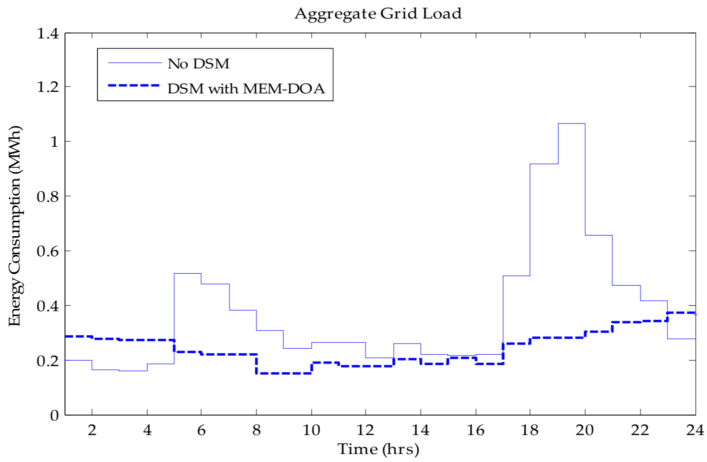

Figure 6. Aggregate grid load profile.

Financial savings and demand dissatisfaction experienced by the four categories of consumers considered are presented in Table 1. The negative dissatisfaction experienced by all the active consumers (Type-B, Type-C and Type-D) showed that integration of DES and DEG into consumer premises with centralised energy storage would offer satisfaction to consumers. The financial savings can also serve as a form of compensation for the initial investments incurred by the active consumers on DEG and DES devices. All the passive consumers (Type-A) would be slightly dissatisfied by an average of 0.121 kWh energy consumption daily, but can reduce their dissatisfaction by trading off financial savings. However, the financial savings observed by all consumers is enhanced by the presence of the centralised MES, DES and DEG devices in the smart microgrid. For instance, the dissatisfaction for Type-B consumers is less than for Energy Scheduling and Distributed Storage (ESDS) consumers in study [11], due to the inclusion of the MES device and arbitrage opportunities in this model, although both consumers possessed only iHES devices locally. Active consumers can, through financial savings, obtain a pay-back in the long-run on their investments on DES devices and DEG. Consumer dissatisfaction was not considered for the PHEVs; however, their charging/discharging profile can affect the residential consumers dissatisfaction in the amount of energy that would need to be purchased from the MES device, energy prices from grid and MES device, and price penalties. A wind turbine-powered Type-C consumer would have better satisfaction and greater profit than a solar-powered Type-C consumer since it has the potential for 24 h generation, because solar radiation is only available during the day and this consumer has no local storage device.

The MES selling prices without penalties ranges from R0.533–R1.056/kWh and R0.639–R1.268/kWh with penalties depending on energy profile within the microgrid. The utility and MES device providers also benefitted from the proposed DSM technique with MEM-DOA by 34% and 37% increases in revenue respectively. The higher increase in revenue by the MES provider could be due to consumers preferring at most times to buy from the MES device rather than the grid because of the price incentive received from contributed storage. In the competitive energy market that the smart grid could turn in to, more incentives are likely to be experienced, which could lead to lower tariffs from electricity utility providers.

Table 1. Average financial savings and dissatisfaction for the Microgrid Energy Management Distributed Optimisation Algorithm (MEM-DOA) smart residential consumers.

Consumer Type	Average Financial Savings	Average Dissatisfaction
Consumer Type-A	18%	0.121 kWh
Consumer Type-B	35%	−1.289 kWh
Consumer Type-C	32%	−0.874 kWh
Consumer Type-D	56%	−2.935 kWh

6. Conclusions

In this work, a DSM technique employing a price-incentivised energy trading in a grid-connected smart microgrid among smart consumers, with a centralised MES and utility grid, was presented. The smart consumers were either passive (no local DEG or DES) or active (with at least one of DEG and DES locally). Distributed optimisation algorithm was employed to enhance scalability, consumer privacy and security. A distributed algorithm called MEM-DOA was designed for each type of participating smart consumer; it would be installed on their smart meters, and EMC for utility and MES providers. The results of the simulations showed increased financial savings and reduced demand dissatisfaction for all participating smart consumers. It further offered a reduced PAR demand and peak demand when compared with the traditional aggregate residential load profile. This algorithm ensures that the active consumers benefit more from energy trading than passive consumers, so they could have faster returns on investment, which is an aspect that many in the literature have not considered. Also, this would increase the penetration of DES and DEG in grid-connected modes. This consequentially would encourage the DES and DEG manufacturing industry, as more consumers will become willing to be patrons. Commercial and industrial consumers can be included in future work.

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