Supplementary Materials

A Novel Vital-Sign Sensing Algorithm for Multiple Subjects Based on 24-GHz FMCW Doppler Radar

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Range estimation using the MUSIC algorithm

The MUSIC algorithm is the high-resolution technique in spectral analysis of signals. The MUSIC algorithm belongs to the subspace-based algorithm using the orthogonality of the signal and noise subspaces. This technique is mainly used for direction of arrival estimation and can be also used for frequency estimation through spectral peak searching.

The received signal is sampled by analogue-to-digital converter with a sampling frequency f_s. Then received signal is modeled as

$$\mathbf{S} = \mathbf{AX} + \mathbf{W}$$
$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi f_{b,1}T_s} & e^{-j2\pi f_{b,2}T_s} & \cdots & e^{-j2\pi f_{b,N}T_s} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi f_{b,1}(M-1)T_s} & e^{-j2\pi f_{b,2}(M-1)T_s} & \cdots & e^{-j2\pi f_{b,N}(M-1)T_s} \end{bmatrix} \begin{bmatrix} F_1 e^{-j2\pi f_{b,1}T_s} \\ F_2 e^{-j2\pi f_{b,2}T_s} \\ \vdots \\ F_N e^{-j2\pi f_{b,N}T_s} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_M \end{bmatrix} \tag{10}$$

where F_k is the complex reflection coefficient for the *k*-th target, W_k is a time sample of the additivie white Gaussian noise with a mean of zero and a variance of σ^2, $T_s = 1/f_s$ is a sampling interval, and M is the number of time samples, which is larger than the number of targets.

The covariance matrix of the received signal can be derived

$$\mathbf{R} = E\left\{\mathbf{SS}^*\right\} = \mathbf{APA}^* + \sigma^2\mathbf{I} \tag{11}$$

where $(\cdot)^*$ denotes the conjugate transpose operator, and **P** is the correlation matrix of the noise-free received signal, which is the diagonal matrix of rank *N* due to the orthogonality characteristic of the exponential function. The covariance matrix can be decomposed of its eigenvectors and eigenvalues using the eigen decomposition. Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ denotes the eigenvalues of the covariance matrix $\mathbf{R}$. The eigenvalue λ_k is given by

$$\begin{cases} \lambda_k > \sigma^2 & \text{for } k = 1, 2, \cdots, p \\ \lambda_k = \sigma^2 & \text{for } k = p+1, \cdots, M \end{cases} \tag{12}$$

It means that $\mathbf{U_S} = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p\}$ are the orthogonal eigenvectors associated with $\{\lambda_1, \lambda_2, \cdots, \lambda_p\}$ as a signal subspace and $\mathbf{U_N} = \{\mathbf{u}_{p+1}, \mathbf{u}_{p+2}, \cdots, \mathbf{u}_M\}$ are the orthogonal eigenvectors associated with $\{\lambda_{p+1}, \lambda_{p+2}, \cdots, \lambda_M\}$ as a noise subspace. Since the eigenvectors of the noise subspace is orthogonal to the eigenvectors of the signal subspace, the beat frequency of the target can be estimated by the pseudospectrum, which is given as

$$P(f) = \frac{1}{\mathbf{a}^*(f)\mathbf{U_N}\mathbf{U_N^*}\mathbf{a}(f)} \tag{13}$$

where $\mathbf{a}(f) = \begin{bmatrix} 1 & e^{-j2\pi f T_s} & \cdots & e^{-j2\pi f(M-1)T_s} \end{bmatrix}$. Then, peak values of $P(f)$ provide beat frequencies $f_{b,n}$ with a high resolution and accuracy.

Vital sign detection using AR method

Using rational transfer function $H(z) = 1/A(z)$, the system model can be expressed as

$$y(t) + \sum_{i=1}^{n} a_i y(t-i) = e(t) \tag{14}$$

where a_i is the AR coefficient. After multiplying (14) by $y^*(t-k)$, the equation can be solved by the Yule-Walker method in matrix form. The Yule-Walker equations can be written as

$$\begin{bmatrix} r(0) & r(-1) & \cdots & r(-n) \\ r(1) & r(0) & \cdots & r(-n+1) \\ \vdots & \vdots & \ddots & \vdots \\ r(n) & r(n-1) & \cdots & r(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{15}$$

Excluding the first row from (15), the AR coefficient can be derived by

$$\mathbf{\Phi} = -\mathbf{R}_n^{-1}\mathbf{r}_n \tag{16}$$

where

$$\mathbf{\Phi} = [a_1 \quad a_2 \quad \cdots \quad a_n], \quad r_n = \left[r(1) \quad r(2) \quad \cdots \quad r(n)\right],$$

$$R_n = \begin{bmatrix} r(0) & r(-1) & \cdots & r(-n+1) \\ r(1) & r(0) & & \vdots \\ \vdots & & \ddots & \\ r(n-1) & \cdots & & r(0) \end{bmatrix}. \tag{17}$$

The principle of the STAR method

The progress of the STAR method is identical with the short-time Fourier transform (STFT). Simply, the time-series signal is multiplied by a window function which has a determined period of time by user. Sequent application of AR method at an interval with a window size can provide the real-time spectrum signal which changes over time at an interval.

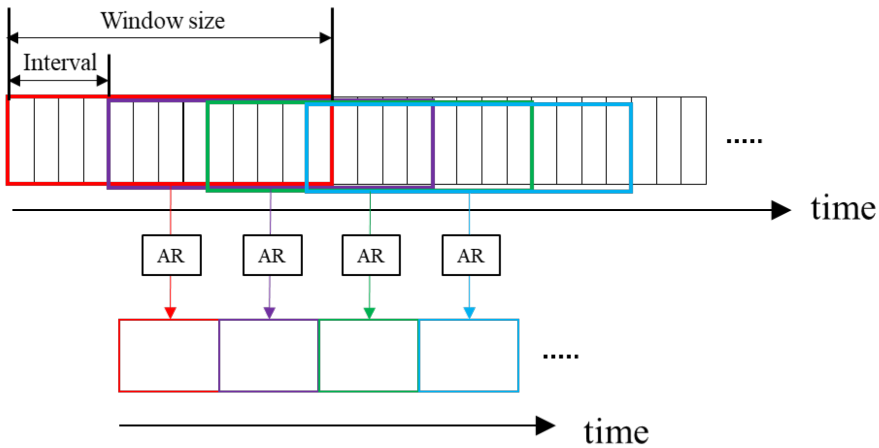

Figure S1. The progress of the STAR