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Analysis of the Characteristics and Ideas of Ancient Urban Land-Use Based on GIS and an Algorithm: A Case Study of Chang'an City in the Sui and Tang Dynasties

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Abstract: As ancient cities are spaces that represent the development of civilization, it is worth exploring and studying their characteristics and conceptions of land use. In this regard, the focus has turned to the issue of how to achieve the efficient mining of massive urban remote sensing data through human–computer collaboration. In this paper, a new intelligent method of analyzing urban land use characteristics and their cultural significance is proposed; it is feasible, effective, accurate, manageable, and portable. The method is based on a geographic information system (GIS) and a specific algorithm. The city plan was calibrated with the help of satellite remote sensing images and sites. By constructing the "urban element area acquisition and analysis model", various operations for areas in the city plan were realized, including an area value calculation, land use structure calculation, area modulus analysis, area ratio analysis between areas, and determination of the cultural significance of numbers and ratios. Taking the Sui and Tang dynasties capital city of Chang'an as an example, we found the existence of a set of urban planning techniques through area modulus (standard area units) for the first time; it took the market area as the modulus A and the area of Daxing Palace as the expanded modulus 2A, made the area of important areas in the city an integer multiplied by the modulus value (for example, the overall scope of the city is 100A, the rectangular urban area is 90A, and the small city area is 10A), and made the key values and numerical ratios have a cultural significance (such as 4.5, 5.5, 10, 25, 30, 100, 12:10, 1.618:1, 9:5, 45:1, 2:1), reflecting the planning and design concept of the capital city, into which the ancient Chinese deliberately integrated "number, shape and meaning". In addition, we carried out supplementary verification with the Roman city of Timgad and the Japanese city of Heijo-kyo, discovering that they also have design methods for area planning. We believe that land use planning can better meet the practical needs of urban resource distribution. Compared with urban form design, it might have chronological precedence. By setting the area modulus and the modulus value of each area, the grid-shaped city achieves the rational distribution of land and the establishment of order in an efficient way, and this thought and operation method greatly contributed to the advancement of ancient civilizations.

Keywords: intelligent land use analysis; urban heritage GIS; area algorithm; area modulus; Chang'an City of the Sui and Tang Dynasties

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1. Introduction

A city is the spatial representation of a particular human civilization. The function and distribution modes of land in the city relate to the quality and efficiency of the city's operations and also highlight the idea of urban planning. Ancient urban heritage is an important aspect of human cultural heritage, and exploring its planning concepts is of great significance. However, as few drawings or detailed texts related to the planning of ancient cities have been preserved, modern people can only study the land use characteristics of ancient cities based on sites or relics and then speculate on the urban planning ideas of the ancients. This creates research difficulties that must be solved using multidisciplinary knowledge.

The study of ancient urban land planning involves a variety of issues, of which two challenges stand out: the extraction and quantitative analysis of data on ancient urban land use, and the intelligent exploration and interpretation of planning ideas based on land use characteristics. These are discussed below.

1.1. The Extraction and Quantitative Analysis of Data on Ancient Urban Land Use

In terms of data extraction, quantity structure, spatial structure, and intensity are the main indicators used to measure urban land use. The quantity structure is fundamentally related to the allocation of related urban resources, and the spatial structure is intuitively correlated to the urban form; meanwhile, there are few research results on land use intensity due to the lack of ancient data. Scholars have mainly extracted relevant data by measuring satellite images (often in conjunction with GISs) [1,2], measuring archaeological sites in the field [3], and searching ancient literature and maps [4,5], and other scholars have worked with algorithms for image recognition to achieve the functional classification of land [6]. In terms of quantitative analysis, scholars have used statistical tables, GISs, and other research tools to conduct analysis. The statistical table is a more traditional tool, but its analytical power is limited and not suitable for the intuitive analysis of large amounts of data [7,8]. The GIS is a good tool that integrates data acquisition and analysis, specific plug-ins can be developed to achieve specific research functions, and there are also analyses that combine GISs with spatial syntax theory [9–17]. This theory abstracts the interrelation between spaces into a connection graph and then carries out the relevant numerical analysis [18,19]. The main variables used for calculation include connectivity, depth, control, integration, and choice. This can effectively transform perceptual visual analysis into rational mathematical analysis, which has certain advantages for the analysis of urban form.

The above tools can be used to analyze the composition and size (area) of each functional land space in the city, obtaining the percentage of each area in the whole city area through a calculation. According to these results, different attribute areas of the city can be compared [20,21].There are also analyses of broader regions (such as provinces) [22–24]. Furthermore, the same functional area can also be compared between different cities, considering questions such as the difference in the proportion of residential land in city A and city B [7]. Ancient and modern comparisons can also be made in order to examine the evolution of specific functional spaces over hundreds of years [8,25], allowing researchers to reach some valuable conclusions. However, due to the limited types of functional zones (generally no more than 10) in a specific city, only some preliminary understanding can be obtained merely by analyzing the share of each area, and it is difficult to carry out in-depth urban data analysis.

At present, the analysis of the internal relationships between various spatial areas in ancient cities is very scarce, and it is also difficult to answer key questions such as whether there is an area modulus (minimum area unit) in urban design. Therefore, in order to solve the problems related to ancient cities, researchers should undertake the necessary optimization according to the specific research needs of urban land use analysis, strengthening the secondary development of existing GISs and other tools to achieve specific research functions.

1.2. The Intelligent Exploration and Interpretation of Planning Ideas Based on Land Use Characteristics

Ancient planning ideas are embodied in land use characteristics, and it is necessary to find suitable methods to reveal the links between characteristic data and planning ideas, and to interpret the relevant ideas as a whole. The characteristics of land use mainly relate to the distribution of land use structures and the ratio between areas; multiple characteristics based on the specific area modulus may also exist, which all involve a number of specific numerical values and numerical ratios. Ancient people tended to select culturally significant numbers or numerical ratios and integrate them into the design to highlight the planning ideas. In most of the current studies, the data (especially the ratio data) are obtained by manual graphing analysis. The comparison and interpretation of data are also done manually; if these data are consistent with the knowledge already obtained by experts, it is concluded that they may represent a certain strain of cultural thought. For example, Andrea Palladio (1570) conducted a graphic analysis of the ratio of ancient cities and gave explanations of the cultural aspects of these ratios [26]. Kostof (1999) studied the land division mode of ancient urbans and analyzed the order reflected in their dimensions and proportions through a number of examples of cities with a grid pattern [27]; Wang Nan (2018) analyzed hundreds of ancient Chinese cities and architectures using the circle and square graphing method and posited that most of these cases fit the ratio of $\sqrt{2}$, 3:2 and $\sqrt{3}$:2 [28] (which he called "the harmonious ratio between heaven and earth"). Fu Xi'nian (1995) found that the length and width of many ancient capital cities in China were proportional to those of the Imperial Cities and explained this as the "emperor-centered" planning idea [29]. Wang Shusheng (2009) found evidence of the use of nine and five to symbolize imperial power (the Chinese called the emperor "Nine-Five Extreme") in Chinese ancient capitals [30]. Peixoto, R. F. (2021) investigated the urban grid of Thourioi and revealed its orthogonal planning technique and thoughts [31]. Park, E. and J. M. Shaikh. (2023) analyzed the spatial framework of the ancient Indian urban space and found that its significance derived from the Indus Script Pictorial Form [32].

Despite the many trends found by the above research, this method also has some limitations, which largely arise from the fact that there are many accidental factors involved in analysis based on expert experience, and there is a lack of intelligent and standardized analysis. For example, the objects in the analysis and the values and ratios of these objects may be deliberately screened rather than comprehensively investigated; when dozens of groups of ratios or values appear in the same city object, there is no clear countermeasure on how to choose between them. There is actually a limited number of important values and ratios in a specific culture, but their boundaries have not yet been defined, and a standard database for repeated use has not been established. In addition, there is no uniform threshold standard for screening the analysis results obtained in previous studies, which greatly affects the scientificity and reliability of the analysis. In fact, most of the values and ratios in a specific culture have been established by academics (in line with the cultural consensus and specific mathematical laws). The process of revealing cultural ideas is the process of comparing the values and ratios discovered in cases where the target values and ratios are established. If there are many consistent phenomena, this may be evidence of some kind of cultural idea. This process can be fully realized by computer intelligence; on this basis, a more in-depth analysis of planning ideas can be carried out. These are the key points to be addressed in this paper.

With regard to these problems, this study aimed to explore a universal intelligent analysis method for ancient urban land use characteristics and planning ideas, to clearly present the area laws of ancient cities, and to reveal their planning ideas and cultural connotations. The Chang'an City of the Sui and Tang Dynasties, an important capital city in ancient China, was selected as a typical case study for the present research. In addition, this study is aimed at providing a new basis for the scientific and efficient analysis of ancient urban land use structure and the deep mining of urban data, as well as providing a reference for solving problems in urban planning, architecture, history, and other areas by using spatial information technology and algorithms.

2. Background

2.1. Overview of Chang'an City of the Sui and Tang Dynasties

Chang'an City of the Sui and Tang Dynasties (hereinafter referred to as "Chang'an City") was located near Xi'an, Shaanxi Province, China, with geographic coordinates of 108.88°–108.99°E, 34.19°–34.31°N. The city (Figures 1 and 2) was founded in 582 AD by Yang Jian (杨坚), Emperor Wen of the Sui Dynasty; Gao Jiong (高颎), Yu Wenkai (宇文恺),

Liu Long (刘龙), and others were responsible for its planning and construction. After its founding, the city was the national capital of the Sui Dynasty. After the collapse of the Sui Dynasty, the Tang Dynasty continued to use it as the national capital. Chang'an City was one of the largest (over 80 square km) capitals in the world from the 6th to the 10th centuries, accommodating up to one million people. It is currently an important heritage site under state protection in China, and the Daming Palace site in the northeast of the city is a world cultural heritage site; therefore, the city has prominent historical and cultural value and research significance [33].

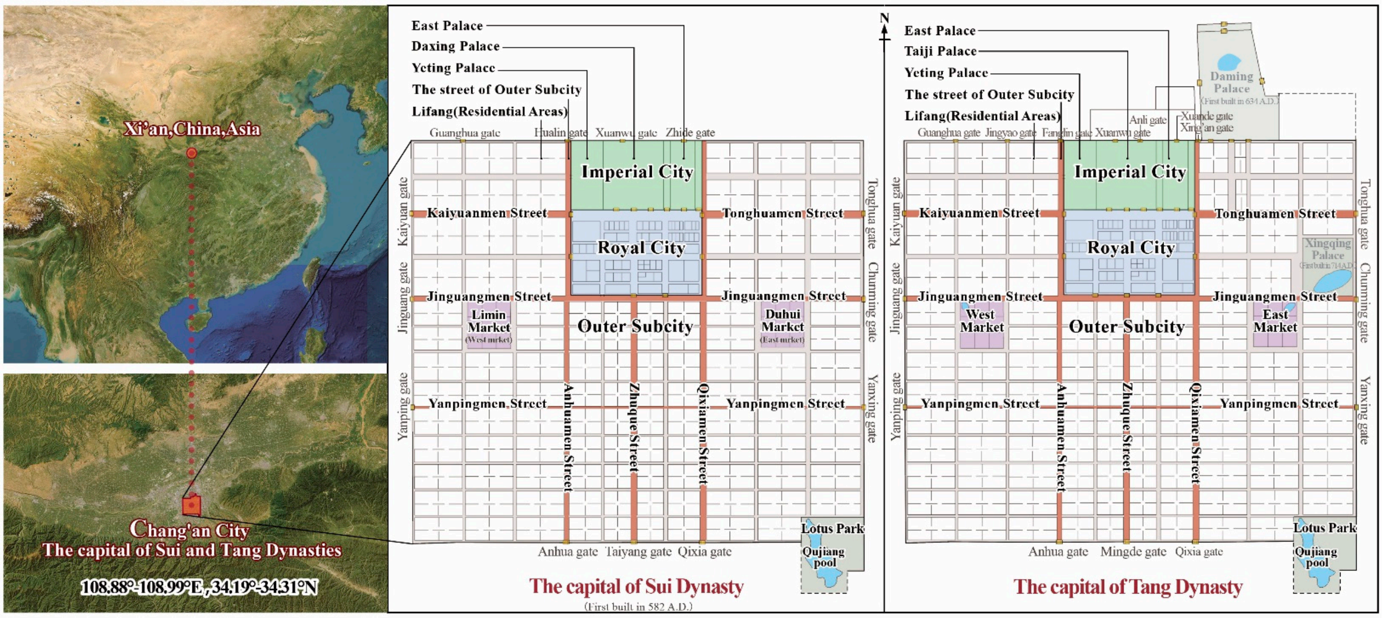


Figure 1. Location Map and ichnography of Chang'an City of the Sui and Tang Dynasties.

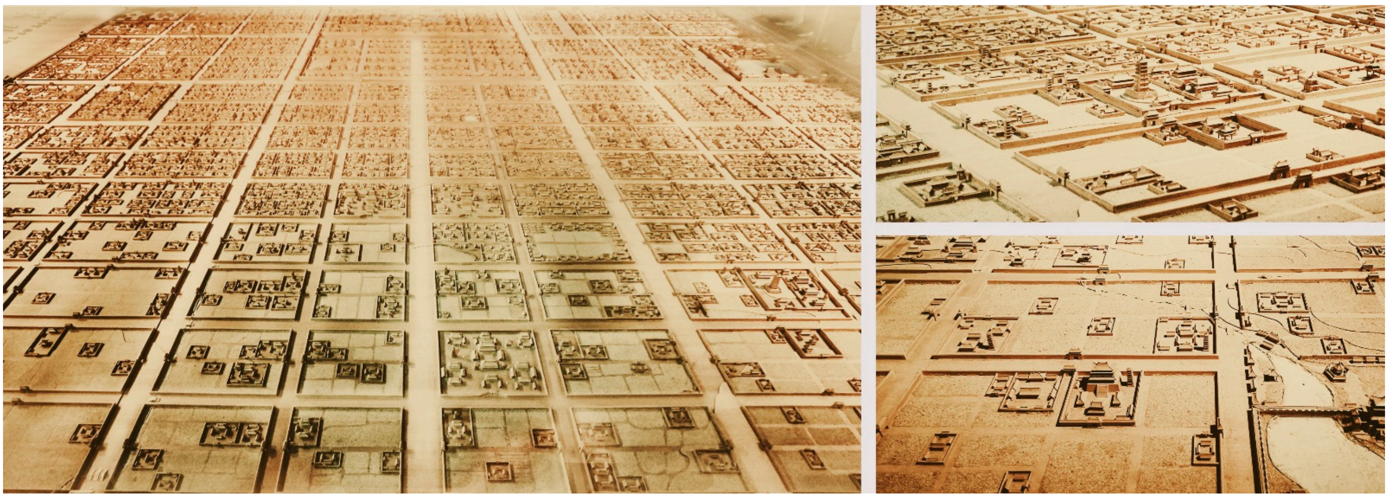


Figure 2. Restored model of Chang'an City of the Sui and Tang Dynasties in Xi'an Museum.

The shape of Chang'an City is basically a regular rectangle, with a transverse width of about 9.6 km and a longitudinal length of about 8.5 km and the ratio of transverse width to longitudinal length is close to 9:8. The Lotus Park in the southeast corner protrudes slightly to the south. If the longitudinal length of the city is calculated according to the south boundary of the Lotus Park, the transverse width is almost equal to the longitudinal length, and the overall urban scope is nearly a standard square. The main body of Chang'an City consists of three parts. In the north of the city is the Imperial City. In the center of the palace is Daxing Palace, which was the residence of the emperor and empress. On the east side is the East Palace, which was the residence of the prince and his subordinates. On the west side is Yeting Palace, which was the residence of the maids and servants; adjacent to it south is the Royal City, which was the office area of government departments. The Imperial City and the Royal City have their own walls, and outside them is a larger Outer Subcity, which is also surrounded by walls and is where the public lives. The East Market and West Market are also located in the Outer Subcity. There are gates on the four sides of the city, with three gates on the east, west, and south sides, and five gates on the north side (at the beginning). The streets in the city are north–south or east–west. The widest street is Zhuque Street, which is about 155 m wide, while other streets vary in width, ranging from 19 to 134 m [33].

Some scholars have found that if Chang'an City is regarded as a regular rectangle, the city is exactly 20 times the size of Daxing Palace [29] and 9 times the size of the Imperial City and the Royal City, and the size of the Imperial City and the Royal City is 5 times that of Daxing Palace [30]. These values indicate that Chang'an City was carefully planned and designed, and this planning deserves further study.

2.2. Research Objects and Current Problems

The main object of this study is Chang'an City of the Sui and Tang Dynasties. The study has two research contents: (1) to quantitatively analyze the land use characteristics of Chang'an City and to reveal its internal laws, and (2) to determine the corresponding cultural meaning of the numerical law of land use in Chang'an City and to explain the planning ideas that underpin the city. The research objective is to establish a feasible, efficient, and propagable analysis method and operation process for accurately revealing the data characteristics and design rules of urban land use, generating further new discoveries and contributing to ancient urban planning research.

There are three difficulties in this study:

1. How does one quantize, quickly extract, and efficiently calculate the massive quantities of land use information in urban objects?
2. How does one establish a clear comparison relation between the calculation results and cultural ideas in order to make intelligent judgments?
3. How does one abstract from specific problems a more universal algorithm that is suitable for multiple urban objects?

3. Research Strategies

3.1. Technical Flow Chart and Technical Defects

The key premise of this study is to digitally process the city and capture key elements based on the GIS, calculate the area value and relevant ratio of elements with specific algorithms, and build a preset database of numbers and numerical ratios based on cultural knowledge. The effective results of high coincidence with the preset values and cultural significance are screened out in the calculation results through intelligent algorithms. We aim to show the general laws and corresponding areas of these results and finally form a comprehensive explanation of the planning concepts using human–computer collaboration.

The research process (Figure 3) is described below:

- In the first stage, collect the relevant data and draw the city plan in AutoCAD.
- In the second stage, import the city plan into the QGIS platform and perform calibration and registration by matching known sites of the same name.
- In the third stage, mark the object attributes in QGIS, determine a number of urban functional areas and designated study areas according to the research needs, and mark all the boundary points of the objects involved in each area.
- In the fourth stage, construct the "urban element area acquisition and analysis model", including the five modules of area value calculation, urban land structure calculation, area modulus analysis, area ratio analysis between areas, and the cultural sig-

nificance determination of numbers and ratios. Then, set the main parameters (coincidence threshold, culturally significant numbers and numerical ratios, etc.) for the model, with full reference to historical literature, cultural concepts, and the academic consensus.

- In the fifth stage, embed the decision model into the platform to run and output visual schematic diagrams and data tables. Based on the results and characteristic rules obtained by the computer, the researchers use professional thinking for human–computer collaborative interpretation and finally form a complete conclusion (i.e., an academic view).

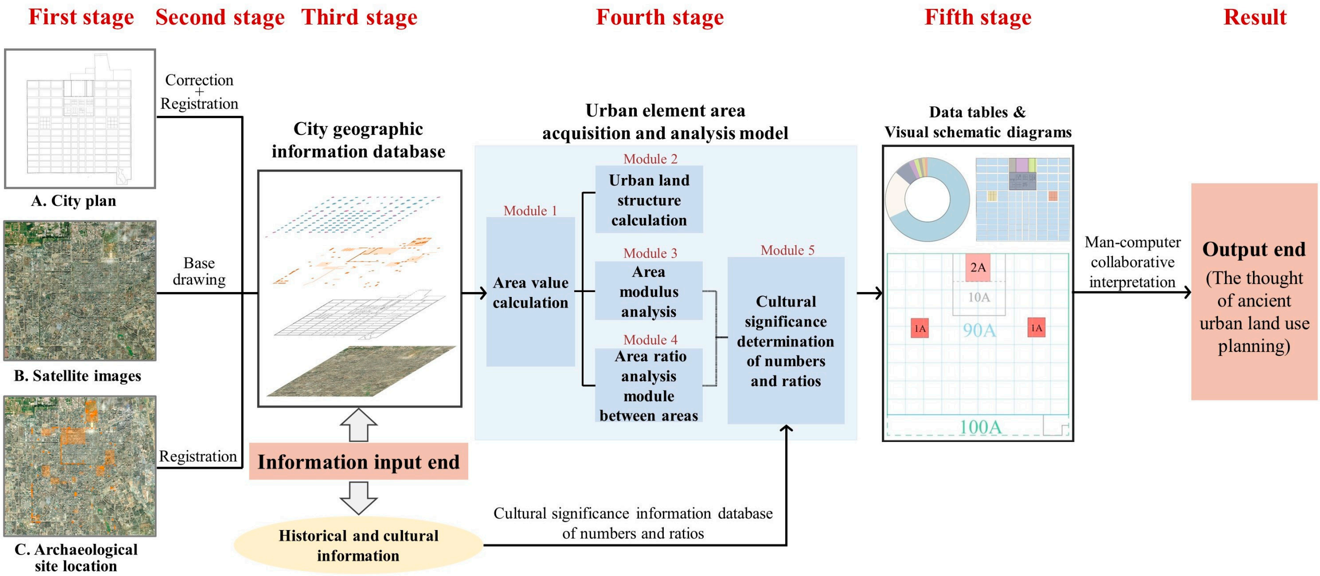


Figure 3. Technology roadmap for the research.

The technical defects of the research are as follows.

The preset key numbers or numerical ratios in the database are derived from historical documents and cultural ideas and need to be selected manually. There will be insignificant but inevitable differences in how each person understands and interprets the results. Having a deeper understanding of history and culture makes it easier to fully and accurately set the database and obtain accurate results. In addition, the method of dividing urban areas and the threshold of high coincidence are adjustable parameters. Improper presetting will produce too many or too few results to discover the truth. However, the dynamic change in the number of results can be observed by appropriately increasing the number of objects and the number of experiments, so as to make up for this deficiency.

3.2. Specific Implementation Processes

3.2.1. Collect the Relevant Data and Draw the City Plan

Open-source and high-resolution Esri satellite images (with a resolution of 0.3 m) were used as the source of basic geographic information data in this study, with the intention that other scholars can refer to this method. The main literature used was the plan of Chang'an City of the Sui and Tang Dynasties in the book *Compilation of Archaeological Data of Chang'an City Site of the Sui and Tang Dynasties* [33]. The paper version of the map was scanned as a digital map and then depicted as a vector map in Adobe Illustrator software (Adobe Systems, San Jose, CA, USA). The Mercator projection coordinate system was used in this study; this projection method is advantageous for the mapping of large-scale objects in mid-latitude areas.

Moreover, books and papers on ancient Chinese urban cultural thoughts [34–36], the meaning of numbers [37–42], and other cultural aspects were also collected for this study, the knowledge of which can help us to interpret urban planning ideas.

3.2.2. Import the City Plan into the GIS Platform for Correction and Registration

The vector map of the city plan was imported into QGIS-OSGeo4W software (an official project of the Open Source Geospatial Foundation), and the accurately located sites were used to correct and register the city plan. The more scattered and numerous the sites are, the more significant the registration effect will be. A total of 82 archaeological sites in the Sui and Tang Dynasties were marked in the study. Because the existing city plan is not very accurate, a "trial registration" of the city plan with the satellite image should first be conducted to identify the inconsistent local areas in the city plan; then, the city plan should be corrected according to the coordinate position of the sites to modify the lines in the plan that do not match up with reality. The two graphs should be registered again until they match perfectly. "Histogram stretching" was used in the registration process, and the transformation type was "linear". After this operation, geometric correction, projection transformation, and unified scale can be realized, so as to obtain the preliminary base map. The specific method can be found in the relevant research carried out previously by this author [43]. Based on the previous research, in this study, some of the size data of Chang'an City of the Sui and Tang Dynasties were slightly modified by investigating the thickness of walls at different locations of Chang'an City and analyzing the relative relationships between sites in detail (Figure 4).

In addition, for important areas where no site remains or where the sites are insufficient to provide overall size data, we made use of previously published archaeological excavation data, such as the size of the East Market and West Market (the East Market is 924 m $\times$ 1000 m, and the West Market is 927 m $\times$ 1031 m in terms of transverse width and longitudinal length [44]) and the east–west width of Yeting Palace (the size is 702.5 m [45]). In addition, some data were inferred by scholars through study. For example, the east–west width of Daxing Palace (that is, Taiji Palace of the Tang Dynasty) was estimated to be 1285 m by scholars according to the symmetry principle and the size of Yeting Palace [45]. However, by investigating the east–west dimension of the axis from the west wall of the Imperial City to the Chengtian Gate of the Imperial City, previous scholars believed that this size was in fact 1345 m. The actual size calibrated with reference to the sites was 1335 m, so the east–west width of Daxing Palace was adjusted to 1265 m (calculation basis: $2 \times (1335\text{ m} - 702.5\text{ m}) = 1265\text{ m}$). The east–west width of the East Palace (previously estimated to be 832.8 m [45]) was also adjusted to 824.5 m (calculation basis: $2792\text{ m} - 702.5\text{ m} - 1265\text{ m} = 824.5\text{ m}$).

3.2.3. Mark the Area Object Information

Using QGIS-OSGeo4W software, important area objects were demarcated based on the city plan. Generally, these objects can be divided into two types: urban functional areas and designated study areas. The two are fundamentally different. In the first type, the objects are mutually exclusive and non-overlapping, and the sum of their land use is the complete total urban land. The second type is the artificially demarcated graphic area, which can either be an urban area divided by streets or the sum of several independent urban functional areas. Using QGIS-OSGeo4W software, the polygon area with the attribute "Polygon" was created for these objects, and the key points were marked to facilitate the subsequent area calculation. When marking the key points, all turning points should be continuously marked clockwise from the first turning position in the upper left corner of the object to form a closed area. If an object contains multiple independent areas, the above operation can be performed for each independent area in turn. The above operation can be completed manually or through image recognition technology with manually assisted inspection.

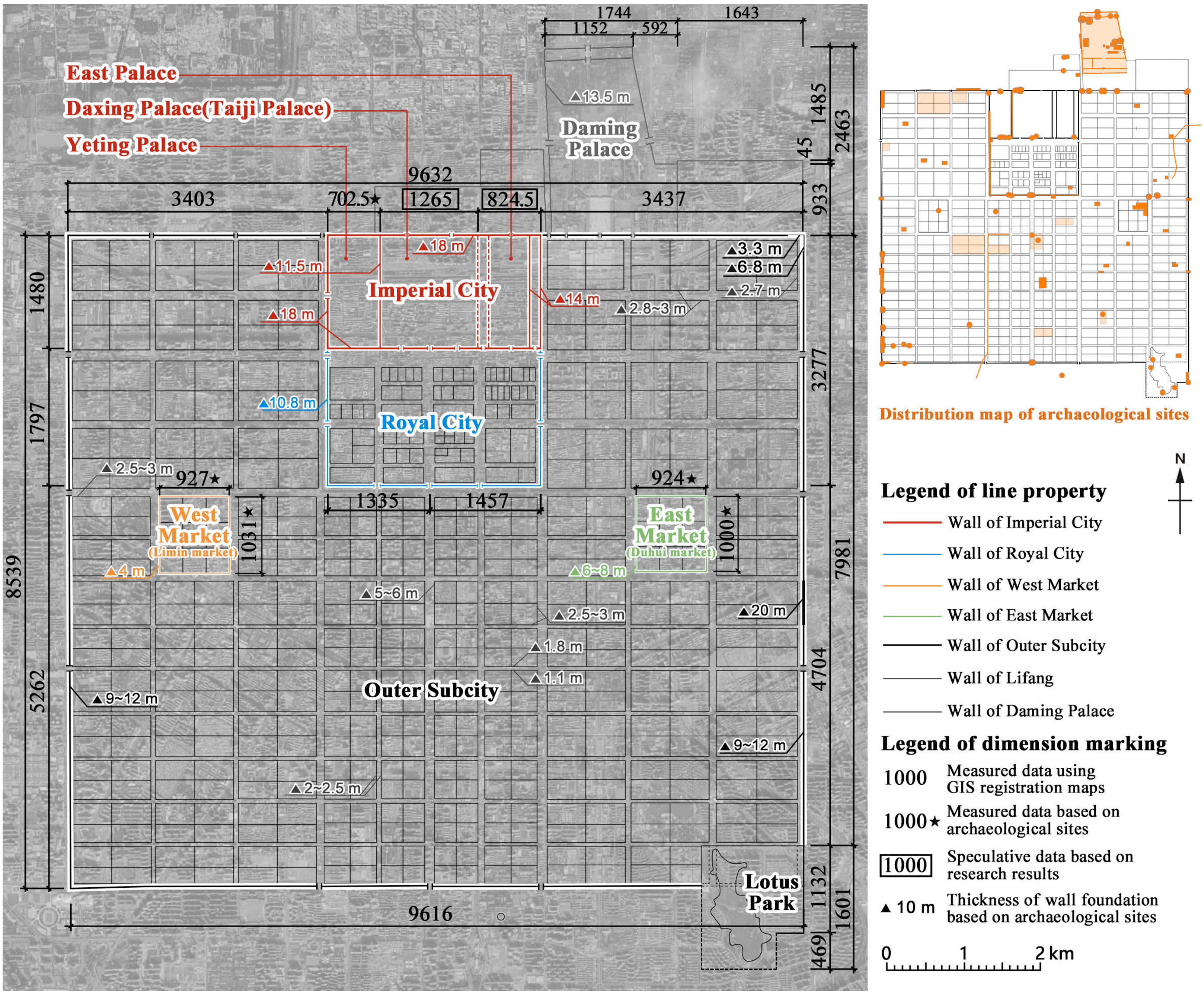


Figure 4. Plan of Chang'an City of the Sui and Tang Dynasties after correction and registration.

In this study, two groups of 11 objects (Table 1) of urban functional areas were created in QGIS, with the variable i as the number of the above objects. It should be pointed out that Daming Palace in the northeast of the capital and Xingqing Palace in the east of the capital were not built until the Tang Dynasty and did not reflect the original intent of the city planners, so the urban functional areas in this study were based on the pattern of the capital when it was initially founded (581 A.D.).

In addition, a total of 38 designated study area objects were created in QGIS (Table 2), with the variable j as the number of the above objects. When demarcating the boundaries of areas, each street can be divided according to the central line or the outer boundary of either side, so there were three situations. The central line of the street was used as the basis for demarcation in this study.

It should be noted that since Lotus Park in the southeast corner of Chang'an City is irregular, the city was regarded as a regular rectangle, and the missing residential areas were supplemented for subsequent calculation and analysis, in order to better determine the original intentions of the ancient planners and to simplify the analysis. The same operation process still works if it is necessary to study the status quo (i.e., the irregular shape).

Table 1. List of urban functional areas in Chang'an City.

Designated Study Area Number (i)	Group	Area Name
1	Group 1 of functional areas	Imperial City
2		Royal City
3		Outer Subcity
4	Group 2 of functional areas	Daxing Palace
5		East Palace
6		Yeting Palace
7		Royal City
8		West Market
9		East Market
10		Residential Area
11		Outer Subcity Street

Table 2. List of designated study area objects in Chang'an City.

Designated Study Area Number (j)	Area Name	Designated Study Area Number (j)	Area Name
1	The outermost enclosed square area of the city (from the southernmost to the south boundary of Lotus Park)	20	Urban area to the east of Qixiamen Street
2	Rectangular urban area (the sum of urban functional areas)	21	Area to the east of Anhuamen Street, the west of Qixiamen Street, and the south of Jinguangmen Street
3	Small city area (Imperial City and Royal City area)	22	Urban area to the north of Yanpingmen Street and the west of Zhuque Street
4	Two markets area	23	Urban area to the north of Yanpingmen Street and the east of Zhuque Street
5	Imperial City, Royal City, two markets area	24	Outer Subcity area to the north of Yanpingmen Street
6	East Palace and Yeting Palace area	25	Outer Subcity area to the north of Yanpingmen Street and the west of Zhuque Street
7	Urban area to the east of Zhuque Street	26	Outer Subcity area to the north of Yanpingmen Street and the east of Zhuque Street
8	Urban area to the west of Zhuque Street	27	Area to the north of Jinguangmen Street and the west of Zhuque Street
9	Outer Subcity area to the east of Zhuque Street	28	Area to the north of Jinguangmen Street and the east of Zhuque Street
10	Outer Subcity area to the west of Zhuque Street	29	Area to the south of Jinguangmen Street and the east of Zhuque Street
11	Urban area to the north of Jinguangmen Street	30	Area to the south of Jinguangmen Street and the west of Zhuque Street

Table 2. *Cont.*

Designated Study Area Number (j)	Area Name	Designated Study Area Number (j)	Area Name
12	Urban area to the south of Jinguangmen Street	31	Area to the south of Yanpingmen Street and the east of Zhuque Street
13	Urban area to the north of Yanpingmen Street	32	Area to the south of Yanpingmen Street and the west of Zhuque Street
14	Urban area to the south of Yanpingmen Street	33	Outer Subcity area to the north of Jinguangmen Street and the west of Zhuque Street
15	Urban area to the west of Anhuamen Street	34	Outer Subcity area to the north of Jinguangmen Street and the east of Zhuque Street
16	Urban area to the east of Anhuamen Street	35	Area to the south of Jinguangmen Street and the north of Yanpingmen Street
17	Outer Subcity area to the east of Anhuamen Street	36	Area to the north of Yanpingmen Street, the south of Jinguangmen Street, and the west of Zhuque Street
18	Outer Subcity area to the west of Qixiamen Street	37	Area to the north of Yanpingmen Street, the south of Jinguangmen Street, and the east of Zhuque Street
19	Urban area to the west of Qixiamen Street	38	Outer Subcity area to the north of Jinguangmen Street

After delimiting the above urban functional areas and designated study areas, the key points involved in these areas were marked. The matching relationship between the key points and their objects was noted to prevent inaccurate or missing marking.

3.2.4. Construct the "Urban Element Area Calculation Analysis and Cultural Significance Determination Model" and Set Parameters

The "urban element area calculation analysis and cultural significance determination model" (Figure 5) was constructed. This model consists of the five modules of area value calculation, urban land structure calculation, area modulus analysis, area ratio analysis between areas, and cultural significance determination of numbers and ratios. Each module involves the setting of several adjustable parameters, which can be debugged and optimized through human–computer collaboration. The important parameters are the preset numbers and numerical ratios for comparative analysis, as well as the coincidence threshold.

The operational rules of the "area value calculation" module used in this study are listed below.

Single objects were selected one by one from the urban functional area and the designated study area, the coordinates of all key points involved in the object were obtained, and its area value (in m^2) was calculated and stored using the ellipsoid area algorithm built in QGIS (the ellipsoid area of any closed figure can be calculated according to the point location coordinate information). The area of the urban functional area object was named A_i, and the area of the designated study area object was named A_j, where i and j are variables representing the object number, as described in Section 3.2.3.

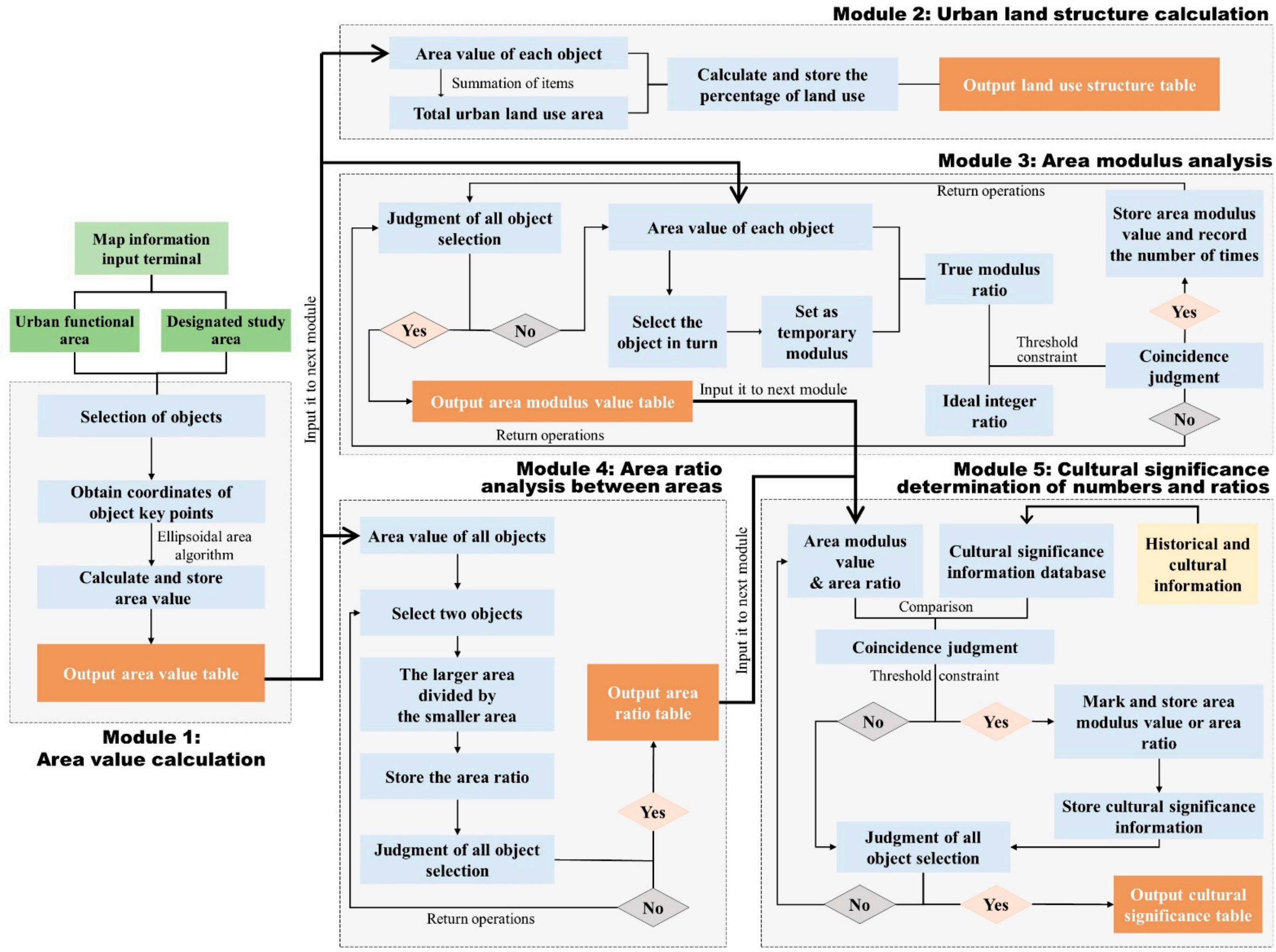


Figure 5. Frame diagram of the urban element area calculation analysis and cultural significance determination model.

The operation rules of the "urban land structure calculation" module are listed below.

Land use structure refers to the proportion obtained by dividing the individual land use area in the city by the total urban land use area. Each object was selected from the urban functional area in turn; the stored area value A_i was read and divided by the total urban land use area A_s to calculate the percentage of land use in the whole city for each functional area, which was then sorted and output as the land use structure table. Two groups of eleven objects are involved in this study, and the sum of the areas of all objects in each group is the total urban land area. The total number of operations of this module is 11.

The operation rules of the "area modulus analysis" module are listed below.

Each object (the object number is the variable I, which is a positive integer between 1 and 11) was selected from the urban functional area in turn. Its area value A_I was read, and it was set as the temporary area modulus m_I (that is, the artificially set calculation unit). The ratio $R_{I\text{–}i}$ of the area A_i to the area modulus m_I of each object in the urban functional area (the object number is the variable i) was calculated (the first part of the corner mark refers to the serial number of the selected area modulus object, and the last part refers to the serial number to calculate the object, and the same is true below). $R_{I\text{–}i}$ must be greater than 1, otherwise it is invalid. Then, the area A_j of each object in the designated study area (the object number is the variable j, which is a positive integer between 1 and 38) was selected, and the ratio $R_{I\text{–}j}$ of A_j to the area modulus m (using the same rule as above) was calculated. When the coincidence between the modulus ratio (R_{i-i} or R_{i-j}) and the nearest positive integer or the number with a mantissa of 0.5 (called half in ancient times) is better than the threshold TH, it means that the modulus ratio is an effective modulus ratio. The true value of the ratio and the adjacent integer value or half value can be stored, and the effective number N_i associated with A_i can be increased by 1.

The modulus ratio is calculated as follows:

$$R_{i\text{-}i} = A_i/m_i$$

$$R_{i\text{-}j} = A_j/m_i$$

The effective modulus ratio of integer multiples is determined as follows:

$$\text{MIN}(R_{i\text{-}i}, \text{ROUND}(R_{i\text{-}i}))/\text{MAX}(R_{i\text{-}i}, \text{ROUND}(R_{i\text{-}i})) \geq \text{TH}$$

$$\text{MIN}(R_{i\text{-}j}, \text{ROUND}(R_{i\text{-}j}))/\text{MAX}(R_{i\text{-}j}, \text{ROUND}(R_{i\text{-}j})) \geq \text{TH}$$

The effective modulus ratio with mantissa of 0.5 is determined as follows:

$$\text{MIN}(2 \times R_{i-i}, \text{ROUND}(2 \times R_{i-i}))/\text{MAX}(2 \times R_{i-i}, \text{ROUND}(2 \times R_{i-i})) \geq \text{TH}$$

$$\text{MIN}(2 \times R_{i-j}, \text{ROUND}(2 \times R_{i-j}))/\text{MAX}(2 \times R_{i-j}, \text{ROUND}(2 \times R_{i-j})) \geq \text{TH}$$

The above operation was repeated. Finally, the different area moduli A_i were displayed in descending order according to their corresponding effective number N_i. The total number of operations Na of this module is shown below:

$$\text{Na} = \text{i} \times (\text{i} + \text{j}) \times 8 = 11 \times (11 + 38) \times 8 = 4312$$

The operation rules of the "area ratio analysis between areas" module are listed below.

Two objects were selected from among all the objects in the urban functional area and the designated study area; then, the stored absolute area value (A_i or A_j) was read, and the smaller of the two values was divided by the larger one. The area ratio was set to R_c (where C is the variable representing the object number) and stored as a decimal. The above operation was repeated until all object combinations were covered. The total number Nc of the area ratio is the combination number related to the sum of i and j. There were 49 objects in the urban functional area and the designated study area, and the total number of operations Nc is as follows:

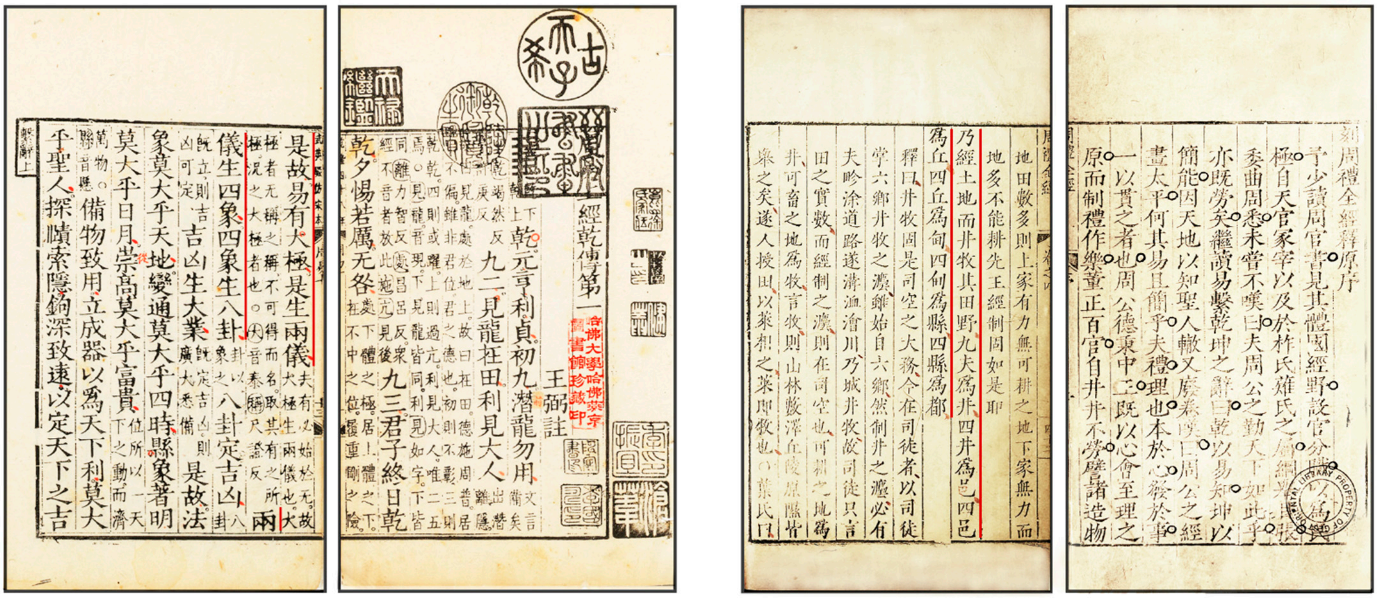
$$\text{Nc} = C_{i+j}^2 = C_{49}^2 = \frac{49!}{2!(49-2)!} = 1176$$

This means that 1176 area ratios (R_c) can be obtained from calculations.

The operation rules of the "cultural significance determination of numbers and ratios" module are listed below.

We constructed a "culturally significant information database", which contains the numbers and numerical ratios with specific cultural significance that the researchers identified in advance. The contents of this database can be dynamically adjusted according to the research needs, and cultural interpretations of specific numbers and values can be added or deleted.

The numbers associated with urban planning and architectural design in ancient China often had a very clear cultural significance. These numbers were mainly recorded in several classical literature from the Zhou dynasty to Han dynasty and were subsequently revered and widely used by successive generations of ancient Chinese. The most famous literature is *The Book of Changes* [39], which involves binary representations (Figure 6a), thus producing many meaningful numbers (such as 2, 4, 8, 16, 32, 64, etc.). It also deals with numbers with specific concepts (for instance, the number of heaven (天数) is 25, the number of earth (地数) is 30, the number of Great Expansion (大衍之数) is 50, the supreme number (至尊之数) is 45, etc.). The second important book is *The Rites of the Zhou* [40], in which the "the square-fields system (井田制)" is a system of dividing square land evenly using a grid. This system involves the expression of quanternary (Figure 6b) and decimalism, thus producing some meaningful numbers (4, 16, 64, 10, 100, etc.). It also discusses numbers related to city planning (for instance, the side of a city is 9 Li, the number of roads in the city is 9, and so on), and it also deals with the relationship between astronomy and numbers (for instance, the heavenly stems (天干) are 10, the earthly branches (地支) are 12, etc.). In addition, there are other books that deal with numbers and their significance, such as the *Chou Pei Suan Ching* [41], which contains many numbers and ratios related to geometry (such as the ratio of the diameter of a circle to the length of its inner square), and the *Huainanzi* [42], which mentions the relationship between music and numbers (for example, the number of Huangzhong (黄钟之数) is 81, etc.). In addition to the original literature, modern scholars have studied and interpreted the above numbers and certain hidden numbers and ratios in more depth [37,38]. Based on our reading and understanding of the relevant documents, 64 commonly used numbers or numerical ratios in ancient times were included in the "Culturally significant information database" in this study. See Table 3 for details.



a. *The Book of Changes*
Havard-Yenching Library collection

b. *The Rites of the Zhou*
Gest Oriental Library collection, Princeton University

Figure 6. *The Book of Changes* and *The Rites of the Zhou*, important books that record ancient Chinese numbers and ratios.

Table 3. Cultural significance information database of ancient Chinese numbers and ratios.

NO (n)	Commonly Used Numbers or Numerical Ratios in Ancient Times (Rn)	Cultural Significance	NO (n)	Commonly Used Numbers or Numerical Ratios in Ancient Times (R_n)	Cultural Significance
1	1 (1:1)	The number of Taiji (太极)	33	60 (60:1)	The number of heavenly stems and earthly branches (天干地支), double number of earth
2	2 (2:1)	The number of Yin (阴) and Yang (阳)	34	64 (64:1)	The number of trigrams (卦) in *The Book of Changes*, the eight–eight multiplier
3	3 (3:1)	The number of three forces (三才)	35	72 (72:1)	Multiplier of Shaoyin and Taiyang
4	4 (4:1)	The number of four images (四象), the two–two multiplier	36	80 (80:1)	The sum of double Chengshu

Table 3. *Cont.*

NO (*n*)	Commonly Used Numbers or Numerical Ratios in Ancient Times (Rn)	Cultural Significance	NO (*n*)	Commonly Used Numbers or Numerical Ratios in Ancient Times (R_n)	Cultural Significance
5	5 (5:1)	The number of five elements (五行)	37	81 (81:1)	The number of Huangzhong (黄钟), the ninety-nine multiplier
6	6 (6:1)	The number of Taiyin (太阴)	38	90 (90:1)	Double Nine-Five Extreme number
7	7 (7:1)	The number of Shaoyang (少阳)	39	100 (100:1)	The number of a hundred years, the ten–ten multiplier
8	8 (8:1)	The number of Shaoyin (少阴)	40	360 (360:1)	The number of circumference of The sum of the number of Qianzhice (乾之策, the number is 216) and Kunzhice (坤之策, the number is 144), the number of the universe (周天)
9	9 (9:1)	The number of Taiyang (太阳), the three–three multiplier	41	1.125 (9:8)	The ratio of Taiyang to Shaoyin
10	10 (10:1)	The number of Heavenly Stems (天干)	42	1.155 (2:$\sqrt{3}$)	The ratio of side length to height of regular triangle, close to the ratio of Shaoyin to Shaoyang
11	12 (12:1)	The number of Earthly Branches (地支)	43	1.167 (7:6)	The ratio of Shaoyang to Taiyin
12	14 (14:1)	Double number of Shaoyang	44	1.2 (12:10)	The ratio of Earthly Branches to Heavenly Stems
13	15 (15:1)	The sum of the number of Shengshu (生数, 1 + 2 + 3 + 4 + 5), the three–five multiplier	45	1.286 (9:7)	The ratio of Taiyang to Shaoyang
14	16 (16:1)	The number of Fuxi (伏羲) trigrams, the four–four multiplier	46	1.414 ($\sqrt{2}$:1)	The ratio of the number of round sky and square earth (ratio of the circle diameter to side length of the inscribed square)
15	18 (18:1)	Double number of Taiyang (太阳)	47	1.5 (3:2)	The ratio of Heaven and Earth, the ratio of Taiyang to Taiyin
16	20 (20:1)	The number of double tens	48	1.6 (16:10)	The number of Fuxi trigrams, the four–four multiplier
17	24 (24:1)	Double number of Earthly Branches (地支)	49	1.618 ($\sqrt{5}$/2–1/2)	Golden ratio
18	25 (25:1)	The number of heaven (天数) in *The Book of Changes* (the sum of the odd numbers of 1–10), the five–five multiplier	50	1.667 (5:3)	The ratio of the number of five elements to the number of three forces, approximate golden ratio
19	27 (27:1)	Triplex number of Taiyang (太阳)	51	1.8 (9:5)	The ratio of the supreme number
20	28 (28:1)	Number of Twenty-eight stars (二十八宿)	52	2.5 (25:10)	The number of heaven in *The Book of Changes*, the five–five multiplier
21	30 (30:1)	The number of earth (地数) in *The Book of Changes* (the sum of the even numbers of 1–10)	53	3.2 (32:10)	Half of the number of trigrams in *The Book of Changes*
22	32 (32:1)	Half of the number of trigrams in *The Book of Changes*	54	3.6 (36:10)	Taixuan number, the sixth–six multiplier
23	36 (36:1)	Taixuan (太玄) number, the six–six multiplier	55	4.2 (42:10)	Multiplier of Taiyin and Shaoyang
24	40 (40:1)	The sum of the numbers of Chengshu (成数, 6 + 7 + 8 + 9 + 10), the five–eight multiplier	56	4.5 (45:10)	Supreme number, the sum of the number of the venation map

Table 3. *Cont.*

NO (*n*)	Commonly Used Numbers or Numerical Ratios in Ancient Times (Rn)	Cultural Significance	NO (*n*)	Commonly Used Numbers or Numerical Ratios in Ancient Times (R_n)	Cultural Significance
25	42 (42:1)	Multiplier of Taiyin and Shaoyang	57	4.9 (49:10)	The used number of Great Expansion, the seven–seven multiplier
26	45 (45:1)	Supreme number (九五至尊之数), the sum of the number of the venation map (洛书)	58	5.4 (54:10)	Multiplier of Taiyin and Taiyang
27	48 (48:1)	Quadruple number of Earthly Branches (地支)	59	5.5 (55:10)	The number of heaven and earth, the sum of the number of the galaxy map
28	49 (49:1)	The used number of Great Expansion (大衍所用之数), the seven–seven multiplier	60	5.6 (56:10)	Multiplier of Shaoyin and Shaoyang
29	50 (50:1)	The number of Great Expansion (大衍之数), double number of heaven	61	6.4 (64:10)	The number of trigrams in *The Book of Changes*, the eight–eight multiplier
30	54 (54:1)	Multiplier of Taiyin and Taiyang	62	7.2 (72:10)	Multiplier of Shaoyin and Taiyang
31	55 (55:1)	The number of heaven and earth (天地之数), the sum of the number of the galaxy map (河图)	63	7.5 (75:10)	Half of the number of Taiyin and Taiyang
32	56 (56:1)	Multiplier of Shaoyin and Shaoyang	64	8.1 (81:10)	The number of Huangzhong, the ninety-nine multiplier

The area modulus value of each object calculated in the "area modulus analysis" module was R_{i-i} or R_{i-j}, and the area ratio of each object calculated in the "area ratio analysis between areas" module was R_c. In the determination stage, R_{i-i}, R_{i-j} and R_c were compared with each preset ratio value R_n (n is the serial number of numbers or numerical ratios entered in the database) in the "cultural significance information database", and the percentage of the smaller one to the larger one was calculated and compared with the threshold value. If the result was lower than the coincidence threshold (TH), this operation is ended and the next operation is entered; if it is better than the coincidence threshold (TH), the area modulus value or the area ratio of objects is determined to be of cultural significance. The area modulus value (R_{i-i} or R_{i-j}) or area ratio (R_c) was marked and output, and the corresponding target ratio R_n and its cultural significance information were output. The above operations were repeated until all numbers and ratios were compared.

The determination formula is shown below:

$$\mathrm{MIN}(R_{i\text{-}i}, R_n)/\mathrm{MAX}(R_{i\text{-}i}, R_n) \geq \mathrm{TH}$$

$$\mathrm{MIN}(R_{i\text{-}j}, R_n)/\mathrm{MAX}(R_{i\text{-}j}, R_n) \geq \mathrm{TH}$$

$$\mathrm{MIN}(R_c, R_n)/\mathrm{MAX}(R_c, R_n) \geq \mathrm{TH}$$

The upper limit Nu for the total number of operations is determined as follows:

$$Nu = \mathrm{n} \times (i \times i + i \times j + \mathrm{N_c}) = 64 \times (11 \times 11 + 11 \times 38 + 1176) = 109{,}760$$

In this study, the globally consistent consistency threshold (TH) was employed, and the values of TH were set to 0.995 (99.5%) and 0.985 (98.5%) to observe how the analysis results changed under different thresholds.

3.2.5. Embed the Decision Model into the Platform for Operation and Human–Computer Collaborative Interpretation

The above "urban element area calculation analysis and cultural significance determination model" was written as code using the Python v3.7 programming language (open-source software, made by Python Software Foundation, State of Delaware, America), and the code (see the attachment to this paper for details) was embedded into the platform through secondary development of the QGIS platform. Researchers only need to enter the geographic information and calculation parameters in QGIS according to the previous steps and then run the QGIS software to realize the automatic analysis, determination, and output of the results of the target city objects with one click. The total operation time of this study was about 7 s. The decision model can produce two types of results at the output end:

The first type consists of numerical results, including the absolute value of the area, the relative value of the area, and the area ratio of each element, as well as the highest potential modulus value, the corresponding area ratio of each area, and the corresponding numerical cultural significance. These results are output in the form of tables and pie charts.

The second type comprises graphic results based on the expression of urban form information, including the land area planning analysis charts (multiple), which are output in the form of a simplified dot-line schematic diagram. This diagram is only given for the convenience of presentation. To improve the aesthetic quality of the final result, Adobe Illustrator software can be used to carry out personalized secondary processing according to different needs, including adjusting the layout, adjusting color matching, adding annotations, adding different base maps, etc.

These two kinds of results can help researchers to better focus on the key information and important laws at work in the object and to expand their research ideas. Computers are not omnipotent, and human–computer collaboration is still necessary. Researchers also need to further think through and screen results according to the calculation results and characteristic laws and systematically explain the objects under exploration, so as to form an academic viewpoint. Therefore, computers act as pathfinders and data collation assistants for academic research, and researchers are the real leaders.

4. Results

The following sections detail the types of analysis results for Chang'an City of the Sui and Tang Dynasties that can be obtained by running the analysis system.

4.1. Urban Land Use Composition Table

This table was made by using the land area indicators of each part of Chang'an City of the Sui and Tang Dynasties and was presented in two groups. See Table 4 and Figure 7 for details. The results of Group 1 show that in the whole city, the Outer Subcity has the largest area, followed by the Imperial City and Royal City. The area of the Outer Subcity comprises nearly 8/9 (88.85%) of the total area, and the combined area of the Imperial City and Royal City constitutes about 1/9 (11.15%) of the total area. The results of Group 2 show that the area of Yeting Palace, the East Palace, the West Market, and the East Market are relatively close, each accounting for 1.1–1.5% of the total area of the city, and the area of Daxing Palace is about twice that of the East Market or West Market.

Table 4. Land use composition in the functional area of Chang'an City of the Sui and Tang Dynasties.

Group	**Name of the Element or Area**	**Measured Area Value**	**Proportion of the Rectangular Urban Area (%)**	**Name of the Element or Area**	**Measured Area Value**	**Proportion of the Irregular Urban Area (Including Lotus Park) (%)**
Group 1	Imperial City	4,132,160 m^2	5.04	Imperial City	4,132,160 m^2	4.96
	Royal City	5,017,224 m^2	6.11	Royal City	5,017,224 m^2	6.02
	Outer Subcity	72,934,520 m^2	88.85	Outer Subcity (including Lotus Park)	74,210,559 m^2	89.02
Group 2	Daxing Palace	1,872,200 m^2	2.28	Daxing Palace	1,872,200 m^2	2.25
	East Palace	1,220,260 m^2	1.49	East Palace	1,220,260 m^2	1.46
	Yeting Palace	1,039,700 m^2	1.27	Yeting Palace	1,039,700 m^2	1.25
	Royal City	5,017,224 m^2	6.11	Royal City	5,017,224 m^2	6.02
	West Market	955,737 m^2	1.16	West Market	955,737 m^2	1.15
	East Market	924,000 m^2	1.13	East Market	924,000 m^2	1.11
	Residential area (regardless of Lotus Park, the vacancy is complemented in a symmetrical position)	56,050,150 m^2	68.28	Residential area (considering the influence of the north of Lotus Park)	55,479,109 m^2	66.55
	Outer Subcity Street	15,004,633 m^2	18.28	Outer Subcity Street (considering the influence of the north of Lotus Park)	14,873,262 m^2	17.84
				Lotus Park area	1,978,451 m^2	2.37
The Overall Chang'an City of the Sui and Tang Dynasties	Regular rectangular city	82,083,904 m^2	100.00	Irregular city (including Lotus Park in the southeast corner of the city)	83,359,943 m^2	100.00
The above data were used in this study.				The above data were not used in this study and are for reference only.		

4.2. Analysis Table of the Modulus and Numerical Cultural Significance of the Urban Area

The area modulus of urban functional areas and the designated study areas was explored. The results show that there are area moduli for the overall design of Chang'an City of the Sui and Tang Dynasties (Table 5). The areas of East Market, Daxing Palace, Yeting Palace, small city area, Royal City, East Palace, West Market, and Imperial City can all be used as the area modulus.

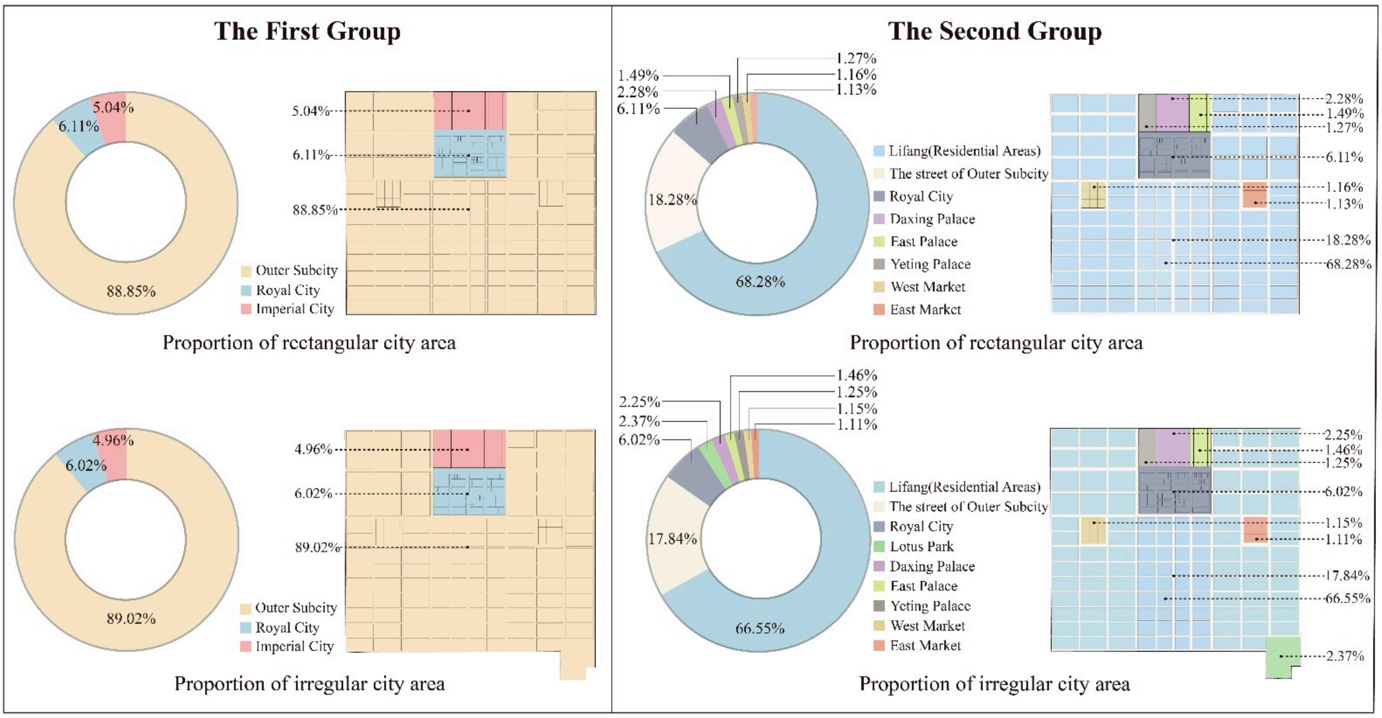


Figure 7. Land use composition in the functional area of Chang'an City of the Sui and Tang Dynasties.

Table 5. List of functional areas that may be used as the area modulus in Chang'an City of the Sui and Tang Dynasties.

Functional Area Name	The Number of Objects Whose Modulus Values Have Cultural Significance under the Threshold of 98.5%	The Number of Objects Whose Modulus Values Have Cultural Significance under the Threshold of 99.5%
East Market	27	8
Daxing Palace	14	9
(called Taiji Palace in the Tang Dynasty)	13	5
Yeting Palace	12	4
Small city area (Imperial City and Royal City area)	10	4
Royal City	10	5
East Palace	9	3
West Market	8	2

After further artificially assisted analysis of the above results, it was found that the East Market and Daxing Palace were most likely taken as the area modulus by ancient designers. This is because the market is the place most closely associated with citizen activities, and it is also the most important sub-area in the Outer Subcity; the imperial palace (Daxing Palace) is the place most closely connected with political activities and also the most important sub-area in the Imperial City. Since the area of the West Market is almost equal to that of the East Market, and both of them are objects with a market function, the area of the East Market, which has more objects, prevails. Under the threshold of 98.5%, when the area of the East Market is taken as the modulus (expressed by A), the area of 27 objects in the city can be expressed as an integer value or a half value in the unit of A (Table 6). When the area of Daxing Palace is taken as the modulus (expressed in A′), the area of 14 objects in the city can be expressed as an integer value or a half value in the unit of A′ (Table 7).

Table 6. Area value and cultural significance of areas in Chang'an City of the Sui and Tang Dynasties with the East Market as the area modulus.

Area Type	Name of the Element or Area	Area Modulus Value (Take the Measured Area of 924,000 m^2 of East Market as Modulus A)	Cultural Significance of the Value	Area Value Converted by the Modulus Value	Measured Area Value	Coincidence (%)
Urban functional area	East Market	1 A	The number of Taiji (太极)	924,000 m^2	924,000 m^2	100.00
	Imperial City	4.5 A	Supreme number (至尊之数), the sum of the number of the venation map (洛书)	4,158,000 m^2	4,132,160 m^2	99.38
	Royal City	5.5 A	The number of heaven and earth (天地之数), the sum of the number of the galaxy map (河图)	5,082,000 m^2	5,017,224 m^2	98.73
	Outer Subcity	80 A	The sum of double Chengshu (成数)	73,920,000 m^2	72,934,520 m^2	98.67
	Daxing Palace	2 A	The number of Yin (阴) and Yang (阳)	1,848,000 m^2	1,872,200 m^2	98.71
	Residential Area	60 A	The number of heavenly stems and earthly branches (天干地支)	55,440,000 m^2	56,050,150 m^2	98.91
	Outer Subcity Street	16 A	The number of Fuxi (伏羲) trigrams, the four–four multiplier	14,784,000 m^2	15,004,633 m^2	98.53
Designated study area	Rectangular urban area (the sum of urban functional areas)	90 A	Double Nine-Five extreme number	83,160,000 m^2	82,083,904 m^2	98.71
	The outermost enclosed square area of the city (from the southernmost to the south boundary of Lotus Park)	100 A	The number of a hundred years, the ten–ten multiplier	92,400,000 m^2	92,448,224 m^2	99.95
	Small city area (Imperial City and Royal City area)	10 A	The number of Heavenly Stems (天干)	9,240,000 m^2	9,149,384 m^2	99.02
	Imperial City, Royal City, two markets area	12 A	The number of Earthly Branches (地支)	11,088,000 m^2	11,029,121 m^2	99.47
	Area to the south of Yanpingmen Street and the east of Zhuque Street	15 A	The sum of the number of Shengshu (生数, 1 + 2 + 3 + 4 + 5), the three–five multiplier	13,860,000 m^2	13,994,917 m^2	99.04
	Area to the east of Anhuamen Street, the west of Qixiamen Street, and the south of Jinguangmen Street	16 A	The number of Fuxi (伏羲) trigrams, the four–four multiplier	14,784,000 m^2	14,979,351 m^2	98.70
	Area to the north of Jinguangmen Street and the east of Zhuque Street	18 A	Double number of Taiyang (太阳)	16,632,000 m^2	16,486,171 m^2	99.12
	Outer Subcity area to the north of Yanpingmen Street and the west of Zhuque Street	24 A	Double number of Earthly Branches (地支)	22,176,000 m^2	22,410,620 m^2	98.95

Table 6. *Cont.*

Area Type	Name of the Element or Area	Area Modulus Value (Take the Measured Area of 924,000 m^2 of East Market as Modulus A)	Cultural Significance of the Value	Area Value Converted by the Modulus Value	Measured Area Value	Coincidence (%)
	Area to the south of Jinguangmen Street and the north of Yanpingmen Street	24 A	Ditto	22,176,000 m^2	22,139,776 m^2	99.84
	Outer Subcity area to the north of Yanpingmen Street and the west of Zhuque Street	25 A	The number of heaven (天数) in *The Book of Changes* (the sum of the odd numbers of 1–10), the five–five multiplier	23,100,000 m^2	22,987,647 m^2	99.51
	Outer Subcity area to the north of Jinguangmen Street	25 A	Ditto	23,100,000 m^2	23,252,712 m^2	99.34
	Area to the south of Jinguangmen Street and the east of Zhuque Street	27 A	Triplex number of Taiyang (太阳)	24,948,000 m^2	25,272,886 m^2	98.71
	Urban area to the south of Yanpingmen Street	30 A	The number of earth (地数) in *The Book of Changes* (the sum of the even numbers of 1–10)	27,720,000 m^2	27,542,067 m^2	99.36
	Urban area to the west of Anhuamen Street	30 A	Ditto	27,720,000 m^2	28,081,209 m^2	98.71
	Urban area to the north of Yanpingmen Street and the east of Zhuque Street	30 A	Ditto	27,720,000 m^2	27,764,140 m^2	99.84
	Urban area to the east of Qixiamen Street	32 A	Half of the number of trigrams in *The Book of Changes*	29,568,000 m^2	29,325,777 m^2	99.18
	Outer Subcity area to the east of Zhuque Street	40 A	The sum of Chengshu (成数, 6 + 7 + 8 + 9 + 10), the five–eight multiplier	36,960,000 m^2	36,982,570 m^2	99.94
	Urban area to the east of Zhuque Street	45 A	Supreme number (九五至尊之数), the sum of the number of venation map (洛书)	41,580,000 m^2	41,759,055 m^2	99.57
	Outer Subcity area to the east of Anhuamen Street	49 A	The used number of Great Expansion (大衍所用之数), the seven–seven multiplier	45,276,000 m^2	44,858,711 m^2	99.08
	Outer Subcity area to the north of Yanpingmen Street	49 A	Ditto	45,276,000 m^2	45,392,480 m^2	99.74
	Urban area to the south of Jinguangmen Street	54 A	Multiplier of Taiyin (太阴) and Taiyang (太阳)	49,896,000 m^2	49,681,837 m^2	99.57

Table 7. Area value and cultural significance of areas in Chang'an City of the Sui and Tang Dynasties with Daxing Palace as the area modulus.

Area Type	Name of the Element or Area	Area Modulus Value (Taking the Area of 1,872,200 m^2 of Daxing Palace as Modulus A′)	Cultural Significance of the Value	Area Value Converted by the Modulus Value	Measured Area Value	Coincidence (%)
Urban functional area	Daxing Palace	1 A′	The number of Taiji (太极)	1,872,200 m^2	1,872,200 m^2	100.00
	Outer Subcity Street	8 A′	The number of Shaoyin (少阴)	14,977,600 m^2	15,004,633 m^2	99.82
	Residential Area	30 A′	The number of earth (地数) in *The Book of Changes* (the sum of the even numbers of 1–10)	56,166,000 m^2	56,050,150 m^2	99.79
Designated study area	Two markets area	1 A′	The number of Taiji (太极)	1,872,200 m^2	1,879,73 7m^2	99.60
	Area to the north of Yanpingmen Street, the south of Jinguangmen Street, and the east of Zhuque Street	6 A′	The number of Taiyin (太阴)	11,233,200 m^2	11,277,969 m^2	99.60
	Area to the south of Yanpingmen Street and the east of Zhuque Street	7.5 A′	Half of the number of Taiyin (太阴) and Taiyang (太阳)	14,041,500 m^2	13,994,917 m^2	99.67
	Area to the east of Anhuamen Street, the west of Qixiamen Street, and the south of Jinguangmen Street	8 A′	The number of Shaoyin (少阴)	14,977,600 m^2	14,979,351 m^2	99.99
	Outer Subcity area to the north of Yanpingmen Street and the west of Zhuque Street	12 A′	The number of Earthly Branches (地支)	22,466,400 m^2	22,410,620 m^2	99.75
	Area to the south of Jinguangmen Street and the north of Yanpingmen Street	12 A′	Ditto	22,466,400 m^2	22,139,776 m^2	98.55
	Urban area to the west of Anhuamen Street	15 A′	The sum of the number of Shengshu (1 + 2 + 3 + 4 + 5), the three–five multiplier	28,083,000 m^2	28,081,209 m^2	99.99
	Area to the south of Yanpingmen Street and the east of Zhuque Street	15 A′	Ditto	28,083,000 m^2	27,764,140 m^2	98.86
	Outer Subcity area to the east of Zhuque Street	20 A′	The number of double tens	37,444,000 m^2	36,982,570 m^2	98.77
	Outer Subcity area to the north of Yanpingmen Street	24 A′	Double number of Earthly Branches (地支)	44,932,800 m^2	45,392,480 m^2	98.99
	Outer Subcity area to the east of Anhuamen Street	24 A′	Ditto	44,932,800 m^2	44,858,711 m^2	99.84
	The outermost enclosed square area of the city (from the southernmost to the south boundary of Lotus Park)	49 A′	The used number of the Great Expansion (大衍所用之数), the seven–seven multiplier	91,737,800 m^2	92,448,224 m^2	99.23

It is worth noting that previous researchers believed that the ratio of the small city area to the Daxing Palace area was 5:1; according to our analysis, however, the coincidence was only 97.74%, lower than the preset threshold of 98.5%, so the conclusion was not reliable, and the proportion relationship was not shown in Table 7. However, the 1265 m transverse width of Daxing Palace was obtained based on our speculation, and the actual value may fluctuate by tens of meters due to construction errors or other factors. If the transverse width of Daxing Palace is slightly reduced and adjusted to 1255 m, the coincidence of the small city area: Daxing Palace area = 5:1 becomes 98.52%. When the area of Daxing Palace with reduced transverse width is taken as the modulus (expressed in A′′), the area of 19 objects in the city can be expressed as an integer value or a half value in the unit of A′′ (Table 8).

4.3. Analysis Table of the Area Ratio and Cultural Significance between Urban Areas

The area ratio between urban areas was analyzed. The results show that there are many groups of area objects with specific area ratios in Chang'an City (Table 9), among which the ratio of Earthly Branches to Heavenly Stems (12:10), the golden ratio (1.618:1), the ratio of the supreme number (9:5), the ratio of the number of Yin and Yang (2:1), the ratio of Taiyang to Shaoyang (9:7), the used number of Great Expansion (49:10), the ratio of Heaven and Earth (3:2), and the ratio of Taiyang to Shaoyang (9:8) are relatively common.

In addition, there are a large number of area ratios with cultural significance between areas in Chang'an City. The number of object groups can reach 475 under the threshold of 98.5% and 170 under the threshold of 99.5%. Based on expert experience, object groups involving important areas (such as the rectangular urban area, the small city area, the Imperial City, and the Royal City) were manually selected; see Table 10 for the area ratio and cultural significance of these object groups. In addition, as the area modulus, the Eastern Market and Daxing Palace have an integral multiple of area ratio to a large number of objects, which is listed in Tables 6 and 7 but is not elaborated herein.

4.4. Validation Results of Other City Cases

In order to further validate the feasibility of the proposed methodology and the validity of the results, as well as to examine a wider range of urban cases in different parts of the world, Timgad in ancient Rome and Heijo-kyo in Japan were selected as additional studies.

Timgad (geographical coordinates 6.46°–6.48°E, 35.48°–35.49°N), located in the Batna Province of Algeria, was built in 100 A.D. as an important Roman military stronghold in southern Africa, following the classic castra model of the Roman period, with an urban form based on a grid [16], which was similar to Chang'an City in the Sui and Tang dynasties. The early shape of the city was a regular rectangle, and there are differing opinions on its dimensions, but this study uses the drawings in Salat Sergea's book [46] as a reference, which were corrected and aligned using QGIS-OSGeo4W software to obtain a more accurate drawing (Figure 10). There are over a hundred blocks in the city that are nearly square in shape, each with a side length of 21.5 to 23 m. In the modal analysis of this study, the block size was set to 22.5 m for calculation, which roughly translates to 15 Roman passus (1 Roman passus is 1.48 m [47]).

4.5. Urban Land Area Planning Analysis Chart

The analysis results are shown in Figures 8 and 9.

Table 8. Area value and cultural significance of areas in Chang'an City of the Sui and Tang Dynasties with Daxing Palace (with corrected size) as the area modulus.

Area Type	Name of the Element or Area	Area Modulus Value (Taking the Area of 1,857,400 m^2 of Daxing Palace as Modulus A″)	Cultural Significance of the Value	Area Value Converted by the Modulus Value	Measured Area Value	Coincidence (%)
Urban functional area	Daxing Palace	1 A″	The number of Taiji (太极)	1,857,400 m^2	1,857,400 m^2	100.00
	Outer Subcity Street	8 A″	The number of Shaoyin (少阴)	14,859,200 m^2	15,004,633 m^2	99.03
	Residential area	30 A″	The number of earth (地数) in *The Book of Changes* (the sum of the even numbers of 1–10)	55,722,001 m^2	56,050,150 m^2	99.41
	Two markets area	1 A″	The number of Taiji (太极)	1,857,400 m^2	1,879,737 m^2	98.81
Designated study area	Small city area (Imperial City and Royal City area)	5 A″	The number of five elements (五行)	9,287,000 m^2	9,149,384 m^2	98.52
	Imperial City, Royal City, Two markets area	6 A″	The number of Taiyin (太阴)	11,144,400 m^2	11,029,121 m^2	98.97
	Area to the north of Yanpingmen Street, the south of Jinguangmen Street, and the east of Zhuque Street	6 A″	Ditto	11,144,400 m^2	11,277,969 m^2	98.82
	Area to the south of Yanpingmen Street and the east of Zhuque Street	7.5 A″	Half of the number of Taiyin (太阴) and Taiyang (太阳)	13,930,500 m^2	13,994,917 m^2	99.54
	Area to the east of Anhuamen Street, the west of Qixiamen Street, and the south of Jinguangmen Street	8 A″	The number of Shaoyin (少阴)	14,859,200 m^2	14,979,351 m^2	99.20
	Area to the north of Jinguangmen Street and the east of Zhuque Street	9 A″	The number of Taiyang (太阳), the three-three multiplier	16,716,600 m^2	16,486,171 m^2	98.62
	Outer Subcity area to the north of Yanpingmen Street and the west of Zhuque Street	12 A″	The number of Earthly Branches (地支)	22,288,800 m^2	22,410,620 m^2	99.46
	Area to the south of Jinguangmen Street and the north of Yanpingmen Street	12 A″	Ditto	22,288,800 m^2	22,139,776 m^2	99.33

Table 8. *Cont.*

Area Type	Name of the Element or Area	Area Modulus Value (Taking the Area of 1,857,400 m^2 of Daxing Palace as Modulus A″)	Cultural Significance of the Value	Area Value Converted by the Modulus Value	Measured Area Value	Coincidence (%)
	Urban area to the south of Yanpingmen Street	15 A″	The sum of the number of Shengshu (生数,1 + 2 + 3 + 4 + 5), the three–five multiplier	27,861,001 m^2	27,542,067 m^2	98.86
	Urban area to the west of Anhuamen Street	15 A″	Ditto	27,861,001 m^2	28,081,209 m^2	99.22
	Urban area to the north of Yanpingmen Street and the east of Zhuque Street	15 A″	Ditto	27,861,001 m^2	27,764,140 m^2	99.65
	Urban area to the east of Qixiamen Street	16 A″	The number of Fuxi (伏羲) trigrams, the four–four multiplier	29,718,401 m^2	29,325,777 m^2	98.68
	Outer Subcity area to the east of Zhuque Street	20 A″	The number of double tens	37,148,001 m^2	36,982,570 m^2	99.55
	Outer Subcity area to the east of Anhuamen Street	24 A″	Double number of Earthly Branches (地支)	44,577,601 m^2	44,858,711 m^2	99.37
	Urban area to the south of Jinguangmen Street	27 A″	Triplex number of Taiyang (太阳)	50,149,801 m^2	49,681,837 m^2	99.07
	The outermost enclosed square area of the city (from the southernmost to the south boundary of Lotus Park)	50 A″	The number of the Great Expansion (大衍之数), double number of heaven	92,870,002 m^2	92,448,224 m^2	99.55

Table 9. Area ratio between important areas in Chang'an City of the Sui and Tang Dynasties and analyses of their cultural significance.

Ratio Value	Cultural Significance of the Ratio	The Number of Objects (Groups) That Meet a Specific Ratio under the Threshold Condition of 98.5%	The Number of Objects (Groups) That Meet a Specific Ratio under the Threshold Condition of 99.5%
1.2 (12:10)	The ratio of Earthly Branches (天干) to Heavenly Stems (地支)	26	10
1.618 ($\sqrt{5}/2$–1/2)	Golden ratio	21	5
1.8 (9:5)	The ratio of the supreme number (至尊之比)	21	12
2 (2:1)	The number of Yin (阴) and Yang (阳)	19	5
1.286 (9:7)	The ratio of Taiyang (太阳) to Shaoyang (少阳)	19	7
4.9 (49:10)	The used number of Great Expansion (大衍所用之数), the seven–seven multiplier	19	6
1.5 (3:2)	The ratio of Heaven to Earth, the ratio of Taiyang (太阳) to Taiyin (太阴)	18	7
1.125 (9:8)	The ratio of Taiyang (太阳) to Shaoyin (少阴)	17	9
1.414 ($\sqrt{2}$:1)	The ratio of the number of round sky and square earth (the ratio of circle diameter to side length of the inscribed square)	12	5
5.5 (55:10)	The number of heaven and earth, the sum of the number of the galaxy map (河图)	9	3

Table 10. Area ratio between important areas in Chang'an City of the Sui and Tang Dynasties and analysis of its cultural significance.

Comparison Objects	Comparison Objects	Measured Area Ratio (The Ratio of the Former to the Latter)	Ideal Area Ratio	Coincidence (%)	Cultural Significance of the Ratio
The outermost enclosed square area of the city	Small city area (Imperial City and Royal City area)	10.10:1	10:1	98.97	The number of Heavenly Stems (天干)
Rectangular urban area	Small city area (Imperial City and Royal City area)	8.97:1	9:1	99.68	The number of Taiyang (太阳), the three–three multiplier
Outer Subcity	Small city area (Imperial City and Royal City area)	7.97:1	8:1	99.64	The number of Shaoyin (少阴)
Urban area to the north of Yanpingmen Street	Small city area (Imperial City and Royal City area)	5.96:1	6:1	99.35	The number of Taiyin (太阴)
Outer Subcity area to the north of Yanpingmen Street	Small city area (Imperial City and Royal City area)	4.96:1	5:1	99.23	The number of five elements (五行)
Urban area to the west of Zhuque Street	Small city area (Imperial City and Royal City area)	4.04:1	4:1	98.96	The number of four images (四象), the two–two multiplier
Urban area to the south of Yanpingmen Street	Small city area (Imperial City and Royal City area)	3.01:1	3:1	99.66	The number of three forces (三才)
Urban area to the south of Jinguangmen Street	Royal City	9.90:1	10:1	99.02	The number of Heavenly Stems (天干)
Outer Subcity area to the north of Yanpingmen Street	Royal City	9.05:1	9:1	99.48	The number of Taiyang (太阳), the three–three multiplier
Outer Subcity Street	Royal City	2.99:1	3:1	99.69	The number of three forces (三才)
Rectangular urban area	Imperial City	19.86:1	20:1	99.32	The number of double tens
Urban area to the south of Jinguangmen Street	Imperial City	12.02:1	12:1	99.81	The number of Earthly Branches (地支)
Urban area to the east of Zhuque Street	Imperial City	10.11:1	10:1	98.95	The number of Heavenly Stems (天干)

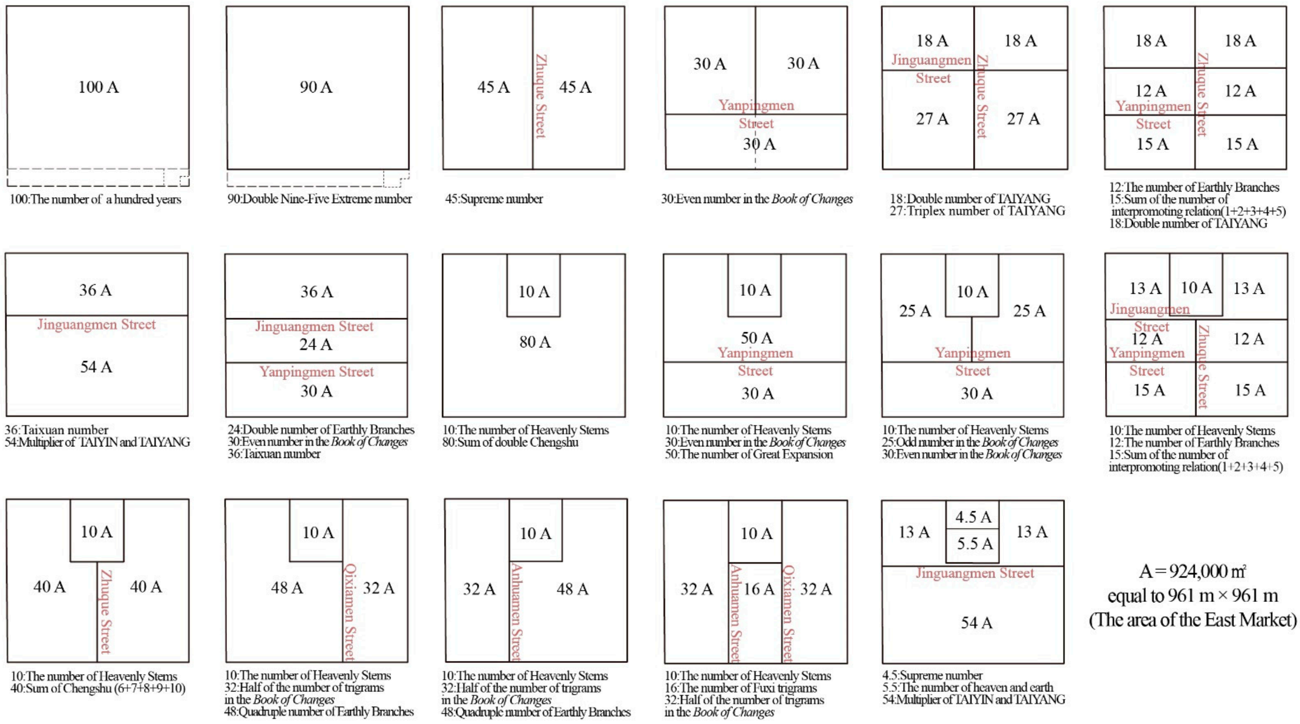


Figure 8. Land use area planning analysis of Chang'an City with the East Market as the area modulus.

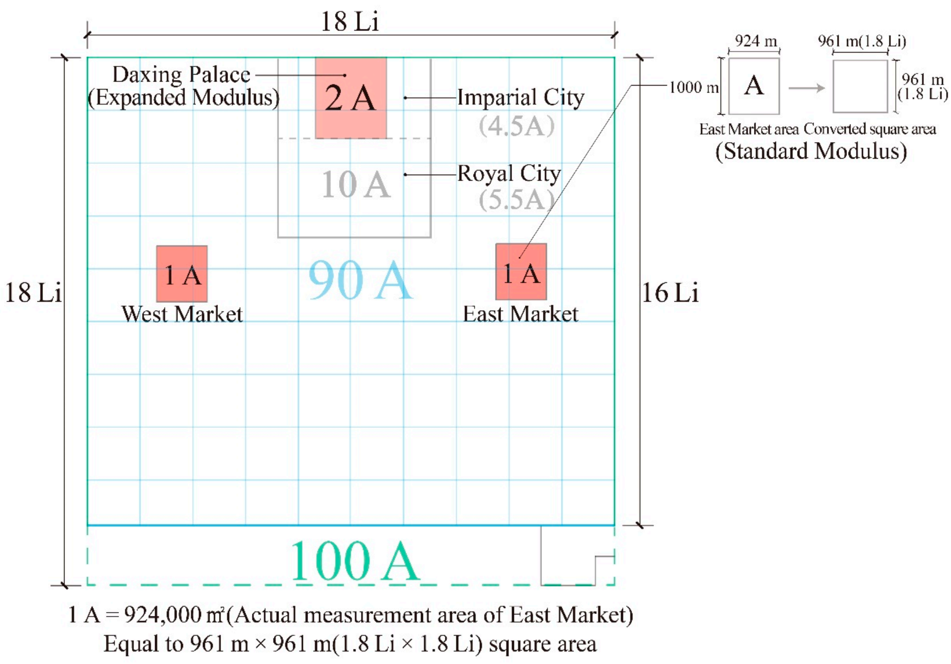
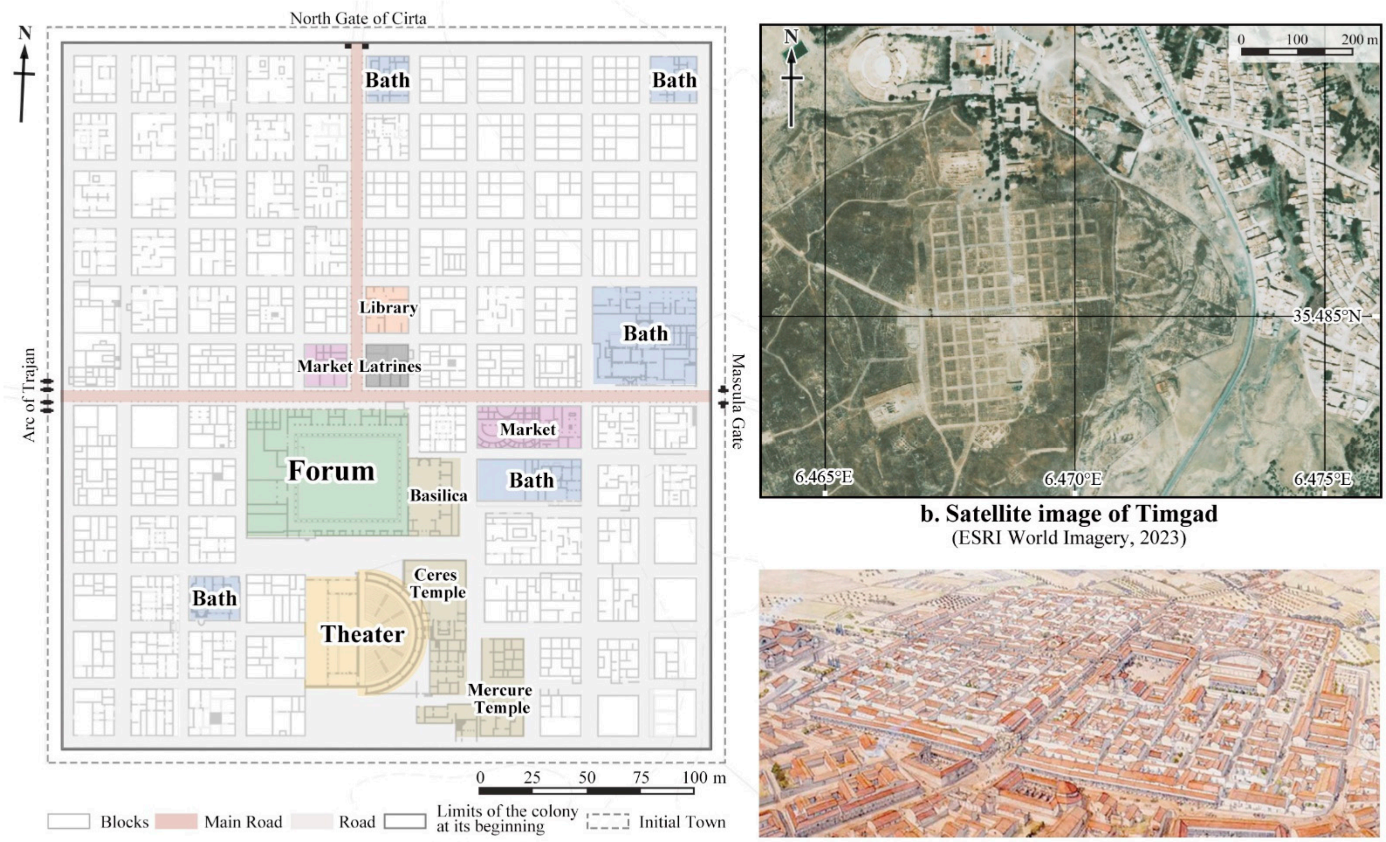


Figure 9. Schematic diagram of the land use planning concepts of Chang'an City based on the area modulus.

Heijo-kyo (geographical coordinates 135.77°–135.85°E, 34.65°–34.70°N), located in Nara, Nara Prefecture, Japan, was built in 708 A.D. as the capital of Japan in imitation of Chang'an city in the Sui and Tang dynasties and bears strong similarities to it in terms of form, layout, elemental composition, and land distribution logic [48]. The Heijo-kyo site is currently heavily damaged, and this study used early archaeological data drawings from the Nara Cultural and Financial Research Institute [49] as a reference, which were corrected and aligned using QGIS-OSGeo4W software to obtain more accurate drawings

(Figure 11), with the city's Heijo Palace, Left City, and Right City forming a larger rectangle; the length is about 4.26 km from east to west and 4.79 km from north to south. The Outer City on its east side is another slightly smaller rectangular block, about 1.60 km from east to west and 2.13 km from north to south. In recent years, archaeologists have discovered a new urban area at the southernmost end of the city [50], extending the length of the city north–south to 5.32 km. However, as the ruins of the city gate are preserved on the original southern boundary of the city, we presume that the location of the gate is the actual city boundary and that the newly discovered area is a suburb of the city. There are dozens of Fang (坊), each with 16 Ting (町) in a near-square shape, within the areas of Left City, Right City, and Outer City. There is a negligible difference in the length of the sides of each Fang, so for the modal analysis in this study, the side length of each Fang was taken as 510 m for calculation purposes. In addition, if the roads around the Fang were included, the length of each Fang was calculated as 532 m, which corresponds to 1500 Dachi (大尺) in ancient Japan (the length of 1 Dachi is 0.354 m [51]).



a. Ichnography of Timgad
(*Integrated Guidelines for Sustainable Neighbourhood Design*, 2021; p.15)

b. Satellite image of Timgad
(ESRI World Imagery, 2023)

c. Picture of restoration of Timgad
(*L'Antiquité Retrouvée*, 2019; p.106)

Figure 10. Ichnography (underlying graph comes from [46]), satellite image, and picture of restoration of Timgad.

The area of Chang'an City in the Sui and Tang Dynasties, Timgad, and Heijo-kyo were approximately 83 km^2, 24 km^2, and 0.1 km^2, respectively, and these three cities represent different scales of cities (large, medium, and small), which has positive significance for understanding urban land use patterns at different scale levels.

Through the method proposed in this paper, we computed and analyzed the two cities mentioned above and still obtained valid results (Figures 12–15). The results reflected some noteworthy characteristic rules that could support subsequent research.

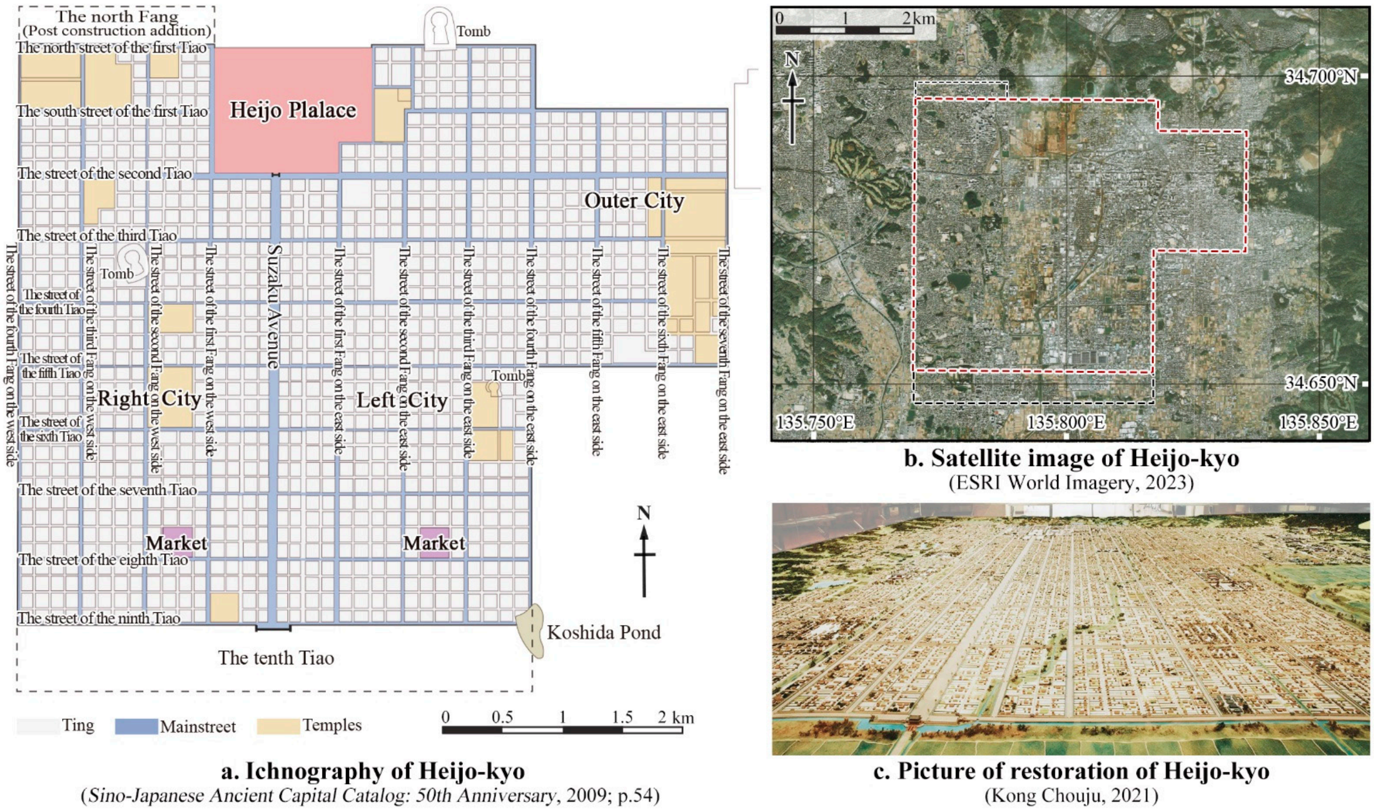


Figure 11. Ichnography (underlying graph comes from [49]), satellite image, and picture of restoration of Heijo-kyo.

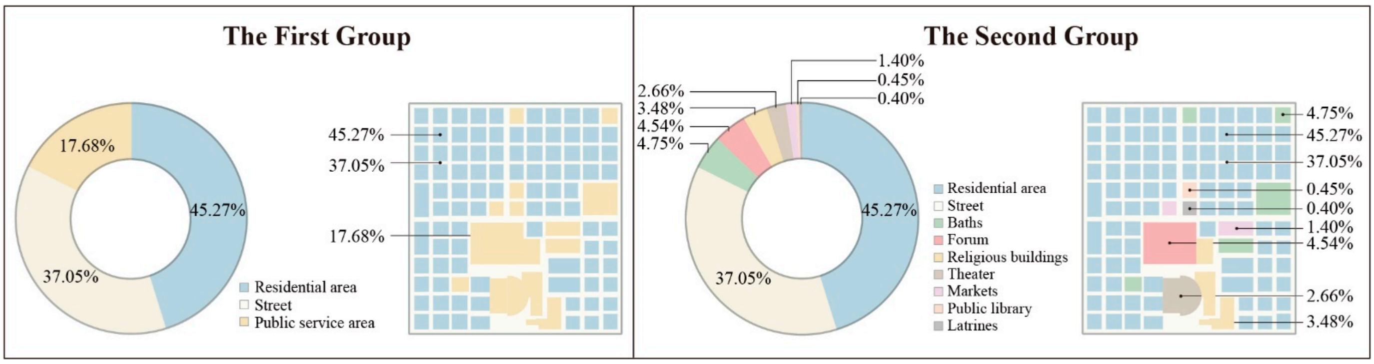


Figure 12. Land use composition in the functional area of Timgad.

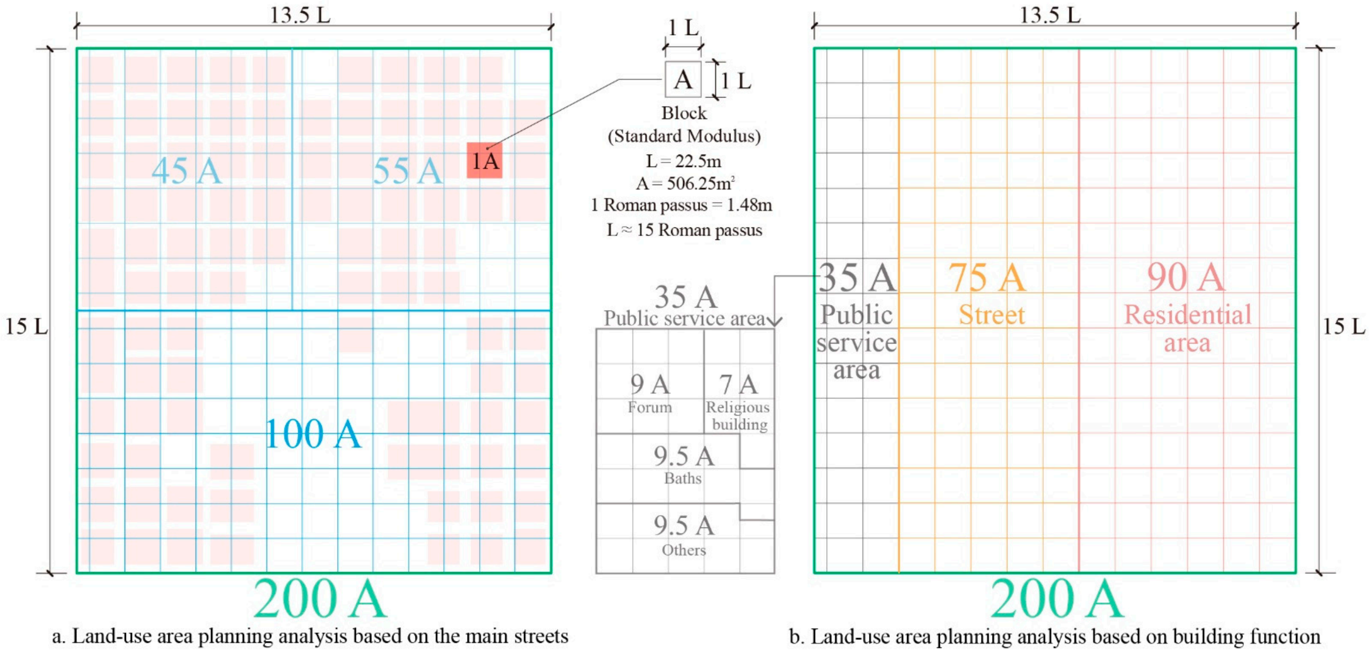


Figure 13. Schematic diagram of the land scale generation logic of Timgad.

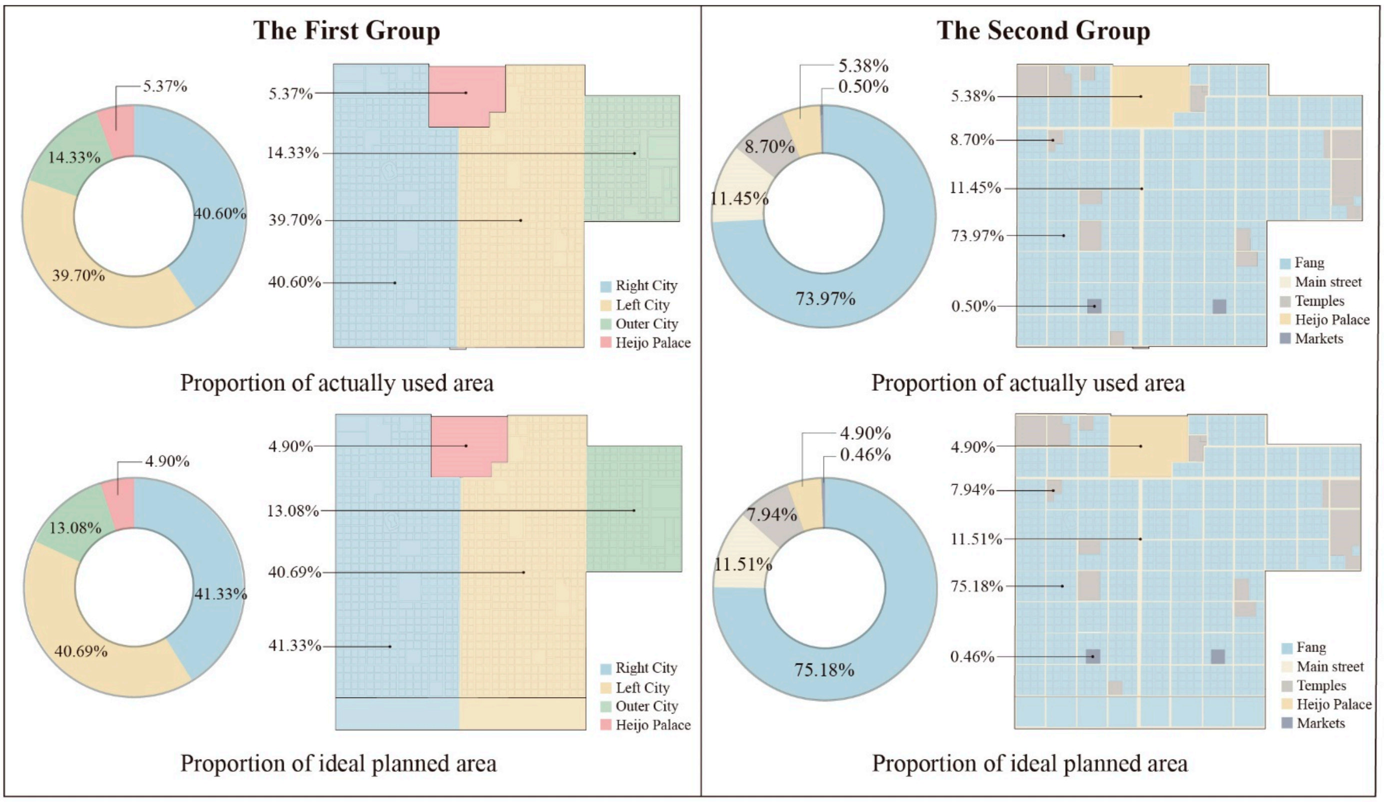


Figure 14. Land use composition in the functional area of Heijo-kyo.

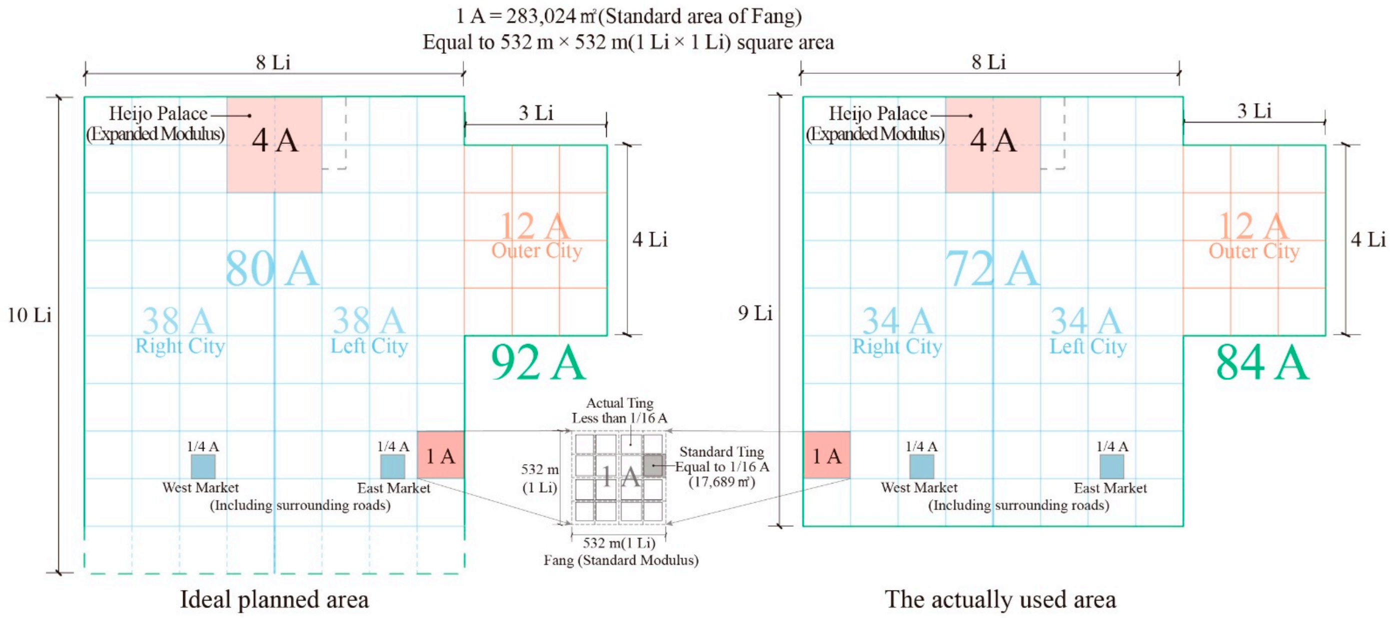


Figure 15. Schematic diagram of the land scale generation logic of Heijo-kyo.

5. Results Discussion

5.1. Land Planning Characteristics and Concepts of Chang'an City of the Sui and Tang Dynasties

The results indicate that Chang'an City of the Sui and Tang Dynasties has significant land use planning characteristics and contains deep-rooted cultural concepts. Specifically:

(1) In terms of land use management in Chang'an City of the Sui and Tang Dynasties, there is a very strict area planning method, which sets the area modulus (minimum unit) and consciously ensures that the area of each area conforms to the integral multiple of the modulus value A. This practice of dividing urban areas may be influenced by the "well field system" (a system of dividing land into several square plots of equal size and distributing them to different people) and related ideas that had developed since the Zhou Dynasty [40]. This is an effective way of controlling urban land resources. Specifically, if the area of the East Market is taken as the modulus, the area of the whole scope of the city is 100A. Meanwhile, the total area of the rectangular urban area is slightly smaller than 90A (88.84A), which is equivalent to removing the southernmost cells from the standard grid of squares. The size of the area modulus A is 924,000 m^2 (equivalent to the square area of 961 m × 961 m); 1 Li (里) in the Sui and Tang Dynasties is generally believed to be 530~540 m, so the area modulus was probably designed in accordance with the square area of 1.8 Li × 1.8 Li, and the whole city area is equivalent to the square grid of 18 Li × 18 Li. If the total area of the rectangular urban area is measured according to the square grid of 18 Li × 16 Li (Figure 7), it is equivalent to 88.84A, which is exactly consistent with the measured value. This shows that the urban designers not only considered the total area value but also accounted for the fact that the city's side length should conform to the length of the integral multiple of the Li. In addition, the objects most likely to be used as the area modulus are the East Market and Daxing Palace, the area ratio of which is very close to 1:2 (98.71%). The ratio of the area of the West Market, which is a little larger than the East Market, to the area of Daxing Palace is also close to 1:2 (coincidence: 97.95%). These phenomena show that in the initial design, the areas of East Market and West Market were intended to be equal, and they were set as the area modulus (A) of the whole capital city. The area of Daxing Palace is twice that of the market as the expanded modulus (2A); the area of other areas of the city should be determined on this basis. Moving from the Eastern Market area (1A) to the Daxing Palace area (2A) to the small city area (10A) is achieved by the constant integer multiple magnification of

the area modulus. After the magnification of different integer multiples for the three areas, the rectangular urban area (90A) and outermost enclosed square area (100A) of the city can be generated, and the area of the urban area divided by main streets (such as 30A, 50A) also emerges, so that the whole city becomes a complete logical system (Figure 16). From the perspective of governance needs and convenient operations, the area planning of ancient capital cities may precede the shape design to solve the key problem of resource allocation; then, the spatial position of each functional block is determined through geometric shape design (the problem can be solved by the square and circular graphic operation method [28]). These findings suggest that the design of Chang'an City is complex and was obtained after comprehensively considering all kinds of issues.

(2) Previous scholars found that Chang'an City of the Sui and Tang Dynasties was formed using the lengths of the Imperial City and the Royal City as the module [29] or the shape of Daxing Palace as the expanded module [30], which proves the dominance and importance of the Imperial City to the whole capital city. The research results show that there may also be a design technique for Chang'an City that takes the market area value as the basic modulus and the Daxing Palace area as the expanded modulus (twice the modulus). Daxing Palace is the most important element in the Imperial City; it is located in the center of the central axis of the capital and represents the royal family. The market is the most important element in the Outer Subcity; it is located on the east and west sides of the central axis of the capital and represents the general public. The study further reveals that the ancients tried to establish a close relationship and mutual transformation between the royal family and the public. Selecting the market as the module perhaps expresses a "people-oriented" concept; however, in order not to reduce the authority of the emperor, the designer consciously made the area of the Imperial City twice the area of the market and placed the Imperial City in the center and the market on both sides, expressing the supremacy of imperial power over civil rights.

(3) There are two types of areas with different attributes in the city, namely the functional area with specific functions, and the designated study area, which is divided by streets. In previous studies, more attention was paid to urban functional areas, while less was paid to non-specific functional areas divided by streets. The results of this study show that the objects in the two areas generally have harmonious mathematical proportional relations, which means that they are effectively unified in the same system. In Chang'an City, the areas divided by main streets, such as Zhuque Street (north–south), Anhuamen Street (north–south), Qixiamen Street (north–south), Jinguangmen Street (east–west), and Yanpingmen Street (east–west), have specific rules in terms of area. They are not only integral multiples of the area modulus (i.e., the area of the East Market), but there is also a specific proportional relationship between the formed areas, indicating that streets are non-negligible elements in the capital and that the positions of streets in the capital were carefully arranged to meet the needs of form and space division.

(4) In the planning of Chang'an City of the Sui and Tang Dynasties, a large number of numerical values and numerical ratios with cultural significance were used. In terms of numbers, these include the supreme number (4.5), the number of heaven and earth (5.5), the number of Heavenly Stems (10), the number of heaven (25), and the number of earth (30) from *The Book of Changes*, as well as the sum of double Chengshu (80), the double Nine-Five Extreme number (90), the number of a hundred years, the ten–ten multiplier (100), etc. In terms of ratios, these include the ratio of Earthly Branches to Heavenly Stems (12:10), the golden ratio (1.618:1), the ratio of the supreme number (9:5, 45:1), the ratio of the number of Yin and Yang (2:1), the ratio of Taiyang to Shaoyang (9:7), the used number of the Great Expansion (49:10), the ratio of Heaven and Earth (3:2), etc. Moreover, the objects with these numerical values or ratios are almost always the key elements in the capital, such as the Imperial City, the Royal

City, and the market. These reflect the fact that that ancient urban planners tried to integrate pure numbers, regular geometric forms, cultural significance, and the actual functions of various objects to form a whole with complex connotations. This fully reflects the design rationality, cultural supremacy, and other characteristics of ancient Chinese capital cities.

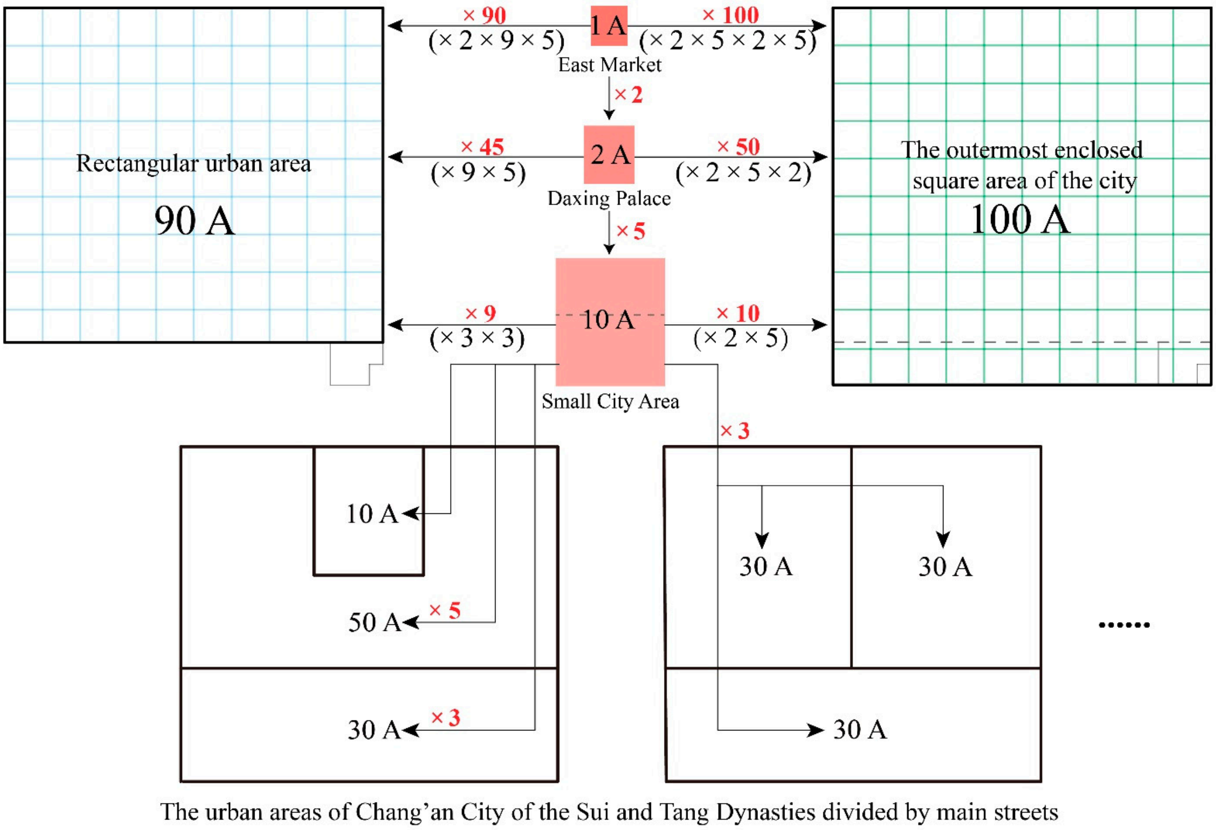


Figure 16. Schematic diagram of the land scale generation logic of Chang'an City of the Sui and Tang Dynasties.

5.2. Comparison of Chang'an City of Sui and Tang Dynasty with Roman Timgad and Japanese Heijo-kyo

For Timgad, the residential area and street occupy a large area of the city, and the public buildings are small but diverse and concentrated in the core of the city (Figure 12). If a block in the residential is taken as a modulus (denoted by A), the whole city can be expressed as an integer value of A, the area divided by the main streets in the city can be expressed as an integer value of A (Figure 13a), and the area of different functional types in the city can also be expressed as an integer value of A (Figure 13b). For Heijo-kyo, the residential area occupies a large area of the city (Figure 14). If an ideal Fang (including its surrounding roads) is taken as a modulus (denoted by A), the area of the whole city and the main objects within the city (Heijo Palace, Left city, Right city, and Outer city) can be expressed as an integer value of A, whether for the ideally planned area or the actually used area (Figure 15) of the city.

We found that the cities of Chang'an city of the Sui and Tang dynasties, Timgad, and Heijo-kyo belong to different civilizations and different periods, but they are still similar from the perspective of land use planning. They are all cities designed under grid-based thinking, achieving rational resource allocation through the area modulus (as the basic unit) and areas of various elements. The object corresponding to the area modulus in Chang'an city of the Sui and Tang dynasties is the market, but its form is not a standard square, nor are there a large number of objects of similar shape and size to it in the city, so it is more like a virtual design. However, the objects corresponding to the area modulus in Timgad and Heijo-kyo (the former being block, the latter being Fang) appear in large quantities in the cities, so they are easier to identify and understand. Although Heijo-kyo was designed in Japan as an imitation of Chang'an City in the Sui and Tang dynasties, it is closer to Timgad in terms of area planning methods. Since Timgad was established the earliest among the three, it is possible that the other two cities were influenced by this type of urban planning thought.

5.3. Feasibility and Transferability of the Research Method

The method proposed in this study has helped the researchers to find many important phenomena and laws that had not been discovered before, and its computational efficiency is high. The algorithm used in this study has low time complexity and space complexity. For Chang'an City of the Sui and Tang Dynasties, the theoretical upper limit of the total number of operations is about 110,000, and the total duration is less than 10 s; it is a similar situation for Timgad and Heijo-kyo. The core data involved in the operation mainly include coordinates of points, area values, ratio values, etc., the storage space of which is almost negligible. Even if the number of objects in an urban area is expanded from the current dozens to thousands, effective results can be obtained in a relatively short time (at the minute level) with the current computing power (the CPU working frequency is counted by the common 2.8 GHz), so it has strong feasibility. In addition, the form of multi-person collaboration can be adopted in the research process, which can significantly improve the efficiency of the analysis and the reliability of the results.

For different research objects, the key algorithms, important parameters, and cultural significance information base used in the study can be easily adjusted according to the research needs. From the research results, it can be seen that the algorithm proposed in this paper is not only effective in solving the case of ancient China, but it is also useful for different cases of other civilizations, such as ancient Rome and Japan. It can conveniently reveal the features of land use and related rules in these cities. Therefore, this model has strong portability and can be widely applied to the urban objects of different times, different regions, and different cultural backgrounds, making it convenient for researchers to analyze the characteristics of land use planning and reveal the potential cultural concepts underpinning these plans. (It should be noted that researchers also need to master the necessary cultural knowledge.)

5.4. Discussion of the Results' Accuracy and Numerical Coincidence

The accuracy of the results in this study is better than the 10 m level, and the actual error can be controlled within 1‰ since the scale of the Chang'an City of the Sui and Tang Dynasties is close to 10 km. For smaller-scale cities such as Timgad and Heijo-kyo, the actual error is somewhat larger, at around 5‰ and 3‰, respectively. For Chang'an City of the Sui and Tang Dynasties, the error mainly comes from three aspects. The first is the position error between the actual measurement point and the ideal object boundary point (or centroid point) in the determination of irregularly shaped sites by RTK, which is about 1~3 m. The second is the drawing error of the area, which cannot be corrected by the position of the site in the city plan; this is about 3~5 m. The third is the visual recognition error generated when manually calibrating the key points of an urban area in GIS, which is about 1~2 m.

Furthermore, it is also worth discussing the coincidence threshold between the calculated results and the ideal values. The inevitable construction deviation in the process of constructing ancient cities, as well as the various changes that arise from historical vicissitude over thousands of years, will lead to incomplete coincidence between the actual situation and the ideal state, so a certain error must be tolerated. The area coincidence threshold in this study was artificially set to 99.5% and 98.5%, which describes the two-dimensional area value. The length coincidence threshold of the single dimension must be better than the square root of the area coincidence threshold. Therefore, the two area coincidence thresholds were converted into length coincidence thresholds, corresponding to 99.75% and 99.25%. The former is equivalent to an allowable error of 2.5 m per 1 km, while the latter is equivalent to an allowable error of 7.5 m per 1 km. According to the current academic consensus, the first threshold is extremely high, and research results based on this threshold can be accepted by almost all scholars. The second threshold is relatively high, and research results based on this threshold can be accepted by most scholars and are strongly persuasive. Additionally, since the result itself has an error of about 10 m, setting a too high threshold has no positive significance for the output of effective research results.

6. Conclusions

In summary, the following conclusions were obtained:

1. An innovative method of urban land use planning analysis based on GIS and an area algorithm was proposed in this study, which can improve researchers' analytical abilities regarding urban land use features and help them reveal the cultural significance and planning ideas reflected by these features. This method is different from those in previous research that relied solely on manual calculations and intuitive interpretation. In essence, it realizes the efficient mining of massive quantities of urban data through human–computer collaboration, which can eliminate a significant amount of the uncertainty generated by previous analysis based solely on scholars' experience, as well as improving the scientificity, accuracy, efficiency, manageability, and portability of relevant research. This method was proven to be feasible and effective based on the analysis and testing of the case of Chang'an City of the Sui and Tang Dynasties. Through the analysis of Timgad in ancient Rome and Heijo-kyo in Japan, it was demonstrated that this method has a wide range of applicability and can be extended to other city case studies.
2. The "urban element area acquisition and analysis model" was constructed for the first time in this study; it can support five important operations, including urban area value calculations, land use structure calculations, area modulus analysis, area ratio analysis between areas, and determining the cultural significance of numbers and ratios. In this model, the object data, core algorithm, and adjustable parameters were managed separately, which makes the original complex research process clear and easy to manage and adjust. A large number of easily overlooked or confused operations were quickly and accurately completed by computer. The two operation behaviors of data entry with low operation difficulty and data analysis with high operation difficulty were separated, making multi-person labor-division collaboration possible.
3. Ancient urban heritage contains wisdom in urban designing and land planning, which is still enlightening to us today. This method pays special attention to the area modulus as the basic unit in the city, which requires the designer to carefully select the objects corresponding to the modulus and set its area values and to intentionally match the area values of major areas within the city to integer multiples of the modulus. Through area planning, the distribution of the city's land resources became reasonable and orderly, and it also carried more cultural significance. For the Chang'an City of the Sui and Tang Dynasties, the area of the market (East Market) was taken as the modulus A (its area was equivalent to the area of a square with a side length of 1.8 Li), the area of Daxing Palace was the expanded modulus 2A, the area of the rectangular urban area was about 90A, and the square area enclosed by the outermost of the capital was 100A; the area of important areas in the city were an integral multiple or half of the modulus value (such as 10A, 30A, 45A). This gave all of the key values (4.5, 5.5, 10, 25, 30, 100, etc.) and numerical ratios (12:10, 1.618:1, 9:5, 45:1, 2:1, etc.) cultural significance (most of them are closely related to cosmology and imperial supremacy), and the corresponding measured values have a high degree of coincidence with the ideal values. These reflect the unique capital design concepts and highly advanced construction abilities of the ancients in the Sui and Tang Dynasties. In addition, there are similar phenomena in some cities around the world. In Timgad in ancient Rome,

the area of a block was taken as the modulus A (its area was the area of a square of land with sides of 15 Roman passus), the area of the whole city was 200A, the area of the residential area was 90A, the area of the streets was 75A, and the area of the public services was 35A (the areas of the forum, bathhouse, religion, and other buildings were all integral multiples or half of the modulus value). In Heijo-kyo in Japan, the area of Fang was taken as the modulus A (its area was the area of a square with a side length of 1 Li), and the area of the whole city was 84A, of which the area of Heijo-kyo as the expanded modulus was 4A, the area of both the Left City and Right City was 34A, and the area of the Outer City was 12A.

4. For a city, both urban form design and land use planning are very important, playing simultaneous roles. The former pays more attention to the style of the city's plan, applying specific geometric shapes (such as squares, rectangles, circles, etc.) and the size modulus (well known by scholars), as well as other related concepts. The latter pays more attention to the method of resource distribution in the city. From our research cases, many ancient cities have the concept of area modulus, and the areas of their main regions also comply with integer multiples of the modulus value, which reflect rational design thought. From a practical perspective, it is necessary for the designers to first determine the overall scale of the city and the land occupation scale of its various elements to meet the real needs of a specific number of people for resources. Therefore, land use planning may have temporal priority compared to urban form design. Additionally, under the same construction cost, the area enclosed by a square of equal side length is larger than that of a rectangular and other irregular shape. The square plot has relatively higher efficiency in resource utilization, which better explains why ancient cities often have a grid-shape form, and why the shapes of their basic units are mostly close to squares. This is an important reflection of how land use planning thinking influences urban form design. Of course, more changes may occur in the subsequent city form design process, and the objects that exist in large quantities in the city may not necessarily be the objects corresponding to the area modulus. For example, in Chang'an city of the Sui and Tang dynasties, there are hundreds of residential areas called "Lifang", which vary in size and are not the objects corresponding to the area modulus. However, in ancient Roman Timgad and Japanese Heijo-kyo, the residential unit plots are indeed the objects corresponding to the area modulus of their respective cities. The grid-shaped cities, by setting area modulus and modulus values of each region, effectively achieved rational land distribution and order establishment, which significantly helped in advancing ancient civilizations. If we fully recognized the role of land use planning, it would have a positive significance for better understanding and protecting the heritage of ancient cities and could also provide insights for solving real urban problems.
5. Remote sensing technology, which is constantly developing, provides important help for the understanding of urban heritage and historical geography. It is often closely combined with GIS, making it possible to carry out detailed academic research on a large spatial scale through clear satellite images and accurate positioning. It is also able to integrate with humanities to give monotonous data a special meaning belonging to different civilizations. Multidisciplinary methods and knowledge create more open and efficient systems to help researchers dig deeper into historical information and ancient ideas, thereby revealing the rich value of urban heritage in a more comprehensive way, strengthening the work related to heritage conservation and display, and promoting the harmonious development of human beings.

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