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Integrated OFDM Waveform Design for RadCom System-Based Signal-to-Clutter Noise Ratio Maximization

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Abstract: Integrated Radar and Communication (RadCom) Systems have become more crucial ingredients in the next generation of networks due to numerous types of characteristics, including reducing the system size, minimizing power consumption, and mitigating spectrum scarcity. In this paper, we propose an integrated Orthogonal Frequency Division Multiplexing (OFDM) waveform design strategy for RadCom system-based Signal-to-clutter Noise Ratio (SCNR) maximization. The multi-carrier OFDM waveform is exploited at the RadCom transceiver to perform the functions of the radar system and communication system concurrently. We aim to improve the radar detection performance by maximizing the SCNR and maintaining the Data Information Rate (DIR) thresholds for communication system metrics. The maximization problem is posed as a convex optimization model and solved analytically using KKT conditions to find an optimal integrated waveform design. The findings obtained via simulation verify that the proposed strategy is capable of solving the waveform design issue for RadCom systems with low complexity. In addition, the proposed strategy effectively improves radar target detection and power consumption for RadCom systems.

Keywords: RadCom System; OFDM; detection probability; signal-to-clutter noise ratio (SCNR); Data Information Rate (DIR)

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Citation: Mohammad, M.A.B.; Cui, G.; Yu, X.; Fakirah, M.; Elhag, N.A.A. Integrated OFDM Waveform Design for RadCom System-Based Signal-to-Clutter Noise Ratio Maximization. *Remote Sens.* **2023**, *15*, 3554. https://doi.org/10.3390/rs15143554

Academic Editor: Dusan Gleich

Received: 12 April 2023
Revised: 1 July 2023
Accepted: 11 July 2023
Published: 15 July 2023

1. Introduction

1.1. Background and Motivation

Recently, with the proliferation of radio frequency devices and the utilization of the radar system for remote sensing, collision avoidance, and other civilian uses, as a result, the resources of the spectrum are becoming congested [1–3]. Hence, the interference and partial interference of the radar system spectrum and the communication system spectrum (The Ultra-high Frequency (UHF) radar system interferes with the second generation (2G) cellular system, and the S-band radar system partially interferes with the worldwide interoperability for microwave access (WiMAX) and Long-term Evolution (LTE) systems.) shed new light on improving the spectrum efficiency rather than bandwidth competition by designing a new integrated system [4–8]. Recently, a number of platforms have been adopted to share hardware and software resources in order to reduce the shortage of spectrum, such as Integrated Radar and Communication (RadCom) Systems, Dual-function Radar-communication Systems (DFRC), and Joint Communication and Radar Systems (JCR). In line with this development, integrated radar and communication systems have drawn significant attention from both academia and industry, including the development of electronic warfare and the intelligent transportation system (ITS). It has also become a key design direction for the Internet of Vehicles (IoV), autonomous driving, the Internet of Things (IoT), and future B5G and 6G networks [8–13]. Moreover, integrated RadCom systems have many features, such as reducing the system size, enhancing system efficiency, reducing power consumption, increasing energy efficiency, improving spectral efficiency, mitigating interference, and lowering the latency and signaling cost compared with conventional individual radar and communication applications [8].

The main challenge for RadCom systems is the waveform design, which is significant for enhancing the performance of radar and communication systems. The waveform design for RadCom systems is divided into two parts: part one relies on the radar waveform, and part two relies on the communication waveform. Linear Frequency Modulation (LFM) waveforms are applied to RadCom systems, which have better performance for radar systems due to the high dynamic range of measurements at the radar receiver when correlation processing is applied. Regrettably, when compared with existing traditional communication systems in the same band, the LFM waveform has a very low rate for communication systems [7]. As a result, the multi-carrier OFDM waveform is proposed for next-generation wireless communication (LTE, 5G New Radio (NR), and 6G) and radar systems [12,14–19]. In order to make the best use of the available resources, OFDM has many benefits, such as flexibility of subcarrier modulation, simple implementation utilizing the Fast Fourier Transform (FFT), and better spectral efficiency. Furthermore, it can deliver great performance for communication and radar systems [7]. The use of the OFDM waveform for target detection at a radar receiver improves detection performance by processing echoes to obtain target information [7,20]. Moreover, the combination of the OFDM waveform and Multiple-input and Multiple-output system (MIMO) improves the data rate and Signal-to-noise Ratio (SNR) for communication systems and also improves the detection performance of MIMO radar systems.

Currently, the waveform design for integrated RadCom systems has gained considerable attention [8,13,21–25]. The waveform design enables full system integration by allowing radar and communication systems to utilize the available resources simultaneously. Many studies have been carried out in order to attain the best waveform design for integrated RadCom systems. In [8], the authors suggested an optimum OFDM waveform design for RadCom systems, using a convex optimization technique to improve the low probability of intercept. The proposed approach primarily took into account mutual information for target parameter estimation as a measure of radar performance while using the Data Information Rate (DIR) as a measure of communication performance. Sturm and Werner proposed an approach for designing single intelligent waveforms for RadCom systems in [7] which is appropriate for carrying out both data transmission and radar sensing simultaneously. Mainly, the proposed approach relies on the phase-coded waveforms used for the existing wireless communication system. They examined the performance of the proposed intelligent waveform for radar systems in a high dynamic range. Several radar signal processing methods were also tested. Finally, the authors presented a full RadCom system in real-world applications to demonstrate the effectiveness of the integrated systems. In [22], the authors suggested robust OFDM integrated radar and communication waveforms. The mono-static radar transceiver was considered for target characterization and a communication transmitter at the same time. They formulated the DIR and Conditional Mutual Information (CMI) for the communication system performance and radar system detection performance based on OFDM integrated radar and communication waveforms. In [26–28], the authors expounded on the direction of future studies to tackle the issue of spectrum scarcity and state the integration level and topology for proposed future systems. Additionally, the achievable inner bound was established for joint radar and communication system performance.

According to the aforementioned studies, SCNR maximization and the DIR have not been considered and need to be investigated to develop an integrated OFDM waveform design for RadCom systems. For this reason, we conducted this study.

1.2. Related Research Review

Generally speaking, integrated RadCom systems can significantly improve the efficiency of radar systems and communication systems. As a result, RadCom systems have gained significant interest from research studies and industries. In [29], Yixuan Huang et al. proposed a novel waveform design algorithm for OFDM-based RadCom systems to decrease the Peak-to-average Power Ratio (PAPR). They considered a flexible and generic

RadCom structure based on a large contiguous spectrum and non-contiguous sub-bands for radar detection and data transmission, respectively. The authors applied a majorization-minimization (MM) optimization method to design an l-norm cyclic algorithm (LNCA). The proposed LNCA algorithm for the RadCom system enhanced the efficiency of the high-power amplifier (HPA) and the probability of detection. In [30], the authors proposed an intelligent waveform design for RadCom systems based on OFDM and circular shift sequences. They utilized the gray code technique to decrease the effect of the Peak-to-mean Envelope Power Ratio (PMERP) and select an optimal cyclic sequence to enhance the peak-to-side lobe ratio (PSLR) in OFDM waveform-based RadCom systems. In [31], Yufeng Chen et al. proposed a joint subcarrier and power allocation for an integrated OFDM waveform in RadCom systems and formulated a non-convex optimization problem to minimize the peak-to-side lobe ratio (PSLR) while satisfying the requirement of the communication data rate (CDR) and preserving a specific range resolution. The power allocation and placement design for OFDM-based integrated radar and communication (RadCom) in automobile systems was investigated in the millimeter wave band. Moreover, the trade-off between the pilot power allocation for the radar system and the placement designs for the communication system was presented and discussed in [32]. In [33], Azka Amin et al. proposed an efficient resource allocation algorithm for D2D communication and cellular systems using Q-learning. They considered the problem of co-channel interference between the D2D and cellular users and proposed an algorithm that maximizes the system throughput by controlling the power of the D2D users while fulfilling the quality of service of the cellular users. The proposed method performed better compared with the existing resource allocation algorithm. In [34], the authors presented an integrated RadCom system to satisfy the requirements of next-generation ITS using a unique platform and phase-coded OFDM waveform. The proposed system utilized the transmitted messages to control the phase sequence of the phase-coded OFDM waveform. Furthermore, a marvelous radar data processing algorithm for a single scatter point target was presented to attain high estimations for the range and velocity along with relatively high transmission data rates simultaneously in a single pulse. In [35], an adaptive OFDM integrated radar and communications waveform design method was proposed to enhance the effectiveness of the spectrum resources. The CMI and DIR of the frequency-selective fading channel were considered performance metrics for the radar system and the communication system, respectively. Finally, the optimization problem was formulated and solved analytically in terms of the CMI and DIR constraints. In [36], the authors proposed waveform design and signal processing methods for RadCom systems using a frequency diversity array (FDA). The proposed waveform is able to provide multi-directional transmission and execute target detection in a single pulse. The performance of the suggested waveform was examined in terms of the ambiguity function, the transmitter and receiver beam patterns, and the bit error rate. In [37], Gu Yabin et al. developed a frequency modulation (FM) approach for RadCom systems based on an orthogonal frequency modulation function. The performance of the proposed scheme was analyzed in terms of the demodulation procedure, ambiguity function, and time-bandwidth product. Finally, the simulation stated the possibility of integration of radar and communication systems in one joint platform. The research in [38] presents a new technique for waveform design for RadCom systems based on the chirped spread spectrum, and the authors also suggested a structure for RadCom systems. The main feature of this system's structure is that many functions can be carried out on the same platform without interfering with each other. A single- and double-robust jamming waveform design strategy was presented to minimize the power consumption of the jamming spectrum. They used optimization techniques to design the jamming waveform and also used the signal-to-interference-plus-noise ratio (SINR) thresholds and the mutual information thresholds as constraints for power control in [39]. In [40], the authors compared the performance of OFDM RadCom systems and universal frequency multi-carrier (UFMC) RadCom systems over two different frequencies (24 GHz and 77 GHz) in order to satisfy adequate wireless solutions and adopt the optimum parametrization for automotive

RadCom systems. In [41], the authors presented a power allocation algorithm for the coexistence of integrating RadCom systems for spectrum sharing with base station users. The low-complexity heuristic approach was devised to produce a suboptimal outcome while minimizing the power for IRCS and BS users. They took into account the radar detection performance, the rate requirement for the base station users, and the latency violation probability for air-to-ground (A2G) and fusion center communication.

While extensive efforts have been made in the study of waveform design for RadCom systems, the debate still continues for achieving the best waveform design that will realize the optimum performance for a radar system and a communication system. The primary aim of this paper is to propose an integrated OFDM waveform design for RadCom systems based on SCNR maximization for radar detection performance while satisfying the DIR requirement for the communication system's performance.

1.3. Major Contributions

This paper contributes to existing knowledge of waveform design for RadCom systems by providing the following:

- An integrated OFDM waveform design for RadCom systems is presented and then posed as an optimization model. The proposed waveform design for RadCom systems is established to maximize the SCNR for the target detection performance of the radar system while fulfilling the specified DIR threshold, which is considered a performance metric for the communication system. As such, the principle concept of this paper is to design an integrated waveform that can improve the detection performance of the radar system.
- We precisely illustrate that the presented integrated OFDM waveform design has been reformulated as an optimization problem. Then, we analytically prove that the objective function is a convex set by deriving the first and second derivatives regarding the integrated transmitted power. The first derivative indicates that the objective function is a monotonically increasing function, while the second derivative states that it is a decreasing function. In addition, the constraints are simplified to be affine functions. Accordingly, the derived optimization problem is convex. Consequently, the solution procedures are simplified.
- We introduce an optimal solution for the convex optimization problem by applying the Lagrangian multipliers technique. Then, we use the Karush-Kuhn-Tucker (KKT) optimality conditions to find an optimal solution and convert the optimization problem to a nonlinear equation problem (aside from the bisection search algorithm) to propose an efficient waveform design with relatively low computational complexity.
- Various simulation results are presented to demonstrate the effectiveness of the proposed integrated OFDM waveform design for RadCom systems. More specifically, the proposed strategy would provide subcarriers with better channel conditions (the subcarriers which have less noise power), more transmit power. Moreover, it is shown that by implementing the proposed integrated OFDM waveform design, the detection performance of the RadCom system is competently enhanced.

The remaining part of this paper proceeds as follows. Section 2 presents the considered RadCom system model, the integrated signal model, and the performance metrics for radar and communication systems. The problem formulation for the proposed integrated OFDM waveform is established and solved analytically based on SCNR maximization in Section 3. The simulation results and performance analysis of the proposed waveform design are shown in Section 4. Finally, the conclusion is drawn in Section 5.

2. System Model and Integrated Signal Model

2.1. System Model

Consider the integrated RadCom systems model shown in Figure 1. It consists of a RadCom transceiver that radiates an OFDM waveform for the intended target and communication receiver concurrently. The RadCom transceiver is able to support both the radar system's and the communication system's functions. The power spectrum of the radar's observational channel and the power spectrum characteristics of the communication channel are assumed to be stationary. It is also supposed that the scattered signal from the target at the communication receiver is much weaker than the direct path transmitted signal, and hence it is ignored here for simplicity.

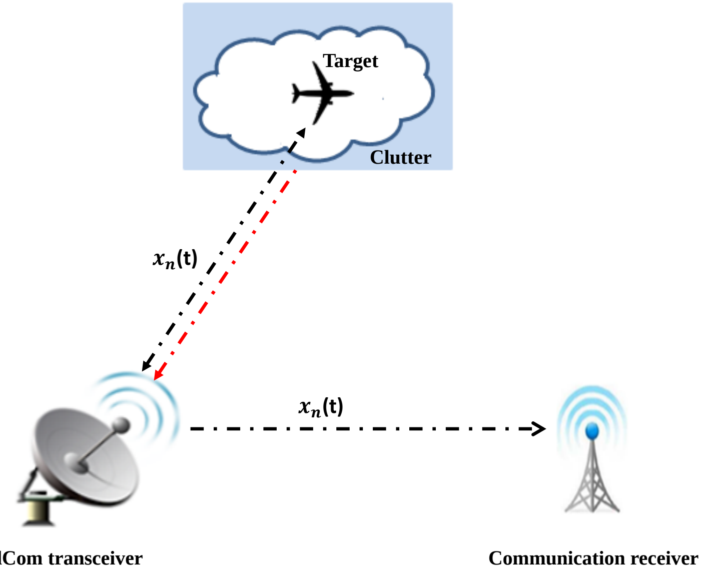


Figure 1. Integrated RadCom systems model.

2.2. Integrated Signal Model

For the RadCom transceiver with N_c subcarriers, the transmitted integrated OFDM waveform $x_n(t)$ is expressed as follows [42,43]:

$$x_n(t) = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} a_n e^{j2\pi(f_o+n\triangle f)t}, \quad 0 \le t < T, \tag{1}$$

where N_c refers to the total number of subcarrier indexes, a_n is the nth subcarrier amplitude, $\triangle f$ denotes the subcarrier spacing, f_o represents the carrier frequency, and T refers to the OFDM symbol time.

The received signal $r(t)$ at the RadCom receiver in the continuous time domain is written as follows:

$$r(t) = x_n(t) \otimes h_{rc}(t) + x_n(t)h_{cc}(t) + n(t), \tag{2}$$

where the first term represents the echo signal from the target due to the RadCom transmitted signal, the second term denotes the signal-dependent clutter due to the RadCom transmitted signal, $h_{cc}(t)$ and $h_{rc}(t)$ represent the responses of the clutter channel and observational channel, respectively, the convolution operation is represented by $\otimes$, and $n(t)$ refers to additive white Gaussian noise.

Hence, the Fourier transform of the received signal $r(t)$ at the RadCom transceiver is represented by [44]

$$r(f) = X_n(f)h_{rc}(f) + X_n(f)h_{cc}(f) + N(n), \tag{3}$$

Thus, the discrete signal spectrum for the received signal $r(f)$ can be written as follows:

$$r[n] = X_n[n]h_{rc}[n] + X_n[n]h_{cc}[n] + N[n], \tag{4}$$

In the end, the spectral variance of the RadCom transceiver's signal $r(t)$ has been expressed as follows:

$$\sigma_r^2[n] = |A[n]|^2\sigma_{rc}^2[n] + |A[n]|^2 P_{cc}[n]L_{rr}[n] + \sigma_v^2[n], \quad (5)$$

where $\sigma_r^2[n]$ represents the probabilistic model for the $r(t)$ ($r[n] \sim \mathcal{CN}(0, \sigma_r^2[n])$), $|A[n]|^2$ refers to the OFDM waveform power, $h_{rc}[n] \sim \mathcal{CN}(0, \sigma_{rc}^2[n])$, $P_{cc}[n]$ is the power spectral density (PSD) of the clutter signal for the n_{th} subcarrier, $\sigma_v^2[n]$ is the AWGN noise variance for the n_{th} subcarrier, and $L_{rr}[n]$ denotes the path attenuation for the observational channel, which can be calculated for the n_{th} subcarrier as follows [44,45]:

$$L_{rr}[n] = \frac{G_t G_r \lambda_n^2}{(4\pi)^3 d_{rc}^4}, \quad (6)$$

where G_t and G_r denote the TX/RX RadCom transceiver antenna gain, λ_n refers to the n_{th} subcarrier wavelength, and d_{rc} is the RadCom transceiver-target distance.

2.3. Performance Metrics for Radar and Communication Systems

Herein, we present the performance metrics for radar and communication systems. We utilized the SCNR and DIR to measure the performance of the radar target detection and the communication system, respectively.

2.3.1. Radar Detection Performance

The SCNR enables the estimation of the target detection at the RadCom transceiver. Hence, the SCNR is an adequate metric for assessing radar target detection performance. According to the RadCom transceiver's received signal (Equation (5)), the SCNR is expressed as follows:

$$\text{SCNR} \triangleq \sum_{n=0}^{N_c-1} \frac{|A[n]|^2\sigma_{rc}^2[n]}{|A[n]|^2 P_{cc}[n]L_{rr}[n] + \sigma_v^2[n]}, \quad (7)$$

At the RadCom transceiver, the radar target detection must be carried out in a noisy environment, and it is uncertain for determining the existence of the target in the received signal. Due to that, we built the binary hypothesis test as follows:

$$\begin{cases} \mathcal{H}_0: & \sigma_r^2[n] = |A[n]|^2 P_{cc}[n]L_{rr}[n] + \sigma_v^2[n], \\ \mathcal{H}_1: & \sigma_r^2[n] = |A[n]|^2\sigma_{rc}^2[n] + \sigma_v^2[n], \end{cases} \quad (8)$$

Suppose that the noise $\sigma_v^2[n]$ is an additive white Gaussian noise with known variance. Hence, for the Neyman–Pearson criterion with a specified detection threshold of T_h, we decide the presence of the target if $\mathcal{H}_1 > T_h$. Furthermore, the detection probability for the Neyman–Pearson detector is given by [46–48]

$$P_D = \frac{1}{2}\text{erfc}\{\text{erfc}^{-1}(2\text{P}_{FA}) - \sqrt{\text{SCNR}}\}, \quad (9)$$

where $\text{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} e^{-t^2}dt$ and P_{FA} represent the complementary error function and the probability of a false alarm, respectively.

2.3.2. Data Information Rate (DIR) Performance

For the wireless communication system, we consider the DIR a figure of merit. In a frequency-selective fading channel, the DIR was enhanced by efficiently allocating the transmitted power in all subcarriers [5,35,43]. Thus, the following is an expression for the DIR in the N_{th} subcarrier:

$$\text{DIR}^n = \log(1 + \frac{|A[n]|^2 L_{cc}[n]}{\sigma_v^2[n]}) \quad nats, \quad (10)$$

where $L_{cc}[n]$ refers to the communication channel path loss, which can be computed for the n_{th} subcarrier by [8]

$$L_{cc}[n] = \frac{G_c^2 \lambda_n^2}{(4\pi)^2 d_c^2}, \tag{11}$$

where G_c is the communication channel's Tx/Rx antenna gain and d_c denotes the distance between the RadCom transceiver and the communication receiver.

3. Problem Formulation of the Proposed OFDM Waveform Design

In this section, the problem formulation for the proposed waveform design strategy is completely described. Then, the optimization problem is posed and solved analytically. Finally, the optimal integrated OFDM waveform design is derived.

3.1. Problem Formulation

The proposed integrated OFDM waveform design for RadCom systems will be addressed by maximizing the optimization problem of the SCNR with respect to the transmitted integrated OFDM waveform power, which is an adaptable parameter related to the SCNR and DIR, while meeting the minimum DIR for the communication receiver and maintaining the power constraint to obtain better RadCom system performance. Mathematically speaking, we can observe from Equation (9) that the probability of detection is a monotonically increasing function of the SCNR, and consequently, maximizing the SCNR improves P_D. Hence, the optimization problem is formulated as a maximization model as follows:

$$\begin{aligned} (\mathbf{P0}) : \max_{|A[n]|^2, n \in N_c} \quad & \sum_{n=0}^{N_c-1} \frac{|A[n]|^2 \sigma_{rc}^2[n]}{|A[n]|^2 P_{cc}[n] L_{rr}[n] + \sigma_v^2[n]} \\ \textbf{s.t.} \begin{cases} \textbf{C1:} & \sum_{n=0}^{N_c-1} \log(1 + \frac{|A[n]|^2 L_{cc}[n]}{\sigma_v^2[n]}) \geq \mathrm{DIR}_{th}, \\ \textbf{C2:} & \sum_{n=0}^{N_c-1} |A[n]|^2 = P_{max,n}, \\ \textbf{C3:} & |A[n]|^2 > 0, \quad 0 \leq n \leq N_c - 1, \end{cases} \end{aligned} \tag{12}$$

where constraint **C1** denotes that the DIR for a communication receiver should be greater than the minimum threshold DIR_{th} and constraints **C2** and **C3** define the upper limit and non-negativity of the power for the RadCom systems, respectively. For the purpose of simplicity, let us define:

$$\begin{cases} |A[n]|^2 = a_n, \\ \sigma_{rc}^2[n] = b_n, \\ P_{cc}[n] L_{rr}[n] = c_n, \\ \sigma_v^2[n] = d_n, \\ ln(x) = log(x), \\ L_{cc}[n] = e_n, \\ m_n = \frac{e_n}{d_n}, \\ P_{min,n} = (\frac{e^{DIR_{th}} - 1}{m_n}), \end{cases}$$

Also, combine constraints **C1** and **C3**. As a result, the recast form of the optimization model **P0** is written as follows:

$$\begin{aligned} (\mathbf{P1}) : \max_{a_n, n \in N_c} \sum_{n=0}^{N_c-1} \frac{a_n b_n}{a_n c_n + d_n} \\ \textbf{s.t.} \begin{cases} \textbf{C1}: & \sum_{n=0}^{N_c-1} a_n = P_{max,n}, \\ \textbf{C2}: & a_n \geq P_{min,n}, \quad 0 \leq n \leq N_c - 1, \end{cases} \end{aligned} \tag{13}$$

The first and second derivatives of the objective function (SCNR) of optimization problem **P1** are written as follows:

$$\frac{\partial}{\partial a_n}\left(\frac{a_n b_n}{a_n c_n + d_n}\right) = \frac{b_n d_n}{(a_n c_n + d_n)^2} > 0, \tag{14}$$

$$\frac{\partial^2}{\partial a_n^2}\left(\frac{a_n b_n}{a_n c_n + d_n}\right) = -2\frac{b_n d_n c_n}{(a_n c_n + d_n)^3} < 0, \tag{15}$$

According to Equations (14) and (7), the objective function is monotonically increasing with regard to a_n. Furthermore, the second derivative in Equation (15) indicates that the objective function of **P1** is concave and decreasing with regard to a_n. Furthermore, that means the objective function is a convex set concerning a_n. Also, the first constraint **C1** and second constraint **C2** are affine functions [49]. As a consequence, **P1** is a convex problem.

3.2. Problem Solution

To address an optimal waveform design for RadCom systems, we utilized the Lagrange multipliers technique, which is a well-known method for solving the constraint convex optimization problem and obtaining the analytical closed-form solution [50,51] which can be given by:

$$L(a_n, \eta, \psi) = -\sum_{n=0}^{N_c-1} \frac{a_n b_n}{a_n c_n + d_n} - \eta_n \sum_{n=0}^{N_c-1} (a_n - P_{min,n}) + \psi\left(\sum_{n=0}^{N_c-1} a_n - P_{max,n}\right) \tag{16}$$

where $\eta = [\eta_1, \eta_2, \ldots, \eta_{Nc}]$ and ψ refers to the Lagrange multipliers with non-negative values related to the constraints of the optimization problem (**P1**).

According to this, the KKT optimality conditions are used to attain the global optimal minimum point for **P1**. The KKT conditions are essential and sufficient conditions for obtaining a_n^*, η_1^* and η_2^* due to the convexity of **P1**. Hence, this can be presented as follows:

$$\frac{\partial \mathcal{L}(a_n, \eta, \psi)}{\partial a_n} = -\frac{b_n d_n}{(a_n^* c_n + d_n)^2} - \eta_n^* + \psi^* = 0, \tag{17a}$$

$$\eta_n^*(a_n^* - P_{min,n}) = 0, \tag{17b}$$

$$a_n^* - P_{min,n} \geq 0, \tag{17c}$$

$$\psi^* \sum_{n=0}^{N_c-1} a_n^* - P_{max,n} = 0, \tag{17d}$$

$$\eta_n^* \geq 0, \quad 0 \leq n \leq N_c - 1, \tag{17e}$$

Based on the KKT optimality conditions, there are two possibilities concerning the power allocation for each subcarrier: $a_n = P_{min,n}$ and $a_n \geq P_{min,n}$. It is apparent from the KKT conditions and the complementary slackness equations in Equations (17a)–(17e) that some cases should be investigated independently for the optimal solution a_n^* as follows:

Case 1: If $\eta_n^* \geq 0$, then according to Equation (17b), we can obtain:

$$a_n^* = P_{min,n}, \tag{18}$$

Case 2: If $\eta_n^* = 0$, then according to Equation (17a), we have:

$$a_n^* = -\frac{d_n}{c_n} + \frac{1}{c_n}\sqrt{\frac{b_n d_n}{\psi^*}}, \tag{19}$$

In order to preserve the feasibility of the optimal solution a_n^*, the condition $\sum_{n=0}^{N_c-1} a_n^* = P_{max}$ must be realized.

Finally, the optimal integrated OFDM waveform design with regard to the optimization problem **P0** that maximizes the SCNR for the RadCom system should satisfy the following equation:

$$a_n^* = \begin{cases} P_{min,n} & d_n \geq \sqrt{\frac{b_n d_n}{\psi^*}} - c_n P_{min,n}, \\ -\frac{d_n}{c_n} + \frac{1}{c_n}\sqrt{\frac{b_n d_n}{\psi^*}} & d_n < \sqrt{\frac{b_n d_n}{\psi^*}} - c_n P_{min,n}, \end{cases} \tag{20}$$

where ψ^* represents the water level, which is specified by the first constraint in problem **P1**:

$$\sum_{n=0}^{N_c-1} a_n^* = P_{max,n}, \tag{21}$$

The KKT optimality conditions transform the problem **P1** to a nonlinear Equation (20). Here, we used the one-dimensional bisection search method to solve it. The integrated OFDM waveform design for RadCom systems based on SCNR maximization and the bisection search method are presented in Algorithms 1 and 2, respectively.

Algorithm 1: Integrated OFDM Waveform Design Strategy

Input: Set DIR_{th}, $P_{max,n}$, N_c and $ite = 1$;
Output: $a_n^*, \forall n \in N_c$
for n=1,2, . . . , N_c **do**
  Calculate a_n^{ite} by solving (20) ;
  Calculate the achevied $a^{ite} \longleftarrow \sum_{n=0}^{N_c-1} a_n^{ite}$;
  Obtain $\psi^{(ite+1)}$ via Algorithm 2;
Update: $a_n^* \leftarrow a_n^{ite}$ for $\forall n$;

Algorithm 2: Bisection Method Algorithm for ψ

Input: $\psi^{(ite)}$, ψ_{min}, ψ_{max}.
Let: $\psi^{(ite)} \leftarrow (\psi_{min} + \psi_{max})/2$;
while $\left|\sum_{n=0}^{N_c-1} a_n^{(ite)} - Pmax,n\right| \neq 0$ **do**
  for $n = 1, 2, \ldots, N_c$, **do**
    if $\sum_{n=0}^{N_c-1} a_{(ite)} < Pmax,n$ **then**
      $\psi_{max} \leftarrow \psi^{(ite)}$;
    else
      $\psi_{min} \leftarrow \psi_{(ite)}$;
    end if
    $\psi^{(ite)} \leftarrow (\psi_{min} + \psi_{max})/2$;
    Calculate a_n^{ite} from (20) and update $a^{(ite)}$;
    $ite \leftarrow ite + 1$;
  Return: $a_n^{(ite)}$;

4. Simulation Results and Performance Analysis

The simulation results and the detection performance analysis for the proposed waveform design are presented in detail in this section, based on the specified system parameters and assumptions and using the MATLAB software program.

4.1. Numerical Set-Up

To obtain the simulation results, we considered the system model depicted in Figure 1, which is composed of an integrated RadCom transceiver, a communication receiver, and the intended target. The integrated OFDM waveform design for RadCom systems that maximizes the SCNR was designed according to the optimization problem **P1** and setting the DIR constraint. Thus, some default specifications were predefined and used in all simulations. For instance, the bandwidth was set to $BW = 512$ MHz, the number of subcarriers was $N_c = 128$, the subcarrier spacing was $\triangle f = 4$ MHz, the carrier frequency was $f_o = 3$ GHz, and the probability of a false alarm was $P_{FA} = 10^{-6}$. Table 1 presents the RadCom system parameters.

As stated earlier, to solve the optimization problem **P1**, it was supposed that the RadCom transceiver knew the power spectral density for the clutter, observational channel, and noise. Therefore, Figures 2 and 3 illustrate the power spectral densities for the observational channel and clutter, respectively.

Table 1. The parameters of the RadCom systems.

Parameter	Value	Parameter	Value
d_{rc}	100 km	d_c	10 km
G_t	30 dB	G_c	30 dB
G_r	30 dB	$P_{max,n}(\forall n)$	600 W

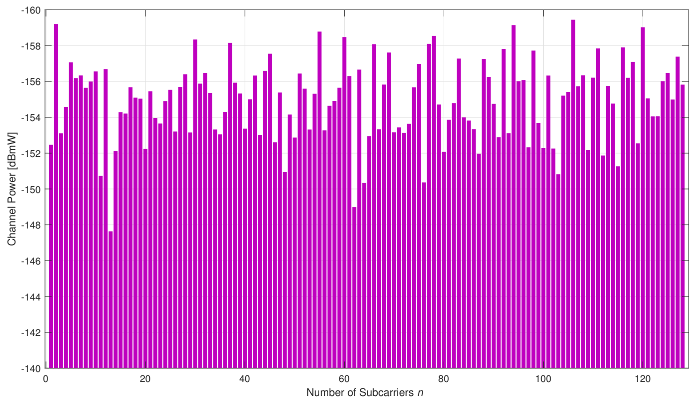


Figure 2. The PSD of the observational channel.

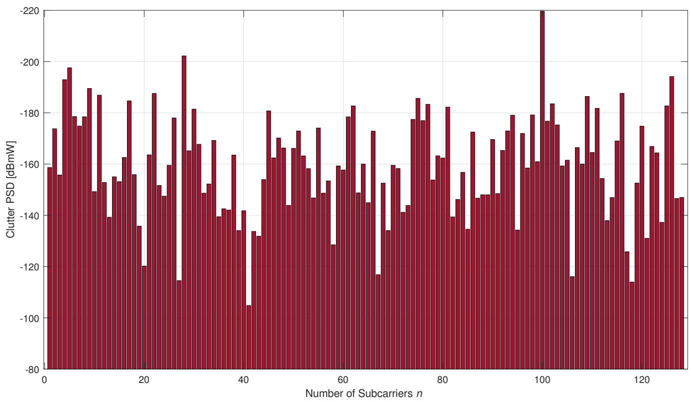


Figure 3. The PSD of clutter.

4.2. Waveform Design Results

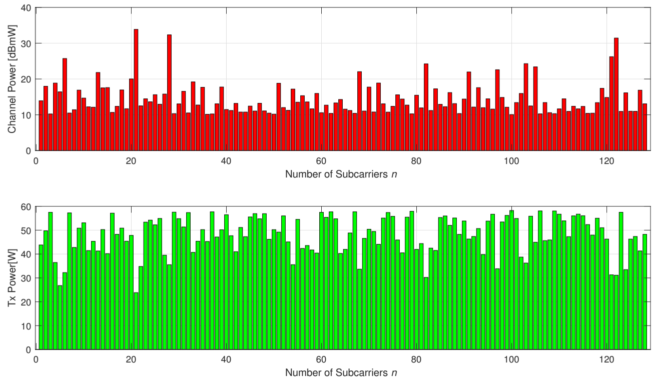
In this subsection, we elaborate upon the simulation findings. Figure 4 shows the waveform design results for RadCom systems with different DIR thresholds, which give insight into the integrated transmitted power allocation for the target detection performance enhancement of RadCom systems. The results clearly show that the integrated OFDM waveform design was specified by the PSD of the noise, clutter and observational channel. Specifically, our proposed integrated waveform design distributes the integrated transmitted power to the subcarrier with less noise power and good channel conditions (see the integrated waveform between subcarriers $0-20$ and $100-120$ at 2.5 nats) so as to guarantee the optimum radar detection performance. Moreover, the integrated transmitted power for the RadCom system was related to the DIR threshold. Mainly, increasing the DIR threshold would monotonically increase the integrated transmitted power according to the optimization problem **P0** constraint (see the integrated waveform design results between the subcarriers from 40 to 100 at DIR = 2.5 nats, DIR = 5 nats, DIR = 7.5 nats and DIR = 10 nats). Hence, the demand for a high DIR requires more transmitted integrated power at the RadCom transmitter to fulfill the desired DIR for the communication system and improve the target detection performance due to an increase in the SCNR.

Finally, the simulation results revealed that, to satisfy the radar detection performance of the radar system and the DIR of the communication system, the RadCom systems efficiently distributed the integrated transmitted power, which indicates that our proposed integrated OFDM waveform design worthily enhanced the probability of detection.

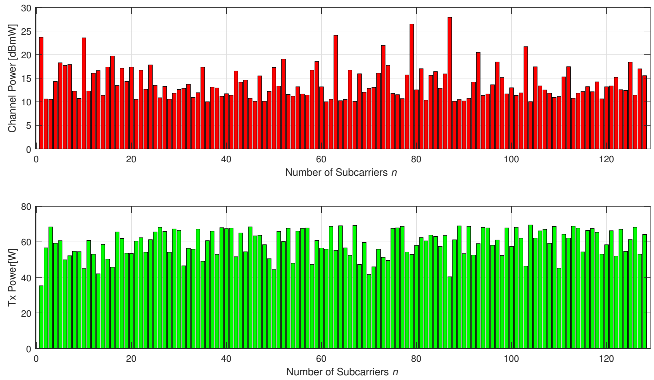
4.3. Detection Performance Analysis

In this subsection, we aim to evaluate the performance of the proposed integrated OFDM waveform design for RadCom systems. Firstly, we consider the uniform power allocation (UPA) for the OFDM waveform compared with the proposed integrated OFDM waveform design to explain the efficiency of the proposed strategy. The UPA distributes the integrated transmitted power uniformly for each subcarrier to satisfy the performance of the radar system and the communication system. More specifically, we compared the total integrated transmitted power for both techniques in Figure 5 and also adjusted the DIR threshold for the presented waveform design and the uniform power allocation to be the same to guarantee fairness. Figure 5 demonstrates that the proposed integrated OFDM waveform design consumes less transmitted power compared with the uniform power allocation technique. Specifically, the proposed integrated waveform design saved the resource power with 2.98 dBW at DIR = 5 nats and 3.2 dBW at DIR = 10 nats. This emphasizes the superiority of our proposed OFDM waveform design for RadCom systems. In addition, Figure 5 reveals that increasing the DIR threshold led to an increase in the total transmitted power to satisfy the constraints of the optimization problem **P0**. Moreover, this result is compatible with the waveform design results in Figure 4.

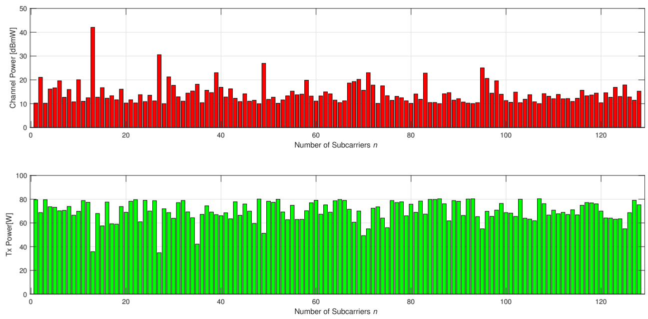
Furthermore, to demonstrate the detection performance improvement for the proposed integrated waveform, we provide a detection probability comparison for different DIR requirements. As stated in Equation (9), the detection probability is a function of the SCNR, and hence the SCNR is a monotonically increasing function of the transmitted power, for which the maximization of the integrated transmitted power enhanced the detection probability for RadCom systems. Figure 6 presents a comparison of the detection probability for different DIR thresholds (DIR = 5 nats and 10 nats). We can notice from Figure 6 that the detection probability improved for the more demanding DIR. Figure 6 shows that to achieve more than 0.9 for the detection probability, the SCNR should be greater than 1.75 dB for a 10 nats information rate and greater than 2.4 dB for a 5 nats information rate, which is in agreement with the waveform design results in Figures 4 and 5.



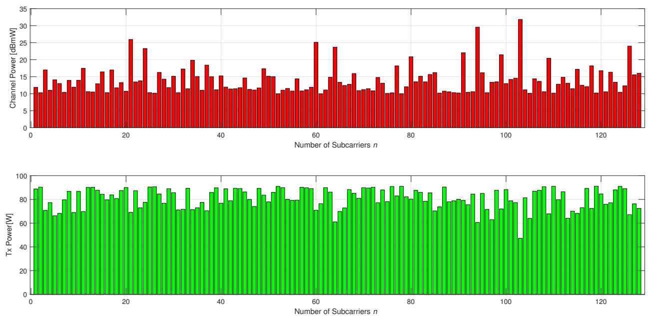
(**a**)



(**b**)



(**c**)



(**d**)

Figure 4. Integrated OFDM waveform design results with different DIR thresholds: (**a**) DIR = 2.5 nats, (**b**) DIR = 5 nats, (**c**) DIR = 7.5 nats and (**d**) DIR = 10 nats.

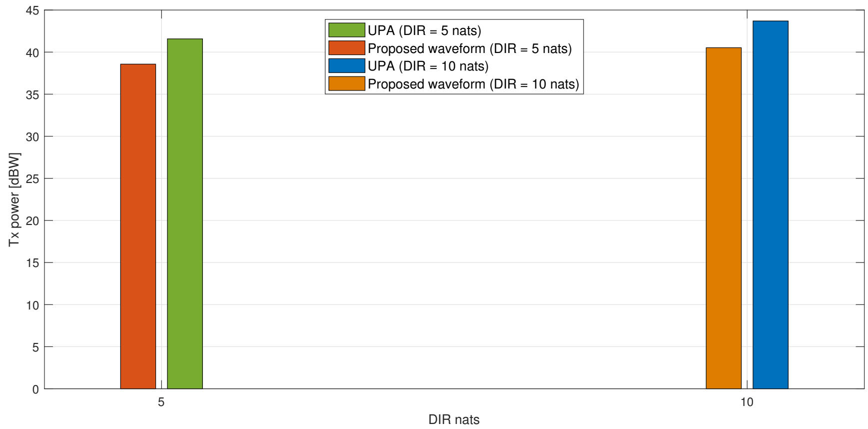


Figure 5. Comparing the total transmit power (dBW) of uniform power allocation with the proposed integrated OFDM waveform design for DIR = 5 nats and DIR = 10 nat.

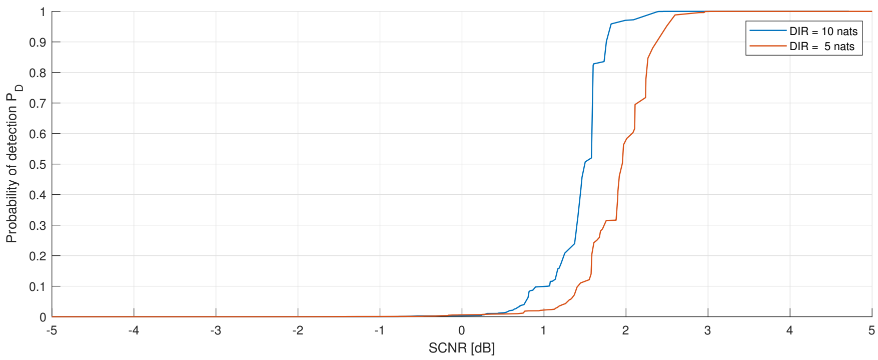


Figure 6. Comparison of the detection performance for the proposed integrated OFDM waveform design at different data information thresholds (DIR = 5 nats and DIR = 10 nats).

4.4. Computation Complexity

The computation complexity of Algorithm 1 depends on the total number of subcarriers N_c and the bisection search method introduced in Algorithm 2. Specifically, in Algorithm 1, the complexity mainly relies on Step 2, which has a time complexity of $\mathcal{O}(n)$. On the other hand, the bisection search method has a complexity of $\mathcal{O}\left[\log(\frac{\psi_{max}-\psi_{min}}{\epsilon})\right]$ [43,52]. Consequently, the total computation complexity of the proposed algorithm can be expressed as $\mathcal{O}\left[N\log(\frac{\psi_{max}-\psi_{min}}{\epsilon})\right]$. This demonstrates significant computational savings compared with the exhaustive search method $\mathcal{O}\left[N(\frac{\psi^{*}-\psi_{min}}{\epsilon})\right]$ [44]. Undoubtedly, the proposed algorithm converged to the optimum solution much faster than the exhaustive search method. Additionally, Algorithm 1 guarantees the achievement of the optimal solution and convergence to the KKT point.

5. Discussion

Based on the results presented in Figures 4–6, our study concludes that the integrated OFDM waveform design strategy is highly attractive for RadCom systems. The proposed integrated OFDM waveform design strategy allocates the transmitted power to subcarriers with respective channel conditions, particularly subcarriers with less noise power. Furthermore, we observed that increasing the DIR threshold increased the overall transmitted power while maintaining the same tendency of power allocation (see Figure 4). In comparison with the uniform power allocation strategy at the predefined DIR threshold (as demonstrated in Figure 5), the proposed waveform design exhibited a significant improvement in total power consumption for the RadCom system. Moreover, the proposed waveform design strategy enhanced the detection performance of the RadCom system as the DIR threshold increased. Figure 6 illustrates that our proposed integrated OFDM waveform design achieved a detection probability of 0.9 at a low-level DIR and SCNR. Notably, for a high DIR threshold, the RadCom system demanded a low SCNR to obtain acceptable detection performance. Finally, the DIR threshold is adjustable, thereby indicating the flexibility of the proposed integrated OFDM waveform design strategy, which can be applied to diverse applications based on varying DIR threshold values.

6. Conclusions

The RadCom system offers several advantages over traditional individual radar and communication systems, including enhanced spectrum efficiency, minimized interference and optimized utilization of hardware resources. However, implementing an effective waveform design for a RadCom system remains a significant challenge. To address this, an integrated OFDM waveform design strategy for RadCom systems based on SCNR maximization is proposed in this paper. The key concept of this strategy is to employ optimization techniques to design an integrated OFDM waveform for a RadCom system to enhance the detection performance of the RadCom system. The problem is formulated as a convex optimization problem, which is then solved analytically using KKT conditions and bisection search methods. Therefore, it is straightforward to obtain the optimal solution that satisfies the requirements of both the radar and communication systems. The obtained results have demonstrated the superior performance of our proposed strategy compared with a uniform power allocation strategy with the same DIR thresholds. More precisely, the simulation results indicated that the proposed strategy allocated more transmit power to the subcarriers with lower noise power. In our future work, the existing model will be expanded to incorporate the off-target scattering signal at communication receivers. Additionally, we will further investigate MIMO RadCom systems to design an integrated OFDM waveform for MIMO RadCom system.

Author Contributions: Conceptualization, M.A.B.M.; validation, M.A.B.M. and G.C.; investigation, M.A.B.M. and X.Y.; writing—original draft preparation, M.A.B.M.; writing—review and editing, X.Y., N.A.A.E. and M.F.; supervision, G.C. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Natural Science Foundation of China under Grant U19B2017, Grant 62101097, Grant 62271126, and Grant 61871080; in part by ChangJiang Scholar Program; and in part by the 111 Project under Grant B17008.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author, Mohammad A. B. Mohammad, upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

UHF	Ultra-high Frequency
LTE	Long-term Evolution
RadCom	Integrated Radar and Communication (RadCom) Systems
DFRC	Dual-function Radar-communication System
JRC	Joint Communication and Radar
MIMO	Multiple-input and Multiple-output
SCNR	Signal-to-clutter Noise Ratio
DIR	Data Information Rate
OFDM	Orthogonal Frequency Division Multiplexing
ITS	Intelligent Transportation System

IoV	Internet of Vehicles
IoT	Internet of Things
LFM	Linear Frequency Modulation
FFT	Fast Fourier Transform
SNR	Signal-to-noise Ratio
PAPR	Peak-to-average Power Ratio
PMERP	Peak-to-mean Envelope Power Ratio
CMI	Conditional Mutual Information

References

1. Griffiths, H.; Blunt, S.; Cohen, L.; Savy, L. Challenge problems in spectrum engineering and waveform diversity. In Proceedings of the 2013 IEEE Radar Conference (RadarCon13), Ottawa, ON, Canada, 29 April–3 May 2013; pp. 1–5.
2. Blunt, S.; Cook, M.; Stiles, J. Embedding information into radar emissions via waveform implementation. In Proceedings of the 2010 International Waveform Diversity And Design Conference, Niagara Falls, ON, Canada, 8–13 August 2010; pp. 195–199.
3. Kumar, S.; Costa, G.; Kant, S.; Flemming, B.; Marchetti, N.; Mogensen, P. Spectrum sharing for next generation wireless communication networks. In Proceedings of the 2008 First International Workshop On Cognitive Radio And Advanced Spectrum Management, Aalborg, Denmark, 14 February 2008; pp. 1–5.
4. Zhou, Y.; Zhou, H.; Zhou, F.; Wu, Y.; Leung, V. Resource allocation for a wireless powered integrated radar and communication system. *IEEE Wirel. Commun. Lett.* **2018**, *8*, 253–256. [CrossRef]
5. Mohammad, M.; Cui, G.; Yu, X.; Ahmed, M.; Gemechu, A. Power allocation strategy for OFDM waveform in RadCom systems. In Proceedings of the 2020 IEEE 11th Sensor Array And Multichannel Signal Processing Workshop (SAM), Hangzhou, China, 8–11 June 2020; pp. 1–5.
6. Shi, C.; Wang, F.; Salous, S.; Zhou, J. Low probability of intercept-based radar waveform design for spectral coexistence of distributed multiple-radar and wireless communication systems in clutter. *Entropy* **2018**, *20*, 197. [CrossRef] [PubMed]
7. Sturm, C.; Wiesbeck, W. Waveform design and signal processing aspects for fusion of wireless communications and radar sensing. *Proc. IEEE* **2011**, *99*, 1236–1259. [CrossRef]
8. Shi, C.; Wang, F.; Salous, S.; Zhou, J. Low probability of intercept-based optimal OFDM waveform design strategy for an integrated radar and communications system. *IEEE Access* **2018**, *6*, 57689–57699. [CrossRef]
9. Liu, F.; Masouros, C.; Ratnarajah, T.; Petropulu, A. On range sidelobe reduction for dual-functional radar-communication waveforms. *IEEE Wirel. Commun. Lett.* **2020**, *9*, 1572–1576. [CrossRef]
10. Yuan, W.; Liu, F.; Masouros, C.; Yuan, J.; Ng, D.; González-Prelcic, N. Bayesian predictive beamforming for vehicular networks: A low-overhead joint radar-communication approach. *IEEE Trans. Wirel. Commun.* **2020**, *20*, 1442–1456. [CrossRef]
11. Wild, T.; Braun, V.; Viswanathan, H. Joint design of communication and sensing for beyond 5G and 6G systems. *IEEE Access* **2021**, *9*, 30845–30857. [CrossRef]
12. Huang, Y.; Ye, Q.; Hu, Z.; Liu, J.; Hu, S.; Zhang, Z. Partially-Reserved Cyclic Algorithm For OFDM-Based RadCom PAPR Reduction. *Procedia Comput. Sci.* **2022**, *202*, 436–448. [CrossRef]
13. Moghaddasi, J.; Wu, K. Multifunctional transceiver for future radar sensing and radio communicating data-fusion platform. *IEEE Access* **2016**, *4*, 818–838. [CrossRef]
14. Saddik, G.; Singh, R.; Brown, E. Ultra-wideband multifunctional communications/radar system. *IEEE Trans. Microw. Theory Tech.* **2007**, *55*, 1431–1437. [CrossRef]
15. Sit, Y.; Zwick, T. MIMO OFDM radar with communication and interference cancellation features. In Proceedings of the 2014 IEEE Radar Conference, Cincinnati, OH, USA, 19–23 May 2014; pp. 265–268.
16. Liu, Y.; Liao, G.; Yang, Z.; Xu, J. Joint range and angle estimation for an integrated system combining MIMO radar with OFDM communication. *Multidimens. Syst. Signal Process.* **2019**, *30*, 661–687. [CrossRef]
17. Liu, Y.; Liao, G.; Yang, Z. Range and angle estimation for MIMO-OFDM integrated radar and communication systems. In Proceedings of the 2016 CIE International Conference On Radar (RADAR), Guangzhou, China, 10–13 October 2016; pp. 1–4.
18. Rahmatallah, Y.; Mohan, S. Peak-to-average power ratio reduction in OFDM systems: A survey and taxonomy. *IEEE Commun. Surv. Tutorials* **2013**, *15*, 1567–1592. [CrossRef]
19. Jankiraman, M.; Wessels, B.; Van Genderen, P. Design of a multifrequency FMCW radar. In Proceedings of the 1998 28th European Microwave Conference, Amsterdam, The Netherlands, 5–9 October 1998; Volume 1, pp. 584–589.
20. Sturm, C.; Zwick, T.; Wiesbeck, W. An OFDM system concept for joint radar and communications operations. In Proceedings of the VTC Spring 2009-IEEE 69th Vehicular Technology Conference, Barcelona, Spain, 26–29 April 2009; pp. 1–5.
21. Rong, J.; Liu, F.; Miao, Y. Integrated Radar and Communications Waveform Design Based on Multi-Symbol OFDM. *Remote Sens.* **2022**, *14*, 4705. [CrossRef]
22. Liu, Y.; Liao, G.; Yang, Z. Robust OFDM integrated radar and communications waveform design based on information theory. *Signal Process.* **2019**, *162*, 317–329. [CrossRef]
23. Ahmed, A.; Zhang, Y.; Hassanien, A. Joint radar-communications exploiting optimized OFDM waveforms. *Remote Sens.* **2021**, *13*, 4376. [CrossRef]

24. Han, L.; Wu, K. Multifunctional transceiver for future intelligent transportation systems. *IEEE Trans. Microw. Theory Tech.* **2011**, *59*, 1879–1892. [CrossRef]
25. Liu, Y.; Liao, G.; Yang, Z.; Xu, J. Design of integrated radar and communication system based on MIMO-OFDM waveform. *J. Syst. Eng. Electron.* **2017**, *28*, 669–680.
26. Paul, B.; Chiriyath, A.; Bliss, D. Survey of RF communications and sensing convergence research. *IEEE Access* **2016**, *5*, 252–270. [CrossRef]
27. Chiriyath, A.; Paul, B.; Jacyna, G.; Bliss, D. Inner bounds on performance of radar and communications co-existence. *IEEE Trans. Signal Process.* **2015**, *64*, 464–474. [CrossRef]
28. Chiriyath, A.; Paul, B.; Bliss, D. Radar-communications convergence: Coexistence, cooperation, and co-design. *IEEE Trans. Cogn. Commun. Netw.* **2017**, *3*, 1–12. [CrossRef]
29. Huang, Y.; Hu, S.; Ma, S.; Liu, Z.; Xiao, M. Designing low-PAPR waveform for OFDM-based RadCom systems. *IEEE Trans. Wirel. Commun.* **2022**, *21*, 6979–6993. [CrossRef]
30. Li, C.; Bao, W.; Xu, L.; Zhang, H.; Huang, Z. Radar communication integrated waveform design based on OFDM and circular shift sequence. *Math. Probl. Eng.* **2017**, *2017*, 9840172. [CrossRef]
31. Chen, Y.; Liao, G.; Liu, Y.; Li, H.; Liu, X. Joint Subcarrier and Power Allocation for Integrated OFDM Waveform in RadCom Systems. *IEEE Commun. Lett.* **2023**, *27*, 253. [CrossRef]
32. Hsu, H.; Lee, M.; Gu, M.; Lin, Y.; Lee, T. Analysis and design for pilot power allocation and placement in OFDM based integrated radar and communication in automobile systems. *IEEE Trans. Veh. Technol.* **2021**, *71*, 1519–1535. [CrossRef]
33. Amin, A.; Liu, X.; Khan, I.; Uthansakul, P.; Forsat, M.; Mirjavadi, S. *A Robust Resource Allocation Scheme for Device-to-Device Communications Based on Q-Learning*; Tech Science Press: Henderson, NV, USA, 2020.
34. Tian, X.; Zhang, T.; Zhang, Q.; Song, Z. Waveform design and processing in OFDM based radar-communication integrated systems. In Proceedings of the 2017 IEEE/CIC International Conference On Communications In China (ICCC), Qingdao, China, 22–24 October 2017; pp. 1–6.
35. Liu, Y.; Liao, G.; Xu, J.; Yang, Z.; Zhang, Y. Adaptive OFDM integrated radar and communications waveform design based on information theory. *IEEE Commun. Lett.* **2017**, *21*, 2174–2177. [CrossRef]
36. Wu, H.; Jin, B.; Xu, Z.; Zhu, X.; Zhang, Z.; Lian, Z. Waveform design and signal processing for integrated radar-communication system based on frequency diversity array. *Digit. Signal Process.* **2023**, *133*, 103839. [CrossRef]
37. Gu, Y.; Zhang, L.; Zhou, Y.; Zhang, Q. Waveform design for integrated radar and communication system with orthogonal frequency modulation. *Digit. Signal Process.* **2018**, *83*, 129–138. [CrossRef]
38. Xie, Y.; Tao, R.; Wang, T. Method of waveform design for radar and communication integrated system based on CSS. In Proceedings of the 2011 First International Conference On Instrumentation, Measurement, Computer, Communication and Control, Beijing, China, 21–23 October 2011; pp. 737–739.
39. Gao, J.; Wu, R.; Zhang, J. An adaptive multi-target jamming waveform design based on power minimization. *Entropy* **2020**, *22*, 508. [CrossRef]
40. Khelouani, I.; Elbahhar, F.; Elassali, R.; Idboufker, N. Performance Evaluation of 5G Waveforms for Joint Radar Communication over 77 GHz and 24 GHz ISM Bands. *Energies* **2022**, *15*, 2049. [CrossRef]
41. Liu, Y.; Wei, Z.; Yan, C.; Feng, Z.; Stüber, G. Effective capacity based power allocation for the coexistence of an integrated radar and communication system and a commercial communication system. *IEEE Access* **2020**, *8*, 58629–58644. [CrossRef]
42. Huang, K.; Bică, M.; Mitra, U.; Koivunen, V. Radar waveform design in spectrum sharing environment: Coexistence and cognition. In Proceedings of the 2015 IEEE Radar Conference (RadarCon), Arlington, VA, USA, 10–15 May 2015; pp. 1698–1703.
43. Shi, C.; Wang, F.; Sellathurai, M.; Zhou, J.; Salous, S. Power minimization-based robust OFDM radar waveform design for radar and communication systems in coexistence. *IEEE Trans. Signal Process.* **2017**, *66*, 1316–1330. [CrossRef]
44. Shi, C.; Salous, S.; Wang, F.; Zhou, J. Low probability of intercept-based adaptive radar waveform optimization in signal-dependent clutter for joint radar and cellular communication systems. *EURASIP J. Adv. Signal Process.* **2016**, *2016*, 1–13. [CrossRef] [PubMed]
45. Turlapaty, A.; Jin, Y. A joint design of transmit waveforms for radar and communications systems in coexistence. In Proceedings of the 2014 IEEE Radar Conference, Cincinnati, OH, USA, 19–23 May 2014; pp. 315–319.
46. Steven, M. *Fundamentals of Statistical Signal Processing*; PTR Prentice-Hall: Englewood Cliffs, NJ, USA, 1993; Volume 10, p. 151045.
47. Tang, B.; Tang, J. Robust waveform design of wideband cognitive radar for extended target detection. In Proceedings of the 2016 IEEE International Conference On Acoustics, Speech And Signal Processing (ICASSP), Shanghai, China, 20–25 March 2016; pp. 3096–3100.
48. Tang, B.; Huang, Z.; Qin, L.; Wang, H. Fundamental limits on detection with a dual-function radar communication system. *arXiv* **2022**, arXiv:2204.04332.
49. Boyd, S.; Boyd, S.; Venberghe, L. *Convex Optimization*; Cambridge University Press: Cambridge, UK, 2004.
50. Zhou, C.; Gu, Y.; He, S.; Shi, Z. A robust and efficient algorithm for coprime array adaptive beamforming. *IEEE Trans. Veh. Technol.* **2017**, *67*, 1099–1112. [CrossRef]

51. Zhou, C.; Gu, Y.; Zhang, Y.; Shi, Z.; Jin, T.; Wu, X. Compressive sensing-based coprime array direction-of-arrival estimation. *Iet Commun.* **2017**, *11*, 1719–1724. [CrossRef]
52. Romero, R.; Shepherd, K. Friendly spectrally shaped radar waveform with legacy communication systems for shared access and spectrum management. *IEEE Access* **2015**, *3*, 1541–1554. [CrossRef]