



# **A** Coherent Integration and Parameter Estimation Method for Constant Radial Acceleration Weak Target via SOKT-IAR-LVD

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**Abstract:** In order to enhance the detection and parameter estimation capacity to the maneuvering target with complex motions, a low complexity coherent integration and parameter estimation method named SOKT-IAR-LVD is proposed in this paper. In SOKT-IAR-LVD, first, the second-order keystone transform (SOKT) is utilized to eliminate the range curvature induced by target acceleration. Second, improved axis rotation (IAR) is applied to regulate the linear range migration by rotating the fast time axis and the target envelope is aligned along the slow time axis with a quadratic phase characteristic. At last, the target signal is coherently integrated via the Lv's Distribution (LVD) transform. The target motion parameters, including range, velocity, and acceleration, are estimated by the IAR and LVD results. The integration gain and computational load of SOKT-IAR-LVD are analyzed. Without needing to estimate the Doppler ambiguity number and target acceleration, the computational burden of SOKT-IAR-LVD is three orders of magnitude lower than that of the Radon-Lv's Distribution (RLVD) method. Simulation results demonstrate that the detection performance of SOKT-IAR-LVD is almost the same as that of RLVD and that the required input SNR of SOKT-IAR-LVD is 17.4 dB lower than that of SOKT-RFT) when the detection threshold is set to 12 dB.

**Keywords:** long-time coherent integration; parameter estimation; improved axis rotation; Lv's distribution; Doppler ambiguity number

# 1. Introduction

With the development of stealth technology, modern ground and aerial targets are able to present the characteristics of high speed, strong maneuverability, long range, and low radar cross-section (RCS) [1]. The traditional moving target detection (MTD) algorithms do not have enough capacity to detect these high-speed maneuvering weak targets. The longtime integration technique is an effective way to increase the signal-to-noise ratio (SNR) and improve radar detection performance [2]. However, when the integration time increases, the high speed and acceleration of the target cause range migration (RM) and Doppler frequency migration (DFM) [3], which limit the performance of classical integration algorithms [4]. Therefore, new approaches to eliminate RM and DFM have been investigated [5].

The typical long-time integration techniques are mainly divided into two categories: incoherent integration and coherent integration. The incoherent integration methods only use the amplitude of echoes to accumulate target signal, which causes poor detection performance in low SNR scenarios [6]. Classical incoherent integration methods include the Hough transform, Radon transform, dynamic programming, and particle filtering methods [7].

Coherent integration performs better than incoherent integration by compensating the phase fluctuation among different sampling pulses. The keystone transform (KT) method corrects the range walk by rescaling the slow time for each range frequency [8]. KT and several improved versions of KT have been widely used, as they can correct the RM effectively without any prior knowledge about the target motion parameter. The Radon Fourier



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). transform (RFT) method eliminates the RM via joint searching along the range and velocity directions of the moving target and integrates coherently via Doppler filtering [9]. The axis rotation MTD (AR-MTD) method eliminates the linear RM by rotating the two-dimensional echoes data plane and realizes coherent integration via the classical MTD algorithm [10]. A fast coherent integration method based on sequence reversing transform was presented in [11], providing a good balance between computational cost and detection ability. By employing the symmetric autocorrelation function and the scaled inverse Fourier transform (SCIFT), a coherent detection algorithm was introduced in [12]; this algorithm can detect high-speed targets without brute-force searching of unknown motion parameters. However, the above methods only consider RM correction, and suffer performance degradation when DFM appears.

KT and second-order KT (SOKT) [13]-based methods have been proposed to regulate both RM and DFM. The KT-matched filtering processing (KT-MFP) method corrects the linear RM via KT and then jointly searches in the fold factor and acceleration domain to remove the residual RM and compensate the DFM. KT-MFP achieves coherent integration through the slow-time Fourier transform (FT) [14]. The SKT-DLVD method uses the segmented keystone transform (SKT) to correct the range walks of targets and then applies the Doppler Lv's transform (DLVD) to estimate the velocities of targets [15]. The KT-LCT method employs KT to eliminate RM. After this, the linear canonical transform (LCT) is applied to compensate DFM and realize coherent integration [16]. In the low-frequency ultra-wideband synthetic aperture radar (SAR), [17] utilizes the first-order KT to correct the range walk and then uses the SOKT to compensate for the range curvature. However, when Doppler ambiguity occurs due to the high speed of the target or limited pulse repetition frequency (PRF), the Doppler ambiguity number has to be estimated before KT and SOKT processing, which increases the computational burden.

In order to integrate coherently under the condition of Doppler ambiguity, the SOKT-RFT method utilizes SOKT for range curvature correction and the improved de-chirping method for DFM compensation. Then, RFT is applied to correct the linear RM. Because SOKT-RFT eliminates RM and DFM in steps, the integration performance can deteriorate due to compensation errors in the previous steps [18]. The SOKT-MFrRT method uses the SOKT to eliminate quadratic range cell migration and the modified fractional Radon transform (MFrRT) to estimate the ambiguity number of Doppler frequency [19].

To eliminate the RM and DFM effects simultaneously, the Radon-fractional Fourier transform (RFRFT) removes the RM effect via three-dimensional searching within the parameter space and realizes integration using the fractional Fourier transform (FRFT) [20]. Inspired by this, the Radon-Lv's distribution (RLVD) method eliminates the RM via jointly searching in the target motion parameter space and achieves coherent integration via Lv's distribution. The RLVD obtains better integration and detection performance than RFRFT [21]. The computational complexity of RLVD is quite large due to the need for multi-dimensional joint searching [22]. In this regard, IAR-FRFT eliminates the linear RM via the improved axis rotation (IAR) transform and realizes coherent integration by FRFT [23]. An approach combining the modified axis rotation transform (MART) and Lv's transform (LVT) is presented in [24]. Compared with RFRFT and RLVD, the computational cost of IAR-FRFT and MART-LVT is decreased; nevertheless, these two methods suffer integration loss because the range curvature induced by the target acceleration is ignored.

For cases of increasing target maneuverability and long observation time, the shorttime generalized radon Fourier transform (STGRFT) method was presented in [25] to detect a maneuvering weak target with multiple motion models. The STGRFT was able to eliminate RM and DFM as well as to estimate the model changing-point time and accumulate the target energy distributed in different motion stages. In the wideband radar scenario, a coherent integration algorithm based on the sub-band keystone transform and extended Lv's distribution (ELVD) was proposed to estimate the motion parameters and reconstruct the high-resolution range profile (HRRP) of a maneuvering weak target [26]. In [27], high-order motion parameter estimation was modeled as an under-estimated linear regression and the complex-field Bayesian compression sensing (BCS) algorithm was designed to resolve the sparse recovery.

In this paper, a low computational complexity coherent integration algorithm named SOKT-IAR-LVD is proposed to eliminate the RM and DFM caused by the constant radial acceleration of the target. First, the SOKT-IAR-LVD employs SOKT to eliminate the range curvature caused by target acceleration and alleviate linear range migration. Second, the IAR is applied to regulate linear range migration by rotating the fast time axis and the target envelope is aligned along the slow time axis with a quadratic phase characteristic. Third, based on the quadratic phase characteristic of the target signal, the LVD is adopted to accumulate the target signal into a well-focused peak in the centroid frequency–chirp rate (CFCR) domain. The motion parameters of the target are estimated by the rotation angle of IAR and the peak position of LVD outputs. The coherent integration gain and computational complexity of SOKT-IAR-LVD are analyzed.

Without needing to estimate the Doppler ambiguity number and target acceleration, SOKT-IAR-LVD possesses a much lower computational cost than RLVD and achieves better integration performance than the IAR-FRFT, SOKT-RFT, and AR-MTD methods.

The rest of this paper is organized as follows. In Section 2, the echo of a constant radial acceleration target is modeled and the phase terms which cause the RM and DFM are analyzed. In Section 3, the procedure of the proposed SOKT-IAR-LVD method is detailed, and the integration gain and computational complexity are analyzed. Section 4 presents simulation results that demonstrate the efficiency of the SOKT-IAR-LVD method.

# 2. Signal Model

Assume that the radar transmitted waveform is a linear frequency modulated (LFM) signal, as follows:

$$s_{\mathrm{T}}(\tau,m) = \mathrm{rect}\left(\frac{\tau}{T_{P}}\right) \exp\left(j\pi K\tau^{2}\right) \exp\left[j2\pi f_{c}(\tau+mT_{r})\right], |\tau| \leq T_{P}/2, m = 0, 1, \dots, M-1$$
(1)

where  $f_c$  is the carrier frequency,  $T_r$  is the pulse repetition interval (PRI),  $T_P$  is the pulse width, B is the bandwidth of the LFM waveform,  $K = B/T_P$  is the frequency modulation rate,  $\tau$  and  $mT_r$  denote the fast time and slow time, respectively, M is the number of transmitted LFM pulses in a coherent processing interval (CPI), and

$$\operatorname{rect}(u) = \begin{cases} 1, |u| \le 1/2 \\ 0, |u| > 1/2. \end{cases}$$

For a target moving towards the radar, the instantaneous range between radar and target is

$$R(\tau, m) = R_0 - vmT_r - \frac{1}{2}a(mT_r)^2 - v\tau$$
(2)

where  $R_0$  is the initial range between the radar and target and v and a represent the radial velocity and acceleration of the target, respectively. For simplicity, we define

$$R(m) = R_0 - vmT_r - \frac{1}{2}a(mT_r)^2.$$
(3)

Because  $|v/c| \ll 1$ , the baseband target echo can be written as

$$s_{\rm R}(\tau,m) = \operatorname{rect}\left[\frac{\tau - 2R(m)/c}{T_P}\right] \exp\left[j\pi K\left(\tau - \frac{2R(m)}{c}\right)^2\right] \exp\left[-j4\pi f_c \frac{R(m)}{c}\right] \exp(j2\pi f_d \tau) \tag{4}$$

where *c* is the speed of light,  $f_d = 2v/\lambda$  is the Doppler frequency, and  $\lambda = c/f_c$  is the carrier wavelength. The matched filtered output of the received target echo is

$$s_{\rm MF}(\tau,m) = \operatorname{sinc}\left[ (B - f_d) \left( \tau - \frac{2R(m)}{c} + \frac{f_d}{K} \right) \right] \exp\left[ j\pi f_d \left( \tau - \frac{2R(m)}{c} + \frac{f_d}{K} \right) \right] \\ \times \exp\left[ -j4\pi (f_c - f_d) \frac{R(m)}{c} \right] \exp\left( -j\pi \frac{f_d^2}{K} \right)$$
(5)

where sinc(x) denotes the sinc function. In (5), the peak position of the target envelope in the *m*th PRI is

$$\tau_m = \frac{2R(m)}{c} - \frac{f_d}{K} \tag{6}$$

which varies with the increase of slow time. Denote  $R_{OFFSET}$  as the range offset during a CPI, which has

$$R_{\text{OFFSET}} = v(M-1)T_r + \frac{1}{2}a[(M-1)T_r]^2.$$
(7)

When  $R_{\text{OFFSET}}$  is less than half of the range resolution, that is,

$$R_{\text{OFFSET}} < \frac{c}{4B},\tag{8}$$

the peak position of the target envelope can be approximately regarded as in the same range cell during the integration time, meaning that the traditional MTD method can be employed for coherent integration. However, the condition of (8) is frequently not met for high-speed maneuvering targets, and the integration performance deteriorates when using the MTD method.

The two range migration terms at the right side of (7) should be eliminated before coherent integration. Applying FT on (5) to the fast time  $\tau$ , the matched filtered output in the range frequency domain is

$$S_{\rm MF}(f,m) = \operatorname{rect}\left(\frac{f - f_d/2}{B - f_d}\right) \exp\left[-j4\pi(f + f_c - f_d)\frac{R_0}{c}\right] \exp\left[j4\pi(f + f_c - f_d)\frac{v}{c}mT_r\right] \\ \times \exp\left[j2\pi(f + f_c - f_d)\frac{a}{c}(mT_r)^2\right] \exp\left(j2\pi f\frac{f_d}{K}\right) \exp\left(-j\pi\frac{f_d^2}{K}\right).$$
(9)

Because of the high target speed or low radar PRF, under-sampling induces ambiguity of the measured target velocity. The velocity of the target is written as

$$v = N_{amb}v_{amb} + v_0 \tag{10}$$

where  $N_{amb}$  is the Doppler ambiguity number,  $v_{amb} = \lambda f_{PRF}/2$  is the blind velocity,  $f_{PRF} = 1/T_r$  is the PRF, and  $v_0 = \text{mod}(v, v_{amb})$  is the measured velocity and satisfies  $|v_0| < v_{amb}/2$ . Because  $|f_d/f_c| \ll 1$ , we can ignore the amount of  $f_d$  in  $f + f_c - f_d$  and write (9) as

$$S_{\rm MF}(f,m) = \operatorname{rect}\left(\frac{f - f_d/2}{B - f_d}\right) \exp\left[-j4\pi(f + f_c)\frac{R_0}{c}\right] \exp\left[j4\pi(f + f_c)\frac{v_0}{c}mT_r\right] \\ \times \exp\left[j4\pi f\frac{N_{amb}v_{amb}}{c}mT_r\right] \exp(j2\pi N_{amb}PRFmT_r) \exp\left[j2\pi(f + f_c)\frac{a}{c}(mT_r)^2\right] \\ \times \exp\left(j2\pi f\frac{f_d}{K}\right) \exp\left(-j\pi\frac{f_d^2}{K}\right)$$
(11)

where  $\exp(j2\pi N_{amb}PRFmT_r) = 1$ . Thus, it has

$$S_{\rm MF}(f,m) = \operatorname{rect}\left(\frac{f - f_d/2}{B - f_d}\right) \exp\left[-j4\pi(f + f_c)\frac{R_0}{c}\right] \exp\left[j4\pi(f + f_c)\frac{v_0}{c}mT_r\right] \\ \times \exp\left(j4\pi f\frac{N_{amb}v_{amb}}{c}mT_r\right) \exp\left[j2\pi(f + f_c)\frac{a}{c}(mT_r)^2\right] \exp\left(j2\pi f\frac{f_d}{K}\right) \\ \times \exp\left(-j\pi\frac{f_d^2}{K}\right).$$
(12)

There are six exponential terms in (12):  $\varphi_1 = \exp[-j4\pi(f + f_c)R_0/c]$  is the initial range term;  $\varphi_2 = \exp[j4\pi(f + f_c)v_0mT_r/c]$  is the Doppler term determined by the measured velocity  $v_0$ , and results in a linear range migration;  $\varphi_3 = \exp(j4\pi f N_{amb}v_{amb}mT_r/c)$  is the phase term induced by the blind velocity, and also causes a linear range migration;  $\varphi_4 = \exp\left[j2\pi(f + f_c)a(mT_r)^2/c\right]$  is the frequency modulation term induced by the target acceleration, and leads to the range curvature and DFM;  $\varphi_5 = \exp(j2\pi f_d f/K)$  indicates the characteristic of the high-speed motion and causes a fixed target envelope offset from its nominal position; and  $\varphi_6 = \exp\left(-j\pi f_d^2/K\right)$  is a constant term.

Here,  $\varphi_2$  and  $\varphi_3$  suffer from the first-order coupling between f and m,  $\varphi_4$  suffers from the second-order coupling between f and m, which causes both range curvature and DFM, and  $\varphi_2$ ,  $\varphi_3$ , and  $\varphi_4$  should be decoupled prior to coherent integration.

# 3. SOKT-IAR-LVD Method

In this section, the SOKT-IAR-LVD method to improve coherent integration performance is detailed. After matched filtering in the range frequency domain, SOKT is employed to eliminate the second-order coupling in  $\varphi_4$  and correct the range curvature caused by the target acceleration. Then, the IAR is used to eliminate the coupling between range frequency and slow time in the terms  $\varphi_2$  and  $\varphi_3$ . At last, the LVD is applied to coherently integrate the target echo. The target motion parameters are estimated by the IAR and LVD results. The details of the SOKT-IAR-LVD method are as follows.

# 3.1. Range Curvature Correction via SOKT

SOKT is utilized to correct the range curvature caused by the target acceleration. SOKT is a process of rescaling the slow time axis for each range frequency. The scaling formula of SOKT is defined as

$$mT_r = \sqrt{\frac{f_c}{f + f_c}} m' T_r \tag{13}$$

where  $m'T_r$  denotes the new slow-time variable.

Substituting (13) into (12), the SOKT output in the fast time frequency domain is

$$S_{\text{SOKT}}(f, m') = S_{\text{MF}}\left(f, \sqrt{\frac{f_c}{f_c + f}}m'\right)$$
  
=  $\operatorname{rect}\left(\frac{f - f_d/2}{B - f_d}\right) \exp\left[-j4\pi(f + f_c)\frac{R_0}{c}\right] \exp\left[j4\pi\sqrt{f_c(f_c + f)}\frac{v_0}{c}m'T_r\right]$   
 $\times \exp\left(j4\pi f\sqrt{\frac{f_c}{f_c + f}}\frac{N_{amb}v_{amb}}{c}m'T_r\right) \exp\left[j2\pi f_c\frac{a}{c}(m'T_r)^2\right]$   
 $\times \exp\left(j2\pi f\frac{f_d}{K}\right) \exp\left(-j\pi \frac{f_d^2}{K}\right).$  (14)

As shown in (14), SOKT removes the second-order coupling in  $\varphi_4$  for range curvature elimination after matched filtering. As  $f \ll f_c$ , the following approximations hold:

$$\begin{cases} \sqrt{f_c(f_c+f)} \approx f_c + f/2\\ f\sqrt{f_c/(f_c+f)} \approx f. \end{cases}$$
(15)

Therefore, (14) is rewritten as

$$S_{\text{SOKT}}(f,m') \approx \operatorname{rect}\left(\frac{f-f_d/2}{B-f_d}\right) \exp\left[-j4\pi(f+f_c)\frac{R_0}{c}\right] \exp\left[j4\pi \frac{f}{c}\left(\frac{v_0}{2}+N_{amb}v_{amb}\right)m'T_r\right] \\ \times \exp\left(j4\pi f_c\frac{v_0}{c}m'T_r\right) \exp\left[j2\pi f_c\frac{a}{c}\left(m'T_r\right)^2\right] \exp\left(j2\pi f\frac{f_d}{K}\right) \exp\left(-j\pi \frac{f_d^2}{K}\right).$$
(16)

As shown in (16), the second-order coupling between f and m in (12) has been eliminated. After performing the inverse Fourier transform (IFT) on (16) into the  $\tau - m'$  domain, we have

$$s_{\text{SOKT}}(\tau, m') = \operatorname{sinc}\left\{ (B - f_d) \left[ \tau - \frac{2}{c} \left( R_0 - \frac{cf_d}{2K} - \left( \frac{v_0}{2} + N_{amb} v_{amb} \right) m' T_r \right) \right] \right\} \\ \times \exp\left\{ j\pi f_d \left[ \tau - \frac{2}{c} \left( R_0 - \frac{cf_d}{2K} - \left( \frac{v_0}{2} + N_{amb} v_{amb} \right) m' T_r \right) \right] \right\} \exp\left( -j4\pi f_c \frac{R_0}{c} \right) \\ \times \exp\left( j4\pi f_c \frac{v_0}{c} m' T_r \right) \exp\left[ j2\pi f_c \frac{a}{c} \left( m' T_r \right)^2 \right] \exp\left( -j\pi \frac{f_d^2}{K} \right).$$

$$(17)$$

The set  $\tau = 2r/c$  (17) can now be rewritten as

$$s_{\text{SOKT}}(r, m') = \operatorname{sinc}\left\{\frac{2(B - f_d)}{c} \left[r - \left(R_0 - \frac{cf_d}{2K} - \left(\frac{v_0}{2} + N_{amb}v_{amb}\right)m'T_r\right)\right]\right\} \\ \times \exp\left\{\frac{2j\pi f_d}{c} \left[r - \left(R_0 - \frac{cf_d}{2K} - \left(\frac{v_0}{2} + N_{amb}v_{amb}\right)m'T_r\right)\right]\right\} \exp\left(-j4\pi f_c \frac{R_0}{c}\right) \\ \times \exp\left(j4\pi f_c \frac{v_0}{c}m'T_r\right) \exp\left[j2\pi f_c \frac{a}{c}(m'T_r)^2\right] \exp\left(-j\pi \frac{f_d^2}{K}\right)$$
(18)

where *r* represents the range corresponding to the fast time  $\tau$ . In (18), the range offset of the target envelope varies linearly with the slow time  $m'T_r$ . In addition, the linear RM caused by the measured velocity  $v_0$  is reduced to half of its value through SOKT processing.

With the fast time sampling frequency  $f_s$ , the discrete form of (18) is

$$s_{\text{SOKT}}(n_r, m') = \operatorname{sinc}\left\{\frac{(B - f_d)}{f_s} \left[n_r - n_0 + \frac{f_d f_s}{K} + \frac{2f_s}{c} \left(\frac{v_0}{2} + N_{amb} v_{amb}\right) m' T_r\right]\right\} \\ \times \exp\left\{\frac{j\pi f_d}{f_s} \left[n_r - n_0 + \frac{f_d f_s}{K} + \frac{2f_s}{c} \left(\frac{v_0}{2} + N_{amb} v_{amb}\right) m' T_r\right]\right\} \exp\left(-j4\pi f_c \frac{R_0}{c}\right) \\ \times \exp\left(j4\pi f_c \frac{v_0}{c} m' T_r\right) \exp\left[j2\pi f_c \frac{a}{c} \left(m' T_r\right)^2\right] \exp\left(-j\pi \frac{f_d^2}{K}\right)$$
(19)

where  $n_r = \text{round}(2rf_s/c)$ ,  $n_0 = \text{round}(2R_0f_s/c)$ , and  $n_r$  and  $n_0$  represent the range cell numbers of r and  $R_0$ , respectively.

## 3.2. Range Migration Correction via IAR

The AR-MTD method concentrates the target echoes in a range cell via the axis rotation transform. However, the Doppler resolution may vary with the axis rotation angle. In the SOKT-IAR-LVD method, the IAR regulates the linear range migration in (19) by rotating the fast time axis. The slow time axis remains unchanged in order to maintain a

constant Doppler resolution. Figure 1 shows the diagram of the IAR transform. In (19), the target signal envelope after SOKT is distributed along a straight line with a slope of  $-2f_sT_r(v_0/2 + N_{amb}v_{amb})/c$  in the coordinate system  $n_r - m'$ . In Figure 1, the angle between the target signal envelope and slow time axis m' is defined as  $\gamma$ , and has

$$\tan \gamma = -\frac{2f_s T_r}{c} \left(\frac{v_0}{2} + N_{amb} v_{amb}\right). \tag{20}$$

The IAR processing is

$$\begin{cases} \tilde{n} = \operatorname{round}(-\sin\beta_i m' + \cos\beta_i n_r) \\ \tilde{m} = m' \end{cases}$$
(21)

where  $\tilde{n} - \tilde{m}$  represents a new coordinate system after axis rotation,  $\beta_i = i\Delta\beta \in [-\pi/2, \pi/2]$  denotes the axis rotation angle, and  $\Delta\beta$  is the angle rotation step. We denote  $N_\beta$  as the number of searching angles. Because the target velocity is frequently limited to a certain range, the angle rotation searching area  $[\beta_{\min}, \beta_{\max}]$  can be predetermined in order to reduce the computational burden when the target's velocity is limited in the region  $[v_{\min}, v_{\max}]$  with

$$v_{\min} = N_{amb,\min} v_{amb} + v_{0,\min} \tag{22}$$

$$v_{\max} = N_{amb,\max} v_{amb} + v_{0,\max}.$$
(23)

According to (20), the angle rotation searching area  $[\beta_{\min}, \beta_{\max}]$  is computed as follows:

$$\beta_{\min} = \operatorname{atan}[-2f_s T_r(v_{0,\min}/2 + N_{amb,\min}v_{amb})/c]$$
(24)

$$\beta_{\max} = \operatorname{atan}[-2f_s T_r(v_{0,\max}/2 + N_{amb,\max}v_{amb})/c].$$
<sup>(25)</sup>



Figure 1. IAR transform for linear range migration elimination.

Substituting (20) and (21) into (19) gives

$$s_{\text{IAR}}(\tilde{n},\tilde{m}) = \operatorname{sinc}\left\{\frac{(B-f_d)}{f_s \cos \beta_i} \left[\tilde{n} - \left(n_0 - \frac{f_d f_s}{K}\right) \cos \beta_i\right] + \frac{(B-f_d)}{f_s} (\tan \beta_i - \tan \gamma)\tilde{m}\right\} \\ \times \exp\left\{\frac{j\pi f_d}{f_s \cos \beta_i} \left[\tilde{n} - \left(n_0 - \frac{f_d f_s}{K}\right) \cos \beta_i\right] + \frac{j\pi f_d}{f_s} (\tan \beta_i - \tan \gamma)\tilde{m}\right\} \\ \times \exp\left(-j4\pi f_c \frac{R_0}{c}\right) \exp\left(j4\pi f_c \frac{v_0}{c}\tilde{m}T_r\right) \exp\left[j2\pi f_c \frac{a}{c}(\tilde{m}T_r)^2\right] \exp\left(-j\pi \frac{f_d^2}{K}\right).$$
(26)

When  $\beta_i = \gamma$ , (26) is written as

$$s_{\text{IAR}}(\tilde{n},\tilde{m})|_{\beta_{i}=\gamma} = \operatorname{sinc}\left\{\frac{(B-f_{d})}{f_{s}\cos\gamma}\left[\tilde{n}-\left(n_{0}-\frac{f_{d}f_{s}}{K}\right)\cos\gamma\right]\right\}$$
$$\times \exp\left\{\frac{j\pi f_{d}}{f_{s}\cos\gamma}\left[\tilde{n}-\left(n_{0}-\frac{f_{d}f_{s}}{K}\right)\cos\gamma\right]\right\}\exp\left(-j4\pi f_{c}\frac{R_{0}}{c}\right)$$
$$\times \exp\left(j4\pi f_{c}\frac{v_{0}}{c}\tilde{m}T_{r}\right)\exp\left[j2\pi f_{c}\frac{a}{c}(\tilde{m}T_{r})^{2}\right]\exp\left(-j\pi\frac{f_{d}}{K}^{2}\right).$$
(27)

As shown in (27), the peak position of the target envelope is concentrated at the range cell  $\tilde{n}_0 = \text{round}[(n_0 - f_d f_s / K) \cos \gamma]$ . The set  $\tilde{n} = \tilde{n}_0$  (27) is simplified as follows:

$$s_{\text{IAR}}(\tilde{m})\big|_{\beta_i=\gamma,\tilde{m}=\tilde{n}_0} = A_1 \exp\left(j2\pi \frac{2f_c v_0}{c}\tilde{m}T_r\right) \exp\left[j\pi \frac{2f_c a}{c}(\tilde{m}T_r)^2\right]$$
(28)

with

$$A_1 = \exp\left(-j4\pi f_c \frac{R_0}{c}\right) \exp\left(-j\pi \frac{f_d^2}{K}\right).$$
(29)

In (28), the linear range migration has been corrected by IAR and the output of IAR is a chirp signal with a chirp rate of  $2f_ca/c$ . Inspired by the excellent performance of LVD in extracting the parameters of chirp signals [28], the LVD method is adopted to integrate the target signal in (28) and then estimate the target motion parameters.

#### 3.3. Coherent Integration and Parameter Estimation with LVD

In LVD processing, the symmetric instantaneous autocorrelation function (SIAF) of (28) is

$$R^{C}(\tilde{m}, l) = s_{\text{IAR}}(\tilde{m} + l)s_{\text{IAR}}^{*}(\tilde{m} - l)$$
  
=  $A_{1}^{2} \exp\left[j\frac{4\pi f_{c}v_{0}}{c}2T_{r}l + j\frac{4\pi f_{c}a}{c}(2T_{r}l)\tilde{m}T_{r}\right]$  (30)

where \* denotes the complex conjugation. Because  $s_{\text{IAR}}(\tilde{m}) = 0$  when  $\tilde{m} < 0$  or  $\tilde{m} \ge M$ , the number of valid elements in the SIAF matrix is  $M^2/4$ .

The variables  $\tilde{m}$  and l in (30) are coupled with each other in the exponential phase term. We set

$$\tilde{m} = \frac{m_b}{2hT_r l} \tag{31}$$

where  $m_b$  is the LVD slow time variable and h is a scaling factor that determines the chirp rate estimation range of  $s_{IAR}(\tilde{m})$ . Substituting (31) into (30), we have

$$R^{C}(m_{b}, l) = A_{1}^{2} \exp\left[j\frac{4\pi f_{c}v_{0}}{c}2T_{r}l + j\frac{4\pi f_{c}a}{hc}T_{r}m_{b}\right]$$
(32)

where the coupling between  $\tilde{m}$  and l has been eliminated. Performing two-dimensional (2D) FT on (32), the output of LVD is

$$L(p,q) = A_2 \sin c \left[ MT_r \left( \frac{p}{2MT_r} - \frac{2f_c v_0}{c} \right) \right] \sin c \left[ MT_r \left( \frac{q}{MT_r} - \frac{2f_c a}{hc} \right) \right]$$
(33)

with

$$A_2 = (MT_r)^2 \exp\left(-j4\pi f_c \frac{R_0}{c}\right)^2 \exp\left(-j\pi \frac{f_d^2}{K}\right)^2$$
(34)

where p - q represents the centroid frequency–chirp rate (CFCR) domain with  $-M/2 \le p, q \le M/2 - 1$ . The target echo is coherently accumulated at  $p = 4MT_rv_0/\lambda$ ,  $q = 2MT_ra/(\lambda h)$ .

Therefore, the velocity and acceleration of target are estimated by the peak position of the LVD results:

$$(\hat{p}, \hat{q}) = \operatorname*{arg\,max}_{p,q} |L(p,q)| \tag{35}$$

$$\hat{v}_0 = \frac{\lambda \hat{p}}{4MT_r} \tag{36}$$

$$\hat{i} = \frac{\lambda h \hat{q}}{2MT_r}.$$
(37)

According to (37), the scaling factor *h* should be no less than  $4T_r a_{max}/\lambda$ , where  $a_{max}$  denotes the maximal target radial acceleration.

Finally, based on the IAR and LVD results, the Doppler ambiguity number  $N_{amb}$  and target velocity v are estimated as follows:

$$\hat{N}_{amb} = \operatorname{round}\left(-\frac{c\tan\gamma + \hat{v}_0 T_r f_s}{2T_r f_s v_{amb}}\right)$$
(38)

$$\hat{v} = \hat{N}_{amb} v_{amb} + \hat{v}_0. \tag{39}$$

# 3.4. Procedure of the SOKT-IAR-LVD Method

Based on the above analysis, the flow chart of the SOKT-IAR-LVD method is shown in Figure 2. The procedure of SOKT-IAR-LVD is divided into the following steps.



Figure 2. Flowchart of the SOKT-IAR-LVD method.

Step 1. Initialize the parameters of the SOKT-IAR-LVD method, including the rotation angle searching area [ $\beta_{min}$ ,  $\beta_{max}$ ], angle search step  $\Delta\beta$ , and scaling factor *h*.

Step 2. Perform matched filtering on the radar echoes in the frequency domain.

Step 3. Apply SOKT to the matched filtered outputs to eliminate the range curvature, then perform range IFT on the SOKT outputs.

Step 4. For each  $\beta_i \in [\beta_{\min}, \beta_{\max}]$ , apply IAR to the SOKT outputs to remove the residual linear RM, then perform LVD transform on the IAR results along the slow time.

Step 5. After computing the LVD results for all rotation angles in  $[\beta_{\min}, \beta_{\max}]$ , search for the peak of the LVD results at each range cell and the corresponding values of  $\beta_i$ ,  $\hat{p}$ , and  $\hat{q}$ .

Step 6. Apply constant false alarm rate (CFAR) detection to the peak of the LVD results. If a target is detected, the range, velocity, and acceleration of the target are estimated using the values of  $\beta_i$ ,  $\hat{p}$ , and  $\hat{q}$  using (35)–(39), respectively.

# 3.5. Integration Gain Analysis

The coherent integration gain of SOKT-IAR-LVD mainly depends on the LVD transform. We denote the output SNR of matched filtering as SNR<sub>MF</sub>, and the output SNR of the IAR transform is the same as SNR<sub>MF</sub>. Adding the complex Gaussian noise to the IAR output in (28) with SNR<sub>MF</sub> =  $A_s^2/\zeta^2$ , we have

$$y(\tilde{m}) = s_{\text{IAR}}(\tilde{m}) + n(\tilde{m}) \tag{40}$$

where  $n(\tilde{m})$  represents the noise with power  $\zeta^2$  and the amplitude  $s_{\text{IAR}}(\tilde{m})$  is  $A_s$ . The SIAF of  $y(\tilde{m})$  is

$$R_{y}^{C}(\tilde{m},l) = y(\tilde{m}+l)y^{*}(\tilde{m}-l)$$
  
=  $s_{IAR}(\tilde{m}+l)s_{IAR}^{*}(\tilde{m}-l) + s_{IAR}(\tilde{m}+l)n^{*}(\tilde{m}-l)$   
+  $n(\tilde{m}+l)s_{IAR}^{*}(\tilde{m}-l) + n(\tilde{m}+l)n^{*}(\tilde{m}-l).$  (41)

In (41), the amplitude of the target signal changes to  $A_s^2$ , while the noise power is  $2A_s^2\zeta^2 + \zeta^4$ . Because the number of valid elements in the SIAF matrix is  $M^2/4$ , the output SNR of LVD is

$$SNR_{LVD} = \frac{M^2 A_s^4}{4(2A_s^2 \zeta^2 + \zeta^4)}.$$
 (42)

Therefore, the integration gain of LVD is

$$G_{\rm LVD} = \frac{\rm SNR_{\rm LVD}}{\rm SNR_{\rm MF}} = 10\log_{10} \left[ M^2 \left/ \left( 8 + \frac{4}{\rm SNR_{\rm MF}} \right) \right] \, \rm dB. \tag{43}$$

The integration gain of the RFT- and FRFT-based methods only depends on the pulse number *M*, which is

$$G_{\rm RFT\ FRFT} = 10\log_{10}M\,\rm{dB}.\tag{44}$$

when the output SNR of matched filtering satisfies

$$SNR_{MF} > 4/(M-8).$$
 (45)

The SOKT-IAR-LVD method achieves a higher integration gain than the RFT and FRFT based methods. Condition (45) frequently holds for long coherent integration scenarios.

#### 3.6. Computation Complexity Analysis

Next, the computational load of SOKT-IAR-LVD is analyzed. Because SOKT and LVD can be realized by Chirp-z transform–inverse fast FT (CZT-IFFT) [29] and scaled FT–IFFT (SFT-IFFT) [30], respectively, the computational complexities of SOKT and IAR are  $O(M_r M \log_2 M)$  and  $O(M_r N_\beta)$ , where  $M_r = f_s T_r$  is the number of range cells in a PRI. The computation complexity of LVD at a rotation angle  $\beta_i$  is  $O(M^2 \log_2 M)$ . Therefore, the overall computational cost of SOKT-IAR-LVD is approximately  $O(N_\beta M_r M^2 \log_2 M)$ .

# 4. Simulation Results

In this section, the performance of SOKT-IAR-LVT is evaluated and compared with that of the RLVD, SOKT-RFT, IAR-FRFT, and AR-MTD methods. Table 1 shows the parameters of the transmitted radar waveform. Table 2 presents the parameters of the SOKT-IAR-LVD method. As shown in Table 2, the searching rotation angles in SOKT-IAR-LVD is  $N_{\beta} = 6573$  and the searching number of acceleration is 512. Accordingly, the searching numbers of velocity and acceleration in RLVD, SOKT-RFT, IAR-FRFT, and AR-MTD are set as 6573 and 512, respectively.

Table 1. Parameters of transmitted radar waveform.

Parameter	Value
Carrier frequency, $f_c$	1 GHz
Bandwidth, B	15 MHz
Pulse width, $T_P$	10 µs
Pulse repetition frequency, $f_{PRF}$	2000 Hz
Sampling rate, $f_s$	60 MSPS
Number of integrated pulses, M	512

Parameter	Value
Velocity measured region	[0 m/s, 5000 m/s]
Acceleration measured region	$[0 \text{ m/s}^2, 450 \text{ m/s}^2]$
Rotation angle searching area $[\beta_{\min}, \beta_{\max}]$	[-46°,0°]
Rotation angle search step, $\Delta\beta$	$0.007^{\circ}$
Scaling factor, h	3

#### Table 2. Parameters of SOKT-IAR-LVD.

# 4.1. Integration Performance with Different Input SNRs

First, the performance of the SOKT-IAR-LVD method is evaluated in a high SNR condition. Consider a constant radial acceleration target moving towards the radar with initial distance  $R_0 = 10$  km, radial velocity v = 1000 m/s, and acceleration a = 100 m/s<sup>2</sup>. The theoretical numbers of cross-range cells caused by the target velocity and acceleration are 102 and 1, respectively. The theoretical slope of the target envelope after SOKT is -0.19, corresponding to a rotation angle of  $\gamma = -10.758^{\circ}$ .

When the SNR of the received target echo in the time domain is -10 dB, Figure 3a shows that the matched filtered output results in a CPI. The range cell migration in Figure 3a is 103, which is consistent with the theoretical value. Figure 3b shows the SOKT outputs for range curvature correction. The theoretical number of cross-range cells removed by SOKT is  $f_s(M-1)T_r[v_0 + a(M-1)T_r]/c \approx 6$ . As shown in Figure 3b, the range cell migration reduces to 97.

When  $\beta_i = -1157\Delta\beta = -10.759^\circ$ , the results of applying the IAR transform to the SOKT results in Figure 3b are shown in Figure 3c. In Figure 3c, the linear range migration has been corrected. The target envelope in every PRI is aligned in the 3930th range cell. The LVD output after applying the LVD transform to the target range cell in Figure 3c is presented in Figure 3d. In Figure 3d, the target signal is coherently integrated at  $\hat{p} = -127$ ,  $\hat{q} = 57$ . The LVD output SNR is 56.28 dB. Therefore, the overall integration gain, including matched filtering and LVD, is 66.28 dB. The theoretical processing gains in matched filtering and LVD are 21.76 dB and 45.01 dB, respectively. The simulated integration gain accurately coincides with the theoretical value.

The peak value of LVD output at each  $\beta_i$  when the rotation angle  $\beta_i$  varies from  $-46^{\circ}$  to  $-0^{\circ}$  is shown in Figure 3e. In Figure 3e, the LVD output achieves a maximum at  $\beta_i = -10.759^{\circ}$ . Using (35)–(39), the estimated range, velocity, and acceleration of the target are 10.001 km, 999.99 m/s, and 100.19 m/s<sup>2</sup>, respectively. SOKT-IAR-LVD achieves high target motion parameter estimation accuracy.

Next, the performance of SOKT-IAR-LVD in the low SNR condition is evaluated. For a constant radial acceleration weak target moving towards the radar with  $R_0 = 15$  km, v = 4500 m/s and a = 350 m/s<sup>2</sup>, the matched filtered outputs are shown in Figure 4a when the input SNR = -35 dB. In Figure 4a, the target signal is submerged by noise and the matched filtered output SNR is very low. After SOKT and IAR transform at  $\beta_i = -41.986^\circ$ , the LVD results are shown in Figure 4b. The weak target signal is integrated with the output SNR=21.10 dB, which is conducive to target detection. The total integration gain is 56.10 dB, which is consistent with the theoretical value of 56.29 dB. The peak position of LVD outputs is at  $\hat{p} = 153$ ,  $\hat{q} = 199$ . The estimated range, velocity, and acceleration of the target are 15.001 km, 4500.02 m/s, and 349.81 m/s<sup>2</sup>, respectively.

Next, the coherent integration gain of SOKT-IAR-LVD at different input SNRs is evaluated. The target motion parameter is the same as in Figure 3. Figure 5 presents the output SNR curves of five methods when the input SNR increases from -45 dB to -20 dB. In Figure 5, when the output SNR after integration is set to 12 dB the required input SNRs are -39.8 dB, -39.6 dB, -36.3 dB, -22.4 dB, and -20.0 dB for the SOKT-IAR-LVD, RLVD, IAR-FRFT, SOKT-RFT, and AR-MTD methods, respectively. The corresponding input SNR of SOKT-IAR-LVD is 2.9 dB, 17.4 dB, and 19.8 dB lower than those of IAR-FRFT, SOKT-RFT, and AR-MTD, respectively. SOKT-IAR-LVD and RLVD achieve a similar

processing gain and possess superior integration ability to SOKT-RFT, IAR-FRFT, and AR-MTD. Because SOKT-RFT estimates the target acceleration by de-chirping and averaging, it inevitably suffers from estimation errors. Therefore, the integration gain of SOKT-RFT is lower than SOKT-IAR-LVD and RLVD. As the target acceleration is not compensated in IAR-FRFT and AR-MTD, the integration performance of these methods is inferior to that of SOKT-IAR-LVD.



**Figure 3.** Target integration via SOKT-IAR-LVD method: (a) matched filtered outputs, (b) results of SOKT, (c) results of IAR transform, (d) LVD outputs in the target range cell, (e) outputs of LVD peaks at different rotation angles.



**Figure 4.** Coherent integration of SOKT-IAR-LVD for a weak target: (**a**) matched filtered output and (**b**) LVD output in the target range cell.



**Figure 5.** Coherent integration performance of various method when input SNR increases from -45 dB to -20 dB.

# 4.2. Integration Performance with Different Motion Parameters

In this subsection, the coherent integration performance for different target motion parameters is evaluated. The input SNR is set as -30 dB. With a constant radial acceleration of  $a = 100 \text{ m/s}^2$ , Figure 6a shows the output SNR when the target velocity increases from 500 m/s to 5000 m/s. Figure 6b presents the output SNR when the target acceleration varies from 50 m/s<sup>2</sup> to 450 m/s<sup>2</sup> and v = 1000 m/s. In Figure 6a,b, SOKT-IAR-LVD and RLVD achieve the highest integration performance. The integration gain of SOKT-IAR-LVD fluctuates slightly around 30.43 dB during the variation of target velocity and acceleration, and is consistent with the theoretical value of 30.55 dB. In Figure 6b, the performance of IAR-FRFT and AR-MTD degrades when the target acceleration increases, which is because the acceleration is not considered in these methods. Suffering from the acceleration estimation error, the performance of SOKT-RFT is stable but inferior to that of SOKT-IAR-LVD.



**Figure 6.** Output SNR results versus target's motion parameters: (**a**) output SNR versus the target's velocity ( $a = 100 \text{ m/s}^2$ ) and (**b**) output SNR versus the target's acceleration (v = 1000 m/s).

# 4.3. Detection Performance

The detection performance of the five methods is evaluated using the cell-averaging CFAR (CA-CFAR) algorithm. The target motion parameter is the same as in Figure 3. The nominal false alarm rate is set to  $P_{f_a} = 10^{-6}$ . The detection probability curves when the input SNR varies from -45 dB to -20 dB are plotted in Figure 7 via 1000 Monte Carlo trials for each SNR. As shown in Figure 7, SOKT-IAR-LVD and RLVD achieve almost the same detection probability and are better than IAR-FRFT, SOKT-RFT, and AR-MTD. At  $P_d = 80\%$ , the input SNR of SOKT-IAR-LVD is about 6.5 dB lower than that of IAR-FRFT and at least 16.5 dB lower than that of SOKT-RFT. The detection ability of SOKT-IAR-LVD for the weak target is superior to IAR-FRFT, SOKT-RFT, and AR-MTD.



Figure 7. Detection probability of the five methods when input SNR increases from -45 dB to -20 dB.

#### 4.4. Computational Complexity Comparison

In this subsection, the computational complexity of the five methods is evaluated. The parameters of radar transmit waveform and SOKT-IAR-LVD are the same as in Tables 1 and 2. As shown in Table 2, the search numbers of rotation angle and acceleration in SOKT-IAR-LVD are selected as  $N_{\beta} = 6573$  and M = 512, respectively. Accordingly, the search numbers of velocity and acceleration in RLVD, IAR-FRFT, SOKT-RFT, and AR-MTD are set as  $N_v = 6573$  and  $N_a = 512$ , respectively. Using the derivation results from Section 3.6, the computation cost of SOKT-IAR-LVD for one range cell is  $O(N_{\beta}M^2\log_2 M)$ , while that of RLVD is  $O(N_vN_aM^2\log_2 M)$  [24]. Because  $N_{\beta} = N_v = 6573$  and  $M = N_a = 512$ , the computational cost of SOKT-IAR-LVD is nearly three orders of magnitude lower than that of RLVD in the theoretical analysis.

Table 3 shows the number of complex multiplications and additions during one range cell coherent integration. The runtime of one hundred range cells integration on a PC equipped with an Intel Core i5-8250U (1.6 GHz) and 8 GB RAM is shown in Table 4.

Table 3. Computational costs of the five methods.

Method	<b>Complex Multiplications</b>	<b>Complex Additions</b>
SOKT-IAR-LVD	$3.7908  imes 10^{10}$	$6.2031  imes 10^{10}$
RLVD	$1.9409  imes 10^{13}$	$3.1760  imes 10^{13}$
IAR-FRFT	$5.5138 imes10^{10}$	$9.3046 imes10^{10}$
SOKT-RFT	$6.0027  imes 10^6$	$8.0958  imes 10^6$
AR-MTD	$1.5144 imes 10^7$	$3.0288  imes 10^7$

Table 4. Runtimes of the five methods.

Method	Runtime (s)
SOKT-IAR-LVD	15.5545
RLVD	6821.4771
IAR-FRFT	19.0411
SOKT-RFT	0.0098
AR-MTD	0.0167

From Tables 3 and 4, it can be seen the computational complexity of SOKT-IAR-LVD is similar to that of IAR-FRFT and nearly three orders of magnitude lower than that of RLVD. Although the computational burdens of SOKT-RFT and AR-MTD are much lower than that of SOKT-IAR-LVD, the detection performance of these methods for the constantly accelerating targets is poor. SOKT-IAR-LVD achieves the best coherent integration performance with a moderate computation cost while estimating the target motion parameter with high accuracy.

# 5. Conclusions

In order to improve the detection performance for the maneuvering targets through long-time integration, the SOKT-IAR-LVD method utilizes the SOKT to eliminate the range curvature induced by target acceleration. Then, it adopts the IAR to remove the linear range migration. At last, the target signal is coherently integrated via the LVD transform. Because estimation of the Doppler ambiguity number and target acceleration is not required, the computational burden of SOKT-IAR-LVD is three orders of magnitude lower than that of RLVD. The detection performance of SOKT-IAR-LVD is almost the same as that of RLVD, and much better than the IAR-FRFT, SOKT-RFT, and AR-MTD methods in the low SNR scenarios. When the detection threshold is set to 12 dB, the required input SNR of SOKT-IAR-LVD is 2.9 dB, 17.4 dB, and 19.8 dB lower than those of IAR-FRFT, SOKT-RFT, and AR-MTD, respectively. Meanwhile, SOKT-IAR-LVD can estimate the target motion parameter accurately.

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