

# An Ensemble Machine Learning Model to Estimate Urban Water Quality Parameters Using Unmanned Aerial Vehicle Multispectral Imagery

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Acronyms and abbreviations used in this study

| Acronym           | Full name                        | Acronym  | Full name                                 |
|-------------------|----------------------------------|----------|---|
| WQPs              | water quality parameters         | TSS      | total suspended solids                    |
| WQP               | water quality parameter          | CDOM     | colored dissolved organic matter          |
| Chla              | chlorophyll-a                    | FC-GA    | feature combination and genetic algorithm |
| SD                | Secchi-Disk depth                | P4M      | Phantom 4 Multispectral                   |
| COD <sub>Mn</sub> | chemical oxygen demand           | ML       | machine learning                          |
| TN                | total nitrogen                   | BRR      | Bayesian ridge regression                 |
| TP                | total phosphorous                | NNR      | k nearest neighbor regression             |
| UAV               | unmanned aerial vehicle          | SVR      | Support Vector Regression                 |
| OAPs              | optically active parameters      | CART     | classification and regression tree        |
| NOAPs             | non-optically active parameters  | RF       | Random Forests                            |
| VIF               | variance inflation factor        | LightGBM | Light Gradient Boosting Machine           |
| R <sub>rs</sub>   | remote sensing reflectance       | MLP      | Multilayer Perceptron                     |
| EML               | ensemble machine learning models |          |   |

## 1 Supplementary Tables

**Table S1** Statistics of information collected from UAV multispectral images.

| ID | Date       | Seasons |
|----|------------|---------|
| 1  | 2022/01/04 | dry     |
| 2  | 2022/04/07 | rainy   |
| 3  | 2022/07/31 | rainy   |
| 4  | 2023/04/26 | rainy   |
| 5  | 2023/05/27 | rainy   |
| 6  | 2023/06/11 | rainy   |

**Table S2** Determination method of water quality parameters

| Parameters                                  | Determination Method   |
|---|--|
| chlorophyll a (Chla)                        | Spectrophotometric method  |
| chemical oxygen demand (COD <sub>Mn</sub> ) | Permanganate index method  |
| total phosphorus (TP)                       | Molybdenum antimony spectrophotometry                                |
| total nitrogen (TN)                         | Alkaline potassium persulfate digestion UV spectrophotometric method |

**Table S3** statistics on the number of available data for WQPs samples.

| WQPs        | COD <sub>Mn</sub> | TP | TN | Chla | SD |
|-------------|-------------------|----|----|------|----|
| sample size | 70                | 64 | 94 | 69   | 69 |

**Table S4** Standard values of the basic items for the Chinese Environmental Quality Standards for Surface Water (GB3838-2002) (units: mg L<sup>-1</sup>)

| Grade     | COD <sub>Mn</sub> | TN  | TP   |
|-----------|-------------------|-----|------|
| Grade I   | 2                 | 0.2 | 0.02 |
| Grade II  | 4                 | 0.5 | 0.1  |
| Grade III | 6                 | 1.0 | 0.2  |
| Grade IV  | 10                | 1.5 | 0.3  |
| Grade V   | 15                | 2.0 | 0.4  |

**Table S5** Descriptive statistics for the whole dataset, training, and validation datasets for COD<sub>Mn</sub>, TP, TN, Chla, and SD. N represents the number of data, and Min and Max represent the maximum and minimum values, respectively. Mean and Std represent the mean and variance, respectively, and CV represents the coefficient of variation. 25%, 50%, and 75% represent the first quartile, the second quartile and the third quartile, respectively.

| WQPs                                       | Dataset    | N  | Mean  | Std   | Min   | 25%   | 50%   | 75%   | Max   | CV      |
|--|------------|----|-------|-------|-------|-------|-------|-------|-------|---------|
| Chla<br>(mg m <sup>-3</sup> )              | whole      | 69 | 10.78 | 5.44  | 2.2   | 6.38  | 9.68  | 15.31 | 23.72 | 50.46%  |
|  | training   | 51 | 10.69 | 5.48  | 2.2   | 6.27  | 10.46 | 15.23 | 23.72 | 51.26%  |
|  | validation | 18 | 11.02 | 5.46  | 4.22  | 6.61  | 8.66  | 15.36 | 21.38 | 49.55%  |
| SD<br>(cm)                                 | all        | 69 | 44.14 | 19.94 | 10.00 | 28.00 | 40.00 | 65.00 | 76.00 | 45.17%  |
|  | training   | 51 | 44.67 | 20.08 | 15.00 | 28.00 | 40.00 | 65.00 | 76.00 | 44.95%  |
|  | validation | 18 | 42.67 | 20.02 | 10.00 | 31.25 | 39.00 | 64.75 | 72.00 | 46.92%  |
| COD <sub>Mn</sub><br>(mg L <sup>-1</sup> ) | whole      | 70 | 2.49  | 0.45  | 1.73  | 2.09  | 2.40  | 2.89  | 3.16  | 18.07%  |
|  | training   | 52 | 2.47  | 0.45  | 1.73  | 2.09  | 2.37  | 2.89  | 3.16  | 18.22%  |
|  | validation | 18 | 2.54  | 0.47  | 1.89  | 2.11  | 2.65  | 2.96  | 3.14  | 18.50%  |
| TN<br>(mg L <sup>-1</sup> )                | whole      | 93 | 0.88  | 0.31  | 0.43  | 0.52  | 0.88  | 1.03  | 1.67  | 35.23%  |
|  | training   | 69 | 0.88  | 0.32  | 0.43  | 0.52  | 0.88  | 1.03  | 1.67  | 36.36%  |
|  | validation | 24 | 0.86  | 0.30  | 0.49  | 0.61  | 0.87  | 1.02  | 1.45  | 34.88%  |
| TP<br>(mg L <sup>-1</sup> )                | whole      | 63 | 0.19  | 0.19  | 0.03  | 0.06  | 0.07  | 0.35  | 0.75  | 100.00% |
|  | training   | 47 | 0.20  | 0.19  | 0.04  | 0.06  | 0.07  | 0.35  | 0.75  | 95.00%  |
|  | validation | 16 | 0.19  | 0.19  | 0.03  | 0.06  | 0.06  | 0.34  | 0.67  | 100.00% |

**Table S6** Statistics on the size of the training dataset's category data before and after oversampling.

| WQPs              | Oversampling | Category |    |     |    |    | Amount |
|-------------------|--------------|----------|----|-----|----|----|--------|
|                   |              | i        | ii | iii | iv | v  |        |
| Chla              | before       | 14       | 13 | 10  | 10 | 4  | 51     |
|                   | after        | 14       | 14 | 14  | 10 | 14 | 66     |
| SD                | before       | 8        | 13 | 11  | 4  | 15 | 51     |
|                   | after        | 15       | 15 | 11  | 15 | 15 | 71     |
| COD <sub>Mn</sub> | before       | 11       | 13 | 4   | 9  | 15 | 52     |
|                   | after        | 15       | 13 | 15  | 15 | 15 | 73     |
| TP                | before       | 30       | 2  | 11  | 2  | 2  | 47     |
|                   | after        | 30       | 30 | 11  | 30 | 30 | 131    |
| TN                | before       | 20       | 22 | 15  | 5  | 7  | 69     |
|                   | after        | 22       | 22 | 22  | 22 | 7  | 95     |

**Table S7** All the combined band formats.

| ID | Format                 | ID  | Format            | ID  | Format           | ID  | Format           |
|----|------------------------|-----|-------------------|-----|------------------|-----|------------------|
| 1  | $a1=R_{rs,450}$        | 271 | $(e1-e2)/(e1+e2)$ | 541 | $(1/a1-1/a2)*a3$ | 811 | $(e1-e2)/e3$     |
| 2  | $a2=R_{rs,560}$        | 272 | $(e1-e3)/(e1+e3)$ | 542 | $(1/a1-1/a2)*a4$ | 812 | $(e1-e2)/e4$     |
| 3  | $a3=R_{rs,650}$        | 273 | $(e1-e4)/(e1+e4)$ | 543 | $(1/a1-1/a2)*a5$ | 813 | $(e1-e2)/e5$     |
| 4  | $a4=R_{rs,730}$        | 274 | $(e1-e5)/(e1+e5)$ | 544 | $(1/a1-1/a3)*a2$ | 814 | $(e1-e3)/e2$     |
| 5  | $a5=R_{rs,840}$        | 275 | $(e2-e3)/(e2+e3)$ | 545 | $(1/a1-1/a3)*a4$ | 815 | $(e1-e3)/e4$     |
| 6  | $b1=1/R_{rs,450}$      | 276 | $(e2-e4)/(e2+e4)$ | 546 | $(1/a1-1/a3)*a5$ | 816 | $(e1-e3)/e5$     |
| 7  | $b2=1/R_{rs,560}$      | 277 | $(e2-e5)/(e2+e5)$ | 547 | $(1/a1-1/a4)*a2$ | 817 | $(e1-e4)/e2$     |
| 8  | $b3=1/R_{rs,650}$      | 278 | $(e3-e4)/(e3+e4)$ | 548 | $(1/a1-1/a4)*a3$ | 818 | $(e1-e4)/e3$     |
| 9  | $b4=1/R_{rs,730}$      | 279 | $(e3-e5)/(e3+e5)$ | 549 | $(1/a1-1/a4)*a5$ | 819 | $(e1-e4)/e5$     |
| 10 | $b5=1/R_{rs,840}$      | 280 | $(e4-e5)/(e4+e5)$ | 550 | $(1/a1-1/a5)*a2$ | 820 | $(e1-e5)/e2$     |
| 11 | $c1=\ln(R_{rs,450})$   | 281 | $f1+f2$           | 551 | $(1/a1-1/a5)*a3$ | 821 | $(e1-e5)/e3$     |
| 12 | $c2=\ln(R_{rs,560})$   | 282 | $f1+f3$           | 552 | $(1/a1-1/a5)*a4$ | 822 | $(e1-e5)/e4$     |
| 13 | $c3=\ln(R_{rs,650})$   | 283 | $f1+f4$           | 553 | $(1/a2-1/a3)*a1$ | 823 | $(e2-e3)/e1$     |
| 14 | $c4=\ln(R_{rs,730})$   | 284 | $f1+f5$           | 554 | $(1/a2-1/a3)*a4$ | 824 | $(e2-e3)/e4$     |
| 15 | $c5=\ln(R_{rs,840})$   | 285 | $f2+f3$           | 555 | $(1/a2-1/a3)*a5$ | 825 | $(e2-e3)/e5$     |
| 16 | $d1=\exp(R_{rs,450})$  | 286 | $f2+f4$           | 556 | $(1/a2-1/a4)*a1$ | 826 | $(e2-e4)/e1$     |
| 17 | $d2=\exp(R_{rs,560})$  | 287 | $f2+f5$           | 557 | $(1/a2-1/a4)*a3$ | 827 | $(e2-e4)/e3$     |
| 18 | $d3=\exp(R_{rs,650})$  | 288 | $f3+f4$           | 558 | $(1/a2-1/a4)*a5$ | 828 | $(e2-e4)/e5$     |
| 19 | $d4=\exp(R_{rs,730})$  | 289 | $f3+f5$           | 559 | $(1/a2-1/a5)*a1$ | 829 | $(e2-e5)/e1$     |
| 20 | $d5=\exp(R_{rs,840})$  | 290 | $f4+f5$           | 560 | $(1/a2-1/a5)*a3$ | 830 | $(e2-e5)/e3$     |
| 21 | $e1=(R_{rs,450})^2$    | 291 | $f1-f2$           | 561 | $(1/a2-1/a5)*a4$ | 831 | $(e2-e5)/e4$     |
| 22 | $e2=(R_{rs,560})^2$    | 292 | $f1-f3$           | 562 | $(1/a3-1/a4)*a1$ | 832 | $(e3-e4)/e1$     |
| 23 | $e3=(R_{rs,650})^2$    | 293 | $f1-f4$           | 563 | $(1/a3-1/a4)*a2$ | 833 | $(e3-e4)/e2$     |
| 24 | $e4=(R_{rs,730})^2$    | 294 | $f1-f5$           | 564 | $(1/a3-1/a4)*a5$ | 834 | $(e3-e4)/e5$     |
| 25 | $e5=(R_{rs,840})^2$    | 295 | $f2-f3$           | 565 | $(1/a3-1/a5)*a1$ | 835 | $(e3-e5)/e1$     |
| 26 | $f1=\sqrt{R_{rs,450}}$ | 296 | $f2-f4$           | 566 | $(1/a3-1/a5)*a2$ | 836 | $(e3-e5)/e2$     |
| 27 | $f2=\sqrt{R_{rs,560}}$ | 297 | $f2-f5$           | 567 | $(1/a3-1/a5)*a4$ | 837 | $(e3-e5)/e4$     |
| 28 | $f3=\sqrt{R_{rs,650}}$ | 298 | $f3-f4$           | 568 | $(1/a4-1/a5)*a1$ | 838 | $(e4-e5)/e1$     |
| 29 | $f4=\sqrt{R_{rs,730}}$ | 299 | $f3-f5$           | 569 | $(1/a4-1/a5)*a2$ | 839 | $(e4-e5)/e2$     |
| 30 | $f5=\sqrt{R_{rs,840}}$ | 300 | $f4-f5$           | 570 | $(1/a4-1/a5)*a3$ | 840 | $(e4-e5)/e3$     |
| 31 | $a1+a2$                | 301 | $f1*f2$           | 571 | $(a1-a2)/a3$     | 841 | $(1/f1-1/f2)*f3$ |
| 32 | $a1+a3$                | 302 | $f1*f3$           | 572 | $(a1-a2)/a4$     | 842 | $(1/f1-1/f2)*f4$ |
| 33 | $a1+a4$                | 303 | $f1*f4$           | 573 | $(a1-a2)/a5$     | 843 | $(1/f1-1/f2)*f5$ |
| 34 | $a1+a5$                | 304 | $f1*f5$           | 574 | $(a1-a3)/a2$     | 844 | $(1/f1-1/f3)*f2$ |
| 35 | $a2+a3$                | 305 | $f2*f3$           | 575 | $(a1-a3)/a4$     | 845 | $(1/f1-1/f3)*f4$ |
| 36 | $a2+a4$                | 306 | $f2*f4$           | 576 | $(a1-a3)/a5$     | 846 | $(1/f1-1/f3)*f5$ |
| 37 | $a2+a5$                | 307 | $f2*f5$           | 577 | $(a1-a4)/a2$     | 847 | $(1/f1-1/f4)*f2$ |
| 38 | $a3+a4$                | 308 | $f3*f4$           | 578 | $(a1-a4)/a3$     | 848 | $(1/f1-1/f4)*f3$ |
| 39 | $a3+a5$                | 309 | $f3*f5$           | 579 | $(a1-a4)/a5$     | 849 | $(1/f1-1/f4)*f5$ |

| ID | Format            | ID  | Format            | ID  | Format           | ID  | Format           |
|----|-------------------|-----|-------------------|-----|------------------|-----|------------------|
| 40 | $a4+a5$           | 310 | $f4*f5$           | 580 | $(a1-a5)/a2$     | 850 | $(1/f1-1/f5)*f2$ |
| 41 | $a1-a2$           | 311 | $f1/f2$           | 581 | $(a1-a5)/a3$     | 851 | $(1/f1-1/f5)*f3$ |
| 42 | $a1-a3$           | 312 | $f1/f3$           | 582 | $(a1-a5)/a4$     | 852 | $(1/f1-1/f5)*f4$ |
| 43 | $a1-a4$           | 313 | $f1/f4$           | 583 | $(a2-a3)/a1$     | 853 | $(1/f2-1/f3)*f1$ |
| 44 | $a1-a5$           | 314 | $f1/f5$           | 584 | $(a2-a3)/a4$     | 854 | $(1/f2-1/f3)*f4$ |
| 45 | $a2-a3$           | 315 | $f2/f3$           | 585 | $(a2-a3)/a5$     | 855 | $(1/f2-1/f3)*f5$ |
| 46 | $a2-a4$           | 316 | $f2/f4$           | 586 | $(a2-a4)/a1$     | 856 | $(1/f2-1/f4)*f1$ |
| 47 | $a2-a5$           | 317 | $f2/f5$           | 587 | $(a2-a4)/a3$     | 857 | $(1/f2-1/f4)*f3$ |
| 48 | $a3-a4$           | 318 | $f3/f4$           | 588 | $(a2-a4)/a5$     | 858 | $(1/f2-1/f4)*f5$ |
| 49 | $a3-a5$           | 319 | $f3/f5$           | 589 | $(a2-a5)/a1$     | 859 | $(1/f2-1/f5)*f1$ |
| 50 | $a4-a5$           | 320 | $f4/f5$           | 590 | $(a2-a5)/a3$     | 860 | $(1/f2-1/f5)*f3$ |
| 51 | $a1*a2$           | 321 | $(f1-f2)/(f1+f2)$ | 591 | $(a2-a5)/a4$     | 861 | $(1/f2-1/f5)*f4$ |
| 52 | $a1*a3$           | 322 | $(f1-f3)/(f1+f3)$ | 592 | $(a3-a4)/a1$     | 862 | $(1/f3-1/f4)*f1$ |
| 53 | $a1*a4$           | 323 | $(f1-f4)/(f1+f4)$ | 593 | $(a3-a4)/a2$     | 863 | $(1/f3-1/f4)*f2$ |
| 54 | $a1*a5$           | 324 | $(f1-f5)/(f1+f5)$ | 594 | $(a3-a4)/a5$     | 864 | $(1/f3-1/f4)*f5$ |
| 55 | $a2*a3$           | 325 | $(f2-f3)/(f2+f3)$ | 595 | $(a3-a5)/a1$     | 865 | $(1/f3-1/f5)*f1$ |
| 56 | $a2*a4$           | 326 | $(f2-f4)/(f2+f4)$ | 596 | $(a3-a5)/a2$     | 866 | $(1/f3-1/f5)*f2$ |
| 57 | $a2*a5$           | 327 | $(f2-f5)/(f2+f5)$ | 597 | $(a3-a5)/a4$     | 867 | $(1/f3-1/f5)*f4$ |
| 58 | $a3*a4$           | 328 | $(f3-f4)/(f3+f4)$ | 598 | $(a4-a5)/a1$     | 868 | $(1/f4-1/f5)*f1$ |
| 59 | $a3*a5$           | 329 | $(f3-f5)/(f3+f5)$ | 599 | $(a4-a5)/a2$     | 869 | $(1/f4-1/f5)*f2$ |
| 60 | $a4*a5$           | 330 | $(f4-f5)/(f4+f5)$ | 600 | $(a4-a5)/a3$     | 870 | $(1/f4-1/f5)*f3$ |
| 61 | $a1/a2$           | 331 | $1/(a1+a2)$       | 601 | $(1/b1-1/b2)*b3$ | 871 | $(f1-f2)/f3$     |
| 62 | $a1/a3$           | 332 | $1/(a1+a3)$       | 602 | $(1/b1-1/b2)*b4$ | 872 | $(f1-f2)/f4$     |
| 63 | $a1/a4$           | 333 | $1/(a1+a4)$       | 603 | $(1/b1-1/b2)*b5$ | 873 | $(f1-f2)/f5$     |
| 64 | $a1/a5$           | 334 | $1/(a1+a5)$       | 604 | $(1/b1-1/b3)*b2$ | 874 | $(f1-f3)/f2$     |
| 65 | $a2/a3$           | 335 | $1/(a2+a3)$       | 605 | $(1/b1-1/b3)*b4$ | 875 | $(f1-f3)/f4$     |
| 66 | $a2/a4$           | 336 | $1/(a2+a4)$       | 606 | $(1/b1-1/b3)*b5$ | 876 | $(f1-f3)/f5$     |
| 67 | $a2/a5$           | 337 | $1/(a2+a5)$       | 607 | $(1/b1-1/b4)*b2$ | 877 | $(f1-f4)/f2$     |
| 68 | $a3/a4$           | 338 | $1/(a3+a4)$       | 608 | $(1/b1-1/b4)*b3$ | 878 | $(f1-f4)/f3$     |
| 69 | $a3/a5$           | 339 | $1/(a3+a5)$       | 609 | $(1/b1-1/b4)*b5$ | 879 | $(f1-f4)/f5$     |
| 70 | $a4/a5$           | 340 | $1/(a4+a5)$       | 610 | $(1/b1-1/b5)*b2$ | 880 | $(f1-f5)/f2$     |
| 71 | $(a1-a2)/(a1+a2)$ | 341 | $1/(a1-a2)$       | 611 | $(1/b1-1/b5)*b3$ | 881 | $(f1-f5)/f3$     |
| 72 | $(a1-a3)/(a1+a3)$ | 342 | $1/(a1-a3)$       | 612 | $(1/b1-1/b5)*b4$ | 882 | $(f1-f5)/f4$     |
| 73 | $(a1-a4)/(a1+a4)$ | 343 | $1/(a1-a4)$       | 613 | $(1/b2-1/b3)*b1$ | 883 | $(f2-f3)/f1$     |
| 74 | $(a1-a5)/(a1+a5)$ | 344 | $1/(a1-a5)$       | 614 | $(1/b2-1/b3)*b4$ | 884 | $(f2-f3)/f4$     |
| 75 | $(a2-a3)/(a2+a3)$ | 345 | $1/(a2-a3)$       | 615 | $(1/b2-1/b3)*b5$ | 885 | $(f2-f3)/f5$     |
| 76 | $(a2-a4)/(a2+a4)$ | 346 | $1/(a2-a4)$       | 616 | $(1/b2-1/b4)*b1$ | 886 | $(f2-f4)/f1$     |
| 77 | $(a2-a5)/(a2+a5)$ | 347 | $1/(a2-a5)$       | 617 | $(1/b2-1/b4)*b3$ | 887 | $(f2-f4)/f3$     |
| 78 | $(a3-a4)/(a3+a4)$ | 348 | $1/(a3-a4)$       | 618 | $(1/b2-1/b4)*b5$ | 888 | $(f2-f4)/f5$     |
| 79 | $(a3-a5)/(a3+a5)$ | 349 | $1/(a3-a5)$       | 619 | $(1/b2-1/b5)*b1$ | 889 | $(f2-f5)/f1$     |

| ID  | Format            | ID  | Format                | ID  | Format           | ID  | Format            |
|-----|-------------------|-----|-----------------------|-----|------------------|-----|-------------------|
| 80  | $(a4-a5)/(a4+a5)$ | 350 | $1/(a4-a5)$           | 620 | $(1/b2-1/b5)*b3$ | 890 | $(f2-f5)/f3$      |
| 81  | $b1+b2$           | 351 | $1/(a1*a2)$           | 621 | $(1/b2-1/b5)*b4$ | 891 | $(f2-f5)/f4$      |
| 82  | $b1+b3$           | 352 | $1/(a1*a3)$           | 622 | $(1/b3-1/b4)*b1$ | 892 | $(f3-f4)/f1$      |
| 83  | $b1+b4$           | 353 | $1/(a1*a4)$           | 623 | $(1/b3-1/b4)*b2$ | 893 | $(f3-f4)/f2$      |
| 84  | $b1+b5$           | 354 | $1/(a1*a5)$           | 624 | $(1/b3-1/b4)*b5$ | 894 | $(f3-f4)/f5$      |
| 85  | $b2+b3$           | 355 | $1/(a2*a3)$           | 625 | $(1/b3-1/b5)*b1$ | 895 | $(f3-f5)/f1$      |
| 86  | $b2+b4$           | 356 | $1/(a2*a4)$           | 626 | $(1/b3-1/b5)*b2$ | 896 | $(f3-f5)/f2$      |
| 87  | $b2+b5$           | 357 | $1/(a2*a5)$           | 627 | $(1/b3-1/b5)*b4$ | 897 | $(f3-f5)/f4$      |
| 88  | $b3+b4$           | 358 | $1/(a3*a4)$           | 628 | $(1/b4-1/b5)*b1$ | 898 | $(f4-f5)/f1$      |
| 89  | $b3+b5$           | 359 | $1/(a3*a5)$           | 629 | $(1/b4-1/b5)*b2$ | 899 | $(f4-f5)/f2$      |
| 90  | $b4+b5$           | 360 | $1/(a4*a5)$           | 630 | $(1/b4-1/b5)*b3$ | 900 | $(f4-f5)/f3$      |
| 91  | $b1-b2$           | 361 | $1/(a1/a2)$           | 631 | $(b1-b2)/b3$     | 901 | $(a1-a2)/(a3+a4)$ |
| 92  | $b1-b3$           | 362 | $1/(a1/a3)$           | 632 | $(b1-b2)/b4$     | 902 | $(a1-a2)/(a3+a5)$ |
| 93  | $b1-b4$           | 363 | $1/(a1/a4)$           | 633 | $(b1-b2)/b5$     | 903 | $(a1-a2)/(a4+a5)$ |
| 94  | $b1-b5$           | 364 | $1/(a1/a5)$           | 634 | $(b1-b3)/b2$     | 904 | $(a1-a3)/(a2+a4)$ |
| 95  | $b2-b3$           | 365 | $1/(a2/a3)$           | 635 | $(b1-b3)/b4$     | 905 | $(a1-a3)/(a2+a5)$ |
| 96  | $b2-b4$           | 366 | $1/(a2/a4)$           | 636 | $(b1-b3)/b5$     | 906 | $(a1-a3)/(a4+a5)$ |
| 97  | $b2-b5$           | 367 | $1/(a2/a5)$           | 637 | $(b1-b4)/b2$     | 907 | $(a1-a4)/(a2+a3)$ |
| 98  | $b3-b4$           | 368 | $1/(a3/a4)$           | 638 | $(b1-b4)/b3$     | 908 | $(a1-a4)/(a2+a5)$ |
| 99  | $b3-b5$           | 369 | $1/(a3/a5)$           | 639 | $(b1-b4)/b5$     | 909 | $(a1-a4)/(a3+a5)$ |
| 100 | $b4-b5$           | 370 | $1/(a4/a5)$           | 640 | $(b1-b5)/b2$     | 910 | $(a1-a5)/(a2+a3)$ |
| 101 | $b1*b2$           | 371 | $1/((a1-a2)/(a1+a2))$ | 641 | $(b1-b5)/b3$     | 911 | $(a1-a5)/(a2+a4)$ |
| 102 | $b1*b3$           | 372 | $1/((a1-a3)/(a1+a3))$ | 642 | $(b1-b5)/b4$     | 912 | $(a1-a5)/(a3+a4)$ |
| 103 | $b1*b4$           | 373 | $1/((a1-a4)/(a1+a4))$ | 643 | $(b2-b3)/b1$     | 913 | $(a2-a3)/(a1+a4)$ |
| 104 | $b1*b5$           | 374 | $1/((a1-a5)/(a1+a5))$ | 644 | $(b2-b3)/b4$     | 914 | $(a2-a3)/(a1+a5)$ |
| 105 | $b2*b3$           | 375 | $1/((a2-a3)/(a2+a3))$ | 645 | $(b2-b3)/b5$     | 915 | $(a2-a3)/(a4+a5)$ |
| 106 | $b2*b4$           | 376 | $1/((a2-a4)/(a2+a4))$ | 646 | $(b2-b4)/b1$     | 916 | $(a2-a4)/(a1+a3)$ |
| 107 | $b2*b5$           | 377 | $1/((a2-a5)/(a2+a5))$ | 647 | $(b2-b4)/b3$     | 917 | $(a2-a4)/(a1+a5)$ |
| 108 | $b3*b4$           | 378 | $1/((a3-a4)/(a3+a4))$ | 648 | $(b2-b4)/b5$     | 918 | $(a2-a4)/(a3+a5)$ |
| 109 | $b3*b5$           | 379 | $1/((a3-a5)/(a3+a5))$ | 649 | $(b2-b5)/b1$     | 919 | $(a2-a5)/(a1+a3)$ |
| 110 | $b4*b5$           | 380 | $1/((a4-a5)/(a4+a5))$ | 650 | $(b2-b5)/b3$     | 920 | $(a2-a5)/(a1+a4)$ |
| 111 | $b1/b2$           | 381 | $\ln/(a1+a2)$         | 651 | $(b2-b5)/b4$     | 921 | $(a2-a5)/(a3+a4)$ |
| 112 | $b1/b3$           | 382 | $\ln/(a1+a3)$         | 652 | $(b3-b4)/b1$     | 922 | $(a3-a4)/(a1+a2)$ |
| 113 | $b1/b4$           | 383 | $\ln/(a1+a4)$         | 653 | $(b3-b4)/b2$     | 923 | $(a3-a4)/(a1+a5)$ |
| 114 | $b1/b5$           | 384 | $\ln/(a1+a5)$         | 654 | $(b3-b4)/b5$     | 924 | $(a3-a4)/(a2+a5)$ |
| 115 | $b2/b3$           | 385 | $\ln/(a2+a3)$         | 655 | $(b3-b5)/b1$     | 925 | $(a3-a5)/(a1+a2)$ |
| 116 | $b2/b4$           | 386 | $\ln/(a2+a4)$         | 656 | $(b3-b5)/b2$     | 926 | $(a3-a5)/(a1+a4)$ |
| 117 | $b2/b5$           | 387 | $\ln/(a2+a5)$         | 657 | $(b3-b5)/b4$     | 927 | $(a3-a5)/(a2+a4)$ |
| 118 | $b3/b4$           | 388 | $\ln/(a3+a4)$         | 658 | $(b4-b5)/b1$     | 928 | $(a4-a5)/(a1+a2)$ |
| 119 | $b3/b5$           | 389 | $\ln/(a3+a5)$         | 659 | $(b4-b5)/b2$     | 929 | $(a4-a5)/(a1+a3)$ |

| ID  | Format            | ID  | Format        | ID  | Format           | ID  | Format            |
|-----|-------------------|-----|---------------|-----|------------------|-----|-------------------|
| 120 | $b4/b5$           | 390 | $\ln/(a4+a5)$ | 660 | $(b4-b5)/b3$     | 930 | $(a4-a5)/(a2+a3)$ |
| 121 | $(b1-b2)/(b1+b2)$ | 391 | $\ln/(a1*a2)$ | 661 | $(1/c1-1/c2)*c3$ | 931 | $(b1-b2)/(b3+b4)$ |
| 122 | $(b1-b3)/(b1+b3)$ | 392 | $\ln/(a1*a3)$ | 662 | $(1/c1-1/c2)*c4$ | 932 | $(b1-b2)/(b3+b5)$ |
| 123 | $(b1-b4)/(b1+b4)$ | 393 | $\ln/(a1*a4)$ | 663 | $(1/c1-1/c2)*c5$ | 933 | $(b1-b2)/(b4+b5)$ |
| 124 | $(b1-b5)/(b1+b5)$ | 394 | $\ln/(a1*a5)$ | 664 | $(1/c1-1/c3)*c2$ | 934 | $(b1-b3)/(b2+b4)$ |
| 125 | $(b2-b3)/(b2+b3)$ | 395 | $\ln/(a2*a3)$ | 665 | $(1/c1-1/c3)*c4$ | 935 | $(b1-b3)/(b2+b5)$ |
| 126 | $(b2-b4)/(b2+b4)$ | 396 | $\ln/(a2*a4)$ | 666 | $(1/c1-1/c3)*c5$ | 936 | $(b1-b3)/(b4+b5)$ |
| 127 | $(b2-b5)/(b2+b5)$ | 397 | $\ln/(a2*a5)$ | 667 | $(1/c1-1/c4)*c2$ | 937 | $(b1-b4)/(b2+b3)$ |
| 128 | $(b3-b4)/(b3+b4)$ | 398 | $\ln/(a3*a4)$ | 668 | $(1/c1-1/c4)*c3$ | 938 | $(b1-b4)/(b2+b5)$ |
| 129 | $(b3-b5)/(b3+b5)$ | 399 | $\ln/(a3*a5)$ | 669 | $(1/c1-1/c4)*c5$ | 939 | $(b1-b4)/(b3+b5)$ |
| 130 | $(b4-b5)/(b4+b5)$ | 400 | $\ln/(a4*a5)$ | 670 | $(1/c1-1/c5)*c2$ | 940 | $(b1-b5)/(b2+b3)$ |
| 131 | $c1+c2$           | 401 | $\ln/(a1/a2)$ | 671 | $(1/c1-1/c5)*c3$ | 941 | $(b1-b5)/(b2+b4)$ |
| 132 | $c1+c3$           | 402 | $\ln/(a1/a3)$ | 672 | $(1/c1-1/c5)*c4$ | 942 | $(b1-b5)/(b3+b4)$ |
| 133 | $c1+c4$           | 403 | $\ln/(a1/a4)$ | 673 | $(1/c2-1/c3)*c1$ | 943 | $(b2-b3)/(b1+b4)$ |
| 134 | $c1+c5$           | 404 | $\ln/(a1/a5)$ | 674 | $(1/c2-1/c3)*c4$ | 944 | $(b2-b3)/(b1+b5)$ |
| 135 | $c2+c3$           | 405 | $\ln/(a2/a3)$ | 675 | $(1/c2-1/c3)*c5$ | 945 | $(b2-b3)/(b4+b5)$ |
| 136 | $c2+c4$           | 406 | $\ln/(a2/a4)$ | 676 | $(1/c2-1/c4)*c1$ | 946 | $(b2-b4)/(b1+b3)$ |
| 137 | $c2+c5$           | 407 | $\ln/(a2/a5)$ | 677 | $(1/c2-1/c4)*c3$ | 947 | $(b2-b4)/(b1+b5)$ |
| 138 | $c3+c4$           | 408 | $\ln/(a3/a4)$ | 678 | $(1/c2-1/c4)*c5$ | 948 | $(b2-b4)/(b3+b5)$ |
| 139 | $c3+c5$           | 409 | $\ln/(a3/a5)$ | 679 | $(1/c2-1/c5)*c1$ | 949 | $(b2-b5)/(b1+b3)$ |
| 140 | $c4+c5$           | 410 | $\ln/(a4/a5)$ | 680 | $(1/c2-1/c5)*c3$ | 950 | $(b2-b5)/(b1+b4)$ |
| 141 | $c1-c2$           | 411 | $\exp(a1+a2)$ | 681 | $(1/c2-1/c5)*c4$ | 951 | $(b2-b5)/(b3+b4)$ |
| 142 | $c1-c3$           | 412 | $\exp(a1+a3)$ | 682 | $(1/c3-1/c4)*c1$ | 952 | $(b3-b4)/(b1+b2)$ |
| 143 | $c1-c4$           | 413 | $\exp(a1+a4)$ | 683 | $(1/c3-1/c4)*c2$ | 953 | $(b3-b4)/(b1+b5)$ |
| 144 | $c1-c5$           | 414 | $\exp(a1+a5)$ | 684 | $(1/c3-1/c4)*c5$ | 954 | $(b3-b4)/(b2+b5)$ |
| 145 | $c2-c3$           | 415 | $\exp(a2+a3)$ | 685 | $(1/c3-1/c5)*c1$ | 955 | $(b3-b5)/(b1+b2)$ |
| 146 | $c2-c4$           | 416 | $\exp(a2+a4)$ | 686 | $(1/c3-1/c5)*c2$ | 956 | $(b3-b5)/(b1+b4)$ |
| 147 | $c2-c5$           | 417 | $\exp(a2+a5)$ | 687 | $(1/c3-1/c5)*c4$ | 957 | $(b3-b5)/(b2+b4)$ |
| 148 | $c3-c4$           | 418 | $\exp(a3+a4)$ | 688 | $(1/c4-1/c5)*c1$ | 958 | $(b4-b5)/(b1+b2)$ |
| 149 | $c3-c5$           | 419 | $\exp(a3+a5)$ | 689 | $(1/c4-1/c5)*c2$ | 959 | $(b4-b5)/(b1+b3)$ |
| 150 | $c4-c5$           | 420 | $\exp(a4+a5)$ | 690 | $(1/c4-1/c5)*c3$ | 960 | $(b4-b5)/(b2+b3)$ |
| 151 | $c1*c2$           | 421 | $\exp(a1-a2)$ | 691 | $(c1-c2)/c3$     | 961 | $(c1-c2)/(c3+c4)$ |
| 152 | $c1*c3$           | 422 | $\exp(a1-a3)$ | 692 | $(c1-c2)/c4$     | 962 | $(c1-c2)/(c3+c5)$ |
| 153 | $c1*c4$           | 423 | $\exp(a1-a4)$ | 693 | $(c1-c2)/c5$     | 963 | $(c1-c2)/(c4+c5)$ |
| 154 | $c1*c5$           | 424 | $\exp(a1-a5)$ | 694 | $(c1-c3)/c2$     | 964 | $(c1-c3)/(c2+c4)$ |
| 155 | $c2*c3$           | 425 | $\exp(a2-a3)$ | 695 | $(c1-c3)/c4$     | 965 | $(c1-c3)/(c2+c5)$ |
| 156 | $c2*c4$           | 426 | $\exp(a2-a4)$ | 696 | $(c1-c3)/c5$     | 966 | $(c1-c3)/(c4+c5)$ |
| 157 | $c2*c5$           | 427 | $\exp(a2-a5)$ | 697 | $(c1-c4)/c2$     | 967 | $(c1-c4)/(c2+c3)$ |
| 158 | $c3*c4$           | 428 | $\exp(a3-a4)$ | 698 | $(c1-c4)/c3$     | 968 | $(c1-c4)/(c2+c5)$ |
| 159 | $c3*c5$           | 429 | $\exp(a3-a5)$ | 699 | $(c1-c4)/c5$     | 969 | $(c1-c4)/(c3+c5)$ |



| ID  | Format            | ID  | Format                  | ID  | Format           | ID   | Format            |
|-----|-------------------|-----|-------------------------|-----|------------------|------|-------------------|
| 160 | $c4*c5$           | 430 | $\exp(a4-a5)$           | 700 | $(c1-c5)/c2$     | 970  | $(c1-c5)/(c2+c3)$ |
| 161 | $c1/c2$           | 431 | $\exp(a1*a2)$           | 701 | $(c1-c5)/c3$     | 971  | $(c1-c5)/(c2+c4)$ |
| 162 | $c1/c3$           | 432 | $\exp(a1*a3)$           | 702 | $(c1-c5)/c4$     | 972  | $(c1-c5)/(c3+c4)$ |
| 163 | $c1/c4$           | 433 | $\exp(a1*a4)$           | 703 | $(c2-c3)/c1$     | 973  | $(c2-c3)/(c1+c4)$ |
| 164 | $c1/c5$           | 434 | $\exp(a1*a5)$           | 704 | $(c2-c3)/c4$     | 974  | $(c2-c3)/(c1+c5)$ |
| 165 | $c2/c3$           | 435 | $\exp(a2*a3)$           | 705 | $(c2-c3)/c5$     | 975  | $(c2-c3)/(c4+c5)$ |
| 166 | $c2/c4$           | 436 | $\exp(a2*a4)$           | 706 | $(c2-c4)/c1$     | 976  | $(c2-c4)/(c1+c3)$ |
| 167 | $c2/c5$           | 437 | $\exp(a2*a5)$           | 707 | $(c2-c4)/c3$     | 977  | $(c2-c4)/(c1+c5)$ |
| 168 | $c3/c4$           | 438 | $\exp(a3*a4)$           | 708 | $(c2-c4)/c5$     | 978  | $(c2-c4)/(c3+c5)$ |
| 169 | $c3/c5$           | 439 | $\exp(a3*a5)$           | 709 | $(c2-c5)/c1$     | 979  | $(c2-c5)/(c1+c3)$ |
| 170 | $c4/c5$           | 440 | $\exp(a4*a5)$           | 710 | $(c2-c5)/c3$     | 980  | $(c2-c5)/(c1+c4)$ |
| 171 | $(c1-c2)/(c1+c2)$ | 441 | $\exp(a1/a2)$           | 711 | $(c2-c5)/c4$     | 981  | $(c2-c5)/(c3+c4)$ |
| 172 | $(c1-c3)/(c1+c3)$ | 442 | $\exp(a1/a3)$           | 712 | $(c3-c4)/c1$     | 982  | $(c3-c4)/(c1+c2)$ |
| 173 | $(c1-c4)/(c1+c4)$ | 443 | $\exp(a1/a4)$           | 713 | $(c3-c4)/c2$     | 983  | $(c3-c4)/(c1+c5)$ |
| 174 | $(c1-c5)/(c1+c5)$ | 444 | $\exp(a1/a5)$           | 714 | $(c3-c4)/c5$     | 984  | $(c3-c4)/(c2+c5)$ |
| 175 | $(c2-c3)/(c2+c3)$ | 445 | $\exp(a2/a3)$           | 715 | $(c3-c5)/c1$     | 985  | $(c3-c5)/(c1+c2)$ |
| 176 | $(c2-c4)/(c2+c4)$ | 446 | $\exp(a2/a4)$           | 716 | $(c3-c5)/c2$     | 986  | $(c3-c5)/(c1+c4)$ |
| 177 | $(c2-c5)/(c2+c5)$ | 447 | $\exp(a2/a5)$           | 717 | $(c3-c5)/c4$     | 987  | $(c3-c5)/(c2+c4)$ |
| 178 | $(c3-c4)/(c3+c4)$ | 448 | $\exp(a3/a4)$           | 718 | $(c4-c5)/c1$     | 988  | $(c4-c5)/(c1+c2)$ |
| 179 | $(c3-c5)/(c3+c5)$ | 449 | $\exp(a3/a5)$           | 719 | $(c4-c5)/c2$     | 989  | $(c4-c5)/(c1+c3)$ |
| 180 | $(c4-c5)/(c4+c5)$ | 450 | $\exp(a4/a5)$           | 720 | $(c4-c5)/c3$     | 990  | $(c4-c5)/(c2+c3)$ |
| 181 | $d1+d2$           | 451 | $\exp((a1-a2)/(a1+a2))$ | 721 | $(1/d1-1/d2)*d3$ | 991  | $(d1-d2)/(d3+d4)$ |
| 182 | $d1+d3$           | 452 | $\exp((a1-a3)/(a1+a3))$ | 722 | $(1/d1-1/d2)*d4$ | 992  | $(d1-d2)/(d3+d5)$ |
| 183 | $d1+d4$           | 453 | $\exp((a1-a4)/(a1+a4))$ | 723 | $(1/d1-1/d2)*d5$ | 993  | $(d1-d2)/(d4+d5)$ |
| 184 | $d1+d5$           | 454 | $\exp((a1-a5)/(a1+a5))$ | 724 | $(1/d1-1/d3)*d2$ | 994  | $(d1-d3)/(d2+d4)$ |
| 185 | $d2+d3$           | 455 | $\exp((a2-a3)/(a2+a3))$ | 725 | $(1/d1-1/d3)*d4$ | 995  | $(d1-d3)/(d2+d5)$ |
| 186 | $d2+d4$           | 456 | $\exp((a2-a4)/(a2+a4))$ | 726 | $(1/d1-1/d3)*d5$ | 996  | $(d1-d3)/(d4+d5)$ |
| 187 | $d2+d5$           | 457 | $\exp((a2-a5)/(a2+a5))$ | 727 | $(1/d1-1/d4)*d2$ | 997  | $(d1-d4)/(d2+d3)$ |
| 188 | $d3+d4$           | 458 | $\exp((a3-a4)/(a3+a4))$ | 728 | $(1/d1-1/d4)*d3$ | 998  | $(d1-d4)/(d2+d5)$ |
| 189 | $d3+d5$           | 459 | $\exp((a3-a5)/(a3+a5))$ | 729 | $(1/d1-1/d4)*d5$ | 999  | $(d1-d4)/(d3+d5)$ |
| 190 | $d4+d5$           | 460 | $\exp((a4-a5)/(a4+a5))$ | 730 | $(1/d1-1/d5)*d2$ | 1000 | $(d1-d5)/(d2+d3)$ |
| 191 | $d1-d2$           | 461 | $(a1+a2)^2$             | 731 | $(1/d1-1/d5)*d3$ | 1001 | $(d1-d5)/(d2+d4)$ |
| 192 | $d1-d3$           | 462 | $(a1+a3)^2$             | 732 | $(1/d1-1/d5)*d4$ | 1002 | $(d1-d5)/(d3+d4)$ |
| 193 | $d1-d4$           | 463 | $(a1+a4)^2$             | 733 | $(1/d2-1/d3)*d1$ | 1003 | $(d2-d3)/(d1+d4)$ |
| 194 | $d1-d5$           | 464 | $(a1+a5)^2$             | 734 | $(1/d2-1/d3)*d4$ | 1004 | $(d2-d3)/(d1+d5)$ |
| 195 | $d2-d3$           | 465 | $(a2+a3)^2$             | 735 | $(1/d2-1/d3)*d5$ | 1005 | $(d2-d3)/(d4+d5)$ |
| 196 | $d2-d4$           | 466 | $(a2+a4)^2$             | 736 | $(1/d2-1/d4)*d1$ | 1006 | $(d2-d4)/(d1+d3)$ |
| 197 | $d2-d5$           | 467 | $(a2+a5)^2$             | 737 | $(1/d2-1/d4)*d3$ | 1007 | $(d2-d4)/(d1+d5)$ |
| 198 | $d3-d4$           | 468 | $(a3+a4)^2$             | 738 | $(1/d2-1/d4)*d5$ | 1008 | $(d2-d4)/(d3+d5)$ |
| 199 | $d3-d5$           | 469 | $(a3+a5)^2$             | 739 | $(1/d2-1/d5)*d1$ | 1009 | $(d2-d5)/(d1+d3)$ |

| ID  | Format          | ID  | Format              | ID  | Format         | ID   | Format          |
|-----|-----------------|-----|---------------------|-----|----------------|------|-----------------|
| 200 | d4-d5           | 470 | (a4+a5)^2           | 740 | (1/d2-1/d5)*d3 | 1010 | (d2-d5)/(d1+d4) |
| 201 | d1*d2           | 471 | (a1-a2)^2           | 741 | (1/d2-1/d5)*d4 | 1011 | (d2-d5)/(d3+d4) |
| 202 | d1*d3           | 472 | (a1-a3)^2           | 742 | (1/d3-1/d4)*d1 | 1012 | (d3-d4)/(d1+d2) |
| 203 | d1*d4           | 473 | (a1-a4)^2           | 743 | (1/d3-1/d4)*d2 | 1013 | (d3-d4)/(d1+d5) |
| 204 | d1*d5           | 474 | (a1-a5)^2           | 744 | (1/d3-1/d4)*d5 | 1014 | (d3-d4)/(d2+d5) |
| 205 | d2*d3           | 475 | (a2-a3)^2           | 745 | (1/d3-1/d5)*d1 | 1015 | (d3-d5)/(d1+d2) |
| 206 | d2*d4           | 476 | (a2-a4)^2           | 746 | (1/d3-1/d5)*d2 | 1016 | (d3-d5)/(d1+d4) |
| 207 | d2*d5           | 477 | (a2-a5)^2           | 747 | (1/d3-1/d5)*d4 | 1017 | (d3-d5)/(d2+d4) |
| 208 | d3*d4           | 478 | (a3-a4)^2           | 748 | (1/d4-1/d5)*d1 | 1018 | (d4-d5)/(d1+d2) |
| 209 | d3*d5           | 479 | (a3-a5)^2           | 749 | (1/d4-1/d5)*d2 | 1019 | (d4-d5)/(d1+d3) |
| 210 | d4*d5           | 480 | (a4-a5)^2           | 750 | (1/d4-1/d5)*d3 | 1020 | (d4-d5)/(d2+d3) |
| 211 | d1/d2           | 481 | (a1*a2)^2           | 751 | (d1-d2)/d3     | 1021 | (b1-b2)/(b3+b4) |
| 212 | d1/d3           | 482 | (a1*a3)^2           | 752 | (d1-d2)/d4     | 1022 | (b1-b2)/(b3+b5) |
| 213 | d1/d4           | 483 | (a1*a4)^2           | 753 | (d1-d2)/d5     | 1023 | (b1-b2)/(b4+b5) |
| 214 | d1/d5           | 484 | (a1*a5)^2           | 754 | (d1-d3)/d2     | 1024 | (b1-b3)/(b2+b4) |
| 215 | d2/d3           | 485 | (a2*a3)^2           | 755 | (d1-d3)/d4     | 1025 | (b1-b3)/(b2+b5) |
| 216 | d2/d4           | 486 | (a2*a4)^2           | 756 | (d1-d3)/d5     | 1026 | (b1-b3)/(b4+b5) |
| 217 | d2/d5           | 487 | (a2*a5)^2           | 757 | (d1-d4)/d2     | 1027 | (b1-b4)/(b2+b3) |
| 218 | d3/d4           | 488 | (a3*a4)^2           | 758 | (d1-d4)/d3     | 1028 | (b1-b4)/(b2+b5) |
| 219 | d3/d5           | 489 | (a3*a5)^2           | 759 | (d1-d4)/d5     | 1029 | (b1-b4)/(b3+b5) |
| 220 | d4/d5           | 490 | (a4*a5)^2           | 760 | (d1-d5)/d2     | 1030 | (b1-b5)/(b2+b3) |
| 221 | (d1-d2)/(d1+d2) | 491 | (a1/a2)^2           | 761 | (d1-d5)/d3     | 1031 | (b1-b5)/(b2+b4) |
| 222 | (d1-d3)/(d1+d3) | 492 | (a1/a3)^2           | 762 | (d1-d5)/d4     | 1032 | (b1-b5)/(b3+b4) |
| 223 | (d1-d4)/(d1+d4) | 493 | (a1/a4)^2           | 763 | (d2-d3)/d1     | 1033 | (b2-b3)/(b1+b4) |
| 224 | (d1-d5)/(d1+d5) | 494 | (a1/a5)^2           | 764 | (d2-d3)/d4     | 1034 | (b2-b3)/(b1+b5) |
| 225 | (d2-d3)/(d2+d3) | 495 | (a2/a3)^2           | 765 | (d2-d3)/d5     | 1035 | (b2-b3)/(b4+b5) |
| 226 | (d2-d4)/(d2+d4) | 496 | (a2/a4)^2           | 766 | (d2-d4)/d1     | 1036 | (b2-b4)/(b1+b3) |
| 227 | (d2-d5)/(d2+d5) | 497 | (a2/a5)^2           | 767 | (d2-d4)/d3     | 1037 | (b2-b4)/(b1+b5) |
| 228 | (d3-d4)/(d3+d4) | 498 | (a3/a4)^2           | 768 | (d2-d4)/d5     | 1038 | (b2-b4)/(b3+b5) |
| 229 | (d3-d5)/(d3+d5) | 499 | (a3/a5)^2           | 769 | (d2-d5)/d1     | 1039 | (b2-b5)/(b1+b3) |
| 230 | (d4-d5)/(d4+d5) | 500 | (a4/a5)^2           | 770 | (d2-d5)/d3     | 1040 | (b2-b5)/(b1+b4) |
| 231 | e1+e2           | 501 | ((a1-a2)/(a1+a2))^2 | 771 | (d2-d5)/d4     | 1041 | (b2-b5)/(b3+b4) |
| 232 | e1+e3           | 502 | ((a1-a3)/(a1+a3))^2 | 772 | (d3-d4)/d1     | 1042 | (b3-b4)/(b1+b2) |
| 233 | e1+e4           | 503 | ((a1-a4)/(a1+a4))^2 | 773 | (d3-d4)/d2     | 1043 | (b3-b4)/(b1+b5) |
| 234 | e1+e5           | 504 | ((a1-a5)/(a1+a5))^2 | 774 | (d3-d4)/d5     | 1044 | (b3-b4)/(b2+b5) |
| 235 | e2+e3           | 505 | ((a2-a3)/(a2+a3))^2 | 775 | (d3-d5)/d1     | 1045 | (b3-b5)/(b1+b2) |
| 236 | e2+e4           | 506 | ((a2-a4)/(a2+a4))^2 | 776 | (d3-d5)/d2     | 1046 | (b3-b5)/(b1+b4) |
| 237 | e2+e5           | 507 | ((a2-a5)/(a2+a5))^2 | 777 | (d3-d5)/d4     | 1047 | (b3-b5)/(b2+b4) |
| 238 | e3+e4           | 508 | ((a3-a4)/(a3+a4))^2 | 778 | (d4-d5)/d1     | 1048 | (b4-b5)/(b1+b2) |
| 239 | e3+e5           | 509 | ((a3-a5)/(a3+a5))^2 | 779 | (d4-d5)/d2     | 1049 | (b4-b5)/(b1+b3) |

| ID  | Format | ID  | Format                | ID  | Format           | ID   | Format            |
|-----|--------|-----|-----------------------|-----|------------------|------|-------------------|
| 240 | e4+e5  | 510 | $((a4-a5)/(a4+a5))^2$ | 780 | $(d4-d5)/d3$     | 1050 | $(b4-b5)/(b2+b3)$ |
| 241 | e1-e2  | 511 | $\sqrt{a1+a2}$        | 781 | $(1/e1-1/e2)*e3$ | 1051 | $(f1-f2)/(f3+f4)$ |
| 242 | e1-e3  | 512 | $\sqrt{a1+a3}$        | 782 | $(1/e1-1/e2)*e4$ | 1052 | $(f1-f2)/(f3+f5)$ |
| 243 | e1-e4  | 513 | $\sqrt{a1+a4}$        | 783 | $(1/e1-1/e2)*e5$ | 1053 | $(f1-f2)/(f4+f5)$ |
| 244 | e1-e5  | 514 | $\sqrt{a1+a5}$        | 784 | $(1/e1-1/e3)*e2$ | 1054 | $(f1-f3)/(f2+f4)$ |
| 245 | e2-e3  | 515 | $\sqrt{a2+a3}$        | 785 | $(1/e1-1/e3)*e4$ | 1055 | $(f1-f3)/(f2+f5)$ |
| 246 | e2-e4  | 516 | $\sqrt{a2+a4}$        | 786 | $(1/e1-1/e3)*e5$ | 1056 | $(f1-f3)/(f4+f5)$ |
| 247 | e2-e5  | 517 | $\sqrt{a2+a5}$        | 787 | $(1/e1-1/e4)*e2$ | 1057 | $(f1-f4)/(f2+f3)$ |
| 248 | e3-e4  | 518 | $\sqrt{a3+a4}$        | 788 | $(1/e1-1/e4)*e3$ | 1058 | $(f1-f4)/(f2+f5)$ |
| 249 | e3-e5  | 519 | $\sqrt{a3+a5}$        | 789 | $(1/e1-1/e4)*e5$ | 1059 | $(f1-f4)/(f3+f5)$ |
| 250 | e4-e5  | 520 | $\sqrt{a4+a5}$        | 790 | $(1/e1-1/e5)*e2$ | 1060 | $(f1-f5)/(f2+f3)$ |
| 251 | e1*e2  | 521 | $\sqrt{a1*a2}$        | 791 | $(1/e1-1/e5)*e3$ | 1061 | $(f1-f5)/(f2+f4)$ |
| 252 | e1*e3  | 522 | $\sqrt{a1*a3}$        | 792 | $(1/e1-1/e5)*e4$ | 1062 | $(f1-f5)/(f3+f4)$ |
| 253 | e1*e4  | 523 | $\sqrt{a1*a4}$        | 793 | $(1/e2-1/e3)*e1$ | 1063 | $(f2-f3)/(f1+f4)$ |
| 254 | e1*e5  | 524 | $\sqrt{a1*a5}$        | 794 | $(1/e2-1/e3)*e4$ | 1064 | $(f2-f3)/(f1+f5)$ |
| 255 | e2*e3  | 525 | $\sqrt{a2*a3}$        | 795 | $(1/e2-1/e3)*e5$ | 1065 | $(f2-f3)/(f4+f5)$ |
| 256 | e2*e4  | 526 | $\sqrt{a2*a4}$        | 796 | $(1/e2-1/e4)*e1$ | 1066 | $(f2-f4)/(f1+f3)$ |
| 257 | e2*e5  | 527 | $\sqrt{a2*a5}$        | 797 | $(1/e2-1/e4)*e3$ | 1067 | $(f2-f4)/(f1+f5)$ |
| 258 | e3*e4  | 528 | $\sqrt{a3*a4}$        | 798 | $(1/e2-1/e4)*e5$ | 1068 | $(f2-f4)/(f3+f5)$ |
| 259 | e3*e5  | 529 | $\sqrt{a3*a5}$        | 799 | $(1/e2-1/e5)*e1$ | 1069 | $(f2-f5)/(f1+f3)$ |
| 260 | e4*e5  | 530 | $\sqrt{a4*a5}$        | 800 | $(1/e2-1/e5)*e3$ | 1070 | $(f2-f5)/(f1+f4)$ |
| 261 | e1/e2  | 531 | $\sqrt{a1/a2}$        | 801 | $(1/e2-1/e5)*e4$ | 1071 | $(f2-f5)/(f3+f4)$ |
| 262 | e1/e3  | 532 | $\sqrt{a1/a3}$        | 802 | $(1/e3-1/e4)*e1$ | 1072 | $(f3-f4)/(f1+f2)$ |
| 263 | e1/e4  | 533 | $\sqrt{a1/a4}$        | 803 | $(1/e3-1/e4)*e2$ | 1073 | $(f3-f4)/(f1+f5)$ |
| 264 | e1/e5  | 534 | $\sqrt{a1/a5}$        | 804 | $(1/e3-1/e4)*e5$ | 1074 | $(f3-f4)/(f2+f5)$ |
| 265 | e2/e3  | 535 | $\sqrt{a2/a3}$        | 805 | $(1/e3-1/e5)*e1$ | 1075 | $(f3-f5)/(f1+f2)$ |
| 266 | e2/e4  | 536 | $\sqrt{a2/a4}$        | 806 | $(1/e3-1/e5)*e2$ | 1076 | $(f3-f5)/(f1+f4)$ |
| 267 | e2/e5  | 537 | $\sqrt{a2/a5}$        | 807 | $(1/e3-1/e5)*e4$ | 1077 | $(f3-f5)/(f2+f4)$ |
| 268 | e3/e4  | 538 | $\sqrt{a3/a4}$        | 808 | $(1/e4-1/e5)*e1$ | 1078 | $(f4-f5)/(f1+f2)$ |
| 269 | e3/e5  | 539 | $\sqrt{a3/a5}$        | 809 | $(1/e4-1/e5)*e2$ | 1079 | $(f4-f5)/(f1+f3)$ |
| 270 | e4/e5  | 540 | $\sqrt{a4/a5}$        | 810 | $(1/e4-1/e5)*e3$ | 1080 | $(f4-f5)/(f2+f3)$ |

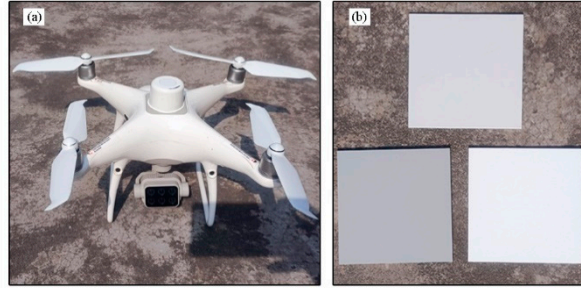
**Table S8** Configuration of the parameters of the genetic algorithm.

| Parameters                  | Details  | Values |
|-----------------------------|--|--------|
| max_features                | The maximum number of features selected                    | 20     |
| n_population                | Number of population                                       | 100    |
| crossover_proba             | Probability of crossover                                   | 0.75   |
| mutation_proba              | Probability of mutation                                    | 0.1    |
| n_generation                | Number of generations                                      | 50     |
| crossover_independent_proba | Independent probability for each attribute to be exchanged | 0.1    |
| mutation_independent_proba  | Independent probability for each attribute to be mutated   | 0.05   |

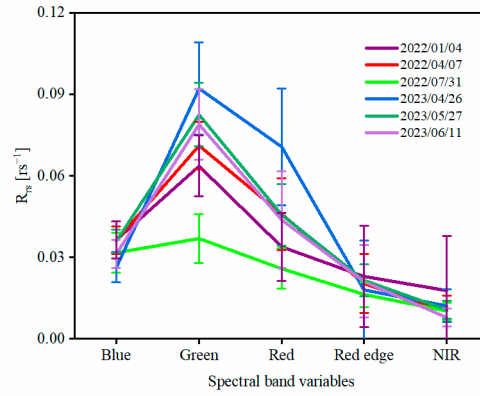
**Table S9** Statistics of the feature band selection results based on the FC-GA for each WQP.

| WQPs              | ML       | Selected bands   |
|-------------------|----------|--|
| Chla              | BRR      | $(d2-d4)/d3, (1/a1-1/a3)*a4, (e2-e3)/e5, b1+b4$  |
|                   | NNR      | $(d1-d2)/(d4+d5), (1/a1-1/a5)*a2, b3/b5$   |
|                   | SVR      | $(e1-e2)/(e4+e5), \exp(a3/a5), (1/f1-1/f2)*f3$   |
|                   | CART     | $(e2-e3)/(e4+e5), (e1-e2)/(e1+e2), e2+e4, (1/c1-1/c5)*c3, b4*b5$   |
|                   | RF       | $(d4-d5)/(d4+d5), (e2-e3)/e1, (f1-f5)/(f2+f4), (1/a1-1/a4)*a5, f1*f5$  |
|                   | LightGBM | $(c4-c5)/(c4+c5), (1/f3-1/f4)*f2, (f4-f5)/(f2-f3), f1/f2, d1-d3$   |
|                   | MLP      | $(1/a1-1/a4)*a2, (1/e1-1/e2)*e3, d1-d5, 1/(a1*a2)$   |
| SD                | BRR      | $(e2-e5)/(e2+e5), (1/c3-1/c5)*c2, \sqrt{(a2*a5)}, (f1-f3)/(f4+f5), c2, b1-b3$                                  |
|                   | NNR      | $a1-a2, (f2-f3)/(f1-f4), a1+a5$  |
|                   | SVR      | $(d1-d2)/d5, \sqrt{(b1/b2)}$   |
|                   | CART     | $(f3-f5)/f1, 1/(a2/a5), (1/d2-1/d3)*d5, \ln(a1/a2), \ln(a3+a5)$  |
|                   | RF       | $(c3-c5)/c1, (f2-f3)/(f1+f4), 1/(a1+a4), \sqrt{((b4-b5)/(b4+b5))}, (c1-c2)/(c4+c5), (1/d2-1/d3)*d5, 1/(a2-a3)$ |
|                   | LightGBM | $e1-e4, b1-b4, (f2-f5)/f3, (d1-d2)/d4, f3, \sqrt{(a1/a3)}$   |
|                   | MLP      | $(f3-f4)*f5, d2*d3, (a1-a4)/a5, \sqrt{(a2-a3)}, (d3-d4)/d5$  |
| COD <sub>Mn</sub> | BRR      | $(1/b1-1/b3)*b4, \exp(a3/a5), b3/b5, (e4-e5)/e2, c2*c3, a2*a4$   |
|                   | NNR      | $1/(a1/a2), (a1-a3)/a4$  |
|                   | SVR      | $(1/f3-1/f5)*f1, (1/f1-1/f3)*f2, (f3-f4)/(f1+f2), (a4-a5)/a1, \sqrt{(a1*a2)}$                                  |
|                   | CART     | $(e3-e5)/e1, (e1-e3)/e2, (f3-f4)/(f1+f2), (1/a4-1/a5)*a1, \sqrt{(a1*a2)}$                                      |
|                   | RF       | $(f2-f5)/(f1+f4), (1/f1-1/f2)*f3, (b1*b3)^2, (a1-a4)/a3$   |
|                   | LightGBM | $a1+a4, (f1-f4)/f5, (b1-b2)/b3, (1/d4-1/d5)*d2$  |
|                   | MLP      | $\sqrt{((a1-a4)/(a1+a4))}, (b4-b4)/b3, (b1-b2)/(b3+b5), e3*e5, (e1-e3)/e4, b3*b4$                              |
| TN                | BRR      | $(1/f2-1/f5)*f4, (a1-a4)/(a3+a5), (c4-c5)/c2, 1/(b1+b3), \sqrt{(a2-a3)}, (1/a3-1/a4)*a2, f1*f3$                |
|                   | NNR      | $a1, (e1-e2)/e3, (1/f4-1/f5)*f1, (d1-d4)/(d2+d3)$  |
|                   | SVR      | $(f3-f5)/(f1+f4), (a1-a3)/a2, (a3/a4)^2, (b1-b3)/(b2+b5), 1/(a4-a5)$   |
|                   | CART     | $(a1-a3)/(a4+a5), (f1-f3)/(f2+f4), (1/a2-1/a3)*a4, (a1+a5)^2, (c2-c5)/(c3+c4), (1/f1-1/f4)*f5$                 |
|                   | RF       | $\sqrt{(a3-a4)}, (f4-f5)/(f2+f3), (c3-c4)/c1, (a1/a3)^2$   |
|                   | LightGBM | $(1/e1-1/e5)*e4, (a1+a4)^2, (1/c1-1/c5)*c2, f1/f2, \sqrt{(a3/a4)}, 1/((a3-a4)/(a3+a4))$                        |
|                   | MLP      | $e1/e3, (e2-e5)/(e1+e4)$   |
| TP                | BRR      | $(1/f1-1/f3)*f4, (f3-f4)/(f1+f2), (d1-d5)/(d1+d5), \exp((a3/a5)), d2/d4$                                       |
|                   | NNR      | $c2/c4, (1/f1-1/f2)*f3, 1/(a2-a3)$   |
|                   | SVR      | $(1/f1-1/f3)*f2, \exp(a3/a5), b1*b4$   |
|                   | CART     | $(b1-b2)/b3, (1/f2-1/f3)*f4, b3, \sqrt{(a1+a2)}, 1/((a2-a3)/(a2+a3))$  |
|                   | RF       | $(b1-b3)/b2, (1/c2-1/c5)*c3, (1/b3-1/b5)*b1, \exp(a2/a4), (c2-c3)/c4, f1*f3$                                   |
|                   | LightGBM | $(f3-f5)/f4, a1/a2, (e1-e5)/e2$  |
|                   | MLP      | $(d1-d2)/d3, 1/(a4-a5), (c4-c5)/c1$  |

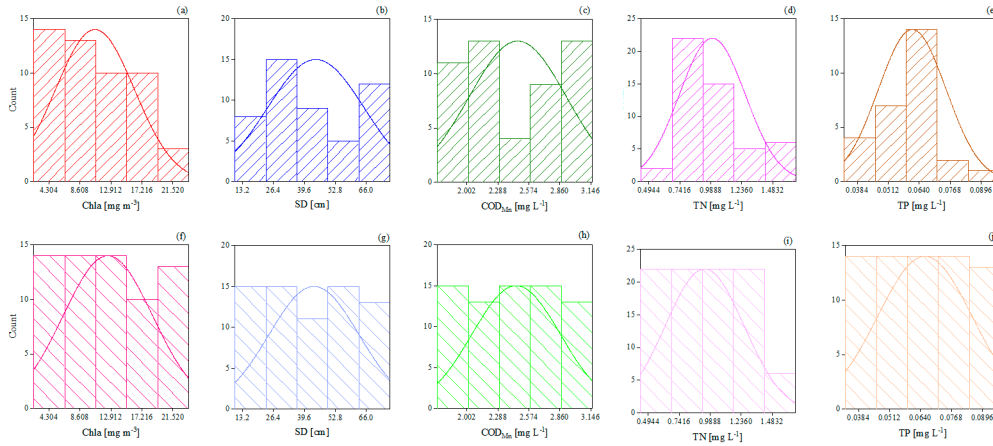
## 2 Supplementary Figures



**Figure S1** Pictures of UAV and gray standard panel. (a) The P4M UAV and (b) three gray standard panels of 50%, 25%, and 75%, respectively.



**Figure S2** Spectral characteristics of six periods at the sampling site of Longdong Reservoir. The error bars represent the degree of dispersion of the Spectral.



**Figure S3** Distribution of five water quality parameters before and after oversampling. (a)~(e) are the distributions of Chla, SD, COD<sub>Mn</sub>, TN, TP before oversampling, and (f)~(j) are the distributions of Chla, SD, COD<sub>Mn</sub>, TN, TP after oversampling, respectively.