

The 1-D (i.e., z-directional), steady-state heat conduction equation can be expressed by:

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{g} = 0 \quad (\text{B-1})$$

The power generation density (\dot{g}) of coil is obtained as

$$\dot{g} = \frac{\dot{Q}}{V} = \frac{I^2 R}{t A_s} = \frac{L_c \times I^2}{t \sigma A_s A_c} \quad (\text{B-2})$$

Here, σ and A_c are electric conductivity, and cross-sectional area of a winding copper, respectively. A_s and t are surface area and thinness of a magnetic-actuated chip for a single coil, respectively.

By integrating Eq.(B-1), it can be performed as

$$T(z) = \frac{\dot{g}}{2k} z^2 + C_1 z + C_2 \quad (\text{B-3})$$

The boundary condition for the center of the chip can be gotten $\left. \frac{dT}{dz} \right|_{z=0} = 0$, thus

$C_1=0$. In addition to, the boundary condition for top and bottom surfaces dissipated by heat convection is expressed by

$$\left. -\frac{1}{2} k \nabla T \right|_{\frac{t}{2}} = h (T_s|_{\frac{t}{2}} - T_\infty) \quad (\text{B-4})$$

Where, k and h are thermal conductivity and heat convection coefficient, respectively.

$$h \left[\left(\frac{\dot{g}}{4k} \left(\frac{t}{2} \right)^2 + C_2 \right) - T_\infty \right] = k \frac{\dot{g}}{2k} \left(\frac{t}{2} \right) \quad (\text{B-5})$$

C_2 can be obtained as $C_2 = T_\infty + \frac{\dot{g}t}{4h} - \frac{\dot{g}t^2}{8k}$. Therefore, Eq.(B-3)

$$T(z) = \frac{\dot{g}}{2k} \left(z^2 - \left(\frac{t}{2} \right)^2 \right) + \frac{\dot{g}t}{4h} + T_\infty \quad (\text{B-6})$$

The surface temperature of the coil chip can be obtained while $z=t/2$.

$$T_s = \frac{L_c}{4\sigma h A_s A_c} I^2 + T_\infty \quad (\text{B-7})$$