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A Stepwise Conflict Analysis Using the Graph Model

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Abstract: Information about decision-makers' preferences is essential for the efficient modeling of a conflict. However, obtaining this information becomes more challenging as the size of the conflict increases. To address this issue, this study proposes a new approach to the option prioritizing method within the graph model for conflict resolution. The approach aims to gather preferences from decision-makers in a more consistent and practical manner. The proposed method involves partitioning the set of conflict options based on their importance, then applying the option prioritizing method and conflict stability analysis to subconflicts, where only the options in each partition set are considered. Additionally, only states that are equilibria in a given step are deemed feasible in subsequent steps. The main findings highlight a reduction in the cognitive effort required from decision-makers and the generation of more effective and consistent solutions that address the core needs of the conflict. By working with subsets of options incrementally, the method offers a more simplified and robust understanding of the problem. To demonstrate the proposed method, a real hydrological conflict was used as a case study.

Keywords: group decisions and negotiations; graph model for conflict resolution; option prioritizing; preference elicitation

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Citation: Mota, R.d.S.; Silva, M.M.; Rêgo, L.C. A Stepwise Conflict Analysis Using the Graph Model. *Games* **2024**, *15*, 39. https://doi.org/10.3390/g15060039

Academic Editors: Kjell Hausken and Ulrich Berge

Received: 15 September 2024
Revised: 24 November 2024
Accepted: 26 November 2024
Published: 27 November 2024

1. Introduction

Strategic conflicts are increasingly prevalent in society's daily life. They arise when multiple parties with decision-making authority have differing preferences regarding potential courses of action. These parties, referred to as decision-makers (DMs), can include individuals, companies, groups, or even countries.

In scenarios where two or more DMs or groups of DMs hold conflicting decision-making powers and interests, a conflict situation arises. To address such situations, conflict modeling has been developed as a mathematical framework to evaluate and analyze the movements and strategies of the DMs involved in the conflict.

In this sense, it is worth highlighting some formal methodologies used for modeling and analyzing conflicts, including game theory [1], metagame analysis [2], conflict analysis [3] and the graph model for conflict resolution (GMCR) [4].

Among the methodologies mentioned for conflict analysis and resolution, the GMCR stands out for its practicality and effectiveness in structuring and analyzing conflicts [5]. Moreover, this approach offers greater flexibility in eliciting DMs' preferences, as it does not require cardinal utilities. As a result, the GMCR proves to be a highly valuable tool for analyzing conflicts and achieving consistent results in conflict resolution.

For the development of this study, the option prioritizing technique will be used, as it is the most commonly applied method in practical problems involving the GMCR. This technique is effective in capturing DMs' preferences in strategic conflicts, whether these preferences are unknown, uncertain, or crisp [6].

In this technique, each DM is asked to provide an ordered list of preference statements in descending order of priority, starting with the most important and ending with the least important. Each preference statement consists of a logical combination of the available options in the conflict. These options represent actions that the DMs can or cannot take during the conflict, and their combinations form the scenarios or states of the conflict.

It is worth mentioning several works that have used the option prioritizing technique to elicit preference structures within the GMCR. Notable examples include: ref. [7], which explores two option prioritizing techniques for obtaining probabilistic preferences in the GMCR; ref. [8], which presents an option prioritizing technique for more efficiently capturing fuzzy preferences; ref. [9], which defines an option prioritizing method for eliciting three-level preferences; and ref. [10], which introduces the option prioritizing method for gray preferences. These methodologies share a common feature: they are adaptations of the classic option prioritizing method designed to handle different preference structures, such as fuzzy, gray, or probabilistic. However, they all share the limitation of being unsuitable for large conflicts, as they require all available options to be considered simultaneously. Moreover, according to [11], while these approaches offer the advantages of simplicity and usability, the evaluation results are highly susceptible to subjective factors such as the cognition, attitudes, and values of DMs, researchers, or experts involved in the conflict.

The option prioritizing method requires DMs to provide an ordered list of preference statements that encompass all options in the conflict, which can be cognitively challenging in conflicts involving a large number of options. The aim of this work is to enhance the preference elicitation process for DMs, particularly in problems with numerous options. To address this, a new approach is proposed that divides both the elicitation of preferences and the stability analysis into multiple steps. The central idea is that large conflicts can be resolved incrementally, following a sequence based on priority. In each step, a sub-conflict is modeled and analyzed, and only the equilibrium states from one step remain feasible in the subsequent steps. By dividing the conflict analysis into steps, it becomes possible to manage conflicts involving a greater number of options, reducing the cognitive effort required from the DMs and thereby enabling more accurate results. The number of steps and the order in which options appear are determined through a pre-negotiation phase with the DMs involved in the conflict.

In many complex conflict situations in real life, it is common for issues to be resolved incrementally. For instance, when voting on a major tax reform in a country, a parliament may choose to vote on individual items of the reform rather than the entire project at once. Similarly, in a war conflict, certain issues, such as humanitarian aid, may be addressed and agreed upon before other matters are resolved.

In line with the proposed study, it is worth mentioning several existing works in the literature that are similar to the current one. The hierarchical graph model also proposes dividing a conflict into sub-conflicts to resolve them effectively. In this area, the study in [12] presents a hierarchical approach to a water distribution problem in China, where conflicts are analyzed separately using the GMCR. Ref. [13] introduces a hierarchical graph model represented by matrices, evaluating two sub-conflicts related to water resource distribution. Finally, ref. [14] demonstrates the use of the hierarchical graph model to analyze a conflict involving two locations in China, where a proposal to divert water from the south to the north is considered. Although hierarchical GMCR models also analyze conflicts separately, the goal in these studies is to assess multiple conflicts that occur simultaneously, where DMs and options may overlap. In contrast, our approach focuses on a single conflict that is analyzed in multiple steps, simplifying both the preference elicitation process and stability analysis.

Generally, the main gap addressed by our proposed methodology is its design for situations where there are numerous issues (options) to be analyzed and resolved among the DMs. In such cases, the cognitive effort required to reach a resolution by considering all options simultaneously is either unfeasible or highly prone to assessment errors due to the Paradox of Choice [15]. The primary benefit of our proposal is that DMs can identify which options should be prioritized for negotiation during a pre-negotiation phase. Naturally, this pre-negotiation phase itself may lead to conflicts of interest, and a conflict resolution method may be needed to generate a prioritized list of options to be negotiated. In this work, we do not address potential conflicts during the pre-negotiation phase; rather, we take the outcome of this phase as the input for our approach. Furthermore, our methodology does not require the conflict to be divided into multiple steps by prioritizing the options; if the DMs agree to analyze all options at once during the pre-negotiation phase, the classic version of the option prioritizing method can still be applied. The key advantage of our approach is that it provides flexibility, allowing DMs to resolve the conflict in parts, starting with the most urgent issues.

To demonstrate the applicability of the method proposed in this study, we used a conflict presented in [16], which describes a water crisis in an irrigated perimeter located in the municipality of Limoeiro do Norte, in the eastern region of the state of Ceará, Brazil. This area is responsible for producing various types of greens and vegetables. The conflict involves five options and four DMs: Government Organizations, Small Farmers, Agribusiness, and Civil Society. At the end of the application, it was found that the proposed approach yielded more consistent results compared to the conventional application of the option prioritizing technique described in [16], which suggested a different equilibrium from the one identified in this study.

The remainder of this paper is organized as follows. Section 2 contains the theoretical framework, including the concepts of the GMCR, the stabilities and the option prioritizing technique. Section 3 details the approach proposed in this study, outlining the formal model for preference elicitation and conflict analysis. Section 4 describes the application of the proposed methodology in a case study involving a water crisis conflict. Finally, Section 5 provides the conclusions of the work.

2. Materials and Methods

In this section, we recall the main concepts necessary for understanding this work.

2.1. GMCR

The GMCR consists of a set of directed graphs, where each graph corresponds to a DM involved in the conflict. Each graph has the same set of vertices, which will constitute the feasible states of the conflict [4].

The GMCR describes the conflict by describing the parties involved in the conflict, which are called DMs, denoted by the set $N = \{1, 2, \ldots, d\}$ and the actions that each DM can or cannot take in the conflict, which are known as options, denoted by $\mathcal{O}_k$, for $k \in N$. It is assumed that $\mathcal{O}_k \neq \emptyset$ and $\mathcal{O}_k \cap \mathcal{O}_j = \emptyset$, if $k \neq j$. Therefore, denote by $\mathcal{O} = \cup_{k \in N} \mathcal{O}_k$ the set of all options available to all DMs in the conflict. Since each option represents a course of action that may or may not be taken in the conflict, a conflict scenario or state is defined as a specification of which options are being chosen in that particular situation.

Formally, a conflict state can be modeled as a function $s : \mathcal{O} \to \{Yes, No\}$, so that for $o \in \mathcal{O}_k$, if $s(o) = Yes$, then DM k takes option o at conflict state s, otherwise $s(o) = No$. In principle, if $||\mathcal{O}|| = m$, there are 2^m possible different states, but not every combination of options can be taken in a particular conflict, making some states unfeasible. The set of feasible states of the conflict is denoted by $S = \{s_1, s_2, \ldots, s_q\}$. Once the set of feasible states is determined, the next step is to model how the DMs can change the conflict states by changing their own options. This is specified through a directed graph (S, A_k), for $k \in N$, such that $(s_p, s_l) \in A_k$ if DM k can in one step take the conflict from s_p to s_l by changing its options. We therefore have the restriction that $s_p(o) = s_l(o), \forall o \notin \mathcal{O}_k$ if $(s_p, s_l) \in A_k$.

To complete the modeling of the conflict, it is necessary to determine a preference relation for each DM over the set of feasible states. In this work, the preferences of a given DM k are represented by a binary preference relation, $\succ_k$, over the set S [5,17]. This preference relation is assumed to be asymmetric, where $s_p \succ_k s_q$ indicates that DM k strictly prefers s_p to s_q. From the strict preference relation $\succ_k$, the weak preference relation $\succsim_k$ is derived, such that $s_p \succsim_k s_q$ if $s_q \not\succ_k s_p$.

2.2. Stability Analysis

After modeling the conflict, a stability analysis is performed to establish effective solutions for the analyzed situation. To conduct this stability analysis in the GMCR, it is essential to first understand that the concepts of stability reflect the strategies of the DMs regarding their view of the conflict and their perceptions of risk [18].

This stability analysis uses a DM k as a reference, known as the focal DM [19]. Intuitively, a state is stable for a DM k if it prefers to remain in that state based on the anticipated reactions of the other participants in the conflict. Since DMs may behave differently in a conflict situation, the possible reactions and counter-reactions lead to different stability concepts. Among the existing stability concepts are: Nash stability (R) [20], general meta-rationality (GMR) [2], symmetric meta-rationality (SMR) [21], sequential stability (SEQ) [21], and symmetric sequential stability (SSEQ) [22].

To formally recall the definition of these stability concepts, we need to introduce some more notation. First, consider the set of all states reachable by DM i from state s given by $R_i(s)$. This set consists of the states to where DM i can move unilaterally (in a single move) from state s. It is formally defined as follows: $R_i(s) = \{s_1 \in S : (s, s_1) \in A_i\}$ [17,23].

Furthermore, we need to identify which states from the set of reachable states by DM i are also preferable to DM i. These moves are known as unilateral improvement moves for DM i from state s [17,23]. The set of all unilateral improvement moves for DM i from state s is defined by: $R_i^+(s) = \{s_1 \in S : (s, s_1) \in A_i \textit{ and } s_1 \succ_i s\}$.

2.2.1. Nash Stability

According to [20], in this stability concept, for any state to which the focal DM can move from a state considered Nash stable for him/her, that state will not be preferable to the initial state.

Definition 1. *Let $i \in N$; state $s \in S$ is Nash stable for DM i, if and only if $R_i^+(s) = \emptyset$.*

2.2.2. GMR Stability

According to [2], in GMR stability, DM i evaluates his/her moves conservatively, believing that by unilaterally moving the conflict, his/her opponent DM j will react in a way that leads to a non-preferable state for DM i compared to the initial state s.

Definition 2. *Let $i \in N$; state $s \in S$ is GMR stable for DM i, if and only if $\forall s_1 \in R_i^+(s)$, there exists at least one state $s_2 \in R_j(s_1)$ such that $s \succsim_i s_2$.*

2.2.3. SMR Stability

Based on study [21], in SMR stability, the focal DM evaluates which move to perform, considers the reactions of their opponent DM j in response to the initial move, and then assesses his/her own counter-reaction to the opponent's response. However, there will be no state that can be reached by DM i and is preferred to the starting state s, if s is SMR stable for DM i.

Definition 3. *Let $i \in N$; state $s \in S$ is SMR stable for DM i if and only if $\forall s_1 \in R_i^+(s)$, there exists at least one state $s_2 \in R_j(s_1)$, such that $s \succsim_i s_2$ and $s \succsim_i s_3, \forall s_3 \in R_i(s_2)$.*

2.2.4. SEQ Stability

As described by [21], in SEQ stability, DM i assumes that DM j will not only attempt to sanction i's unilateral improvements, but will also aim to achieve his/her own improvements. In other words, DM j unilaterally moves the conflict to a state that is preferable for him/her, but not preferable for DM i compared to the initial state.

Definition 4. *Let $i \in N$; state $s \in S$ is SEQ stable for DM i if and only if $\forall s_1 \in R_i^+(s)$, there exists at least one state $s_2 \in R_j^+(s_1)$, such that $s \succsim_i s_2$.*

2.2.5. SSEQ Stability

As proposed by [22], SSEQ stability incorporates aspects of both SEQ and SMR stability. In this case, DM i assumes that DM j is not solely focused on sanctioning DM i's unilateral improvements, but also seeks his/her own improvements. Furthermore, DM i cannot escape the sanction imposed by DM j's move.

Definition 5. *Let $i \in N$; state $s \in S$ is SSEQ stable for DM i if and only if $\forall s_1 \in R_i^+(s)$, there exists at least one state $s_2 \in R_j^+(s_1)$, such that $s \succsim_i s_2$ and $s \succsim_i s_3, \forall s_3 \in R_i(s_2)$.*

2.3. Option Prioritizing

One of the main challenges in conflict modeling is obtaining preferences. To address this, one of the most widely used techniques in GMCR analysis is the option prioritizing technique [24]. This method was developed based on the preference tree concept introduced in [25,26]. The option prioritizing technique involves eliciting, for each DM, an ordered list of preference statements, ranked from highest to lowest priority. Each preference statement is a logical formula that involves Boolean combinations of the conflict options. Consequently, each state may or may not satisfy a given preference statement.

As previously described, the options in a conflict are actions that DMs may choose to take or not take during the dispute. The set of conflict options available to DM k is denoted by

$$\mathcal{O}_k = \{o_1^k, o_2^k, \ldots, o_{m_k}^k\}.$$

Moreover, preference statements are denoted by $\psi(\mathcal{O})$, taking a corresponding truth value, either "True" (T) or "False" (F), based on whether the associated options are taken or not in a given state. According to [24], preference statements can be categorized into three types: non-conditional, conditional, or biconditional.

Thus, an unconditional preference statement refers to a combination of available options and logical connectives, such as: negation ("not" or -), conjunction ("and" or "&") and disjunction ("or" or "|"). Parentheses ("(" and ")") are also used to control the priority of the connectives in a preference statement. On the other hand, a conditional (or biconditional) preference statement involves two preference statements connected by the logical operator "IF" (or "IFF", respectively).

According to the option prioritizing technique, the states that satisfy the first statements of the ordered list are preferable to those that do not. Formally, if $\Psi_k(\mathcal{O}) = (\psi_k^1(\mathcal{O}), \psi_k^2(\mathcal{O}), \ldots, \psi_k^{l_k}(\mathcal{O}))$ is the ordered list of DM k's preference statements, state s is preferred to s' if there exists $1 \leq t^* \leq l_k$ such that $\psi_k^t(\mathcal{O})(s) = \psi_k^t(\mathcal{O})(s')$, for all $t < t^*$, $\psi_k^{t^*}(\mathcal{O})(s) = T$ and $\psi_k^{t^*}(\mathcal{O})(s') = F$. In other words, the above definition establishes that state s is preferable to state s' by DM k if, according to the ordered sequence of preference statements, state s is the first that uniquely satisfies a preference statement.

Among the three approaches for obtaining DMs' preferences in a conflict (option weighting, option prioritizing, and direct ranking), option prioritizing is considered the most flexible. It is regarded as the most suitable for most models, as it only requires DMs to provide preference statements about specific options being selected or not, in descending order of priority [6].

Several studies have explored the application of the option prioritizing method within the GMCR framework. For instance, ref. [27] proposed a methodology incorporating fuzzy truth values into the option prioritizing technique to achieve a more realistic preference ordering of feasible states.

Similarly, ref. [28] introduced enhancements to the option prioritizing technique in GMCR by proposing a score function based on a confidence level function and utilizing a preference tree. These improvements aim to quantitatively express differences in DMs' preferences across various states.

Ref. [29] introduced a dynamic conflict model that incorporates the evolving attitudes of DMs using the option prioritizing technique within the GMCR framework. This model was applied to analyze the planning of an urban transport system project in Pakistan. Additionally, ref. [30] proposed an alternative approach that involves third-party interventions within the GMCR framework. Their method aims to achieve satisfactory agreements by promoting minimal adjustments in the prioritization of preference statements.

Thus, the option prioritizing method stands out for its applicability, flexibility, and the advantages it brings to both the structuring and analysis of conflicts. It contributes to improved understanding and ensures more consistent information. However, when dealing with conflicts involving a large number of options, it becomes cognitively challenging to generate an ordered sequence of preference statements that accurately reflects the DMs' preferences. To address this issue, this work introduces a stepwise approach to option prioritizing.

3. Stepwise Option Prioritizing

This new approach aims to facilitate the process of eliciting DMs' preferences in conflicts involving a large number of options. The strategy for managing these conflicts involves dividing the available options based on their importance and resolving the conflicts incrementally. Once resolutions are determined for the most important options, the feasible outcomes in subsequent steps must align with the equilibria established in earlier steps. The overall conflict resolution is achieved in the final steps, where all available options are considered.

In general, this approach aims to adapt option prioritization by dividing the elicitation process into multiple steps, rather than requiring preference statements for the entire set of options at once. At each step, DMs provide preference statements for a subset of the available options, prioritizing the most important options first. Below, we formally describe the stepwise option prioritization method.

Let $\mathcal{O}^* = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, where $\alpha_i \subseteq \mathcal{O}$ is the subset of options to be negotiated at the i-th conflict step, $\alpha_i \subsetneq \alpha_{i+1}$, for $i = 1, 2, \ldots, n-1$ and $\alpha_n = \mathcal{O}$.

In this approach, the preference elicitation process for DMs is divided into multiple steps. In the first step, the preference statements involve only the options within the set α_1. In the i-th step, new preference statements are added to the list, involving only the options within the set α_i. The number of steps and the order in which the options appear at each step result from a pre-negotiation phase. This phase must consider the number of options in the conflict, the DMs' priorities, and the interdependencies between the options. In this work, we assume that this pre-negotiation phase has already been completed, and the sets α_i's are provided as input data for our problem.

The idea is that, at each step, new preference statements are added to the end of the list for each DM. These new statements involve the additional options introduced in that step, capturing the DMs' preferences for those options. Since not all options are present in every step, conflict states are described partially by the corresponding partial states at each step. Formally, a partial state at the i-th step is a function $s : \alpha_i \to \{Y, N\}$. The set of all feasible partial states at the i-th step is denoted by S_{α_i}. These states involve only the options in α_i that are being negotiated, subject to the restriction that the agreements made in previous steps must be upheld.

In this context, during the first step, the options initially prioritized by the DMs are described in α_1, resulting in a set of states S_{α_1}. Thus, for $\alpha_i, \alpha_j \in \mathcal{O}^*$, with $j > i$, we have $\alpha_i \subsetneq \alpha_j$, so S_{α_j} is considered more refined than S_{α_i}, in that the partial states in the set S_{α_j} reflect all the options described in the partial states in S_{α_i} and more. Formally this is denoted by $S_{\alpha_j} \geq S_{\alpha_i}$.

As the name suggests, partial states provide partial descriptions of conflict states, such that these states are interconnected. We will utilize the concept of projection, as proposed in [31], to model this relationship. For $\alpha_i \subseteq \alpha_j$ and a state $s \in S_{\alpha_j}$, we define the projection of this state into S_{α_i} to be $r^{\alpha_j}_{\alpha_i}(s) \in S_{\alpha_i}$ so that $r^{\alpha_j}_{\alpha_i}(s)(o) = s(o)$, for all $o \in \alpha_i$.

Therefore, the same options in α_i are taken in $s \in S_{\alpha_j}$ and in $r^{\alpha_j}_{\alpha_i}(s) \in S_{\alpha_i}$. Furthermore, we assume that $S_{\alpha_i} = \{r^{\alpha_n}_{\alpha_i}(s) : s \in S\}$, i.e., the set of feasible states at the i-th step is the set of projections of feasible conflict states considering the set of options available in the n-th step.

In this way, let $N_i \subseteq N$ represent the set of DMs participating in the i-conflict step. In this i-th step of the conflict analysis, we will obtain, by the options prioritization method, the preferences $\succ^i_k$, for $k \in N_i$, on the set of partial states S_{α_i}. However, when carrying out the stability analysis of the i-th step, not all states in S_{α_i} will be considered, since the states that do not satisfy the notion of equilibrium in the previous step will be disregarded in the next step. Accessibility sets at each step should also take into account which option changes are allowed in the original conflict. Next, we formally describe each step of conflict modeling and analysis:

Step 1. In this first step, we perform the usual stability analysis considering the GMCR $(S_{\alpha_1}, \{A^1_k\}_{k \in N^1}, \{\succ^1_k\}_{k \in N^1})$, where $(s^1_p, s^1_l) \in A^1_k$ if and only if $(s_p, s_l) \in A_k$, $s^1_p = r^{\alpha_n}_{\alpha_1}(s_p)$ and $s^1_l = r^{\alpha_n}_{\alpha_1}(s_l)$. Moreover, if $\Psi_k(\alpha_1) = (\psi^1_k(\alpha_1), \psi^2_k(\alpha_1), \ldots, \psi^{l^1_k}_k(\alpha_1))$ is the ordered list of preference statements for DM k at Step 1, then $s^1_p \succ^1_k s^1_l$ if there exists $1 \le t^* \le l^1_k$, such that $\psi^t_k(\alpha_1)(s^1_p) = \psi^t_k(\alpha_1)(s^1_l)$ for $t < t^*$, $\psi^{t^*}_k(\alpha_1)(s^1_p) = T$, and $\psi^{t^*}_k(\alpha_1)(s^1_l) = F$. For $STAB \in \{Nash, GMR, SMR, SEQ$ and $SSEQ\}$, let S^{STAB}_1 be the subset of S_{α_1} consisting of all states that satisfy the equilibrium notion $STAB$ at Step 1.

Step i. For Step i, the set of states considered in the analysis is given by the subset of S_{α_i} whose states have projection in $S_{\alpha_{i-1}}$ within the set S^{STAB}_{i-1}. Formally, $\Omega^{STAB}_i = \{s \in S_{\alpha_i} : r^{\alpha_i}_{\alpha_{i-1}}(s) \in S^{STAB}_{i-1}\}$. The accessibility set for the DM k in the i-th step, $A^i_k \subseteq \Omega^{STAB}_i \times \Omega^{STAB}_i$ will be given by $(s^i_p, s^i_l) \in A^i_k$ if and only if $(s_p, s_l) \in A_k$, where $s^i_p = r^{\alpha_n}_{\alpha_i}(s_p)$ and $s^i_l = r^{\alpha_n}_{\alpha_i}(s_l)$. Furthermore, if

$$\begin{aligned}\Psi_k(\alpha_i) &= \Psi_k(\alpha_{i-1}) \circ (\phi^1_k(\alpha_i), \phi^2_k(\alpha_i), \ldots, \phi^{n^i_k}_k(\alpha_i)) \\ &= (\psi^1_k(\alpha_i), \psi^2_k(\alpha_i), \ldots, \psi^{l^i_k}_k(\alpha_i)),\end{aligned}$$

where $\circ$ is the concatenation of two lists of preference statements, $l^i_k = l^{i-1}_k + n^i_k$ and $\Psi_k(\alpha_i)$ is the ordered list of preference statements DM k at Step i, then $s^i_p \succ^i_k s^i_l$ if there exists $1 \le t^* \le l^i_k$ such that $\psi^t_k(\alpha_i)(s^i_p) = \psi^t_k(\alpha_i)(s^i_l)$ for $t < t^*$, $\psi^{t^*}_k(\alpha_i)(s^i_p) = T$, and $\psi^{t^*}_k(\alpha_i)(s^i_l) = F$. Finally, a usual stability analysis is made considering the GMCR $(\Omega^{STAB}_i, \{A^i_k\}_{k \in N^i}, \{\succ^i_k\}_{k \in N^i})$. For $STAB \in \{Nash, GMR, SMR, SEQ$ and $SSEQ\}$, define by S^{STAB}_i the subset of Ω^{STAB}_i consisting of the states satisfying the equilibrium notion $STAB$ at Step i.

Definition 6. *State $s \in S$ is a stepwise equilibrium according to the stability notion $STAB$ if $s \in S^{STAB}_n$.*

4. Application

4.1. Conflict Description

The conflict used to demonstrate the applicability of the method proposed in this work is presented in [16]. This is a conflict related to the water scarcity of an irrigated perimeter located in the municipality of Limoeiro do Norte, eastern region of the state of Ceará, Brazil, which is responsible for the production of various types of vegetables. Table 1 presents the DMs involved in the conflict.

Table 1. DMs in the conflict. Source: [16].

DMs	Description
Governmental Organizations (DM1)	COGERH, DNOCS, SOHIDRA.
Small Farmers (DM2)	Members of local families owners of small farms.
Agribusiness (DM3)	Members of big agribusiness companies settler in the irrigated perimeter.
Civil Society (DM4)	Members of community entities, technical-scientific associations (UFC, IFCE) and professional associations.

The DMs, described in Table 1 can make decisions based on the choice of options that best suit their preferences. These options are shown in Table 2.

Table 2. Conflict options. Source: [16].

Options	Description
Governmental Organizations (DM1)	
o_1	Demand the implementation of a water reuse system by large companies.
o_2	Increase water pumping tariffs for producers.
Small Farmers (DM2)	
o_3	Drill deep water wells.
Agribusiness (DM3)	
o_4	Grow crops that consume less water.
Civil Society (DM4)	
o_5	Provide political training and social mobilization in support of small farmers.

In line with study [16], Government Agencies appear to have two potential options to address the water crisis in the Jaguaribe-Apodi irrigated perimeter of Chapada do Apodi. These options, aimed at reducing water consumption, are outlined in Table 2. However, they are not favored by other decision-makers (DMs), such as Agribusiness, Small Farmers, and Civil Society, as implementing these measures would result in increased costs for them.

In addition, since there is groundwater in the region of the irrigated perimeter, farmers can drill water wells to increase their access to water resources. However, this option involves higher costs. From the perspective of agribusiness, there is the possibility of adopting crops that require less water, but this is not desirable due to the accompanying changes in market dynamics. Regarding civil society, promoting political training and social mobilization to support small farmers is a viable strategy.

Therefore, as shown in Table 3, the conflict described in [16] comprises 32 possible states, as there are no infeasible states in this scenario. Additionally, each state includes options that decision-makers (DMs) can either adopt or reject, represented by Yes (Y) and No (N), respectively.

Table 3. Conflict states. Source: [16].

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
DM1																
o_1	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
o_2	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
DM2																
o_3	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
DM3																
o_4	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
DM4																
o_5	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}	s_{25}	s_{26}	s_{27}	s_{28}	s_{29}	s_{30}	s_{31}	s_{32}
DM1																
o_1	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
o_2	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
DM2																
o_3	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
DM3																
o_4	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
DM4																
o_5	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Next, we apply the method proposed in this work to model and analyze this conflict.

4.2. Stepwise Option Prioritizing Technique Application

The elicitation and stability analysis of the conflict in the irrigated perimeter of Chapada do Apodi is divided into two steps. This division aims to simplify the elicitation process, enabling DMs to analyze and evaluate their preferences more effectively by considering a smaller number of options at each step. In the first step, we focus on a subset of options or logical combinations of options deemed more relevant to the problem, based on the preferences of the DMs involved. Even though the conflict consists of only 32 states, this approach remains valuable for simplifying the process of capturing preferences.

After completing the stability analysis in the first step, the second step focuses on eliciting preference statements related to options not considered in the first step. However, the preference statements established in the first step take precedence over those elicited in the second step. The analysis was conducted using the GMCR+ software. For the SSEQ analysis, the relationships with other stability concepts, as presented in [22], were utilized.

4.2.1. First Step

In the first step of conflict modeling and analysis, the options with the greatest impact are selected. Only the decision-makers (DMs) controlling these selected options are included in this initial step. Specifically, the focus is on analyzing the conflict involving DM1 and DM3. This is because DM1 represents the public agency responsible for making decisions to address the water crisis, while DM3 represents large agribusiness companies with high water consumption demands. For the present case, three options were considered at first, namely: options o_1 and o_2 for Governmental Organizations (DM1), and option o_4 for Agribusiness (DM3). The preferences of the DMs, as shown in Table 4, were determined through option prioritization. For this study, the list of preference statements was constructed by considering only the statements presented in [16] that involve the selected options o_1, o_2, and o_4, maintaining their original order of priority.

Table 4. List of preference statements—1st step.

DM1	DM3
o_1 & $(-o_4)$	$(-o_1)$ & $(-o_2)$
o_4	$-o_4$
o_1	o_4 if $(o_1$ & $o_2)$
o_2	$-o_2$
o_1 & o_2	

As shown in the proposed methodology, this subset of options leads to a set of partial states, which are described in Table 5.

Table 5. Conflict states—1st step.

	s_1^1	s_2^1	s_3^1	s_4^1	s_5^1	s_6^1	s_7^1	s_8^1
DM1								
o_1	N	Y	N	Y	N	Y	N	Y
o_2	N	N	Y	Y	N	N	Y	Y
DM3								
o_4	N	N	N	N	Y	Y	Y	Y

Using the option prioritization technique, the preferences of the decision-makers (DMs) over the set of partial states were determined and are presented in Table 6. It is noted that the most preferred state for DM1 is state s_4^1, in which the options o_1 and o_2 are taken, but the option o_4 is not. This represents a scenario in which Governmental Organizations initially aim to implement a water reuse system for large companies, followed by an increase in pumping tariffs, with the goal of optimizing water resource management.

From the point of view of DM3, state s_4^1 is not among its most preferred since options o_1 and o_2 demand high costs. For DM3, it is observed that the most preferable state is state s_1^1, in which both DMs do not take any action to change, followed by state s_5^1, where DM3 adopts new operating formats for its enterprise that reduce water consumption. However, in this state, DM1 does not perform actions to demand the mandatory implementation of water reuse systems or to increase the pumping fee.

Table 6. DMs' preference rankings—1st step.

DMs	Preference Rankings
DM1	$s_4^1 \succ s_2^1 \succ s_8^1 \succ s_6^1 \succ s_7^1 \succ s_5^1 \succ s_3^1 \succ s_1^1$
DM3	$s_1^1 \succ s_5^1 \succ s_2^1 \succ s_3^1 \succ s_4^1 \succ s_6^1 \succ s_7^1 \sim s_8^1$

Once the DMs' preferences over the states are determined, the stability analysis can be performed to find the equilibria states. Table 7 shows the result of this analysis for the first step of analysis.

Table 7. Conflict equilibria—1st step.

Stabilities	s_1^1	s_2^1	s_3^1	s_4^1	s_5^1	s_6^1	s_7^1	s_8^1
Nash				X				
GMR		X		X				
SMR		X		X				
SEQ				X				
SSEQ				X				

At the end of the first stage, some equilibria for the conflict are identified. However, since the evaluated states are partial, these equilibria do not represent a complete resolution to the conflict. Instead, they indicate partial resolutions concerning the options considered in this step of the conflict analysis. Therefore, states in the next stage that do not satisfy the equilibrium conditions from the first stage will be considered unfeasible. Since there are various notions of stability and, consequently, different types of equilibria, the unfeasible states in the second stage depend on the stability notion applied in the first stage. In this conflict, if we consider Nash, SEQ or SSEQ stability, only state s_4^1 will be equilibrium indicating that only states in which options o_1 and o_2 are taken and option o_4 is not taken remain feasible. On the other hand, if we consider GMR or SMR stability, states s_2^1 and s_4^1 are equilibria, indicating that in the feasible states of the second step, option o_1 should be taken, but not the option o_4. However, there is no restriction on option o_2, as there is an equilibrium in which it is taken (s_4^1) and another in which it is not (s_2^1). These two possibilities will be considered below.

4.2.2. Second Step—Nash, SEQ or SSEQ Case

In the second step, the unfeasible conditions that do not satisfy the equilibrium state resulting from the first step are incorporated. Table 8 illustrates the states of this second step, considering that the Nash, SEQ or SSEQ equilibria remained from the first step.

Table 8. Conflict states—2nd step; Nash, SEQ or SSEQ case.

	$s_1^{2.1}$	$s_2^{2.1}$	$s_3^{2.1}$	$s_4^{2.1}$
DM1				
o_1	Y	Y	Y	Y
o_2	Y	Y	Y	Y
DM2				
o_3	N	Y	N	Y
DM3				
o_4	N	N	N	N
DM4				
o_5	N	N	Y	Y

For constructing the list of preference statements in the second step, the preference statements previously expressed by each DM in the first step are retained at the beginning of the list. Following these statements, new ones are added, involving the new options introduced in the second step. Table 9 presents the lists of preference statements considered in this second step. In this study, the new declarations were also derived from [16], maintaining their priority order, with the preference statements from the first step taking precedence.

Table 9. Preference statement lists—2nd step; Nash, SEQ or SSEQ case.

DM1	**DM2**	**DM3**	**DM4**
o_1 & $(-o_4)$	o_3 & o_5	$(-o_1)$ & $(-o_2)$	o_1
o_4	$(-o_2)$ & o_3	$-o_4$	o_4
o_1	$(o_1\&o_3)$ if o_2	o_4 if $(o_1\&o_2)$	$(o_1\&o_2)$ if o_3
o_2	o_1 if $(-o_3)$	$-o_2$	o_3
o_1 & o_2	o_3	$-o_3$	o_5
o_4 & $(-o_3)$	$-o_2$	$(-o_2)\&(-o_3)$	
o_2 iff & $[(-o_1)$ & o_3			
$(o_1\&o_2)$ iff $[o_3/[(-o_4)$ & o_5			
$(-o_3)$			

According to Table 10, the most preferable states by DMs 1 and 2 are ($s_3^{2.1}$) and ($s_4^{2.1}$), respectively, while for DMs 3 and 4 are ($s_1^{2.1}$ and $s_3^{2.1}$) and ($s_3^{2.1}$ and $s_4^{2.1}$), respectively. In comparison with the results presented in [16], it is noted that the options taken by some DMs remained equivalent in each state, with the exception of DM1. In this sense, in the conflict modeled in [16], the states considered preferable by DM1 were those in which it did not take either of his two options; contrary to the results from the application of the approach of this study, where the preferable state for it was state $s_3^{2.1}$ in which it takes the options o_1 and o_2, which makes even more sense, since, this DM represent the Government agencies responsible for resolving and mediating the conflict in order to resolve it.

Moreover, according to Table 10, state $s_4^{2.1}$ is the most preferable for DM2, having the support of DM4 taking option o_5, i.e., providing political training and instructions for social mobilization. On the other hand, the best states for DM3 are $s_1^{2.1}$ and $s_3^{2.1}$. These states differ only in respect to option o_5, which does not affect DM3, thus justifying the indifference. In these states, neither the small farmers drill deep water wells nor the agribusiness has to change its crops.

Finally, the best states for DM4 are $s_3^{2.1}$ and $s_4^{2.1}$. In these states, DM4 provides training for DM2, and it does not care whether or not DM2 decides to drill deep water wells.

Table 10. DMs' preference rankings—2nd step; Nash, SEQ or SSEQ case.

DMs	Preference Rankings
DM1	$s_3^{2.1} \succ s_2^{2.1} \sim s_4^{2.1} \succ s_1^{2.1}$
DM2	$s_4^{2.1} \succ s_2^{2.1} \succ s_1^{2.1} \sim s_3^{2.1}$
DM3	$s_1^{2.1} \sim s_3^{2.1} \succ s_2^{2.1} \sim s_4^{2.1}$
DM4	$s_3^{2.1} \sim s_4^{2.1} \succ s_1^{2.1} \sim s_2^{2.1}$

As illustrated by Table 11, state $s_4^{2.1}$ is suggested as a potential resolution for the conflict according to all stability concepts. State $s_4^{2.1}$ represents the scenario in which Government Organizations (DM1) require the implementation of a water reuse system to reduce water consumption by agribusiness and increase the water pumping tariff in order to induce lower consumption by producers. As for small farmers (DM2), it is suggested the drilling of deep water wells and for agribusiness (DM3), that it does not cultivate crops that consume less water, and finally, that civil society (DM4) continues to offer political training and social mobilization for small farmers.

Table 11. Conflict equilibria—2nd step; Nash, SEQ or SSEQ case.

Stabilities	$s_1^{2.1}$	$s_2^{2.1}$	$s_3^{2.1}$	$s_4^{2.1}$
Nash				X
GMR				X
SMR				X
SEQ				X
SSEQ				X

In comparison with the approach taken in [16], the equilibrium suggested at the end of this analysis as a possible conflict resolution was different. The difference was with respect to the choice of option o_2 by DM1, because in the work of [16] this action of increasing the pumping tariff in order to induce a lower consumption of water by the producers was not chosen. This was the case because the decision regarding this option had already been made in the first step of analysis, where Government Agencies reached an agreement with agribusinesses. Under this agreement, the Government decided to increase the pumping fee in order to maintain control over water usage, without requiring agribusinesses to change their crops. Note that, according to the priority specified in this study, DMs 2 and

4 do not participate in this part of the negotiation, as their options were considered of low priority.

4.2.3. Second Step—GMR or SMR Case

As explained earlier, unlike the Nash, SEQ, or SSEQ cases, no restriction is imposed on option o_2, since there is an equilibrium from the first step in which it is adopted and another in which it is not.

Based on this assumption, some contributions were observed solely from the perspective of GMR and SMR stabilities. Initially, it was noted that the number of states in the second step remained the same as in the first step (eight states), which was greater than the previous approach for the Nash or SEQ cases, where only four states were identified in the second step.

Also, as illustrated by Table 12, the states most preferred by DMs 1 and 2 are ($s_6^{2.2}$) and ($s_7^{2.2}$), respectively, and the most preferred for DMs 3 and 4 are ($s_1^{2.2}$ and $s_5^{2.2}$) and ($s_5^{2.2}$, $s_6^{2.2}$ and $s_8^{2.2}$), respectively. In comparison with the results shown in Table 11 for the Nash, SEQ, or SSEQ cases, it was observed that the actions prioritized by some decision-makers (DMs) differed. Specifically, for DMs 2, 3, and 4, the rankings were different from the previous case, mainly due to the exclusion of option o_2 in the leading states of these rankings, similar to what was presented in [16]. On the other hand, DM1's preference ranking remained unchanged.

Table 12. DMs' preference rankings—2nd step; GMR or SMR case.

DMs	Preference Rankings
DM1	$s_6^{2.2} \succ s_4^{2.2} \sim s_8^{2.2} \succ s_2^{2.2} \succ s_1^{2.2} \succ s_5^{2.2} \succ s_3^{2.2} \sim s_7^{2.2}$
DM2	$s_7^{2.2} \succ s_8^{2.2} \succ s_3^{2.2} \succ s_4^{2.2} \succ s_1^{2.2} \sim s_5^{2.2} \succ s_2^{2.2} \sim s_6^{2.2}$
DM3	$s_1^{2.2} \sim s_5^{2.2} \succ s_3^{2.2} \sim s_7^{2.2} \succ s_2^{2.2} \sim s_6^{2.2} \succ s_4^{2.2} \sim s_8^{2.2}$
DM4	$s_5^{2.2} \sim s_6^{2.2} \sim s_8^{2.2} \succ s_1^{2.2} \sim s_2^{2.2} \sim s_4^{2.2} \succ s_7^{2.2} \succ s_3^{2.2}$

As for the states suggested as possible conflict resolutions, it was observed from Table 13 that state $s_8^{2.2}$, in which the same options are taken as in state $s_4^{2.1}$ of the previous case, is an equilibrium according to all stability notions. However, in this new approach to the second step, state $s_4^{2.2}$ emerged as another suggestion of conflict equilibrium (Table 14) from the perspective of GMR and SMR stabilities. State $s_4^{2.2}$ differs from state $s_8^{2.2}$ due to the non-choice of option o_5, i.e., in this state $s_4^{2,2}$, DM4 does not provide support for small farmers, unlike state $s_8^{2.2}$.

Table 13. Conflict states—2nd step; GMR or SMR case.

	$s_1^{2.2}$	$s_2^{2.2}$	$s_3^{2.2}$	$s_4^{2.2}$	$s_5^{2.2}$	$s_6^{2.2}$	$s_7^{2.2}$	$s_8^{2.2}$
DM1								
o_1	Y	Y	Y	Y	Y	Y	Y	Y
o_2	N	Y	N	Y	N	Y	N	Y
DM2								
o_3	N	N	Y	Y	N	N	Y	Y
DM3								
o_4	N	N	N	N	N	N	N	N
DM4								
o_5	N	N	N	N	Y	Y	Y	Y

Table 14. Conflict equilibria—2nd step; GMR or SMR case.

Stabilities	$s_1^{2.2}$	$s_2^{2.2}$	$s_3^{2.2}$	$s_4^{2.2}$	$s_5^{2.2}$	$s_6^{2.2}$	$s_7^{2.2}$	$s_8^{2.2}$
Nash								X
GMR				X				X
SMR				X				X
SEQ								X

5. Conclusions

This work proposes a method to divide the option prioritizing elicitation process of the DMs' preferences into into multiple steps. The proposed method enables a more consistent handling of conflicts, particularly when dealing with a larger number of options that would otherwise demand significant cognitive effort to produce an ordered list of preference statements all at once.

The proposed method was applied to the conflict of the irrigated perimeter of Chapada do Apodi, as presented in [16]. For this purpose, the conflict was divided into two steps. Compared to the original approach described in [16], a difference in the suggested conflict resolution can be observed. Specifically, in the solution proposed in the original work [16], the Governmental Organizations did not implement option o_2, which involves increasing water pump tariffs for producers. However, in the present approach, the Governmental Organizations decided to increase these tariffs as an outcome of the first step of negotiation. This decision followed the agribusiness sector's choice not to change the type of crops it grows. Thus, the novel approach demonstrates its benefits, even in relatively small conflicts such as the one discussed in this paper.

Although the same preference information is used in both the original work by [16] and this study, the present work incorporates additional information regarding the urgency or priority of resolving certain options. This extra information results in outcomes that better align with the decision-makers' preferences.

The new methodological proposal seeks to address the Paradox of Choice [15], which suggests that having too many options can hinder decision-making due to cognitive overload. In conflicts involving numerous options, providing an ordered list of preference statements requires many decisions, potentially reducing the quality of the responses. To mitigate this issue, decision-makers may find it more manageable to resolve the conflict in stages, focusing first on the most critical issues (options).

Therefore, using the proposed approach in conjunction with the *GMCR+* software, it was confirmed that applying the stepwise option prioritizing technique, integrated with the GMCR method, significantly enhanced the systematic analysis of the conflict described in [16]. This proposal stands out by making the elicitation process more comprehensive and illustrative for decision-makers (DMs), enabling them to better understand and assess the consequences of their actions. As a result, it provides a more robust approach to support and generate consistent outcomes for decision-making in conflicts. For future research, it is recommended to conduct further comparative studies to explore additional advantages of this approach.

Author Contributions: Conceptualization, M.M.S. and L.C.R.; Methodology, L.C.R.; Formal analysis, R.d.S.M.; Writing—original draft, R.d.S.M.; Writing—review & editing, M.M.S. and L.C.R.; Supervision, M.M.S. and L.C.R.; Funding acquisition, M.M.S. All authors have read and agreed to the published version of the manuscript.

Funding: National Council for Scientific and Technological Development: 05792/2017-2; National Council for Scientific and Technological Development: 428325/2018-1; National Council for Scientific and Technological Development: 310729/2020-3; National Council for Scientific and Technological Development: 308980/2021-2; National Council for Scientific and Technological Development: 306523/2023-0; National Council for Scientific and Technological Development: 406697/2023-0.

Data Availability Statement: The authors confirm that all data generated or analyzed during this study are included in this published article.

Conflicts of Interest: The authors declare no conflicts of interest.

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