Strategy Constrained by Cognitive Limits, and the Rationality of Belief-Revision Policies

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Abstract: Strategy is formally defined as a complete plan of action for every contingency in a game. Ideal agents can evaluate every contingency. But real people cannot do so, and require a belief-revision policy to guide their choices in unforeseen contingencies. The objects of belief-revision policies are beliefs, not strategies and acts. Thus, the rationality of belief-revision policies is subject to Bayesian epistemology. The components of any belief-revision policy are credences constrained by the probability axioms, by conditionalization, and by the principles of indifference and of regularity. The principle of indifference states that an agent updates his credences proportionally to the evidence, and no more. The principle of regularity states that an agent assigns contingent propositions a positive (but uncertain) credence. The result is rational constraints on real people's credences that account for their uncertainty. Nonetheless, there is the open problem of non-evidential components that affect people's credence distributions, despite the rational constraint on those credences. One non-evidential component is people's temperaments, which affect people's evaluation of evidence. The result is there might not be a proper recommendation of a strategy profile for a game (in terms of a solution concept), despite agents' beliefs and corresponding acts being rational.

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1. Introduction

Ken Binmore [1] demarcates two modes of analysis in game theory. The eductive mode (comprised of deduction, induction, and adduction) analyzes social behavior according to the strict idealizations of rational choice theory. The evolutive mode analyzes social behavior according to dynamic models that relax said idealizations. I substitute the disciplines of epistemic and evolutionary game theory for Binmore's modes of analysis. Epistemic game theory provides the epistemic foundations for rational justification of social behavior. Conversely, evolutionary game theory establishes social behavior that is evolutionarily fit and stable. It is descriptive and provides for the causal explanation of social behavior.

A recent discipline—I denote it 'naturalistic game theory'—splits the difference between epistemic and evolutionary game theory. The disciplines of neuroeconomics and behavioral game theory [2], the application of heuristics from an adaptive toolbox to real-world uncertainty [3,4], and the study of optimization under cognitive constraints [5,6] are three converging approaches to a more realistic basis for game theory. All foundational concepts are analyzed vis-à-vis real people with cognitive limits. Work has started on common knowledge, convention, and salience; I contribute to this discipline with my analysis of strategy and the rationality of belief-revision policies.

I argue that the very concept of strategy is best understood vis-à-vis real people's cognitive limits. A strategy is traditionally defined as a complete plan of action, which evaluates every contingency in a game of strategy. However, only ideal agents can evaluate every contingency. Real people

require policies to revise their beliefs and guide their choices when confronted with unforeseen contingencies. The very concept of strategy requires belief-revision policies. Yet, in epistemic game theory, belief-revision policies are part of those idealizations (the game's structure, the agents' rationality, and the game's epistemic context) stipulated to formally prove conditional results about the game's outcomes. The rationality of strategies or acts depends on belief-revision policies, but the rationality of belief-revision policies remains unanalyzed.

The rationality of belief-revision policies is the subject of Bayesian epistemology. The objects of belief-revision policies are doxastic or epistemic states (I, henceforth, use 'epistemic' to refer to both types of states). Hence, the objects of analysis of belief-revision policies are epistemic states, not strategies and acts. Whereas traditional epistemology studies three basic judgments (belief, disbelief, or suspension of belief), Bayesian epistemology studies degrees of belief as credences, which indicate the confidence that is appropriate to have in various propositions. I extend the epistemic foundations of game theory to issues in Bayesian epistemology. I construct an arbitrary belief-revision policy by establishing its components: agents' credences are constrained by the probability axioms, by conditionalization, and by the principles of indifference and of regularity. I also consider non-evidential factors that affect credence distributions, despite those credences being rationally constrained.

I explain the general program of naturalistic game theory in Section 2. The main claim is that foundational concepts are best understood vis-à-vis real people's cognitive limits, which explains why analyzing said concepts in highly abstract and artificial circumstances is problematic. I then argue the same case for strategy and belief-revision policies in Section 3. Indeed, the charge of incoherence to backward induction is best explained by naturalism. However, this leaves open the question of how we analyze the rationality of belief-revision policies. I suggest doing so through Bayesian epistemology in Sections 4 and 5. I place the epistemic foundations of game theory into a more foundational epistemology.

2. Naturalistic Game Theory

Naturalistic game theory can be encapsulated in one claim: concepts foundational to game theory require realism. Realism provides understanding of said concepts that better matches our common sense and intuitions, while keeping in rigor. Remove realism and the foundational concepts are confronted by potentially irredeemable difficulties. Common knowledge, convention, rationality, and salience are excellent candidates for requiring realism.

I use Zachary Ernst's [5] argument form: an inference to the best explanation. The literature has recorded that the aforementioned concepts succumb to conceptual difficulties. One explanation is that the proper formal analysis has yet to be developed to account for said difficulties. But the favored explanation for naturalism is that these concepts are heuristics optimized for a range of realistic circumstances. The conceptual difficulties recorded in the literature are actually markers for the concepts' limits of application, and it is unsurprising they fail in extremely abstract and artificial circumstances. I provide two examples below: (Section 2.1) principles of rationality fail to prescribe the proper choice of action in specific abstract or artificial circumstances; and (Section 2.2) ideal agents cannot act in circumstances in which real people easily do so. This is counterintuitive. Ideal agents are *ideal*, because they have all of the information and cognitive ability to evaluate any choice and act on it in any circumstance.

2.1. Newcomb's Problem

Newcomb's Problem [7] is specifically constructed to contrast recommendations between the principles of expected-utility maximization and of dominance. There are two boxes, one opaque and the other transparent. An agent chooses between the contents of the opaque box (one-boxing) or the contents of *both* the opaque and transparent boxes (two-boxing). The transparent box contains one-thousand dollars. The opaque box is either empty or contains one-million dollars, depending on a

perfect prediction made about the agent's choice. If the prediction is that the agent will one-box, the opaque box contains one-million dollars. If the prediction is that the agent will two-box, the opaque box is empty. If the agent chooses one-boxing, the prediction of one-boxing is guaranteed. If the agent chooses two-boxing, the prediction of two-boxing is guaranteed. Hence, one-boxing's expected utility is greater than two-boxing's expected utility. The principle of expected-utility maximization prescribes one-boxing. However, the outcome of two-boxing is better by one-thousand dollars than the outcome of one-boxing, regardless of the prediction. The principle of dominance prescribes two-boxing ([8], Section 2.1).

The principles of expected-utility maximization and of dominance are equally reasonable, but individually prescribe conflicting choices of action. There are two proposals ([8], Section 2.2). One proposal is to argue for a particular choice and its corresponding principle. Simon Burgess [9] recommends one-boxing by partitioning the problem into two stages. The agent firmly commits to one-boxing in the first stage before the prediction. In the second stage, the agent does not waver from his choice and acts on it. The other proposal is to reconcile both principles of rationality. Allan Gibbard and William Harper [10] demarcate between causal and evidential decision theory. Causal decision theory shows that two-boxing's expected utility is greater than one-boxing's expected utility. Hence, both the principles of expected-utility maximization and of dominance prescribe two-boxing. However, this depends on whether the agent's choice causes the prediction (perhaps there is retro-causality).

There is ongoing debate over the viability of these proposals. Naturalistic game theory does not argue that Newcomb's Problem can never be solved. Perhaps there is a proper formal analysis that further and further refines decision theory to provide a consistent solution. However, the including of more and more 'machinery' into an analysis can count against the reasonability of said analysis. The history of science is replete with moments where including more and more machinery into explanations has counted against the reasonability of said explanations. Naturalistic game theory instead infers that principles of expected-utility maximization and of dominance evolved in real-time as heuristics to optimize choices in a range of realistic circumstances. Newcomb's Problem marks the limits of those principles.

2.2. Common Knowledge

Robert Aumann's [11] common knowledge is standard in epistemic game theory: (every agent in a group knows that)^n p, where n is the degree of 'every agent in a group knows that'. The group has mutual knowledge of p just in case $n = 1$. The group has common knowledge of p just in case n is arbitrary. Aumann's common knowledge imposes strong epistemic conditions on agents, conditions that real people cannot satisfy. Common knowledge thus requires ideal agents with no cognitive limits. However, ideal agents are beset with difficulties. For example, there are many versions of the Email Game in which agents with common knowledge cannot coordinate their actions [12]. The original game is a coordination problem where two agents attempt to coordinate their actions by sending each other emails with instructions, but each email has a small probability of not reaching its intended target. Other versions involve two generals coordinating their attacks through messengers, who might die traveling to their intended target; and two spies coordinating their risky package-exchange with clandestine signals. The agents require some level of mutual knowledge to successfully coordinate their actions. Yet the reasoning that justifies one level of mutual knowledge is equally required to justify an additional level of mutual knowledge, and an additional level of mutual knowledge, ad infinitum. The result is an impossibility for ideal agents to coordinate their risky actions, because they never obtain justification. This is counterintuitive. Ideal agents ought to obtain any level of justification—hence, the idealization.

Zachary Ernst [5] and Robin Cubit and Robert Sugden [13,14] explain that David Lewis's common knowledge is radically different from Aumann's. Lewis's book *Convention* [15] is, first and foremost, a defense of analyticity (a sentence is true in virtue of its words' meaning) against regress arguments. Suppose language is true by convention. Then speakers must know how the convention applies to that

language. However, preexisting knowledge of the convention is required to know how the convention applies to the language. This creates a vicious regress, and language is not true by convention.

Lewis defends analyticity by developing a concept of convention that allows for meaning of words to be conventional, and for agents to be part of the convention, without requiring those agents to have explicit knowledge of the convention. He develops an account of common knowledge with minimal epistemic conditions so agents can be part of a convention. The agents know how to apply the convention but this knowledge does not require inordinate levels of mutual knowledge. Lewis instead emphasizes states of affairs as sources of common knowledge: the occurrence of a state of affairs about *p* indicates to agents in a group that they have a reasonable degree of mutual knowledge about *p*. Lewis's common knowledge emphasizes agents' environments much more than their cognitive capacities.

Again, naturalistic game theory does not argue that Aumann's common knowledge can never be reconciled in coordination problems such as the Email Game. However, Lewis provides a realistic account of common knowledge that real people can obtain and regularly act on, despite being unable to reason to inordinate levels of mutual knowledge. Real people do not require strong justification, and can obtain common knowledge with little reasoning. Lewis's realism also explains the fragility of common knowledge and convention [16].

3. Strategy for Ideal Agents and for Real People

The concept of strategy has the same fate as rationality and common knowledge: strategy is best conceived vis-à-vis real people's cognitive limits. Indeed, I argue that the charge of incoherence to backward induction is an excellent example of the limits of the traditional concept of strategy.

The charge of incoherence is presented as either one of two problems. The first problem is that backward induction requires agents to evaluate all contingencies in a game to settle on the backward induction profile, but certain contingencies cannot be evaluated without violating backward induction [17–23]. The second problem is that backward induction 'seems logically inescapable, but at the same time it is intuitively implausible' ([24], p. 171).

A game theorist analyzes a game from an outside perspective to provide a game's complete analysis, which is its theory [18]. The game theorist specifies the game's structure, the set of agents' strategies, and their payoffs. He defines the agents' rationality, and then specifies the game's epistemic context, which is the agents' epistemic states concerning their circumstances. The game theorist then proves conditional results about the game's outcomes as profiles of agents' strategies [25–28]. Backward induction applies to the formation of strategies.

For any sequential game with perfect (and complete) information, a strategy is a function that assigns an action at every stage the agent can possibly reach, including those stages not reached during gameplay ([29], p. 108). Consider Robert Stalnaker's [30,31] sequential game in Figure 1:
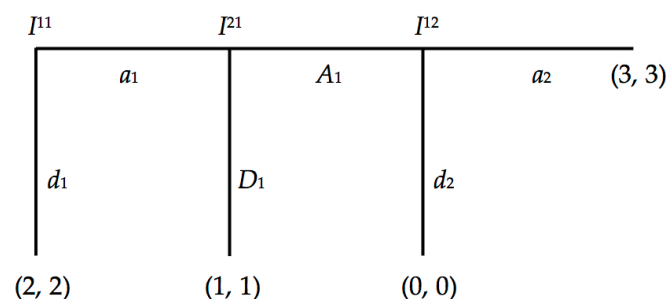

Figure 1. I^{ij} is the *j*-th information set ($j \geq 1$) of agent *i* ($i = 1, 2$). Robert Stalnaker's game has two pure strategies at each information set: Either 'Down' ending the game, or 'Across' allowing the other agent to choose an action. The agents' payoffs are represented at the tree's endpoints, the left member being the first agent's payoff, the right member being the second agent's payoff. The backward induction profile is (a_1a_2, A_1).

Suppose we differentially distribute the agents' epistemic states across the game's stages and consider each stage as an agent's information state such that:

(1)　The first agent is rational at I^{12}.
(2)　The second agent is rational at I^{21} and knows (1).
(3)　The first agent is rational at I^{11} and knows (2).

The agents are rational insofar as they maximize their expected payoffs. Then the distributed epistemic states are all that are required to construct the agents' strategies through backward induction. Beginning at the third stage and reasoning our way backwards to the first stage, backward induction settles the first agent's strategy (a_1, a_2) as the best response to the second agent's strategy (A_1), and vice versa. The outcome (a_1a_2, A_1) is the sub-game perfect equilibrium and the game's solution entailed by (1), (2) and (3). Though it is not the case in this game, an important condition of the formal definition of strategy is that agents assign actions for those stages that will not be reached by the agent during gameplay.

Robert Aumann argues that common knowledge of rationality implies the backward induction profile in sequential games with perfect information [32–34]. Backward induction requires that there are no stages where it is unclear whether a choice maximizes an agent's expected payoff. Aumann introduces clarity by providing agents with the right sort of rationality throughout the game. An agent is substantively rational just in case every choice maximizes his expected payoffs at all stages the agent can possibly reach. Substantive rationality includes those stages that an agent will not actually reach during gameplay. This is the sort of rationality Aumann associates with backward induction. This is different from evaluating acts that have occurred. An agent is materially rational just in case every choice the agent actually acts on maximizes his expected payoffs. Notice the trivial case that if an agent never acts in a game, that agent is materially rational.

Consider the following case from Stalnaker [31] with the profile (d_1a_2, D_1). This profile is a deviation from the backward induction profile, and Stalnaker uses it to show the difference between substantive and material rationality. Suppose the first agent knows the second agent is following strategy D_1. The first agent's act d_1 maximizes his expected payoff, given D_1. And the first agent's act a_2 maximizes his expected payoff, given the game's structure and payoffs. The first agent is substantively rational. Now suppose the second agent knows the first agent is following strategy d_1a_2. Thus, the second agent knows that the first agent initially chooses d_1, and, if the first agent reaches the third stage, the first agent chooses a_2. Stalnaker explains that the second agent is materially rational, since the second agent never acts on his strategy. His material rationality is preserved through inaction. Is the second agent, nonetheless, substantively rational? Aumann argues that the second agent is not substantively rational, because his choice D_1 does not maximize his payoff at the second stage. However, Stalnaker argues that there is insufficient information about what the second agent knows about the first agent, to assess the second agent's choice D_1 at the second stage.

Stalnaker explains that substantive rationality conflates two types of conditionals, given deviations from the backward induction profile. The first conditional is causal: if the second stage were reached, the second agent only evaluates subsequent stages and believes the first agent would choose a_2, since the first agent is rational. The causal conditional expresses a belief about an opponent's disposition to act in a contingency that will not arise. The second conditional is epistemic: suppose there is a deviation from the backward induction profile. The second agent revises his beliefs about his opponent, *given* that the second stage *is reached*. The epistemic conditional concerns the second agent's belief-revision policy. The second agent is not required to maintain his current beliefs about his opponent upon learning there is a deviation from the backward induction profile. Therefore, if the formal definition of strategy requires agents to evaluate all contingencies in a game to settle on the backward induction profile, and if deviations cannot be evaluated without violating backward induction, then a belief-revision policy is required to evaluate deviations without violating backward induction. The epistemic conditional

is the conditional relevant to evaluating deviations and circumventing the charge of incoherence to backward induction.

I argue, however, that the charge of incoherence to backward induction presumes a concept of strategy that requires an *explicit* choice of action for every contingency, which further presumes an ideal agent. Dixit, et al. [35] provide a simple test to determine whether a strategy is a complete plan of action. A strategy:

> . . . should specify how you would play a game in such full detail—describing your action in every contingency—that, if you were to write it all down, hand it to someone else, and go on vacation, this other person acting as your representative could play the game just as you would have played it. He would know what to do on each occasion that could conceivably arise in the course of play, without ever needing to disturb your vacation for instructions on how to deal with some situation that you have not foreseen ([35], p. 27).

The test is whether your strategy is sufficient for your representative to play the game just as you would without requiring further instruction. If so, your strategy is a complete plan of action. The representative test requires an ideal agent. An ideal agent has no cognitive limits, can perform any deduction, knows all logical and mathematical truths, and has all information (traditionally in the form of common knowledge) required to choose rationally. Only an ideal agent can evaluate and then assign a choice of action for every contingency.

However, does an ideal agent have *use* for a belief-revision policy to account for deviations from the backward induction profile? I argue 'no'. An ideal agent is provided with all the information about his opponent to properly strategize. The ideal agent knows his opponent. An ideal agent can consider counterfactual circumstances about his opponent (supposing he were mistaken about his opponent or circumstances). However, the ideal agent does not need to strategize based on possible mistakes, because those mistakes are not matter-of-fact. Indeed, an ideal agent maintains his current beliefs about his opponent. Hence, strategizing precludes any deviations from expected gameplay. If an ideal agent has little need for a belief-revision policy, then an ideal agent has little need for strategizing with epistemic conditionals. The ideal agent will take his circumstances and evaluate them as they are.

Stalnaker anticipates my objection.

> Now it may be that if I am absolutely certain of something, and have never even imagined the possibility that it is false, then I will not have an explicit policy in mind for how to revise my beliefs upon discovering that I am wrong. But I think one should think of belief-revision policies as dispositions to respond, and not necessarily as consciously articulated policies. Suppose Bob is absolutely certain that his wife Alice is faithful to him—the possibility that she is not never entered his mind. And suppose he is right—she is faithful, and he really knows that she is. We can still ask how Bob would revise his beliefs if he walked in one day and found his wife is bed with another man. We need not assume that Bob's absolute certainty implies that he would continue to believe in his wife's faithfulness in these circumstances, and we need not believe that these circumstances are logically impossible. We can make sense of them, as a logical possibility, even if they never occur to Bob. And even if Bob never thought of this possibility, it might be true that he would react to the situation in a certain way, revising his beliefs in one way rather than another ([31], pp. 49–50).

However, what Bob knows about Alice is crucial to how he evaluates his choices in this case. What is relevant to Bob is how he will *actually* interact with Alice, and how he will actually interact with Alice depends on what he knows about her. Bob can consider a hypothetical case of his wife's infidelity and still know Alice is faithful—they are not logically incompatible. However, Bob will choose how he interacts with his wife based on his knowledge of her faithfulness, not on a hypothetical case of infidelity. Bob can even devise the most sophisticated belief-revision policy to account for

myriad hypothetical cases of Alice's infidelity. But that policy is based on nothing more than fictions: epistemic conditionals with false antecedents. Bob does not include his belief-revision policy in his decision-making when interacting with his wife, because of what he knows about her. Indeed, I can make sophisticated decisions about how I will interact in Harry Potter's world. I can devise a belief-revision policy to account for myriad magical circumstances, a policy that considers that I am mistaken about not existing in Harry Potter's world, but it is a fictional world and it does not affect how I will act in this world.

Choices track efficacy, not auspiciousness ([8], Section 1). Evidential conditionals have false antecedents. Hence, an ideal agent will not consider deviations, because they are not efficacious, regardless of their auspiciousness. The information provided to an ideal agent is information that tracks efficacy. Hence, an ideal agent evaluates his choices' causal influence on an outcome. An ideal agent's belief-revision policy just accounts for a fictional world.

Therefore, ideal agents do not require belief-revision policies. If strategy does require belief-revision policies, it is for real people with cognitive limits. Real people can only foresee some contingencies and require a strategy with a belief-revision policy to address unforeseen contingencies. Their plan of action is provisional and open-ended. Real people do not know their choice's efficacy. Moreover, the representative test is not intelligible to real people. It is not simply a listing of instructions of the form: 'If contingency C occurs, choose action A'. A strategy includes the complete reasoning behind the forming of instructions, and that reasoning includes what is known about every contingency. A complete plan of action is incomprehensible and too complex for real people. It takes an ideal agent to make sense of the representative test.

I propose a more realistic understanding of the representative test. A realistic strategy can pass the test without the need of providing a complete plan of action. A representative might play the game in your favor by following a simple strategy in the form of an algorithm that can still evaluate any unforeseen contingency. The strategy Tit-For-Tat passes the representative test. The instructions might be written as:

Step 1: Cooperate at the first stage of the game.
Step 2: If the opponent cooperates at stage $n - 1$ (for $n > 2$), cooperate at stage n. Otherwise, defect at stage n.

The strategy is simple, yet provides sufficient information for a representative to play the game exactly as you specify, knowing what to do in any contingency without needing to contact you for further instructions given an unforeseen situation. Notice the strategy does not assign *specific* actions for every contingency in a sequential game. There is no need for such specification. This simple strategy ably passes the representative test, because it has an implicit belief-revision policy.

The implicit belief-revision policy (without formally stating it for the moment) includes epistemic dependence between an opponent's prior acts and an agent's future choice of action. I shall address this more in the next section. Belief-revision polices are crucial to the evaluation of real people's strategies and acts, but only if belief-revision policies themselves are rational. How do we assess a policy's rationality?

4. The Fitness of Belief-Revision Policies

I shall briefly discuss the potential role of evolutionary game theory for investigating the rationality of belief-revision policies. It can evaluate belief-revision policies by testing which polices are 'better' than others through evolutionary fitness. Robert Axelrod's tournaments [36] are well-known for directly testing strategies for evolutionary fitness. These same tournaments can indirectly test belief-revision policies for fitness, because strategies in said tournaments have corresponding belief-revision policies.

The strategy Tit-For-Tat has the following implicit policy:

Tit-For-Tat Policy: Cooperate at the first stage of the game. If the opponent cooperates at stage $n - 1$ (for $n > 2$), treat the opponent as a cooperator in future stages and choose cooperation at stage n. Otherwise, treat the opponent as a defector in future stages and choose defection at stage n.

The strategy calls for an agent to initially cooperate, and then consider his opponent's previous act, and then revise his belief about his opponent's subsequent act. It is a policy that has epistemic dependence between an opponent's prior acts and the agent's future choice of action.

There is nothing preventing us from devising strategies and corresponding policies with as much or as little detail as we desire. We simply include Robert Stalnaker's condition of epistemic dependence between prior acts and future choice of action. Any belief-revision policy can be represented as follows:

Generic Belief-Revision Policy: If an opponent does action a_j at stage $n - 1$ (for $n > 2$), treat the opponent with disposition D and respond with action A_j at stage n. Otherwise, continue with current set of beliefs and choice of action at stage n.

The response simply depends on the agent's belief-revision policy, which attributes a disposition to the opponent (for example, the opponent's hand trembled, is irrational, is a cooperator, etc.).

This program is fruitful and necessary: it explains the fitness of belief-revision policies. However, it is limited. It provides only as much information about which policy is fittest, as it does about which strategy is fittest. Kristian Lindgren and Mats Nordahl [37] have studied numerous strategies in Axelrod tournaments and found that there is no 'optimal', 'best', or outright 'winning' strategy. A strategy such as Tit-For-Tat performs better than other strategies for a period of time, before another strategy replaces it. The same might be said about corresponding belief-revision policies.

5. Constructing Belief-Revision Policies

An ideal agent with no cognitive limits can settle on a strategy for every contingency in a game. He has no need for a belief-revision policy as Robert Stalnaker envisions. However, real people with cognitive limits can only consider some contingencies. They do require a belief-revision policy for contingencies they have not considered. Thus, if a strategy requires a belief-revision policy, the concept is best understood vis-à-vis real people's cognitive limits. I shall focus on this realism in this section.

Epistemic game theory has developed Bayesian analyses of agents revising their beliefs during gameplay. However, the discipline has not fully considered some foundational issues in Bayesian epistemology. The most significant issue is that virtually any rational credence distribution can be assigned to a set of propositions (without changing the evidential circumstances). The aim is to compute a unique credence distribution for a set of propositions, given the evidential circumstances. I provide insight into the rationality of belief-revision policies by introducing their components through an example. The aim is for a unique credence distribution that is actionable. belief-revision policies have evidential and non-evidential components. The evidential components are Bayesian principles that guide how agents ought to revise their beliefs during gameplay. The non-evidential component that I consider is agents' temperaments. I show that evidential components can pare down an agent's credences into a unique distribution, but temperaments change credence distributions between agents—a concern that might have to be accepted about belief-revision policies.

Traditional epistemology treats three sorts of judgments: Belief, disbelief, and suspension of belief [38]. Bayesian epistemology instead treats credences, which indicate the differences in an agent's confidence in a set of propositions. Bayesian epistemology provides rational constraints on credences. The two fundamental rational constraints are synchronic and diachronic [39]. The synchronic thesis states that an agent's credences are rational at a given moment just in case those credences comply with Andrey Kolmogorov's [40] probability axioms. The diachronic thesis states that an agent's credences are rational across time just in case those credences update through conditionalization in response to evidence. Both the synchronic and diachronic theses are typically justified by Dutch Book arguments.

The credence function *Cr* maps a set of propositions to real numbers. Conditionalization applies Bayes's theorem to a conditional credence $Cr(H \mid E) = (Cr(E \mid H)Cr(H))/Cr(E)$, given that $Cr(E) > 0$. $Cr(H)$ is the unconditional credence in the hypothesis. $Cr(E \mid H)$ is the likelihood of the hypothesis on the total evidence. An unconditional credence distribution is computed for a set of propositions from the conditional credence distribution for those propositions, given the total evidence. Conditionalization provides rational closure. If an unconditional credence distribution is constrained by the probability axioms and updates by conditionalization, the resulting credence distribution is constrained by the probability axioms, too.

An arbitrary belief-revision policy is thus far composed of credences constrained by the probability axioms and by conditionalization. In general, I apply credences to finitely repeated and sequential (two-agent) games with perfect (but incomplete) information. Propositions are about agents' strategies. Agents have cognitive limits, but they know certain features of a game. They have perfect information: they know all acts that occur during gameplay. This might not be a fair specification for real people (real people have imperfect memories), but it tracks conditionalization. Agents also know the structure of the game, its payoffs, and the set of available strategies. However, they do not know each other's chosen strategy or immediate acts. Indeed, I specify that agents do not settle on a complete plan of action before gameplay. They instead settle on an immediate choice of action. An agent applies conditionalization on his opponent's acts during gameplay. Furthermore, they do not know each other's rationality. I specify that each agent is rational insofar as they each maximize their expected payoffs, but they do not know about each other's rationality. Each agent has some initial credence that his opponent is irrational, given that some of his opponent's available strategies might not maximize expected payoffs.

I use the Centipede Game in Figure 2 as an example. Agents are not endowed with any evidence about each other's strategies before gameplay. The first agent has 2^3 available strategies: $(a_1a_2a_3)$, $(a_1a_2d_3)$, $(a_1d_2a_3)$, $(a_1d_2d_3)$, $(d_1a_2a_3)$, $(d_1a_2d_3)$, $(d_1d_2a_3)$, and $(d_1d_2d_3)$. The second agent has 2^2 available strategies: (A_1A_2), (A_1D_2), (D_1A_2), and (D_1D_2).
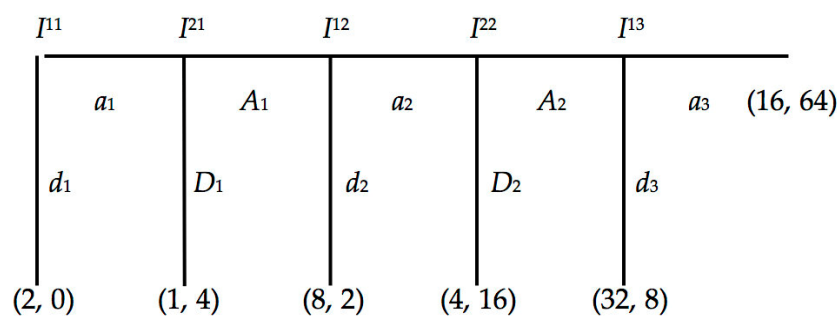

Figure 2. I^{ij} is the *j*-th information set ($j \geq 1$) of agent *i* ($i = 1, 2$). This Centipede Game has two pure strategies at each information set: Either 'Down' ending the game, or 'Across' allowing the other agent to choose an action. The agents' payoffs are represented at the tree's endpoints, the left member being the first agent's payoff, the right member being the second agent's payoff. The backward induction profile is $(d_1d_2d_3, D_1D_2)$.

Agents are thus far not precluded from assigning $Cr = 1$ or $Cr = 0$ to contingent propositions. Agents might have certainty or complete unbelief about contingent propositions. But it is more plausible that real people with cognitive limits are uncertain about opponents' strategies. Any evidence about opponents' strategies accumulated during gameplay will provide some credence about those strategies. Thus, a credence distribution assigns each strategy some credence. I thus include the principle of regularity into any belief-revision policy. It states that a contingent proposition in a rational credence distribution receives $1 > Cr > 0$ [41]. The principle of regularity is an additional rational

constraint to the probability axioms and conditionalization. It pares down credence distributions that can be assigned to a set of contingent propositions.

However, it does not, together with the probability axioms and conditionalization, compute a unique credence distribution. Suppose each agent begins the game with a $Cr = 1/n$ for a number n of an opponent's available strategies. The second agent is assigned $Cr = 1/8$, while first agent is assigned $Cr = 1/4$ for each of his opponent's available strategies. Suppose the first agent chooses and acts on a_1 at I^{11}, and the second agent must decide whether to choose D_1 or A_1 at I^{21}, which depends on whether the first agent will choose d_2 or a_2 at I^{12}. The second agent's choice is between his opponent's four remaining available strategies: $(a_1 a_2 a_3)$, $(a_1 a_2 d_3)$, $(a_1 d_2 a_3)$, $(a_1 d_2 d_3)$. It is important to recognize that the act a_1 at I^{11} is evidence that alone does not favor any particular strategy over another in the remaining set of available strategies. The second agent, thus, applies conditionalization on said evidence and evenly assigns credences to those strategies (the evidence does not favor one strategy over another). This is a reasonable application of the principle of indifference. It states that an agent updates his credences proportionally to the evidence, and no more [42]. The second agent applies conditionalization upon the current evidence of a_1 at I^{11}, and has $Cr = 1/4$ in strategies $(a_1 a_2 a_3)$, $(a_1 a_2 d_3)$, $(a_1 d_2 a_3)$, $(a_1 d_2 d_3)$. Removing acts that have occurred, he has $Cr = 1/4$ in $(a_2 a_3)$, $(a_2 d_3)$, $(d_2 a_3)$, $(d_2 d_3)$. The principle of indifference guides the second agent's judgment of his opponent's future choice of action. It is an additional rational constraint to regularity, to the probability axioms, and to conditionalization.

How does the second agent choose an action, given his current credence distribution? The second agent has a simplified decision problem at I^{21}. He has two choices between D_1 for the immediate payoff of 4, or A_1 for the chance of a payoff of 16 at I^{22}. This depends on the likelihood of the first agent choosing a_2 at I^{12}. We can compute a credence threshold for the second agent. His expected payoff for A_1 at I^{21} is $Cr(a_2 \text{ at } I^{12})(16) + (1 - Cr(a_2 \text{ at } I^{12}))(2)$, which is set equal to 4. Apply some algebra and the second agent has a threshold $Cr(a_2 \text{ at } I^{12}) = 1/7$. If the second agent's credence at I^{21} surpasses his threshold, he maximizes his expected payoff by choosing A_1 at I^{22}. The second agent's credence in my example is $1/2$ that a_2 at I^{12}.

Now, suppose the first agent chooses a_2 at I^{12}, and the second agent must decide whether to choose D_2 or A_2 at I^{22}. This depends on whether the first agent will choose d_3 or a_3 at I^{13}. Again, the act a_2 at I^{12} is evidence that does not alone favor any particular strategy over another in the remaining set of available strategies. The second agent thus updates his credences proportionally to the evidence, and no more. The second agent applies conditionalization upon the current evidence of a_2 at I^{12}, and has $Cr = 1/2$ in strategies $(a_1 a_2 a_3)$, $(a_1 a_2 d_3)$. Removing acts that have occurred, he has $Cr = 1/2$ in a_3 and d_3. The second agent's credence threshold that a_3 at I^{13} is $1/7$, as computed above. He maximizes his expected payoff by choosing A_2 at I^{22}. The reader might find this result surprising, because it is stipulated that each agent is rational—the first agent will choose d_3 at I^{13}. However, remember that agents only know about their own rationality, not about each other's.

The principle of indifference is an additional rational constraint to regularity, to the probability axioms, and to conditionalization. It pares down credence distributions that can be assigned to a set of contingent propositions. However, there are objections to the principle that it recommends different credences for the same evidence, depending on how we partition a set of propositions. Here is a standard example. I explain that my ugly sweater is some color. You consider whether my ugly sweater is red, and partition the choices to red and not-red. The principle of indifference recommends that $Cr(R) = 1/2$, and that $Cr(\sim R) = 1/2$. However, you then consider whether my ugly sweater is one of the primary colors. Then the principle recommends that $Cr(R) = 1/3$, and that $Cr(\sim R) = 2/3$. The principle recommends different credences depending on how you partition the choice of colors, given the same evidence.

I argue, however, that the problem is not with the principle of indifference. Evidence is both subject to evidential and non-evidential components, and this objection to the principle of indifference is an example of the influence of non-evidential components. Consider that epistemology has two

comprehensive goals. One goal is to minimize the amount of falsehoods an agent obtains. The other goal is to maximize the amount of truth an agent obtains. No goal is guaranteed, and realistically an agent balances between both in evaluating evidence. Thus, some agents have a skeptical temperament. They cautiously accept evidence, which is reflected by low credences in particular propositions. Some agents have a credulous temperament. They too willingly accept evidence, which is reflected by high credences in particular propositions. Temperaments affect how agents partition a set of propositions, or agents' confidence in the evidence they evaluate. This non-evidential component combines with evidential components to establish credence distributions. Non-evidential components make the difference between two agents applying conditionalization with the same evidence, resulting in different credence distributions, despite their rational constraint.

How is epistemic game theory to account for non-evidential components? Epistemic game theory is traditionally concerned with the use of epistemic states to rationally justify a game's outcome as a profile of agents' strategies—the solution to the game. A Bayesian approach to belief-revision policies (for agents with cognitive limits, where temperaments affect their credences) might have to concede that there is no rational prescription for games as an outcome with a unique profile of strategies—no ultimate solution. It might prescribe a set of profiles where many of a game's outcomes will satisfy that set of profiles. But this is simply saying that a game can have many outcomes real people might rationally settle on. This is not to say their beliefs and corresponding actions are not rational. If there is a lesson to learn from Bayesianism, agents' beliefs and their corresponding acts can be rational while simultaneously not settling on an optimal outcome.

As a side note, naturalistic game theory does investigate non-evidential components. The study of heuristics [3,4] is a study of temperaments, or, more specifically, cognitive biases. The term 'cognitive bias' has a negative connotation, but biases can provide better outcomes in specific circumstances than more sophisticated tools, especially in uncertainty. The study of biases as heuristics is a study of the environment in which the biases evolved; and there is ongoing investigation into how agents' temperaments (non-evidential components) are environmentally constrained.

6. Conclusions

The motivation behind the recent formation of naturalistic game theory does stem from dissatisfaction with the current debates over foundational concepts. However, the intent is not to replace epistemic and evolutionary game theory. How then does naturalistic game theory split the difference between said disciplines? Naturalistic game theory purports that foundational concepts are confronted by potentially irredeemable difficulties. Epistemic game theory provides a valuable role by testing the limits of those concepts foundational to the discipline. Once those limits are established, the principal aim for naturalistic game theory is to provide guidance for real people's judgments in their realistic circumstances. Thus, I suggest that epistemic game theory has two aims. One aim is to derive conditional results based on the game theorist's desired idealizations and stipulations. The second aim is to clarify the conceptual limits of those idealizations and stipulations. This latter aim extends to more traditional epistemological concerns about those idealizations foundational to analysis. Some idealizations are innocuous, such as a game's structure and its payoffs. However, some idealizations are not innocuous—the concepts of strategy and of belief-revision policies are two examples.

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