



# *Article* **Capillary Flows of Nematic Liquid Crystal**

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**Abstract:** In this paper we report the new experimental results on the rise of a liquid crystal in flat capillaries with inner photosensitive surfaces. The capillaries with different surface orientations were prepared by the use of the photo-alignment technique. Such a surface treatment makes it possible to eliminate the noncontrollable influence of a nanorelief on the wetting process, which takes place in the rubbing treatment technique previously used in similar experiments. The dynamics of the capillary rise of a nematic liquid crystal 5CB (4-cyano-4'-pentylbiphenyl) in vertical plane capillaries with photo-aligned substrates were studied for the first time. It was found that the stationary value of a contact angle weakly depends on the direction of a planar surface orientation relative to the direction of a capillary rise. It has been shown that the application of strong electric fields resulted in a decreasing of the contact angle. The results, obtained for the nematic liquid crystal, are compared with the results of an investigation of the capillary flow in a shock-free ferroelectric smectic phase.

**Keywords:** capillary flow; photo-alignment; electric field; anisotropic shear viscosities; contact angle; liquid crystal

## **1. Introduction**

The investigation of the wetting of solid surfaces by different liquids is an interesting issue, both from an academic and a technological point of view [\[1,](#page-14-0)[2\]](#page-14-1). It is important to explore the dynamics of the director reorientation in a nematic liquid crystal (LC) phase confined to thin or ultrathin channels or capillaries [\[1](#page-14-0)[,3\]](#page-14-2). These processes were investigated using different experimental methods, for example by the study of liquid (isotropic and anisotropic) drops spreading over both isotropic and anisotropic surfaces  $[4-8]$  $[4-8]$ , including those with the micro (nano) relief [\[7\]](#page-14-5). In turn, the alternative well-established technique is based on the arising of liquids in vertical capillaries with different cross sections. This technique allows the determination of static parameters (surface tension coefficient, static contact angle) and the investigation of a specific decay capillary flow. It should be noted that both experimental and theoretical studies of the capillary rise of isotropic liquids have a long history [\[9–](#page-14-6)[12\]](#page-14-7). At the same time, the dynamics of the capillary decay flow of liquid crystals are not well explored. In particular, the roles of anisotropic surface interactions and the bulk anisotropy of the nematic phase in this process are still unclear. In the presence of external and internal forces, the relaxation of the director  $\mathbf{n}(r,t)$  into its equilibrium orientation  $\mathbf{n}_{eq}(\mathbf{r})$  is governed by elastic, electric, viscous, and anchoring torques, as well as by forces exerted per unit of LC volume [\[13\]](#page-14-8). The intrinsic connection between flow gradients and the overall orientation of LC molecules described in terms of the director

field **n**(**r**,t) is responsible for the non-Newtonian character of liquid crystal flows. The fact that in the case of a Poiseuille flow of LC through capillaries of different cross sections the effective shear viscosity η*e*ff depends on the flow rate, the size of capillaries and the orientation of the boundaries were experimentally established and explained in the framework of the Ericksen–Leslie hydrodynamic theory [\[14\]](#page-14-9). It is also established that a strong magnetic field **B** applied to the flowing liquid crystals has a stabilizing effect on the equilibrium director orientation  $\mathbf{n}_{eq}(\mathbf{r}) = \mathbf{n}_0$ , which ultimately provides the Newtonian-like behavior of an effective shear viscosity  $\eta_{\text{eff}} = \eta_{\text{eff}}(\mathbf{n}_0)$  [\[13\]](#page-14-8). For the flow through the flat capillary of a gap *h* that is much smaller than the width A, the three principal shear viscosity coefficients (Miesowicz viscosities) are  $\eta_1 = \eta_1(\mathbf{n}_0 \, \text{He}_z)$  when  $\mathbf{n}_0$  is parallel to  $\mathbf{e}_z$ ,  $\eta_2 = \eta_2(\mathbf{n}_0 \, \text{IIv})$  when  $\mathbf{n}_0$  is parallel to **v**, and  $\eta_3 = \eta_3(\mathbf{n}_0 \mathbf{I} \mathbf{e}_{\mathbf{z}}, \mathbf{v})$  when **n** is orthogonal to both **v** and  $\mathbf{e}_{\mathbf{z}}$  [\[14\]](#page-14-9), respectively. Here,  $\mathbf{e}_{\mathbf{z}}$  is the unit vector oriented along the normal to the boundaries of the flat capillary. The precise measurements of the Miesowicz viscosities [\[15\]](#page-14-10) can provide the most reliable information about dissipative and reactive parameters (the so-called Leslie coefficients  $\alpha_i$ , i = 1  $\dots$  6) that are used in Ericksen–Leslie theory [\[16\]](#page-14-11).

Previously, it was shown that the stabilizing effect of the magnetic field in shear viscosity measurements can be replaced by surface induced orientation [\[17\]](#page-14-12). In this case, the three Miesowicz viscosities can be obtained by means of different surface treatments of the inner surfaces of the flat capillary. Such a technique is more suitable for the us of newly synthesized LC materials because it demands small amounts of LC material. Experimental realization of this idea was achieved via the use of a decay Poiseuille flow, arising in a flat capillary under the difference in hydrostatic pressure ∆*P*(*t*) = ρ*g*∆*H*(*t*). Here, ρ is the density of the LC phase, *g* is the gravitational constant, and ∆*H*(*t*) is the difference between the levels in two vertical open tubes connected with a capillary. In this case, the decay time of the flow  $\tau_d$  is proportional to the effective viscosity coefficient  $\eta_{eff}$ , and can be used to derive the three principal viscosities corresponding to the abovementioned geometries. Later, an additional modification of this method accounting for the electric field and photo-controlled surfaces is proposed [\[18](#page-14-13)[,19\]](#page-14-14).

In turn, the current investigations are also based on the usage of the decay flow of the LC phase through the flat vertically oriented capillary. Contrary to the abovementioned flow, the current case is controlled by three forces:

- i. first, the capillary force **Fc**, which induces the rise of a meniscus;
- ii. second, the gravity force **Fg**, which is proportional to the height of the rise of the LC phase and acts against the capillary force until the total stop of a meniscus motion;
- iii. third, the viscous force **Fv**, proportional to the velocity of a meniscus motion and also acting against the capillary force.

The investigation of such types fulfilled with isotropic liquids turned out to be useful for the study of both the bulk (viscosity) and the surface (wetting) phenomena [\[10\]](#page-14-15). The latter are controlled by the physical mechanisms acting on both the mesoscopic and microscopic lengths, including the absorption–desorption processes during the motion of the precursor film. Previously, such a phenomena of spreading the LC droplets over the anisotropic surfaces with submicron relief has been investigated [\[7\]](#page-14-5). The main aim of our experiments is to clarify the effects of both the surface and the bulk properties of the liquid crystal phase on the wetting processes, which are responsible for the decay of the capillary flow of anisotropic liquids.

A significant difference between our research and numerous experiments on the capillary rise of isotropic liquids in capillaries with a circular cross section [\[10\]](#page-14-15) is the use of flat capillaries with photo-controlled surfaces. The latter provides surfaces with essentially identical properties, except for surface orientation (easy axes direction), which defines, in combination with the flow and electric field, the orientational structure in the bulk of the LC layer. This approach is proposed here, to our knowledge, for the first time.

#### **2. Materials and Methods**

The construction of the sandwich-like LC cells of the first type (cells 1 and 2) with a constant gap is shown in Figure [1a](#page-2-0). It consists of the two glass plates (of thickness 1.1 mm) coated with ITO (indium tin oxide) transparent electrodes. In our experiments, the ITO coating on one of the glass substrates was partly deleted to provide the application of alternating electric voltage U of frequency f to one of the two local capillaries with the same surface orientation. This revealed the effect of the electric field on the wetting and viscosity properties of liquid crystals. The inner surfaces of the plates were thoroughly cleaned to establish strongly water-wet glass surfaces and spin coated by 0.5% solution of azo sulphiric dye (SD-1) in dimethyl formamide (DMF) and baked at 90 ◦C for a 1 h to provide a thin nanometer layer of a dye. *Crystals* **2020**, *10*, x FOR PEER REVIEW 4 of 16

<span id="page-2-0"></span>

**Figure 1.** Designs of experimental cells. (**a**)—design of the cells 1 and 2 with a constant gap, shaded areas indicate the presence of ITO electrodes; (b)—design of the cell 3 with a variable gap; (c)—scheme illustrating the capillary flow of LC under the effect of capillary and viscous forces; (**d**)—structural illustrating the capillary flow of LC under the effect of capillary and viscous forces; (**d**)—structural formula of the azo dye SD-1; (**e**)—structural formula of the liquid crystal 5CB. formula of the azo dye SD-1; (**e**)—structural formula of the liquid crystal 5CB.

Afterwards, the plates were stacked with the help of spacers (nylon thread of the given diameter d) and fixed by clue to form the three or two flat channels of the gap *h*, width *A* and length L (see Table [1\)](#page-2-1). Each of the channels were treated by ultraviolet (UV) linear polarized irradiation ( $\lambda$  = 345 nm) of the same exposure dose ( $D = 0.45$  W/cm<sup>2</sup>) but different orientation of a polarization plane. In accordance with the photoalignment technique, previously used in photonics [\[20](#page-14-16)[,21\]](#page-14-17), this procedure provides different orientation of azo dye molecules without changes in the nano scaled relief of the surface and corresponding surface orientation of LC, described by the surface director  $n_s$ , after filling the cell. Contrary to the similar experiments [\[22\]](#page-14-18) where the rubbing technique was used, it allows to study the net effect of anisotropy on wetting and capillary phenomena.

<span id="page-2-1"></span>



diameters  $(d_1 \text{ and } d_2)$  fixed at the distance *A* (see Table [1\)](#page-2-1). This allowed us to study the capillary flow with a continuous change in the thickness *h* of the LC layer in the direction normal to the flow direction. The photoalignment technique, described above provided the initial surface orientation of LC layer normal to the flow direction. The small value of the wedge angle (about 0.1 deg) gave reason to consider the wedge-like capillary as a large number of independent parallel capillaries with a constant gap. Thus, the theoretically predicted dependences of the flow parameters on the thickness of the LC layer can be verified experimentally.

After filling the cell with a liquid crystal, the values of a capillary gap *h* were additionally determined by measuring the electrically induced birefringence changes as polarized laser radiation propagated through the LC layer. These values, presented in Table [1](#page-2-1) slightly exceeded the spacers' diameters most likely due to insufficient compression of the cell during cluing.

After preparation, the vertically oriented cell was quickly immersed in the rectangular vessel filled with LC, which led to the appearance of a capillary flow recorded by a digital camera. The use of two crossed polarizers, oriented at an angle of 45 degrees to the flow direction, provided control of orientational structure of LC. Digital image processing allowed us to identify the time dependence of the contact line displacement *l*(*t*) in different parts of the cell.

In our experiments we used the standard nematic liquid crystal 5CB (4-cyano-4'-pentylbiphenyl) with a measured clearing temperature  $T_c = 35 \pm 0.2$  °C. All measurements in the nematic phase were carried out at temperature T = 24  $\pm$  0.5 °C for cell 1 and T = 27  $\pm$  0.5 °C for cells 2 and 3. The experiments were carried out in a thermostatically controlled chamber. Before the start of the experiment, the empty cell and the vessel with LC were placed in a thermostat for half an hour to minimize temperature gradients. The temperature deviation from the given value did not exceed  $\pm 0.5$  K during the entire experimental period.

The choice of 5CB was dictated by the available information on electrical, optical, viscous and elastic properties [\[13](#page-14-8)[,14\]](#page-14-9). In addition, the large positive value of  $\Delta \varepsilon = 11$  for 5CB provided the effective electrical control of the rheological behavior of this nematic in the capillary flow. In particular, the use of a strong electric field stabilized the orientation of the LC, which made it possible to use the approximation of a conditionally Newtonian fluid in the analysis of flow phenomena.

#### **3. Results and Discussion**

The typical images of the cell with a constant gap obtained at different times passed after beginning of the capillary flow are shown in Figure [2.](#page-4-0) One can see a slight difference in the instantaneous values of capillary rise *l*(*t*) recorded for the channels with different surface orientation. At the same time, applying of the electric field resulted in pronounced slowing of the capillary flow. This effect can be attributed to a significant increase in the viscous forces originating due to the reorientation of LC layers from the initial planar towards the quasi homeotropic structure. The dark fields in the images of the electrically controlled channels, shown in Figures [2](#page-4-0) and [3](#page-4-1) and as well as the absence of interference stripes indicate that the real orientational structure in the bulk of LC layers is close to the hometropic one, with the exception of boundary quasi planar layers with the thickness less than the visible light wavelength (about 0.5 µm). At the same time, it was found that the use of a high electric voltage (150 V) induces the pronounced increase in the  $\Delta l^{(e)}$  of the stationary levels *l*<sub>0</sub> of the capillary rise. The values of the ratio ∆*l*<sup>(e)</sup>/*l*<sub>0</sub> were approximately 0.1 for two types of surface orientation. So, this increase can obviously be assigned to the electrowetting phenomenon well studied for isotropic liquids [\[1\]](#page-14-0). Such a phenomenon is probably also responsible for the field induced transformation of the profile of the contact line profile shown in Figure [3.](#page-4-1)

<span id="page-4-0"></span>

Figure 2. Snapshots showing the movement of the contact line in capillaries with different surface orientations in the cell 1 (h = 110  $\mu$ m). The polarizer and analyzer are crossed and oriented at  $45^{\circ}$ relatively to the flow direction. (1)—n<sub>s</sub> parallel to the flow velocity and perpendicular to the flow velocity gradient; (2)—n<sub>s</sub> perpendicular to the flow velocity and the flow velocity gradient; (3)—n<sub>s</sub> perpendicular to the flow velocity and the flow velocity gradient. The applied electric voltage is  $U = 100$  V,  $f = 5$  kHz, respectively.

<span id="page-4-1"></span>

Figure 3. Snapshots showing the movement of a contact line in capillaries in the cell 2 (h =  $95 \mu m$ ). Polarizer and analyzer are crossed and oriented at  $45^{\circ}$  relatively to the flow direction. (1)— $\mathbf{n}_s$  parallel to the flow velocity and perpendicular to the flow velocity gradient;  $(2)$ —n<sub>s</sub> parallel to the flow velocity and perpendicular to the flow velocity gradient. The applied electric voltage is  $U = 150$  V,  $f = 1$  kHz, respectively.

The dependences of  $l(t)$ , obtained as a result of digital image processing, are shown in Figures [4](#page-5-0) an[d 5](#page-5-1). The registered decrease in the instantaneous velocity  $d/dt$  of the contact line movement can be explained by the increase in the gravitational and viscous forces that slow down the capillary flow over time. The application of the electric field leads to a significant decrease in the instant rate  $d/dt$  in the comparison with this parameter for the field-free motion of the contact line.

Analysis of similar photos of the wedge-like cell made it possible to record the instant lateral profiles of the contact line at different times. The examples of such profiles are shown in Figure [6.](#page-6-0) A significant difference can be observed in the shape of the profile recorded at the initial and final stages of the capillary flow. The quasi stationary profile  $l_0(x)$ , obtained in the region of high local values of *h* after almost complete decay of the flow is close to the linear dependence. The time dependences of the capillary rise *l*(*t*) at different local values of the gap, obtained via processing of the instant profiles are shown in Figure [7.](#page-6-1)

<span id="page-5-0"></span>

Figure 4. Time dependences of the meniscus displacement  $l(t)$  for the channels with the initial surface orientation  $n_s$  perpendicular to the flow velocity and the flow velocity gradient in the absence (cell 1(2), red circles) and presence (cell 1 (3), black squares) of the electric voltage  $U = 100 V$ ,  $f = 5$  kHz. Dashed lines show the corresponding stationary levels (the level reached by the meniscus 10 h after the start of the experiment).

<span id="page-5-1"></span>

**Figure 5.** Time dependences of the meniscus displacement  $l(t)$  for the channels with the initial surface orientation  $\mathbf{n_s}$  parallel to the flow velocity and perpendicular to the flow velocity gradient in the absence (cell 1 (1), red circles) and presence (cell 2 (2), black squares) of the electric voltage  $U = 150 V$ ,  $f = 1 kHz$ . Dashed lines show the corresponding stationary levels (the level reached by the meniscus 10 h after the start of the experiment).

<span id="page-6-0"></span>

<span id="page-6-1"></span>Figure 6. Experimentally obtained instant lateral profiles in cell 3 at different times. Dashed line shows the stationary level (the level reached by the meniscus 10 h after the start of the experiment), perpendicular to the flow velocity and the flow velocity gradient. **ns** perpendicular to the flow velocity and the flow velocity gradient. perpendicular to the flow velocity and the flow velocity gradient.



**Figure 7.** Time dependences of the meniscus displacement  $l(t)$  at different local values of the gap  $h$  in cell 3 with the initial surface orientation  $n_s$  perpendicular to the flow velocity and the flow velocity gradient. Dashed lines shows the corresponding stationary levels (the level reached by the meniscus 10 h after the start of the experiment, red dashed line (2) for  $h = 79 \mu m$ , black dashed line (1) for  $h = 106 \mu m$ , respectively.

Analysis of the experimental data, presented above, reveals the experimental data above, reveals the features o<br>Analysis of a capital data and developed the features of a capital description of the dumention of conjillary anisotropic liquids. It shows that the noted that the three that the theoretical description of the dynamics of  $\alpha$ rise is quite difficult even in the case of isotropic liquids. To explain some features of this properties of  $\eta$ pour microscopic and hydrodynamic approaches with microscopic and hydrodynamic approaches with empirical to the Analysis of the experimental data, presented above, reveals the features of a capillary flow of Analysis of the experimental data, presented above, reveals the features of a capillary flow of anisotropic liquids. It should be noted that the theoretical description of the dynamics of capillary anisotropic liquids. It should be noted that the theoretical description of the dynamics of capillary rise is quite difficult even in the case of isotropic liquids. To explain some features of this phenomenon, both microscopic and hydrodynamic approaches were used together with empirical equations [\[10\]](#page-14-15). This applies, in particular, to the time dependence of the contact angle, are observed experimentally.

 $\epsilon$  applies, in particular, i.e. the time dependence of the contact angle, are observed by  $\epsilon$  and the contact angle. experimentally. flow characteristics of liquid crystals [\[13\]](#page-14-8). It should be noted that this approach was previously used The experimental results were analyzed using a continuum approach that takes into account the to describe the dynamics of the rise of the isotropic Newtonian liquids in the vertical capillaries of

various cross-sections and sizes [\[10\]](#page-14-15). Now we used the well-known Poiseullie Equation, describing the volumetric flow rate *Q*

$$
Q = \frac{dV}{dt} \tag{1}
$$

induced by the pressure difference ∆*P* in the capillary of length *l*. Here *V* is the pumped volume. For the plane capillary with the large value of the aspect ratio *R* = *A*/*h* (here *A* and *h* are the width and the gap of the capillary) the Poiseullie Equation is written as:

$$
\frac{dV}{dt} = \frac{h^3 A}{12\eta} \sum \frac{\Delta P}{l} \tag{2}
$$

where ∑∆P is the resulting pressure difference, which includes both the hydrostatic

$$
P_g = \rho g l \tag{3}
$$

and the capillary

$$
P_{\sigma} = \frac{2\sigma\cos\theta}{h} \tag{4}
$$

pressures, respectively.

In these equations the density  $\rho$  and the shear viscosity  $\eta$  are the parameters describing the bulk properties of the liquid, whereas both the surface tension coefficient  $\sigma$  and the contact angle  $\theta$  are the parameters which characterize the interaction of the liquid phase with the air and the solid surface, respectively. Taking into account the expression for the pumping volume

$$
dV = hAdl \tag{5}
$$

and using the Equations (1)–(4), one can get the next differential equation for a description of the motion of the contact line:

$$
\frac{dl}{dt} = \frac{\sigma h \cos \theta}{6\eta l} - \frac{h^2 \rho g}{12\eta} \tag{6}
$$

This equation can be considered as the simplified (and modified) version of the well-known Washburn Equation [\[10\]](#page-14-15) proposed for a describing the motion of the contact line in the circular capillary.

In our calculation the contact angle  $\theta$  was chosen to be independent on time and equal to the static value. This approximation is confirmed by the estimation of the time dependent contribution to the value of the contact angle  $\theta$ , made in accordance with the semi empirical expression [\[10\]](#page-14-15), which is read as:

$$
\cos \theta_d = \cos \theta_e - 2(1 + \cos \theta_e)Ca^{\frac{1}{2}}
$$
\n(7)

where  $\theta_d$  is the dynamic contact angle,  $\theta_e$  is the static contact angle,

$$
Ca = \frac{\eta v}{\sigma} \tag{8}
$$

is the capillary number, whereas  $v = d/dt$  is the velocity of a contact line.

The maximal value of the dynamical contribution to  $cos(\theta_d)$  takes place at the initial stage of the flow and does not exceed 15 . . . 20% at typical values  $\eta \approx 0.05$  Pa\*s,  $\sigma = 0.033$  N/m and registered values of  $v \leq 2$  cm/s.

We also neglected the possible effect of the roughness of the internal surfaces on the capillary flow [\[23](#page-14-19)[,24\]](#page-14-20). The latter seems to be reasonable, since UV irradiation of the azo dye layer does not lead to any changes in the surface relief.

The solution of Equation (6) for the constant value of the contact angle  $\theta_e$  with the zero initial condition  $(l = 0$  at  $t = 0)$  is defined by the next expression:

$$
y = \frac{l_0 - l}{l_0} \exp\left(\frac{l}{l_0}\right) = \exp\left(-\frac{t}{\tau}\right) \tag{9}
$$

where the maximal value of the capillary rise  $l_0$  and characteristic decay time  $\tau$  of the flow are expressed as:

$$
l_0 = \frac{2\sigma\cos\theta}{\rho g h} \tag{10}
$$

and

$$
\tau = \frac{24\sigma\eta\cos\theta}{h^3(\rho g)^2} = \frac{12\eta l_0}{h^2\rho g}
$$
\n(11)

respectively.

Thus, the parameter  $l_0$  depends on both the surface tension coefficient  $\sigma$  and the contact angle  $\theta$ . Each of these parameters may depend on a surface orientation of the LC, but the surface orientation on the boundary LC/air is fixed and does not depend on the orientation on the boundary LC/dye, with the exception of the wall area.

Comparison of the experimental results with the predictions of the theoretical model includes analysis of both the static and dynamic characteristics of the capillary rise. In particular, the maximal values of the capillary rise  $l_0$  measured in different cells were used in accordance with expression (10), to calculate the corresponding contact angles (see Table [2\)](#page-8-0).

<span id="page-8-0"></span>**Table 2.** The static parameters of the capillary rise when there is no electric voltage for cells 1 and 3. Cell 1 (2) and the cell 3—**ns** perpendicular to the flow velocity and the flow velocity gradient; cell 1 (1)—**ns** parallel to the flow velocity and perpendicular to the flow velocity gradient. The contact angle is calculated using Equation (10).

		Cell <sub>3</sub>					Cell 1 (2)	Cell 1 (1)
Capillary gap, $h(\mu m)$	80	85	90	95	100	105	110	110
Stationary level, $l_0$ (mm)	64.9	62.9		60.6 57.5	54.6	50.8	48.2	49.9
Contact angle, $\theta$ (deg)	37	36	33	33	33	35	36	33

The results obtained for 5CB and for the different directions of surface orientation (cell 1) show a weak (about 10%) anisotropy of the contact angle, which is comparable to the error (about 5%) when determining this parameter from the stationary meniscus profile in the wedge-like cell 3 shown in Figure [6.](#page-6-0)

The approximation of the experimental dependences by the function (see Equation (9)) for different directions of surface orientation, both with and without accounting for the electric field is presented in Figures [8](#page-9-0) and [9.](#page-9-1)

The parameters of approximation of the experimentally obtained data by Equation (9) for different values of the surface orientation, both in the absence (cell 1) and presence of the electric field are shown in Table [3.](#page-8-1)

<span id="page-8-1"></span>**Table 3.** The parameters for approximating experimental data in accordance with Equation (9) are shown in Figures [8](#page-9-0) and [9](#page-9-1) and the values of effective shear viscosities calculated by using Equation (11).

	Cell $1(1)$	Cell 2 (2)	Cell 1 (2)	Cell 1 (3)
$\tau$ (sec.)	185	680	181	375
$l_0$ (mm)	49.9	59.9	45.3	48.2
$\eta$ (Pa <sup>*</sup> s)	0.037	0.086	0.038	0.085

<span id="page-9-0"></span>

**Figure 8.** Approximation by Equation (9) from experimental data with the relaxation time *τ* as the **Figure 8.** Approximation by Equation (9) from experimental data with the relaxation time τ as the fitting parameter in the absence and presence of electric voltage  $U = 100$  V,  $f = 5$  kHz for the initial surface orientation n<sub>s</sub> perpendicular to the flow velocity and the flow velocity gradient. Black squares correspond to the results obtained for the cell 1 (3), red circles—for the cell 1 (2). The values of  $\tau$  and  $l_0$ *l*0 are presented in Ta[ble](#page-8-1) 3. are presented in Table 3.  $\mathbf{L}$ . *η* (Pa s) 0.037 0.086 0.038 0.085

<span id="page-9-1"></span>

**Figure 9.** Approximation by Equation (9) from experimental data with the relaxation time τ as the **Figure 9.** Approximation by Equation (9) from experimental data with the relaxation time τ as the fitting parameter in the absence and presence of electric voltage  $U = 150$  V,  $f = 1$  kHz for the initial surface orientation  $n_s$  parallel to the flow velocity and perpendicular to the flow velocity gradient. Black squares correspond to the results obtained for the cell 2 (2), red circles—for the cell 1 (1). The values of  $\tau$  and  $l_0$  are presented in Table [3.](#page-8-1)

The best agreement between the theory and the experiment in the absence of electric field occurs for the channel with the surface orientation parallel to the flow direction. At the same time, the value of the shear viscosity coefficient obtained using the least squares approximation significantly exceeds the values of the Miesowicz coefficient  $\eta_2$  [\[15\]](#page-14-10) which corresponds to the homogeneous orientation of the director along the flow direction. Some difference can be assigned to the flow induced declination  $\theta_{\text{fl}}$ of the polar angle  $\theta$ , which describes the bulk orientation from the planar structure ( $\theta = 0$ ), stabilized by surfaces. The maximal value of  $\theta_{\text{fl}}$ , corresponding to the case of intensive flows can be calculated from the well-known expression

$$
tg^2\theta_{fl} = \frac{\alpha_3}{\alpha_2} \tag{12}
$$

where  $\alpha_3$  and  $\alpha_2$  are the Leslie's coefficients, which are equal to 0.0036 Pa\*s and 0.0812 Pa\*s, respectively, for 5CB [\[14\]](#page-14-9). Using these values for the Leslie coefficients, one can estimate, according to Equation (12), the value of  $\theta_{fl}$ , which is equal to  $\approx 12^\circ$ . The maximal difference  $\Delta \eta$  between the effective shear viscosity measured in the intensive flow and the Miesowicz viscosity  $\eta_2$  can be estimated by using the dependence of the shear viscosity on the polar  $\theta$  and the azimuthal  $\varphi$  angles [\[13\]](#page-14-8):

$$
\eta(\theta,\varphi) = (\eta_1 + \eta_{12}sin^2\theta)cos^2\theta cos^2\varphi + \eta_2 sin^2\theta + \eta_3 cos^2\theta sin^2\varphi \tag{13}
$$

where the viscosity  $\eta_{12}$  is essentially smaller than  $\eta_1$ . The estimate of  $\Delta\eta$  in accordance with Equation (13) for the case of variation of the director in the flow plane ( $\varphi = 0$ ) at values  $\eta_1 = 0.105$  Pa\*s and  $\eta_2$  = 0.0204 Pa\*s at T = 26 °C [\[15\]](#page-14-10) gives the value  $\Delta \eta / \eta_2 \approx 0.18$ . It partly explains the above mentioned deviation of the effective shear viscosity from Miesowicz viscosity  $\eta_2$ .

The agreement of the theoretical curve with the experimental results becomes essentially significant for the case of surface orientation normal to the flow plane (Figure [8\)](#page-9-0). This may indicate a deviation of the capillary flow from the Newtonian behavior. The latter may occur due to the well-known hydrodynamic instabilities resulting in the rotation of the bulk director from the initially planar orientation, stabilized by surfaces and normal to the flow plane, towards the flow plane [\[25,](#page-14-21)[26\]](#page-14-22). The threshold pressure gradient is determined by the critical value of the Ericksen number [\[27\]](#page-14-23):

$$
Er = \frac{\Delta P/\Delta x}{\eta_3} \sqrt{\frac{\alpha_2 \alpha_3}{K_{11} K_{22}}} \left(\frac{h}{2}\right)^3 = \overline{s_c} \sqrt{\frac{\alpha_2 \alpha_3}{K_{11} K_{22}}} \left(\frac{h}{2}\right)^3 \tag{14}
$$

where  $\alpha_2$  and  $\alpha_3$  are the Leslie coefficients,  $K_{11}$  and  $K_{22}$  are the splay and twist Frank elastic constants, respectively,  $\eta_3$  is the shear viscosity coefficient (Miesowicz viscosity), corresponding to the initial planar orientation, normal to the flow plane, and  $\overline{s_c}$  is the average value of the velocity gradient.

In accordance with the theoretical estimates [\[25\]](#page-14-21), the critical value of the Ericksen number, corresponding to the threshold of the abovementioned instability is defined as  $Er_c = 17.3$  [\[25\]](#page-14-21).

The experimental value of the instantaneous pressure gradient can be easily expressed as

$$
\frac{\Delta P}{\Delta l} = \frac{\rho g (l_0 - l)}{l} \tag{15}
$$

what allows us to calculate of the Ericksen number at different stages of the flow. The estimates of the Ericksen number show that this parameter varies from about 120, at *l* = 1cm, to 25, at *l* = 3 cm, respectively. Thus, the flow at the initial stage is relatively strong and orients the LC presumably close to the flow direction, and only at the final stage of the flow a significant influence of surfaces can be observed, leading to the reverse transition of the LC layer to a uniform plane orientation. This explains the fact that the effective value of the shear viscosity (Table [3\)](#page-8-1) is intermediate between the Miesowicz viscosities  $\eta_2 = 0.0221$  Pa\*s and  $\eta_3 = 0.0358$  Pa\*s, at corresponding temperature T = 24 °C.

The experimental time dependences obtained by applying the electric field to LC layers with the planar surface orientation parallel (or normal) to the flow direction are in a good agreement with the time dependences obtained using the Equation  $(9)$  $(9)$  (see Figures [8](#page-9-0) and 9). This indicates the Newtonian nature of the capillary flow decay. Nevertheless, the calculated values of the effective shear viscosity  $\eta_{\text{eff}}$  are lower than the maximal value of the Miesowicz viscosity  $\eta_1 = 0.1215$  (0.0991) Pa\*s, for T = 25(27)  $\degree$ C [\[14\]](#page-14-9), respectively. This can be partly explained by the effect of thin surface layers with presumably flat orientation on viscous losses and the orientation effect of the flow. The thickness of the surface layers can be estimated by the calculating the electric coherence length ξ*<sup>E</sup>* in accordance with the well-known expression [\[13\]](#page-14-8)

$$
\xi_E = \sqrt{\frac{K_{ii}}{\varepsilon_0 \Delta \varepsilon}} \frac{1}{E} \tag{16}
$$

where  $K_{ii} = K_{11}$  or  $K_{22}$  for the surface orientation along the flow direction or perpendicular to the flow plane. Using the material parameters of 5CB [\[13\]](#page-14-8)  $K_{11} = 6.2$  pN and  $K_{22} = 6.2$  pN, respectively, one can get for the electric voltages  $U = 150$  V and  $U = 100$  V, applied to the 100  $\mu$ m capillaries, with different surface orientation the values  $ξ<sub>E</sub> = 0.16 \mu m$  and 0.19  $\mu m$ , respectively. This values of  $ξ<sub>E</sub>$  is essentially smaller than the thickness of yhe LC layer. This means that the electric field effectively suppresses the orientation effect of surfaces, with the exception of near-surface layers and can induce a slight decrease δη in the effective shear viscosity η relative to the Miesowicz viscosity  $\eta_1$  due to the slipping effect. The details of estimation of this decreasing was described in Reference [\[28\]](#page-15-0). In particular, it was shown that for the case of surface orientation along the flow direction, the next expression for  $\delta\eta$  is valid:

$$
\frac{\delta \eta}{\eta} = \frac{\eta_1 - \eta_{eff}}{\eta_1} \approx 6 \left( \frac{\xi_E}{h} \right) \left( \frac{\eta_1}{\eta_2} - 1 \right) \tag{17}
$$

The similar expression with a substitution of  $\eta_3$  instead of  $\eta_2$  can be obtained for the second type of surface orientation. In both cases, the above-mentioned decreasing of the effective shear viscosity is of about 5% at voltages, used in experiments. Taking into account this effect, one can conclude that there is a satisfactory correspondence between the effective shear viscosity obtained for the initial surface orientation parallel to the flow and the Miesowicz coefficient  $\eta_1$ .

Outside the surface layers, the maximal anglular deviation  $\theta_m$  of the director from the initially homeotropic orientation stabilized by the strong electric field can be estimated in accordance with the next expression [\[13\]](#page-14-8):

$$
\theta_m = \frac{-\alpha_2}{2\eta_1} \frac{G}{\varepsilon_0 \Delta \varepsilon E^2} h \tag{18}
$$

The estimation of the last parameter corresponding to 5CB [\[14\]](#page-14-9) leads to the value  $\theta_m \approx 0.05$  rad. This was done for the pressure gradient  $G \approx 10^5$  Pa/m, corresponding to the initial stage ( $l = 0.5$  cm) of the capillary rise. According to Equation (14), this results in the negligible influence of the flow. Thus, in this case, the electric field effectively suppresses the orientation effect of both the surface and the flow, which is also confirmed by the digital images presented above (see Figures [2](#page-4-0) and [3\)](#page-4-1). In accordance with the above conclusion, the electrically stabilized LC behaves as a conventionally Newtonian liquid with the constant value of the effective shear viscosity close to the Miesowicz coefficient  $\eta_1$ . The last fact is confirmed by the results of the analysis of the experimental data presented above.

The rigorous description of experimental results, in the absence of the electric field, is a very difficult task, which is beyond the scope of this article. In particular, it is interesting to understand how the abovementioned hydrodynamic instability at the initial planar surface orientation normal to the flow direction can play any role in the decay of 5CB capillary flow. Our estimates made in accordance with Equation (14), using the exact values of 5CB material parameters, lead to the threshold values of the pressure gradient *G*<sub>*c*</sub> =  $(\Delta P/\Delta x)_c$  = 1.27 × 10<sup>4</sup> Pa/m for *h* = 100 μ, which corresponds to the value of the capillary rise  $l_c = 25$  mm and time  $t_c \approx 50$  s, respectively. At the same time, the declination of the experimental dependence from the theoretically obtained data, shown in Figure [8,](#page-9-0) takes place only for the time *t* > 100 s. Nevertheless, one has to take into account the rather slow motion of the director in the absence of the electric field, which may lead to additional time required for the director to return to the initial planar state, normal to the flow direction. The typical time scale for such a motion can be defined by a well-known expression [\[13\]](#page-14-8):

$$
\tau_n = \frac{\gamma_1 h^2}{\pi^2 K_{22}} \tag{19}
$$

which gives for 5CB, the value of  $\tau_n$  equal to be  $\approx 20$  s. It is also known [\[13\]](#page-14-8), that the typical times of reverse relaxation of the strongly deformed orientation structure induced by hydrodynamic instabilities<br>between the above mentioned the above mentioned the above mentioned theoretical data and the above mentioned t to its initial state can be significantly longer than those, defined by Equation (19). It may be responsible<br>. for the difference between the experimental data and the abovementioned theoretical estimate.<br>
Some additional integration of liquid crystals can be extracted from the shear viscosity of liquid crystals c which gives to  $5C$ , the value of  $t_n$  equation  $b \in \infty$  zo s. It is also known  $[15]$ , that the typical three

Some additional information on the shear viscosity of liquid crystals can be extracted from the shear values of the experimental values of the experimental values of the experimental values of the experimental values of th analysis of the experimental dependences  $l(t)$  (see Figure [7\)](#page-6-1) obtained for various local values of the gap in the wedge-like cell. In particular, at the initial stage of the capillary rise, one can neglect by influence<br>Contact line of the hydrostatic pressure difference on the motion of the contact line. The solution (9) in this case is simplified and expressed as:

$$
l^2(t) = \left(\frac{h\sigma\cos\theta}{3\eta}\right)t
$$
\n(20)

<span id="page-12-0"></span>The comparison of the experimental results with this law at different values of the capillary gap The comparison of the experimental results with this law at different values of the capillary gap at the beginning of flow (0-30 s.) is presented in Figure [10.](#page-12-0) The slope of straight lines provides a determination of the effective shear viscosity of LC. The obtained values of this parameter are intermediate between Miesowicz viscosities η<sup>2</sup> and η3, which confirms the possibility of estimating of intermediate between Miesowicz viscosities *η*2 and *η*3, which confirms the possibility of estimating of the shear viscosity using such a simple law. the shear viscosity using such a simple law.



**Figure 10.** Time dependences of the function  $l^2(t)/h$ , for a number of values of the capillary gap h: 79  $\mu$ m (squares,  $\eta = 0.037 \text{ Pa}^*$ s), 70  $\mu$ m (circles,  $\eta = 0.030 \text{ Pa}^*$ s), and 49  $\mu$ m (triangles  $\eta = 0.02 \text{ Pa}^*$ s), respectively.

Recently, this approach has been applied to describe the capillary flow of the shock free ferroelectric smectic  $C^*$  liquid crystal in a flat capillary with the gap in 23  $\mu$ m and with homeotropic boundary conditions [\[29\]](#page-15-1). The corresponding geometry of the experiment is shown in Figure [11a](#page-13-0). The horizontal position of the capillary made it possible to neglect the hydrostatic pressure difference. The result of the comparing the experimental data with data obtained by the theoretical law of Equation (20), is shown in Figure [11b](#page-13-0). It is interesting, that contrary to nematics, the capillary flow resulted in an arising of long living defects, which occupied only a small part of the LC cell. The good agreement between the experiment and the theory in this case confirms the perspectives of using the capillary flow to study the viscous properties of both the nematic and smectic C\* phases.

<span id="page-13-0"></span>

Figure 11. (a) Experimental setup scheme. (b) Time dependence of the function  $l^2(t)$ , for the value of  $\eta_{\text{eff}} = 0.47 \text{ Pa}^* \text{s}.$ 

# **4. Conclusions 4. Conclusions**

The first experimental studies of liquid crystal (LC) rise in flat capillaries with internal The first experimental studies of liquid crystal (LC) rise in flat capillaries with internal photosensitive surfaces have been performed. The capillaries with different surface orientations were prepared using the photoalignment technique. The influence of the surface and bulk orientation of LC molecules on the dynamics of a decay capillary flow in the absence and presence of an electric field has been experimentally studied, analyzed and compared with the previously obtained The results, obtained for the nematic liquid crystal (5CB) are compared with the results of an results. The results, obtained for the nematic liquid crystal (5CB) are compared with the results of an investigation of a capillary flow in shock-free ferroelectric smectic  $\mathrm{C}^*$  phase. In summary, we note the following results: following results:

1. The dynamics of capillary rise of nematic liquid crystal 5CB in the vertical plane of capillaries 1. The dynamics of capillary rise of nematic liquid crystal 5CB in the vertical plane of capillaries with photo-oriented substrates is studied for the first time. It was found, that the stationary value of with photo-oriented substrates is studied for the first time. It was found, that the stationary value of the contact angle weakly depends on the direction of the planar surface orientation relatively to the the contact angle weakly depends on the direction of the planar surface orientation relatively to the direction of the capillary rise. Application of the strong electric fields resulted in a decreasing of the direction of the capillary rise. Application of the strong electric fields resulted in a decreasing of the contact angle. contact angle.

2. The theoretical model of the capillary flow, describing the capillary rise of isotropic Newtonian 2. The theoretical model of the capillary flow, describing the capillary rise of isotropic Newtonian liquids, was applied to study the dynamics of the capillary flow of anisotropic liquids. liquids, was applied to study the dynamics of the capillary flow of anisotropic liquids.

3. It was established, that application of the strong electric field provided the best agreement the experimental data with the theoretically obtained results describing the capillary flow of the of the experimental data with the theoretically obtained results describing the capillary flow of the conventionally Newtonian anisotropic liquid with the constant value of the shear viscosity. conventionally Newtonian anisotropic liquid with the constant value of the shear viscosity.

4. In the case when the initial surface orientation was normal compared to the flow plane, experimental results indicated on the flow induced an orientational transition between homogeneous the experimental results indicated on the flow induced an orientational transition between homogeneous planar structure and distorted twisted configuration described by the Guion—Pieranski mechanism. planar structure and distorted twisted configuration described by the Guion—Pieranski mechanism.

5. The comparison of the results of the investigation of capillary flows in nematics and shock-5. The comparison of the results of the investigation of capillary flows in nematics and shock-free ferroelectric smectic  $C^*$  phases revealed the perspectives of usage of such flows for determination of anisotropic shear viscosities of liquid crystals.

**Author Contributions:** D.V.S., S.S.K., V.A.B. and T.P.T. performed the experiments, analyzed the optical patterns<br>and orientational etmography of L.C. D.V.S., S.V.P. E.P.P. and A.V.Z. supervised the study. All authors una reviewed the manuscript. All authors have read and agreed to the published version of the manuscript. and orientational structures of LC. D.V.S., S.V.P., E.P.P. and A.V.Z. supervised the study. All authors wrote and

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