crystals

MDPI

Article

The Griffith Crack and the Interaction between Screw Dislocation and Semi-Infinite Crack in Cubic Quasicrystal Piezoelectric Materials

Jiandong Pi [1,2] and Lianhe Li [1,2,*]

[1] College of Mathematics Science, Inner Mongolia Normal University, Huhhot 010022, China
[2] Inner Mongolia Center for Applied Mathematics, Inner Mongolia Normal University, Huhhot 010022, China
* Correspondence: nmglilianhe@163.com

Abstract: The Griffith crack problem and the interaction between screw dislocation and semi-infinite crack in cubic quasicrystal piezoelectric materials are studied by using the complex variable function method. The stress intensity factors and electric displacement intensity factors are obtained. The effects of the linear force and coupling elastic coefficient on the stress intensity factor of phonon field and phason fields are discussed in detail. By numerical examples, it is found that the linear force and the coupling elastic constant have a significant effect on the stress intensity factor.

Keywords: cubic quasicrystal piezoelectric materials; crack; screw dislocation; complex variable function method

check for updates

Citation: Pi, J.; Li, L. The Griffith Crack and the Interaction between Screw Dislocation and Semi-Infinite Crack in Cubic Quasicrystal Piezoelectric Materials. *Crystals* **2022**, *12*, 1250. https://doi.org/10.3390/cryst12091250

Academic Editors: Ulrich Prahl, Sergey Guk and Faisal Qayyum

Received: 14 August 2022
Accepted: 30 August 2022
Published: 2 September 2022

1. Introduction

Since solid quasicrystals were reported in 1984, their physical properties have attracted extensive attention. In terms of the piezoelectric effect of quasicrystals, researchers have also done a lot of research. Hu et al. [1] derived the piezoelectric constants of two-dimensional (2D) and three-dimensional (3D) quasicrystals using thermodynamics and group representation theory. Altay and Dökmeci [2] studied the basic equation of quasicrystal piezoelectricity and gave the basic equation of the quasi-static electric effect of 3D quasicrystal elastic materials through differential and variational forms. Li et al. [3] studied the basic solution of the one-dimensional (1D) hexagonal quasicrystal piezoelectric problem, and expressed the basic solution by five quasi-harmonic functions using operator theory and generalized Almansi theorem. Yu et al. [4] derived the governing equations of each crystal system of 1D quasicrystal piezoelectric materials, and gave the general solutions of the governing equations by using the operator method and complex method.

Since the establishment of the elastic theory of quasicrystal piezoelectric materials, although many achievements have been made, most of the research has been focused on 1D and 2D quasicrystal piezoelectric materials [5–13]; there have been few studies on the fracture of 3D quasicrystal piezoelectric materials. Because the elasticity of quasicrystal piezoelectric materials is similar to that of quasicrystal materials, the defect problem of 3D cubic quasicrystal piezoelectric materials can be studied by means of the defect problem of 3D cubic quasicrystal materials. However, due to the piezoelectric effect, the defects of quasicrystal piezoelectric materials are more complicated. For cubic quasicrystal materials, some research results have focused on the defects problem. Zhou et al. [14] studied the anti-plane problem and mode III crack problem of cubic quasicrystals, resulting in new developments in the elastic theory of cubic quasicrystals. Zhang [15] studied the anti-plane conjugate crack problem; general solutions for the stress and strain of conjugate cracks in cubic quasicrystal were obtained. Gao et al. [16] studied the elliptical hole or crack problem, and obtained analytical expressions for both entire and asymptotic fields. Suo et al. [17] studied the effect of T stress on the cross-type cracks in cubic quasicrystals.

These studies focused on cubic quasicrystal materials, and the methods involved can be used to solve the defect problems of cubic quasicrystal piezoelectric materials. For cubic quasicrystal piezoelectric materials, Zhang et al. [18] studied the free vibration of plates, producing one of the few research studies focusing on the defect problem of cubic quasicrystal piezoelectric materials. In this paper, the Griffith crack problem and the interaction between screw dislocation and semi-infinite crack in 3D cubic quasicrystal piezoelectric materials are studied. The main factors affecting the stress intensity factor will be discussed.

2. Basic Equations of Cubic Quasicrystal Piezoelectric Material

In the spatial rectangular coordinate system $x_i(i = 1, 2, 3)$, the basic equations of the anti-plane elasticity problem for cubic quasicrystal piezoelectric materials can be expressed as follows:

The constitutive equation [19]

$$\begin{bmatrix} \sigma_{yz} \\ \sigma_{zx} \\ H_{yz} \\ H_{zx} \\ D_x \\ D_y \end{bmatrix} = \begin{bmatrix} C_{44} & 0 & R_3 & 0 & -e_{14} & 0 \\ 0 & C_{44} & 0 & R_3 & 0 & -e_{14} \\ R_3 & 0 & K_{44} & 0 & -d_{123} & 0 \\ 0 & R_3 & 0 & K_{44} & 0 & -d_{123} \\ e_{14} & 0 & d_{123} & 0 & \lambda_{11} & 0 \\ 0 & e_{14} & 0 & d_{123} & 0 & \lambda_{11} \end{bmatrix} \begin{bmatrix} 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \\ 2\omega_{yz} \\ 2\omega_{zx} \\ E_x \\ E_y \end{bmatrix}, \tag{1}$$

the equilibrium equation (regardless of body force) [20]

$$\sigma_{iz,i} = 0, H_{iz,i} = 0, D_{i,i} = 0 \ (i = x, y, z), \tag{2}$$

and the geometric equation

$$\begin{array}{c} \varepsilon_{yz} = \frac{1}{2}\frac{\partial u_z}{\partial y}, \varepsilon_{zx} = \frac{1}{2}\frac{\partial u_z}{\partial x}, \omega_{yz} = \frac{1}{2}\frac{\partial w_z}{\partial y}, \\ \omega_{zx} = \frac{1}{2}\frac{\partial w_z}{\partial x}, E_x = -\frac{\partial \Phi}{\partial x}, E_y = -\frac{\partial \Phi}{\partial y}, \end{array} \tag{3}$$

where u_z represents phonon field displacement, w_z represents phason field displacement, and Φ represents electric potential. The σ_{ij}, ε_{ij} represent the stress and strain of phonon field, respectively; H_{ij}, ω_{ij} represent the stress and strain of phason field, respectively; D_{ij} represents electric displacement; E_i represents electric field strength; C_{44} is the elastic coefficient of the phonon field; K_{44} is the elastic coefficient of the phason field; R_3 is the coupling coefficient between the phonon field and phason field; e_{14} and d_{123} are the piezoelectric constant; and λ_{11} is the dielectric constant.

By Equations (1)–(3), the governing equations expressed by displacement and electric potential can be obtained as follows:

$$\begin{array}{c} C_{44}\nabla^2 u_z + R_3\nabla^2 w_z + 2e_{14}\frac{\partial^2 \Phi}{\partial x \partial y} = 0, \\ R_3\nabla^2 u_z + K_{44}\nabla^2 w_z + 2d_{123}\frac{\partial^2 \Phi}{\partial x \partial y} = 0, \\ 2e_{14}\frac{\partial^2 u_z}{\partial x \partial y} + 2d_{123}\frac{\partial^2 w_z}{\partial x \partial y} - \lambda_{11}\nabla^2 \Phi = 0, \end{array} \tag{4}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator.

Thus, the anti-plane elasticity problem of cubic quasicrystal piezoelectric materials is reduced to solving the system of partial differential Equation (4) under appropriate boundary conditions.

3. Basic Solution for the Fracture Problem of Cubic Quasicrystal Piezoelectric Materials

Equation (4) can be rewritten [4] as follows:

$$A\boldsymbol{U} = 0 \tag{5}$$

where $\boldsymbol{U} = (u_z, w_z, \Phi)^T$ and A is a matrix of differential operators

$$A = \begin{bmatrix} C_{44}\nabla^2 & R_3\nabla^2 & 2e_{14}\frac{\partial^2}{\partial x\partial y} \\ R_3\nabla^2 & K_{44}\nabla^2 & 2d_{123}\frac{\partial^2}{\partial x\partial y} \\ 2e_{14}\frac{\partial^2}{\partial x\partial y} & 2d_{123}\frac{\partial^2}{\partial x\partial y} & -\lambda_{11}\nabla^2 \end{bmatrix}. \tag{6}$$

From Equation (6), the determinant of A can be given as follows:

$$|A| = a\frac{\partial^6}{\partial y^6} + b\frac{\partial^2}{\partial x^2}\frac{\partial^4}{\partial y^4} + b\frac{\partial^4}{\partial x^4}\frac{\partial^2}{\partial y^2} + a\frac{\partial^6}{\partial x^6}, \tag{7}$$

in which

$$a = \lambda_{11}\left(R_3^2 - C_{44}K_{44}\right), b = 4\left(2e_{14}d_{123}R_3 - d_{123}^2C_{44} - e_{14}^2K_{44}\right) + 3\lambda_{11}\left(R_3^2 - C_{44}K_{44}\right).$$

Now, we introduce a function F that satisfies the equation

$$\nabla_1^2\nabla_2^2\nabla_3^2F = 0 \tag{8}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{t_i^2}\frac{\partial^2}{\partial y^2}(i = 1, 2, 3)$ and t_i^2 are the three characteristic roots of equation $at^6 - bt^4 + bt^2 - a = 0$.

According to the operator theory, the general solution of Equation (5) can be expressed as follows:

$$u_z = A_{i1}F, w_z = A_{i2}F, \Phi = A_{i3}F, (i = 1, 2, 3). \tag{9}$$

Let $i = 2$, then the cofactor of the matrix A can be expressed as follows:

$$\begin{aligned} A_{21} &= \lambda_{11}R_3\frac{\partial^4}{\partial x^4} + 2(\lambda_{11}R_3 + 2e_{14}d_{123})\frac{\partial^4}{\partial x^2\partial y^2} + \lambda_{11}R_3\frac{\partial^4}{\partial y^4}, \\ A_{22} &= -\lambda_{11}C_{44}\frac{\partial^4}{\partial x^4} - 2\left(\lambda_{11}C_{44} + 2e_{14}^2\right)\frac{\partial^4}{\partial x^2\partial y^2} - \lambda_{11}C_{44}\frac{\partial^4}{\partial y^4}, \\ A_{23} &= 2(e_{14}R_3 - d_{123}C_{44})\left(\frac{\partial^4}{\partial x^3\partial y} + \frac{\partial^4}{\partial x\partial y^3}\right). \end{aligned} \tag{10}$$

Substituting Equation (10) into Equation (9), one has the following:

$$u_z = \lambda_{11}R_3\frac{\partial^4 F}{\partial x^4} + 2(\lambda_{11}R_3 + 2e_{14}d_{123})\frac{\partial^4 F}{\partial x^2\partial y^2} + \lambda_{11}R_3\frac{\partial^4 F}{\partial y^4}, \tag{11}$$

$$w_z = -\lambda_{11}C_{44}\frac{\partial^4 F}{\partial x^4} - 2\left(\lambda_{11}C_{44} + 2e_{14}^2\right)\frac{\partial^4 F}{\partial x^2\partial y^2} - \lambda_{11}C_{44}\frac{\partial^4 F}{\partial y^4}, \tag{12}$$

$$\Phi = 2(e_{14}R_3 - d_{123}C_{44})\left(\frac{\partial^4 F}{\partial x^3\partial y} + \frac{\partial^4 F}{\partial x\partial y^3}\right). \tag{13}$$

Assuming that the form of the complex function $F(x, y)$ is $F(x, \mu y)$, then μ must satisfy the following characteristic equation [7]:

$$a\mu^6 + b\mu^4 + b\mu^2 + a = 0 \tag{14}$$

Equation (14) has six pure imaginary roots

$$\mu_1 = is_1 = i, \mu_2 = is_2 = i\frac{\sqrt{b+a}+\sqrt{b-3a}}{2\sqrt{a}}, \mu_3 = is_3 = i\frac{\sqrt{b+a}-\sqrt{b-3a}}{2\sqrt{a}},$$

$$\mu_4 = -\mu_1, \mu_5 = -\mu_2, \mu_6 = -\mu_3, \left(i = \sqrt{-1}, s_1 = 1, \mu_2\mu_3 = -1(or\ s_2s_3 = 1)\right)$$

According to the properties of the analytic function and Equation (8), the complex representation of the displacement function F can be given as follows:

$$F = 2Re\sum_{k=1}^{3} F_k(z_k), \tag{15}$$

where Re represents the real part and $F_k(z_k)$ represents three arbitrary analytic functions with the argument $z_k = x + \mu_k y$ $(k = 1, 2, 3)$.

By Equations (1), (3), (11)–(13), and (15), the complex representations of the stresses and electric potential can be obtained as follows:

$$\sigma_{yz} = -4e_{14}(d_{123}C_{44} - e_{14}R_3)Re\sum_{k=1}^{3}\left(\mu_k - \mu_k^3\right)F_k^{(5)}(z_k), \tag{16}$$

$$\sigma_{zx} = 4e_{14}(d_{123}C_{44} - e_{14}R_3)Re\sum_{k=1}^{3}\left(\mu_k^2 - \mu_k^4\right)F_k^{(5)}(z_k), \tag{17}$$

$$H_{yz} = Re\sum_{k=1}^{3}\left(l\mu_k + m\mu_k^3 + n\mu_k^5\right)F_k^{(5)}(z_k), \tag{18}$$

$$H_{zx} = Re\sum_{k=1}^{3}\left(n + m\mu_k^2 + l\mu_k^4\right)F_k^{(5)}(z_k), \tag{19}$$

$$D_x = 2\lambda_{11}(d_{123}C_{44} - e_{14}R_3)Re\sum_{k=1}^{3}\left(\mu_k - \mu_k^5\right)F_k^{(5)}(z_k), \tag{20}$$

$$D_y = -2\lambda_{11}(d_{123}C_{44} - e_{14}R_3)Re\sum_{k=1}^{3}\left(1 - \mu_k^4\right)F_k^{(5)}(z_k), \tag{21}$$

in which

$$\begin{aligned} l &= 2\lambda_{11}\left(R_3^2 - C_{44}K_{44}\right) - 4d_{123}^2C_{44} + 4e_{14}d_{123}R_3, \\ m &= 4\lambda_{11}\left(R_3^2 - C_{44}K_{44}\right) - 4d_{123}^2C_{44} - 8e_{14}^2K_{44} + 12e_{14}d_{123}R_3, \\ n &= 2\lambda_{11}\left(R_3^2 - C_{44}K_{44}\right). \end{aligned}$$

4. Griffith Crack Problem

Now, we study the Griffith crack problem with a crack length of $2a$, which is subjected to a pair of linear forces and charges at the distancefrom the origin of the coordinates. The linear force intensity of the phonon field and the phason field per unit length are Q_1 and Q_2, respectively, and the linear charge density is q, as shown in Figure 1.

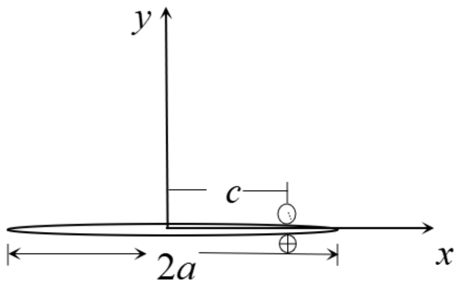

Figure 1. Griffith crack in cubic quasicrystal piezoelectric materials.

The complex function ${F_k}^{(5)}(z_k)$ is assumed as follows:

$$ {F_k}^{(5)}(z_k) = \frac{A_k}{(z_k - c)\sqrt{z_k^2 - a^2}}, \tag{22} $$

the constants $A_k = \alpha_k + \beta_k i$ $(k = 1, 2, 3)$ can be determined by the boundary conditions.

When $y = 0$, $|x| > a$, it is required to $u_z = 0, w_z = 0, \Phi = 0$, then

$$ 2\lambda_{11} R_3 Re \sum_{k=1}^{3} A_k + 4(\lambda_{11} R_3 + 2e_{14} d_{123}) Re \sum_{k=1}^{3} \mu_k^2 A_k + 2\lambda_{11} R_3 Re \sum_{k=1}^{3} \mu_k^4 A_k = 0, \tag{23} $$

$$ -2\lambda_{11} C_{44} Re \sum_{k=1}^{3} A_k - 4\left(\lambda_{11} C_{44} + 2e_{14}^2\right) Re \sum_{k=1}^{3} \mu_k^2 A_k - 2\lambda_{11} C_{44} Re \sum_{k=1}^{3} \mu_k^4 A_k = 0, \tag{24} $$

$$ -4(d_{123} C_{44} - e_{14} R_3)\left(Re \sum_{k=1}^{3} \mu_k A_k + Re \sum_{k=1}^{3} \mu_k^3 A_k \right) = 0. \tag{25} $$

and then we can deduce the following:

$$ \begin{array}{l} \sum\limits_{k=1}^{3} \left(2\lambda_{11} R_3 - 4(\lambda_{11} R_3 + 2e_{14} d_{123}) s_k^2 + 2\lambda_{11} R_3 s_k^4\right)\alpha_k = 0, \\ \sum\limits_{k=1}^{3} \left(-2\lambda_{11} C_{44} + 4\left(\lambda_{11} C_{44} + 2e_{14}^2\right) s_k^2 - 2\lambda_{11} C_{44} s_k^4\right)\alpha_k = 0, \\ \sum\limits_{k=1}^{3} \left(s_k^3 - s_k\right)\beta_k = 0. \end{array} \tag{26} $$

The integration along a small semi-circle around the point of action of a linear force and charge on $x = c$, one has

$$ \begin{array}{c} \sum_{k=1}^{3} \left(s_k + s_k^3\right)\beta_k = \frac{Q_1 \sqrt{a^2 - c^2}}{4\pi e_{14}(C_{44} d_{123} - e_{14} R_3)}, \\ \pi\lambda_{11}\left(R_3^2 - C_{44} K_{44}\right) \sum\limits_{k=1}^{3} \left(-\frac{1}{s_k} + s_k + s_k^3 - s_k^5\right)\beta_k \\ +4\pi e_{14}(d_{123} R_3 - e_{14} K_{44}) \sum\limits_{k=1}^{3} \left(s_k + s_k^3\right)\beta_k = Q_2 \sqrt{a^2 - c^2}, \\ \sum\limits_{k=1}^{3} \left(s_k^4 - 1\right)\alpha_k = \frac{q\sqrt{a^2 - c^2}}{2\pi\lambda_{11}(C_{44} d_{123} - e_{14} R_3)}. \end{array} \tag{27} $$

Combining Equations (26) and (27), the expressions of the complex constant A_k can be obtained as follows:

$$ \begin{array}{c} A_1 = i\frac{Q_1 \sqrt{a^2 - c^2}\left(4e_{14}(e_{14} K_{44} - d_{123} R_3) s_3^2 - \left(C_{44} K_{44} - R_3^2\right)\left(-1 + s_3^2\right)^2 \lambda_{11}\right)}{8\pi e_{14}(-C_{44} d_{123} + e_{14} R_3)\left(C_{44} K_{44} - R_3^2\right)\left(-1 + s_3^2\right)^2 \lambda_{11}} \\ -i\frac{Q_2 \sqrt{a^2 - c^2} s_3^2}{2\pi\left(C_{44} K_{44} - R_3^2\right)\left(-1 + s_3^2\right)^2 \lambda_{11}}, \end{array} $$

$$\begin{aligned} A_2 = & \frac{q\sqrt{a^2-c^2}s_3^4}{4\pi(C_{44}d_{123}-e_{14}R_3)\left(-1+s_3^2\right)\left(1+s_3^2\right)\lambda_{11}} \\ & +i\frac{\sqrt{a^2-c^2}(Q_1(e_{14}K_{44}-d_{123}R_3)+Q_2(C_{44}d_{123}-e_{14}R_3))s_3^5}{2\pi(C_{44}d_{123}-e_{14}R_3)\left(C_{44}K_{44}-R_3^2\right)\left(-1+s_3^2\right)^2\left(1+s_3^2\right)\lambda_{11}}, \\ A_3 = & \frac{q\sqrt{a^2-c^2}}{4\pi(C_{44}d_{123}-e_{14}R_3)\left(-1+s_3^4\right)\lambda_{11}} \\ & +i\frac{\sqrt{a^2-c^2}(Q_1(e_{14}K_{44}-d_{123}R_3)+Q_2(C_{44}d_{123}-e_{14}R_3))s_3}{2\pi(C_{44}d_{123}-e_{14}R_3)\left(C_{44}K_{44}-R_3^2\right)\left(-1+s_3^2\right)^2\left(1+s_3^2\right)\lambda_{11}}. \end{aligned} \tag{28}$$

Thus, the complete elastic field can be obtained. After derivation, the corresponding stress and electrical displacement intensity factors can be expressed as follows:

$$K_\sigma = \frac{Q_1\sqrt{a+c}}{\sqrt{a-c}\sqrt{\pi a}}, \tag{29}$$

$$K_H = \frac{Q_2\sqrt{a+c}}{\sqrt{a-c}\sqrt{\pi a}}, \tag{30}$$

$$K_{D_y} = \frac{q\sqrt{a+c}}{\sqrt{a-c}\sqrt{\pi a}}. \tag{31}$$

Now take $a = 5 \times 10^{-3}$ m to analyze the influence of linear force Q_1 on the phonon field stress intensity factor K_σ (both the phason field stress intensity factor and the electric displacement intensity factor have similar variation rules with the phonon field stress intensity factor).

It can be seen from Figure 2 that the stress intensity factor of the phonon field increases with the increase of c, indicating that the farther the action point is from the origin, the greater the stress intensity factor. Furthermore, the stress intensity factor increases with the increase of linear force Q_1, and the farther the action point is from the origin, the greater the influence of the linear force.

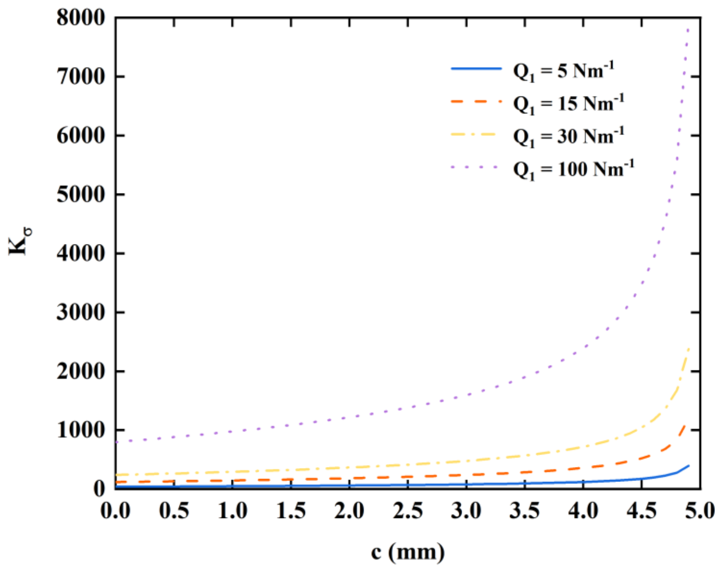

Figure 2. Influence of linear force Q_1 on the phonon field stress intensity factor K_σ.

5. Interaction between Screw Dislocation and Semi-Infinite Crack

In this section, the interaction between a semi-infinite crack and a screw dislocation with the Burgers vector $B = (b_1, b_2)$ is studied. Suppose that the dislocationi s located at the distance c from the semi-infinite crack tip, as shown in Figure 3.

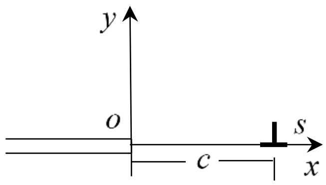

Figure 3. A screw dislocation near a semi-infinite crack.

Suppose the analytical formula F_k has the following form:

$$F_k{}^{(5)}(z_k) = \frac{B_k}{(z_k - c)\sqrt{z_k}}, (k = 1, 2, 3). \tag{32}$$

the constants $B_k = \gamma_k + \delta_k i$ $(k = 1, 2, 3)$.

When $x < 0$, make $\sigma_{yz} = 0, H_{yz} = 0, D_y = 0$, we have the following:

$$\begin{gathered}\sum_{k=1}^{3}\left(s_k + s_k^3\right)\gamma_k = 0,\\ \sum_{k=1}^{3}\left(ls_k - ms_k^3 + ns_k^5\right)\gamma_k = 0,\\ \sum_{k=1}^{3}\left(1 - s_k^4\right)\delta_k = 0.\end{gathered} \tag{33}$$

when $x = c$, we have the following:

$$\begin{gathered}2\pi\sum_{k=1}^{3}\left[-\lambda_{11}R_3 + 2(\lambda_{11}R_3 + 2e_{14}d_{123})s_k^2 - \lambda_{11}R_3 s_k^4\right]\delta_k = b_1\sqrt{c},\\ 2\pi\sum_{k=1}^{3}\left[\lambda_{11}C_{44} - 2\left(\lambda_{11}C_{44} + 2e_{14}^2\right)s_k^2 + \lambda_{11}C_{44}s_k^4\right]\delta_k = b_2\sqrt{c},\\ \sum_{k=1}^{3}\left(s_k^3 - s_k\right)\gamma_k = 0.\end{gathered} \tag{34}$$

The following can be obtained from Equations (33) and (34):

$$\gamma_1 = \gamma_2 = \gamma_3 = 0, \tag{35}$$

$$\delta_1 = \frac{\sqrt{c}\left(b_1\left(4e_{14}^2 s_3^2 - C_{44}\left(-1 + s_3^2\right)^2\lambda_{11}\right) + b_2\left(4d_{123}e_{14}s_3^2 - R_3\left(-1 + s_3^2\right)^2\lambda_{11}\right)\right)}{8\pi e_{14}(-C_{44}d_{123} + e_{14}R_3)\left(-1 + s_3^2\right)^2\lambda_{11}}, \tag{36}$$

$$\delta_2 = \frac{\sqrt{c}(b_2 d_{123} + b_1 e_{14})s_3^4}{4\pi(C_{44}d_{123} - e_{14}R_3)\left(-1 + s_3^2\right)^2\lambda_{11}}, \tag{37}$$

$$\delta_3 = \frac{\sqrt{c}(b_2 d_{123} + b_1 e_{14})}{4\pi(C_{44}d_{123} - e_{14}R_3)\left(-1 + s_3^2\right)^2\lambda_{11}}. \tag{38}$$

Substituting Equations (35)–(38) into Equations (16) and (18), the analytical expressions of the stresses were obtained, as follows:

$$\begin{gathered}\sigma_{yz} = 4e_{14}(d_{123}C_{44} - e_{14}R_3)\sum_{k=1}^{3}\frac{\left(s_k + s_k^3\right)\delta_k}{(x-c)\sqrt{x}},\\ H_{yz} = \sum_{k=1}^{3}\frac{\left(-ls_k + ms_k^3 - ns_k^5\right)\delta_k}{(x-c)\sqrt{x}}.\end{gathered} \tag{39}$$

Substituting Equation (39) into the expressions of the stress intensity factors for phonon field and phason field, respectively, and introducing

$$\begin{aligned} K_\sigma &= \lim_{x\to 0}\sqrt{2\pi x}\sigma_{yz}, \\ K_H &= \lim_{x\to 0}\sqrt{2\pi x}H_{yz}. \end{aligned}$$

then one has the following:

$$K_\sigma = -\frac{\sqrt{2}\left(b_1\left(2e_{14}^2 s_3 + C_{44}(1+s_3)^2\lambda_{11}\right) + b_2\left(2d_{123}e_{14}s_3 + R_3(1+s_3)^2\lambda_{11}\right)\right)}{\sqrt{\pi c}(1+s_3)^2\lambda_{11}}, \tag{40}$$

$$\begin{aligned} K_H = b_1\Bigg(& \frac{2\left(2e_{14}^2(e_{14}K_{44}-d_{123}R_3)s_3 - C_{44}d_{123}R_3(1+s_3)^2\lambda_{11}\right)}{\sqrt{2\pi c}(C_{44}d_{123}-e_{14}R_3)(1+s_3)^2\lambda_{11}} \\ & - \frac{e_{14}\left(C_{44}K_{44}(-1+s_3)^2 - R_3^2\left(1+s_3^2\right)\right)}{\sqrt{2\pi c}(C_{44}d_{123}-e_{14}R_3)s_3}\Bigg) \\ & + b_2\Bigg(\frac{2e_{14}R_3\left(-2d_{123}^2 s_3 + K_{44}(1+s_3)^2\lambda_{11}\right)}{\sqrt{2\pi c}(C_{44}d_{123}-e_{14}R_3)(1+s_3)^2\lambda_{11}} \\ & + \frac{\left(d_{123}\left(4e_{14}^2K_{44}s_3^2 + (1+s_3)^2\left(R_3^2(-1+s_3)^2 - C_{44}K_{44}\left(1+s_3^2\right)\right)\lambda_{11}\right)\right)}{\sqrt{2\pi c}(C_{44}d_{123}-e_{14}R_3)s_3(1+s_3)^2\lambda_{11}}\Bigg). \end{aligned} \tag{41}$$

The influence of coupled elastic coefficient on stress intensity factor will be discussed in detail by numerical examples. To this end, the material properties have been chosen by referring to the previous work [3], such as follows:

$$\begin{aligned} C_{44} = 50\times 10^9\ \text{Nm}^{-2},\ R_3 = 0.5\times 10^9\ \text{Nm}^{-2},\ K_{44} = 0.3\times 10^9\ \text{Nm}^{-2}, \\ e_{14} = -0.138\ \text{Cm}^{-2}, d_{123} = -0.16\ \text{Cm}^{-2}, \lambda_{11} = 82.6\times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}. \end{aligned}$$

Other parameters are $x = 1\times 10^{-3}$ m, $b_1 = 1.6\times 10^{-9}$ m, $b_2 = 10.7\times 10^{-9}$ m.

It can be seen from Figures 4 and 5 that the coupling elastic coefficient R_3 has a significant impact on both the phonon and phason filed stress intensity factors. When the distance between the dislocation and the crack tip is determined, it can be found that the stress intensity factor decreases with the increase of R_3.

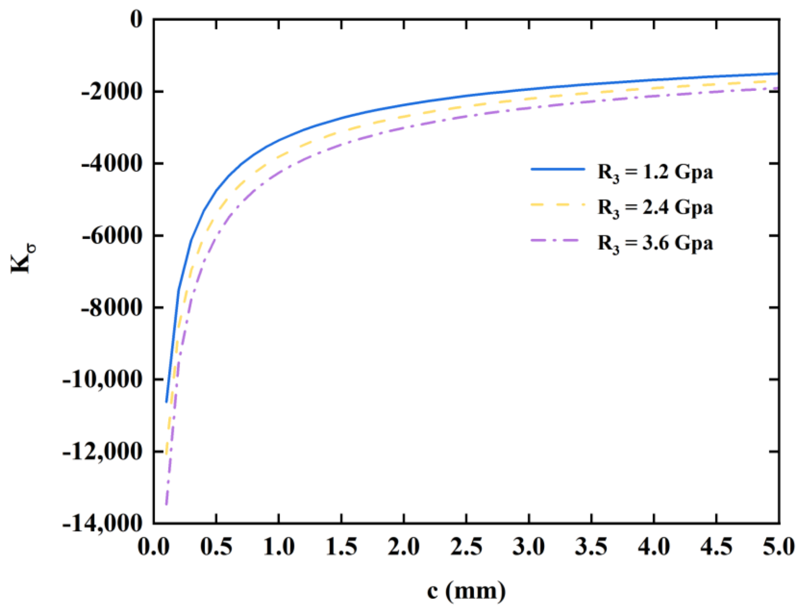

Figure 4. Influence of coupling elastic coefficient R_3 on the phonon filed stress intensity factor.

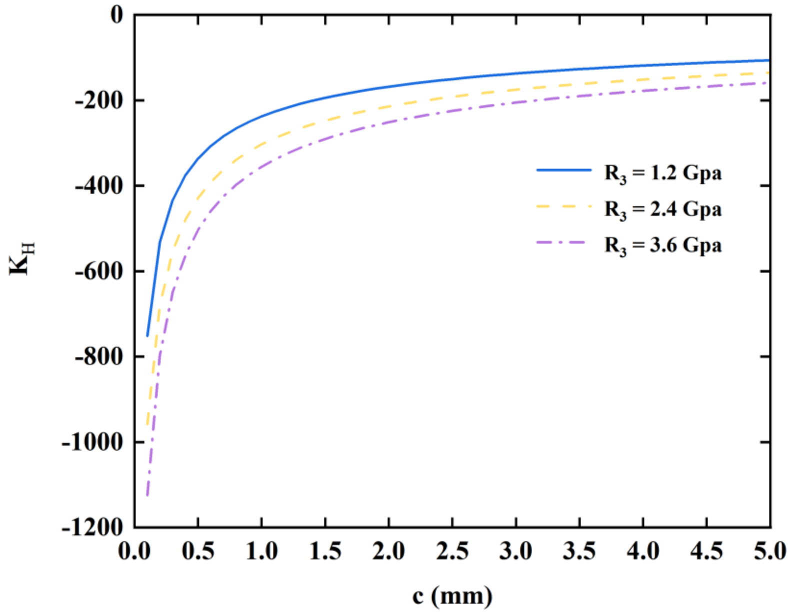

Figure 5. Influence of coupling elastic coefficient R_3 on the phason field stress intensity factor.

6. Conclusions

The Griffith crack problem and the interaction between screw dislocation and semi-infinite crack in cubic quasicrystal piezoelectric materials were studied. With the help of complex function theory, the analytical expression of stress intensity factor was obtained. Numerical examples showed that for the Griffith crack problem, the magnitude of the linear force and the distance from the crack tip have obvious effects on stress intensity factor, while for the interaction between screw dislocation and semi-infinite crack problem, the coupling elastic constant of the phonon and phason fields has a significant effect on the stress intensity factor.

Author Contributions: Project administration, Writing—review & editing, J.P.; Conceptualization, Methodology, L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This study was supported by the National Natural Science Foundation of China (Nos. 11962026, 12002175, 12162027 and 62161045), the Natural Science Foundation of Inner Mongolia (Nos. 2020MS01018, 2021MS01013) and the Fundamental Research Funds for Inner Mongolia Normal University (Nos. 2022JBYJ020, 2022JBZD010).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare that they have no conflict of interest with this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

References

1. Hu, C.Z.; Wang, R.H.; Ding, D.H.; Yang, W.G. Piezoelectric effects in quasicrystals. *Phys. Rev. B* **1997**, *56*, 2463–2468. [CrossRef]
2. Altay, G.; Dökmeci, M.C. On the fundamental equations of piezoelasticity of quasicrystal media. *Int. J. Solids Struct.* **2012**, *49*, 3255–3262. [CrossRef]
3. Li, X.Y.; Li, P.D.; Wu, T.H.; Shi, M.X.; Zhu, Z.W. Three-dimensional fundamental solutions for one-dimensional hexagonal quasicrystal with piezoelectric effect. *Phys. Lett. A* **2014**, *378*, 826–834. [CrossRef]
4. Yu, J.; Guo, J.H.; Pan, E.N.; Xing, Y.M. General solutions of plane problem in one-dimensional quasicrystal piezoelectric materials and its application on fracture mechanics. *Appl. Math. Mech. (Engl. Ed.)* **2015**, *36*, 793–814. [CrossRef]
5. Zhou, Y.B.; Liu, G.T. Electroplastic analysis of anti-plane type III crack in one-dimensional hexagonal quasicrystal piezoelectric materials. *Chin. J. Solid Mech.* **2015**, *36*, 63–68.

6. Guo, J.h.; Zhang, Z.Y.; Xing, Y.M. Antiplane analysis for an elliptical inclusion in 1D hexagonal piezoelectric quasicrystal composites. *Philos. Mag.* **2016**, *96*, 349–369. [CrossRef]
7. Cui, X.W.; Li, L.H. Anti-plane problem of finite large one-dimensional hexagonal quasicrystal piezoelectric wedge with screw dislocation. *Chin. J. Appl. Mech.* **2019**, *36*, 1058–1062.
8. Zhou, Y.B.; Liu, G.T.; Li, L.H. Effect of T-stress on the fracture in an infinite one-dimensional hexagonal piezoelectric quasicrystal with a Griffith crack. *Eur. J. Mech.—A/Solids* **2021**, *86*, 104184. [CrossRef]
9. Loboda, V.; Sheveleva, A.; Komarov, O.; Lapusta, Y. An interface crack with mixed electrical conditions at it faces in 1D quasicrystal with piezoelectric effect. *Mech. Adv. Mater. Struct.* **2022**, *29*, 3334–3344. [CrossRef]
10. Jiang, L.J.; Liu, G.T. The interaction between a screw dislocation and a wedge-shaped crack in one-dimensional hexagonal piezoelectric quasicrystals. *Chin. Phys. B* **2017**, *26*, 249–255. [CrossRef]
11. Zhang, F.; Li, X. The Analytic Solutions of a Circular Hole with Four Cracks of One-Dimensional Hexagonal Piezoelectric Quasicrystals. *J. Jiangxi Norm. Univ. (Natural Sci. Ed.)* **2015**, *39*, 50–54.
12. Bai, Q.M.; Ding, S.H. Anti-plane problem of hexagonal hole edge crack in one-dimensional hexagonal quasicrystal piezoelectric. *Appl. Math. Mech.* **2019**, *40*, 1071–1080.
13. Pi, J.D.; Zhao, Y.; Li, L.H. Interaction between a ScrewDislocation and Two Unequal Interface racks Emanating from an Elliptical Hole in One Dimensional Hexagonal Piezoelectric Quasicrystal Bi-Material. *Crystals* **2022**, *12*, 314. [CrossRef]
14. Zhou, W.M.; Fan, T.Y.; Yin, S.Y.; Wang, N.P. Anti-Plane Elasticity Problem and Mode III Crack Problem of Cubic Quasicrystal. *J. Beijing Inst. Technol. (Engl. Ed.)* **2001**, *10*, 250–254.
15. Zhang, L. Stress Intensity of Antiplane Conjugate Cracks in Cubic Quasicrystal. *J. Southwest JT. Univ. (Engl. Ed.)* **2008**, *16*, 285–289.
16. Gao, Y.; Ricoeur, A.; Zhang, L. Plane problems of cubic quasicrystal media with an elliptic hole or a crack. *Phys. Lett. A* **2011**, *375*, 2775–2781. [CrossRef]
17. Suo, Y.R.; Zhou, Y.B.; Liu, G.T. Effect of T stress on the presence of cross-shaped cracks in an optically shaped quasicrystal. *Acta Mech. Solida Sin.* **2022**, *43*, 95–110.
18. Zhang, J.M.; Mao, Z.H.; Feng, X.; Zhang, L.L.; Gao, Y. Free Vibration of Three-Dimensional Piezoelectric Cubic Quasicrystal Plates. In Proceedings of the 2020 15th Symposium on Piezoelectrcity, Acoustic Waves and Device Applications (SPAWDA), Zhengzhou, China, 16–19 April 2021.
19. Wang, R.H.; Hu, C.Z.; Gui, J.N. *Quasicrystal Physics*; Science Press: Beijing, China, 2004.
20. Gao, Y. Governing equations and general solutions of plane elasticity of cubic quasicrystals. *Phys. Lett. A* **2009**, *8*, 885–889. [CrossRef]