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Analytical Solution of the Interference between Elliptical Inclusion and Screw Dislocation in One-Dimensional Hexagonal Piezoelectric Quasicrystal

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Abstract: This study examines the interference problem between screw dislocation and elliptical inclusion in one-dimensional hexagonal piezoelectric quasicrystals. The general solutions are obtained using the complex variable function method and the conformal transformation technique. When the screw dislocation is located outside or inside the elliptical inclusion, the perturbation method and Laurent series expansion are employed to derive explicit analytical expressions for the complex potentials in the elliptical inclusion and the matrix, respectively. Considering four types of far-field force and electric loading conditions, analytical solutions for various specific cases are obtained by using matrix operations. Expressions for the phonon field stress, phason field stress, and electric displacement are given for special cases, including the absence of a dislocation, the presence of an elliptical hole, and the interference between a screw dislocation and circular inclusion, as well as the case of a circular hole. The design and analysis of quasicrystal inclusion structures can benefit from the results of this work.

Keywords: piezoelectric quasicrystal; inclusion; dislocation; complex variable function method; analytical solutions

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1. Introduction

As a new type of functional and structural material, quasicrystals can be widely used in engineering applications [1–6]. Different kinds of defects, such as dislocations, cracks, and inclusions, greatly affect their properties and coupling behavior under loading [7–11]. Exploration of the mechanisms controlling the interaction between inclusions and dislocations in quasicrystal materials can improve our understanding of the deformation strengthening and failure mechanisms of components. Therefore, it is important to study the interference of dislocations and inclusions in quasicrystals under the piezoelectric effect.

For elastic materials, Eshelby [12] asserted that there are interior and exterior elastic fields for ellipsoidal inclusions with eigenstrains. When the eigenstrain or external loading is uniform, the elastic field inside the inclusion is also uniform, which is a classic axiom of inclusion research. Smith [13] studied the interference between screw dislocations located in a matrix and elliptical holes or rigid elliptical inclusions and obtained a complex solution for the potential of a corresponding elastic field. Gong and Meguid [14] studied the interference between dislocations and elastic elliptical inclusions, although they assessed the force of dislocations at specific positions. Meguid and Zhong [15] analyzed the electric and elastic fields of piezoelectric elliptical inclusions. Deng and Meguid [16] studied the electroelastic coupling between elliptical inclusions and screw dislocations in piezoelectric materials.

The mechanics of quasicrystal materials with inclusions or dislocations have also attracted the attention of scholars [17–23]. Using analytical continuation and conformal mapping methods, Wang [24] studied Eshelb's problem of two-dimensional (2D) inclusions with arbitrary shapes contained in 2D decagonal quasicrystals on a plane or half-plane. Shi [25] studied the problem of collinear periodic cracks/rigid inclusions in sliding modes in one-dimensional (1D) hexagonal quasicrystals. Using the displacement function method, Gao and Ricoeurb [26] studied the 3D problem of ellipsoidal inclusions in an infinite body of 2D quasicrystals. Yang et al. [27] used the generalized Stroh formula to obtain the electroelastic field induced by straight dislocations parallel to the periodic axis of 1D quasicrystals. Guo et al. [28] used a conformal mapping technique to analyze the problem of elliptical inclusions in an infinite 1D hexagonal piezoelectric quasicrystal matrix. Li and Liu [29] employed the Stroh formula to analyze the electroelasticity of icosahedral quasicrystals with straight dislocations. Fan et al. [30] deduced a basic solution for extended dislocations in 1D hexagonal piezoelectric quasicrystals. Lou et al. [31] studied a thin elastic inclusion in infinite 1D hexagonal quasicrystals using a hypersingular integral equation. Zhang et al. [32] studied the infinite bodies of 1D hexagonal piezoelectric quasicrystals with ellipsoidal inclusions. By selecting a suitable potential function, the analytical solutions for the electric displacement, phonon field stress, and phason field stress in a matrix and inclusion were obtained. They also analyzed special cases for ellipsoidal voids and coin-shaped cracks. Hu et al. [33] extended the Eshelby tensor from elastic isotropic inclusions to piezoelectric quasicrystal inclusions. By introducing eigenstrain and Green's function, a simple explicit expression of the 1D Eshelby tensor was obtained. Other studies [34,35] examined partially debonded circular inclusions and cylindrical inclusions in piezoelectric quasicrystal materials. Zhai et al. [36] studied the planes of 2D decagonal quasicrystals with rigid arc inclusions under the action of infinite tension and concentrated force.

The presence or evolution of inclusions has a strong perturbation effect on the surrounding media that is counteracted by dislocations, microcracks, holes, and heterogeneous materials in the matrix. This interaction can be used to analyze the relationship between the material strength, modulus, plasticity, and toughness. It can also be used to better understand the strengthening or hardening mechanism of a material and further explain the failure mechanism to improve the processing and service performance of the material. Hu et al. [37] used a complex variable function to study the interference between screw dislocations and circular inclusions in 1D hexagonal quasicrystal materials and obtained boundary conditions represented by the complex potential function and the analytical expression between the stress field and the dislocation force. They also discussed how different dislocation positions and material parameters affect the dislocation force and equilibrium position. Li and Liu [38] studied the interactions between dislocations and elliptical holes in icosahedral quasicrystals. Zhao [39] studied the interactions between screw dislocations and wedge-shaped cracks in 1D hexagonal piezoelectric quasicrystal bimaterials. Lv and Liu [40] used complex variable function theory and the conformal transformation method to study the interaction between multiple parallel dislocations and wedge-shaped cracks in 1D hexagonal piezoelectric quasicrystals and their collective response to the applied generalized stress. Pi et al. [41] studied the interactions between screw dislocations in 1D hexagonal piezoelectric quasicrystal bimaterials and two unequal interfacial cracks with elliptical shapes.

Quasicrystal materials are characterized by a light weight, high brittleness, high hardness, and low friction, and they are very sensitive to defects such as dislocations and inclusions. These materials can be used to describe inclusions simplified as elliptical shapes with various 2D scale ratios (circular and linear), including cracks and rigid line inclusions, which can all be degenerated from elliptical inclusions. In addition, the function describing an elliptical shape is relatively simple, and it is easier to perform various operations for elliptical shapes than for arbitrary shapes to obtain a closed-form solution to a problem. Therefore, this study investigates the interaction between screw dislocation and elliptical inclusion in 1D hexagonal piezoelectric quasicrystals and reduces the problem to several special cases, obtaining the analytical solutions for the corresponding problems.

2. Basic Equations

For a 1D hexagonal piezoelectric quasicrystal, the anti-plane phonon field displacement u_z and phason field displacement w_z are coupled with the electric fields E_x and E_y

in the plane and are irrelevant to the vertical co-ordinate z, i.e., $u_z = u_z(x, y)$, $w = w(x, y)$, $E_x = E_x(x, y)$, and $E_y = E_y(x, y)$. The basic equation is as follows [8,28,30].

The equilibrium equation can be expressed as follows:

$$\frac{\partial}{\partial x}\mathbf{\Lambda}_1 + \frac{\partial}{\partial y}\mathbf{\Lambda}_2 = 0, \tag{1}$$

where:

$$\mathbf{\Lambda}_1 = [\sigma_{zx}\ H_{zx}\ D_x]^{\mathrm{T}},$$

$$\mathbf{\Lambda}_2 = \left[\sigma_{zy}\ H_{zy}\ D_y\right]^{\mathrm{T}},$$

Here, $\sigma_{mn}(m = x, y, z; n = x, y, z)$ is the phonon field stress, H_{mn} is the phason field stress, and D_n is the electric displacement.

The relationship between generalized strain and displacement expressed by the displacement and electric potential is as follows:

$$\begin{aligned} [2\varepsilon_{zx}\ \omega_{zx}\ \ -E_x]^{\mathrm{T}} &= \tfrac{\partial}{\partial x}[u_z\ w\ \phi]^{\mathrm{T}} = \tfrac{\partial}{\partial x}\mathbf{u}, \\ \left[2\varepsilon_{zy}\ \omega_{zy}\ \ -E_y\right]^{\mathrm{T}} &= \tfrac{\partial}{\partial y}[u_z\ w\ \phi]^{\mathrm{T}} = \tfrac{\partial}{\partial y}\mathbf{u}, \end{aligned} \tag{2}$$

where:

$$\mathbf{u} = [u_z\ w\ \phi]^{\mathrm{T}},$$

Here, ε_{mn} is the phonon field strain, ω_{mn} is the phason field strain, u_z is the phonon field displacement, w is the phason field displacement, E_n is the electric field, and ϕ is the electric potential.

If we ignore the effect of the generalized body force, then the generalized stress–strain relationship of a 1D hexagonal piezoelectric quasicrystal is as follows:

$$\begin{aligned} \mathbf{\Lambda}_1 &= \mathbf{D}[\varepsilon_{zx}\ \omega_{zx}\ E_x]^{\mathrm{T}}, \\ \mathbf{\Lambda}_2 &= \mathbf{D}\left[\varepsilon_{zy}\ \omega_{zy}\ E_y\right]^{\mathrm{T}}, \end{aligned} \tag{3}$$

where:

$$\mathbf{D} = \begin{bmatrix} 2C_{44} & R_3 & -e_{15} \\ 2R_3 & K_2 & -d_{15} \\ 2e_{15} & d_{15} & \lambda_{11} \end{bmatrix},$$

Here, C_{44} is the elastic constant of the phonon field, R_3 is the elastic constant of the phason field, K_2 is the coupling elastic constant of the phonon and phason fields, e_{15} and d_{15} are the piezoelectric coefficients, and λ_{11} is the dielectric coefficient.

By substituting Equation (2) into Equation (3), the constitutive relation represented by displacement and electric potential can be obtained as follows:

$$\begin{aligned} \mathbf{\Lambda}_1 &= \mathbf{C}\left[\tfrac{\partial u_z}{\partial x}\ \tfrac{\partial w}{\partial x}\ \tfrac{\partial \phi}{\partial x}\right]^{\mathrm{T}} = \mathbf{C}\tfrac{\partial}{\partial x}\mathbf{u}, \\ \mathbf{\Lambda}_2 &= \mathbf{C}\left[\tfrac{\partial u_z}{\partial y}\ \tfrac{\partial w}{\partial y}\ \tfrac{\partial \phi}{\partial y}\right]^{\mathrm{T}} = \mathbf{C}\tfrac{\partial}{\partial y}\mathbf{u}, \end{aligned} \tag{4}$$

where:

$$\mathbf{C} = \begin{bmatrix} C_{44} & R_3 & e_{15} \\ R_3 & K_2 & d_{15} \\ e_{15} & d_{15} & -\varepsilon_{11} \end{bmatrix}.$$

Substituting Equation (4) into Equation (1) gives:

$$\frac{\partial}{\partial x}\mathbf{\Lambda}_1 + \frac{\partial}{\partial y}\mathbf{\Lambda}_2 = C\nabla^2\mathbf{u} = 0, \tag{5}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the 2D Laplace operator.

$|\mathbf{C}| \neq 0$ and $||$ are matrix determinants. Thus, Equation (5) can be written as:

$$\nabla^2 u_z = 0,\ \nabla^2 w = 0,\ \nabla^2 \phi = 0. \tag{6}$$

If u_z, w, and ϕ are selected as the real parts of the analytic function, Equation (6) can be satisfied.

By introducing $\Psi(t)$, $\Theta(t)$, and $\Phi(t)$ as analytic functions, one obtains:

$$\mathbf{u} = \mathbf{C}_1[\mathrm{Re}\Psi(t)\ \mathrm{Re}\Theta(t)\ \mathrm{Re}\Phi(t)]^{\mathrm{T}}, \tag{7}$$

Here, $t = x + \mathrm{i}y$ is the complex variable, and $\mathrm{i}^2 = -1$ and i are imaginary units. Re represents the real part of the complex variable function. $\mathbf{C}_1$ is given in Appendix A (Equation (A1)).

Substituting Equation (7) into Equation (4) yields:

$$\begin{aligned} \mathbf{\Lambda}_1 &= \tfrac{1}{2}\mathbf{C}\mathbf{C}_1[\Psi'(t)\ \Theta'(t)\ \Phi'(t)]^{\mathrm{T}}, \\ \mathbf{\Lambda}_2 &= \tfrac{\mathrm{i}}{2}\mathbf{C}\mathbf{C}_1[\Psi'(t)\ \Theta'(t)\ \Phi'(t)]^{\mathrm{T}}. \end{aligned} \tag{8}$$

According to Equations (2) and (7), the electric field represented by the analytic function $\Phi(t)$ is given by:

$$\left[E_x\ E_y\right]^{\mathrm{T}} = -\frac{1}{\varepsilon_{11}}\left[\frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\right]^{\mathrm{T}}\mathrm{Re}\Phi(t) = -\frac{1}{2\varepsilon_{11}}[1\ \mathrm{i}]^{\mathrm{T}}\Phi'(t). \tag{9}$$

Hence, the phonon field stress, phason field stress, electric displacement, and electric field intensity can be expressed as follows:

$$\begin{aligned} &\mathbf{\Lambda}_1 - \mathrm{i}\mathbf{\Lambda}_2 = \mathbf{C}\mathbf{C}_1[\Psi'(t)\ \Theta'(t)\ \Phi'(t)]^{\mathrm{T}}, \\ &E_x - \mathrm{i}E_y = -\tfrac{1}{\varepsilon_{11}}\Phi'(t), \end{aligned} \tag{10}$$

where the apostrophe ′ denotes the derivative of the analytic function with respect to independent variable t.

With Equation (8), the resultant force of phonon field stress and phason field stress along the integral curve AB and the integral value of the normal component of electric displacement can be calculated as follows:

$$\begin{aligned} \Sigma &= \left[\int_A^B(\sigma_{zx}dy - \sigma_{zy}dx)\int_A^B(H_{zx}dy - H_{zy}dx)\int_A^B(D_x dy - D_y dx)\right]^{\mathrm{T}} \\ &= \mathbf{C}\mathbf{C}_1\left[[\mathrm{Im}\Psi'(t)]_A^B\ [\mathrm{Im}\Theta'(t)]_A^B\ [\mathrm{Im}\Phi'(t)]_A^B\right]^{\mathrm{T}}, \end{aligned} \tag{11}$$

where Im denotes the imaginary part of the complex variable function and $[\,]_A^B$ is the changing value of the function within the bracket along the integral curve from point A to point B.

3. Problem Description

Considering the presence of an elliptical inclusion in the 1D hexagonal piezoelectric quasicrystal, the major axis of the elliptical inclusion is $2a$, and the minor axis is $2b$, as shown in Figure 1. The area occupied by the matrix is Ω_{I}, and that occupied by the inclusion is Ω_{II}. L is the elliptical interface between the elliptical inclusion and the matrix. Assume that the matrix and the inclusion are well bonded at the interface. In the Cartesian co-ordinate system (x, y, z), the atomic arrangement of the matrix and the inclusion is quasiperiodic along the axis of z, while the atoms are arranged periodically on the $x - y$ plane. The generalized screw dislocation is located at an arbitrary point z_0 in the matrix. The Burgers vector is $\mathbf{B} = \begin{bmatrix} 0\; 0\; b_z\; b_\perp\; b_\phi \end{bmatrix}^T$, which is linear and infinitely extended along the z-axis. b_z is the screw dislocation of the phonon field, $b_\perp$ is the screw dislocation of the phason field, and b_ϕ is the dislocation of electric potential. The superscript 'T' indicates the transpose of the vector or matrix. This paper considers two cases, where the dislocation is located outside the inclusion and inside the inclusion. First, we consider the first case.

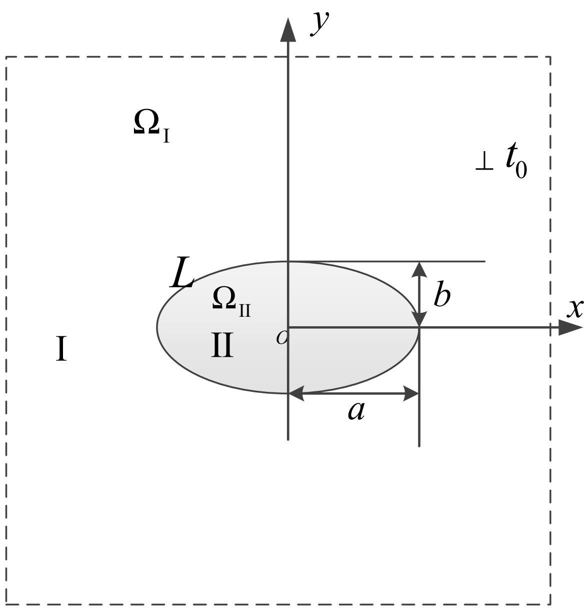

Figure 1. Schematic diagram of a screw dislocation in a 1D hexagonal piezoelectric quasicrystal matrix.

The following mapping function [16] is introduced:

$$t = \Omega(\zeta) = \frac{c}{2}\left(R\zeta + \frac{1}{R\zeta}\right), \tag{12}$$

where:

$$R\zeta = \frac{1}{c}\left(t + \sqrt{t^2 - c^2}\right), \frac{1}{R\zeta} = \frac{1}{c}\left(t - \sqrt{t^2 - c^2}\right), \zeta = \xi + \mathrm{i}\eta,$$

$$R = \sqrt{\frac{a+b}{a-b}} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}, c = \sqrt{a^2 - b^2} = a\sqrt{1-\varepsilon^2}, \varepsilon = \frac{b}{a},$$

In Equation (12), the area Ω_{I} on the $t-$plane is mapped as the external area Γ_{I} of the unit circle $\Gamma_1(\rho = 1)$ on the $\zeta-$plane, and the area Ω_{II} is mapped as the circular area Γ_{II} composed of circle $\Gamma_2(\rho = 1/R)$ and unit circle Γ_1. Γ_2 indicates cutting from $x = -c$ to $+c$ on the $z-$ plane when $y = 0$. Figure 2 presents the conformal mapping plane where the screw dislocation is located in the matrix.

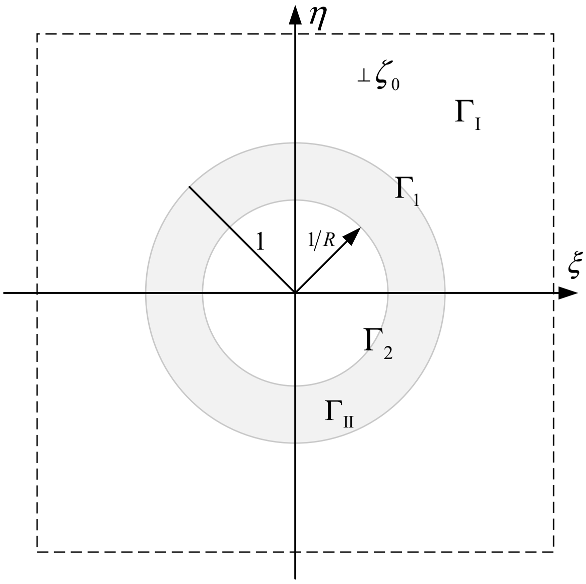

Figure 2. Conformal mapping plane for screw dislocation in a 1D hexagonal piezoelectric quasicrystal matrix.

Substituting Equation (12) into Equations (7) and (11) yields:

$$\mathbf{u} = \mathbf{C}_1[\mathrm{Re}\Psi(\zeta)\ \mathrm{Re}\Theta(\zeta)\ \mathrm{Re}\Phi(\zeta)]^{\mathrm{T}}, \tag{13}$$

and:

$$\Sigma = \mathbf{CC}_1\left[[\mathrm{Im}\Psi'(\zeta)]_A^B\ [\mathrm{Im}\ \Theta\,\prime(\zeta)]_{\mathrm{A}}^{\mathrm{B}}\ [\mathrm{Im}\Phi'(\zeta)]_A^B\right]^{\mathrm{T}}, \tag{14}$$

where:

$$[\Psi(\zeta)\ \Theta(\zeta)\ \Phi(\zeta)]^{\mathrm{T}} = [\Psi(\Omega(\zeta))\ \Theta(\Omega(\zeta))\ \Phi(\Omega(\zeta))]^{\mathrm{T}}.$$

According to perturbation theory [42], the general solution of Equation (13) in the matrix can be expressed as follows:

$$\mathbf{u}_{\mathrm{I}} = \mathbf{C}_1^I\begin{bmatrix}\mathrm{Re}(\Psi_0(\zeta)+\Psi_{\mathrm{I}}(\zeta))\\ \mathrm{Re}(\Theta_0(\zeta)+\Theta_{\mathrm{I}}(\zeta))\\ \mathrm{Re}(\Phi_0(\zeta)+\Phi_{\mathrm{I}}(\zeta))\end{bmatrix}, \zeta\in\Gamma_{\mathrm{I}}. \tag{15}$$

The general solutions of the generalized displacement and electric potential within the inclusion are:

$$\mathbf{u}_{\mathrm{II}} = \mathbf{C}_1^{\mathrm{II}}[\mathrm{Re}\Psi_{\mathrm{II}}(\zeta)\ \mathrm{Re}\Theta_{\mathrm{II}}(\zeta)\ \mathrm{Re}\Phi_{\mathrm{II}}(\zeta)]^{\mathrm{T}}, \zeta\in\Gamma_{\mathrm{II}}. \tag{16}$$

where $\mathbf{C}_1^{\mathrm{I}}$ and $\mathbf{C}_1^{\mathrm{II}}$ are given in Appendix A (Equation (A2)).

In the matrix, the resultant force of the phonon field stress and phason field stress along the integral curve AB and the integral value of the normal component of electric displacement can be represented as follows:

$$\Sigma_{\mathrm{I}} = \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix}[\mathrm{Im}\Psi_0(\zeta)+\mathrm{Im}\Psi_{\mathrm{I}}(\zeta)]_A^B\\ [\mathrm{Im}\Theta_0(\zeta)+\mathrm{Im}\Theta_{\mathrm{I}}(\zeta)]_A^B\\ [\mathrm{Im}\Phi_0(\zeta)+\mathrm{Im}\Phi_{\mathrm{I}}(\zeta)]_A^B\end{bmatrix}, \zeta\in\Gamma_{\mathrm{I}}. \tag{17}$$

Within the inclusion, the resultant force of the phonon field stress and phason field stress along the integral curve AB and the integral value of the normal component of electric displacement can be represented as follows:

$$\Sigma_{\mathrm{II}} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{I}}\left[[\mathrm{Im}\Psi_{\mathrm{II}}(\zeta)]_A^B\ [\mathrm{Im}\Theta_{\mathrm{II}}(\zeta)]_A^B\ [\mathrm{Im}\Phi_{\mathrm{II}}(\zeta)]_A^B\right]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{II}}. \tag{18}$$

where $\mathbf{C}_1^{\mathrm{I}}$ and $\mathbf{C}_1^{\mathrm{II}}$ can be found in Appendix A (Equation (A3)).

The subscripts (or superscripts) I and II indicate that the material constants and physical quantities come from the area $\Omega_{\mathrm{I}}(\Gamma_{\mathrm{I}})$ of the matrix and the area $\Omega_{\mathrm{II}}(\Gamma_{\mathrm{II}})$ of the inclusion, respectively. $\Psi_0(\zeta)$, $\Theta_0(\zeta)$, and $\Phi_0(\zeta)$ represent the field potentials of the stress and electric potential in the matrix without inclusion, which are not disturbed by inclusion. The whole area can be analytically described, except for the singular point. $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, and $\Phi_{\mathrm{I}}(\zeta)$ are the field potentials of the stress and electric potential resulting from the influence of inclusion in the matrix. They are analytically described in area Γ_{I}. $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$, representing the field potentials of stress and electric potential within the inclusion, are analytically described in area Γ_{II}.

Assume the interface Γ_1 is completely bonded, without any free charge and stress, and the normal components of displacement, electric potential, stress, and electric displacement passing through the elliptical interface are continuous. The condition of continuity can be represented as follows:

$$\mathbf{u}_{\mathrm{I}} = \mathbf{u}_{\mathrm{II}}, \zeta \in \Gamma_1\left(\zeta = \sigma = e^{\mathrm{i}\theta}\right), \tag{19}$$

and:

$$\Sigma_{\mathrm{I}} = \Sigma_{\mathrm{II}}, \zeta \in \Gamma_1\left(\zeta = \sigma = e^{\mathrm{i}\theta}\right). \tag{20}$$

Substituting Equations (15) and (16) into Equation (19), one has:

$$\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix}\mathrm{Re}\Psi_0(\sigma)+\mathrm{Re}\Psi_{\mathrm{I}}(\sigma)\\ \mathrm{Re}\Theta_0(\sigma)+\mathrm{Re}\Theta_{\mathrm{I}}(\sigma)\\ \mathrm{Re}\Phi_0(\sigma)+\mathrm{Re}\Phi_{\mathrm{I}}(\sigma)\end{bmatrix} = \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix}\mathrm{Re}\Psi_{\mathrm{II}}(\sigma)\\ \mathrm{Re}\Theta_{\mathrm{II}}(\sigma)\\ \mathrm{Re}\Phi_{\mathrm{II}}(\sigma)\end{bmatrix}. \tag{21}$$

Substituting Equations (17) and (18) into Equation (20) yields:

$$\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix}\mathrm{Im}\Psi_0(\sigma)+\mathrm{Im}\Psi_{\mathrm{I}}(\sigma)\\ \mathrm{Im}\Theta_0(\sigma)+\mathrm{Im}\Theta_{\mathrm{I}}(\sigma)\\ \mathrm{Im}\Phi_0(\sigma)+\mathrm{Im}\Phi_{\mathrm{I}}(\sigma)\end{bmatrix} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix}\mathrm{Im}\Psi_{\mathrm{II}}(\sigma)\\ \mathrm{Im}\Theta_{\mathrm{II}}(\sigma)\\ \mathrm{Im}\Phi_{\mathrm{II}}(\sigma)\end{bmatrix}. \tag{22}$$

Additionally, on the interface Γ_2, the following conditions are satisfied:

$$\left[\Psi_{\mathrm{II}}\left(\frac{\sigma}{R}\right)\ \Theta_{\mathrm{II}}\left(\frac{\sigma}{R}\right)\ \Phi_{\mathrm{II}}\left(\frac{\sigma}{R}\right)\right]^{\mathrm{T}} = \left[\Psi_{\mathrm{II}}\left(\frac{\overline{\sigma}}{R}\right)\ \Theta_{\mathrm{II}}\left(\frac{\overline{\sigma}}{R}\right)\ \Phi_{\mathrm{II}}\left(\frac{\overline{\sigma}}{R}\right)\right]^{\mathrm{T}}. \tag{23}$$

On the $t-$plane, when $y = 0$, points σ/R and $\overline{\sigma}/R$ within the range from $x = -c$ to $+c$ are the same point. Notably, both force and electric loads are uniform in the far field. When the screw dislocation is located at point $t_0 = \Omega(\zeta_0)$, then $\Psi_0(\zeta)$, $\Theta_0(\zeta)$, and $\Phi_0(\zeta)$ can be represented as follows:

$$[\Psi_0(t)\ \Theta_0(t)\ \Phi_0(t)]^{\mathrm{T}} = \mathbf{C}_2^{\mathrm{I}}\mathbf{B}\frac{1}{2\pi\mathrm{i}}\ln(t-t_0) + [p_0\ g_0\ q_0]^{\mathrm{T}}t. \tag{24}$$

Substituting Equation (12) into Equation (24), one obtains:

$$[\Psi_0(\zeta)\ \Theta_0(\zeta)\ \Phi_0(\zeta)]^{\mathrm{T}} = \mathbf{C}_2^{\mathrm{I}}\mathbf{B}\frac{1}{2\pi\mathrm{i}}\ln(\Omega(\zeta)-\Omega(\zeta_0)) + [p_0\ g_0\ q_0]^{\mathrm{T}}\Omega(\zeta), \tag{25}$$

where $\mathbf{C}_2$, $\mathbf{C}_2^{\mathrm{I}}$, and $\mathbf{C}_2^{\mathrm{II}}$ are provided in Appendix A (Equation (A4)).

Here, p_0, g_0, and q_0 are complex constants that can be determined by the force and electric loading in the far field or are equivalent to far-field force and electric fields as follows:

$$\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}} = [p_0\ g_0\ q_0]^{\mathrm{T}}. \tag{26}$$

According to Ref. [43], there are four possible combinations of far-field force and electric loading:

Combination 1: far-field phonon field strain $\varepsilon_{zx\mathrm{I}}^{\infty}$ and $\varepsilon_{zy\mathrm{I}}^{\infty}$, far-field phason field strain $\omega_{zx\mathrm{I}}^{\infty}$ and $\omega_{zy\mathrm{I}}^{\infty}$, and far-field electric field intensity $E_{x\mathrm{I}}^{\infty}$ and $E_{y\mathrm{I}}^{\infty}$.

According to Equation (15), one obtains:

$$\begin{aligned} \left[2\varepsilon_{zx\mathrm{I}}^{\infty}\ \omega_{zx\mathrm{I}}^{\infty}\ E_{x\mathrm{I}}^{\infty}\right]^{\mathrm{T}} &= \tfrac{1}{2}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ -\Phi'_0(\infty)\right]^{\mathrm{T}}, \\ \left[2\varepsilon_{zy\mathrm{I}}^{\infty}\ \omega_{zy\mathrm{I}}^{\infty}\ E_{y\mathrm{I}}^{\infty}\right]^{\mathrm{T}} &= \tfrac{\mathrm{i}}{2}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ -\Phi'_0(\infty)\right]^{\mathrm{T}}. \end{aligned} \tag{27}$$

Substituting Equation (26) into Equation (27) and performing matrix operations, one obtains:

$$[p_0\ g_0\ q_0]^{\mathrm{T}} = \left(\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\left[2\varepsilon_{zx\mathrm{I}}^{\infty} - 2\mathrm{i}\varepsilon_{zy\mathrm{I}}^{\infty}\ \omega_{zx\mathrm{I}}^{\infty} - \mathrm{i}\omega_{zy\mathrm{I}}^{\infty}\ -E_{x\mathrm{I}}^{\infty} + \mathrm{i}E_{y\mathrm{I}}^{\infty}\right]^{\mathrm{T}}.$$

Combination 2: far-field phonon field stress $\sigma_{zx\mathrm{I}}^{\infty}$ and $\sigma_{zy\mathrm{I}}^{\infty}$, far-field phason field stress $H_{zx\mathrm{I}}^{\infty}$ and $H_{zy\mathrm{I}}^{\infty}$, and far-field electric field intensity $D_{x\mathrm{I}}^{\infty}$ and $D_{y\mathrm{I}}^{\infty}$.

According to Equations (8) and (15), one gives:

$$\begin{aligned} \left[\sigma_{zx\mathrm{I}}^{\infty}\ H_{zx\mathrm{I}}^{\infty}\ D_{x\mathrm{I}}^{\infty}\right]^{T} &= \tfrac{1}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}}, \\ \left[\sigma_{zy\mathrm{I}}^{\infty}\ H_{zy\mathrm{I}}^{\infty}\ D_{y\mathrm{I}}^{\infty}\right]^{T} &= \tfrac{\mathrm{i}}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}}. \end{aligned} \tag{28}$$

Substituting Equation (26) into Equation (28) and performing matrix operations, the following can be obtained:

$$[p_0\ g_0\ q_0]^{\mathrm{T}} = \left(\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\left[\sigma_{zx\mathrm{I}}^{\infty} - \mathrm{i}\sigma_{zy\mathrm{I}}^{\infty}\ H_{zx\mathrm{I}}^{\infty} - \mathrm{i}H_{zy\mathrm{I}}^{\infty}\ D_{x\mathrm{I}}^{\infty} - \mathrm{i}D_{y\mathrm{I}}^{\infty}\right]^{\mathrm{T}}.$$

Combination 3: far-field phonon field strain $\varepsilon_{zx\mathrm{I}}^{\infty}$ and $\varepsilon_{zy\mathrm{I}}^{\infty}$, far-field phason field strain $\omega_{zx\mathrm{I}}^{\infty}$ and $\omega_{zy\mathrm{I}}^{\infty}$, and far-field electric displacement $D_{x\mathrm{I}}^{\infty}$ and $D_{y\mathrm{I}}^{\infty}$.

According to Equations (8) and (15), one obtains:

$$\begin{aligned} \left[2\varepsilon_{zx\mathrm{I}}^{\infty}\ \omega_{zx\mathrm{I}}^{\infty}\ D_{x\mathrm{I}}^{\infty}\right]^{\mathrm{T}} &= \tfrac{1}{2}\mathbf{C}_3^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}}, \\ \left[2\varepsilon_{zy\mathrm{I}}^{\infty}\ \omega_{zy\mathrm{I}}^{\infty}\ D_{y\mathrm{I}}^{\infty}\right]^{\mathrm{T}} &= \tfrac{\mathrm{i}}{2}\mathbf{C}_3^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}}, \end{aligned} \tag{29}$$

where $\mathbf{C}_3$ and $\mathbf{C}_3^{\mathrm{I}}$ are provided in Appendix A (Equation (A5)).

Substituting Equation (26) into Equation (29) and performing matrix operations, the following can be obtained:

$$[p_0\ g_0\ q_0]^{\mathrm{T}} = \left(\mathbf{C}_3^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\left[2\varepsilon_{zx\mathrm{I}}^{\infty} - 2\mathrm{i}\varepsilon_{zy\mathrm{I}}^{\infty}\ \omega_{zx\mathrm{I}}^{\infty} - \mathrm{i}\omega_{zy\mathrm{I}}^{\infty}\ D_{x\mathrm{I}}^{\infty} - \mathrm{i}D_{y\mathrm{I}}^{\infty}\right]^{\mathrm{T}}.$$

Combination 4: far-field phonon field stress $\sigma_{zx\mathrm{I}}^{\infty}$ and $\sigma_{zy\mathrm{I}}^{\infty}$, far-field phason field stress $H_{zx\mathrm{I}}^{\infty}$ and $H_{zy\mathrm{I}}^{\infty}$, and far-field electric field intensity $E_{x\mathrm{I}}^{\infty}$ and $E_{y\mathrm{I}}^{\infty}$.

According to Equations (8) and (15), one obtains:

$$\left[\sigma^{\infty}_{zx\mathrm{I}}\ H^{\infty}_{zx\mathrm{I}}\ E^{\infty}_{x\mathrm{I}}\right]^{\mathrm{T}} = \frac{1}{2}\mathbf{C}^{\mathrm{I}}_4\mathbf{C}^{\mathrm{I}}_1\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}},$$
$$\left[\sigma^{\infty}_{zy\mathrm{I}}\ H^{\infty}_{zy\mathrm{I}}\ E^{\infty}_{y\mathrm{I}}\right]^{\mathrm{T}} = \frac{\mathrm{i}}{2}\mathbf{C}^{\mathrm{I}}_4\mathbf{C}^{\mathrm{I}}_1\left[\Psi'_0(\infty)\ \Theta'_0(\infty)\ \Phi'_0(\infty)\right]^{\mathrm{T}}, \quad (30)$$

where $\mathbf{C}_4$ and $\mathbf{C}^{\mathrm{I}}_4$ are provided in Appendix A (Equation (A6)).

Substituting Equation (26) into Equation (30) yields:

$$[p_0\ g_0\ q_0]^{\mathrm{T}} = \left(\mathbf{C}^{\mathrm{I}}_4\mathbf{C}^{\mathrm{I}}_1\right)^{-1}\left[\sigma^{\infty}_{zx\mathrm{I}} - \mathrm{i}\sigma^{\infty}_{zy\mathrm{I}}\ H^{\infty}_{zx\mathrm{I}} - \mathrm{i}H^{\infty}_{zy\mathrm{I}}\ E^{\infty}_{x\mathrm{I}} - \mathrm{i}E^{\infty}_{y\mathrm{I}}\right]^{\mathrm{T}}.$$

Using the mapping function (12), the following relation exists:

$$\ln(1-\zeta) = -\sum_{k=1}^{\infty}\frac{\zeta^k}{k}, |\zeta| < 1.$$

Equation (25) can be expanded into the general form of a Laurent series.

$$\begin{bmatrix}\Psi_0(\zeta)\\ \Theta_0(\zeta)\\ \Phi_0(\zeta)\end{bmatrix} = \begin{bmatrix}\sum\limits_{k=0}^{\infty}\left(a_{k0}\zeta^{k+1} + b_{k0}\frac{1}{\zeta^{k+1}}\right)\\ \sum\limits_{k=0}^{\infty}\left(c_{k0}\zeta^{k+1} + d_{k0}\frac{1}{\zeta^{k+1}}\right)\\ \sum\limits_{k=0}^{\infty}\left(e_{k0}\zeta^{k+1} + f_{k0}\frac{1}{\zeta^{k+1}}\right)\end{bmatrix}, 1 \le |\zeta| < |\zeta_0|. \quad (31)$$

The constant terms of the corresponding field potential and rigid displacement are ignored. Coefficients a_{k0}, b_{k0}, c_{k0}, d_{k0}, e_{k0}, and f_{k0} can be represented as follows:

$$a_{k0} = \begin{cases} -\frac{b_z C^{\mathrm{I}}_{44}}{2\pi\mathrm{i}}\frac{1}{\zeta_0} + \frac{p_0 c}{2}R,\ k=0,\\ -\frac{1}{k+1}\frac{b_z C^{\mathrm{I}}_{44}}{2\pi\mathrm{i}}\frac{1}{\zeta_0^{k+1}},\ k=1,2,\cdots, \end{cases} \quad b_{k0} = \begin{cases} -\frac{b_z C^{\mathrm{I}}_{44}}{2\pi\mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{p_0 c}{2R},\ k=0,\\ -\frac{1}{k+1}\frac{b_z C^{\mathrm{I}}_{44}}{2\pi\mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}},\ k=1,2,\cdots, \end{cases}$$
$$c_{k0} = \begin{cases} -\frac{b_\perp K^{\mathrm{I}}_2}{2\pi\mathrm{i}}\frac{1}{\zeta_0} + \frac{g_0 c}{2}R,\ k=0,\\ -\frac{1}{k+1}\frac{b_\perp K^{\mathrm{I}}_2}{2\pi\mathrm{i}}\frac{1}{\zeta_0^{k+1}},\ k=1,2,\cdots, \end{cases} \quad d_{k0} = \begin{cases} -\frac{b_\perp K^{\mathrm{I}}_2}{2\pi\mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{g_0 c}{2R},\ k=0,\\ -\frac{1}{k+1}\frac{b_\perp K^{\mathrm{I}}_2}{2\pi\mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}},\ k=1,2,\cdots, \end{cases} \quad (32)$$
$$e_{k0} = \begin{cases} -\frac{b_\varphi \varepsilon^{\mathrm{I}}_{11}}{2\pi\mathrm{i}}\frac{1}{\zeta_0} + \frac{q_0 c}{2}R,\ k=0,\\ -\frac{1}{k+1}\frac{b_\varphi \varepsilon^{\mathrm{I}}_{11}}{2\pi\mathrm{i}}\frac{1}{\zeta_0^{k+1}},\ k=1,2,\cdots, \end{cases} \quad f_{k0} = \begin{cases} -\frac{b_\varphi \varepsilon^{\mathrm{I}}_{11}}{2\pi\mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{q_0 c}{2R},\ k=0,\\ -\frac{1}{k+1}\frac{b_\varphi \varepsilon^{\mathrm{I}}_{11}}{2\pi\mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}},\ k=1,2,\cdots. \end{cases}$$

Then, it is necessary to determine the complex potentials Ψ_j, Θ_j, and $\Phi_j (j = \mathrm{I}, \mathrm{II})$ so that they can meet the conditions of continuity (19), (20) and (23).

On the $\zeta-$plane, $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, and $\Phi_{\mathrm{I}}(\zeta)$ are analytic functions in area Γ_{I}, and $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$ are analytic functions in area Γ_{II}; thus, they can be represented by the Laurent expansion as follows:

$$[\Psi_{\mathrm{I}}(\zeta)\ \Theta_{\mathrm{I}}(\zeta)\ \Phi_{\mathrm{I}}(\zeta)]^{\mathrm{T}} = \left[\sum_{k=0}^{\infty} b^{\mathrm{I}}_k\frac{1}{\zeta^{k+1}}\ \sum_{k=0}^{\infty} d^{\mathrm{I}}_k\frac{1}{\zeta^{k+1}}\ \sum_{k=0}^{\infty} f^{\mathrm{I}}_k\frac{1}{\zeta^{k+1}}\right]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{I}}, \quad (33)$$

and:

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \\ \Phi_{\mathrm{II}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(a_k^{\mathrm{II}}\zeta^{k+1} + b_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(c_k^{\mathrm{II}}\zeta^{k+1} + d_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(e_k^{\mathrm{II}}\zeta^{k+1} + f_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}. \tag{34}$$

Substituting Equation (34) into Equation (23) yields:

$$\left[a_k^{\mathrm{II}}\ c_k^{\mathrm{II}}\ e_k^{\mathrm{II}}\right]^{\mathrm{T}} = R^{2(k+1)}\left[b_k^{\mathrm{II}}\ d_k^{\mathrm{II}}\ f_k^{\mathrm{II}}\right]^{\mathrm{T}}.$$

Thus, Equation (34) is rephrased as follows:

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \\ \Phi_{\mathrm{II}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty} a_k^{\mathrm{II}}\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty} c_k^{\mathrm{II}}\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty} e_k^{\mathrm{II}}\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}. \tag{35}$$

where $\zeta = \sigma = 1/\overline{\sigma}$ is on the unit circle Γ_1. According to Equations (31), (33) and (35), Equation (21) can be represented as follows:

$$\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} a_{k0}^{\mathrm{I}} + \overline{b}_{k0}^{\mathrm{I}} + \overline{b}_k^{\mathrm{I}} \\ c_{k0}^{\mathrm{I}} + \overline{d}_{k0}^{\mathrm{I}} + \overline{d}_k^{\mathrm{I}} \\ e_{k0}^{\mathrm{I}} + \overline{f}_{k0}^{\mathrm{I}} + \overline{f}_k^{\mathrm{I}} \end{bmatrix} = \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} a_k^{\mathrm{II}} + \frac{1}{R^{2k+2}}\overline{a}_k^{\mathrm{II}} \\ c_k^{\mathrm{II}} + \frac{1}{R^{2k+2}}\overline{c}_k^{\mathrm{II}} \\ e_k^{\mathrm{II}} + \frac{1}{R^{2k+2}}\overline{e}_k^{\mathrm{II}} \end{bmatrix}.$$

According to the theory of analytic function, $\Psi(\zeta)$, $\Theta(\zeta)$, and $\Phi(\zeta)$ are analytic functions in area Γ_{I}, and $\overline{\Psi(1/\overline{\zeta})}$, $\overline{\Theta(1/\overline{\zeta})}$, and $\overline{\Phi(1/\overline{\zeta})}$ are analytic functions in area Γ_{II}. Therefore, if Γ_0 represents the internal area of the circle Γ_2, $\Gamma_0 + \Gamma_{\mathrm{II}}$ indicates the entire area inside the unit circle, and $[\theta_1(\zeta)\ \theta_2(\zeta)\ \theta_3(\zeta)]^{\mathrm{T}}$ can be represented as follows:

$$\begin{bmatrix} \theta_1(\zeta) \\ \theta_2(\zeta) \\ \theta_3(\zeta) \end{bmatrix} = \mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\overline{b}_k^{\mathrm{I}}\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\overline{d}_k^{\mathrm{I}}\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\overline{f}_k^{\mathrm{I}}\zeta^{k+1} \end{bmatrix} - \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}a_k^{\mathrm{II}}\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}c_k^{\mathrm{II}}\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}e_k^{\mathrm{II}}\zeta^{k+1} \end{bmatrix} - \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\overline{a}_k^{\mathrm{II}}\frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty}\overline{c}_k^{\mathrm{II}}\frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty}\overline{e}_k^{\mathrm{II}}\frac{\zeta^{k+1}}{R^{2k+2}} \end{bmatrix}$$
$$+\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(a_{k0}^{\mathrm{I}} + \overline{b}_{k0}^{\mathrm{I}}\right)\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\left(c_{k0}^{\mathrm{I}} + \overline{d}_{k0}^{\mathrm{I}}\right)\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\left(e_{k0}^{\mathrm{I}} + \overline{f}_{k0}^{\mathrm{I}}\right)\zeta^{k+1} \end{bmatrix}, \zeta \in \Gamma_0 + \Gamma_{\mathrm{II}}, \tag{36a}$$

$$\begin{bmatrix} \theta_1(\zeta) \\ \theta_2(\zeta) \\ \theta_3(\zeta) \end{bmatrix} = \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}a_k^{\mathrm{II}}\frac{1}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}c_k^{\mathrm{II}}\frac{1}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}e_k^{\mathrm{II}}\frac{1}{R^{2k+2}\zeta^{k+1}} \end{bmatrix} + \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\overline{a}_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\overline{c}_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\overline{e}_k^{\mathrm{II}}\frac{1}{\zeta^{k+1}} \end{bmatrix} - \mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}b_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}d_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}f_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}} \end{bmatrix} \tag{36b}$$

$$-\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix}\sum_{k=0}^{\infty}\left(\overline{a}_{k0}^{\mathrm{I}}+b_{k0}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}}\\ \sum_{k=0}^{\infty}\left(\overline{c}_{k0}^{\mathrm{I}}+d_{k0}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}}\\ \sum_{k=0}^{\infty}\left(\overline{e}_{k0}^{\mathrm{I}}+f_{k0}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}}\end{bmatrix},\zeta\in\Gamma_I.$$

Equations (36a) and (36b) are holomorphic and single-valued on the whole plane. Thus, $[\theta_1(\zeta)\ \theta_2(\zeta)\ \theta_3(\zeta)]^{\mathrm{T}}\equiv 0$ can be obtained according to the Liouville theorem. With this result, the following can be obtained from Equation (36a,b):

$$\begin{bmatrix}b_k^{\mathrm{I}}\\ d_k^{\mathrm{I}}\\ f_k^{\mathrm{I}}\end{bmatrix}=\mathbf{D}_3\begin{bmatrix}a_{k0}^{\mathrm{I}}\\ c_{k0}^{\mathrm{I}}\\ e_{k0}^{\mathrm{I}}\end{bmatrix}+\mathbf{D}_4\begin{bmatrix}\overline{a}_{k0}^{\mathrm{I}}\\ \overline{c}_{k0}^{\mathrm{I}}\\ \overline{e}_{k0}^{\mathrm{I}}\end{bmatrix}-\begin{bmatrix}b_{k0}^{\mathrm{I}}\\ d_{k0}^{\mathrm{I}}\\ f_{k0}^{\mathrm{I}}\end{bmatrix}. \quad (37)$$

Similarly, we can obtain the following based on the condition of continuity in Equation (20):

$$\begin{bmatrix}a_k^{\mathrm{II}}\\ c_k^{\mathrm{II}}\\ e_k^{\mathrm{II}}\end{bmatrix}=\mathbf{D}_5\begin{bmatrix}a_{k0}^{\mathrm{I}}\\ c_{k0}^{\mathrm{I}}\\ e_{k0}^{\mathrm{I}}\end{bmatrix}+\mathbf{D}_6\begin{bmatrix}\overline{a}_{k0}^{\mathrm{I}}\\ \overline{c}_{k0}^{\mathrm{I}}\\ \overline{e}_{k0}^{\mathrm{I}}\end{bmatrix}, \quad (38)$$

where $\mathbf{D}_i(i=1,2,\cdots,6)$ is given in Appendix A (Equation (A7)).

Let

$$\mathbf{D}_3=\begin{bmatrix}I_{k2}&K_{k2}&M_{k2}\\ I_{k4}&K_{k4}&M_{k4}\\ I_{k6}&K_{k6}&M_{k6}\end{bmatrix},\ \mathbf{D}_4=\begin{bmatrix}J_{k2}&L_{k2}&N_{k2}\\ J_{k4}&L_{k4}&N_{k4}\\ J_{k6}&L_{k6}&N_{k6}\end{bmatrix},\ \mathbf{D}_5=\begin{bmatrix}I_{k1}&K_{k1}&M_{k1}\\ I_{k3}&K_{k3}&M_{k3}\\ I_{k5}&K_{k5}&M_{k5}\end{bmatrix},\ \mathbf{D}_6=\begin{bmatrix}J_{k1}&L_{k1}&N_{k1}\\ J_{k3}&L_{k3}&N_{k3}\\ J_{k5}&L_{k5}&N_{k5}\end{bmatrix}. \quad (39)$$

The next step is to determine the coefficients of the expanded complex series. For a given k, six systems of linear equations with six unknowns a_k^{II}, c_k^{II}, e_k^{II}, b_k^{I}, d_k^{I}, and f_k^{I} can be obtained according to Equations (37) and (38). These unknown coefficients to be solved are represented by specific coefficients a_{k0}, b_{k0}, c_{k0}, d_{k0}, e_{k0}, and f_{k0}:

$$\begin{bmatrix}a_k^{\mathrm{II}}\\ c_k^{\mathrm{II}}\\ e_k^{\mathrm{II}}\end{bmatrix}=\begin{bmatrix}I_{k1}a_{k0}+K_{k1}c_{k0}+M_{k1}e_{k0}+J_{k1}\overline{a}_{k0}+L_{k1}\overline{c}_{k0}+N_{k1}\overline{e}_{k0}\\ I_{k3}a_{k0}+K_{k3}c_{k0}+M_{k3}e_{k0}+J_{k3}\overline{a}_{k0}+L_{k3}\overline{c}_{k0}+N_{k3}\overline{e}_{k0}\\ I_{k5}a_{k0}+K_{k5}c_{k0}+M_{k5}e_{k0}+J_{k5}\overline{a}_{k0}+L_{k5}\overline{c}_{k0}+N_{k5}\overline{e}_{k0}\end{bmatrix}, \quad (40)$$

and:

$$\begin{bmatrix}b_k^{\mathrm{I}}\\ d_k^{\mathrm{I}}\\ f_k^{\mathrm{I}}\end{bmatrix}=\begin{bmatrix}I_{k2}a_{k0}+K_{k2}c_{k0}+M_{k2}e_{k0}+J_{k2}\overline{a}_{k0}+L_{k2}\overline{c}_{k0}+N_{k2}\overline{e}_{k0}-b_{k0}\\ I_{k4}a_{k0}+K_{k4}c_{k0}+M_{k4}e_{k0}+J_{k4}\overline{a}_{k0}+L_{k4}\overline{c}_{k0}+N_{k4}\overline{e}_{k0}-d_{k0}\\ I_{k6}a_{k0}+K_{k6}c_{k0}+M_{k6}e_{k0}+J_{k6}\overline{a}_{k0}+L_{k6}\overline{c}_{k0}+N_{k6}\overline{e}_{k0}-f_{k0}\end{bmatrix}. \quad (41)$$

By substituting Equation (32) into Equations (40) and (41), all coefficients in the series expansion Equations (33) and (34), i.e., $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, $\Phi_{\mathrm{I}}(\zeta)$, $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$, can be determined. Then, the problem is solved.

4. Typical Case of Dislocation Located Outside Inclusion

In special cases, the series solutions (33) and (34) can be given in a simpler form or their expressions can be obtained by summing. In this section, we address and discuss some special cases: without considering the dislocation or electric field, considering the interference between the screw dislocation and elliptical hole, considering the interference between the screw dislocation and circular inclusion, and considering the interference between the screw dislocation and circular hole.

4.1. No Dislocation

Without considering dislocation, i.e., the Burgers vector $\mathbf{B} = [0\ 0\ 0\ 0\ 0]^T$, the problem degenerates into a 1D hexagonal piezoelectric quasicrystal with an elliptical inclusion.

In the matrix, one has:

$$\begin{aligned}
\mathbf{\Lambda}_{1\mathrm{I}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{I}} &= -\tfrac{1}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left(R^2\mathbf{D}_3 - \mathbf{E}\right)\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix}\tfrac{t}{\sqrt{t^2-c^2}} - \tfrac{1}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}R^2\mathbf{D}_4\begin{bmatrix} \overline{p}_0 \\ \overline{g}_0 \\ \overline{q}_0 \end{bmatrix}\tfrac{t}{\sqrt{t^2-c^2}} \\
&\quad + \tfrac{1}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left(R^2\mathbf{D}_3 + \mathbf{E}\right)\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix} + \tfrac{1}{2}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}R^2\mathbf{D}_4\begin{bmatrix} \overline{p}_0 \\ \overline{g}_0 \\ \overline{q}_0 \end{bmatrix}, t \in \Omega_{\mathrm{I}}, \\
E_{x\mathrm{I}} - \mathrm{i}E_{y\mathrm{I}} &= \tfrac{R^2}{2\varepsilon_{11}^{\mathrm{I}}}\left(p_0 I_{06} + g_0 K_{06} + q_0 M_{06} + \overline{p}_0 J_{06} + \overline{g}_0 L_{06} + \overline{q}_0 N_{06}\right)\left(\tfrac{t}{\sqrt{t^2-c^2}} - 1\right) \\
&\quad - \tfrac{q_0}{2\varepsilon_{11}^{\mathrm{I}}}\left(\tfrac{t}{\sqrt{t^2-c^2}} + 1\right), t \in \Omega_{\mathrm{I}}.
\end{aligned}$$

In the inclusion, one obtains:

$$\mathbf{\Lambda}_{1\mathrm{II}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{II}} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\mathbf{D}_5[p_0\ g_0\ q_0]^{\mathrm{T}} + \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\mathbf{D}_6[\overline{p}_0\ \overline{g}_0\ \overline{q}_0]^{\mathrm{T}},$$

$$E_{x\mathrm{II}} - \mathrm{i}E_{y\mathrm{II}} = -\tfrac{1}{\varepsilon_{11}^{\mathrm{II}}}\left(I_{05}p_0 + K_{05}g_0 + M_{05}q_0 + J_{05}\overline{p}_0 + L_{05}\overline{g}_0 + N_{05}\overline{q}_0\right).$$

where $\mathbf{D}_i (i = 3, 4, 5, 6)$ is provided in Appendix A (Equation (A8)).

It can be discerned that the phonon field stress, phason field stress, electric field intensity, and electric displacement in the elliptical inclusion are uniform.

4.2. Interference between the Screw Dislocation and Elliptical Hole

The elliptical inclusion can be reduced to an elliptical hole, i.e., $C_{44}^{\mathrm{II}} = R_3^{\mathrm{II}} = K_2^{\mathrm{II}} = d_{15}^{\mathrm{II}} = e_{15}^{\mathrm{II}} = 0$ and $\varepsilon_{11}^{\mathrm{II}} = \varepsilon_0$.

The coefficients a_k^{II}, b_k^{I}, c_k^{II}, d_k^{I}, e_k^{II}, and f_k^{I} can be represented as follows:

$$\begin{aligned}
\begin{bmatrix} b_k^{\mathrm{I}} \\ d_k^{\mathrm{I}} \\ f_k^{\mathrm{I}} \end{bmatrix} &= \begin{bmatrix} -\frac{4R^{2k+2}\left(R^{4k+4}+1\right)\left(R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}-e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}\right)M_4}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)\left(R^{4k+4}-1\right)} \\ \frac{4R^{2k+2}\left(R^{4k+4}+1\right)\left(R_3^{\mathrm{I}}e_{15}^{\mathrm{I}}-C_{44}^{\mathrm{I}}d_{15}^{\mathrm{I}}\right)M_4}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)\left(R^{4k+4}-1\right)} \\ \frac{4R^{2k+2}\varepsilon_0\left(R_3^{\mathrm{I}2}-C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\right)M_3}{M_1{}^2R^{4k+4}-M_2{}^2} \end{bmatrix} e_{k0}^{\mathrm{I}} - \begin{bmatrix} b_{k0}^{\mathrm{I}} \\ d_{k0}^{\mathrm{I}} \\ f_{k0}^{\mathrm{I}} \end{bmatrix} \\
&+ \begin{bmatrix} \overline{a}_{k0}^{\mathrm{I}} + \frac{2C_{44}^{\mathrm{I}}\varepsilon_0\left(R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}-K_2^{\mathrm{I}}e_{15}^{\mathrm{I}}\right)\left(2\varepsilon_0M_5R^{4k+4}+M_1R^{8k+8}+M_2\right)}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)\left(R^{4k+4}-1\right)}\overline{e}_{k0}^{\mathrm{I}} \\ \overline{c}_{k0}^{\mathrm{I}} + \frac{2K_2^{\mathrm{I}}\varepsilon_0\left(R_3^{\mathrm{I}}e_{15}^{\mathrm{I}}-C_{44}^{\mathrm{I}}d_{15}^{\mathrm{I}}\right)\left(2\varepsilon_0M_5R^{4k+4}+M_1R^{8k+8}+M_2\right)}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)\left(R^{4k+4}-1\right)}\overline{e}_{k0}^{\mathrm{I}} \\ -\frac{M_1M_2\left(R^{4k+4}-1\right)}{M_1{}^2R^{4k+4}-M_2{}^2}\overline{e}_{k0}^{\mathrm{I}} \end{bmatrix},
\end{aligned} \tag{42}$$

and:

$$\begin{bmatrix} a_k^{\mathrm{II}} \\ c_k^{\mathrm{II}} \\ e_k^{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{2R^{4k+4}\varepsilon_0M_1M_3}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)}e_{k0}^{\mathrm{I}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{2R^{2k+2}\varepsilon_0M_2M_3}{\varepsilon_{11}^{\mathrm{I}}\left(M_1{}^2R^{4k+4}-M_2{}^2\right)}\overline{e}_{k0}^{\mathrm{I}} \end{bmatrix}. \tag{43}$$

where $M_i (i = 1, 2, \cdots, 5)$ is provided in Appendix A (Equation (A9)).

If Equation (32) is substituted into Equations (42) and (43), all coefficients in the series expansion Equations (33) and (34), i.e., $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, $\Phi_{\mathrm{I}}(\zeta)$, $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$, can be determined. Then, the problem is solved.

Equation (43) shows that the phonon field stress and phason field stress in the elliptical hole are equal to zero.

4.3. Interference between the Screw Dislocation and Circular Inclusion

The elliptical inclusion can be reduced to a circular hole, i.e., $a = b$.

In the matrix, the expressions of the phonon field stress, phason field stress, electric field intensity, and electric displacement are as follows:

$$
\begin{aligned}
\mathbf{\Lambda}_{1\mathrm{I}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{I}} &= \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix} + \frac{1}{2\pi \mathrm{i}}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} \\ b_\perp K_2^{\mathrm{I}} \\ b_\varphi \varepsilon_{11}^{\mathrm{I}} \end{bmatrix}\frac{1}{t-t_0} - \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} J_{02}\overline{p}_0 + L_{02}\overline{g}_0 + N_{02}\overline{q}_0 \\ J_{04}\overline{p}_0 + L_{04}\overline{g}_0 + N_{04}\overline{q}_0 \\ J_{06}\overline{p}_0 + L_{06}\overline{g}_0 + N_{06}\overline{q}_0 \end{bmatrix}\frac{a^2}{t^2} \\
&+ \frac{\mathrm{i}}{2\pi}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} J_{02} + b_\perp K_2^{\mathrm{I}} L_{02} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{02} \\ b_z C_{44}^{I} J_{04} + b_\perp K_2^{\mathrm{I}} L_{04} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{04} \\ b_z C_{44}^{\mathrm{I}} J_{06} + b_\perp K_2^{I} L_{06} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{06} \end{bmatrix}\frac{a^2}{t(\overline{t}_0 t - a^2)},\ t \in \Omega_{\mathrm{I}}, \\
E_{x\mathrm{I}} - \mathrm{i}E_{y\mathrm{I}} &= -\frac{1}{\varepsilon_{11}^{\mathrm{I}}}q_0 - \frac{b_\varphi}{2\pi \mathrm{i}}\frac{1}{t-t_0} + \frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(J_{06}\overline{p}_0 + L_{06}\overline{g}_0 + N_{06}\overline{q}_0\right)\frac{a^2}{t^2} \\
&- \frac{\mathrm{i}}{2\pi}\frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(b_z C_{44}^{\mathrm{I}} J_{06} + b_\perp K_2^{\mathrm{I}} L_{06} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{06}\right)\frac{a^2}{t(\overline{t}_0 t - a^2)},\ t \in \Omega_{\mathrm{I}}.
\end{aligned}
$$

In the circular inclusion, one has:

$$
\begin{aligned}
\mathbf{\Lambda}_{1\mathrm{II}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{II}} &= -\frac{\mathrm{i}}{2\pi}\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} I_{01} + b_\perp K_2^{\mathrm{I}} K_{01} + b_\varphi \varepsilon_{11}^{\mathrm{I}} M_{01} \\ b_z C_{44}^{\mathrm{I}} I_{03} + b_\perp K_2^{\mathrm{I}} K_{03} + b_\varphi \varepsilon_{11}^{\mathrm{I}} M_{03} \\ b_z C_{44}^{\mathrm{I}} I_{05} + b_\perp K_2^{\mathrm{I}} K_{05} + b_\varphi \varepsilon_{11}^{\mathrm{I}} M_{05} \end{bmatrix}\frac{1}{t-t_0} \\
&+ \frac{1}{a}\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} I_{01}p_0 + K_{01}g_0 + M_{01}q_0 \\ I_{03}p_0 + K_{03}g_0 + M_{03}q_0 \\ I_{05}p_0 + K_{05}g_0 + M_{05}q_0 \end{bmatrix},\ t \in \Omega_{\mathrm{II}}, \\
E_{x\mathrm{II}} - \mathrm{i}E_{y\mathrm{II}} &= \frac{\mathrm{i}}{2\pi}\frac{1}{\varepsilon_{11}^{\mathrm{II}}}\left(b_z C_{44}^{\mathrm{I}} I_{05} + b_\perp K_2^{\mathrm{I}} K_{05} + b_\varphi \varepsilon_{11}^{\mathrm{I}} M_{05}\right)\frac{1}{t-t_0} \\
&- \frac{1}{a\varepsilon_{11}^{\mathrm{II}}}\left(I_{05}p_0 + K_{05}g_0 + M_{05}q_0\right),\ t \in \Omega_{\mathrm{II}}.
\end{aligned}
$$

Clearly, the phonon field stress, phason field stress, and electric field intensity within the inclusion all show typical screw dislocation behavior, i.e., a singularity of $1/(t - t_0)$ at point $t = t_0$.

4.4. Interference between Screw Dislocation and Circular Hole

If the circular inclusion reduces into a circular hole, then $a = b$, $C_{44}^{\mathrm{II}} = R_3^{\mathrm{II}} = K_2^{\mathrm{II}} = d_{15}^{\mathrm{II}} = e_{15}^{\mathrm{II}} = 0$, and $\varepsilon_{11}^{\mathrm{II}} = \varepsilon_0$.

In the matrix, the expressions of the phonon field stress, phason field stress, electric field intensity, and electric displacement are as follows:

$$
\mathbf{\Lambda}_{1\mathrm{I}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{I}} = \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix} + \frac{1}{2\pi \mathrm{i}}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} \\ b_\perp K_2^{\mathrm{I}} \\ b_\varphi \varepsilon_{11}^{\mathrm{I}} \end{bmatrix}\frac{1}{t-t_0} - a^2\overline{q}_0 N_{k6}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\frac{1}{t^2},\ t \in \Omega_{\mathrm{I}},
$$

$$E_{x\mathrm{I}} - \mathrm{i}E_{y\mathrm{I}} = -\frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(q_0 + \frac{b_\varphi \varepsilon_{11}^{\mathrm{I}}}{2\pi\mathrm{i}}\frac{1}{t-t_0} - a^2\overline{q}_0 N_{k6}\frac{1}{t^2}\right), t \in \Omega_{\mathrm{I}}.$$

In the circular hole, the expressions of the phonon field stress, phason field stress, electric field intensity, and electric displacement are as follows:

$$\mathbf{\Lambda}_{1\mathrm{II}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{II}} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}[0\ 0\ M_{k5}q_0]^{\mathrm{T}}, t \in \Omega_{\mathrm{II}},$$

$$E_{x\mathrm{II}} - \mathrm{i}E_{y\mathrm{II}} = -\frac{1}{\varepsilon_{11}^{\mathrm{II}}} M_{k5}q_0, t \in \Omega_{\mathrm{II}}.$$

where M_{k5} and N_{k6} are provided in Appendix A (Equation (A10)).

Apparently, the phonon field stress, phason field stress, and electric field intensity in the matrix are all affected by the dislocation, equivalent far-field phonon field, phason field, and electric field. The phonon field stress and phason field stress in the circular hole are equal to zero, and the electric field strength is affected by the equivalent far-field electric field q_0 rather than dislocation. In addition, the electric field strength and electric displacement in the circular hole are uniform.

4.5. No Electric Field

Without considering the electric field, the problem reduces to the interference between screw dislocation and elliptical inclusion in the matrix in a 1D hexagonal quasicrystal. The elastic constants of the 1D hexagonal quasicrystal are shown in Appendix B (Equations (A11) and (A12)).

By substituting the elastic constants of the 1D hexagonal quasicrystal, the series expansion coefficients of $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, $\Psi_{\mathrm{II}}(\zeta)$, and $\Theta_{\mathrm{II}}(\zeta)$ are as follows:

$$\begin{bmatrix} \Psi_{\mathrm{I}}(\zeta) \\ \Theta_{\mathrm{I}}(\zeta) \end{bmatrix} = -\frac{1}{2\pi\mathrm{i}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\frac{1}{k+1}\left(b_z C_{44}^{\mathrm{I}}\left(\frac{I_{k2}}{\zeta_0^{k+1}} + \frac{J_{k2}}{\overline{\zeta}_0^{k+1}} + \frac{1}{(R^2\zeta_0)^{k+1}}\right) + b_\perp K_2^{\mathrm{I}}\left(\frac{K_{k2}}{\zeta_0^{k+1}} + \frac{L_{k2}}{\overline{\zeta}_0^{k+1}}\right)\right)\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\frac{1}{k+1}\left(b_z C_{44}^{\mathrm{I}}\left(\frac{I_{k4}}{\zeta_0^{k+1}} + \frac{J_{k4}}{\overline{\zeta}_0^{k+1}} + \frac{1}{(R^2\zeta_0)^{k+1}}\right) + b_\perp K_2^{\mathrm{I}}\left(\frac{K_{k4}}{\zeta_0^{k+1}} + \frac{L_{k4}}{\overline{\zeta}_0^{k+1}}\right)\right)\frac{1}{\zeta^{k+1}} \end{bmatrix}$$

$$+\frac{c}{2}\begin{bmatrix} R(I_{02}p_0 + J_{02}\overline{p}_0 + K_{02}g_0 + L_{02}\overline{g}_0) - \frac{p_0}{R} \\ R(I_{04}p_0 + J_{04}\overline{p}_0 + K_{04}g_0 + L_{04}\overline{g}_0) - \frac{g_0}{R} \end{bmatrix}\frac{1}{\zeta}, \zeta \in \Gamma_{\mathrm{I}}.$$

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \end{bmatrix} = -\frac{1}{2\pi\mathrm{i}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\frac{1}{k+1}\left(b_z C_{44}^{\mathrm{I}}\left(\frac{I_{k1}}{\zeta_0^{k+1}} + \frac{J_{k1}}{\overline{\zeta}_0^{k+1}}\right) + b_\perp K_2^{\mathrm{I}}\left(\frac{K_{k1}}{\zeta_0^{k+1}} + \frac{L_{k1}}{\overline{\zeta}_0^{k+1}}\right)\right)\left(\zeta^{k+1} + \frac{1}{(R^2\zeta)^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\frac{1}{k+1}\left(b_z C_{44}^{\mathrm{I}}\left(\frac{I_{k3}}{\zeta_0^{k+1}} + \frac{J_{k3}}{\overline{\zeta}_0^{k+1}}\right) + b_\perp K_2^{\mathrm{I}}\left(\frac{K_{k3}}{\zeta_0^{k+1}} + \frac{L_{k3}}{\overline{\zeta}_0^{k+1}}\right)\right)\left(\zeta^{k+1} + \frac{1}{(R^2\zeta)^{k+1}}\right) \end{bmatrix}$$

$$+\begin{bmatrix} \left(I_{01}\frac{p_0 c}{2}R + J_{01}\frac{\overline{p}_0 c}{2}R + K_{k1}\frac{g_0 c}{2}R + L_{k1}\frac{\overline{g}_0 c}{2}R\right)\left(\zeta + \frac{1}{R^2\zeta}\right) \\ \left(I_{k3}\frac{p_0 c}{2}R + J_{k3}\frac{\overline{p}_0 c}{2}R + K_{k3}\frac{g_0 c}{2}R + L_{k3}\frac{\overline{g}_0 c}{2}R\right)\left(\zeta + \frac{1}{R^2\zeta}\right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}.$$

Then, the problem is solved.

5. Dislocation Located within an Inclusion

Now let us consider the second case: a generalized screw dislocation with Burgers vector $\mathbf{B} = \left[0\ 0\ b_z\ b_\perp\ b_\phi\right]^T$ is located at point $t = t_0$ in the elliptical inclusion, as shown in Figure 3.

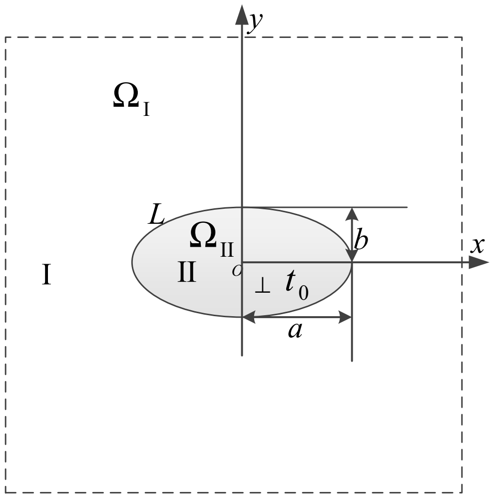

Figure 3. Schematic diagram for a screw dislocation located within an inclusion.

The perturbation technique [42] can be employed to represent the displacement in the matrix as follows:

$$\mathbf{u}_{\mathrm{I}} = \mathbf{C}_1^{\mathrm{I}}[\mathrm{Re}\Psi_{\mathrm{I}}(\zeta)\ \mathrm{Re}\Theta_{\mathrm{I}}(\zeta)\ \mathrm{Re}\Phi_{\mathrm{I}}(\zeta)]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{I}}. \tag{44}$$

In the inclusion, the displacement can be expressed as follows:

$$\mathbf{u}_{\mathrm{II}} = \mathbf{C}_1^{\mathrm{II}}[\mathrm{Re}\Psi_{\mathrm{II}}(\zeta)\ \mathrm{Re}\Theta_{\mathrm{II}}(\zeta)\ \mathrm{Re}\Phi_{\mathrm{II}}(\zeta)]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{II}}. \tag{45}$$

In the matrix, the complex potentials of stress are:

$$\Sigma_{\mathrm{I}} = \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\left[[\mathrm{Im}\Psi_{\mathrm{I}}(\zeta)]_A^B\ [\mathrm{Im}\Theta_{\mathrm{I}}(\zeta)]_A^B\ [\mathrm{Im}\Phi_{\mathrm{I}}(\zeta)]_A^B\right]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{I}}. \tag{46}$$

In the inclusion, the complex potentials of stress are:

$$\Sigma_{\mathrm{II}} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{I}}\left[[\mathrm{Im}\Psi_{\mathrm{II}}(\zeta)]_A^B\ [\mathrm{Im}\Theta_{\mathrm{II}}(\zeta)]_A^B\ [\mathrm{Im}\Phi_{\mathrm{II}}(\zeta)]_A^B\right]^{\mathrm{T}}, \zeta \in \Gamma_{\mathrm{II}}. \tag{47}$$

In area Γ_I, complex potentials $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, and $\Phi_{\mathrm{I}}(\zeta)$ are analytic functions, while, in area Γ_{II}, the analytic functions include $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$.

Substituting Equations (44) and (45) into Equation (19) yields:

$$\mathbf{C}_1^{\mathrm{I}}[\mathrm{Re}\Psi_{\mathrm{I}}(\sigma)\ \mathrm{Re}\Theta_{\mathrm{I}}(\sigma)\ \mathrm{Re}\Phi_{\mathrm{I}}(\sigma)]^{\mathrm{T}} = \mathbf{C}_1^{\mathrm{II}}[\mathrm{Re}\Psi_{\mathrm{II}}(\sigma)\ \mathrm{Re}\Theta_{\mathrm{II}}(\sigma)\ \mathrm{Re}\Phi_{\mathrm{II}}(\sigma)]^{\mathrm{T}}.$$

Substituting Equations (46) and (47) into Equation (20) yields:

$$\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix}\mathrm{Im}\Psi_{\mathrm{I}}(\sigma)\\ \mathrm{Im}\Theta_{\mathrm{I}}(\sigma)\\ \mathrm{Im}\Phi_{\mathrm{I}}(\sigma)\end{bmatrix} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix}\mathrm{Im}\Psi_{\mathrm{II}}(\sigma)\\ \mathrm{Im}\Theta_{\mathrm{II}}(\sigma)\\ \mathrm{Im}\Phi_{\mathrm{II}}(\sigma)\end{bmatrix}.$$

In the matrix, the complex potential can be expanded to the Laurent series as follows:

$$\begin{bmatrix}\Psi_{\mathrm{I}}(\zeta)\\ \Theta_{\mathrm{I}}(\zeta)\\ \Phi_{\mathrm{I}}(\zeta)\end{bmatrix} = \begin{bmatrix}\sum\limits_{k=0}^{\infty}\left(a_{k0}^{\mathrm{I}}\zeta^{k+1} + b_{k0}^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\right)\\ \sum\limits_{k=0}^{\infty}\left(c_{k0}^{\mathrm{I}}\zeta^{k+1} + d_{k0}^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\right)\\ \sum\limits_{k=0}^{\infty}\left(e_{k0}^{\mathrm{I}}\zeta^{k+1} + f_{k0}^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\right)\end{bmatrix} + \begin{bmatrix}\sum\limits_{k=0}^{\infty}b_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\\ \sum\limits_{k=0}^{\infty}d_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\\ \sum\limits_{k=0}^{\infty}f_k^{\mathrm{I}}\frac{1}{\zeta^{k+1}}\end{bmatrix}, \zeta \in \Gamma_{\mathrm{I}}. \tag{48}$$

In the inclusion, the complex potentials are expanded to the Laurent series as follows:

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \\ \Phi_{\mathrm{II}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(a_{k0}^{\mathrm{II}}\zeta^{k+1} + b_{k0}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(c_{k0}^{\mathrm{II}}\zeta^{k+1} + d_{k0}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(e_{k0}^{\mathrm{II}}\zeta^{k+1} + f_{k0}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \end{bmatrix} + \begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(a_{k}^{\mathrm{II}}\zeta^{k+1} + b_{k}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(c_{k}^{\mathrm{II}}\zeta^{k+1} + d_{k}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(e_{k}^{\mathrm{II}}\zeta^{k+1} + f_{k}^{\mathrm{II}}\frac{1}{\zeta^{k+1}}\right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}. \quad (49)$$

The constant terms of the corresponding potential field and rigid displacement are ignored. Coefficients a_{k0}^{I}, b_{k0}^{I}, c_{k0}^{I}, d_{k0}^{I}, e_{k0}^{I}, and f_{k0}^{I} can be represented as follows:

$$a_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_z C_{44}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0} + \frac{p_0 c}{2}R, k=0, \\ -\frac{1}{k+1}\frac{b_z C_{44}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=1,2,\cdots, \end{cases} \quad b_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_z C_{44}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{p_0 c}{2R}, k=0, \\ -\frac{1}{k+1}\frac{b_z C_{44}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=1,2,\cdots \end{cases}$$
$$c_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_\perp K_2^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0} + \frac{g_0 c}{2}R, k=0, \\ -\frac{1}{k+1}\frac{b_\perp K_2^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=1,2,\cdots, \end{cases} \quad d_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_\perp K_2^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{g_0 c}{2R}, k=0, \\ -\frac{1}{k+1}\frac{b_\perp K_2^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=1,2,\cdots, \end{cases} \quad (50)$$
$$e_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_\varphi \varepsilon_{11}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0} + \frac{q_0 c}{2}R, k=0, \\ -\frac{1}{k+1}\frac{b_\varphi \varepsilon_{11}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=1,2,\cdots, \end{cases} \quad f_{k0}^{\mathrm{I}} = \begin{cases} -\frac{b_\varphi \varepsilon_{11}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^2\zeta_0} + \frac{q_0 c}{2R}, k=0, \\ -\frac{1}{k+1}\frac{b_\varphi \varepsilon_{11}^{\mathrm{I}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=1,2,\cdots. \end{cases}$$

Coefficients a_{k0}^{II}, b_{k0}^{II}, c_{k0}^{II}, d_{k0}^{II}, e_{k0}^{II}, and f_{k0}^{II} can be represented as follows:

$$a_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_z C_{44}^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=0,1,2,\cdots, b_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_z C_{44}^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=0,1,2,\cdots,$$
$$c_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_\perp K_2^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=0,1,2,\cdots, d_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_\perp K_2^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=0,1,2,\cdots, \quad (51)$$
$$e_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_\varphi \varepsilon_{11}^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{\zeta_0^{k+1}}, k=0,1,2,\cdots, f_{k0}^{\mathrm{II}} = -\frac{1}{k+1}\frac{b_\varphi \varepsilon_{11}^{\mathrm{II}}}{2\pi \mathrm{i}}\frac{1}{R^{2k+2}\zeta_0^{k+1}}, k=0,1,2,\cdots.$$

Then, complex potentials Ψ_j, Θ_j, and $\Phi_j (j = \mathrm{I}, \mathrm{II})$ need to be determined so that they can meet the conditions of continuity in Equations (19), (20) and (23).

Substituting Equation (49) into Equation (23) yields: $\left[a_{k0}^{\mathrm{II}} + a_k^{\mathrm{II}}\ c_{k0}^{\mathrm{II}} + c_k^{\mathrm{II}}\ e_{k0}^{\mathrm{II}} + e_k^{\mathrm{II}}\right]^{\mathrm{T}} = R^{2(k+1)}\left[b_{k0}^{\mathrm{II}} + b_k^{\mathrm{II}}\ d_{k0}^{\mathrm{II}} + d_k^{\mathrm{II}}\ f_{k0}^{\mathrm{II}} + f_k^{\mathrm{II}}\right]^{\mathrm{T}}$.

Thus, Equation (49) is rephrased as follows:

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \\ \Phi_{\mathrm{II}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(a_{k0}^{\mathrm{II}} + a_k^{\mathrm{II}}\right)\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(c_{k0}^{\mathrm{II}} + c_k^{\mathrm{II}}\right)\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \\ \sum\limits_{k=0}^{\infty}\left(e_{k0}^{\mathrm{II}} + e_k^{\mathrm{II}}\right)\left(\zeta^{k+1} + \frac{1}{R^{2k+2}\zeta^{k+1}}\right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}. \quad (52)$$

According to Equations (48) and (52), there is $\zeta = \sigma = 1/\overline{\sigma}$ on the unit circle Γ_1.

With Equation (19), on the elliptical interface, the conditions of continuity for displacement and potential can be represented as follows:

$$\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} a_{k0}^{\mathrm{I}} + \overline{b}_{k0}^{\mathrm{I}} + \overline{b}_k^{\mathrm{I}} \\ c_{k0}^{\mathrm{I}} + \overline{d}_{k0}^{\mathrm{I}} + \overline{d}_k^{\mathrm{I}} \\ e_{k0}^{\mathrm{I}} + \overline{f}_{k0}^{\mathrm{I}} + \overline{f}_k^{\mathrm{I}} \end{bmatrix} = \mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} a_{k0}^{\mathrm{II}} + a_k^{\mathrm{II}} \\ c_{k0}^{\mathrm{II}} + c_k^{\mathrm{II}} \\ e_{k0}^{\mathrm{II}} + e_k^{\mathrm{II}} \end{bmatrix} + \frac{1}{R^{2k+2}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \overline{a}_{k0}^{\mathrm{II}} + \overline{a}_k^{\mathrm{II}} \\ \overline{c}_{k0}^{\mathrm{II}} + \overline{c}_k^{\mathrm{II}} \\ \overline{e}_{k0}^{\mathrm{II}} + \overline{e}_k^{\mathrm{II}} \end{bmatrix}.$$

According to the theory of analytic function, in area Γ_{I}, $\Psi(\zeta)$, $\Theta(\zeta)$, and $\Phi(\zeta)$ are analytic functions, and, in area Γ_{II}, $\overline{\Psi(1/\overline{\zeta})}$, $\overline{\Theta(1/\overline{\zeta})}$, and $\overline{\Phi(1/\overline{\zeta})}$ serve as analytic functions.

If Γ_0 represents the internal area of the circle Γ_2, then $\Gamma_0 + \Gamma_{\text{II}}$ represents the entire area inside the unit circle. We can obtain a function vector defined in the whole area $[\theta_1(\zeta)\ \theta_2(\zeta)\ \theta_3(\zeta)]^{\text{T}}$:

$$\begin{bmatrix} \theta_1(\zeta) \\ \theta_2(\zeta) \\ \theta_3(\zeta) \end{bmatrix} = \mathbf{C}_1^{\text{I}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \overline{b}_k^{\text{I}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \overline{d}_k^{\text{I}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \overline{f}_k^{\text{I}} \zeta^{k+1} \end{bmatrix} + \mathbf{C}_1^{\text{I}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \left(a_{k0}^{\text{I}} + \overline{b}_{k0}^{\text{I}}\right) \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \left(c_{k0}^{\text{I}} + \overline{d}_{k0}^{\text{I}}\right) \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \left(e_{k0}^{\text{I}} + \overline{f}_{k0}^{\text{I}}\right) \zeta^{k+1} \end{bmatrix} - \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \overline{a}_k^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty} \overline{c}_k^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty} \overline{e}_k^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \end{bmatrix}$$
$$-\mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} a_{k0}^{\text{II}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} c_{k0}^{\text{II}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} e_{k0}^{\text{II}} \zeta^{k+1} \end{bmatrix} - \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \overline{a}_{k0}^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty} \overline{c}_{k0}^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \\ \sum\limits_{k=0}^{\infty} \overline{e}_{k0}^{\text{II}} \frac{\zeta^{k+1}}{R^{2k+2}} \end{bmatrix} - \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} a_k^{\text{II}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} c_k^{\text{II}} \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} e_k^{\text{II}} \zeta^{k+1} \end{bmatrix}, \zeta \in \Gamma_0 + \Gamma_{\text{II}}, \tag{53a}$$

$$\begin{bmatrix} \theta_1(\zeta) \\ \theta_2(\zeta) \\ \theta_3(\zeta) \end{bmatrix} = \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{a_k^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{c_k^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{e_k^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \end{bmatrix} + \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{\overline{a}_k^{\text{II}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{c}_k^{\text{II}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{e}_k^{\text{II}}}{\zeta^{k+1}} \end{bmatrix} - \mathbf{C}_1^{\text{I}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{b_k^{\text{I}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{d_k^{\text{I}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{f_k^{\text{I}}}{\zeta^{k+1}} \end{bmatrix}$$
$$+\mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{a_{k0}^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{c_{k0}^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{e_{k0}^{\text{II}}}{R^{2k+2}\zeta^{k+1}} \end{bmatrix} + \mathbf{C}_1^{\text{II}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{\overline{a}_{k0}^{\text{II}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{c}_{k0}^{\text{II}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{e}_{k0}^{\text{II}}}{\zeta^{k+1}} \end{bmatrix} - \mathbf{C}_1^{\text{I}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{\overline{a}_{k0}^{\text{I}} + b_{k0}^{\text{I}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{c}_{k0}^{\text{I}} + d_{k0}^{\text{I}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{\overline{e}_{k0}^{\text{I}} + f_{k0}^{\text{I}}}{\zeta^{k+1}} \end{bmatrix}, \zeta \in \Gamma_I. \tag{53b}$$

With Equations (20), (48) and (52), on the elliptical interface, the conditions of continuity for the normal components of stress and electric displacement can be ex-pressed as follows:

$$\mathbf{C}_1^{\text{I}} \mathbf{C}_1^{\text{I}} \begin{bmatrix} \overline{b}_{k0}^{\text{I}} + \overline{b}_k^{\text{I}} - a_{k0}^{\text{I}} \\ \overline{d}_{k0}^{\text{I}} + \overline{d}_k^{\text{I}} - c_{k0}^{\text{I}} \\ \overline{f}_{k0}^{\text{I}} + \overline{f}_k^{\text{I}} - e_{k0}^{\text{I}} \end{bmatrix} = \frac{1}{R^{2k+2}} \mathbf{C}^{\text{II}} \mathbf{C}_1^{\text{II}} \begin{bmatrix} \overline{a}_{k0}^{\text{II}} + \overline{a}_k^{\text{II}} \\ \overline{c}_{k0}^{\text{II}} + \overline{c}_k^{\text{II}} \\ \overline{e}_{k0}^{\text{II}} + \overline{e}_k^{\text{II}} \end{bmatrix} - \mathbf{Cl}^{\text{II}} \mathbf{C}_1^{\text{II}} \begin{bmatrix} a_{k0}^{\text{II}} + a_k^{\text{II}} \\ c_{k0}^{\text{II}} + c_k^{\text{II}} \\ e_{k0}^{\text{II}} + e_k^{\text{II}} \end{bmatrix}.$$

Similarly, from the conditions of continuity for stress and electric displacement, a function vector equation $[\theta_4(\zeta)\ \theta_5(\zeta)\ \theta_6(\zeta)]^{\text{T}}$ defined in the whole area can also be obtained as follows:

$$\begin{bmatrix} \theta_4(\zeta) \\ \theta_5(\zeta) \\ \theta_6(\zeta) \end{bmatrix} = \mathbf{C}^{\text{I}} \mathbf{C}_1^{\text{I}} \begin{bmatrix} \sum\limits_{k=0}^{\infty} \left(\overline{b}_{k0}^{\text{I}} + \overline{b}_k^{\text{I}} - a_{k0}^{\text{I}}\right) \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \left(\overline{d}_{k0}^{\text{I}} + \overline{d}_k^{\text{I}} - c_{k0}^{\text{I}}\right) \zeta^{k+1} \\ \sum\limits_{k=0}^{\infty} \left(\overline{f}_{k0}^{\text{I}} + \overline{f}_k^{\text{I}} - e_{k0}^{\text{I}}\right) \zeta^{k+1} \end{bmatrix} \tag{54a}$$

$$
-\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(\frac{1}{R^{2k+2}}\left(\overline{a}_{k0}^{\mathrm{II}}+\overline{a}_{k}^{\mathrm{II}}\right)-\left(a_{k0}^{\mathrm{II}}+a_{k}^{\mathrm{II}}\right)\right)\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\left(\frac{1}{R^{2k+2}}\left(\overline{c}_{k0}^{\mathrm{II}}+\overline{c}_{k}^{\mathrm{II}}\right)-\left(c_{k0}^{\mathrm{II}}+c_{k}^{\mathrm{II}}\right)\right)\zeta^{k+1} \\ \sum\limits_{k=0}^{\infty}\left(\frac{1}{R^{2k+2}}\left(\overline{e}_{k0}^{\mathrm{II}}+\overline{e}_{k}^{\mathrm{II}}\right)-\left(e_{k0}^{\mathrm{II}}+e_{k}^{\mathrm{II}}\right)\right)\zeta^{k+1} \end{bmatrix}, \zeta\in\Gamma_0+\Gamma_{\mathrm{II}},
$$

$$
\begin{bmatrix}\theta_4(\zeta)\\ \theta_5(\zeta)\\ \theta_6(\zeta)\end{bmatrix} = -\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(\overline{a}_{k0}^{\mathrm{I}}-b_{k0}^{\mathrm{I}}-b_{k}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\left(\overline{c}_{k0}^{\mathrm{I}}-d_{k0}^{\mathrm{I}}-d_{k}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\left(\overline{e}_{k0}^{\mathrm{I}}-f_{k0}^{\mathrm{I}}-f_{k}^{\mathrm{I}}\right)\frac{1}{\zeta^{k+1}} \end{bmatrix}
$$

$$
+\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}\begin{bmatrix} \sum\limits_{k=0}^{\infty}\left(\left(\overline{a}_{k0}^{\mathrm{II}}+\overline{a}_{k}^{\mathrm{II}}\right)-\frac{1}{R^{2k+2}}\left(a_{k0}^{\mathrm{II}}+a_{k}^{\mathrm{II}}\right)\right)\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\left(\left(\overline{c}_{k0}^{\mathrm{II}}+\overline{c}_{k}^{\mathrm{II}}\right)-\frac{1}{R^{2k+2}}\left(c_{k0}^{\mathrm{II}}+c_{k}^{\mathrm{II}}\right)\right)\frac{1}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty}\left(\left(\overline{e}_{k0}^{\mathrm{II}}+\overline{e}_{k}^{\mathrm{II}}\right)-\frac{1}{R^{2k+2}}\left(e_{k0}^{\mathrm{II}}+e_{k}^{\mathrm{II}}\right)\right)\frac{1}{\zeta^{k+1}} \end{bmatrix}, \zeta\in\Gamma_I. \tag{54b}
$$

Equations (53a), (53b), (54a) and (54b) are holomorphic and single-valued on the whole plane.

$[\theta_1(\zeta)\ \theta_2(\zeta)\ \theta_3(\zeta)]^{\mathrm{T}}\equiv 0$ can be obtained according to the Liouville theorem. With this result, we can obtain the following from Equations (53a) and (53b):

$$
\begin{bmatrix} b_k^{\mathrm{I}}\\ d_k^{\mathrm{I}}\\ f_k^{\mathrm{I}}\end{bmatrix} = \mathbf{D}_3\begin{bmatrix} a_{k0}^{\mathrm{I}}\\ c_{k0}^{\mathrm{I}}\\ e_{k0}^{\mathrm{I}}\end{bmatrix} + \mathbf{D}_4\begin{bmatrix} \overline{a}_{k0}^{\mathrm{I}}\\ \overline{c}_{k0}^{\mathrm{I}}\\ \overline{e}_{k0}^{\mathrm{I}}\end{bmatrix} - \begin{bmatrix} b_{k0}^{\mathrm{I}}\\ d_{k0}^{\mathrm{I}}\\ f_{k0}^{\mathrm{I}}\end{bmatrix}. \tag{55}
$$

$[\theta_4(\zeta)\ \theta_5(\zeta)\ \theta_6(\zeta)]^{\mathrm{T}}\equiv 0$ can also be obtained according to the Liouville theorem. With this result, we can obtain the following from Equations (54a) and (54b):

$$
\begin{bmatrix} a_k^{\mathrm{II}}\\ c_k^{\mathrm{II}}\\ e_k^{\mathrm{II}}\end{bmatrix} = \mathbf{D}_5\begin{bmatrix} a_{k0}^{\mathrm{I}}\\ c_{k0}^{\mathrm{I}}\\ e_{k0}^{\mathrm{I}}\end{bmatrix} + \mathbf{D}_6\begin{bmatrix} \overline{a}_{k0}^{\mathrm{I}}\\ \overline{c}_{k0}^{\mathrm{I}}\\ \overline{e}_{k0}^{\mathrm{I}}\end{bmatrix} - \begin{bmatrix} a_{k0}^{\mathrm{II}}\\ c_{k0}^{\mathrm{II}}\\ e_{k0}^{\mathrm{II}}\end{bmatrix}, \tag{56}
$$

The next step is to determine the coefficients of the expanded complex series. For the given k, six systems of linear equations with six unknowns a_k^{II}, c_k^{II}, e_k^{II}, b_k^{I}, d_k^{I}, and f_k^{I} can be obtained according to Equations (55) and (56). These unknown coefficients that can be solved are represented by specific coefficients a_{k0}^{I}, b_{k0}^{I}, c_{k0}^{I}, d_{k0}^{I}, e_{k0}^{I}, and f_{k0}^{I}:

$$
\begin{bmatrix} a_k^{\mathrm{II}}\\ c_k^{\mathrm{II}}\\ e_k^{\mathrm{II}}\end{bmatrix} = \begin{bmatrix} I_{k1}a_{k0}^{\mathrm{I}}+J_{k1}\overline{a}_{k0}^{\mathrm{I}}+K_{k1}c_{k0}^{\mathrm{I}}+L_{k1}\overline{c}_{k0}^{\mathrm{I}}+M_{k1}e_{k0}^{\mathrm{I}}+N_{k1}\overline{e}_{k0}^{\mathrm{I}}-a_{k0}^{\mathrm{II}} \\ I_{k3}a_{k0}^{\mathrm{I}}+J_{k3}\overline{a}_{k0}^{\mathrm{I}}+K_{k3}c_{k0}^{\mathrm{I}}+L_{k3}\overline{c}_{k0}^{\mathrm{I}}+M_{k3}e_{k0}^{\mathrm{I}}+N_{k3}\overline{e}_{k0}^{\mathrm{I}}-c_{k0}^{\mathrm{II}} \\ I_{k5}a_{k0}^{\mathrm{I}}+J_{k5}\overline{a}_{k0}^{\mathrm{I}}+K_{k5}c_{k0}^{\mathrm{I}}+L_{k5}\overline{c}_{k0}^{\mathrm{I}}+M_{k5}e_{k0}^{\mathrm{I}}+N_{k5}\overline{e}_{k0}^{\mathrm{I}}-e_{k0}^{\mathrm{II}} \end{bmatrix}, \tag{57}
$$

and:

$$
\begin{bmatrix} b_k^{\mathrm{I}}\\ d_k^{\mathrm{I}}\\ f_k^{\mathrm{I}}\end{bmatrix} = \begin{bmatrix} I_{k2}a_{k0}^{\mathrm{I}}+J_{k2}\overline{a}_{k0}^{\mathrm{I}}+K_{k2}c_{k0}^{\mathrm{I}}+L_{k2}\overline{c}_{k0}^{\mathrm{I}}+M_{k2}e_{k0}^{\mathrm{I}}+N_{k2}\overline{e}_{k0}^{\mathrm{I}}-b_{k0}^{\mathrm{I}} \\ I_{k4}a_{k0}^{\mathrm{I}}+J_{k4}\overline{a}_{k0}^{\mathrm{I}}+K_{k4}c_{k0}^{\mathrm{I}}+L_{k4}\overline{c}_{k0}^{\mathrm{I}}+M_{k4}e_{k0}^{\mathrm{I}}+N_{k4}\overline{e}_{k0}^{\mathrm{I}}-d_{k0}^{\mathrm{I}} \\ I_{k6}a_{k0}^{\mathrm{I}}+J_{k6}\overline{a}_{k0}^{\mathrm{I}}+K_{k6}c_{k0}^{\mathrm{I}}+L_{k6}\overline{c}_{k0}^{\mathrm{I}}+M_{k6}e_{k0}^{\mathrm{I}}+N_{k6}\overline{e}_{k0}^{\mathrm{I}}-f_{k0}^{\mathrm{I}} \end{bmatrix}. \tag{58}
$$

By substituting Equations (50) and (51) into Equations (57) and (58), all coefficients of $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, $\Phi_{\mathrm{I}}(\zeta)$, $\Psi_{\mathrm{II}}(\zeta)$, $\Theta_{\mathrm{II}}(\zeta)$, and $\Phi_{\mathrm{II}}(\zeta)$ can be determined. Then, the problem is solved.

6. Typical Case of Dislocation Located within an Inclusion

In some special cases, the series solutions in Equations (48) and (49) can be provided in a simpler form or their expressions can be obtained by summing. In this section, special cases such as not considering a dislocation, considering an elliptical hole, and not considering the electric field are solved and discussed.

6.1. Elliptical Hole

If an elliptical inclusion reduces into an elliptical hole, then $C_{44}^{\mathrm{II}} = R_3^{\mathrm{II}} = K_2^{\mathrm{II}} = d_{15}^{\mathrm{II}} = e_{15}^{\mathrm{II}} = 0$, $\mathbf{B} = [0\ 0\ 0\ 0\ 0]^T$, and $\varepsilon_{11}^{\mathrm{II}} = \varepsilon_0$.

In the matrix, the following can be obtained:

$$\begin{aligned}
\mathbf{\Lambda}_1^{\mathrm{I}} - \mathrm{i}\mathbf{\Lambda}_2^{\mathrm{I}} &= \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix}\frac{t+\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}} + R^2\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} \overline{p}_0 \\ \overline{g}_0 \\ \overline{q}_0 \end{bmatrix}\frac{t-\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}} \\
&+\frac{\varepsilon_{11}^{\mathrm{I}}}{\varepsilon_0}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} 0 \\ 0 \\ (M_{05}q_0 + N_{05}\overline{q}_0) + (M_{05}\overline{q}_0 + N_{05}q_0)R^2 \end{bmatrix}\frac{t-\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}}, \\
E_x^{\mathrm{I}} - \mathrm{i}E_y^{\mathrm{I}} &= -\frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(q_0\frac{t+\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}} + R^2\overline{q}_0\frac{t-\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}}\right) \\
&-\frac{1}{\varepsilon_0}\left((M_{05}q_0 + N_{05}\overline{q}_0) + (M_{05}\overline{q}_0 + N_{05}q_0)R^2\right)\frac{t-\sqrt{t^2-c^2}}{\sqrt{t^2-c^2}}.
\end{aligned}$$

In the elliptical hole, one has:

$$\mathbf{\Lambda}_1^{\mathrm{II}} - \mathrm{i}\mathbf{\Lambda}_2^{\mathrm{II}} = \mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}[0\ 0\ M_{05}\ q_0 + N_{05}\overline{q}_0]^{\mathrm{T}},$$

$$E_x^{\mathrm{II}} - \mathrm{i}E_y^{\mathrm{II}} = -\frac{1}{\varepsilon_0}(M_{05}q_0 + N_{05}\overline{q}_0).$$

where M_{05} and N_{05} are provided in Appendix C (Equation (A13)).

The phonon field stress and phason field stress in the elliptical hole are equal to zero, and the electric field intensity and electric displacement are uniform.

6.2. Interference between Screw Dislocation and Circular Inclusion

An elliptical inclusion can be reduced to a circular hole, i.e., $a = b$.

In the matrix, the exact solutions for the phonon field stress, phason field stress, electric field intensity, and electric displacement are as follows:

$$\begin{aligned}
\mathbf{\Lambda}_{1\mathrm{I}} - \mathrm{i}\mathbf{\Lambda}_{2\mathrm{I}} &= \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix} + \frac{1}{2\pi\mathrm{i}}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} \\ b_\perp K_2^{\mathrm{I}} \\ b_\varphi \varepsilon_{11}^{\mathrm{I}} \end{bmatrix}\frac{1}{t-t_0} - \mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} J_{02}\overline{p}_0 + L_{02}\overline{g}_0 + N_{02}\overline{q}_0 \\ J_{04}\overline{p}_0 + L_{04}\overline{g}_0 + N_{04}\overline{q}_0 \\ J_{06}\overline{p}_0 + L_{06}\overline{g}_0 + N_{06}\overline{q}_0 \end{bmatrix}\frac{a^2}{t^2} \\
&+\frac{1}{2\pi\mathrm{i}}\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\begin{bmatrix} b_z C_{44}^{\mathrm{I}} J_{02} + b_\perp K_2^{\mathrm{I}} L_{02} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{02} \\ b_z C_{44}^{\mathrm{I}} J_{04} + b_\perp K_2^{\mathrm{I}} L_{04} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{04} \\ b_z C_{44}^{\mathrm{I}} J_{06} + b_\perp K_2^{\mathrm{I}} L_{06} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{06} \end{bmatrix}\frac{a^2}{t\left(\overline{t}_0 t - a^2\right)}, t \in \Omega_{\mathrm{I}}, \\
E_{x\mathrm{I}} - \mathrm{i}E_{y\mathrm{I}} &= -\frac{1}{\varepsilon_{11}^{\mathrm{I}}}q_0 - \frac{b_\varphi}{2\pi\mathrm{i}}\frac{1}{t-t_0} + \frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(J_{06}\overline{p}_0 + L_{06}\overline{g}_0 + N_{06}\overline{q}_0\right)\frac{a^2}{t^2} \\
&-\frac{1}{2\pi\mathrm{i}}\frac{1}{\varepsilon_{11}^{\mathrm{I}}}\left(b_z C_{44}^{\mathrm{I}} J_{06} + b_\perp K_2^{\mathrm{I}} L_{06} + b_\varphi \varepsilon_{11}^{\mathrm{I}} N_{06}\right)\frac{a^2}{t\left(\overline{t}_0 t - a^2\right)}, t \in \Omega_{\mathrm{I}}.
\end{aligned}$$

The exact solutions for each field in the circular inclusion are:

$$\begin{aligned}
\mathbf{\Lambda}_{1\text{II}} - \mathrm{i}\mathbf{\Lambda}_{2\text{II}} &= \frac{1}{2\pi \mathrm{i}} \mathbf{C}^{\text{II}} \mathbf{C}_1^{\text{II}} \begin{bmatrix} b_z C_{44}^{\text{I}} I_{01} + b_\perp K_2^{\text{I}} K_{01} + b_\varphi \varepsilon_{11}^{\text{I}} M_{01} \\ b_z C_{44}^{\text{I}} I_{03} + b_\perp K_2^{\text{I}} K_{03} + b_\varphi \varepsilon_{11}^{\text{I}} M_{03} \\ b_z C_{44}^{\text{I}} I_{05} + b_\perp K_2^{\text{I}} K_{05} + b_\varphi \varepsilon_{11}^{\text{I}} M_{05} \end{bmatrix} \frac{1}{t - t_0} \\
&+ \mathbf{C}^{\text{II}} \mathbf{C}_1^{\text{II}} \begin{bmatrix} I_{01} p_0 + K_{01} g_0 + M_{01} q_0 \\ I_{03} p_0 + K_{03} g_0 + M_{03} q_0 \\ I_{05} p_0 + K_{05} g_0 + M_{05} q_0 \end{bmatrix}, t \in \Omega_{\text{II}}, \\
E_{x\text{II}} - \mathrm{i}E_{y\text{II}} &= -\frac{1}{2\pi \mathrm{i}} \frac{1}{\varepsilon_{11}^{\text{II}}} \left(b_z C_{44}^{\text{I}} I_{05} + b_\perp K_2^{\text{I}} K_{05} + b_\varphi \varepsilon_{11}^{\text{I}} M_{05} \right) \frac{1}{t - t_0} \\
&- \frac{1}{\varepsilon_{11}^{\text{II}}} (I_{05} p_0 + K_{05} g_0 + M_{05} q_0), t \in \Omega_{\text{II}}.
\end{aligned}$$

Clearly, the phonon field stress, phason field stress, and electric field intensity within the inclusion all show typical screw dislocation behavior, i.e., the singularity of $1/(t - t_0)$ at point $t = t_0$.

6.3. Circular Hole

An inclusion can be reduced to a circular hole, i.e., $C_{44}^{\text{II}} = R_3^{\text{II}} = K_2^{\text{II}} = d_{15}^{\text{II}} = e_{15}^{\text{II}} = 0$, $\mathbf{B} = [0\ 0\ 0\ 0\ 0]^T$, and $\varepsilon_{11}^{\text{II}} = \varepsilon_0$. Next, it is necessary to solve the problem of a 1D hexagonal piezoelectric quasicrystal with a circular hole.

In area Ω_{I}, the expressions of complex potentials $\Psi_{\text{I}}(t)$, $\Theta_{\text{I}}(t)$, and $\Phi_{\text{I}}(t)$ can be obtained as follows:

$$\begin{bmatrix} \Psi_{\text{I}}(t) \\ \Theta_{\text{I}}(t) \\ \Phi_{\text{I}}(t) \end{bmatrix} = t \begin{bmatrix} p_0 \\ g_0 \\ q_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ N_{k6} \frac{a^2 \bar{q}_0}{t} \end{bmatrix}, t \in \Omega_{\text{I}}.$$

In area Ω_{II}, the expressions of complex potentials $\Psi_{\text{II}}(t)$, $\Theta_{\text{II}}(t)$, and $\Phi_{\text{II}}(t)$ can be obtained as follows:

$$\begin{bmatrix} \Psi_{\text{II}}(t) \\ \Theta_{\text{II}}(t) \\ \Phi_{\text{II}}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ M_{k5} q_0 t \end{bmatrix}, t \in \Omega_{\text{II}}.$$

where M_{k5} and N_{k6} are provided in Appendix C (Equation (A14)).

It is obvious that, in the circular hole, both the phonon field stress and phason field stress are zero, the electric potential is a linear function of the independent variable t, and the electric field is uniform.

6.4. No Electric Field

Without considering the electric field, the problem reduces to the interference between an elliptical inclusion and a screw dislocation in a 1D hexagonal quasicrystal, and the dislocation is located in the inclusion.

In the matrix, the following can be obtained:

$$\begin{bmatrix} \Psi_{\text{I}}(\zeta) \\ \Theta_{\text{I}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty} \left(a_{k0}^{\text{I}} \zeta^{k+1} + \frac{b_{k0}^{\text{I}}}{\zeta^{k+1}} \right) \\ \sum\limits_{k=0}^{\infty} \left(c_{k0}^{\text{I}} \zeta^{k+1} + \frac{d_{k0}^{\text{I}}}{\zeta^{k+1}} \right) \end{bmatrix} + \begin{bmatrix} \sum\limits_{k=0}^{\infty} \frac{I_{k2} a_{k0}^{\text{I}} + K_{k2} c_{k0}^{\text{I}} + J_{k2} \bar{a}_{k0}^{\text{I}} + L_{k2} \bar{c}_{k0}^{\text{I}} - b_{k0}^{\text{I}}}{\zeta^{k+1}} \\ \sum\limits_{k=0}^{\infty} \frac{I_{k4} a_{k0}^{\text{I}} + K_{k4} c_{k0}^{\text{I}} + J_{k4} \bar{a}_{k0}^{\text{I}} + L_{k4} \bar{c}_{k0}^{\text{I}} - d_{k0}^{\text{I}}}{\zeta^{k+1}} \end{bmatrix}, \zeta \in \Gamma_{\text{I}}.$$

In the inclusion, one has:

$$\begin{bmatrix} \Psi_{\mathrm{II}}(\zeta) \\ \Theta_{\mathrm{II}}(\zeta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{k=0}^{\infty} \left(I_{k1} a_{k0}^{\mathrm{I}} + K_{k1} c_{k0}^{\mathrm{I}} + J_{k1} \overline{a}_{k0}^{\mathrm{I}} + L_{k1} \overline{c}_{k0}^{\mathrm{I}} \right) \left(\zeta^{k+1} + \frac{1}{(R^2\zeta)^{k+1}} \right) \\ \sum\limits_{k=0}^{\infty} \left(I_{k3} a_{k0}^{\mathrm{I}} + K_{k3} c_{k0}^{\mathrm{I}} + J_{k3} \overline{a}_{k0}^{\mathrm{I}} + L_{k3} \overline{c}_{k0}^{\mathrm{I}} \right) \left(\zeta^{k+1} + \frac{1}{(R^2\zeta)^{k+1}} \right) \end{bmatrix}, \zeta \in \Gamma_{\mathrm{II}}.$$

Thus, the problem mentioned above is solved, and the complex potentials $\Psi_{\mathrm{I}}(\zeta)$, $\Theta_{\mathrm{I}}(\zeta)$, $\Psi_{\mathrm{II}}(\zeta)$, and $\Theta_{\mathrm{II}}(\zeta)$ are irrelevant to the coefficients a_{k0}^{II}, b_{k0}^{II}, c_{k0}^{II}, and d_{k0}^{II}.

7. Conclusions

This study uses a complex variable function and the conformal transformation technique to investigate the interference between screw dislocation and elliptical inclusion in 1D hexagonal piezoelectric quasicrystals. The anti-plane elastic equation of a 1D hexagonal piezoelectric quasicrystal is represented in matrix form, which simplifies the expression. The unknown variables are solved by applying matrix operations. For the dislocation in the matrix or inclusion, a general series solution for a corresponding field in the matrix and inclusion is given using the perturbation method. Special cases are addressed and discussed, such as the absence of dislocation, the interference between an elliptical hole, a screw dislocation, and a circular inclusion, the presence of a circular hole, and the absence of an electric field. The results show that the electric potential inside a circular hole is a linear function of the independent variable. Regarding the interference between a screw dislocation and circular inclusion, the phonon field stress, phason field stress, and electric field intensity in the inclusion show typical screw dislocation behavior, i.e., a $1/(t-t_0)$ singularity at point $t = t_0$. The results of this study can reveal the mechanism of the interaction between inclusion and dislocation, providing an important theoretical basis for exploring the strengthening and hardening mechanisms of quasicrystal components.

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Appendix A

$$\mathbf{C}_1 = \begin{bmatrix} \frac{1}{C_{44}} & 0 & 0 \\ 0 & \frac{1}{K_2} & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{11}} \end{bmatrix}, \tag{A1}$$

$$\mathbf{C}_1^{\mathrm{I}} = \begin{bmatrix} \frac{1}{C_{44}^{\mathrm{I}}} & 0 & 0 \\ 0 & \frac{1}{K_2^{\mathrm{I}}} & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{11}^{\mathrm{I}}} \end{bmatrix}, \mathbf{C}_1^{\mathrm{II}} = \begin{bmatrix} \frac{1}{C_{44}^{\mathrm{II}}} & 0 & 0 \\ 0 & \frac{1}{K_2^{\mathrm{II}}} & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{11}^{\mathrm{II}}} \end{bmatrix}. \tag{A2}$$

$$\mathbf{C}^{\mathrm{I}} = \begin{bmatrix} C_{44}^{\mathrm{I}} & R_3^{\mathrm{I}} & e_{15}^{\mathrm{I}} \\ R_3^{\mathrm{I}} & K_2^{\mathrm{I}} & d_{15}^{\mathrm{I}} \\ e_{15}^{\mathrm{I}} & d_{15}^{\mathrm{I}} & -\varepsilon_{11}^{\mathrm{I}} \end{bmatrix}, \mathbf{C}^{\mathrm{II}} = \begin{bmatrix} C_{44}^{\mathrm{II}} & R_3^{\mathrm{II}} & e_{15}^{\mathrm{II}} \\ R_3^{\mathrm{II}} & K_2^{\mathrm{II}} & d_{15}^{\mathrm{II}} \\ e_{15}^{\mathrm{II}} & d_{15}^{\mathrm{II}} & -\varepsilon_{11}^{\mathrm{II}} \end{bmatrix}. \tag{A3}$$

$$\mathbf{C}_2 = \begin{bmatrix} C_{44} & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & \varepsilon_{11} \end{bmatrix}, \mathbf{C}_2^{\mathrm{I}} = \begin{bmatrix} C_{44}^{\mathrm{I}} & 0 & 0 \\ 0 & K_2^{\mathrm{I}} & 0 \\ 0 & 0 & \varepsilon_{11}^{\mathrm{I}} \end{bmatrix}, \mathbf{C}_2^{\mathrm{II}} = \begin{bmatrix} C_{44}^{\mathrm{II}} & 0 & 0 \\ 0 & K_2^{\mathrm{II}} & 0 \\ 0 & 0 & \varepsilon_{11}^{\mathrm{II}} \end{bmatrix}, \tag{A4}$$

$$\mathbf{C}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ e_{15} & d_{15} & -\varepsilon_{11} \end{bmatrix}, \mathbf{C}_3^{\mathrm{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ e_{15}^{\mathrm{I}} & d_{15}^{\mathrm{I}} & -\varepsilon_{11}^{\mathrm{I}} \end{bmatrix}. \tag{A5}$$

$$\mathbf{C}_4 = \begin{bmatrix} C_{44} & R_3 & e_{15} \\ R_3 & K_2 & d_{15} \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{C}_4^{\mathrm{I}} = \begin{bmatrix} C_{44}^{\mathrm{I}} & R_3^{\mathrm{I}} & e_{15}^{\mathrm{I}} \\ R_3^{\mathrm{I}} & K_2^{\mathrm{I}} & d_{15}^{\mathrm{I}} \\ 0 & 0 & -1 \end{bmatrix}. \tag{A6}$$

Appendix B

$$\begin{aligned}
\mathbf{D}_1 &= \tfrac{1}{R^{2k+2}}\left(\left(\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}} - \left(\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}_1^{\mathrm{II}}\right), \\
\mathbf{D}_2 &= \left(\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}_1^{\mathrm{II}} + \left(\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}, \\
\mathbf{D}_3 &= -\tfrac{4}{R^{2k+2}}\left(\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}_1^{\mathrm{II}}\left(\mathbf{D}_1^2 - \mathbf{D}_2^2\right)^{-1}\left(\mathbf{C}^{\mathrm{I}}\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}^{\mathrm{II}}\mathbf{C}_1^{\mathrm{II}}, \\
\mathbf{D}_4 &= -\left(2\left(\mathbf{C}_1^{\mathrm{I}}\right)^{-1}\mathbf{C}_1^{\mathrm{II}}\left(\mathbf{D}_1^2 - \mathbf{D}_2^2\right)^{-1}\left(\tfrac{1}{R^{2k+2}}\mathbf{D}_1 + \mathbf{D}_2\right) + \mathbf{E}\right), \\
\mathbf{D}_5 &= -2\left(\mathbf{D}_1^2 - \mathbf{D}_2^2\right)^{-1}\mathbf{D}_2, \\
\mathbf{D}_6 &= -2\left(\mathbf{D}_1^2 - \mathbf{D}_2^2\right)^{-1}\mathbf{D}_1,
\end{aligned} \tag{A7}$$

Here, **E** is the third-order identity matrix.

$$\begin{aligned}
\mathbf{D}_3 &= \begin{bmatrix} I_{02} & K_{02} & M_{02} \\ I_{04} & K_{04} & M_{04} \\ I_{06} & K_{06} & M_{06} \end{bmatrix}, \mathbf{D}_4 = \begin{bmatrix} J_{02} & L_{02} & N_{02} \\ J_{04} & L_{04} & N_{04} \\ J_{06} & L_{06} & N_{06} \end{bmatrix}, \\
\mathbf{D}_5 &= \begin{bmatrix} I_{01} & K_{01} & M_{01} \\ I_{03} & K_{03} & M_{03} \\ I_{05} & K_{05} & M_{05} \end{bmatrix}, \mathbf{D}_6 = \begin{bmatrix} J_{01} & L_{01} & N_{01} \\ J_{03} & L_{03} & N_{03} \\ J_{05} & L_{05} & N_{05} \end{bmatrix}.
\end{aligned} \tag{A8}$$

$$\begin{aligned}
M_1 &= \left(\varepsilon_0 + \varepsilon_{11}^{\mathrm{I}}\right)\left(R_3^{\mathrm{I}\,2} - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\right) + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - C_{44}^{\mathrm{I}}d_{15}^{\mathrm{I}\;2} - K_2^{\mathrm{I}}e_{15}^{\mathrm{I}\;2}, \\
M_2 &= \left(\varepsilon_0 - \varepsilon_{11}^{\mathrm{I}}\right)\left(R_3^{\mathrm{I}\,2} - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\right) - 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} + C_{44}^{\mathrm{I}}d_{15}^{\mathrm{I}\;2} + K_2^{\mathrm{I}}e_{15}^{\mathrm{I}\;2}, \\
M_3 &= \left(R_3^{\mathrm{I}\,2} - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\right)\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - C_{44}^{\mathrm{I}}d_{15}^{\mathrm{I}\;2} - K_2^{\mathrm{I}}e_{15}^{\mathrm{I}\;2}, \\
M_4 &= R_3^{\mathrm{I}\,2}\varepsilon_0^2 C_{44}^{\mathrm{I}} - C_{44}^{\mathrm{I}\;2}\varepsilon_0^2 K_2^{\mathrm{I}}, \\
M_5 &= R_3^{\mathrm{I}\,2} - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}.
\end{aligned} \tag{A9}$$

$$\begin{aligned}
M_{k5} &= \frac{2\varepsilon_0\left(d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}\right)}{\varepsilon_{11}^{\mathrm{I}}\left(d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0 - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}\right)}, \\
N_{k6} &= \frac{4R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0}{d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0 - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}}.
\end{aligned} \tag{A10}$$

The elastic constant of a 1D hexagonal quasicrystal can be represented by a matrix as follows:

$$\mathbf{C} = \begin{bmatrix} C_{44} & R_3 \\ R_3 & K_2 \end{bmatrix}, \mathbf{C}^{\mathrm{I}} = \begin{bmatrix} C_{44}^{\mathrm{I}} & R_3^{\mathrm{I}} \\ R_3^{\mathrm{I}} & K_2^{\mathrm{I}} \end{bmatrix}, \mathbf{C}^{\mathrm{II}} = \begin{bmatrix} C_{44}^{\mathrm{II}} & R_3^{\mathrm{II}} \\ R_3^{\mathrm{II}} & K_2^{\mathrm{II}} \end{bmatrix}, \tag{A11}$$

$$\mathbf{C}_1 = \begin{bmatrix} \frac{1}{C_{44}} & 0 \\ 0 & \frac{1}{K_2} \end{bmatrix}, \mathbf{C}_1^{\mathrm{I}} = \begin{bmatrix} \frac{1}{C_{44}^{\mathrm{I}}} & 0 \\ 0 & \frac{1}{K_2^{\mathrm{I}}} \end{bmatrix}, \mathbf{C}_1^{\mathrm{II}} = \begin{bmatrix} \frac{1}{C_{44}^{\mathrm{II}}} & 0 \\ 0 & \frac{1}{K_2^{\mathrm{II}}} \end{bmatrix}. \tag{A12}$$

Appendix C

$$M_{05} = \frac{M_1}{M_2}, N_{05} = \frac{N_1}{N_2}, M_2 = N_2, \tag{A13}$$

where:

$$\begin{aligned}
M_1 \; &= -2R^4\varepsilon_0\big(\big({R_3^{\mathrm{I}}}^2 - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\big)\varepsilon_{11}^{\mathrm{I}} - C_{44}^{\mathrm{I}}\big({d_{15}^{\mathrm{I}}}^2 + K_2^{\mathrm{I}}\varepsilon_0\big) + {R_3^{\mathrm{I}}}^2\varepsilon_0 + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big) \\
&*\big(\big({R_3^{\mathrm{I}}}^2 - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\big)\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}} - C_{44}^{\mathrm{I}}{d_{15}^{\mathrm{I}}}^2\big) \\
&*\big({R_3^{\mathrm{I}}}^2\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - \big({d_{15}^{\mathrm{I}}}^2 + \varepsilon_{11}^{\mathrm{I}}K_2^{\mathrm{I}}\big)C_{44}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big)^2, \\
N_1 \; &= -2R^2\varepsilon_0\big(\big(C_{44}^{\mathrm{I}}K_2^{\mathrm{I}} - {R_3^{\mathrm{I}}}^2\big)\varepsilon_{11}^{\mathrm{I}} + C_{44}^{\mathrm{I}}\big({d_{15}^{\mathrm{I}}}^2 - K_2^{\mathrm{I}}\varepsilon_0\big) + {R_3^{\mathrm{I}}}^2\varepsilon_0 - 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} + {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big) \\
&*\big(\big({R_3^{\mathrm{I}}}^2 - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\big)\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}} - C_{44}^{\mathrm{I}}{d_{15}^{\mathrm{I}}}^2\big) \\
&*\big({R_3^{\mathrm{I}}}^2\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - \big({d_{15}^{\mathrm{I}}}^2 + \varepsilon_{11}^{\mathrm{I}}K_2^{\mathrm{I}}\big)C_{44}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big)^2, \\
M_2 \; &= \varepsilon_{11}^{\mathrm{I}}\big(\big({R_3^{\mathrm{I}}}^2 - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\big)\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}} - C_{44}^{\mathrm{I}}{d_{15}^{\mathrm{I}}}^2\big)^2 \\
&*\big(\big({d_{15}^{\mathrm{I}}}^2 + \varepsilon_{11}^{\mathrm{I}}K_2^{\mathrm{I}} - \varepsilon_0K_2^{\mathrm{I}}\big)C_{44}^{\mathrm{I}} + {R_3^{\mathrm{I}}}^2\varepsilon_0 - {R_3^{\mathrm{I}}}^2\varepsilon_{11}^{\mathrm{I}} - 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} + {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big)^2 \\
&-R^4\varepsilon_{11}^{\mathrm{I}}\big(\big({R_3^{\mathrm{I}}}^2 - C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\big)\varepsilon_{11}^{\mathrm{I}} - \big({d_{15}^{\mathrm{I}}}^2 + \varepsilon_0K_2^{\mathrm{I}}\big)C_{44}^{\mathrm{I}} + {R_3^{\mathrm{I}}}^2\varepsilon_0 + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big)^2 \\
&*\big({R_3^{\mathrm{I}}}^2\varepsilon_{11}^{\mathrm{I}} + 2R_3^{\mathrm{I}}d_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}} - \big({d_{15}^{\mathrm{I}}}^2 + \varepsilon_{11}^{\mathrm{I}}K_2^{\mathrm{I}}\big)C_{44}^{\mathrm{I}} - {e_{15}^{\mathrm{I}}}^2K_2^{\mathrm{I}}\big)^2.
\end{aligned}$$

$$\begin{aligned}
M_{k5} &= \frac{2\varepsilon_0\big(d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}\big)}{\varepsilon_{11}^{\mathrm{I}}\big(d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0 - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}\big)}, \\
N_{k6} &= \frac{4R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0}{d_{15}^{\mathrm{I}}d_{15}^{\mathrm{I}}C_{44}^{\mathrm{I}} + C_{44}^{\mathrm{I}}K_2^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_{11}^{\mathrm{I}} - R_3^{\mathrm{I}}R_3^{\mathrm{I}}\varepsilon_0 - 2e_{15}^{\mathrm{I}}R_3^{\mathrm{I}}d_{15}^{\mathrm{I}} + e_{15}^{\mathrm{I}}e_{15}^{\mathrm{I}}K_2^{\mathrm{I}}}.
\end{aligned} \tag{A14}$$

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