



# *Communication* **Empirical Analysis of Stability of An+1BnO3n+1 Ruddlesden–Popper Phases Using Reciprocal** *n***-Values**

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**Abstract:** Layered  $A_{n+1}B_nO_{3n+1}$  ( $n = 1... \infty$ ) Ruddlesden–Popper (RP) phases are a promising system for a variety of applications. Within the RP family, the thermodynamic properties of the phases are essentially additive with variation in the n value, but at present, there are no general approaches that would allow one to evaluate the individual stability of the RP phases and the possibility of their interconversion. The aim of this paper is to present a novel concept for performing a thermodynamic analysis of RP phases using the reciprocal values of the index *n*. We present an empirical equation  $\Delta G_{1/n}$  =  $\Delta G_P$  + B<sub>1</sub>/*n* + B<sub>2</sub>/*n*<sup>2</sup>, where  $\Delta G_{1/n}$  and  $\Delta G_P$  are the molar Gibbs energies of formation of the Ruddlesden–Popper (RP) phase  $(AO)_{1/n}ABO_3$  and the parent  $ABO_3$  perovskite, respectively, and *n* is a stoichiometry index of  $A_{n+1}B_nO_{3n+1}$  RP phase. The correlation was validated using available thermodynamic data for the systems Sr-Ti-O, Ca-Ti-O, Sr-Zr-O, La-Ni-O, and La-Co-O. For all A-B combinations, the equation quantitatively describes the Gibbs energy of RP phase formation. Predicted values for the non-linear approximation lie within the experimental uncertainty in determining  $\Delta G_{1/n}$ . The proposed correlation was used to analyze the relative stability of the RP phases and to determine the feasibility of synthesizing new compounds.

**Keywords:** perovskites; Ruddlesden–Popper phases; Gibbs free energy; thermodynamics

## **1. Introduction**

Compounds with the  $ABO<sub>3</sub>$  composition and perovskite structure, as well as layered perovskite-like  $A_{n+1}B_nO_{3n+1}$  Ruddlesden–Popper (RP) phases, are among the most attractive and interesting mixed oxides due to their outstanding functional properties [\[1\]](#page-6-0). In the last decade, RP phases based on rare earth oxides and transition metals Mn, Fe, and Co have attracted particular interest as functional materials for solid fuel elements [\[2](#page-6-1)[,3\]](#page-6-2), oxygenpermeable materials [\[4\]](#page-6-3) and electrocatalysts as a single-phase [\[5,](#page-6-4)[6\]](#page-6-5) or perovskite-based composites [\[7\]](#page-6-6).

 $A_{n+1}B_nO_{3n+1}$  RP phases are a homologous series of layered substances which originate from perovskite ABO3. The structure of the *n*-th member of the RP family can be visualized as alternating AO layers with a rock salt structure, separated by *n* layers of ABO<sub>3</sub> with a perovskite structure. The stoichiometry of such phases is usually written as  $A_{n+1}B_nO_{3n+1}$ or AO(ABO<sub>3</sub>)<sub>n</sub> ( $n = 1... \infty$ ). Thus, the first member of the series is the compound A<sub>2</sub>BO<sub>4</sub>  $(n = 1)$ , and the last is perovskite ABO<sub>3</sub>  $(n = \infty)$ .

A theoretical study of the electronic structure of  $Sr_{n+1}Ti_nO_{3n+1}$  ( $n = 1-3$ ,  $\infty$ ) shows that with increasing index *n*, the  $A_{n+1}B_nO_{3n+1}$  phases become unstable with respect to dissociation, the predicted limit of phase stability being  $n \approx 3$  [\[8](#page-6-7)[,9\]](#page-6-8). This is in line with experimental published data for Sr-Ti-O [\[10\]](#page-6-9), Sr-Ru-O [\[11,](#page-6-10)[12\]](#page-6-11), Ca-Ti-O [\[13\]](#page-6-12), La-Ni-O [\[14,](#page-6-13)[15\]](#page-6-14) series; compounds with  $n = 1-3$  were fabricated using standard high-temperature methods of ceramic synthesis. Nevertheless, the high  $n$  members of the RP  $Sr_{n+1}Ti_nO_{3n+1}$  family were prepared using molecular beam epitaxy and pulsed laser deposition methods for



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 $n = 4-6$  [\[16\]](#page-6-15),  $n = 1-5$  and 10 [\[17\]](#page-6-16), or an unprecedented  $n = 20$  RP phase, AO·(ATiO<sub>3</sub>)<sub>20</sub>  $(A = Ba_{0.6}Sr_{0.4})$  [\[18\]](#page-6-17).

An analysis of the literature has shown that, at present, very few studies have been published in which the thermodynamic properties of RP phases are systematized, making it possible to quantitatively predict their stability when the value of *n* changes. The thermodynamic properties of  $A_{n+1}B_nO_{3n+1}$  ( $n=1-4$ ,  $\infty$ ) compounds for Ca-Ti-O, Sr-Ti-O, Sr-Zr-O, Sr-Mo-O, and Sr-W-O systems have been analyzed to examine the stabilities of perovskites and RP phases [\[19\]](#page-6-18). The chemical potential diagrams (a log[*a*(A)/*a*(B)] vs. log  $P_{O2}$ ) were constructed using a computer program and the available estimated or experimental thermodynamic data. A thermodynamic analysis was performed on the dissociation reaction of  $A_2MO_4$  compounds into perovskites  $AMO_3$  and alkaline earth oxides (AOs) using empirical correlations between stabilization energy and the tolerance factor. It was shown that the correlation between the tolerance factor and enthalpy of formation among perovskites was found to be very useful to predict an absence of some perovskite phase in certain systems. Thermodynamics of layered perovskites was systematically considered in [\[20\]](#page-6-19). Based on thermodynamic data for perovskites and RP phases of various chemical compositions, the paper demonstrates that thermodynamic layer values are substantially additive, thus permitting prediction of the properties for materials with unknown values. Similar conclusions were drawn in paper [\[21\]](#page-6-20). Based on dissolution enthalpies of  $La<sub>n+1</sub>N<sub>in</sub>O<sub>3n+1</sub>$  complex oxides, the thermochemical characteristics of these oxides from binary oxides were calculated. It was found that there is a linear dependence (1) between the enthalpy of formation of any complex oxide in the RP series and the value of *n*:

$$
\Delta H = 30.9 - 971n \text{ (kJ/mol La)}\tag{1}
$$

These approaches, based on the additivity of properties and their linear correlation with the *n* value, have two significant drawbacks. Firstly, strict additivity implies full reversibility of a transformation. For example, the reversibility of PR phase dissociation, Equation (2), results in a failure to synthesize a pure  $AO(ABO<sub>3</sub>)<sub>n</sub>$  phase. Therefore, it is necessary to take into account deviations from linearity (additivity), the magnitude of which, despite their small values, will most likely determine the stability of the products.

$$
2 \text{ AO}(ABO_3)_n = \text{AO}(ABO_3)_{n-1} + \text{AO}(ABO_3)_{n+1} \tag{2}
$$

Secondly, when analyzing the dependence  $F = f(n)$ , where F is any property and  $n = 1$ ... $\infty$ , the ABO<sub>3</sub> perovskite ( $n = \infty$ ) falls out of consideration.

In this paper, we consider a new approximation to describe the stability of RP phases using reciprocal values of index  $1/n = 1$ ,  $1/2$ ,  $1/3$ ...0. The aim of the study is to formulate general relationships covering a wide range of RP phases and test these relations using published thermodynamic experimental data for different sets of A and B cations that can form layered perovskite-like phases.

### **2. Materials and Methods**

Here, and below, the abbreviation RPn is used for an *n*-th member of the Ruddlesden– Popper family  $(A_{n+1}B_nO_{3n+1}$ , AO·(ABO<sub>3</sub>)<sub>n</sub> or  $(AO)_{1/n}$ ·ABO<sub>3</sub>,  $n = 1... \infty$ ).

Gibbs energies of formation ( $\Delta G_n$ ) were used as a measure of RPn phase stability. The thermodynamic quantities of the RPn-forming systems ( $n = 1-3, \infty$ ) were collected from the appropriate publications (Sr-Ti-O [\[19](#page-6-18)[,22\]](#page-7-0); Ca-Ti-O [\[23\]](#page-7-1); Sr-Zr-O [\[19\]](#page-6-18); La-Ni-O [\[15\]](#page-6-14); La-Co-O [\[24,](#page-7-2)[25\]](#page-7-3)). Values of  $\Delta G_n$  at 298 K were used without any transformation as they appear in the parent papers. High temperature values of  $\Delta G_n$  were calculated for a reference temperature 1200 K using the equations given in the original sources. Values of  $\Delta G_n$ for stoichiometry  $A_{n+1}B_nO_{3n+1}$  were converted to the reciprocal scale  $1/n$  (stoichiometry  $(AO)_{1/n}$ ·ABO<sub>3</sub>,  $1/n = 1$ ,  $1/2$ ,  $1/3$ , 0) using the obvious expression (3):

$$
\Delta G_n(A_{n+1}B_nO_{3n+1}) = n \cdot \Delta G_{1/n}((AO)_{1/n}ABO_3)
$$
\n(3)

#### **3. Results and Discussion**

We will now construct the *n*-th member of the RP family by stacking *n* layers of perovskite on an AO layer. The work to create such a structure can be written as (4), and dividing both sides of the equation by *n*, we obtain (4a).  $v_{\text{max}}$  boundary of the equation by  $n$ , we obtain  $\text{max}$ .

$$
\Delta G_n = n\Delta G_P + \Delta G_{AO} \qquad \text{for AO}(ABO_3)_n \text{ phase} \tag{4}
$$

$$
\Delta G_{1/n} = \Delta G_n / n = \Delta G_P + \Delta G_{AO} / n \quad \text{for (AO)}_{1/n} \text{ABO}_3 \text{ phase} \tag{4a}
$$

where ΔG<sub>n</sub>, ΔG<sub>1/n</sub>—molar Gibbs energies of AO(ABO<sub>3</sub>)<sub>n</sub> and (AO)<sub>1/n</sub>ABO<sub>3</sub> formation, accordingly;

 $\Delta G_P$ ,  $\Delta G_{AO}$ —increments corresponding to the addition of the perovskite or AO layer.

 $\overline{\text{A}}$  and  $\overline{\text{A}}$  do not depend on *n*. This means that the expression (4a) is valid for any *n* and that the values of  $\Delta P$  do not depend on *n*. of ΔG<sub>1/n</sub> (molar Gibbs energy of (AO)<sub>1/n</sub>ABO<sub>3</sub> formation) is a linear function of 1/*n*  $\Delta G_{1/n} = A' + B_1'/n$ . In this equation, A' (y-intercept) is equal to  $\Delta G_P$  (the Gibbs energy of  $\Delta G_1$ ) of ABO<sub>3</sub> perovskite formation), since  $\Delta G_1/n = \Delta G_P$  at  $n = \infty$  and  $B_1$ ' is a slope (empirical fitting parameter). fitting parameter).  $I_{AB}$  and  $I_{AB}$  in this equation,  $A'$  (y-intercept) is equal to  $\Delta V_{AB}$  check for  $\Delta V_{AB}$ 

Using available thermodynamic data from [\[15,](#page-6-14)[19](#page-6-18)[,22](#page-7-0)[–25\]](#page-7-3), the values of ∆G1/n were cal-Using available thermodynamic data from [15,19,22–25], the values of ΔG*1/n* were calculated for different A-B combinations, and the results were plotted against  $1/n$ (Figure [1a](#page-2-0)). It can be seen that the experimental points for all systems satisfactorily correspond to the linear dependence; for all cases, Pearson product–moment correlation coefficients were found to be not less than  $0.998$ . Figure [1b](#page-2-0) shows the approximation of the points for  $(SrO)_{1/n}SrTiO_3$  by a linear function (4a) in more detail; the residual plot for the regression of  $\Delta G_{1/n}$  vs.  $1/n$  is shown in Figure [1b](#page-2-0), inset (red line). A pronounced non-random U-shaped pattern suggests a better fit for a non-linear model (5), where  $\rm B_{1}^{\,\prime}$  and B<sub>2</sub>' are empirical fitting parameters. To assess correspondence of the linear (4a)/non-linear quadratic model (5) to the data, a successive F-test was employed to check whether the  $1/n^2$ term results in a significant improvement. It shows that at a 0.05 significance level, model (5) is more likely to be correct ( $F_{exp}$  = 1035.9 >  $F_{crit}(0.95,1,1)$  = 18.5). It should be pointed out that in the case of  $\rm{(SrO)}_{1/n}\rm{SrTiO}_3$ , predicted values for the non-linear approximation lie within the experimental uncertainty in determining  $\Delta G_{1/n}$  and the same regularities are  $\alpha$  valid for the rest of the A-B cations under consideration (Figure [1a](#page-2-0)). come available thermodynamic dia from  $[10,10,22-20]$ , the values of  $\Delta q_{1/n}$  were c

$$
\Delta G_{1/n} = \Delta G_P + B_1' / n + B_2' / n^2 \tag{5}
$$

<span id="page-2-0"></span>

 $\mathbf{F}^{(n)}$  (a) Molar Gibbs energies of ( $\Lambda$ O)<sup>1</sup>  $\Lambda$ BO<sub>3</sub> formation ( $\Lambda G$ <sub>1</sub>) for various  $\Lambda$ -B combinations **Figure 1. (a)** Molar Gibbs energies of  $(AO)_{1/n}ABO_3$  formation  $(\Delta G_{1/n})$  for various A-B combinations as a function of  $1/n$ ; (**b**) Linear (dashed, red) and quadratic (solid, blue) approximation of  $\Delta G_{1/n}$ as a function of  $1/n$  for  $(SrO)_{1/n}$ SrTiO<sub>3</sub>. Inset: **residual plot** (observed-predicted) vs.  $1/n$  for linear (open, red) and non-linear (closed, blue) regression models. The whiskers show the experimental uncertainties of  $\Delta G_{1/n}$  taken from [\[22\]](#page-7-0).

The existence of dependence (5) gives rise to a number of significant and unanticipated consequences.

- 1. As follows from Figure [1b](#page-2-0) (inset), the accuracy of the approximation of  $\Delta G_{1/n}$  by Equation (4) is higher than the uncertainty of its experimental determination. Thus, Equation (5) can be used to estimate the Gibbs energy of formation of any member of the RP family, which cannot be determined experimentally, and to verify the values obtained experimentally.
- 2. Let us consider the possibility of the disproportionation reaction of an RPn phase into  $RP(n-1)$  and  $RP(n+1)$  phases by Equations (2) and (6):

$$
(AO)1/nABO3 = (n - 1)/(2n) (AO)1/(n-1)ABO3 + (n + 1)/(2n) (AO)1/(n+1)ABO3 (6)
$$

It is easy to show that the change in the Gibbs energy  $\Delta G_{r,n}$  of reaction (6) is reduced to expression (7) if correlation (5) is valid. Given that  $B'_{2} > 0$  (Figure [1b](#page-2-0)), this means that for any  $n > 1$ ,  $\Delta G_{r,n}$  is positive. Therefore, there is no inherent instability of PRn phases which was predicted in theoretical studies for  $n \geq 3$  ([\[8](#page-6-7)[,9\]](#page-6-8)).

$$
\Delta G_{\rm r,n} = \frac{B_2'}{n^2(n^2 - 1)}\tag{7}
$$

3. The absolute value of the change in the Gibbs free energy of reaction (6) of interconversion of the RP phases is small. In particular, in the case of strontium titanates, it does not exceed 281 J/mol at 1200 K. This is exactly what causes the difficulty of synthesizing the RP phases  $A_{n+1}B_nO_{3n+1}$  with a high *n* value. On the other hand, low values of ∆Gr,n make it possible to influence the relative stability of the RP phases and, accordingly, to shift the equilibrium of their interconversion, using a factor that affects the thermodynamics of compounds with different *n* to varying extents. Oxygen non-stoichiometry, for instance, can act as such a factor for systems based on transition metals (Co, Ni, Fe). It is known that double perovskites  $(AA')MO<sub>3</sub>$  and the RP phases  $(A, A'$  = rare and alkaline earth metals,  $M = Co$ , Ni, Fe), are capable of reversibly adding/eliminating oxygen (8) to form non-stoichiometric compounds depending on the temperature and partial pressure of oxygen.

$$
A_{n+1}M_nO_{3n+1} = A_{n+1}M_nO_{3n+1-\delta} + \delta/2 O_2
$$
\n(8)

To date, it has not been possible to find systematic experimental data in the literature on the influence of oxygen non-stoichiometry δ on the thermodynamic quantities (enthalpy, entropy, Gibbs energy) of  $A_{n+1}M_nO_{3n+1}$  compounds with different *n* values. Therefore, it is not possible to quantitatively confirm the assumption made. However, a qualitative illustration of the relationship between non-stoichiometry and stability of the RP phase with a certain value of the index *n* is the observation of the sequential disproportionation of perovskite LaCoO<sub>3</sub> to La<sub>4</sub>Co<sub>3</sub>O<sub>10</sub>, La<sub>2</sub>CoO<sub>4</sub> and CoO at 1100–1400 K with a decrease in the partial pressure of oxygen from  $10^5$  to  $10^{-10}$  Pa [\[25\]](#page-7-3). It has also been suggested that direct decomposition of La<sub>3</sub>Ni<sub>2</sub>O<sub>7−δ</sub> leads to the formation of La<sub>2</sub>NiO<sub>4−δ</sub> and La<sub>4</sub>Ni<sub>3</sub>O<sub>10−δ</sub> under the process of electrochemical oxygen removal at 1400 K [\[26\]](#page-7-4). The formation and development of oxygen and ruthenium vacancies by adjusting the oxygen partial pressure was shown to facilitate a phase transition from  $SFRuO<sub>3</sub>$  to  $Sr<sub>3</sub>Ru<sub>2</sub>O<sub>7</sub>$  [\[27\]](#page-7-5). An additional indication of the validity of this assumption is the reversible interconversion (9) of the type  $ABO<sub>3</sub>+A<sub>2</sub>BO<sub>4</sub> \leftrightarrow A<sub>3</sub>B<sub>2</sub>O<sub>7</sub>$  for the system 0.3  $Gd<sub>2</sub>O<sub>3</sub> - 1.2$  SrO – CoO, occurring at 1243–1473 K with oxygen pressure variations in the range from 2 $\cdot 10^4$  to 1 Pa [\[28\]](#page-7-6).

$$
(Gd_x Sr_{1-x})CoO_{3-\delta p} + (Gd_y Sr_{1-y})_2 CoO_{4-\delta 1} \leftrightarrow (Gd_{0.2} Sr_{0.8})_3 Co_2 O_{7-\delta 2} + (\delta_2 - \delta_1 - \delta_p)/2 O_2
$$
\n(9)

4. A further generalization of the approximation (5) can be obtained by reducing the equation to a dimensionless form (10), where  $B_1 = B_1' / \Delta G_P$  and  $B_2 = B_2' / \Delta G_P$ .

$$
G_{n/P} = (\Delta G_{1/n} / \Delta G_P - 1) = (\Delta G_{1/n} - \Delta G_P) / \Delta G_P = B_1 / n + B_2 / n^2
$$
 (10)

This transformation allows us to describe the energetics of the RP phases in all A-B-O series based on a minimum number of empirical parameters. Figure 2 shows the experimental data transformed according to Equation (10); the lines are fitted experimental values of  $(ΔG<sub>1/n</sub>/ΔG<sub>P</sub> - 1)$  to the reciprocal index *n*. Table 1 shows the esti[ma](#page-4-1)tes of parameter  $B_1$  and  $B_2$  which were obtained using a non-linear least squares estimation procedure. The dimensionless form (10) of Equation (5) represents the most complete generalization of the implicit dependence of the Gibbs energy of formation of a  $A_{n+1}B_nO_{3n+1}$ member of the RP family on the index  $n$ . Transformation of the experimental points using Equation (10) results in a set of closely spaced curves passing through the origin, individual differences associated with the nature of the A and B cations being much less pronounced (Figures [1a](#page-2-0) and 2). Moreover, a comparison of the curves at 298 K and 1200 K for the Sr<sub>n+1</sub>Ti<sub>n</sub>O<sub>3n+1</sub> system (Figure [2;](#page-4-0) ■ and  $\Delta$  points) demonstrates that the influence of temperature is effectively neutralized.

<span id="page-4-0"></span>

**Figure 2.** Comparing fitted (lines, Equation (10)) and experimental values of  $(\Delta G_{1/n}/\Delta G_P - 1)$  as a function of the mating sol in drugs tion of the reciprocal index *n*. function of the reciprocal index *n*.

<span id="page-4-1"></span>**Table 1.** Number of  $(AO)_{1/n}ABO_3$  phases (N) included in calculation (fitting), their indexes (*n*) and estimates of parameter  $B_1$  and  $B_2$ , Equation (10). System **N df** *n* **B**<sub>1</sub> **B**<sub>2</sub>

System	N	df	$\boldsymbol{n}$	B <sub>1</sub>	B <sub>2</sub>
$Sr_{n+1}Ti_nO_{3n+1}$ (1200K)	4	2	$1-3.\infty$	$0.34985 + 0.00046$	$-0.02873 \pm 0.00051$
$Ca_{n+1}Ti_nO_{3n+1}$ (1200K)	3		$2,3,\infty$	$0.17996 + 1 \times 10^{-16}$ *	$-0.15427 \pm 2 \times 10^{-16}$ *
$La_{n+1}Co_nO_{3n+1}$ (1200K)	3		$1,3,\infty$	$0.90318 + 6 \times 10^{-16}$ *	$-0.13991 \pm 7 \times 10^{-16}$ *
$Sr_{n+1}Ti_nO_{3n+1}$ (298K)	4	2	$1-3.\infty$	$0.37907 + 0.00085$	$-0.01143 \pm 0.00097$
$Sr_{n+1}Zr_nO_{3n+1}$ (298K)	4		$1-3.\infty$	$0.34264 + 0.00028$	$-0.0056 \pm 0.0003$
$La_{n+1}Ni_nO_{3n+1}$ (298K)	4		$1-3.\infty$	$0.69749 + 0.0015$	$-0.01505 + 0.00169$

 $*$  extremely small value of standard error results from insufficient number of experimental points (N = 3) and degrees of freedom  $(df = 1)$ .

It is noteworthy that no significant correlation was found between  $B_1$  and  $B_2$  coefficients at p < 0.05 (Pearsonʹs correlation coefficient 0.0736, Spearman rank order correlation cients at *p* < 0.05 (Pearson's correlation coefficient 0.0736, Spearman rank order correlation  $\rho = 0.2619$ ). We can therefore treat the terms  $B_1/n$  and  $B_2/n^2$  as independent and ascribe to them a reasonable, simple physical interpretation. It seems that there are at least three main factors that could potentially influence the stability of the  $AO(ABO<sub>3</sub>)<sub>n</sub>$  phase structure. These factors could be (i) the nature of the parent perovskite  $ABO<sub>3</sub>$ , (ii) the dimensionality of the perovskite block *n*, and (iii) the modulation of the crystal structure by AO layers. Thus,

the term  $G_{n/P} = (\Delta G_{1/n}/\Delta G_P - 1)$  in Equation (10) is related to the thermodynamics of the parent perovskite ABO<sub>3</sub> and corresponds to the relative excess of the molar Gibbs energy of the RP phase  $(AO)_{1/n}ABO_3$  compared to that of  $ABO_3$ . The term  $B_1/n$  reflects the additive properties of the structure and accounts for the progressive decrease in the contribution of the insertion of the AO layer to the Gibbs energy with increasing dimension  $n$  of the perovskite block. The negative non-linear term  $B_2/n^2$  corresponds to the contribution to the stabilization of the structure. This can be attributed to the interaction of the layers of rock salt with each other.

The present paper does not aim to define the exact physical meaning of B coefficients. Instead, it introduces the conception of performing thermodynamic analysis using the reciprocal value of *n*. At this stage, the coefficients  $B_1$  and  $B_2$  are empirical fitting parameters and are determined from experimental data. Further development of Equation (10) may involve a theoretical or semi–empirical evaluation of these coefficients, which will allow the calculation of the Gibbs free energy values. Figure  $3$  clearly shows the obvious correlation between the coefficient  $B_1$  and Goldschmidt's tolerance factor (TFper), which provides evidence that such assessments are possible. This dependence is quite logical, as TFper is an indicator for the stability and distortion of perovskite structures, and, as mentioned above, the coefficient  $B_1$  can be related to the dimensionality of the perovskite block. The relationship of the  $B_2$  coefficient and specific physical and chemical characteristics is more problematic. However, the theoretical calculation of these parameters is beyond the scope of this paper and is the subject of future research.

<span id="page-5-0"></span>

**Figure 3. Figure 3. Parameter plotted against the respective to respect**ive to the respective to  $\frac{1}{2}$ perovskite. The tolerance factor is derived from 12 coordination number radii of A ions [29]. The perovskite. The tolerance factor is derived from 12 coordination number radii of A ions [\[29\]](#page-7-7). The solid line is a guide to the eye. solid line is a guide to the eye. **Figure 3.** Values of fitted  $B_1$  parameter plotted against the respective tolerance factor (TFper) of ABO<sub>3</sub>

#### **4. Conclusions**

The proposed dependence (5) permits for the calculation of the molar Gibbs energies of the formation of the Ruddlesden–Popper phases  $(AO)_{1/n}ABO_3$ , wherein the reciprocal value of the index *n* is employed as a variable. The calculation is based on the molar Gibbs energy of formation of the parent perovskite ABO<sub>3</sub> and incorporates two supplementary empirical parameters. The parent person of the parent person incorporates two supplementary  $\sim$ 

This dependence (5) can be transformed into a dimensionless Equation (10), which, when employed, translates the experimental points into a set of analogous curves that pass through the origin. In this case, the first parameter  $(B_1)$  is responsible for the slope of the curve, and the second  $(B_2)$  determines its curvature. While the exact physical interpretation of  $B_1$  and  $B_2$  parameters is currently unclear, it is reasonable to assume that the first one is associated with the description of the additive changes in Gibbs energy with an increase in the perovskite block dimension, while the second (non–linear) relates to stabilization of the layered structure.

We believe that the approach associated with the description of the thermodynamic properties of RP phases in the space of reciprocal values of the index *n* is promising for the search of the most general regularities, which will yield an extensive set of possibilities for thermodynamic prediction within and across layered perovskite–like RP phases, and possibly can be extended to other layered perovskites (Aurivillius and Dion–Jacobson phases).

**Author Contributions:** Conceptualization, S.V.; methodology, S.V.; writing—original draft preparation, S.V. and V.D.; writing—review and editing, S.V.; formal analysis, V.D.; data curation, V.D. All authors have read and agreed to the published version of the manuscript.

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