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Elastic Wave Propagation of Two-Dimensional Metamaterials Composed of Auxetic Star-Shaped Honeycomb Structures

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Abstract: In this paper, the wave propagation in phononic crystal composed of auxetic star-shaped honeycomb matrix with negative Poisson's ratio is presented. Two types of inclusions with circular and rectangular cross sections are considered and the band structures of the phononic crystals are also obtained by the finite element method. The band structure of the phononic crystal is affected significantly by the auxeticity of the star-shaped honeycomb. Some other interesting findings are also presented, such as the negative refraction and the self-collimation. The present study demonstrates the potential applications of the star-shaped honeycomb in phononic crystals, such as vibration isolation and the elastic waveguide.

Keywords: phononic crystal; auxetic structure; star-shaped honeycomb structure; wave propagation

1. Introduction

Phononic crystals are artificial crystals composed of a periodic alternation of at last two different materials. These materials were first studied by Sigalas and Economou et al. [1] and Kushwaha et al. [2,3], in which one of the most important properties in the crystals, the band gap, was investigated. They found that for some periodic arrangement, the waves could be stopped by phononic crystals. This unique property was found to have potential to be applied in isolating incident waves, acoustics and vibrations in many studies [4–6]. Sigalas [7] studied the elastic wave propagation in phononic crystals by using the plane wave expansion method. They investigated the defect effects inside the crystal system. The analysis of the band gaps suggested that the wave propagation in phononic crystals can be controlled by the defect states. Other research regarding the defect effects on the wave propagation in phononic crystals was also presented in the literature [8,9]. These studies suggested that their design of defect modes can be used as high-Q narrow band pass acoustical filters.

The concept of locally resonant phononic crystal was first proposed by Liu et al. [10]. The materials, based on the simple realization and composited with locally resonant structural units, can exhibit effective negative elastic constants at certain frequency ranges in their study. The model and experiments showed the spectral gaps with lattice constants two orders of magnitude smaller than the relevant sonic wavelength. Their findings suggested that the locally resonant crystal is the key factor to control the wave propagation with large wave length by small-size phononic crystals. Liu [11] and Wang [12] performed systematic studies to analyze the factors that affect the locally resonant phononic crystals. They also found that the band gap can be manipulated by tuning the elastic modulus. The suitable range of the elastic modulus can be obtained through the design of the local structures in the periodic crystal [13–15]. For the locally resonant phononic crystals, the wave length is much larger than the lattice constant. Therefore, the crystals can be modelled as a homogeneous medium with effective elastic modulus and effective Poisson's ratio [16,17].

Some of the above mentioned phononic crystals are the cellular structures, which have properties such as high stiffness and low density. Some cellular structures have unique properties, such as negative Poisson ratio. The material with the negative Poisson ratio is called the auxetics. Gibson et al. [18] investigated that some cellular structures, such as re-entrance honeycomb, have a negative effective Poisson ratio. Then, Almgren [19] proposed a 3D structure composed of honeycomb structure with a Poisson ratio of −1. The negative poisson's ratios in composites with star-shaped inclusions by numerical homogenization approach was presented by Panagiotopoulus et al. [20]. The wave propagations in the auxetic materials were also investigated. Gonella and Ruzzene [21] considered the plane wave propagation in re-entrant and hexagonal honeycomb structures. Two-dimensional dispersion relations were analyzed to reveal peculiar properties of the re-entrant honeycomb structures. Liebold-Ribeiro and Körner [22] performed the eigenmode analysis to various periodic cellular materials. Then, the star-shaped honeycomb with varied Poisson ratio as presented by Meng et al. [23] and the wave propagation behaviors and the band gaps were also analyzed in their studies. Wojciechowski et al. [24] presented some investigations about the auxetics and other systems of anomalous characteristics. The abovementioned papers showed that the auxetic material with negative values of Poisson ratio have significant effects on the wave motion in elastic solid.

Self-collimation is another important phenomenon of wave propagation in phononic crystals. Qiu et al. [25] proposed a design of a highly-directional acoustic source, formed by placing a line acoustic source inside a planar cavity of two-dimensional phononic crystals. The propagation characteristics of elastic transverse waves emitted by line sources embedded inside two-dimensional phononic crystals were studied by Liu and Su [26]. Cicek and his co-workers [27,28] studied numerically the wave propagation characteristics and self-collimation phenomenon of phononic crystals. Besides this, some experimental results have been performed to validate this phenomenon [29–31].

Recently, we published some research regarding wave propagation in phononic crystals [32,33], in which we performed the wave propagation analysis and had some interesting findings, including self-collimation and negative reflection. In this paper, we propose a novel auxetic material composed of star-shaped honeycomb structure. The Poisson's ratio with various negative value of the structure can be obtained by adjusting the star angle θ. The dispersion of the phononic crystal composed of auxetic star-shaped honeycomb will be analyzed by eigenmode analysis. The self-collimation characteristic of the star-shaped honeycomb is also investigated.

2. Materials and Methods

The dynamic equilibrium equations for an isotropic elastic solid are expressed as [34]:

$$(\lambda + \mu)u_{j,ji} + \mu u_{i,jj} = \rho \ddot{u}_i \tag{1}$$

where u_i is the displacement vector component, λ and μ are the Lame constant, and ρ is the mass density, respectively. The Lame constants can be written in terms of Young's modulus E and Poisson's ratio ν, i.e., $\lambda = E\nu/(1+\nu)(1-2\nu)$ and $\mu = E/2(1-\nu)$. The constitutive equation for a linear isotropic elastic material is denoted as:

$$\tau_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{2}$$

in which, τ_{ij}, ε_{ij}, and δ_{ij} are stress, strain tensor, and the unitary tensor, respectively. Then, the strain-displacement relation is as follows:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

The phononic crystal composed of square lattice of steel cylinder in an auxetic matrix is a periodic medium. The displacement field of the periodic structure can be expressed as follows by applying the Bloch theorem.

$$\boldsymbol{u}(\boldsymbol{r}) = \boldsymbol{u}_{\boldsymbol{k}}(\boldsymbol{r})e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} \tag{4}$$

where $\boldsymbol{u}(\boldsymbol{r})$ is the displacement vector, ω is the natural frequency, $\boldsymbol{r}$ is the position vector, $\boldsymbol{k}$ is the wave vector in the first Brillouin zone, and $\boldsymbol{u}_{\boldsymbol{k}}(\boldsymbol{r})$ is the periodic vector function of $\boldsymbol{r}$, respectively. The phononic crystal lattice and $\boldsymbol{u}_{\boldsymbol{k}}$ have the same periodicity. The wave propagation in the periodic structure can be solved by using Equations (1) and (2). In this paper, the solution procedures are done by the finite element method. The boundary conditions of the unit cell is:

$$\boldsymbol{u}(\boldsymbol{r}+\boldsymbol{a}) = e^{i(\boldsymbol{k}\cdot\boldsymbol{a})}\boldsymbol{u}(\boldsymbol{r}) \tag{5}$$

Thus, we can obtain the discrete from of finite element eigenvalue system in the unit cell as following form by applying the boundary conditions:

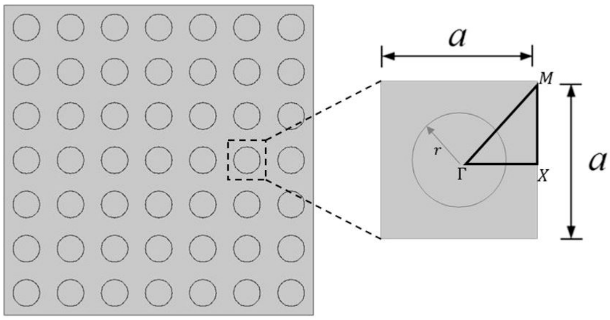
$$\left(\boldsymbol{K} - \omega^2\boldsymbol{M}\right)\boldsymbol{u} = \boldsymbol{0} \tag{6}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are stiffness and mass matrix of the unit cell and $\boldsymbol{a}$ is the lattice constant, respectively.

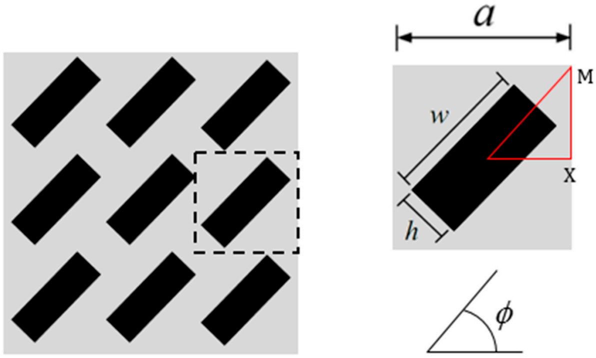
3. Results

In this section, numerical results will be presented to demonstrate the band structure of the phononic crystal composed of auxetic materials. The two types of phononic crystal systems in this paper are shown in Figure 1a,b, respectively. The lattice constant of phononic crystal is $\boldsymbol{a} = \mathbf{0.8\ m}$. The material used for the inclusion is steel with material properties $\boldsymbol{E_s} = \mathbf{200\ GPa}$, $\boldsymbol{\rho_s} = \mathbf{7850\ kg/m^3}$ and $\boldsymbol{\nu_s} = \mathbf{0.33}$. Two shapes of the cross sections of the inclusion, the circular shape with radius $\boldsymbol{r} = \mathbf{0.3}\boldsymbol{a}$ and rectangular shape with width $\boldsymbol{w} = \mathbf{14}\boldsymbol{a}/\mathbf{15}$ and height $\boldsymbol{h} = \boldsymbol{a}/\mathbf{5}$, are considered, as shown in Figure 1a. For rectangular shaped cross section as shown in Figure 1b, the incline angle $\boldsymbol{\phi}$ of the inclusion with rectangular cross section is also considered as a parameter. Therefore, we can realize an auxetic material by using the star-shaped honeycomb structure. The effective Young's modulus and effective Poisson's ratio of the star-shaped honeycomb structure are considered as the material properties of the auxetic matrix.

Figure 2 presents band structures of the elastic wave in the phononic crystals with cylinder inclusions and auxetic matrix, of which three different Poisson's ratios, $\boldsymbol{\nu} = -\mathbf{0.5}$, $-\mathbf{0.7}$, and $-\mathbf{0.9}$, are considered. The numerical results show that as the absolute value of the Poisson's ratio, $|\boldsymbol{\nu}|$, increases, the center frequencies of the band gaps increase. The materials with negative Poisson's ratio are rare in the natural materials, especially for high strength materials. Thus, the re-entrant and star-shaped honeycomb structures as shown in Figure 3 are proved to have negative values of Poisson's ratio [23]. In this study, the shar-shaped honeycomb structure is adopted as the matrix in the phononic crystals. The scheme of the phononic crystal, composed of periodic steel inclusion with circular cross section and the star-shaped honeycomb matrix, is shown in Figure 3a. Figure 3b shows the unit cell of the star-shaped honeycomb, the material properties of which is steel, same as the material used for inclusions. The length of the ligament of the star is **0.05 m**, the width of the beam is one-tenth of the length, and lattice constant of the star shaped unit cell is $\boldsymbol{L} = \mathbf{0.16\ m}$. Then, the effective material properties of the matrix are listed in Table 1 [35] and it can be observed that the Poisson's ratio varies with star angle $\boldsymbol{\theta}$. For star angles smaller than $\mathbf{65}^\circ$, the Poisson's ratio is negative, and the matrix can be considered as an auxetic material.



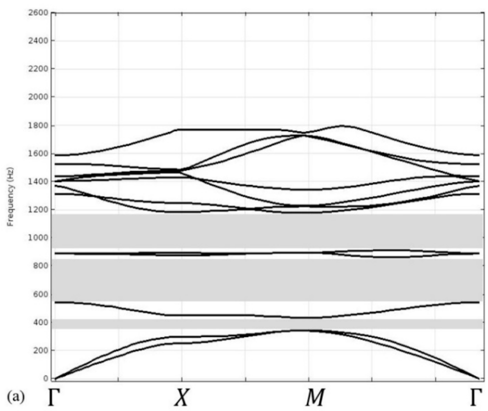
(a)



(b)

Figure 1. The illustration of phononic crystal. Two cross sections of the inclusions are considered. They are (**a**) circular cross section and (**b**) rectangular cross section.

(a)



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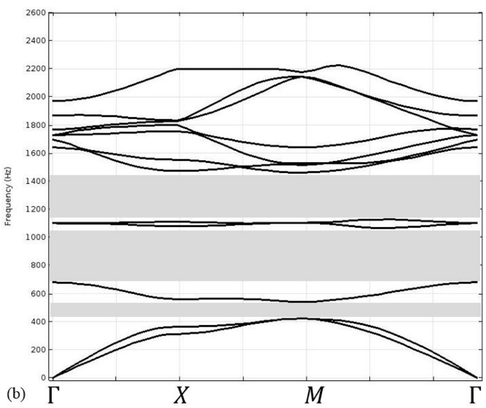


Figure 2. *Cont*.

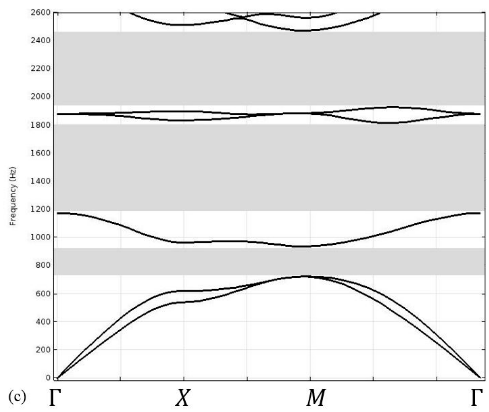


Figure 2. The band structure of the elastic wave in the phononic crystal with auxetic matrix (**a**) $\nu = -0.5$ (**b**) $\nu = -0.7$ (**c**) $\nu = -0.9$.

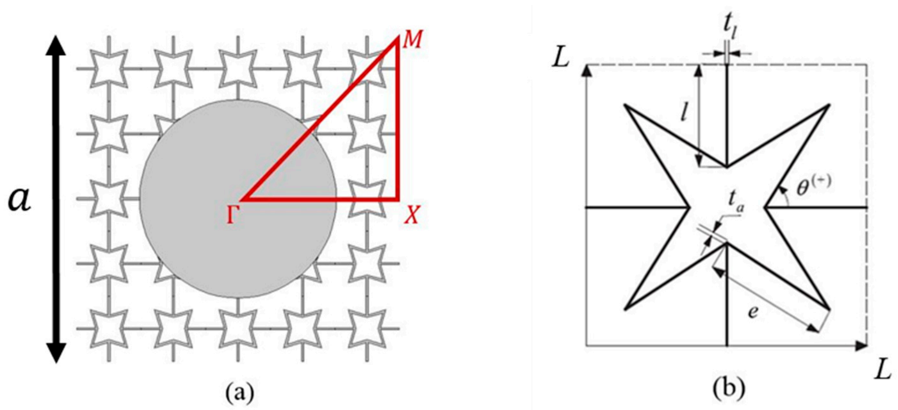


Figure 3. The scheme of the auxetic phononic crystals. (**a**) The unit cell of the phononic crystals composed of star-shaped honeycomb structure and circular shaped inclusion; (**b**) the unit cell structure of the star-shaped honeycomb. The parameter of star-shaped honeycomb is $l = 0.05$ m, $e = 0.05$ m, $t_l = 0.005$ m and $t_a = 0.005$ m. The filling ratio of cylinder are $r = 0.3a$.

Table 1. The effective material properties of steel star-shaped honeycomb, calculated by COMSOL FEM software [35].

STAR ANGLE θ	55°	60°	65°	70°
Young's module E (Gpa)	0.5124	0.3123	0.2305	0.1806
Poisson's ratio ν	−0.53	−0.35	−0.15	0.05
Mass density ρ (kg/m^3)	921	921	921	921

Figure 4 shows the effects of the star angles on the band structures of the phononic crystal composed of steel cylinders in the star-shaped honeycomb matrix. Three different star angles, $\boldsymbol{\theta} = \mathbf{55}°$, **60**°, and **70**° are calculated and the corresponding effective Poisson's ratios are −**0.53**, −**0.35** and −**0.15**, respectively. The band structures of the phononic crystal system are computed and obtained by the finite element software ***COMSOL***$^{(r)}$. Three band gaps are found in the frequency range from **0 Hz** to **1.3 kHz**. The frequency of the band gap increases if the absolute value of Poisson's ratio decreases. Additionally, it is also observed that the widths of the band gap increase with the decrease of Poisson's ratio. According to the figure, it is evident that the band structures strongly depend on the auxeticity of the matrix.

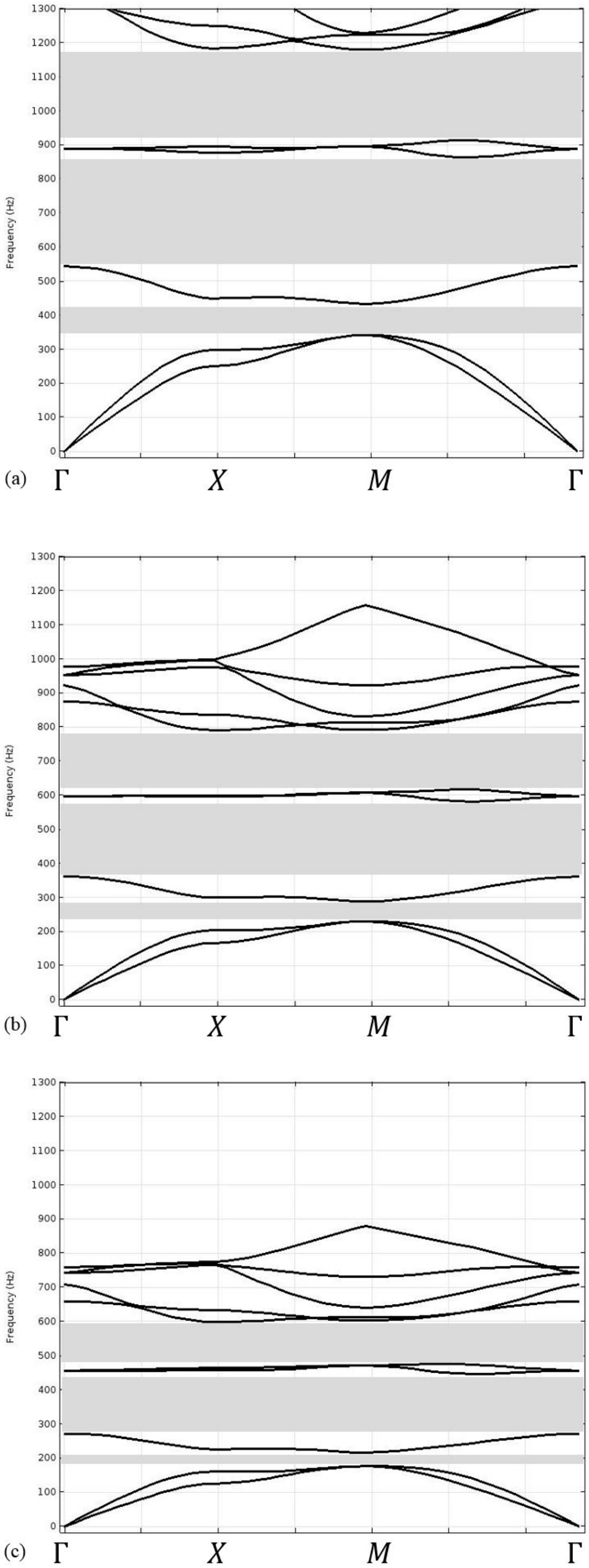


Figure 4. The band structure of phononic crystals composed of steel cylinders and star-shaped honeycombs. (**a**) $\theta = 55^\circ$, $\nu = -0.53$ (**b**) $\theta = 60^\circ$, $\nu = -0.35$ (**c**) $\theta = 65^\circ$, $\nu = -0.15$.

Then, the equi-frequency contour (EFC) of the third band of the star-shaped unit cell for the case of $\boldsymbol{\theta} = \mathbf{55}^\circ$ and $\boldsymbol{\nu} = -\mathbf{0.53}$ is plotted in Figure 5. The dashed black line denotes the construction line and the red arrow indicates the gradient of the contour, which is the direction of the elastic wave propagation in the phononic crystal. It indicates that the negative refraction occurs at this band. The full wave finite element simulation result is shown in Figure 6. A plane elastic wave of **500 Hz** is incident from the left side of the phononic crystal composed of star-shaped honeycomb matrix with star angle $\boldsymbol{\theta} = \mathbf{55}^\circ$. The width of the plane wave and the incident angle are $\mathbf{4a}$ and $\mathbf{30}^\circ$, respectively. The simulation of the phononic crystal system is also computed and obtained by the finite element software ***COMSOL***$^{(r)}$.The negative refraction is shown clearly in Figure 6. The refraction angle is about $-\mathbf{45}^\circ$ and the refraction ratio is about $-\mathbf{0.707}$.

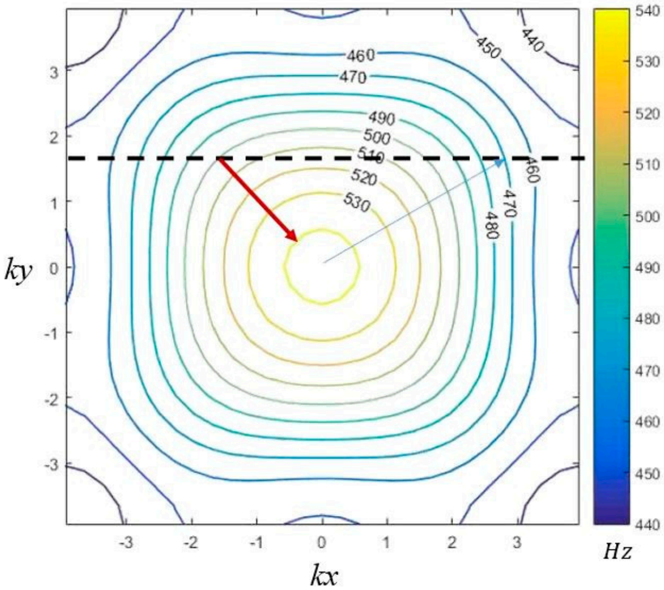


Figure 5. The equi-frequency contour of the third band in Figure 4a. The star angle and the effective Poisson's ratio are $\theta = 55^\circ$ and $\nu = -0.53$, respectively.

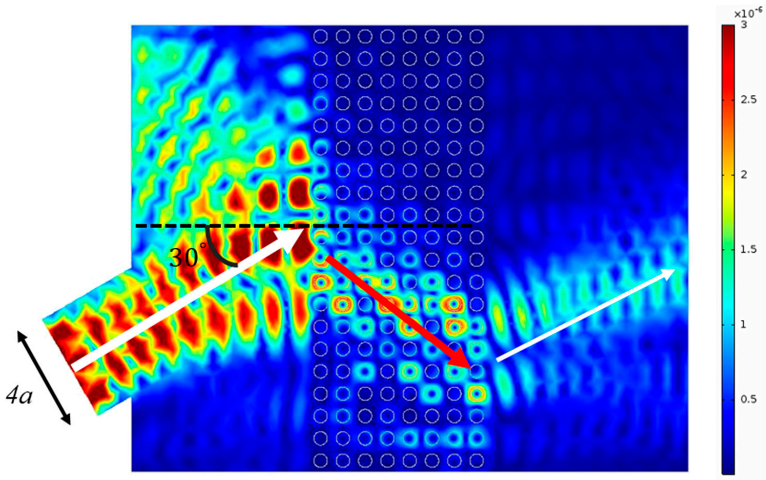
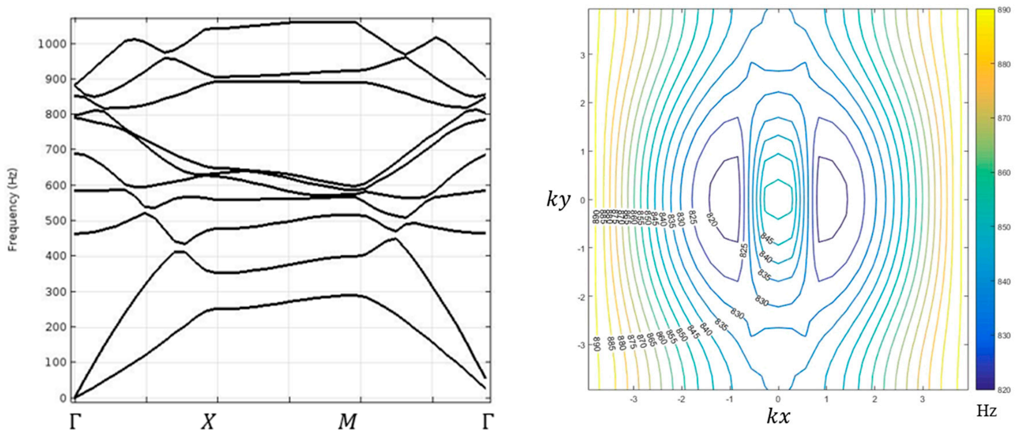


Figure 6. The simulation of wave propagation for negative refraction. A plane elastic wave of 500 Hz is incident from the left side of phononic crystal. The width of the wave and the incident angle are $4a$ and 30°, respectively. The refraction angle is about -45° and the refraction ratio is about -0.707. The material on both sides of the structure is steel.

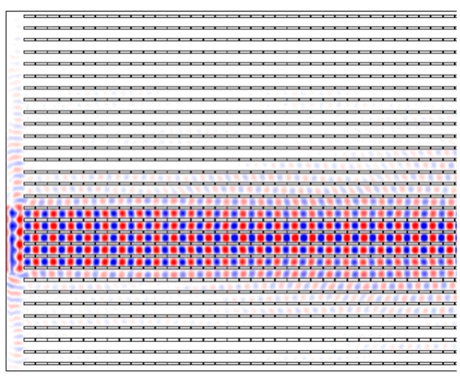
Figure 7 presents the numerical results of wave propagation of the phononic crystal, composed of inclusion with rectangular shape and the star-shaped honeycomb as the matrix. The star angle is $\boldsymbol{\theta} = \mathbf{55}^\circ$, and Poisson's ratio of the matrix is $\boldsymbol{\nu} = -\mathbf{0.53}$, respectively. The geometric parameters of the inclusions are $\boldsymbol{w} = \mathbf{14a/15}$, $\boldsymbol{h} = \boldsymbol{a}/\mathbf{5}$, and $\boldsymbol{\phi} = \mathbf{0}^\circ$. Figure 7a is the band structure, in which the $\mathbf{X} - \mathbf{M}$

region exhibit approximately horizontal lines. The equi-frequency contours are shown in Figure 7b by taking the 8th band as an example. The frequency range of the 8th band is from **820 Hz** to **900 Hz**. In the case that the eigen-frequency is greater than **870 Hz**, the equi-frequency contours are more like the vertical straight lines. It suggests that the self-collimation phenomenon occurs with the wide range of the wave incident into the phononic crystal. After that, the simulation of the wave propagation for three different orientation angles of the rectangular inclusions is presented in Figure 8 and the finite element model is a $\mathbf{50} \times \mathbf{40}$ array of lattices. The incident wave with **885 Hz** is from the left boundary and the incident angle is fixed horizontally for all cases. It is obvious that the wave propagation direction aligns the orientation angle $\boldsymbol{\phi}$. In addition, Figure 9 shows that the simulation of the wave propagation for the model consists of gradually varied orientation angles. The orientation angle is $\boldsymbol{\phi} = \mathbf{0}^\circ$ at the left boundary, $\boldsymbol{x} = \mathbf{0}$, and linearly increases to $\boldsymbol{\phi} = \mathbf{50}^\circ$ at the right boundary, $\boldsymbol{x} = \mathbf{100}\boldsymbol{a}$. The simulation results reveal that when the wave is horizontally incident from the left boundary to the phononic crystal, the wave propagation direction is automatically collimated along the orientation angles of the rectangular inclusions, resulting in a curved path of the wave propagation. The numerical results shown in Figures 7 and 8 validate the self-collimation phenomenon of the phononic crystal with star-shaped honeycomb matrix and rectangular inclusions.



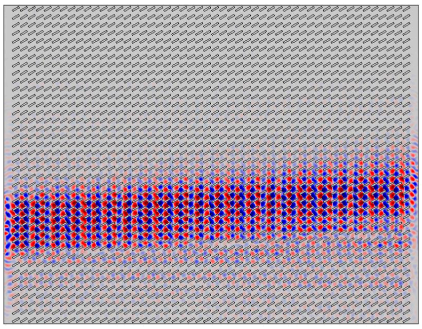
(a) (b)

Figure 7. The numerical results of the wave propagation of the phononic crystals composed of inclusions with rectangular cross section and star-shaped honeycomb matrix. (**a**) the band structures; (**b**) the equi-frequency contour of the 8th band. The star angle is $\theta = 55^\circ$, Poisson's ratio of the matrix is $\nu = -0.53$. The geometric parameters of the inclusions are $w = 14a/15$, $h = a/5$ and $\phi = 0^\circ$.



(a)

Figure 8. *Cont.*



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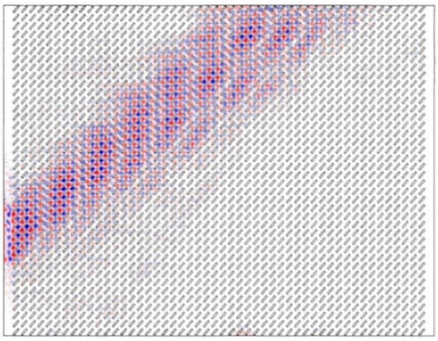

(c)

Figure 8. The displacement field simulations of the wave propagation in phononic crystals composed of star-shaped honeycomb and rectangular inclusions for (**a**) $\phi = 0°$, (**b**) $\phi = 25°$ and (**c**) $\phi = 60°$. The star angle of the matrix is $\theta = 55°$ and the dimensions of the rectangular inclusions are $w = 14/15a$, $h = a/5$.

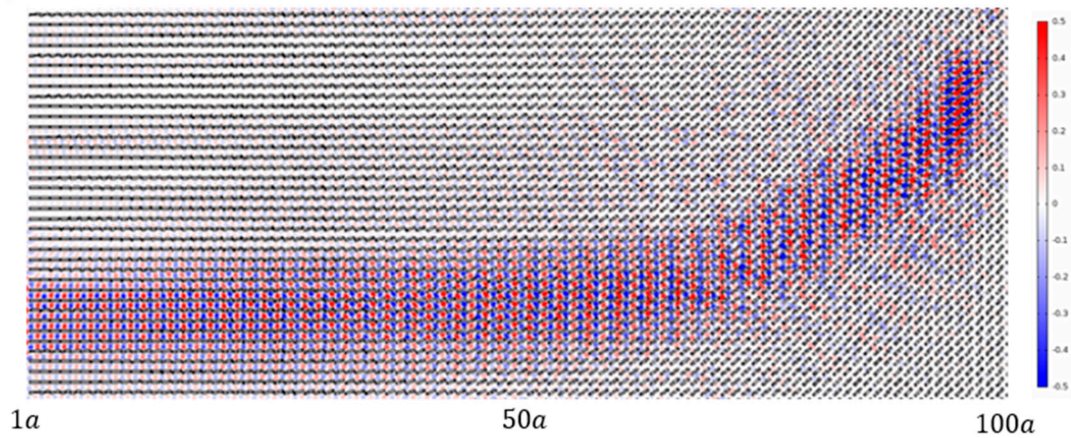

Figure 9. The displacement field simulation of the wave propagation in the phononic crystal composed of star-shaped honeycomb matrix and rectangular inclusions with gradually varied orientations in the x-direction. The orientation angle is $\phi = 0°$ at the left boundary, $x = 0$, and linearly increases to $\phi = 50°$ at the right boundary, $x = 100a$. The orientation angles ϕ are linearly varied by $\phi(x) = (x/100a)50°$.

4. Discussion and Conclusions

In this paper, the elastic wave propagations in phononic crystals composed of steel inclusions and auxetic material matrix are investigated. The star-shaped honeycomb structures are utilized and have been proven to have negative Poisson's ratio. Two types of inclusions with circular and rectangular cross sections are considered and discussed. The band structures of the phononic crystals are calculated and obtained by the finite element method.

The auxeticity of the star-shaped honeycomb is found to have significant effect on the band structure of the phononic crystal. The bandwidths and the mid-frequencies of the band gap increase as the Poisson's ratio decreases. In addition to this, the negative refraction is also found in the phononic crystals for the star-shaped honeycomb as the matrix. The self-collimation phenomenon is also studied. The wide angle and wide band collimation are observed in the phononic crystal with rectangular inclusions. The present study demonstrates the potential applications of the star-shaped honeycomb in phononic crystals, such as vibration isolation and the elastic waveguide.

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Conflicts of Interest: The authors declare no conflict of interest.

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