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Mechanism Design with Singularity Avoidance of Crystal-Inspired Deployable Structures

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Abstract: Although deployable structures have important applications in various fields, developing a new form of structural configuration faces some scientific challenges. Furthermore, kinematic singularity frequently exists in these structures, which has a negative impact on deployment performance and stiffness. To deal with these problems, this paper obtains inspiration from crystals on two-dimensional (2D) space, and aims at developing symmetric deployable structures assembled by identical link members and periodic units. Mobility and compatibility conditions of crystal-inspired deployable structures are given, and a detailed design for novel joints with bevels gears is proposed to avoid singularity of these symmetric structures. According to feasible solutions to the compatibility conditions, several types of deployable structures are developed and verified to be mobile with a single degree of freedom. The results show that the proposed joint with bevel gears has a satisfactory singularity avoidance capability, and the assembled structures exhibit a good deployment performance. Because a crystal-inspired deployable structure can be gradually deployed to cover a large area, it has a potential engineering application as a macroscopic or mesoscale structure.

Keywords: crystal; deployable structure; folding; singularity; internal mechanism; symmetry

1. Introduction

Deployable structures have certain modes of internal mechanism, and they are capable of transforming from compactly stowed states into deployed states [1–3]. Thus, they have been widely studied and obtained many engineering applications. For instance, deployable structures can be adopted for retractable roof structures [4], reconfigurable mechanisms and robotics [5], foldable solar and masts [6], and self-deployable stents [7]. It is important to develop innovative deployable structures, which faces a few scientific challenges. During mechanism design, the link member connected by revolute joints at its ends is always utilized as basic units for assembling a large scale deployable structure [8–11]. However, these mechanisms have lesser mobility and redundant constraints, which will result in a sudden change of structural configuration and kinematic singularity.

In fact, kinematic singularity frequently exists in deployable structures, especially when the adjacent links become coplanar. Admittedly, singularity has a critical effect on accuracy, deployment performance, and structural stiffness. To overcome these difficulties, some researchers have investigated singularity of deployable structures. Kumar and Pellegrino [12] introduced the singular value decomposition technique to study the motion path and singularity of two-dimensional (2D) pin-jointed mechanisms. Recent studies [13,14] have pointed out that a deployable structure has new mechanism modes at the singular points along the motion path, which potentially leads the structure transforms into a specific bifurcation path. Lee and Park [15] proposed a double parallel mechanism, which can reduce the interference between links and avoid singularity through constraining the motion. Bandyopadhyay and Ghosal [16] proposed a method for avoiding singularity by recreating a non-singular path near the singular point and maintaining external forces on certain joints. Wei et al. [17] and Ding et al. [2] proposed different types of polyhedral linkages, and proposed different types of deployable structures and further explored kinematic mobility and bifurcation behavior of these symmetric deployable structures. Recently, based on group representation theory, Chen et al. [18] utilized symmetry to investigate singularity of deployable structures, and extracted new mechanisms with lower-order symmetries. To identify the feasibility of the bifurcation paths, they improved the prediction–correction algorithm to follow the structures transforming into expected bifurcation paths. Nevertheless, limited literature discussed how to avoid the negative influence of the kinematic singularity. Importantly, deployable structures for engineering applications should be reasonably designed to exhibit regularity and symmetry. Their link members and joints should be easy to be fabricated and assembled.

On the other hand, crystals (such as snowflakes, diamonds, and table salt) are arranged in highly ordered microscopic structures in 2D or 3D space [19]. They have a long-range translational order, characterized by a periodic spacing of unit cells [20], and exhibit periodic symmetry. Thus, crystals can provide important inspiration for developing the connectivity patterns of the members of innovative deployable structures. Accordingly, the atoms or molecules of the crystal structures can represent connecting joints of deployable structures. Inspired from crystals on 2D space, this study aims at dealing with developing large-scale and symmetric deployable structures assembled by identical link members, and proposing an effective approach for avoiding kinematic bifurcation by replacing the revolute joints that introduce singularity with novel joints. Different from the conventional approach, which generally relies on redundant actuation, the novel joints with bevel gears are simple and the applications of crystal-inspired deployable structures are promising.

2. Materials and Methods

On the basis of the mobility rule proposed by Hunt [21] and extended by Guest and Fowler [22], the generalized mobility criterion of an over-constrained structure starting in T-dimensional space is

$$m = (T + R) \cdot (n - 1) - (T + R)g + \sum_{i=1}^{g} f_i \quad (1)$$

where m is the relative mobility of the structure and $m > 0$ for a deployable structure, n is the number of members, g is the number of joints, and f_i denotes the number of the relative freedom permitted by a joint $i \in [1, g]$. Notably, in Equation (1), T modes of rigid-body translation and R modes of rigid-body rotations have been excluded, because the structure is generally freestanding.

When a structure is symmetric [18,23], involved mobility analysis can be significantly simplified using group theory, and fruitful insights can be obtained from certain symmetry representations [3,24]. Importantly, a necessary condition for guaranteeing the mobility of a symmetric deployable structure is that the structure must retain internal mechanism mode with full symmetry. That is,

$$\Gamma_m \supset \Gamma^{(1)} \quad (2)$$

where Γ_m denotes the symmetry representation of the relative mobility [18,24], $\Gamma^{(1)}$ indicates full symmetry in a symmetry group, and $\Gamma^{(1)}$ can be directly read from group theory tables [25].

Inspired by crystals on 2D space [20], deployable structures should be neatly designed to exhibit regularity and periodic symmetry, which are beneficial to the involved fabrication, assembly, and cost. Here, inspired by different types of Bravais lattices in two-dimensional space [20,25], crystal-inspired deployable structures assembled by the link members with identical lengths and less than three types of connecting joints are concerned. Thereafter, a general link member of the structure is connected to two specific joints, which are respectively designed for connecting n_1 links and n_2 links. For example, Figure 1 shows two illustrative unit cells of deployable structures assembled by different types of joints.

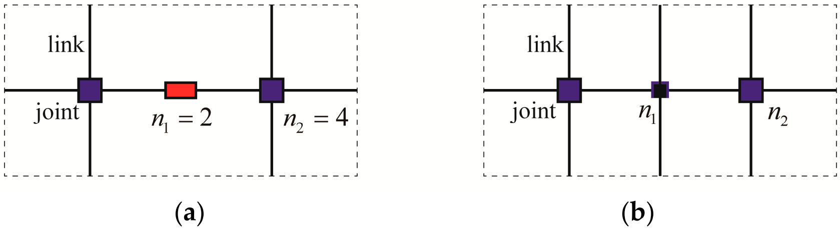


Figure 1. Unit cells of deployable structures assembled by different types of joints, which are designed to respectively connect to n_1 and n_2 links: (**a**) $n_1 = 2$, and $n_2 = 4$; (**b**) $n_1 = n_2 = 4$.

Based on the compatibility conditions of the members in the fully folded and deployed states, it satisfies

$$\frac{(n_2 - 2)\pi}{n_2} \leq \frac{2\pi}{n_1} \tag{3}$$

where the right term in Equation (3) describes the angle between the adjacent members connected by the joints with n_1 links, and the left term in Equation (3) denotes the inner angle of an n_2-sided regular polygon. Then, Equation (3) can be rewritten as

$$0.5n_1n_2 \leq n_1 + n_2 \tag{4}$$

On the other hand, to maintain full symmetry and overcome the singularity of a deployable structure, all the n_1 or n_2 links connected to the same joint should be synchronously folded and deployed during transformations. In other words, the rotation angles of adjacent links are similar, and, thus, the compatibility equation for the joint can be established. For example, as far as the joint connected by n_1 or $n_2 = 4$ links is concerned, the corresponding geometric constraint equation is given by

$$\boldsymbol{J} \cdot \boldsymbol{d} = \begin{bmatrix} \boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{X}_0^{\mathrm{T}} & -\boldsymbol{X}_0^{\mathrm{T}} & 0 & 0 \\ \boldsymbol{X}_{23}^{\mathrm{T}} & 0 & \boldsymbol{X}_0^{\mathrm{T}} & -\boldsymbol{X}_0^{\mathrm{T}} & 0 \\ \boldsymbol{X}_{34}^{\mathrm{T}} & 0 & 0 & \boldsymbol{X}_0^{\mathrm{T}} & -\boldsymbol{X}_0^{\mathrm{T}} \\ \boldsymbol{X}_{41}^{\mathrm{T}} & -\boldsymbol{X}_0^{\mathrm{T}} & 0 & 0 & \boldsymbol{X}_0^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} d\boldsymbol{X}_0 \\ d\boldsymbol{X}_1 \\ d\boldsymbol{X}_2 \\ d\boldsymbol{X}_3 \\ d\boldsymbol{X}_4 \end{bmatrix} = 0 \tag{5}$$

where $\boldsymbol{X}_i$ and $d\boldsymbol{X}_i$ are the nodal vector and displacement increment of the node $i \in [0, 4]$, node 0 is the intersected joint, and nodes 1–4 are the connected joints. In Equation (5), the vector $\boldsymbol{X}_{12}^{\mathrm{T}} = \boldsymbol{X}_2 - \boldsymbol{X}_1$, and the other items can be explained in a similar way.

3. Results

Because of the symmetry requirements on each connecting joint and the integers $n_1 \geq 2$ and $n_2 \geq 3$, limited feasible solutions to the compatibility conditions given by Equations (3) and (4) were obtained and shown in Figure 2. That is:

$$\begin{cases} n_2 = 3, & n_1 = 6,\ 3,\ 2 \\ n_2 = 4, & n_1 = 4,\ 2 \\ n_2 = 6, & n_1 = 3 \end{cases} \tag{6}$$

Figure 2 shows that limited feasible solutions exist for the compatibility conditions. Through these solutions, a few crystal-inspired deployable structures with different configurations and symmetry can be obtained. For instance, when $n_1 = n_2 = 3$ or $n_1 = n_2 = 4$, only one type of connecting joint is adopted for the structures. Otherwise, two different types of connecting joints are needed for assembling the desired deployable structure. Note that the case $n_1 = 6$, $n_2 = 3$ and the case $n_1 = 3$, $n_2 = 6$ reveal the same type of deployable structures.

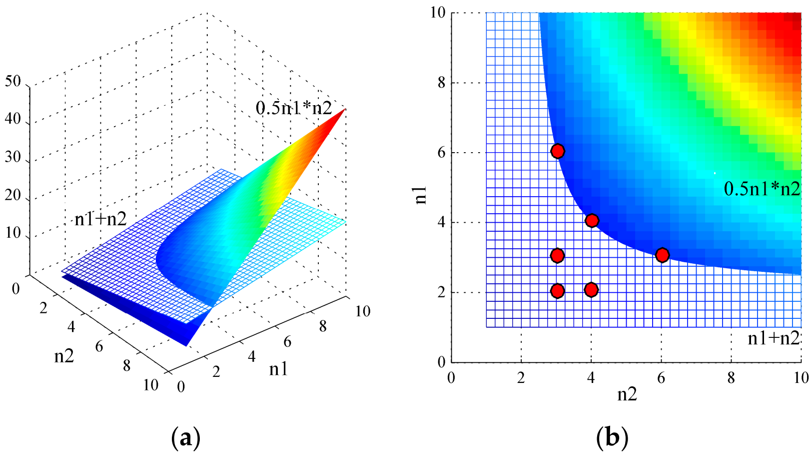


Figure 2. Feasible solutions to the compatibility conditions of the members given by Equations (3) and (4): (**a**) Three-dimensional (3D) view; (**b**) typical solutions marked by the dots.

3.1. Feasible Deployable Structures with Different Types of Joints and Configurations

3.1.1. Case I: $n_1 = n_2 = 4$

To verify the feasibility of the obtained solutions, four links connected by a common joint is taken as a basic unit, whereas $n_1 = n_2 = 4$. Then, according to the periodic symmetry, a simple deployable structure with 2 × 2 basic units can be assembled by repeating and combining two basic units along both directions of a 2D space. This structure holds four-fold symmetry. Mobility analysis shows that this structure is deployable with one degree of freedom ($m = 1$). Typical configurations during deployment of the structure are shown in Figure 3. It verifies that the structure keeps its original symmetry during transformations.

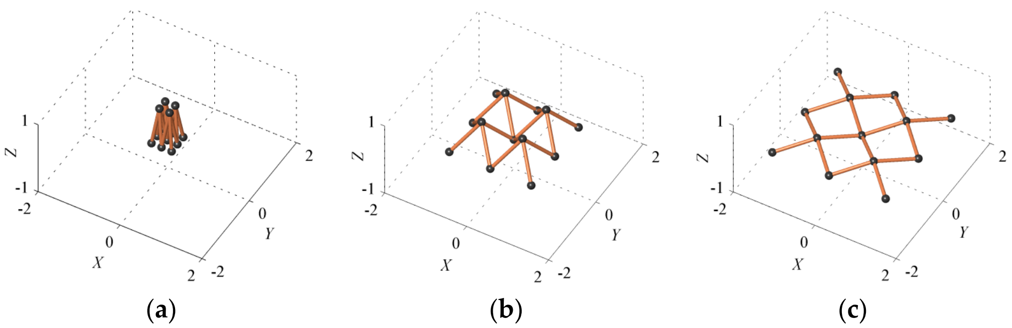
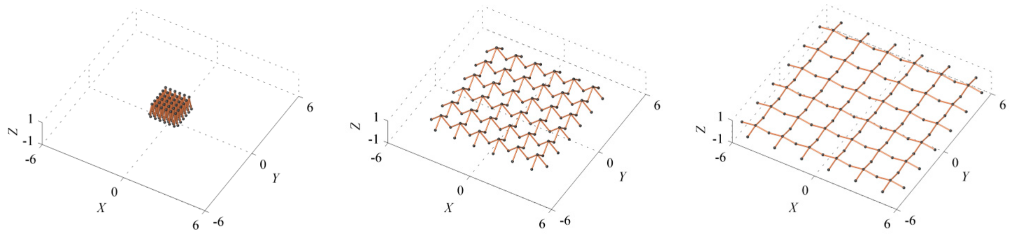


Figure 3. Motion of a deployable structure assembled by 2 × 2 basic units, with $n_1 = n_2 = 4$: (**a**) Folded configuration; (**b**) partially deployed configuration; (**c**) deployed configuration.

3.1.2. Case II: $n_1 = 2$ and $n_2 = 4$

Note that a joint will be connected to only two links when $n_1 = 2$. Then, this type of joint is equivalent to the traditional revolute joint for deployable structures, where both of the two connected links rotate in the same plane.

For the case with $n_1 = 2$ and $n_2 = 4$, the basic unit consists of four straight links, an intersecting joint for connecting four links, and four joints for connecting two adjacent links. For example, Figure 4 shows a crystal-inspired deployable structure assembled by 6 × 6 basic units. It keeps four-fold symmetry, and shows smooth transformations along the motion path. With the mobility $m = 1$, this structure exhibits a satisfactory folding ratio. This can be observed from Figure 4 that the structure is compactly folded and then deployed into a much larger-scale structure.

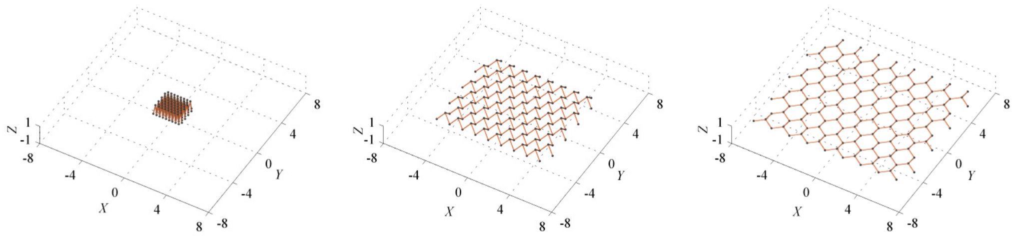


(**a**) (**b**) (**c**)

Figure 4. Motion of a deployable structure assembled by 6 × 6 basic units, with n_1 = 2 and n_2 = 4: (**a**) Folded configuration; (**b**) partially deployed configuration; (**c**) deployed configuration.

3.1.3. Case III: $n_1 = n_2 = 3$

Another basic unit is formed by connecting three links to a common joint, on the condition that $n_1 = n_2 = 3$. This type of structures shows three-fold symmetry, where each joint is connected to three links. For example, a symmetric deployable structure is shown in Figure 5, which is assembled by 8 × 8 basic units. This structure exhibits strong regularity, and can be smoothly deployed from a compacted state. Kinematic analysis indicates that such a type of deployable structures has a single mode of finite mechanism ($m = 1$), and thus it is a feasible deployable structure.

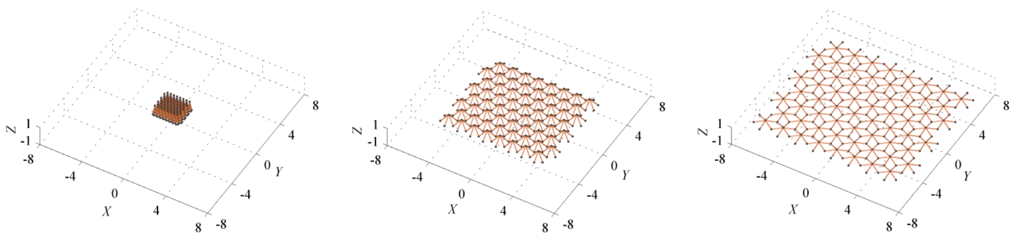


(**a**) (**b**) (**c**)

Figure 5. Motion of a deployable structure assembled by 8 × 8 basic units, with $n_1 = n_2 = 3$: (**a**) Folded configuration; (**b**) partially deployed configuration; (**c**) deployed configuration.

3.1.4. Case IV: $n_1 = 3$ and $n_2 = 6$

For the fourth case with $n_1 = 3$ and $n_2 = 6$, the structural configuration is complex and composed of many more link members. The corresponding basic unit is formed by six links, which are intersected at the same joint, where the other end of each link is connected to two different links. For instance, Figure 6 shows typical configurations of a deployable structure with $n_1 = 3$ and $n_2 = 6$, which is also assembled by 8 × 8 basic units.



(**a**) (**b**) (**c**)

Figure 6. Motion of a deployable structure assembled by 8 × 8 basic units, with n_1 = 3 and n_2 = 6: (**a**) Folded configuration; (**b**) partially deployed configuration; (**c**) deployed configuration.

As expected, this structure exhibits three-fold symmetry and strong regularity. Notably, Figure 6 shows that this symmetric deployable structure can be smoothly transformed from the folded state to the fully deployed state, and the motion process is reversible. and can be smoothly deployed from a compacted state. During transformation, the joints attached on the two ends of a link member rotate on two different planes, where the rotation axes are parallel to each other. Moreover, because of the many more connected links, this type of structure shows much stronger stiffness than the structures presented above.

3.2. Avoiding Singularity by Novel Joints with Bevel Gears

It is important to note that these crystal-inspired deployable structures can be singular when the links intersected at the same joint become coplanar [2,18,26]. Thereafter, the compatibility matrix and the Jacobian matrix shown in Equation (5) become singular, and some additional mechanism modes are induced. Consequently, it is difficult to maintain synchronous motion of adjacent link members. Then, involved bifurcation paths lead these structures to get into singular configurations, which is known as the singularity of a deployable structure [18,27].

To avoid singularity of these deployable structures, a novel type of connecting joints with bevel gears is designed. Figure 7a illustrates an example of the joint with four pairs of bevel gears, which can ensure four connected links maintain synchronous motion. For clarity, Figure 6b describes the assembly of the joint.

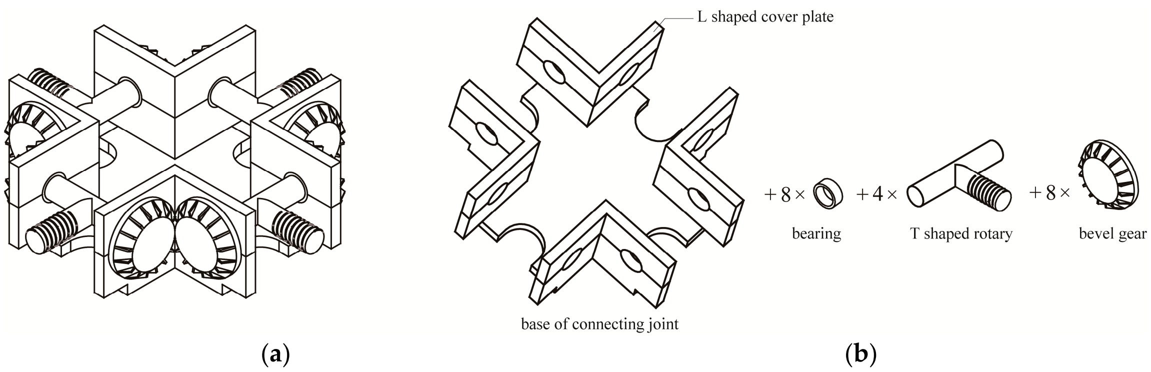


Figure 7. Design of a joint with four pairs of bevel gears and connected to $n_1 = 4$ link members: (**a**) Sketch of the joint model; (**b**) basic components, including base, cover plate, bearing, rotary, and bevel gears.

As shown in Figure 7, this type of novel joint is generally assembled by the main base of the joint, n_1 couples of L-shaped cover plates, n_1 T-shaped rotary, and n_1 pairs of bevel gears. The integer $n_1 \geq 2$ denotes the number of links connected to the joint. The n_1 couples of L-shaped cover plates keep n-fold symmetry. Each T-shaped rotary is located between the two couples of L-shaped cover plates. In addition, each pair of the bevel gears meshes tightly, where the dihedral angle between the top surfaces of the gears is $\theta = 2\pi/n_1$. In Figure 7, $n_1 = 4$ and $\theta = 0.5\pi$. Notably, to allow the connected links to be compactly folded, a total of n_1 U-shaped notches are symmetrically set on the base of the connecting joint.

To verify the feasibility of the design of these novel joints, a specific joint connected by four links is assembled and evaluated, as shown in Figure 8. It turns out that the members maintain synchronous motion due to the intersecting joint with bevel gears, and the whole system keeps four-fold symmetry. The connected members are able to smoothly transform from one folded state to the other folded state. The whole process is reversible and repeatable, without singularity induced. These deployable structures are robust, and can obtain potential applications for retractable roof structures, medical devices, solar panels, and masts [1,28]. In addition, they can guide the construction and destruction process for engineering structures (e.g., cable domes and frame structures) [29,30].

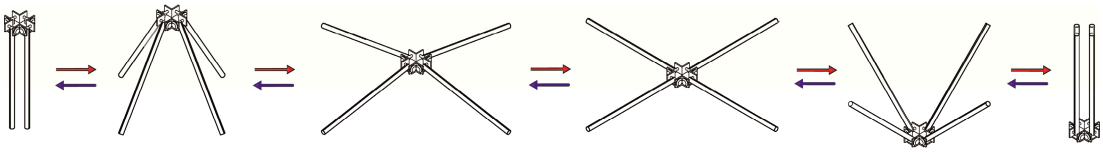


Figure 8. Synchronous and symmetric motion of n_1 = 4 links connected the novel joint with bevel gears.

4. Discussion

This study presents an innovative design of a variety of periodic deployable structures, inspired by the geometry of typical Bravais lattices in 2D space. It should be explained that there are many kinds of crystals (e.g., the known cubic ice and hexagonal ice crystals) [20,25], which include regular or nonregular geometry in 2D/3D space. On the condition that the complexity of joints or the regularity is weakened, certain types of joints and members with different lengths can be included in the unit cells of a deployable structure. Thereafter, many more crystal-inspired deployable structures can be proposed. However, a key point is that most of the joints should be under constraint and connected to a limited number of links [5,24] to guarantee the mobility of the proposed structures.

On the other hand, Section 3.2 briefly illustrates the concept design of novel joints with a number of pairs of bevel gears. Importantly, each joint is not limited to being connected by four or six links, as this can be adjusted for different joints. For example, by modifying the base of the joint (see Figure 9), the angle between the bevel gears, and the other assembly, the joint can be designed to respectively connect three and five link members. Through comprehensive comparisons among the above-mentioned configurations, the structure with n_1 = 3 and n_2 = 6 has a better structural stiffness and redundancy. However, the difficulty of fabricating its joints and folding its links slightly increases. From practicality and feasibility points of view, the structure with n_1 = n_2 = 4 is favorable, as it exhibits satisfactory folding performance and rigidity.

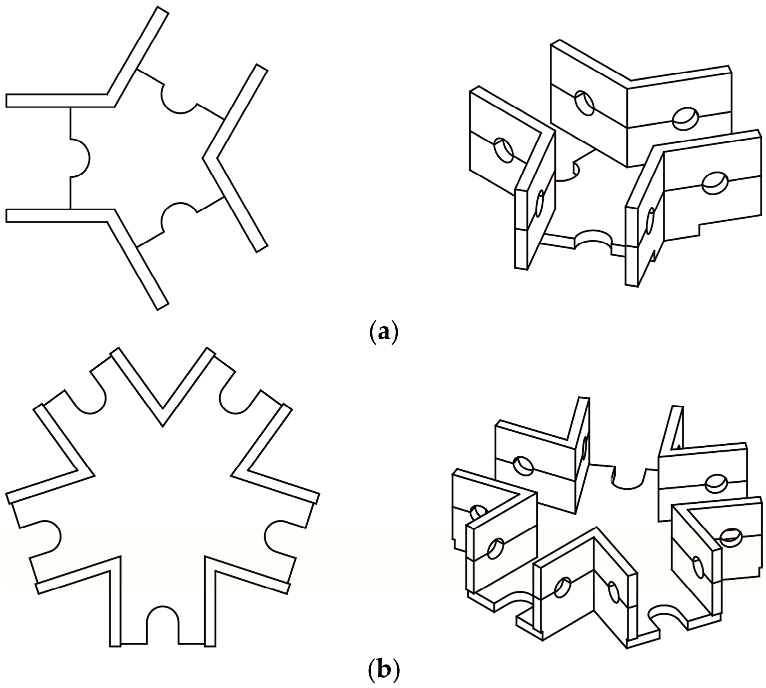


Figure 9. Plan view and 3D view of the base of novel joints with bevel gears: (**a**) Connected to three link members; (**b**) connected to five link members.

In comparison with conventional joints for deployable structures, the presented joints have certain advantages. Because of the bevel gears, the connected link members can maintain synchronous rotations along the rotation axes. Then, potential singularity induced by the coplanarity of the links can be prevented. In addition, the U-shaped notches are helpful for improving the folding ratio of the structures. However, when the motion process calls for high precision, the accuracy of the bevel gear should be deliberately improved for the synchronous motion.

5. Conclusions

This study demonstrated the design of novel revolute joints with bevel gears to realize singularity avoidance and better deployment performance for crystal-inspired deployable structures. We showed that a number of innovative deployable structures can be developed by considering regular and symmetry, and adopting the geometry of typical 2D crystals. Singularity occurs frequently along the motion process of a deployable structure, which leads to bifurcated configurations. To avoid singularity, some connecting joints with bevel gears were presented. The obtained results verified that the proposed joint design has a satisfactory singularity avoidance capability and deployment performance. These crystal-inspired deployable structures can potentially play a positive role in design and engineering applications for deployable structures.

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