

# Supplementary Material for “XCMAX4: A Robust X Chromosomal Genetic Association Test Accounting for Covariates”

**Table S1** Estimated type I error rate ( $\times 10^{-4}$ ) at the nominal significance level  $1 \times 10^{-4}$  for XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> against  $q_f$ ,  $q_m$ ,  $\alpha_2$ , and  $\alpha_3$  based on 1,000,000 replicates when  $F = 0.05$ .

$q_f$	$q_m$	$\alpha_3$	$\alpha_2 = 0.4005$					$\alpha_2 = -0.4005$				
			XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
0.1	0.1	0.5	0.86	1.10	0.91	0.86	0.91	0.84	1.05	0.95	0.87	1.01
	0.2		0.84	0.93	0.83	0.82	0.95	0.97	1.02	1.08	1.02	0.93
	0.3		1.12	1.03	0.93	1.13	1.02	1.01	1.03	1.16	0.85	1.02
0.2	0.1	0.5	0.90	0.94	0.95	0.80	0.91	1.01	0.99	1.08	0.87	1.03
	0.2		0.82	0.93	0.94	0.91	0.90	0.96	1.00	1.05	1.05	1.04
	0.3		0.83	0.96	0.85	0.89	0.98	0.89	1.08	1.05	1.04	1.05
0.3	0.1	0.5	0.80	0.98	0.93	1.02	0.87	0.95	0.97	0.88	0.97	0.85
	0.2		0.84	0.93	0.94	1.13	1.01	1.16	1.26	1.16	1.22	1.13
	0.3		0.94	1.08	0.94	0.99	1.04	0.88	0.96	0.95	1.06	0.94
0.1	0.1	1.5	0.87	0.87	0.97	0.82	0.99	0.98	0.98	0.95	0.85	1.03
	0.2		0.97	1.13	1.03	0.98	1.03	0.84	0.88	1.04	0.81	0.91
	0.3		0.86	0.78	0.92	0.87	0.75	1.03	0.99	0.96	1.03	0.98
0.2	0.1	1.5	0.91	0.88	1.01	0.80	1.00	1.00	1.08	1.03	0.88	1.01
	0.2		0.96	0.99	0.94	0.77	1.10	0.85	0.99	1.04	0.89	0.99
	0.3		1.06	0.81	1.10	1.02	0.97	0.82	0.94	1.00	1.06	0.88
0.3	0.1	1.5	0.93	1.14	0.98	1.15	0.93	1.00	1.15	0.92	0.98	0.92
	0.2		1.07	1.21	1.07	1.14	1.13	1.03	1.33	1.07	1.02	1.18
	0.3		0.91	1.04	1.09	0.90	1.11	0.99	1.03	0.98	1.10	1.05

**Table S2.** Estimated type I error rate ( $\times 10^{-3}$ ) at the nominal significance level  $1 \times 10^{-3}$  for XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> against  $q_f$ ,  $q_m$ ,  $\alpha_2$ , and  $\alpha_3$  based on 1,000,000 replicates under HWE.

$q_f$	$q_m$	$\alpha_3$	$\alpha_2 = 0.4005$					$\alpha_2 = -0.4005$				
			XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
0.1	0.1	0.5	0.89	0.96	0.96	0.97	0.94	0.90	1.01	0.98	0.97	0.97
	0.2		0.84	0.92	0.95	0.91	0.90	0.84	0.95	0.95	0.92	0.94
	0.3		0.92	0.97	1.00	0.99	0.97	0.97	1.12	1.09	1.03	1.10
0.2	0.1	0.5	0.95	1.01	1.02	0.97	1.02	0.83	0.92	0.88	0.90	0.90
	0.2		0.87	0.96	0.98	0.93	0.95	0.81	0.96	0.93	0.93	0.95
	0.3		0.91	1.00	1.02	0.98	1.01	0.88	0.99	1.00	0.96	0.98
0.3	0.1	0.5	0.87	0.96	0.96	0.92	0.94	0.89	1.00	0.96	0.93	0.94
	0.2		0.89	1.00	1.01	0.97	1.00	0.87	1.00	1.02	0.94	1.01
	0.3		0.85	0.94	0.95	0.96	0.95	0.96	1.03	1.03	1.09	1.03
0.1	0.1	1.5	0.90	0.99	0.95	0.92	0.97	0.85	0.97	0.93	0.92	0.95
	0.2		0.90	1.00	0.97	0.96	0.99	0.88	0.93	0.94	0.96	0.94
	0.3		0.89	0.96	1.03	1.03	0.97	0.89	1.00	0.99	0.97	1.00
0.2	0.1	1.5	0.88	0.94	0.93	0.89	0.91	0.81	0.96	0.94	0.95	0.95
	0.2		0.86	0.96	0.97	0.95	0.96	0.86	0.95	0.97	0.95	0.99
	0.3		0.88	1.05	1.02	0.98	1.03	0.92	0.99	1.04	1.01	1.02
0.3	0.1	1.5	0.91	1.00	1.01	0.97	1.00	0.89	1.04	1.01	0.91	0.99
	0.2		0.83	0.95	0.94	0.93	0.94	0.82	0.93	0.92	0.94	0.92
	0.3		0.88	1.01	0.98	1.01	0.99	0.88	1.03	1.03	1.00	1.03

**Table S3.** Estimated type I error rate ( $\times 10^{-5}$ ) at the nominal significance level  $1 \times 10^{-5}$  for XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> against  $q_f$ ,  $q_m$ ,  $\alpha_2$ , and  $\alpha_3$  based on 1,000,000 replicates under HWE.

$q_f$	$q_m$	$\alpha_3$	$\alpha_2 = 0.4005$					$\alpha_2 = -0.4005$				
			XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
0.	0.1	0.5	0.8	0.9	1.0	0.9	1.0	0.6	0.8	0.5	0.4	0.6
1	0.2		0.5	0.5	1.0	0.5	0.6	0.6	0.5	0.5	0.6	0.5
	0.3		0.9	0.7	1.0	0.4	0.8	0.5	0.3	0.8	0.4	0.2
0.	0.1		0.8	0.8	0.6	0.6	0.6	0.6	0.8	0.8	0.5	0.8
2	0.2		1.2	1.1	0.8	0.8	1.1	0.7	0.7	0.8	1.4	0.7
	0.3		0.8	0.6	1.2	0.6	0.8	0.5	0.4	0.8	0.8	0.9
0.	0.1		0.8	0.6	0.9	0.9	0.8	1.1	1.2	1.1	1.1	1.1
3	0.2		1.2	1.5	1.2	1.5	1.3	0.9	1.2	1.1	1.2	1.1
	0.3		1.1	1.0	1.0	1.0	0.9	1.0	1.2	1.1	1.1	0.8
0.	0.1	1.5	0.4	0.4	0.8	1.1	0.6	1.1	0.8	0.8	0.7	1.0
1	0.2		0.3	0.3	0.6	0.3	0.3	0.9	0.3	0.7	0.9	0.6
	0.3		0.6	1.0	0.5	0.9	1.1	1.4	1.6	1.4	1.4	1.4
0.	0.1		0.7	0.9	1.0	0.8	0.9	1.1	1.2	1.2	0.8	1.1
2	0.2		1.3	1.1	0.7	1.7	1.0	0.9	1.0	1.3	0.9	1.1
	0.3		1.0	1.2	1.0	1.3	1.4	1.2	0.9	1.1	1.4	0.8
0.	0.1		0.7	1.5	0.9	0.4	0.8	0.8	1.2	0.8	0.6	0.6
3	0.2		0.8	1.1	0.7	1.1	1.0	0.5	0.9	1.0	0.8	0.9
	0.3		0.4	0.5	0.3	0.4	0.3	0.8	0.7	0.9	1.2	0.6

**Table S4.** Estimated type I error rate ( $\times 10^{-3}$ ) at the nominal significance level  $1 \times 10^{-3}$  for XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> against  $q_f$ ,  $q_m$ ,  $\alpha_2$ , and  $\alpha_3$  based on 1,000,000 replicates when  $F = 0.05$ .

$q_f$	$q_m$	$\alpha_3$	$\alpha_2 = 0.4005$					$\alpha_2 = -0.4005$				
			XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
0.1	0.1	0.5	0.83	0.93	0.95	0.89	0.93	0.82	0.93	0.90	0.89	0.93
	0.2		0.90	0.98	1.02	0.92	0.98	0.88	0.99	0.96	1.02	0.99
	0.3		0.91	1.02	1.04	0.99	1.01	0.87	0.97	0.96	1.01	0.97
0.2	0.1		0.90	0.97	0.98	0.96	0.97	0.96	1.02	1.09	0.99	1.07
	0.2		0.85	0.99	0.94	0.93	0.97	0.85	0.98	0.95	0.99	0.94
	0.3		0.90	0.97	0.98	1.02	0.97	0.90	1.00	0.99	0.96	1.01
0.3	0.1		0.90	0.98	0.96	0.97	0.95	0.88	1.01	1.00	0.93	0.98
	0.2		0.83	0.95	0.94	0.99	0.97	0.81	0.88	0.89	0.95	0.89
	0.3		0.91	1.05	1.06	1.02	1.04	0.80	0.95	0.90	0.94	0.93
0.1	0.1	1.5	0.89	0.95	1.00	0.95	1.00	0.84	0.91	0.93	0.89	0.93
	0.2		0.93	1.03	1.00	0.98	1.04	0.90	1.00	0.97	0.97	0.99
	0.3		0.84	0.94	0.93	0.94	0.93	0.88	1.00	1.01	1.04	1.00
0.2	0.1		0.91	0.99	1.04	0.91	1.02	0.89	1.02	1.00	0.92	0.99
	0.2		0.83	1.00	0.94	0.92	0.94	0.86	0.94	0.99	0.95	0.97
	0.3		0.90	1.01	0.97	1.00	1.01	0.87	0.97	0.99	0.96	0.95
0.3	0.1		0.87	0.94	0.94	0.92	0.93	0.90	0.98	1.04	0.98	1.03
	0.2		0.91	1.05	1.01	1.04	1.00	0.90	1.01	1.00	0.97	0.98
	0.3		0.87	1.02	0.98	0.98	1.01	0.87	1.02	0.98	1.00	0.97

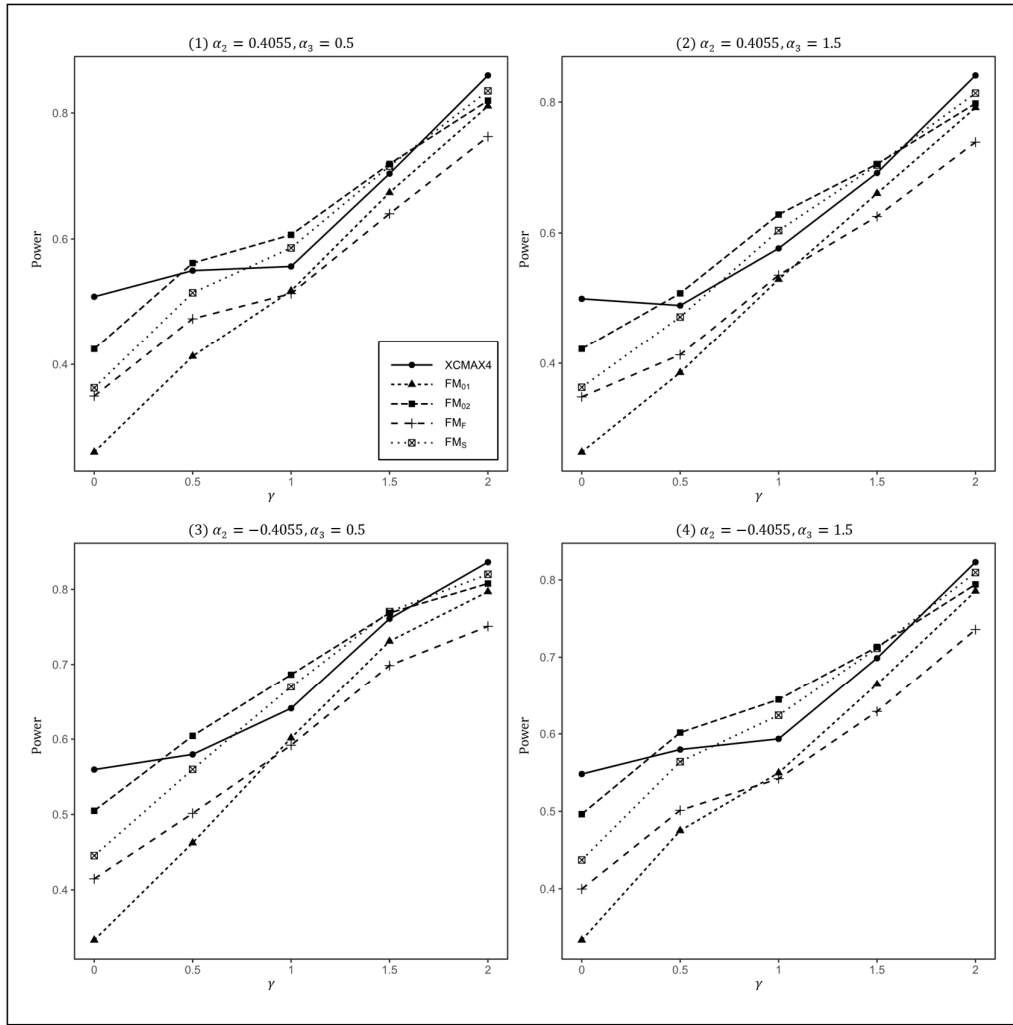
**Table S5.** Estimated type I error rate ( $\times 10^{-5}$ ) at the nominal significance level  $1 \times 10^{-5}$  for XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> against  $q_f$ ,  $q_m$ ,  $\alpha_2$ , and  $\alpha_3$  based on 1,000,000 replicates when  $F = 0.05$ .

$q_f$	$q_m$	$\alpha_3$	$\alpha_2 = 0.4005$					$\alpha_2 = -0.4005$				
			XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
0.1	0.1	0.5	1.0	0.8	1.4	0.8	1.1	1.2	0.4	0.8	0.5	0.5
	0.2		0.8	0.7	0.8	1.2	0.7	0.8	0.7	1.0	1.0	0.9
	0.3		0.6	1.1	0.9	1.0	1.1	1.4	1.1	1.2	0.6	1.1
0.2	0.1		1.1	1.5	0.9	1.0	0.9	0.9	0.8	1.1	0.4	0.9
	0.2		2.0	1.1	0.6	0.9	1.1	1.0	1.0	1.3	1.5	1.0
	0.3		0.4	0.5	0.7	0.8	0.6	0.7	0.5	0.9	0.4	1.0
0.3	0.1		0.8	1.1	0.7	0.3	0.7	0.8	0.8	1.2	0.6	1.3
	0.2		0.8	0.8	0.7	1.2	0.8	1.7	2.2	2.2	2.1	1.8
	0.3		1.3	1.1	1.1	1.8	1.4	0.8	1.5	1.2	1.2	1.1
0.1	0.1	1.5	1.0	0.9	0.7	1.4	0.7	1.0	1.0	0.7	0.7	0.9
	0.2		1.4	1.3	1.0	1.1	1.1	1.0	1.1	1.0	0.3	1.0
	0.3		0.6	0.6	0.5	0.7	0.6	0.9	1.3	0.8	1.2	1.3
0.2	0.1		1.0	0.6	0.7	1.1	0.7	1.3	0.7	1.1	1.1	1.1
	0.2		0.7	1.2	1.1	0.7	1.0	1.2	1.4	1.2	0.9	1.4
	0.3		1.5	1.0	1.1	1.1	0.8	0.8	0.8	1.0	1.1	1.2
0.3	0.1		1.2	1.1	1.1	0.9	0.8	0.8	1.5	1.0	0.9	1.2
	0.2		1.3	1.4	1.2	1.5	1.2	2.0	1.9	1.7	1.2	1.9
	0.3		0.8	0.9	1.0	1.1	0.9	1.5	1.3	0.9	1.7	1.2

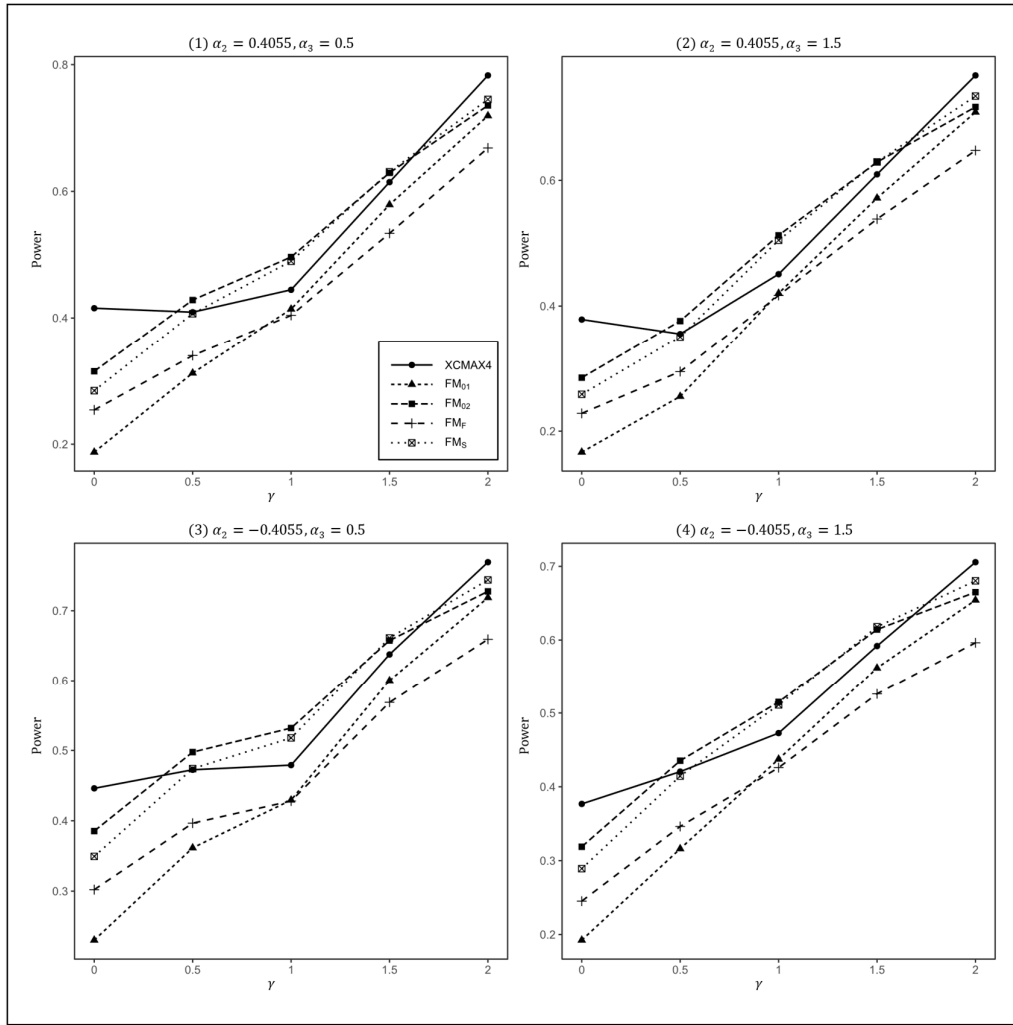
**Table S6** Time used to test 2000 SNPs with a sample size of 5000\*

Time (second)	XCMAX4	FM <sub>01</sub>	FM <sub>02</sub>	FM <sub>F</sub>	FM <sub>S</sub>
User	9.79	19.39	19.28	19.36	19.35
System	0.00	0.00	0.00	0.02	0.02
Elapsed	9.79	19.29	19.29	19.38	19.36

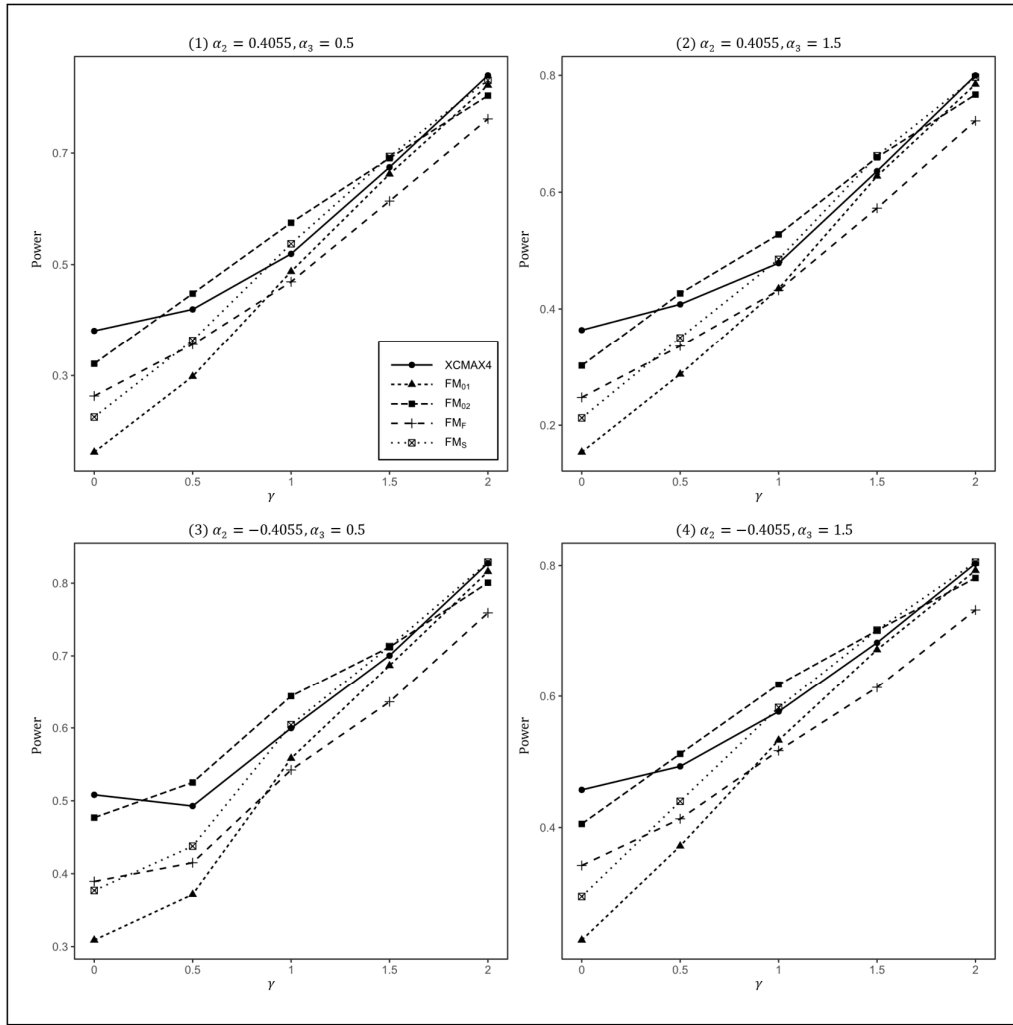
\* The fitted model is the same as that in our simulation study. The results are obtained in R 4.1.2, running on a laptop with CPU: AMD Ryzen 5 5600H and RAM: 16G.



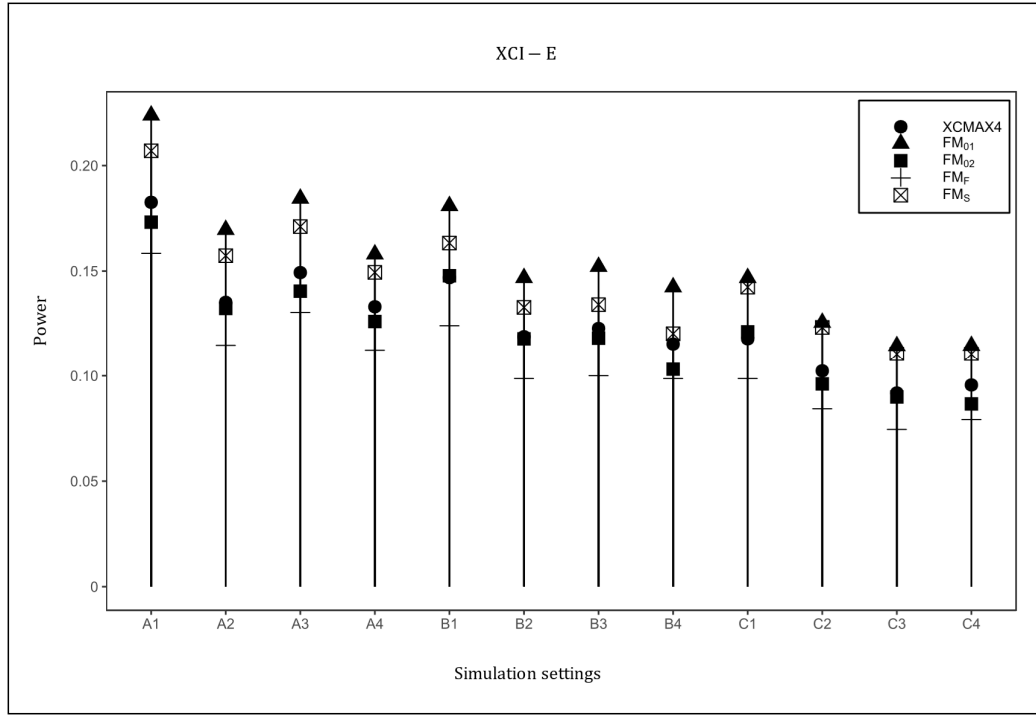
**Figure S1.** Estimated powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.15$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ , and  $q_f = q_m = 0.3$ .



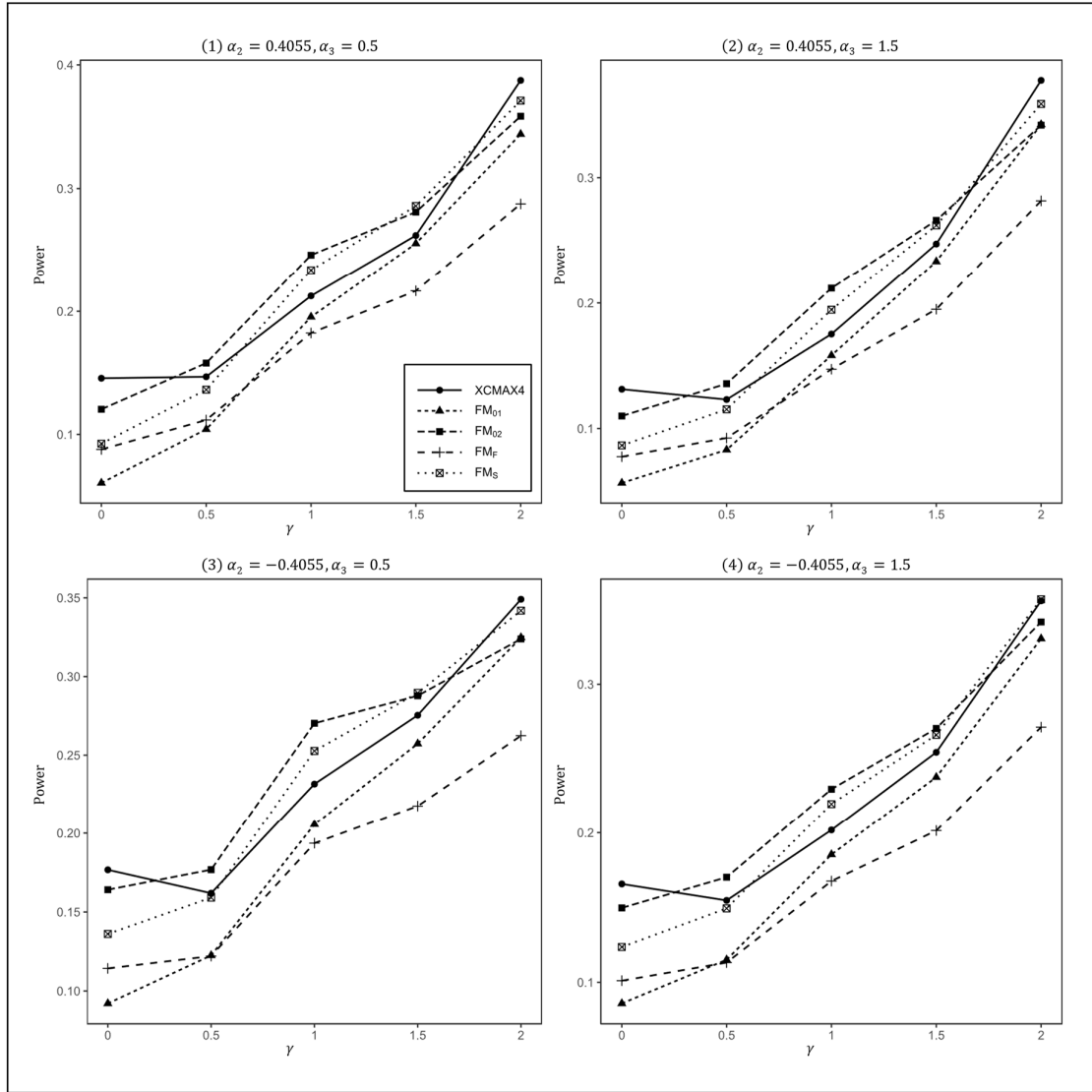
**Figure S2.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.15$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.3$ , and  $q_m = 0.2$ .



**Figure S3.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.15$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.2$ , and  $q_m = 0.3$ .

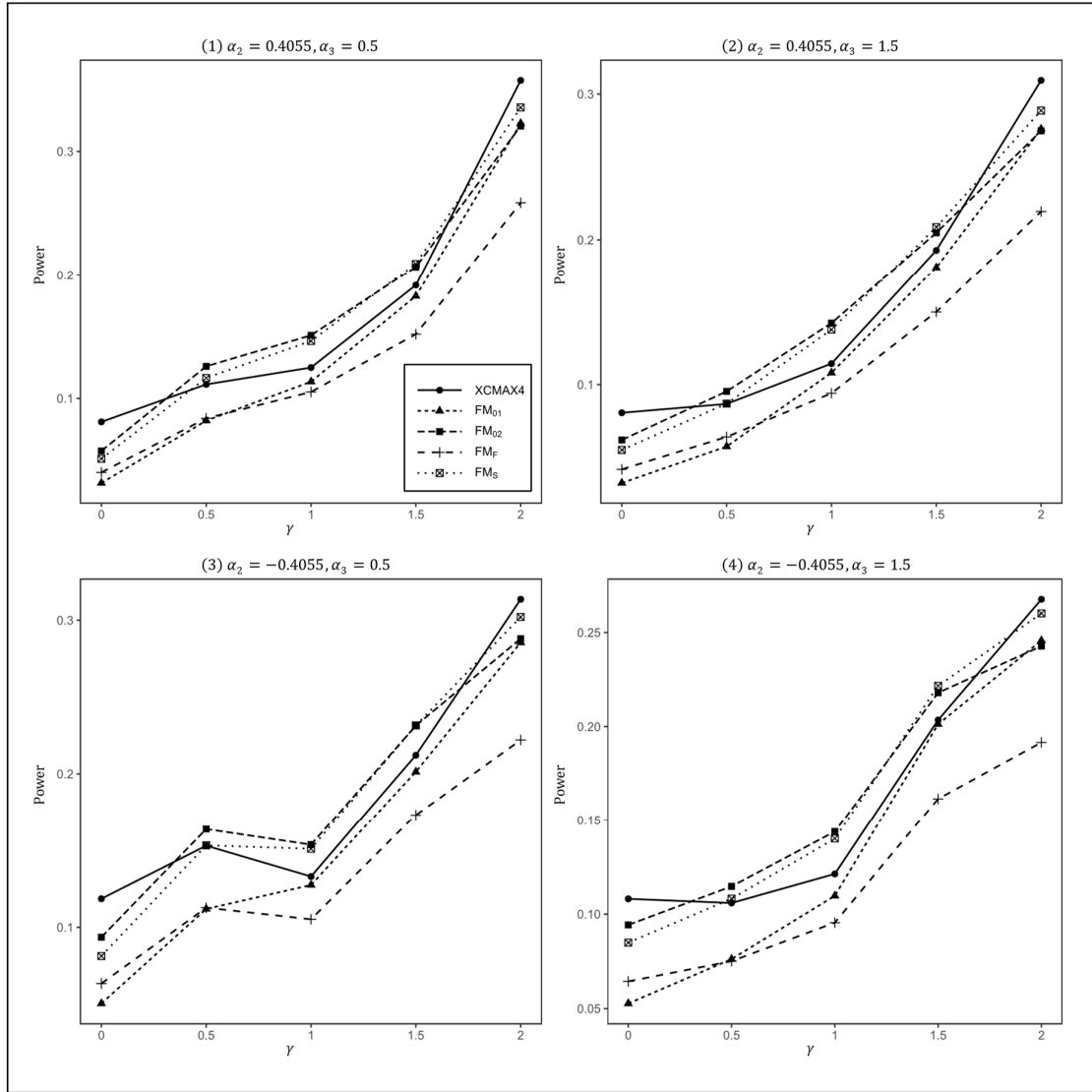


**Figure S4.** Powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under the XCI-E pattern. The simulation is based on 10,000 replicates with  $\beta = 0.15$ ,  $\alpha_1 = -5$ , and  $F = 0.05$ . In the horizontal coordinates, "A", "B" and "C" represent three combinations of  $(q_f, q_m)$ : (0.3,0.3), (0.3,0.2), and (0.2,0.3), respectively. The numbers 1-4 represent four combinations of  $(\alpha_2, \alpha_3)$ : (0.4055,0.5), (0.4055,1.5), (-0.4055,0.5), and (-0.4055,1.5), respectively.

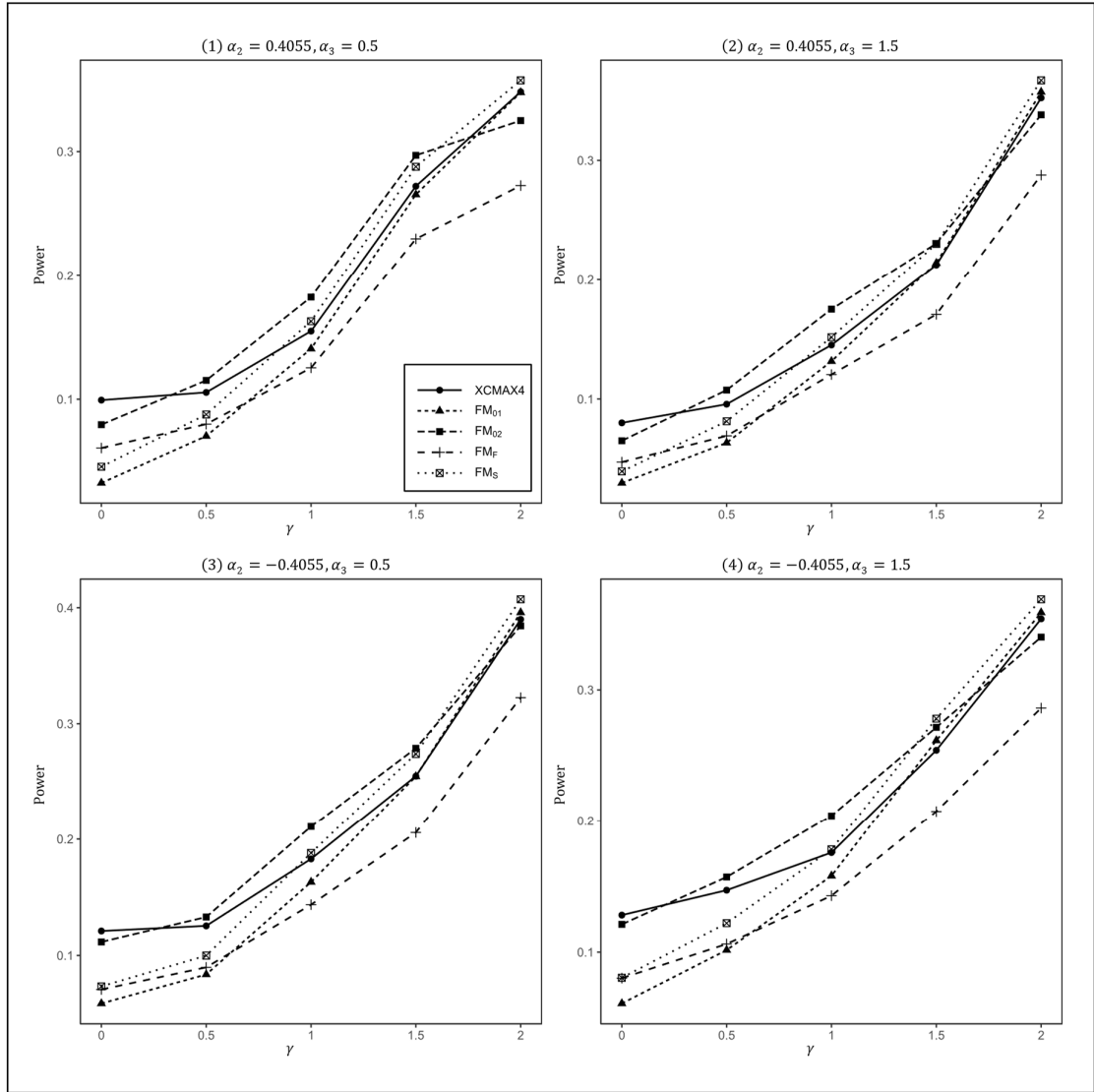


**Figure S5.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0$ , and  $q_f = q_m = 0.3$ .

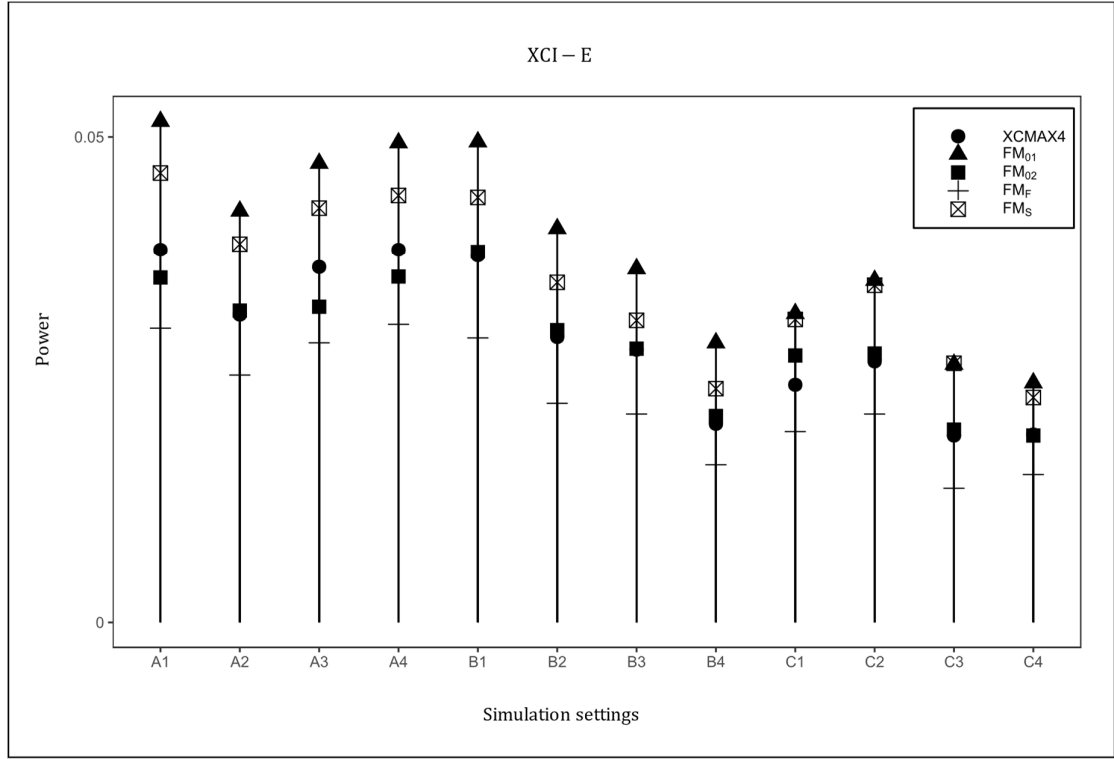




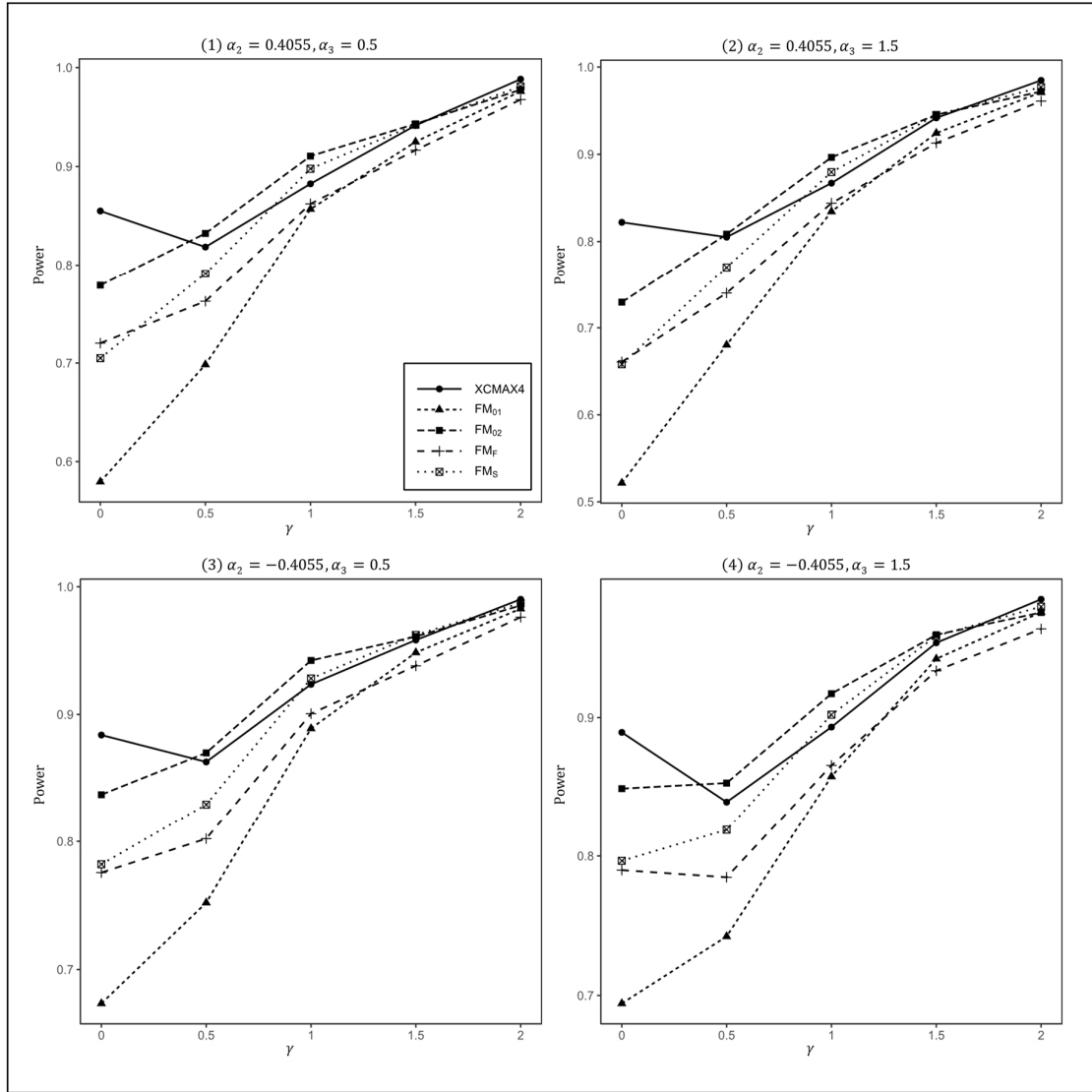
**Figure S6.** Estimated powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0$ ,  $q_f = 0.3$ , and  $q_m = 0.2$ .



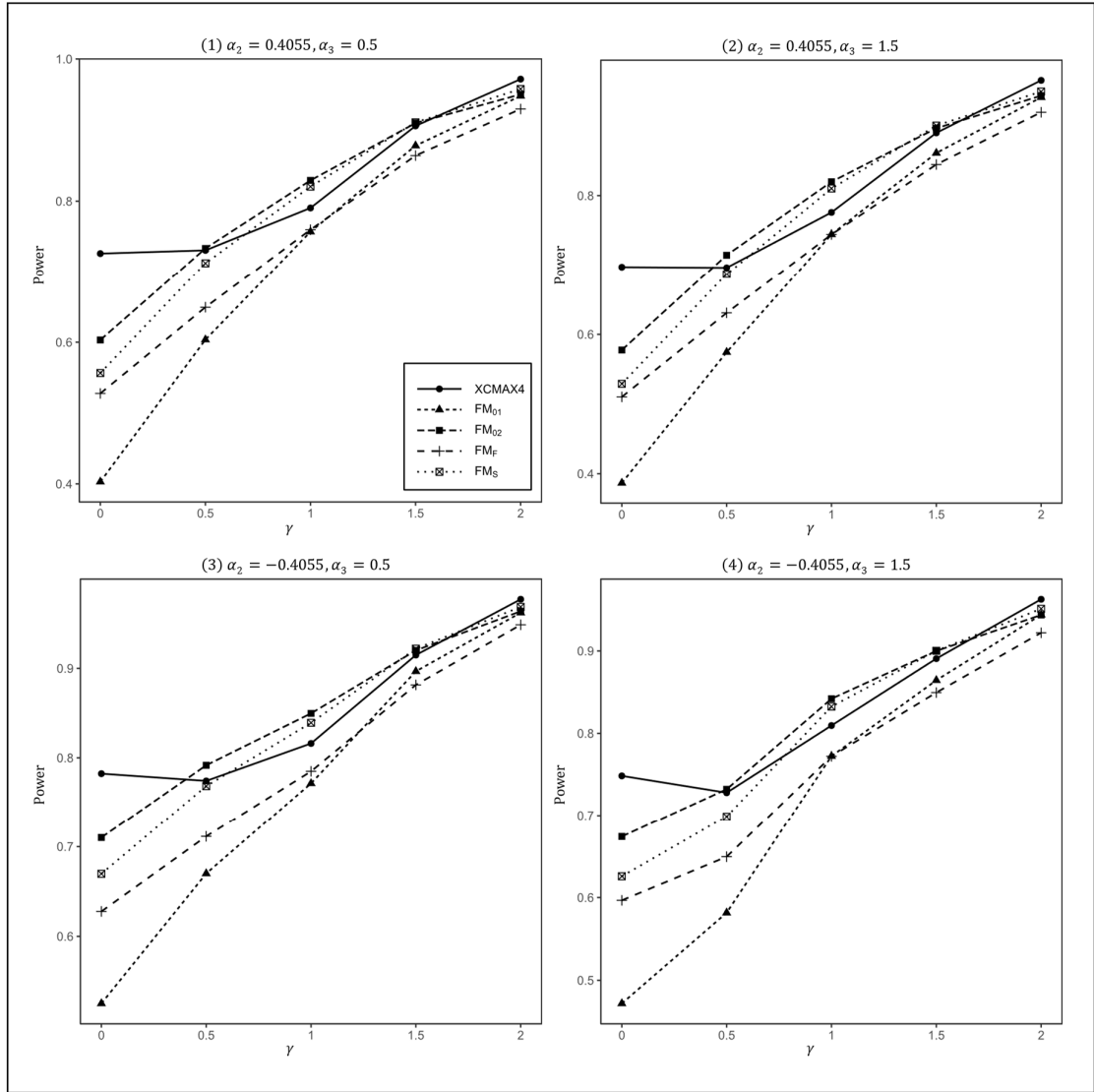
**Figure S7.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0$ ,  $q_f = 0.2$ , and  $q_m = 0.3$ .



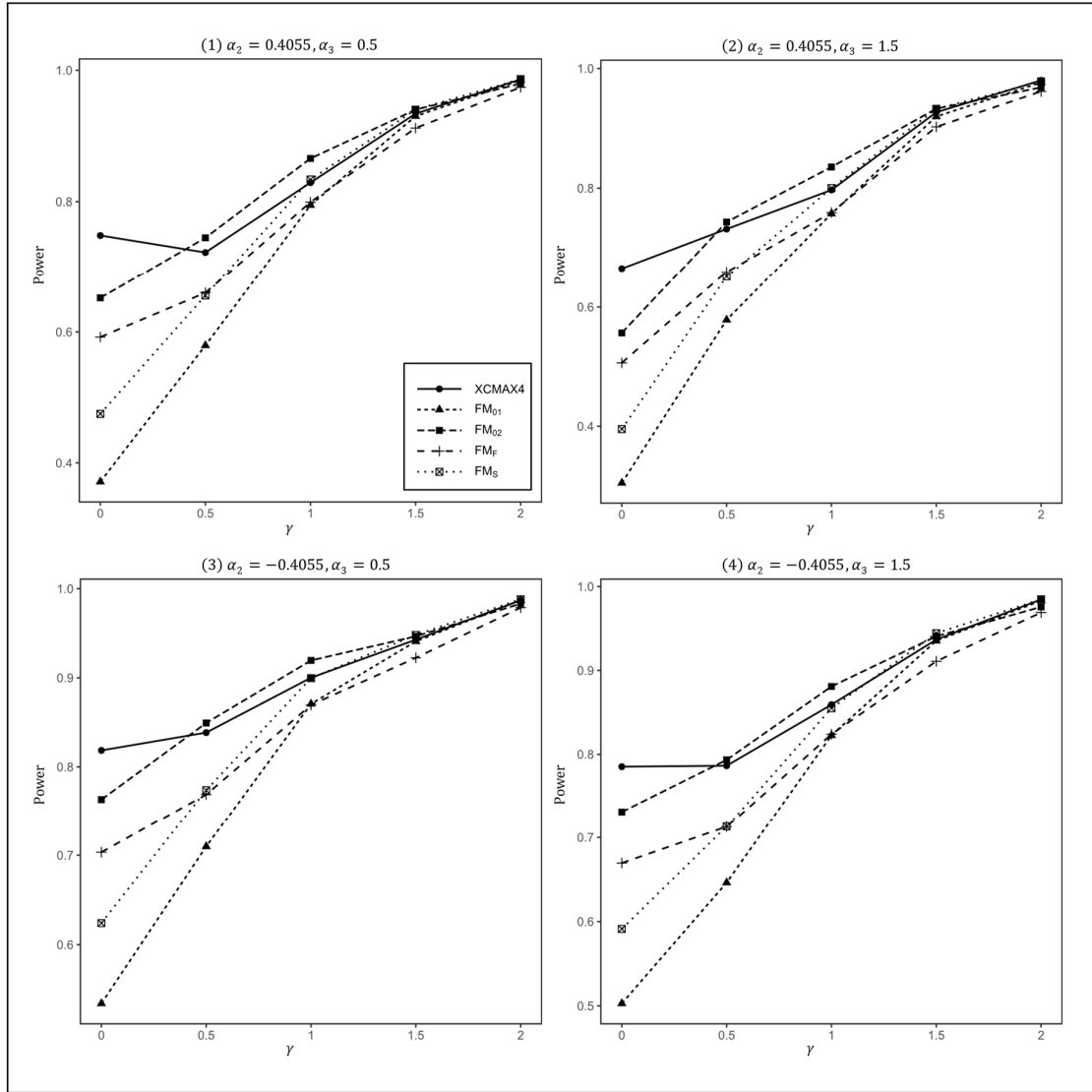
**Figure S8.** Powers of XCMAx4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under XCI-E. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ , and  $F = 0$ . In the horizontal coordinates, "A", "B" and "C" represent three combinations of  $(q_f, q_m)$ : (0.3,0.3), (0.3,0.2), and (0.2,0.3), respectively, and the numbers 1-4 represent four combinations of  $(\alpha_2, \alpha_3)$ : (0.4055,0.5), (0.4055,1.5), (-0.4055,0.5), and (-0.4055,1.5), respectively.



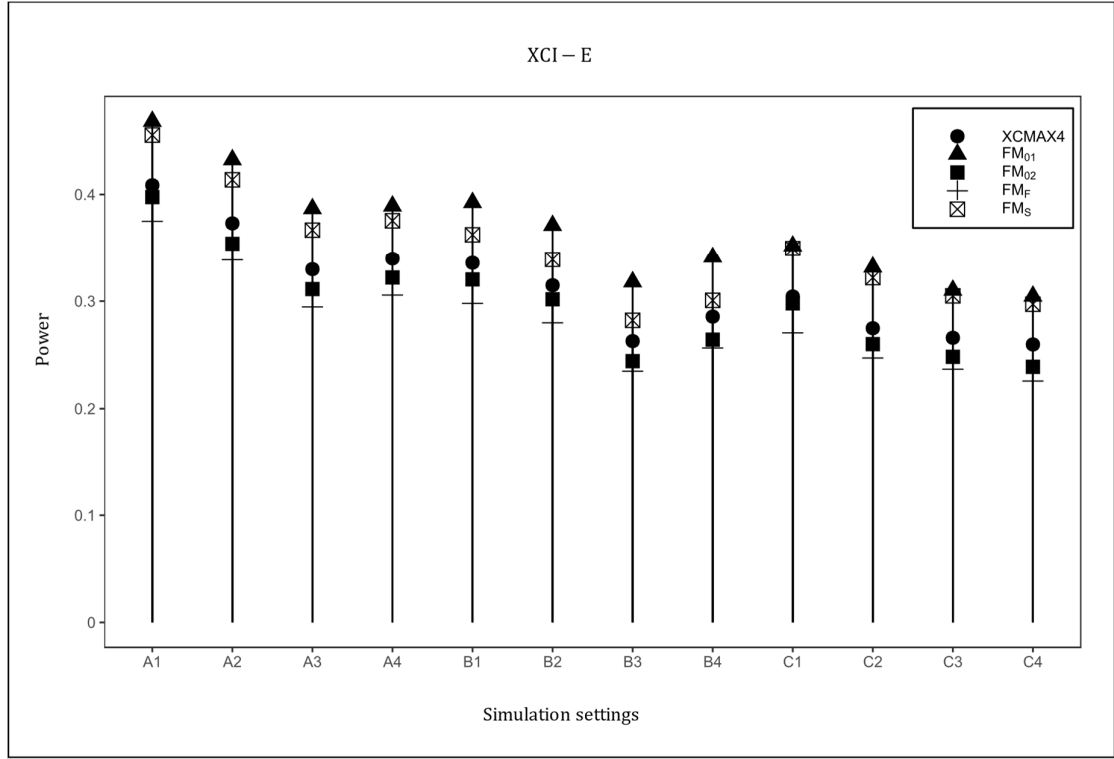
**Figure S9.** Estimated powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0$ , and  $q_f = q_m = 0.3$ .



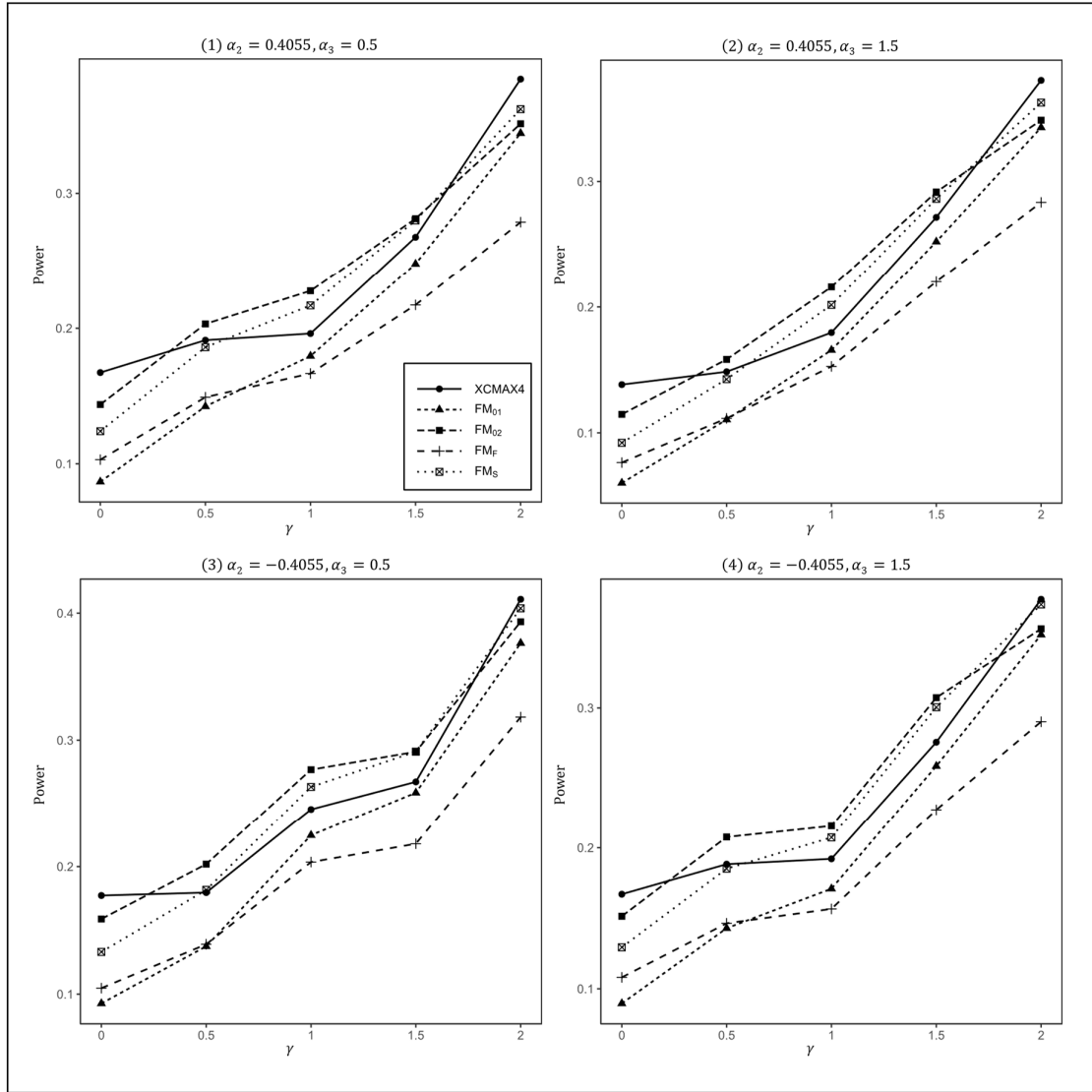
**Figure S10.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0$ ,  $q_f = 0.3$ , and  $q_m = 0.2$ .



**Figure S11.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0$ ,  $q_f = 0.2$ , and  $q_m = 0.3$ .

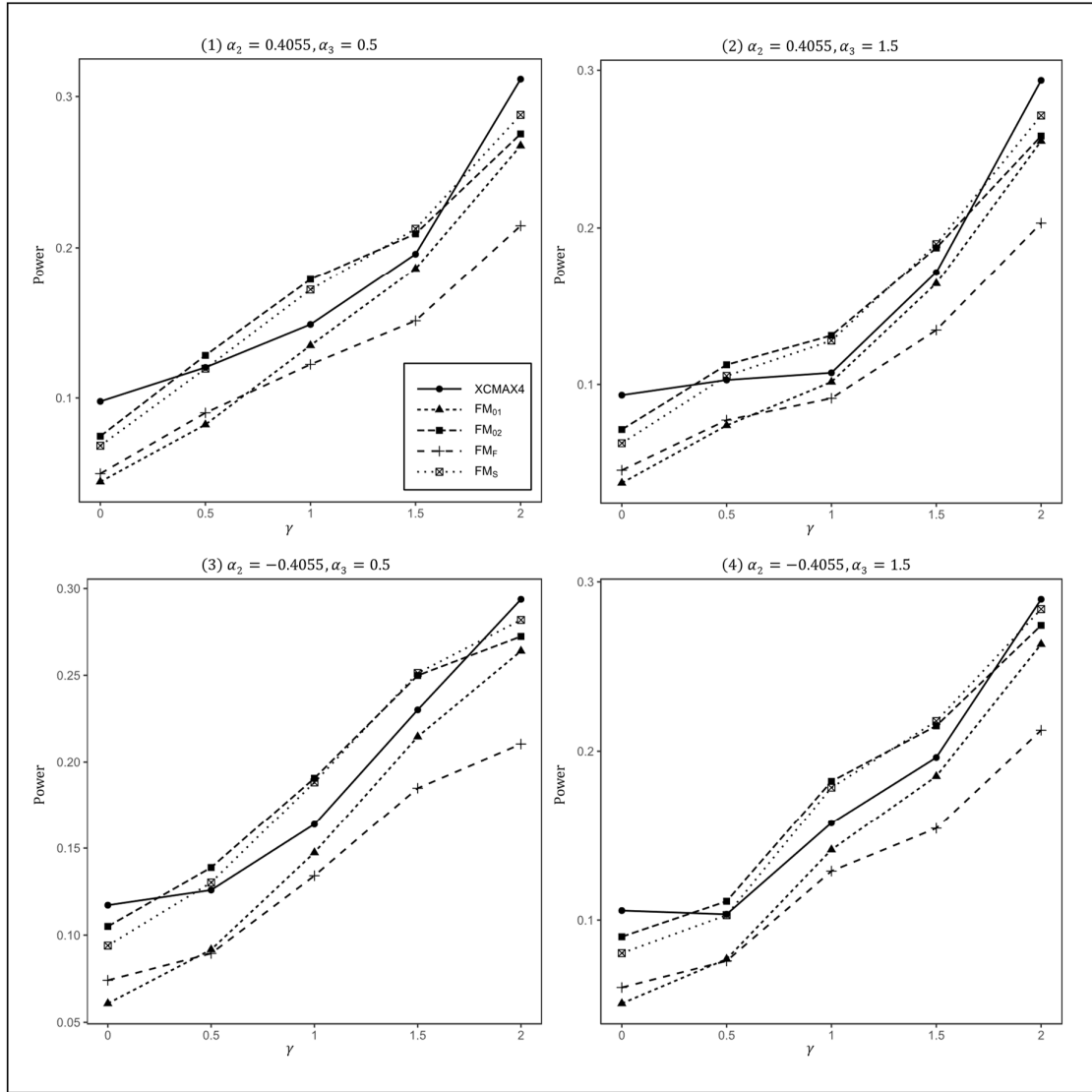


**Figure S12.** Powers of XCMAx4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under XCI-E. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ , and  $F = 0$ . In the horizontal coordinates, "A", "B" and "C" represent three combinations of  $(q_f, q_m)$ : (0.3,0.3), (0.3,0.2), and (0.2,0.3), respectively, and the numbers 1-4 represent four combinations of  $(\alpha_2, \alpha_3)$ : (0.4055,0.5), (0.4055,1.5), (-0.4055,0.5), and (-0.4055,1.5), respectively.

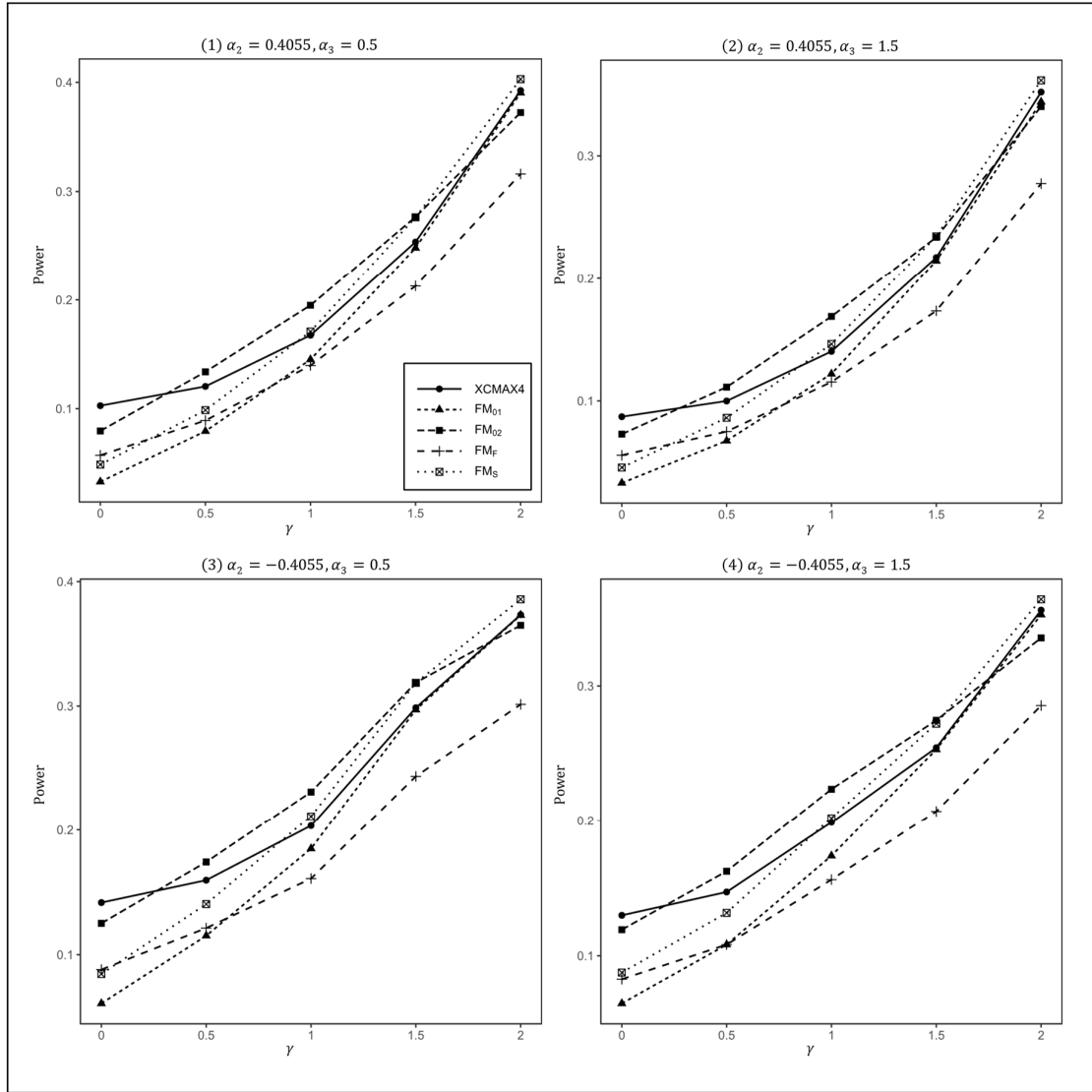


**Figure S13.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ , and  $q_f = q_m = 0.3$ .

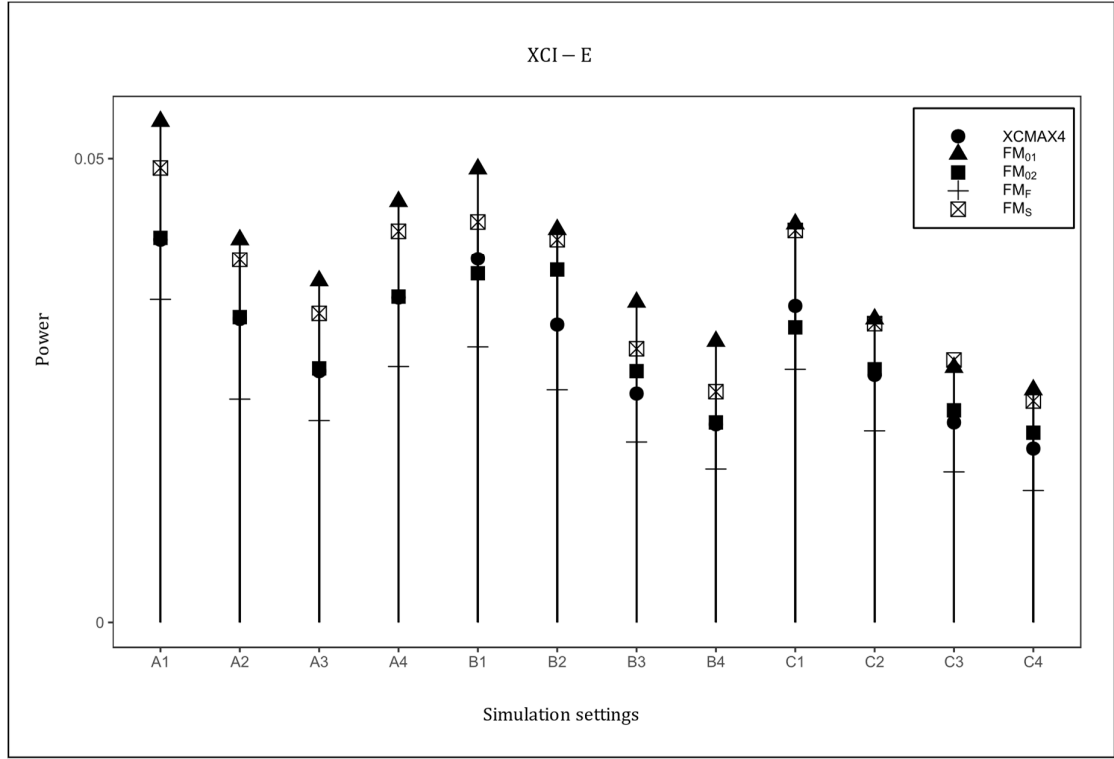




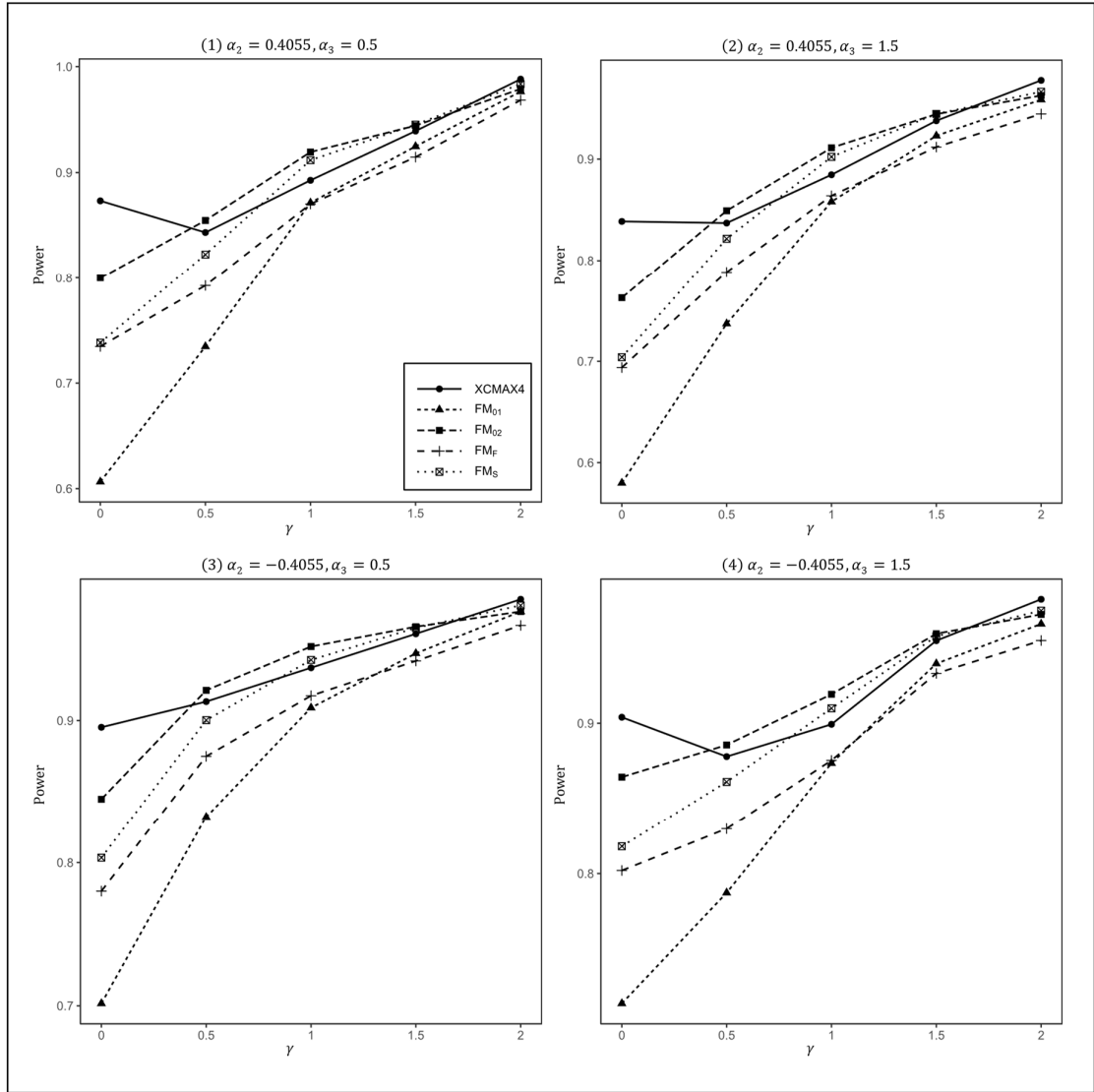
**Figure S14.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.3$ , and  $q_m = 0.2$ .



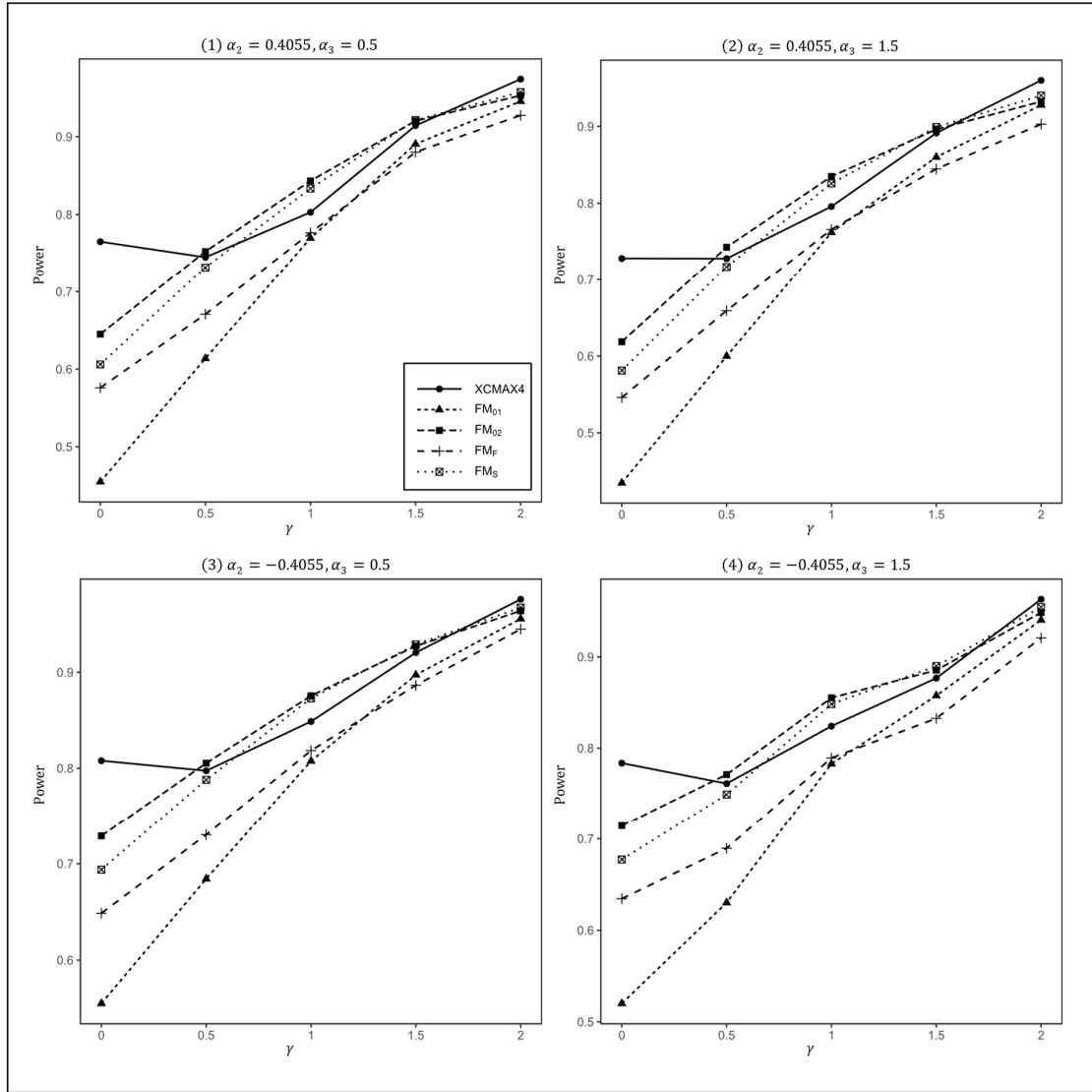
**Figure S15.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.2$ , and  $q_m = 0.3$ .



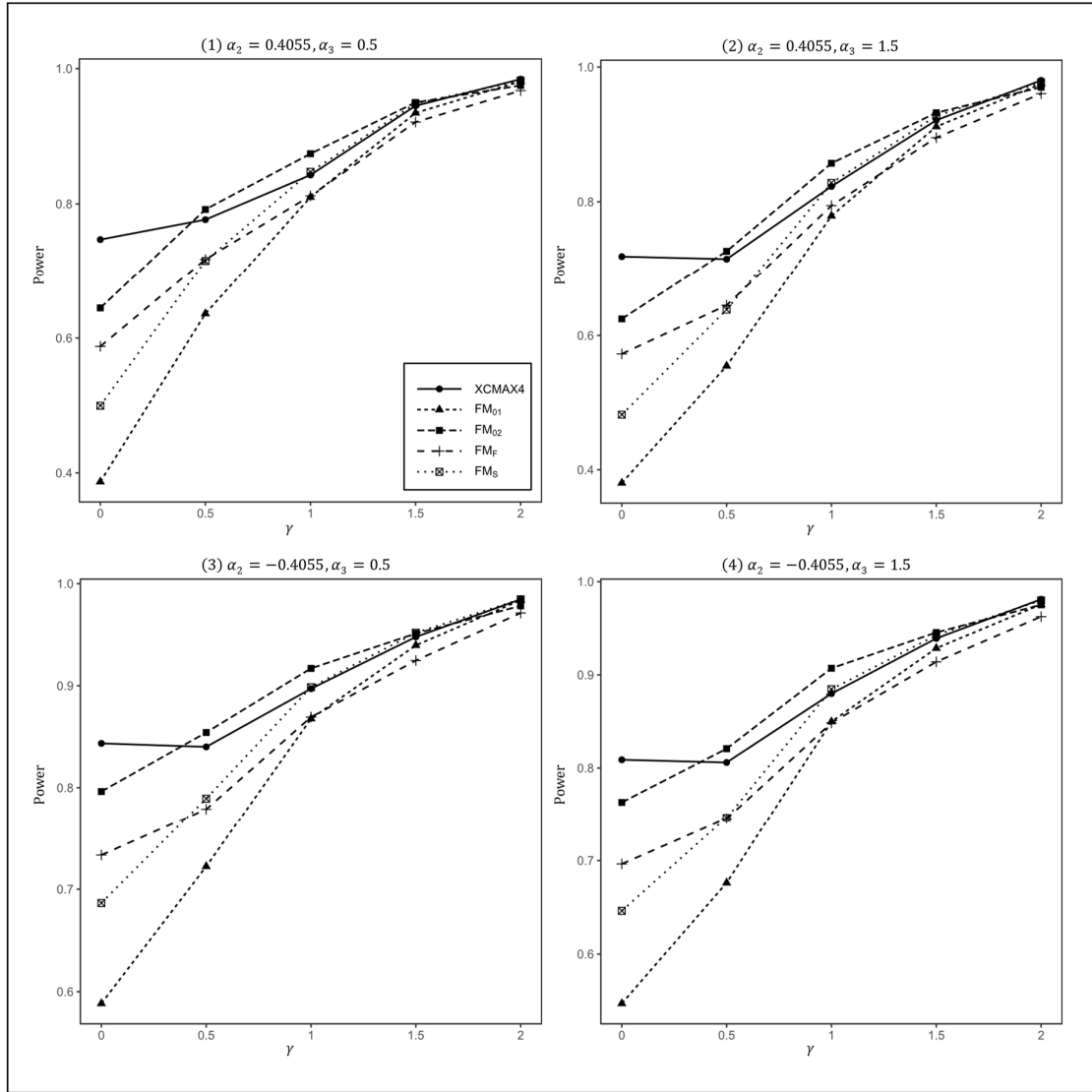
**Figure S16.** Powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under XCI-E. The simulation is based on 10,000 replicates with  $\beta = 0.1116$ ,  $\alpha_1 = -5$ , and  $F = 0.05$ . In the horizontal coordinates, "A", "B" and "C" represent three combinations of  $(q_f, q_m)$ : (0.3,0.3), (0.3,0.2), and (0.2,0.3), respectively, and the numbers 1-4 represent four combinations of  $(\alpha_2, \alpha_3)$ : (0.4055,0.5), (0.4055,1.5), (-0.4055,0.5), and (-0.4055,1.5), respectively.



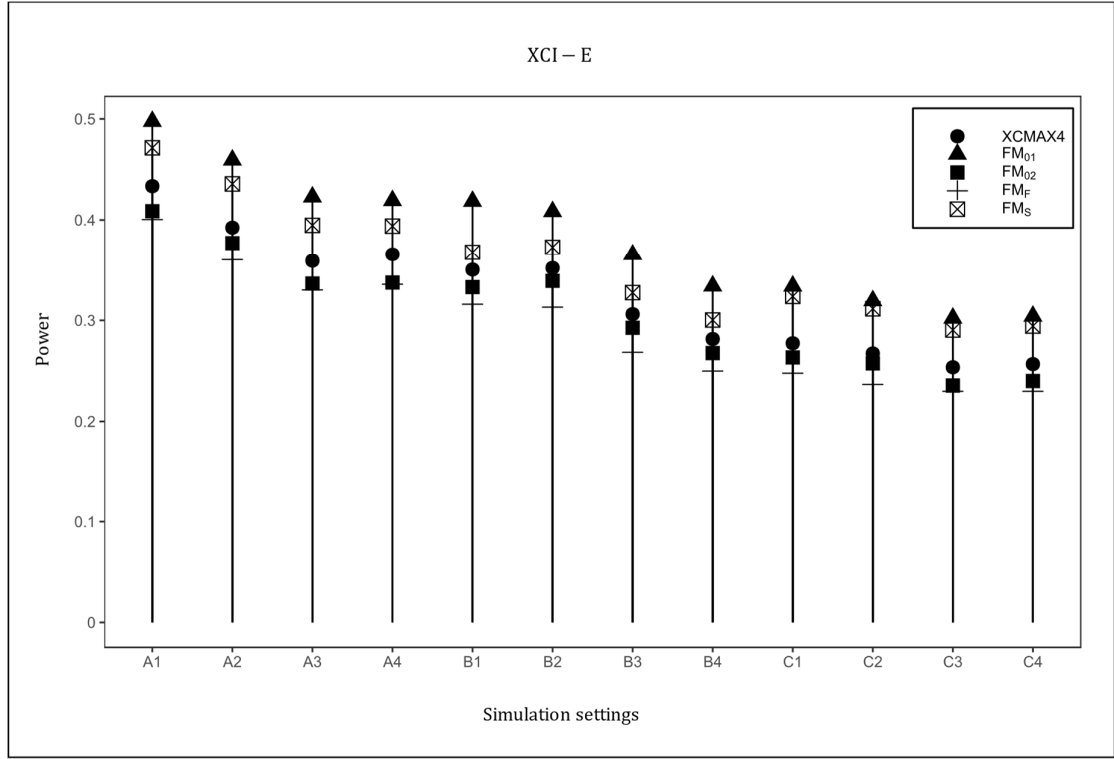
**Figure S17.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ , and  $q_f = q_m = 0.3$ .



**Figure S18.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.3$ , and  $q_m = 0.2$ .



**Figure S19.** Estimated powers of XCMAX4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under various XCI models. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ ,  $F = 0.05$ ,  $q_f = 0.2$ , and  $q_m = 0.3$ .



**Figure S20.** Powers of XCMA4, FM<sub>01</sub>, FM<sub>02</sub>, FM<sub>F</sub>, and FM<sub>S</sub> under XCI-E. The simulation is based on 10,000 replicates with  $\beta = 0.1858$ ,  $\alpha_1 = -5$ , and  $F = 0.05$ . In the horizontal coordinates, "A", "B" and "C" represent three combinations of  $(q_f, q_m)$ : (0.3,0.3), (0.3,0.2), and (0.2,0.3), respectively, and the numbers 1-4 represent four combinations of  $(\alpha_2, \alpha_3)$ : (0.4055,0.5), (0.4055,1.5), (-0.4055,0.5), and (-0.4055,1.5), respectively.