Supplementary

Exploring the Origin of the Two-Week Predictability Limit: A Revisit of Lorenz's Predictability Studies in the 1960s

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Supplementary Materials: Comments on the Doubling Time of 2-3 days Reported in Lorenz's 1969 studies

After Lorenz's studies in the 1960s, people became aware of the impact of nonlinear instability (i.e., chaos) on predictability. In addition to saturation time, doubling times were estimated by Lorenz to determine the predictability of atmosphere using all the three approaches discussed in the main text. As reported in Lorenz (1969b [31], 1969e [33]), Lorenz was able to obtain a doubling time of 2-3 days for all three approaches. In the 1960s and 1970s, such a consistent result was, indeed, encouraging. Nevertheless, subsequent progress in predictability research and the inclusion of cautionary statements within the original studies (e.g., as shown in the captions of Figure 5) have prompted us to approach the interpretation of results with care. In line with this perspective, several remarks are presented below.

To aid in discussions, Table S1 presents the changing views of Lorenz on predictability limits, supplementing the information in Table 1 of the main text. Figure S1 is provided to address the statements near the end of Section 2.1, as follows:

> *Indeed, if we consider a doubling time of five days, reducing initial error amplitudes by half could potentially extend the predictability range by an additional five days. As a result, continuous extensions of predictability horizons become possible by minimizing initial errors, aligning with Arakawa's viewpoint.*

In Lorenz's (1969d) [29] influential work, Lorenz indicated that the doubling time of 5 days may be reasonablefor errors confined to the larger scales but may be unrealistic for errors in smaller scales. Contrasting with a focus on doubling time, saturation times were employed in order to assess predictability at different scales. However, the remaining five studies by Lorenz in 1969 and the study by Charney et al. (1966) [17] did not explicitly address the connection between doubling time and saturation time. In fact, as discussed below, a doubling time and saturation time from the same numerical experiments could yield different estimates for predictability limits.

Based on the Mintz–Arakwa model, the middle panel of Figure 2 indicated a doubling time of five days, but RMS errors in the same panel did not suggest saturation during a 30-day simulation. While the doubling time of 5 day was applied to estimate a predictability limit of two weeks, a saturation time, if determined, suggests a longer predictability. Similarly, in a recent investigation expanding the Lorenz 1965 28-variable model (Lorenz 1965) [53] to a 1,640-variable model (Krishnamurthy 2019 [10]), as shown in Figure S2, Krishnamurthy reported a comparable doubling time of 2.9 days alongside a saturation time of 100 days.

While employing the analogue approach, Lorenz (1969c) [32] obtained a doubling time of 8 days. However, the logistic differential equation he employed yielded an estimated doubling time of 2.5 days. In order to address the inconsistencies, Lorenz replaced the quadratic term (X^2) with a cubic term (X^3), resulting in a different doubling time of 5 days. Nevertheless, Lorenz acknowledged the lesser justification for choosing the cubic term. In fact, in a later work (Lorenz, 1982 [50]), it was suggested that results obtained using the quadratic hypothesis are "reasonable but not readily verifiable". Discrepancies in predictability estimates can also arise not only from employing different nonlinear terms (such as quadratic and cubic terms) but also from utilizing different types of ODEs (with the same initial growth rate), including first- and second-order ODEs (as shown in Figure 5 of Paxson and Shen 2022 [106]).

A recent reanalysis of the Lorenz 1969 model emphasized that the linear 1969 model produces three types of solutions, showing different growth rates and doubling times (e.g., Shen et al. 2022a , 2023a [16, 55]). As reported by Lewis (2005) [49], Arakawa did not believe that the predictability limit should be a fixed number. As such, doubling time should also not be a fixed number. In fact, the concept of attractor coexistence in recent studies (e.g., within the classical Lorenz 1963 model in Shen et al. (2021b) [109] and the generalized Lorenz model in Shen et al. (2021a) [51]) suggests distinct predictability, thereby different doubling times. In reality, the occurrence of different weather regimes (e.g., Lorenz 1984b [54]) also indicates different doubling times. In addition, as shown in Figure S3, a dependence of growth rates on geographical regions (e.g., tropics vs. midlatitudes) was reported (Reynolds et al. 1994 [107]; Kalnay 2002 [108]). All of the above discussions suggest the dependence of doubling times on different types of solutions, different weather systems, or different geographical regions. Thus, a comprehensive exploration of the relationship between doubling time, saturation time, and predictability limit warrants careful investigation.

Table S1: Evolution of Lorenz's Views on Predictability Horizons from the 1960s.

	Approaches/Tools	Limit	Remarks
1969a	analogues		a doubling time of 8 days
1969b	The L69 model	16.8 days	
1969c	A review of 1969a, b	"3 weeks"	*"the atmosphere possesses an intrinsic range of predictability, of perhaps three weeks."*
1972	a revisit of the 1969b study	20.6 days	An impact of a spectral gap
1973	analogues	at least 12 days	
1982	descriptions	No specific limit	*"...it does not reveal the range at which the uncertainty in prediction must become large."*
1984a	a review of Charney et al. (1966)	~ 2 weeks	*"good two-week forecasts might eventually become a reality"*
1984b	a revisit of the 1969b study	better than 16.8 days	Discrepancies in estimates using the L69 and ECMWF models
1985	a revisit of the 1969b study	20.6 days	Same as 1972
1993	a review of Charney et al. (1966)	~ 2 weeks	
1996	a many mode model	~ 2 weeks	
2007 (Reeves 2014)	an interview	No specific limit	*"It didn't tell you whether they were a week or year or what."*

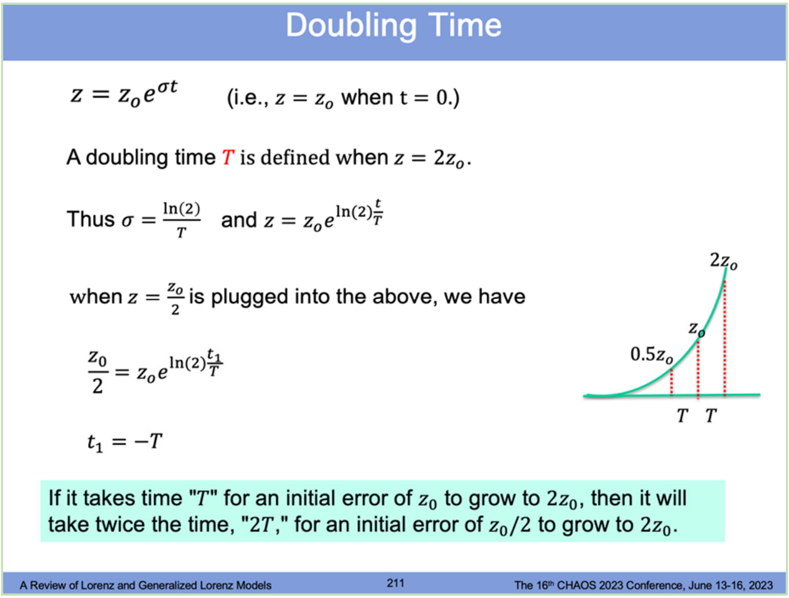
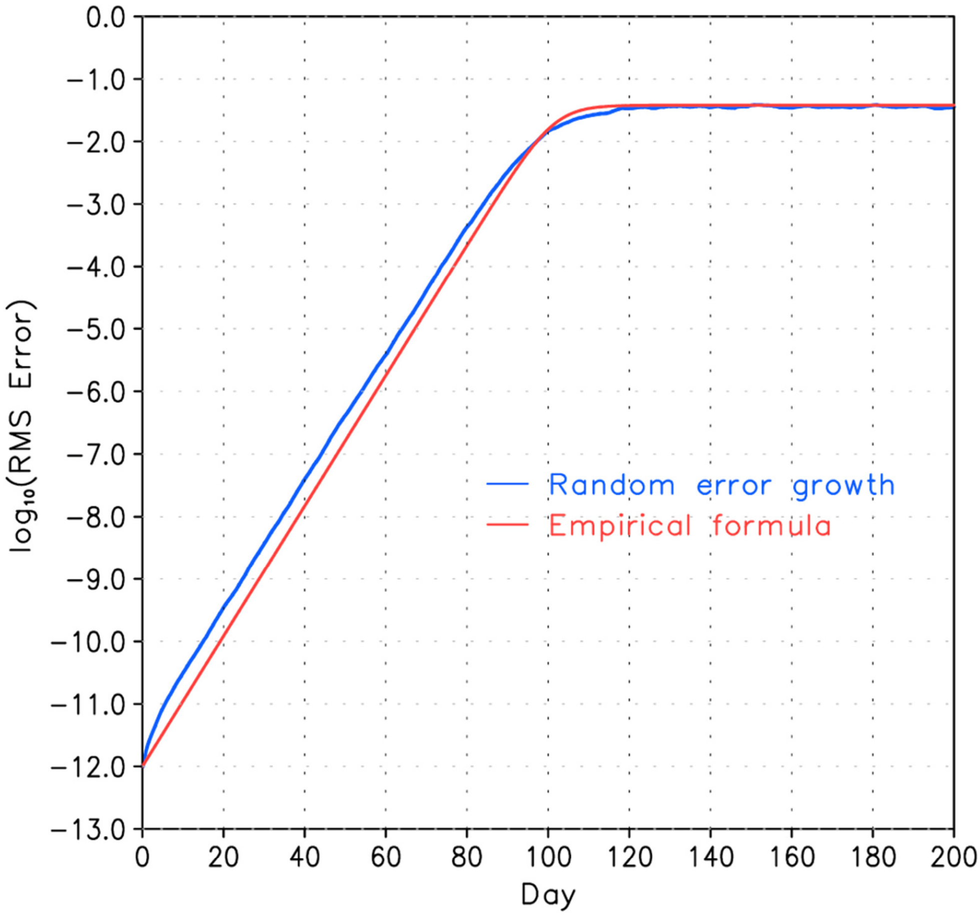
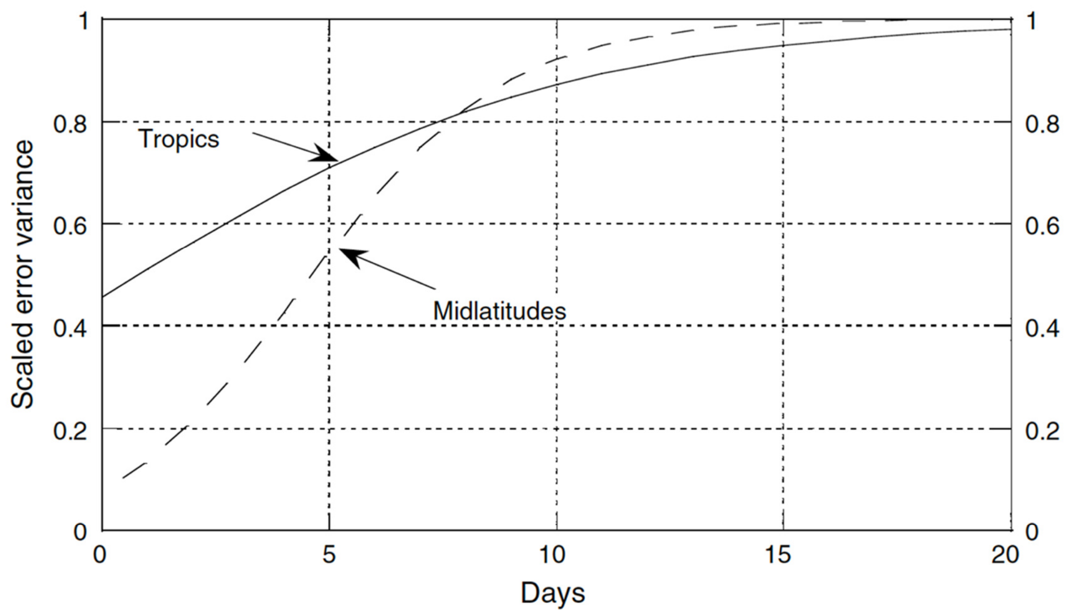


Figure S1: Predictability estimates using a doubling time. This figure shows that if we consider a doubling time of five days, reducing initial error amplitudes by half could potentially extend the predictability range by an additional five days



Root mean square (RMS) error (blue) growth of initial random errors introduced at 5,000 points in the attractor of the two-layer quasi-geostrophic model and the fit (red) of nonlinear empirical formula of equation (3).

Figure S2. Root mean square (RMS) errors obtained from the utilization of a 1,640-variable model (Krishnamurthy 2019 [10]). Outcomes indicate a doubling time of 2.9 days and a saturation time of 100 days.



Parameterization of scaled forecast error variance in the presence of model deficiencies with values of the growth rate due to model deficiencies s and to instabilities b appropriate for mid-latitudes (($b = 0.4$, $s = 0.05$) and the tropics ($b = 0.1$, $s = 0.1$).

Figure S3: The dependence of error growth rates on geographical locations (tropical vs. midlatitude regions) (Reynolds et al. 1994 [107]; Kalnay 2002 [108]).