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The Deep Physics Hidden within the Field Expressions of the Radiation Fields of Lightning Return Strokes

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Abstract: Based on the electromagnetic fields generated by a current pulse propagating from one point in space to another, a scenario that is frequently used to simulate return strokes in lightning flashes, it is shown that there is a deep physical connection between the electromagnetic energy dissipated by the system, the time over which this energy is dissipated and the charge associated with the current. For a given current pulse, the product of the energy dissipated and the time over which this energy is dissipated, defined as *action* in this paper, depends on the length of the channel, or the path, through which the current pulse is propagating. As the length of the channel varies, the action plotted against the length of the channel exhibits a maximum value. The location of the maximum value depends on the ratio of the length of the channel to the characteristic length of the current pulse. The latter is defined as the product of the duration of the current pulse and the speed of propagation of the current pulse. The magnitude of this maximum depends on the charge associated with the current pulse. The results show that when the charge associated with the current pulse approaches the electronic charge, the value of this maximum reaches a value close to $h/8\pi$ where h is the Plank constant. From this result, one can deduce that the time-energy uncertainty principle is the reason for the fact that the smallest charge that can be detected from the electromagnetic radiation is equal to the electronic charge. Since any system that generates electromagnetic radiation can be represented by a current pulse propagating from one point in space to another, the result is deemed valid for electromagnetic radiation fields in general.

Keywords: lightning; electromagnetic fields; accelerating charges; return stroke models; time-energy uncertainty principle

1. Introduction

One of the most famous mathematical models used to simulate return strokes in lightning flashes is the transmission line model [1]. In this model, the return stroke is represented as a current pulse that propagates from ground to cloud with constant speed and without attenuation. The model has been used successfully in the remote sensing of the currents in lightning return strokes [2]. This model is modified by several scientists by relaxing its simplifying assumptions. For example, in the model introduced by Nucci *et al.* [3], the current amplitude decreases exponentially along the channel and in the one introduced by Rakov and Dulzon [4], the current amplitude decreases linearly. Cooray and Orville [5] introduced both current attenuation and dispersion while allowing the return stroke speed to vary along the channel.

Even though the transmission line model or the physical scenario associated with it (*i.e.*, propagation of a current pulse from one point in space to another with constant speed and without attenuation) is used mainly in lightning research, it can actually be used to represent other electromagnetic radiation systems as well. For example, if the duration of the current pulse is much longer than the time necessary for the current pulse to travel the length of the path, then the physical system and the resulting radiation fields become identical to those of a short dipole [6,7]. Since the electromagnetic radiation is created by the acceleration of charges associated with the spatial and temporal variations of electric currents [8], this model should be able to represent any radiating system if we relax the assumptions of constant speed and absence of attenuation of the current waveform. For example, references given in the previous section (*i.e.*, [3–5]) show that by introducing attenuation and dispersion into this model, one can utilize it to model a more realistic return stroke.

In this paper, the above-mentioned model is used to study the relationship between the electromagnetic radiation, the time over which this electromagnetic radiation is generated and the charge associated with the current. It will be shown that the product of the magnitude of the emitted radiation and the time over which it is emitted is deeply connected to the fact that the minimum charge that can be transported by the current pulse is equal to the electronic charge. However, the first step of our study is to evaluate the electromagnetic energy generated by the system under consideration. This evaluation is conducted by utilizing the electric fields of accelerating charges as presented by Cooray and Cooray [7,9]. The relevant field equations are given in the following sections.

2. Electromagnetic Fields Generated by A Current Pulse that Moves From One Point in Space to Another along A Straight Line with Uniform Velocity and without Attenuation

The geometry under consideration is shown in Figure 1. A current pulse originates at point S1 and travels with uniform speed u without attenuation or dispersion towards S2. At S2, the current is terminated. As shown by Cooray and Cooray [7,9] the total electric field at point P, generated by this process, has five components. They are as follows: (i) the radiation field generated from S1 during the acceleration of charge when the current is initiated, (ii) the radiation field generated from S2 during the charge deceleration as the current is terminated, (iii) the electrostatic field generated by the negative charge accumulated at S1 when the positive charge travels towards S2, (iv) the electrostatic field generated by the accumulation of positive charge at S2, and (v) the velocity field generated as the current pulse moves along the channel. The magnetic field generated by the current flow consists of three terms, namely, two radiation fields generated at S1 and S2, and the velocity field generated as the current pulse propagates along the path. Let us now write down the expressions obtained by Cooray and Cooray [7,9] for these field components.

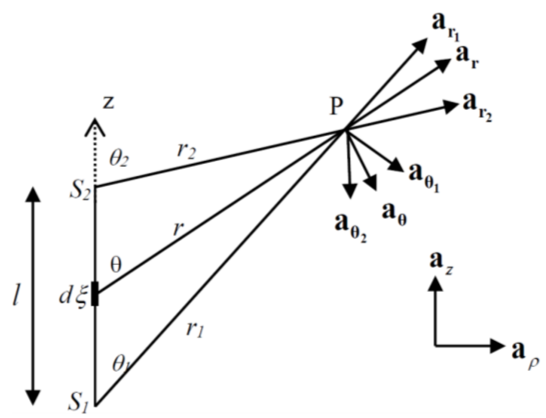


Figure 1. Geometry used in deriving the electromagnetic fields of a propagating current pulse.

2.1. The Electric Radiation Field Generated From S1

Let us assume that the current pulse leaving S1 can be represented by $i(t)$. The radiation field generated due to the acceleration of charges at point P is given by Cooray and Cooray [7,9].

$$\mathbf{e}_{rad,S_1} = \frac{i(t - r_1/c)u\sin\theta_1}{4\pi\varepsilon_o c^2 r_1} \frac{1}{\left[1 - \dfrac{u\cos\theta_1}{c}\right]} \mathbf{a}_{\theta_1} \tag{1}$$

2.2. The Electric Radiation Field Generated From S2

The radiation field generated due to the deceleration of charges at S2 is given by

$$\mathbf{e}_{rad,S_2} = -\frac{i(t - l/u - r_2/c)u\sin\theta_2}{4\pi\varepsilon_o c^2 r_2} \frac{1}{\left[1 - \dfrac{u\cos\theta_2}{c}\right]} \mathbf{a}_{\theta_2} \tag{2}$$

2.3. The Static Field Generated by the Accumulation of Negative Charge at S1

The negative charge accumulation at S1 is equal to the integral of the current, and the field component generated by the charges is given by

$$\mathbf{e}_{stat,S_1} = -\frac{\int\limits_{t-r_1/c}^{t} i(\xi)d\xi}{4\pi\varepsilon_o r_1^2} \mathbf{a}_{\mathbf{r_1}} \qquad t > r_1/c \tag{3}$$

2.4. The Static Field Generated by the Accumulation of Positive Charge at S2

The component of the static field generated by the accumulation of positive charge at S2 is given by

$$\mathbf{e}_{stat,S_1} = \frac{\int\limits_{t-l/u-r_2/c}^{t} i(\xi)d\xi}{4\pi\varepsilon_o r_2^2} \mathbf{a}_{\mathbf{r_2}} \qquad t > l/u + r_2/c \tag{4}$$

2.5. The Velocity Field Generated as the Current Pulse Propagates along the Channel Element

The component attributable to the velocity field generated as the current pulse propagates along the channel can be written as [7,9].

$$\mathbf{e}_{vel} = \int\limits_0^l \frac{i(t - \xi/u - r/c)\left\{1 - \dfrac{u^2}{c^2}\right\}}{4\pi\varepsilon_o r^2 \left[1 - \dfrac{u}{c}\cos\theta\right]^2} \left[\frac{\mathbf{a_r}}{u} - \frac{\mathbf{a_z}}{c}\right] d\xi \tag{5}$$

Since the vector $\mathbf{a}_\mathrm{r}$ varies as one moves along the channel, the above equation can be decomposed into components along vertical (z-direction) and horizontal (ρ-direction) as follows:

$$\mathbf{e}_{vel} = \int\limits_0^l \frac{i(t - \xi/u - r/c)\left\{1 - \dfrac{u^2}{c^2}\right\}}{4\pi\varepsilon_o r^2 \left[1 - \dfrac{u}{c}\cos\theta\right]^2} \left[\frac{\cos\theta\mathbf{a}_z}{u} + \frac{\sin\theta\mathbf{a}_\rho}{u} - \frac{\mathbf{a_z}}{c}\right] d\xi \tag{6}$$

2.6. Magnetic Radiation Field Generated from S1

The magnetic radiation field generated from S1 is given by

$$\mathbf{b}_{rad,S_1} = \frac{i(t - r_1/c)u\sin\theta_1}{4\pi\varepsilon_o c^3 r_1} \frac{1}{\left[1 - \dfrac{u\cos\theta_1}{c}\right]} \mathbf{a}_{\boldsymbol{\varphi}} \tag{7}$$

Note that the magnetic field is in the azimuthal direction.

2.7. Magnetic Radiation Field Generated from S2

The magnetic radiation field generated from S2 is given by

$$\mathbf{b}_{rad,S_2} = -\frac{i(t - l/u - r_2/c)u\sin\theta_2}{4\pi\varepsilon_o c^3 r_2} \frac{1}{\left[1 - \dfrac{u\cos\theta_2}{c}\right]} \mathbf{a}_{\boldsymbol{\varphi}} \tag{8}$$

2.8. Magnetic Velocity Field Generated as the Current Pulse Propagate along the Channel Element

The magnetic velocity field generated as the current pulse propagates along the channel is given by

$$\mathbf{b}_{vel} = \int_0^l \frac{i(t - \xi/u - r/c)\left\{1 - \dfrac{u^2}{c^2}\right\}\sin\theta}{4\pi\varepsilon_o r^2 c^2 \left[1 - \dfrac{u}{c}\cos\theta\right]^2} \mathbf{a}_{\boldsymbol{\varphi}} dl \tag{9}$$

The field components given by Equations (1)–(9) provide a complete description of the electric and magnetic fields generated by the current pulse propagating with uniform velocity and without attenuation.

The set of equations given above can be utilized in various applications in physics and lightning research. Even though the equations are given for a pulse propagating with uniform speed without attenuation, they can be utilized to estimate electromagnetic fields in situations where the attenuation and dispersion of the current pulse take place. In such cases, the channel can be divided into small sections and in each section the pulse is assumed to propagate with uniform speed and without distortion. By summing up the contribution from all the elements, one can calculate the electromagnetic fields from any event where the speed, current amplitude and the current shape vary in time and space. As one can see from the equations, when the current pulse is propagating at the speed of light, the fields become pure radiation, except for the close static fields associated with the charge accumulated at the ends of the channel. Let us now consider a specific example and illustrate the features of the radiation fields generated by the above physical system. We will consider a current pulse propagating with speed u along a channel of length l. We assume that the current waveform injected into the channel has the form of a half sine wave with a duration $\boldsymbol{\tau}$. That is, the injected current waveform is given by

$$i(t) = I_0\sin(\pi t/\tau) \; t \leqslant \tau \tag{10}$$

The charge transported by the current waveform, q, is

$$q = \int_0^\tau I_0\sin(\pi t/\tau)dt \tag{11}$$

After performing this integration we obtain

$$q = \frac{2I_0\tau}{\pi} \tag{12}$$

Thus, the expression for the current waveform can also be given as

$$i(t) = \frac{q\pi}{2\tau}\sin(\pi t/\tau)\ t \leqslant \tau \tag{13}$$

Now, in order to illustrate the radiation fields let us consider the case $u = c$ where c is the speed of light in free space and the duration of the current pulse τ is equal to l/c. The problem under consideration is depicted pictorially in Figure 2. The current pulse starts at point S1 and after travelling the length l it is absorbed at S2. The radiation field generated by the initiation of the current pulse at point S1 and the absorption of the current pulse at S2 at a distant location P where the distance r is much greater than the channel length l is given by (the relevant geometry is shown in Figure 3).

$$E_{rad} = \frac{\sin\theta}{4\pi\varepsilon_o c r_1}\frac{I_0}{[1-\cos\theta]}\left(\sin\frac{\pi c}{l}(t-r/c)\right)\mathbf{a}_\theta \quad r/c < t < r/c + l/c \tag{14}$$

$$E_{rad} = \frac{\sin\theta}{4\pi\varepsilon_o c r_1}\frac{I_0}{[1-\cos\theta]}\left(\sin\frac{\pi c}{l}(t-r/c) - \sin\frac{\pi c}{l}(t-r/c-l/c)\right)\mathbf{a}_\theta \quad t > r/c + l/c \tag{15}$$

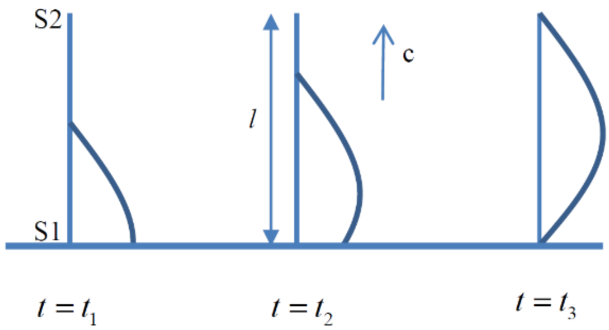


Figure 2. The figure illustrates pictorially the propagation of a current pulse in the shape of a half sine wave with a duration l/c propagating along the channel of length l with speed c. Note that t is the time and $t_1 < t_2 < t_3$. Note that the duration of the current is the same as the time required for the current front to traverse the channel.

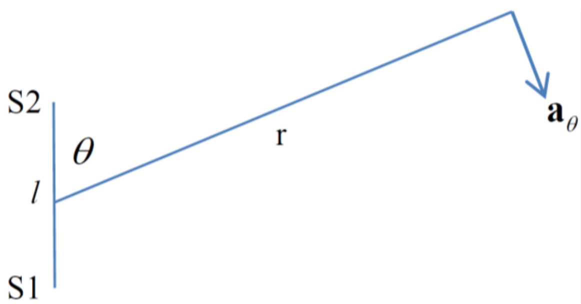


Figure 3. The geometry relevant to the Equations (14) and (15).

The field given in Equation (14) (and the first term in 15) is generated during the initiation of the current at S1 and the second term in 15 is generated during the termination of the current at S2. From these expressions, one can see that the shape of the radiation field is a function of the angle θ. The radiation field at a distance of 100 km for a channel length of 1.5 km is shown in Figure 4. In this example I_0 = 30 kA. Note how the shape of the radiation field becomes narrower as the angle θ increases. This shows that the high frequency content of the radiation field increases with decreasing angle of elevation.

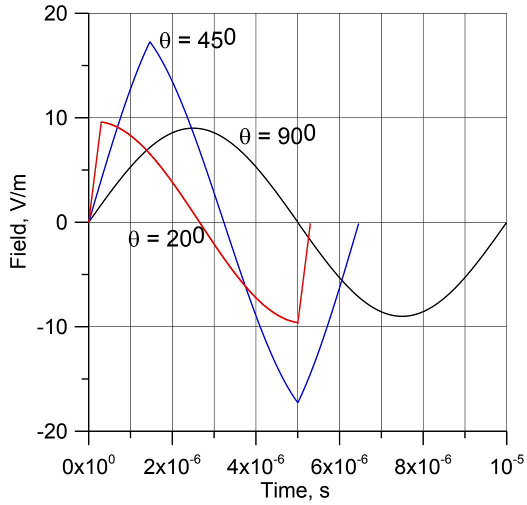


Figure 4. Electric radiation field for r = 100 km for three values of angle θ. The angle θ and distance r are defined in Figure 3. The peak current associated with the pulse, I_0, is equal to 30 kA.

3. Energy Dissipated by A Current Pulse that Moves from One Point in Space to Another along A Straight Line with Uniform Velocity and without Attenuation

Consider the example of a current pulse moving from S1 to S2 with constant speed u without attenuation (Figure 1). As before, let us denote the distance between S1 and S2 by l. The electric radiation fields generated by this physical system are given in Equations (1), (2), (14) and (15). At far distances the parameters r_1 and r_2 can be replaced by r, and θ_1 and θ_2 can be replaced by θ (see Figure 3). The Poynting vector associated with these fields are given by

$$\mathbf{S}(\theta,t) = \frac{i(t-r/c)^2u^2\sin^2\theta}{(4\pi)^2\,\varepsilon_0c^3r^2}\frac{1}{\left[1-\dfrac{u}{c}\cos\theta\right]^2}\mathbf{a_r}\ \text{ for } r/c<t<r/c+l/u \tag{16}$$

$$\mathbf{S}(\theta,t) = \frac{u^2\sin^2\theta}{(4\pi)^2\,\varepsilon_0c^3r^2}\frac{[i(t-r/c)-i(t-r/c-l/u)]^2}{\left[1-\dfrac{u}{c}\cos\theta\right]^2}\mathbf{a_r}\ \text{ for } t>r/c+l/u \tag{17}$$

These expressions are identical to those derived by Krider [10]. The power dissipated by the system is then given by

$$P(t) = \int_0^{2\pi}\int_0^{\pi} S(\theta,t)r^2\sin\theta\,d\theta\,d\phi \tag{18}$$

Since the system is located symmetrically with respect to the variable φ, this can be written as

$$P(t) = 2\pi\int_0^{\pi} S(\theta,t)r^2\sin\theta\,d\theta \tag{19}$$

The total energy dissipated is then given by

$$U = \int_0^{\infty} P(t)dt \tag{20}$$

Let us again consider the current waveform described by Equation (10). Note that since the power dissipation is proportional to ${I_0}^2$ from Equation (12) we find that the power is proportional to q^2.

Actually, one can show that this is true for any current waveform. This is the case since for any given current waveform the amplitude is proportional to the total charge associated with the current. Thus, the total energy dissipated by the current pulse is also proportional to q^2. We can write therefore that

$$U = q^2 F(\tau, l, u) \tag{21}$$

In the above equation F(τ, l, u) is a function of the parameters τ, l and u. In order to illustrate the effect of velocity of propagation on the energy dissipated by the propagating current, let us consider the same current waveform as given by Equation (10) with a duration equal to 100 μs. In our calculation we consider l = 5 km. Furthermore, we select the peak amplitude of the current waveform in such a way that the total charge carried by the current waveform is 5 C (*i.e.*, $I_0 = q\pi/2\tau$ with $q = 5C$). These parameters are typical for a lightning return stroke.

The energy transmitted radially across a unit area located at a distance of 100 km as a function of the elevation angle is shown in Figure 5. The data show that for a given angle of elevation, the energy transmitted increases with increasing speed of the current pulse. For a given speed, the energy transmitted increases as the elevation angle increases, reaches a peak, and then starts to decrease with the angle. The angle at which the transmitted energy reaches a peak moves towards smaller elevation angles as the return stroke speed increases. The total energy radiated by the system as a function of the charge carried by the current and the speed of propagation of the current are shown in Figure 6 (plot a). For comparison purposes, the electromagnetic energy generated by a current pulse that can be represented by the equation $i(t) = i_0 e^{-t/\tau}$ (with τ = 50 μs), which is a better approximation for the return stroke current, is also shown in the figure (plot b). Note again that the total energy dissipated increases with increasing charge and with increasing speed of propagation. Results show that for q = 5 C and $u = 10^8$ m/s, the radiated energy is about 5×10^5 J. The values used in the calculation represent typical charge and speed of return strokes in lightning flashes

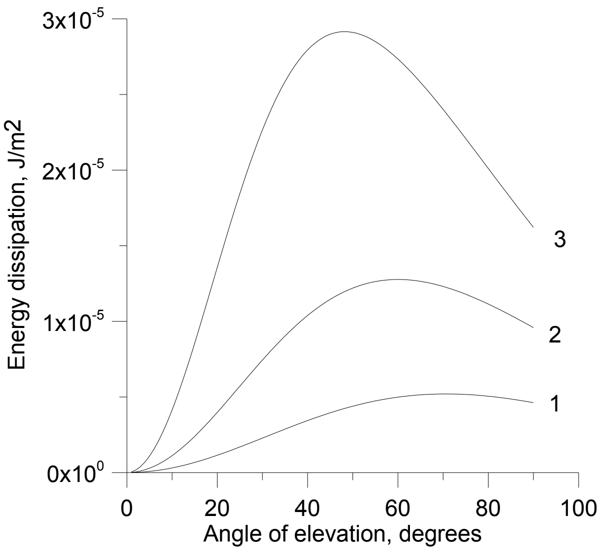
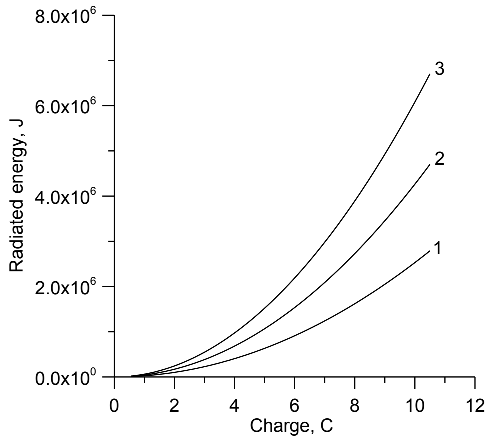
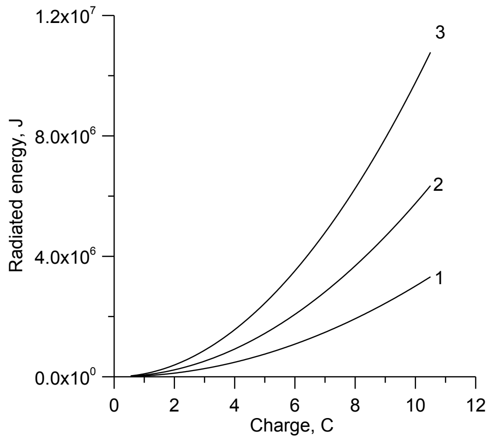


Figure 5. The energy transmitted radially across a unit area at different elevation angles at a distance of 100 km by a half sinusoidal current waveform carrying a charge of 5 C. The results are shown for three current propagation speeds. (1) $u = 1.0 \times 10^8$ m/s. (2) $u = 1.5 \times 10^8$ m/s. (3) $u = 2.0 \times 10^8$ m/s.



(**a**)



(**b**)

Figure 6. The total energy radiated by the current pulse as a function of the associated charge. The results are shown for three speeds of propagation. Plot (**a**) corresponds to half sine waveform and plot; (**b**) corresponding to exponential current. (1) $u = 1.0 \times 10^8$ m/s; (2) $u = 1.5 \times 10^8$ m/s; (3) $u = 2.0 \times 10^8$ m/s.

4. The Connection between the Energy Emitted by a Current Pulse that Moves from One Point in Space to Another along A Straight Line with Uniform Velocity and without Attenuation and the Time of Emission of Radiation

The next step of our investigation is to study the energy radiated by the current pulse as a function of the duration of the current and the charge associated with it. In order to make the analysis as broad as possible, we will consider the duration of the current pulse as a variable parameter. This is done as follows. Since the highest radiation for a given charge is produced when the speed of propagation is equal to the speed of light in free space, we assume that the current pulse propagates at this speed. Now, the time of travel of the current pulse from S1 to S2 is given by l/c. We assume that the duration of the current pulse is given by kl/c where k is a positive constant. As k varies from +0 to infinity, the duration of the current pulse will take all possible values pertinent to the system under consideration. The example where $k = 1$ is shown in Figure 2 for one particular current waveform having a half sine wave shape (defined by Equation (10)). By changing the value of k, one would be able to study the effect of current pulses of different durations on the emitted radiation. When k is much smaller than 1, the current duration is much shorter than the time necessary for the current pulse to propagate along the path and when k is much larger than 1 the current duration is much longer than this time. First, we will obtain results pertinent to a current waveform having the shape of a half sine wave. Later, we will study the effect of changing the current wave shape on the results.

Figure 7 shows the energy dissipated by the current pulses having a half sine wave shape carrying different charges as a function of the parameter k. Note how the energy dissipated varies with k. For a given charge the dissipated energy increases as the duration of the current pulse decreases. As one can see from Figure 4, the radiation appears as two bursts, one caused by the initiation of the current pulse and the other by the termination of the current at the opposite channel end. In some regions of space these two pulses appear separately and in other regions they merge into each other. As the duration of the current pulse decreases, the spatial region where the radiation field appears as two bursts increases. The duration of each burst is equal to the duration of the current pulse. Thus, the duration of the emitted radiation, Δt, is about $2kl/c$. Now, the product of the energy radiated by the system and the duration over which it is emitted has the units of action. Let us call this parameter the "action". The action associated with the emitted radiation, A, is given by

$$A = U\frac{2kl}{c} \tag{22}$$

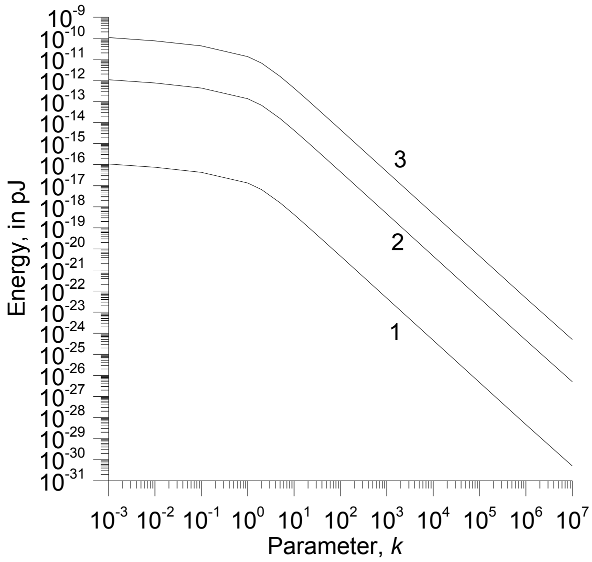


Figure 7. Energy dissipated by current pulses having a half sine wave shape carrying different charges as a function of the parameter k. (1) $q = 1e$; (2) $q = 100e$; (3) $q = 1000e$. Here e is the electronic charge.

The action, as a function of k, for different values of charges is shown in Figure 8. Observe that the action has a maximum value close to the value $k = 1$. Recall that at $k = 1$, the duration of the current pulse matches the time of travel along the path. The value of this maximum decreases with decreasing charge and the lowest value corresponds to the lowest possible charge in the current waveform, namely, the electronic charge. The question is whether the presence of this maximum is unique to the particular current waveform used in the calculation or whether it is a general result. In order to check this point, the calculation is repeated with several other shapes for the current waveform and the results were observed to hold irrespective of the shape of the current waveform. The results obtained for the following two current waveshapes are presented as an example here.

$$i(t) = i_0 \exp(-t/\tau_1) \tag{23}$$

$$i(t) = i_0 t \exp(-t/\tau_2) \tag{24}$$

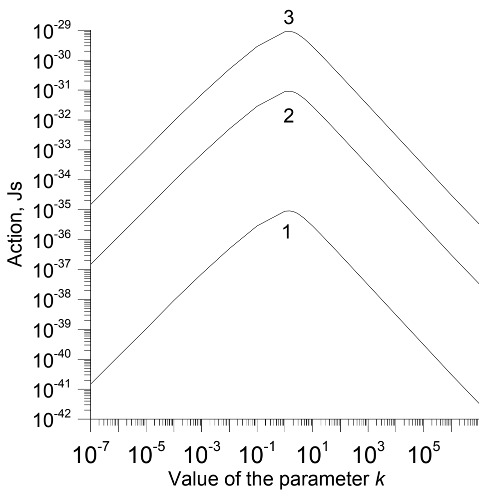


Figure 8. The action, as a function of k, for different values of charges. The current used in the calculation has the half sine waveshape. (1) $q = 1e$; (2) $q = 100e$; (3) $q = 1000e$. Note that e is the electronic charge.

The values of τ_1 and τ_2 were selected in such a way so that the total duration of these current waveforms, *i.e.*, the time over which the current waveform decays to 99% of its peak amplitude, is equal to kl/c. The results obtained are shown in Figures 9a and 9b. The most interesting observation that is valid for all the current waveforms is the fact that there is a maximum in the action for a given charge when the duration of the current waveform almost matches the time of propagation. This maximum value varies only slightly from one current waveform to another. Since one electron is the minimum charge that can be associated with the current pulse, the maximum value associated with the curve corresponding to one electron is the smallest maximum that can occur in nature. We have studied the magnitude of this maximum and found out that it is in some way connected to the Plank's constant. For example, the curves corresponding to one electronic charge depicted in Figures 8 and 9 were re-plotted in Figure 10 by dividing the numbers on the y-axis by $h/8\pi$. Note that the maximum value of the action when the charge in the current waveform is of the order of an electron is of the order of magnitude of $h/8\pi$. As we can see from the data presented, the result is more or less the same irrespective of what current waveform is used in the analysis. This forced us to conclude that the result is a general one and is independent (as far as order of magnitude is concerned) of the shape of the current waveform. This result seems to indicate that when the charge that is being associated with the propagation current pulse approaches that of an electron, even the classical fields show some features that are connected to the quantum nature. The result can also be interpreted as follows. First, note that electronic charge is the minimum that can occur in nature. Therefore, in order to generate an action that is larger than or equal to $h/8\pi$, the charge associated with any current pulse has to be larger than or equal to that of an electron. In other words, one can write

$$U\Delta t \geqslant \frac{h}{8\pi} \Rightarrow q \geqslant e \tag{25}$$

where q is the charge associated with the current pulse, Δt is the duration of radiation emission and e is the electronic charge. Note that the reciprocal of this relationship is not valid because a charge larger than e can still generate an action that is less than $h/8\pi$ depending on the duration of the current waveform. This can be seen very clearly from the data depicted in Figures 8 and 9.

The physical system that we have considered is very general and therefore one can consider the above result also to be a general condition valid for classical electromagnetic fields. This result shows that the radiation fields that have been utilized to study the lightning return strokes can reveal fundamental information that is important both in physics and in systems when the charges involved are close to the electronic charge.

In the derivation given earlier we assumed that the speed of propagation of the current pulses is equal to the speed of light. As we have seen before, the energy radiated by the system increases with increasing speed and it reaches its peak value when the speed is equal to the speed of light. Since the speed of light is the maximum speed that can be achieved by the propagating current pulse, the condition given by Equation (25) remains valid even for lower speeds. In our derivation we have also assumed that the current waveform propagates along the path without attenuation. At distances where the field is pure radiation, the dimension of the source can be neglected in comparison to the distance to the observation point. That means the source appears as a point source from the observation point. Even in this case, whether the current attenuates stepwise along the channel or whether it attenuates suddenly at the end of the channel will make a difference to the total energy radiated by the current. One can show that the effect of attenuation is to reduce the magnitude of the radiated energy. Therefore, this assumption does not change the results obtained either. These considerations show that the case that we have considered is a very general one and the relationship given by Equation (25) can be considered generally valid for classical radiation fields.

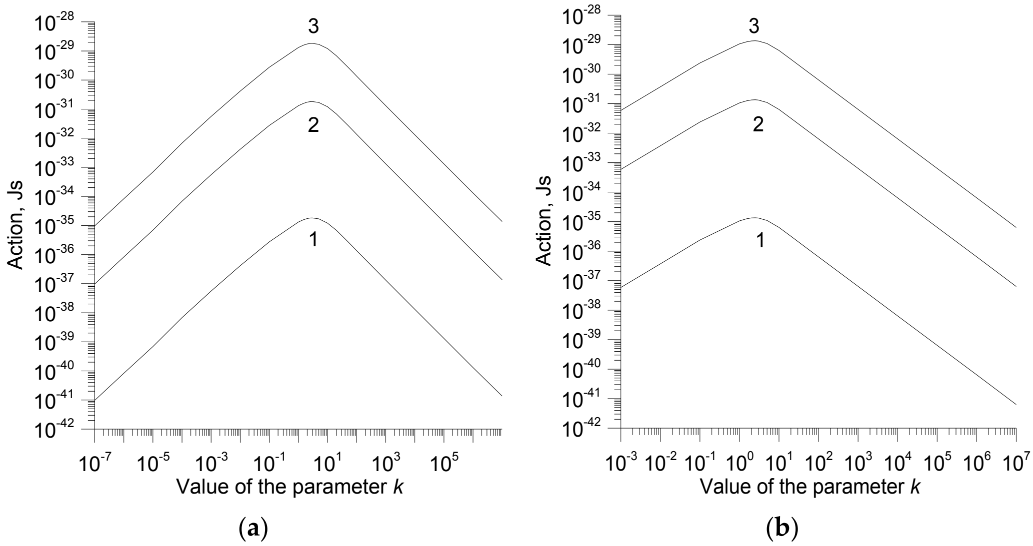


Figure 9. Same as Figure 8 except that the current waveform is given by (**a**) Equation (23); (**b**) Equation (24).

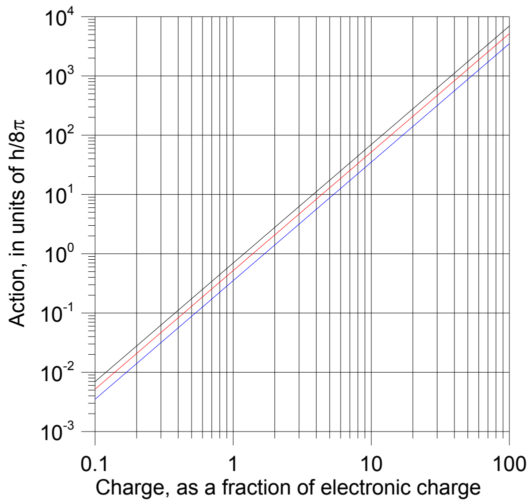


Figure 10. The value of the maximum observed in Figures 8 and 9 as a function of the charge associated with the current.

5. Discussion

In this paper we have used the parameter that we have called 'action' to study the connection between the electromagnetic energy dissipated by the system, the duration of its emission and the electric charge associated with the current. As pointed out by one of the referees of this paper, it is of interest to study whether this parameter could also be used to characterize lightning return strokes and correlate the action to the effects of the return stroke electromagnetic fields on electrical systems. One interesting fact that we have observed in this paper is that the action is maximum when the duration of the current waveform matches the time required for the current waveform to propagate along the channel. Further studies are needed to correlate the effects of electromagnetic fields, such as induced voltages in power systems, with the action of the electromagnetic fields. Another area that the concept of action is of interest is the interaction of electromagnetic fields with the upper atmosphere. In this case too, the influence of the charge particles in the ionosphere by the electromagnetic fields may depend on the action of the electromagnetic fields. Of course one need not limit oneself to return strokes in applying this concept to study the effects of electromagnetic fields. This concept could also be used to study the effect of electric and magnetic fields caused by electrostatic discharges on miniature electronic circuits.

The main result that we have obtained in this paper is summarized in Equation (25). In order to show its physical significance, let us continue as follows. It is a known fact that the electric charge is quantized and the smallest charge freely available in nature is the electronic charge. Consider an experiment where an attempt is made to measure the charge transported by the current from the dissipated energy using Equation (21). Under the best conditions, the measurement of charge can be performed to an accuracy of one electron. When the charge associated with the current approaches that of an electron (*i.e.*, e) the uncertainty in the measurement of the charge, say Δe, becomes comparable to the charge e itself. Thus, for the electronic charge the relationship given in Equation (25) can be written as

$$e\Delta e F(\tau, l, c)\Delta t = \frac{h}{8\pi} \tag{26}$$

where we have substituted for U from Equation (21) and replaced e^2 by $e\Delta e$ (note that in this case $q = e$). In the case of charges larger than the electronic charge, $q > e$ and $\Delta q \geqslant e$ (note that Δq is the uncertainty in measurement), the above equation can be written as

$$q\Delta q F(\tau, l, c)\Delta t \geqslant \frac{h}{8\pi} \tag{27}$$

From Equation (21) one can also see that $\Delta U = 2q\ \Delta q\ \mathrm{F}(\tau,l,c)$ and the above equation can be written as

$$\Delta U \Delta t \geqslant \frac{h}{4\pi} \tag{28}$$

In this equation ΔU is the uncertainty in the electromagnetic energy and Δt is the duration of the emission of the radiation. Interestingly, the above equation is identical to the earliest proposed version of the time-energy uncertainty principle [11]. This shows that the time-energy uncertainty principle is governing the fact that the smallest charge that can be detected from the electromagnetic radiation is equal to the electronic charge. One can also conclude that the application of time-energy uncertainty principle to the electromagnetic radiation leads to the fact that the smallest detectable charge in nature is the electronic charge. Of course, the calculations presented in this paper are based purely on the classical electromagnetic fields and it is of interest to look at the same problem from a quantum mechanical point of view.

6. Conclusions

The expressions for the electromagnetic fields generated by a current pulse moving from one point in space to another are utilized to extract important information concerning the physical nature of the 'action' of the electromagnetic fields defined as the product of the emitted energy and the time of emission. For currents transporting charges on the order of an electron, the action has a maximum whose magnitude is on the order of $h/8\pi$ where h is the Plank constant. From this result one can come to the conclusion that the time-energy uncertainty principle is closely related to the fact that the smallest observed charge in nature is the electronic charge. Since the source of the radiation fields used in the study is very general, one can infer that the results presented here are valid for any system radiating in free space.

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References

1. Uman, M.A.; McLain, D.K. Magnetic field of lightning return stroke. *J. Geophys. Res.* **1969**, *74*, 6899–6910. [CrossRef]
2. Willett, J.C.; Idone, V.P.; Orville, R.E.; Leteinturier, C.; Eybert-Berard, A.; Barret, L.; Krider, E.P. An experimental test of the "transmission-line model" of electromagnetic radiation from triggered lightning return strokes. *J. Geophys. Res.* **1988**, *20*, 3867–3878. [CrossRef]
3. Nucci, C.A.; Mazzetti, C.; Rachidi, F.; Ianoz, M. On lightning return stroke models for LEMP calculations. In Proceedings of the 19th International Conference on Lightning Protection, Graz, Austria, 25–29 April 1988.
4. Rakov, V.A.; Dulzon, A.A. A modified transmission line model for lightning return stroke field calculation. In Proceedings of the 9th International Symposium on Electromagnetic Compatibility, Zurich, Switzerland, 12–14 March 1991; pp. 229–235.
5. Cooray, V.; Orville, R.E. The effects of variation of current amplitude, current risetime and return stroke velocity along the return stroke channel on the electromagnetic fields generated by return strokes. *J. Geophys. Res.* **1990**, *95*, D11. [CrossRef]
6. McLain, D.K.; Uman, M.A. Exact expression and moment approximation for the electric field intensity of the lightning return stroke. *J. Geophys. Res.* **1971**, *76*, 2101–2105. [CrossRef]
7. Cooray, G.; Cooray, V. Electromagnetic fields of a short electric dipole in free space—Revisited. *Prog. Electromagn. Res.* **2012**, *131*, 357–373. [CrossRef]
8. Panofsky, W.K.H.; Phillips, M. *Classical electricity and magnetism*; Addison-Wesley publishing company: London, UK, 1962.
9. Cooray, V.; Cooray, G. The Electromagnetic fields of an accelerating charge: Applications in lightning return-stroke models. *IEEE Proc.* **2010**, *52*, 944–955. [CrossRef]
10. Krider, E.P. On the electromagnetic fields, poynting vector and peak power radiated by lightning return strokes. *J. Geophys. Res.* **1992**, *97*, 15913–15917. [CrossRef]
11. Busch, P. *The Time–Energy Uncertainty Relation, in Time in Quantum Mechanics*, 2nd ed.; Springer-Verlag: Berlin, Germany, 2009.