Supplementary Information

SI 1. INCA-PEco supplementary equations documentation

INCA-PEco is a semi-distributed model, which simulates the flow of water, sediment, phosphorus, dissolved oxygen and biochemical oxygen demand through the terrestrial system, to river reaches, differentiated by landuse type. In-stream simulation of river flow, phosphorus (dissolved and particulate), sediment, dissolved oxygen, biochemical oxygen demand and phytoplankton are made by accounting for inputs from land, upstream, sewage effluent, sediment interactions and biological processes. Mass fluxes are solved as systems of linked first order differential equations (ODEs), through one or more sub-catchments comprised of one or more different land use types, up to a network of multiple reaches. A mass balance is imposed at all sub-catchment and river reach boundaries, operating on multiple scales: a cell level for each individual land use; a sub catchment level for multiple land use inputs and transport of nutrient fluxes along river reaches; and a network scale for total loads from all sub-catchments.

INCA-PEco is driven primarily by a time series input of soil moisture deficit (SMD), hydrologically effective rainfall (HER), air temperature ($\mathrm{Temp_{air}}$) and precipitation (PPTN). In the model, HER is converted to a flux ($\mathrm{HER_f}$) (m^3 s^{-1} km^{-2}) using the equation:

$$\mathrm{HERf} = \frac{10^3}{86{,}400} \times HER \qquad \text{Eqn 1}$$

It must be considered, however, that not all precipitation falling to the ground is immediately available in liquid form, and that delayed additions of water from snowmelt to the soil can significantly impact hydrology in some regions. The liquid portion of PPTN (rainfall plus snowmelt, or $\mathrm{PPTN_{r+s}}$) is therefore calculated by taking into account the proportion of precipitation falling as snow ($\mathrm{Snow_{Ppt}}$) , the proportion of snow which is melting (SM) and a user defined rainfall correction factor ($\mathrm{R_{RCorrect}}$).

$$\mathrm{PPTN_{r+s}} = \frac{10^3}{86{,}400} \times \left(\mathrm{R_{RCorrect}}\left(\mathrm{PPTN} - \mathrm{Snow_{Ppt}}\right)\right) + SM \qquad \text{Eqn 2}$$

$\mathrm{Snow_{Ppt}}$ is calculated by defining a temperature threshold ($\mathrm{R_{Rtemp}}$), above which all precipitation falls as rain; and a separate temperature threshold ($\mathrm{R_{Stemp}}$) below which all precipitation falls as snow:

$$if\ \mathrm{Temp_{air}} \geq \mathrm{R_{Rtemp}} : \mathrm{Snow_{Ppt}} = 0 \qquad \text{Eqn 3}$$

$$if\ \mathrm{R_{Stemp}} < \mathrm{Temp_{air}} < \mathrm{R_{Rtemp}} : \mathrm{Snow_{Ppt}} = \mathrm{R_{SCorrect}} \times \mathrm{PPTN}\left(\frac{\mathrm{R_{Rtemp}} - \mathrm{Temp_{air}}}{\mathrm{R_{Rtemp}} - \mathrm{R_{Stemp}}}\right) \qquad \text{Eqn 4}$$

$$if\ \mathrm{Temp_{air}} \leq \mathrm{R_{Stemp}} : \mathrm{Snow_{Ppt}} = \mathrm{R_{SCorrect}} \times \mathrm{PPTN} \qquad \text{Eqn 5}$$

Snowmelt (SM) is calculated as the minimum of either water lost from the snowpack through snowmelt, or the snow water depth equivalent ($\mathrm{Snow_{Deqv}}$):

$$SM = minimum\left(\mathrm{R_{sdeg}}(\mathrm{Temp_{air}} - \mathrm{R_{melt}}), \mathrm{Snow_{Deqv}}\right) \qquad \text{Eqn 6}$$

Where $\mathrm{Snow_{Deqv}}$ is calculated as that from the previous timestep ($\mathrm{Snow_{Deqv0}}$); the quantity of precipitation falling as snow ($\mathrm{Snow_{Ppt}}$), snow melt (SM), and evaporation from the snowpack ($\mathrm{Snow_{Evap}}$).

$$\mathrm{Snow_{Deqv}} = \mathrm{Snow_{Deqv0}} + \mathrm{Snow_{Ppt}} - SM - \mathrm{Snow_{Evap}} \qquad \text{Eqn 7}$$

And $\mathrm{Snow}_{\mathrm{Evap}}$ is determined as the minimum of precipitation falling as snow ($\mathrm{Snow}_{\mathrm{Ppt}}$), and the potential snow evaporation rate ($\mathrm{R}_{\mathrm{PotEvap}}$), which itself is a user defined constant.

$$\mathrm{Snow}_{\mathrm{Evap}} = minimum\left(\mathrm{Snow}_{\mathrm{Ppt}}, \mathrm{R}_{\mathrm{PotEvap}}\right) \quad \text{Eqn 8}$$

Terrestrial equations

SI 1.1. Hydrology

INCA-PEco enables the user to define any number of terrestrial 'land use' classes, each of which supports three primary water stores; quickflow (QF), soil water (SW) and groundwater (GW). Flow out of a terrestrial store is equal to the store volume divided by the residence time constant for the store. Water moves between water stores through the processes of infiltration excess, saturation excess and percolation. Total flow from each landuse class to the stream is calculated by multiplying outflow rates from each store ($\mathrm{m}^3\ \mathrm{s}^{-1}\ \mathrm{km}^{-2}$) by the proportion of the subcatchment area made up of that landuse class (LU%) and the subcatchment area (Ca). Total flow to the stream is calculated by summing the totals from each store.

$$\mathrm{Q}_{\mathrm{SW\_land}} = \sum^{LUclasses} Ca\ \frac{LU\%}{100} \times (1 - BFI) \times Q_{SW} \quad \text{Eqn 9}$$

$$\mathrm{Q}_{\mathrm{GW\_land}} = \sum^{LUclasses} Ca\ \frac{LU\%}{100} \times Q_{GW} \quad \text{Eqn 10}$$

$$\mathrm{Q}_{\mathrm{QF\_land}} = \sum^{LUclasses} Ca\ \frac{LU\%}{100} \times Q_{QF} \quad \text{Eqn 11}$$

Where BFI is the user defined base flow index (0-1), as developed at the Centre for Ecology and Hydrology (Gustard et al., 1992). The BFI quantifies the proportion of stream flow derived from groundwater. Fluxes of water, sediment, phosphorus, dissolved oxygen and biochemical oxygen demand are transferred directly from their landscape unit to their associated river reach; but cannot move between landscape units.

SI 1.1.1 Soil water drainage and retention

In INCA-PEco, the soil box is split into three volumes depending on whether they are able to contribute to runoff; that which drains and percolates through to the groundwater, that which contributes to runoff (i.e. that which is above a saturation excess threshold); and that which is retained and stored as soil water. The user may set a minimum outflow discharge of soil water which the model then sustains at all times ($\mathrm{sust}_{\mathrm{SW}}$), although the use of this minimum flow does break the water balance.

a) Soil Drainage

Change in soil water flow over time, including saturation excess discharge is calculated as a function of incoming $\mathrm{HER}_{\mathrm{f}}$, minus soil outflow ($\mathrm{Q}_{\mathrm{SW\_out}}$) and the soilwater residence time constant (T_{SW}) ;

$$\frac{d\mathrm{Q}_{\mathrm{SW\_out}}}{dt} = \frac{\mathrm{HER}_f - \mathrm{Q}_{\mathrm{SW\_out}}}{\mathrm{T}_{\mathrm{SW}}} \quad \text{Eqn 12}$$

Where $\mathrm{Q}_{\mathrm{SW\_out}} = maximum\left(sust_{sw}, \mathrm{Q}_{\mathrm{SW\_out}}\right)$ Eqn 13

And the change in total soil drainage volume over time is calculated as:

$$\frac{d\mathrm{V}_{\mathrm{SW\_drainage}}}{dt} = 86{,}400\left(\mathrm{HER}_f - \mathrm{Q}_{\mathrm{SW\_out}}\right) \qquad \text{Eqn 14}$$

Drainage flow from the soil water box is then input to groundwater flow; although this flow (Q_{SW}) is a measure of soil water *after* losses from saturation excess have be taken into account. This adjusted soil flow is calculated through the user defined parameter (Sat_{max}), which depicts the threshold soil water flow at which saturation excess occurs; or in other words depicts the field capacity of the soil zone.

$$Q_{SW} = minimum\left(\mathrm{Q}_{\mathrm{SW\_out}}, \mathrm{Sat}_{max}\right) \qquad \text{Eqn 15}$$

b) *Soil Retention*

The volume of water retained ($V_{SW\_retention}$) within the soil is calculated using the soil moisture deficit timeseries.

$$\mathrm{V}_{\mathrm{SW\_retention}} = 10^3 \times \mathrm{R}_{\mathrm{SW\_retention}}(\mathrm{SMD}_{\mathrm{max}} - \mathrm{SMD}) \qquad \text{Eqn 16}$$

From this, the total volume of water present in the soil can be calculated as the sum of the volume draining, and that retained:

$$\mathrm{V}_{\mathrm{SW}} = \mathrm{V}_{\mathrm{SW\_drainage}} + \mathrm{V}_{\mathrm{SW\_retention}} \qquad \text{Eqn 17}$$

SI 1.1.2 Quickflow and infiltration excess, and saturation excess

When precipitation falls onto the ground at a rate faster than it can infiltrate (infiltration excess) , or when water volumes within a soil box exceed its total holding capacity (saturation excess) , surface overland flow is generated; termed 'quickflow' in INCA-PEco. Quickflow rates are therefore controlled by the quick box residence time constant (T_{QF}) and inputs from infiltration excess overland flow (Q_{IE}) and saturation excess (Q_{SE}), and by outputs from overland flow (Q_{QF}) to the river:

$$\frac{d\mathrm{Q}_{\mathrm{QF}}}{dt} = \frac{\mathrm{Q}_{SE} + \mathrm{Q}_{\mathrm{IE}} - \mathrm{Q}_{\mathrm{QF}}}{\mathrm{T}_{QF}} \qquad \text{Eqn 18}$$

Or, change in quickflow volume (V_{QF}) as:

$$\frac{d\mathrm{V}_{\mathrm{QF}}}{dt} = 86{,}400\left(\mathrm{Q}_{SE} + \mathrm{Q}_{\mathrm{IE}} - \mathrm{Q}_{\mathrm{QF}}\right) \qquad \text{Eqn 19}$$

The input time series of HER (converted to HER_f) is used to determine the infiltration rate; although $PPTN_{r+s}$ or a hybrid timeseries may alternatively be used. Infiltration is calculated as:

$$\mathrm{Q}_{\mathrm{IE}} = \mathrm{IE\%}\left(HER_f - Q_{infiltration}\right) \qquad \text{Eqn 20}$$

Where Infiltration rate ($Q_{Infiltration}$) = $\mathrm{I}_{max}\,\frac{1000}{86{,}400}\left(1 - e^{\frac{-86.4\,x\,HER_f}{I_{max}}}\right)$ Eqn 21

The excess proportion of water moving from the soil through to direct runoff via saturation excess is calculated using the user defined parameter ($Sat_{max;}$ Eqn 15), which sets the threshold of soil water flow for saturation excess.

$$Q_{sat} = maximum\left(Q_{SW\_out} - \mathrm{Sat}_{max}\right), 0 \qquad \text{Eqn 22}$$

SI 1.1.3 Groundwater and percolation

All groundwater inputs are derived from percolation from the soilwater box. Groundwater flows are controlled by the residence time constant (T_{GW}). Similar to soilwater, thresholds of groundwater flow can be set to sustain a specified minimum constant (Sust_{GW}), although again the decision to set a minimum flow will alter the water balance. Water flowing out from the groundwater zone inflows to the river.

$$\frac{d\mathrm{Q_{GW}}}{dt} = \frac{(\mathrm{BFI\ x\ Q_{SW}}) - Q_{GW}}{T_{GW}} \qquad \text{Eqn 23}$$

Where $\mathrm{Q_{GW}} = maximum\ (\mathrm{Sust}_{GW},\ \mathrm{Q_{GW}})$ Eqn 24

And change in groundwater volume (V_{GW}) is calculated as:

$$\frac{dV_{\mathrm{GW}}}{dt} = 86{,}400\ \big((\mathrm{BFI\ x\ Q_{SW}})\ - Q_{GW}\big) \qquad \text{Eqn 25}$$

SI 1.2 Soil erosion

Equations for soil movement in INCA-PEco are derived from INCA-Sed (Jarritt and Lawrence 2007, Lazar et al., 2010) . The model initiates sediment movement through splash detachment and flow erosion processes, and subsequently transports the sediment via direct runoff.

SI 1.2.1 Splash detachment

The mass of material made available via splash detachment during a timestep ($M_{SplashDet}$) can be either directed by the user (Eqn 26), or calculated by the model. In the user specified equation, $M_{SplashDet}$ is calculated using mass of rainfall flux (HER_f), user-specified scaling coefficients (R_{SSP}), user-specified soil splash erosion potential ($R_{SplashPot}$) and a vegetation cover coefficient ($R_{vegetation}$);

$$\mathrm{M_{SplashDet}} = \mathrm{R_{SSP}}\ \times \left(HER_f \times 10^6\right)\left(\mathrm{R_{SplashPot}}\right)^{\left(\frac{1}{R_{vegetation}}\right)} \times 86400 \times 10^{10} \qquad \text{Eqn 26}$$

where $R_{vegetation}$ is a value between 0 - 10; with 0 representing bare soil, and 10 indicating the soil has a complete and continuous vegetation cover. The user specifies growing seasons for vegetation in the model (Eqn 51), outside of which it is assumed that vegetation cover is zero ($R_{vegetation} = 0$). Controls are placed upon the equations ($R_{vegetation}$) to ensure that an $R_{vegetation}$ of 0 cannot occur during plant growth periods (Eqn 51) or under conditions of low soil splash erosion potential ($R_{splashpot}$), and that splash detatchment cannot occur during growth periods when $R_{vegetation}$ is 10, or when the ground is covered in snow.

SI 1.2.2 Transport capacity

The maximum mass of material that can be transported from the terrestrial store to the river is calculated in equation 27. This term is important as it controls the flow erosion rate. To perform the calculation, the total pathway length over which sediment is transported must first be defined, using the reach length (L) and the subcatchment area (Ca):

$$M_{TC} = R_{TCsf}\ \times \left(\frac{Ca \times Q_{QF}}{L} - R_{TCmax}\right)^{\mathrm{R_{TCcoeff}}} \times 86{,}400 \qquad \text{Eqn 27}$$

SI 1.2.3 Flow erosion

The rate at which sediment is detached from the soil store due to flow erosion ($M_{FlowErosion}$) is driven by sediment transport capacity (M_{TC}), sediment splash detachment ($M_{SplashDet}$) and a function (k) of soil erosion potential (SEP).

$$\mathrm{M_{FlowErosion}} = \left(\frac{k}{1+\frac{k}{M_{TC}}}\right)\left(1 - \frac{\mathrm{M_{SplashDet}}}{M_{TC}}\right) \quad \text{Eqn 28}$$

$$\text{Where } k = \mathrm{R_{FEsf}} \times \mathrm{R_{FEpot}} \left(\frac{Ca \times \mathrm{Q_{Quick}}}{L} - \mathrm{R_{FEmax}}\right)^{\mathrm{R_{FEcoeff}}} \times 86{,}400 \quad \text{Eqn 29}$$

When the mass of sediment available in the sediment store ($M_{Sedstore}$) is greater than zero ($\frac{dM_{\mathrm{SedStore}}}{dt}>0$) or where the sediment store equals zero and that mobilized through splash erosion exceeds the transport capacity of the overland flow ($\frac{dM_{\mathrm{SedStore}}}{dt} \leq 0 \; AND \; \mathrm{M_{SplashDet}} > M_{TC}$), the mass of sediment carried to the stream is equal to the sediment transport capacity (M_{TC}). In this case, sediment changes in the sediment store are calculated as:

$$\frac{dM_{\mathrm{SedStore}}}{dt} = \mathrm{M_{SplashDet}} - M_{TC} \quad \text{Eqn 30}$$

And here, the mass of sediment transported to the stream is calculated as:

$$M_{SedOut} = M_{TC} \quad \text{Eqn 31}$$

Otherwise, where the transport capacity is greater than the sediment mobilized in splash detachment, there is no change in the sediment store:

$$\frac{dM_{\mathrm{SedStore}}}{dt} = 0 \quad \text{Eqn 32}$$

In these conditions, $M_{SedOut} = M_{\mathrm{SplashDet}} + M_{\mathrm{FlowErosion}}$ Eqn 33

The total mass of sediment transported to the reach from all different landuse classes is calculated by summing outputs from all classes:

$$M_{TotalSedOut} = \sum^{LU\ classes} Ca \, \frac{LU\%}{100} \, M_{SedOut} \quad \text{Eqn 34}$$

SI 1.2.4 Grain sizes

Five soil grain size categories are modelled in INCA-PEco; clay (0-0.002mm); silt (0.002-0.06 mm); fine sand (0.06-0.2mm); medium sand (0.2-0.6mm) and coarse sand (0.6 – 2mm). In the terrestrial phase, for each landuse type the user enters size grain size distributions (%s) across the 5 categories. The size distributions are combined to calculate a weighted average soil texture for the entire subcatchment, and used to estimate the median grain size distribution of the output of sediment mass. This makes the assumption that the transported soil aggregates have the same particle size distribution as the parent soils.

SI 1.3 Phosphorus

There have been multiple iterations of the INCA-P model, but each had a common theme whereby phosphorus is simulated in a sum of ‘dissolved’ and ‘particulate’ forms; or in the case of soil and groundwater, a combination of labile and inactive. Dissolved phosphorus may move between quickflow, soil and groundwater stores in hydrological drainage. Particulate phosphorus is simulated only within the quickflow store but may be moved laterally through sediment erosion and transport pathways; and labile phosphorus is simulated only within groundwater and soil stores. Phosphorus may move between dissolved and particulate (or labile) phases within each store through processes of desorption and sorption, controlled by Freundlich isotherms.

Total phosphorus (TP) is input into each of the terrestrial stores in either a solid or liquid form, by user-specified inputs of fertilizer and atmospheric deposition. To simulate these, periods of fertilization (start: F_{start}) and duration (F_{dur}) are set by the user; specific to each landuse type. Where the model date occurs within the range of $F_{start} + F_{dur}$, solid, phosphorus inputs (kg ha^{-1} day^{-1}) to the model include:

$$M_{solP} = R_{sMan} + R_{sFert} + R_{res} + \frac{R_{sDep}}{365} \quad \text{Eqn 35}$$

And $M_{liqP} = R_{lMan} + R_{lfert} + M_{wDep}$ Eqn 36

Where $M_{wDep} = \frac{HER_f}{HER_{fannual}} R_{wDepAnnual}$ Eqn 37

For dates occuring outside of the fertilisation period, inputs are simply:

$$M_{solP} = R_{res} + \frac{R_{sDep}}{365} \quad \text{Eqn 38}$$

And $M_{liqP} = M_{wDep}$ Eqn 39

Solid inputs of P are added to the solid soil store, and liquid inputs to the soil water.

SI 1.3.1 Phosphorus in soil and soilwater

Soil phosphorus is divided into two components: a) that which is water soluble or ‘labile’, and which is available for sorption reactions; and b) the unavailable inactive mineral or recalcitrant organic form. The store of labile P can change over time, due to inputs of solid P (M_{solP}), adsorption of phosphorus from the soil water, and breakdown of inactive mineral or organic P over time. Outputs occur through erosion and transport to the river reach, or through immobilization back to inactive P.

$$\frac{dM_{labile}}{dt} = 100 \times M_{solP} + \frac{dM_{sorbed}}{dt} + R_{weather} C_t M_{inactive} - M_{SedOut} \frac{M_{labile}}{M_{soil}} - R_{immob} C_t M_{labile} \quad \text{Eqn 40}$$

A user-defined saturation threshold may be set for the maximum concentration of labile phosphorus permitted within the soil (R_{labsat}), where:

If $\frac{M_{labile}}{M_{soil}} \geq R_{labsat} : M_{labile} = R_{labsat} \times M_{soil}$ Eqn 41

In this case, the excess P is added to the soil water TDP (equation 44). In turn, the inactive P store receives inputs from immobilization of labile soil P; with outputs from erosion and weathering.

$$\frac{dM_{inactive}}{dt} = R_{immob} C_t M_{inactive} - M_{SedOut} \frac{M_{inactive}}{M_{soil}} - R_{weather} C_t M_{labile} \qquad \text{Eqn 42}$$

In the liquid phase, phosphorus is represented as total dissolved phosphorus (TDP). Changes in TDP mass are calculated as total liquid inputs (M_{liqP}), minus outputs of portions which are sorbed to the soil labile P mass, those taken up by plant growth and those removed through hydrological flow pathways (i.e. soil drainage). TDP drained from the soil may transfer to direct runoff through saturation excess flow, or to groundwater via percolation.

When the volume of water in the soil water store is >0, then:

$$\frac{dM_{TDP\_sw}}{dt} = 100 \times M_{liqP} - \frac{dM_{sorbed}}{dt} - P_{uptake} - \frac{86400 \times Q_{SW\_out} \times M_{TDP\_sw}}{V_{SW}} \qquad \text{Eqn 43}$$

However where there is no water in the store, TDP cannot be removed through drainage, and the change in TDP mass is simply:

$$\frac{dM_{TDP\_sw}}{dt} = 100 \times M_{liqP} - \frac{dM_{sorbed}}{dt} - P_{uptake} \qquad \text{Eqn 44}$$

If the mass of labile P in the soil is greater than the user-defined saturation threshold (R_{labsat}), then total mass of TDP in the soil water is calculated as :

$$M_{TDP\_sw} = M_{TDP\_sw(0)} + \left(M_{labile} - (R_{labsat} \times M_{soil})\right) \qquad \text{Eqn 45}$$

It is here that the additional labile P (from equation 40) is added to the soilwater store as TDP. The concentration of TDP (mg/l) in the soilwater is calculated from the mass as:

$$C_{TDP\_sw} = 10^3 \times \frac{M_{TDP\_sw}}{V_{SW}} \qquad \text{Eqn 46}$$

a) Soil P sorption processes

The change in mass of P sorbed to the soil over time (M_{sorbed}), is calculated as being proportional to the difference between the soil water TDP concentration and the concentration at which no adsorption or desorption occurs; otherwise known as its equilibrium phosphorus concentration zero value, or EPC_0 Calculating this change in mass is achieved using a version of the Freundlich isotherm derived from House and Denison (2000):

$$\frac{dM_{sorbed}}{dt} = 10^{-3} \times R_{ssc} \left(C_{TDP\_sw}{}^{\frac{1}{FI_{SW}}} - EPC_{0,SW}{}^{\frac{1}{FI_{SW}}} \right) V_{SW} \qquad \text{Eqn 47}$$

It has been found that the EPC_0 of soils and sediments are dynamic, and that affected by long term accumulation or loss of labile P stores. In INCA-PEco therefore, a dynamic EPC_0 is calculated, which is dependent upon the mass of P in the labile store:

If the M_{soil}>0 and M_{labile} >0 and soil P sorption coefficient (R_{scsoil}) is > 0

$$EPC_{0,SW} = \left(\frac{10^6}{R_{SCsoil}} \times \frac{M_{labile}}{M_{soil}}\right)^{FI_{SW}} \quad \text{Eqn 48}$$

Otherwise $EPC_{0,SW} = C_{TDP\_sw}$ Eqn 49

b) Plant growth and temperature dependency of processes

Within the soil water, TDP can be taken up by plants, which requires specification of growth start dates (G_{start}) and duration (G_{dur}), growth rates and nutrient requirements. User-specified annual maximum daily or annual uptake values (kg m^{-2} day^{-1}) are provided for plant growth, which the plants may not exceed. TDP uptake calculated for a date occurring outside of the growth period, or for a period after which the maximum uptake quantity has been exceeded, is calculated as zero. In this event, excess P would remain in the soil water as TDP. Otherwise:

$$P_{uptake} = min\left(10^6 \times R_{uptake} C_t \times \left(\frac{SMD_{max} - SMD}{SMD_{max}}\right) I_{growth} \times \frac{M_{TDP\_sw}}{V_{SW}}, 100 \times R_{maxup}\right) \quad \text{Eqn 50}$$

Where I_{growth} is a seasonal plant growth index, defined as:

$$I_{growth} = R_{offset} + R_{amp}\, sin\left(\frac{2\pi}{365}(DOY - G_{start})\right) \quad \text{Eqn 51}$$

A temperature factor ‘Ct’ is used in plant growth, and in chemical immobilisation and mineralisation equations. This factor increases reaction rates under conditions of increasing temperature.

$$C_t = Temp_{Q10}^{\frac{Temp_{soil0} - Temp_{Q10a}}{10}} \quad \text{Eqn 52}$$

C_t is therefore dependent on a time-series of soil temperature, which is calculated from air temperature using the Rankinen et al (2004) model.

$$Temp_{soil} = \left(Temp_{soil0} + \frac{86400\, R_{therm}}{10^6\, Heat_{cap}\, (2d_{soiltemp})^2}\,(Temp_{air0} - Temp_{soil0})\right) e^{-R_{damp} d_{snow}} \quad \text{Eqn 53}$$

Where $d_{soiltemp}$ is the depth at which soil temperature is calculated (0.2m), and $Heat_{Cap}$ is the combined specific heat capacity from freezing, thawing and soil:

If $Temp_{soil0} > 0$: Ca = K_{soil}; otherwise: Ca = $K_{soil} + R_{freeze}$ Eqn 54

And where d_{snow} is the snow depth (cm) calculated from the snow water equivalent (equation 7) using a user defined water equivalent factor’ (R_{weqv})

$$d_{snow} = \frac{1}{R_{weqv}} \times \frac{Snow_{deqv}}{10} \quad \text{Eqn 55}$$

A minimum air temperature threshold can also be set for the process of immobilisation. Should the air temperature falls below this value, $C_t = 0$

SI 1.3.2 Phosphorus in groundwater

TDP is input to groundwater through desorption from the aquifer matrix, and via percolation from the soil water. Similar to processes occurring in the soil water phase, adsorption and desorption are governed by the dynamic EPCo and Freundlich isotherm. The aquifer mass involved in P sorption reactions is calculated as:

$M_{aquifer} = 10^9 R_{Maquifer}$ Eqn 56

And the change in P sorbed in groundwater over time as:

$$\frac{dM_{sorbed\_GW}}{dt} = 10^{-3} \times R_{GWssc}\left(C_{TDP\_GW}{}^{\frac{1}{FI_{GW}}} - EPC_{0,GW}{}^{\frac{1}{FI_{GW}}}\right)V_{GW} \qquad \text{Eqn 57}$$

Where both the groundwater mass of sorbed P, and the groundwater P sorption coefficient are greater than zero, the $EPC_{0,GW}$ is defined as:

$$EPC_{0,GW} = \left(\frac{10^6}{R_{GW\_coeff}}\frac{M_{sorbed\_GW}}{M_{aquifer}}\right)^{FI_{GW}} \qquad \text{Eqn 58}$$

Otherwise: $EPC_{0gw} = C_{TDPgw}$

And again, similar to within the soil, a saturation excess threshold can be set within the groundwater aquifer (R_{aqsat}), where the excess P remains as TDP in the groundwater aquifer.

If $\frac{M_{sorbed\_GW}}{M_{aquifer}} \geq R_{aqsat}$: $M_{sorbedgw} = R_{aqsat} \times M_{aquifer}$ Eqn 59

Therefore the total change in TDP mass within the groundwater is calculated as that which has percolated down from the soilwater, minus that which has sorbed (and temporarily inactivated) to the aquifer, minus that which has drained out of the aquifer:

$$\frac{dM_{TDP\_GW}}{dt} = \frac{86400\,BFI \times Q_{SW} \times M_{TDP\_sw}}{V_{SW}} - \frac{dM_{sorbed\_GW}}{dt} - \frac{86400\,Q_{GW} \times M_{TDP\_GW}}{V_{GW}} \qquad \text{Eqn 60}$$

And If $\frac{M_{sorbed\_GW}}{M_{aquifer}} \geq R_{aqsat}$: $M_{TDP\_GW} = M_{TDP\_GW(0)} + \left(M_{sorbed\_GW} - R_{aqsat} \times M_{aquifer}\right)$ Eqn 61

The final TDP concentration in the groundwater water column is therefore:

$$C_{TDP\_GW} = 10^3\,\frac{M_{TDP\_GW}}{V_{GW}} \qquad \text{Eqn 62}$$

SI 1.3.3 Phosphorus in quickflow

Uniquely in direct runoff compartment of the model, no transformation between dissolved or solid bound P-phases are simulated. Instead in this phase phosphorus is modelled as dissolved (TDP) and particulate (PP) phosphorus, each in isolation of one another.

1.3.4.1 Dissolved phosphorus:

TDP inputs to the quickflow compartment come only from saturation excess flow, originating from the soilwater. An assumption is made that the TDP concentration of the saturation excess water is equal to that of the soilwater. Although water from infiltration excess is added to the quickflow zone, no phosphorus concentration is included in this flow input; and as a result, any IE water additions effectively dilute the saturation excess concentrations. Change in TDP quickflow mass is calculated as:

$$\frac{dM_{TDP\_QF}}{dt} = 86400\left(\frac{Q_{sat} \times M_{TDP\_sw}}{V_{SW}} - \frac{Q_{QF} \times M_{TDP\_QF}}{V_{QF}}\right) \qquad \text{Eqn 63}$$

Where the concentration of TDP is calculated as $C_{TDP\_QF} = 10^3 \, \frac{M_{TDP\_QF}}{V_{QF}}$ and TDP is output directly to the stream.

The fluxes of TDP exported to the river from each landuse class (kg km^{-2}) is calculated by summing exports from soilwater, quickflow and groundwater:

$$M_{TDPclass} = 86400\left(\frac{Q_{SW} \times M_{TDP\_sw}}{V_{SW}} + \frac{Q_{GW} \times M_{TDP\_GW}}{V_{GW}} + \frac{Q_{QF} \times M_{TDP\_QF}}{V_{QF}}\right) \quad \text{Eqn 64}$$

And the total TDP mass exported to the river reach is calculated by summing totals from each landuse class within each subcatchment:

$$M_{TDPtotal} = \sum^{LU\ classes} Ca \, \frac{LU\%}{100} \, M_{TDPclass} \qquad \text{Eqn 65}$$

1.3.4.2 Particulate phosphorus

The mass of particulate phosphorus (PP) carried in the quickflow compartment is characterized from the mass of sediment, labile phosphorus and inactive phosphorus eroded from the soil water (equation 39). As fine-grained particles will typically hold higher quantities of P than coarse grained ones, an 'enrichment' factor allows the user to characterize their PP concentrations by soil type. Within each landuse class, the flux of particulate phosphorus (kg km^{-2} day^{-1}) is calculated as:

$$M_{PPclass} = R_{er} \times M_{SedOut} \frac{(M_{labile} + M_{inactive})}{M_{soil}} \qquad \text{Eqn 66}$$

The total mass of PP exported from the land to the stream is calculated by summing the total sediment exported from each landuse class:

$$M_{PPtotal} = \sum^{LU\ classes} Ca \, \frac{LU\%}{100} \, M_{PPclass} \qquad \text{Eqn 67}$$

SI 1.4 Biological oxygen demand

A key new feature of INCA-PEco is the ability to simulate both biochemical oxygen demand (BOD) and dissolved oxygen (DO) in terrestrial and aquatic phases. For each landuse category, the user enters an initial soil water BOD concentration (mg/l). The soil water BOD concentration can be reduced by a user-specified daily decay rate, and through inputs of organic material onto the land, e.g., manure and plant residue, at user specified times and durations throughout the year.

Change in soil water BOD mass (M_{BOD_SW}, g km^{-2}) is therefore calculated as BOD additions associated with organic material, fertiliser ($R_{BODfert}$, g km^{-2} d^{-1}), a BOD decay rate, and advective transport out of the soil water box:

$$\frac{dM_{BOD\_SW}}{dt} = R_{BODfert} - \left(R_{SW\_BODdecay} \times rT \times M_{BOD\_SW}\right) - 86400 \times Q_{SW\_out} \times \left(\frac{M_{BOD\_SW}}{V_{SW}}\right) \quad \text{Eqn 68}$$

Where rT is a soil temperature factor, similar to that in equation 51; which increases the rate of BOD decay under higher temperatures:

$$rT = 1.07^{(Temp_{soil}-20)} \qquad \text{Eqn 69}$$

Similarly, in the groundwater store, initial conditions are specified by the user. Change in groundwater BOD mass (M_{BOD_GW}, g km^{-2} day^{-1}) is equal to BOD additions from percolation of soil water, decay of BOD and advective transport out of the groundwater phase. Where BOD percolating from the soilwater is calculated as:

$$M_{BODperc} = BFI \times Q_{SW} x\, M_{BOD\_SW} \times 86400 \qquad \text{Eqn 70}$$

And BOD groundwater flux from each landuse category:

$$\frac{dM_{BOD\_GW}}{dt} = \frac{M_{BODperc}}{V_{SW}} - \left(R_{GW\_BODdecay} \times rT \times M_{BOD\_GW}\right) - \left(\frac{Q_{GW} \times M_{BOD\_GW} \times 86400}{V_{GW}}\right) \qquad \text{Eqn 71}$$

Biological oxygen demand is not simulated in direct runoff as it is assumed that any surface runoff will be sufficiently aerated to have a BOD of zero.

Fluxes of BOD exported to the river from each landuse class (kg km^{-2}) are calculated by summing exports from soilwater and groundwater:

$$M_{BODLUclass} = 86400\left(\frac{Q_{SW} \times M_{BOD\_SW}}{V_{SW}} + \frac{Q_{GW} \times M_{BOD\_GW}}{V_{GW}}\right) \qquad \text{Eqn 72}$$

And the total BOD mass exported to the river reach calculated by summing totals from each landuse class within each subcatchment:

$$M_{BODtotal} = \sum^{LU\ classes} Ca\ \frac{LU\%}{100}\ M_{BODLUclass} \qquad \text{Eqn 73}$$

SI 1.5 Dissolved oxygen

Unlike for phosphorus and BOD, the user simply specifies a 'DO concentration constant' for soil water (R_{DO_SW}), groundwater (R_{DO_GW}) and quickflow (R_{DO_QF}) boxes. Daily flux within a landuse class is therefore associated with advective flow:

$$\frac{dM_{DO\_sw}}{dt} = R_{DO\_SW} \times 86400 \times Q_{SW\_out} \times \left(\frac{M_{DO\_SW}}{V_{SW}}\right) \qquad \text{Eqn 74}$$

$$\frac{dM_{DO\_GW}}{dt} = R_{DO\_GW} \times 86400 \times Q_{GW} \times \left(\frac{M_{DO\_GW}}{V_{GW}}\right) \qquad \text{Eqn 75}$$

$$\frac{dM_{DO\_QF}}{dt} = R_{DO\_QF} \times 86400 \times Q_{QF} \times \left(\frac{M_{DO\_QF}}{V_{QF}}\right) \qquad \text{Eqn 76}$$

The flux of DO exported to the river from each landuse class (kg km^{-2}) is calculated by summing exports from soilwater, groundwater and quickflow:

$$M_{DOLUclass} = 86400\left(\frac{Q_{SW} \times M_{DO\_sw}}{V_{SW}} + \frac{Q_{GW} \times M_{DO\_GW}}{V_{GW}} + \frac{Q_{QF} \times M_{DO\_QF}}{V_{QF}}\right) \qquad \text{Eqn 77}$$

And the total DO mass exported to the river reach calculated by summing totals from each landuse class within each subcatchment:

$$M_{DOtotal} = \sum^{LU\ classes} Ca\ \frac{LU\%}{100}\ M_{DOLUclass} \qquad \text{Eqn 78}$$

Aquatic equations

SI 1.6 in-stream hydrology

Within each reach, the change in flow, volume, depth and width of a water body are tracked using a version of the Manning's equation. The use of this method in the INCA suite of models is unique to INCA-PEco and INCA (ON)-THE and is adapted from recent changes made to the PERSiST model (Futter et al., *this issue*).

$$v = n^{-1} \times HR^{2/3} \times S^{1/2} \qquad \text{Eqn 79}$$

Where n (Manning's roughness) and S (stream bed slope) are user defined parameters, and HR (hydraulic radius) is calculated as reach cross sectional area (A_r) divided by reach wetted perimeter (P_r). The cross sectional area is determined using the stream depth (D), and average width of the channel taken at the stream bed (W_{sed}) and at the stream surface (W_{sur})

$$A_r = D\ \left(\frac{W_{sur}+W_{sed}}{2}\right) \qquad \text{Eqn 80}$$

And reach Pr is calculated as a function of W_{sed}, W_{sur}, and D:

$$P_r = W_{sed} + 2\ \left(\left(\frac{W_{sur}-W_{sed}}{2}\right)^2 + D^2\right)^{\frac{1}{2}} \qquad \text{Eqn 81}$$

A constant relationship between channel depth and $Q_{reachout}$ is determined through user-defined values of 'c' and 'f':

$$D = c\ \times Q_{reachout}{}^{f} \qquad \text{Eqn 82}$$

And similarly between $Q_{reachout}$ and stream surface width using constants a and b:

$$W_{sur} = a \times Q_{reachout}{}^{b} \qquad \text{Eqn 83}$$

Present day in-stream water storage (S_{reach}) is calculated using the previous day's volume (S_{reach0}), and net infow ($Q_{reachIn}$), accounting for reach residence time. $Q_{reachIn}$ is the sum of upstream inputs, terrestrial inputs from quickflow, groundwater and the soilzone; and effluent discharge minus abstraction:

$$S_{reach} = S_{reach0}e^{\frac{-86400\ \times v}{L}} + RT_{reach} \times Q_{reachIn} \times \left(1 - e^{\frac{-86400\ \times v}{L}}\right) \qquad \text{Eqn 84}$$

Where $RT_{reach} = \frac{L}{86400\ \times v}$ Eqn 85

Reach outflow (Qout) is therefore calculated as:

$$Q_{reachout} = \frac{S_{reach-1} + Q_{reachIn} - S_{reach}}{86400} \qquad \text{Eqn 86}$$

SI 1.7 In-stream sediment

Inputs of suspended sediment to the river reach include surface runoff (eqn 34), from direct point-source inputs such as sewage treatment works, or from optional additional bank erosion (termed 'background sediment release' in the model). Depending on the balance between stream bed entrainment factors, shear velocity and the grain size of the available sediments; suspended sediments may settle out and accumulate on the stream bed, or be entrained from the bed into the water column. Previous versions of INCA made assumptions that flow was governed only by gravity (i.e. slope); that the channel cross section was rectangular, and that river reaches were uniformly mixed. With the integration of the new Manning's formula, the model is no longer reliant upon this first assumption.

SI 1.7.1 bed sediment entrainment

Sediment within the river is tracked independently by grain size (clay, silt and fine-, medium-, and coarse- sand). First the maximum grain size which can be entrained (G_{max}) is determined, using the reach shear velocity (V_{shear}), which is determined by acceleration through gravity (g), water depth (D), channel slope (S), and a user-specified deviation from the 'ideal' channel characteristic (V_{dev}).

Where $G_{max} = 9.9941\ (V_{shear})^{2.5208}$ Eqn 87

And $V_{shear} = \sqrt{gDV_{dev}S}$ Eqn 88

The G_{max} is used to calculate the proportion of each size class that can be entrained for a given shear velocity. Entrainment is considered impossible where G_{max} is less than the smallest diameter of a particular size class ($Ent_{\%} = 0$). If $G_{max} >$ the largest grain diameter of the class, all sediment of that class is movable ($Ent_{\%} = 1$). The grain size distributions within a class are assumed to be uniform; and where G_{max} is between the smallest and largest grain size, proportional entrainment within that class range is calculated. The moveable bed sediment mass (M_{mov}) is therefore determined as:

$$M_{mov} = M_{bed} \times Ent_{\%} \qquad \text{Eqn 89}$$

Where M_{bed} is the total bed sediment mass (kg) available on that particular day.

Moveable bed mass and the amount of sediment which is entrained are not however the same thing. Calculations of entrainment additionally take into consideration stream power (S_p m^{-2}) and bed friction (r_f):

$$M_{ent} = R_{entsf} \times M_{mov} \times S_p \times r_f \times 86400 \qquad \text{Eqn 90}$$

Where Sp is calculated as a function of water density, acceleration due to gravity, stream depth, velocity and slope:

$$S_p = Density_{H20} \times g \times D \times v \times S \qquad \text{Eqn 91}$$

Where R_{entsf} is a user defined scaling factor, designed to enable the scaling up of processes, and bed friction (r_f) is a function of stream hydraulic radius , compared to that of a perfect pipe

$r_f = \frac{HRsed}{HR_{max}}$ Eqn 92

Where hydraulic radius used in the bed friction equation (HRsed) is defined as:

$$HR_{sed} = \frac{2 \times W_{sed}}{2D \times W_{sed}}$$

And where $HR_{max} = \frac{\pi \left(\frac{W_{sed}}{2}\right)^2}{\pi W_{sed}}$ Eqn 93

SI 1.7.2 bed sediment deposition

A terminal settling velocity (v_{term}) determines the point at which suspended sediment settles from the water column to the river bed. This velocity is dependent upon gravity, density of sediment (2650 kg m^{-3}) the median particle diameter of suspended sediment (G_{med}) and fluid viscosity (f_v):

$v_{term} = \frac{(Density_{sed} - Density_{H20}) \times g \times G_{med}{}^2}{18 f_v}$ Eqn 94

A combination of the terminal settling velocity and the total mass of suspended sediment available are used to calculate the mass of sediment deposited (M_{sedDep}).

Where $M_{sedDep} = v_{term} \times \frac{M_{sus}}{S_{reach}} \times 86400$ Eqn 95

Where M_{sus} is the mass of sediment available at the start of the time-step.

SI 1.7.3 Background sediment release

During low flow conditions, trapped sediment may still be released from vegetation patches, from between larger pebbles and boulders, and from depressions in the stream bed. The INCA-PEco model has a 'background sediment release' function to allow the user to simulate such release of sustained suspended sediment concentrations during low flow conditions. As this function is designed to characterize fine grained sediments observed during baseflow, it releases only particles within the clay grain-size fraction. Two user-defined parameters, termed 'background scaling factor' (Bsf) and 'background non-linear coefficient' (Bnl) are used to release clay as a function of reach flow:

$M_{background} = Bsf \times Q_{reachOut}{}^{Bnl}$ Eqn 96

Concurrent mass balance accounting is performed for each grain size category at each time step, by considering the entrainment and deposition of each grain size class; so that the bed sediment flux is:

$\frac{d\,M_{bed}}{dt} = M_{sedDep} - M_{ent}$ Eqn 97

And the total flux in suspended sediment is dependent on inputs into and out of the reach, entrainment, sedimentation, background sediment release, point source effluent, and reach residence time:

$$\frac{d\,M_{sus}}{dt} = M_{TotalSedOut} + M_{PS} + \Big((L \times W_{sed})M_{ent} - M_{sedDep} + M_{background}\Big) + M_{up} - \frac{M_{sus}}{S_{reach}} \times Q_{reachOut} \times 86400$$

Eqn 98

SI 1.8 Instream Phosphorus

In the aquatic phase, phosphorus is represented as TDP and PP, and is simulated separately in the water column (WC) and the stream bed (SB). Exchange may occur between the two stores (stream bed and water column) either through direct movement of TDP from bed sediment pore-water into the overlying water column (pore-water flushing); or through mobilization and settling of PP between stream bed sediments and the water column (particle suspension and deposition). Within each store, phosphorus may also transform between TDP and PP using Freundlich isotherms. Equations are solved independently for each reach.

SI 1.8.1 Phosphorus in the water column

a) Dissolved phosphorus

TDP is added to a reach from upstream and terrestrial and inputs (eqn 65), and from user-defined sewage effluent. Outputs occur via abstractions and reach outflow.

$$\frac{d\,M_{TDP,WC}}{dt} = M_{TDP\_US} + M_{TDPtotal} + M_{TDPeff} + M_{TDPex} - M_{TDPabs} - M_{TDPout} - \frac{dM_{sorb,WC}}{dt}$$

Eqn 99

A user defined saturation threshold is included however (C_{Psatwc}), to prevent P from adsorbing to suspended sediment *ad infinitum*. Should the concentration of PP in the water column (as a fraction of total suspended sediment mass) exceed this threshold, then the excess P is added to the water column as:

$$\frac{d\,M_{TDP,WC}}{dt} = M_{TDP\_WC(0)} + \left(M_{PP\_WC} - \left(C_{Psat\_WC} \times M_{sus}\right)\right)$$

Eqn 100

The concentration of TDP in the water column (mg/l) is expressed as:

$$C_{TDP\_WC} = 10^{3} \times \frac{M_{TDP\_WC}}{S_{reach}}$$

Eqn 101

Where TDP outflow from the reach (M_{TDPout}) is calculated as a function of advection:

$$M_{TDPout} = 86400 \times \frac{M_{TDP\_WC} \times Q_{reachOut}}{S_{reach}}$$

Eqn 102

The change in mass of P adsorbed to suspended sediments in the water column is calculated in a similar manner to that in the terrestrial phase (Eqn 47). The theory applied is that change in mass of P sorbed to suspended sediments over time (M_{sorb_WC}), is proportional to the difference between the water column TDP concentration and the water column EPC0 value.

$$\frac{dM_{sorb\_WC}}{dt} = 10^{-3} \times R_{SF\_WC}\left(C_{TDP\_WC}{}^{\frac{1}{FI_{wc}}} - EPC_{0,wc}{}^{\frac{1}{FI_{wc}}}\right)S_{reach}$$

Eqn 103

And where if the $M_{sus}>0$ and $M_{PP\_WC}>0$ and the water column P sorption coefficient (R_{SF_WC}) is > 0

$$EPC_{0,wc} = \left(\frac{10^{6}}{R_{SC\_WC}} \times \frac{M_{PP\_WC}}{M_{sus}}\right)^{FI_{wc}}$$

Eqn 104

Otherwise $EPC_{0,wc} = C_{TDP\_WC}$ Eqn 105

Finally, the exchange of TDP between the water column and stream bed pore waters (M_{TDPex}) is calculated as:

$$M_{TDPex} = 10^{-3} \times R_{Ex\%} \times S_{reach} \left(C_{TDP\_SB} - C_{TDP\_WC}\right) \qquad \text{Eqn 106}$$

b) Particulate phosphorus

PP is input to a river reach from the terrestrial phase (Eqn 65), upstream reaches, sewage effluent, and entrainment from stream bed PP (Eqn 84). It is removed via abstraction, particle settling, de-sorption, abstraction and through outflow from the reach.

$$\frac{d\,M_{PP\_wc}}{dt} = M_{PP\_US} + M_{PPtotal} + M_{PPeff} + \frac{dM_{sorb\_wc}}{dt} + M_{PPent} - M_{PPdep} - M_{PPabs} - M_{PPout} \qquad \text{Eqn 107}$$

Again, that saturation threshold (C_{Psatwc}) for PP cannot be exceeded however, and therefore:

$$\text{if } \frac{M_{PP\_WC}}{Msus} \geq C_{Psat\_WC}: M_{PP\_WC} = C_{Psat\_WC} \times M_{sus} \qquad \text{Eqn 108}$$

Both entrainment and deposition of PP are controlled by reach sediments, where entrainment and deposition are assumed to be zero where the mass of bed sediment and suspended sediment (respectively) are nil. Otherwise:

$$M_{PPent} = M_{ent} \frac{M_{PP\_SB}}{M_{\_SB}} L \times w_{bed} \qquad \text{Eqn 109}$$

$$\text{And } M_{PPdep} = M_{sedDep} \frac{M_{PP\_WC}}{M_{sus}} L \times w_{bed} \qquad \text{Eqn 110}$$

The mass of PP output from the reach is calculated as:

$$M_{PPout} = \frac{86400 \times M_{PP\_WC} \times Q_{reachOut}}{S_{reach}} \qquad \text{Eqn 111}$$

And the final concentration of PP in the water column is expressed as:

$$C_{PPwc} = 10^{3} \frac{M_{PP\_wc}}{S_{reach}} \qquad \text{Eqn 112}$$

SI 1.8.2 Phosphorus in the bed sediment

a) Dissolved phosphorus

TDP exchange between stream bed sediment (SB) and the water column (WC) may move in either direction, depending on the concentration of pore water and concentrations in the overlying water column (Eqn 106). This exchange, in addition to the change in P sorbed to the stream bed determines the bed sediment TDP concentration:

$$\frac{d\,M_{TDP\_SB}}{dt} = -M_{TDPex} - \frac{dM_{sorbed}}{dt} \qquad \text{Eqn 113}$$

And, as in the water column, if the mass of PP in the stream bed (as a fraction of total bed sediment mass) is greater than the user-defined maximum allowable phosphorus: sediment in the stream bed (kg P/kg sed), then:

$$M_{TDP\_SB} = M_{TDP\_SB(0)} + \left(M_{PP\_SB} - \left(C_{Psat\_SB} \times M_{bed}\right)\right) \qquad \text{Eqn 114}$$

P sorption in the stream bed is calculated through Freundlich isotherms and EPC0 concentrations, in an identical fashion to that applied to the water column sorption reactions (Eqn 103-195):

$\frac{dM_{sorbbed}}{dt} = 10^{-3} \times R_{SF\_SB} \left(C_{TDP\_SB}{}^{\frac{1}{FI_{SB}}} - EPC_{0,SB}{}^{\frac{1}{FI_{SB}}} \right) S_{reach}$ Eqn 115

Where if the M_{bed}>0 and M_{PP_SBbed} >0 and the stream bed P sorption scaling factor (R_{SF_SB}) is > 0

$EPC_{0SB} = \left(\frac{10^6}{R_{SC\_SB}} \times \frac{M_{PP\_SB}}{M_{bed}} \right)^{FI_{SB}}$ Eqn 116

Otherwise $EPC_{0,SB} = C_{TDP\_SB}$ Eqn 117

The concentration of TDP in the porewater (C_{TDP_SB}) is calculated taking into account the porewater volume (V_{pw}), where:

$V_{pw} = w_{sed} \times L \times R_{bedDepth} \times R_{bed\emptyset}$ Eqn 118

So that:

$C_{TDP\_SB} = 10^3 \times \frac{M_{TDP\_SB}}{V_{pw}}$ Eqn 119

b) Particulate phosphorus

In the stream bed, PP may be lost through entrainment or de-sorption, and gained through deposition or adsorption:

$\frac{dM_{PP\_SB}}{dt} = \frac{dM_{sorbed}}{dt} + M_{PPdep} - M_{PPent}$ Eqn 120

Again here the saturation threshold limits the amount of P which can be adsorbed onto bed sediments:

Where $\frac{M_{PP\_SB}}{Mbed} \geq C_{Psat\_SB}: M_{PP\_SB} = C_{Psat\_SB} \times M_{bed}$ Eqn 121

SI 1.8.3 Soluble reactive phosphorus in the water column

While only two fractions of phosphorus are explicitly modelled in INCA (PP, TDP), and of course the sum of their parts TP; INCA-PEco does have a function with which to derive a more bioavailable component, soluble reactive phosphorus (SRP). Several options are available, although all assume a constant relationship over time with another (proxy) parameter.

Either $C_{SRP\_WC} = Grad_{SRP:TDP} \times C_{TDP\_WC} + Y_{SRP:TDP}$ Eqn 122

Or $C_{SRP\_WC} = C_{TDP\_WC} + Rat_{SRP:DOC} \times Obs_{DOC}$ Eqn 123

Planned future improvements in INCA-PEco will explicitly model SRP, DHP, and organic matter, to include time-varying relationships between TDP and SRP; mobilization of SRP from the stream bed through microbial communities, and interactions between Fe and anoxia.

SI 1.9 Instream biochemical oxygen demand:

Organic compounds which are present in the water column may reduce concentrations of dissolved oxygen (DO) as they decay and oxidise. The demand that this decaying organic matter puts upon the DO concentration is known as the biochemical oxygen demand. Determination of changes in BOD mass within the water column (M_{BOD_WC}) are therefore essential for determination of water column DO. Essentially they represent the amount of oxygen used in respiration by micro-organisms during their consumption of organic matter (Cox, 2002). The rate at which biochemical oxidation occurs is assumed to be proportional to the amount of organic matter remaining in the water; therefore, BOD can be used as a proxy for a waterbody's organic matter content (Cox, 2002). In INCA-PEco inputs of BOD are from upstream reaches, the land phase (Equation 70), wastewater treatment effluent (M_{BODww}) and contribution of dead phytoplankton to BOD (BOD_{phyto}). Reach outputs are via settling of BOD (BOD_{settle}) and advection:

$$\frac{d\,M_{BOD\_WC}}{dt} = M_{BODus} + M_{BODtotal} + M_{BODww} + BOD_{phyto} - BOD_{settle} - \left(\frac{86400\,M_{BOD\_WC} \times Q_{reachOut}}{S_{reach}}\right)$$

Eqn 124

As phytoplankton die, the oxidation of their decaying matter takes up oxygen from the water column. In INCA-PEco this mass of phytoplankton oxygen demand is included in the BOD as 'phytoplankton BOD' (g BOD km^{-2}). Where $R_{PhytoBOD}$ is a user defined parameter of the dead algae contribution to the BOD (mg O_2 µg chl a^{-1} day^{-1}) and $C_{PhytoDeath}$ is concentration of phytoplankton dying per day (µg chl a day^{-1}).

$$BOD_{phyto} = R_{phytoBOD}\left(C_{phytoDeath}\frac{1000}{S_{reach}}\right) \qquad \text{Eqn 125}$$

As organic matter settles and is buried within the bed sediment, it is deactivated as a source of oxygen demand. It is possible however that the decaying organic matter could later be re-suspended, and re-integrated as a BOD input. Therefore, in INCA-PEco, the amount of BOD which has been buried (g O_2 km^{-2} day^{-1}) is calculated using a *net* settling velocity (burial minus resuspension) of m day^{-1} which varies with reach depth

$$BOD_{settle} = \frac{R_{netv}}{D}\,M_{BOD\_WC} \qquad \text{Eqn 126}$$

Where R_{netv} is a user defined parameter.

SI 1.10 Dissolved oxygen

Within the water column, changes in mass of DO (g km-^{2}) are calculated by taking into consideration the major sources and sinks of DO in rivers (Cox; 2002) including influx from upstream reaches (M_{DOus} g km^{-2}) and the land phase ($M_{DOtotal}$ g km-^{2}), phytoplankton oxygen contributions (DO_{phytox}, g O_2 km^{-2} day^{-1}) , and re-aeration (DO_{atmox}, g O_2 km^{-2} day^{-1}). Outputs of DO are from uptake through BOD ($M_{BOD\_decay\_WC}$, g km-^{2}) sediment oxygen demand (DO_{sod}, g O_2 km^{-2} day^{-1}), and advection. Temperature is perhaps the most significant driver of DO concentration in water, and as such is a key factor in each equation.

$$\frac{M_{DO\_WC}}{dt} = M_{DOus} + M_{DOtotal} + DO_{phytox} - DO_{sod} + DO_{atmox} - M_{BOD\_decay\_WC} - \left(\frac{86400 \times M_{DO\_WC} \times Q_{reachOut}}{S_{reach}}\right) \qquad \text{Eqn 127}$$

And the concentration of dissolved oxygen expressed as

$$C_{DO\_WC} = 10^3 \times \frac{M_{DO\_WC}}{S_{reach}} \qquad \text{Eqn 128}$$

To ensure that BOD cannot cause the DO to become negative, oxygen loss is set to zero if there is insufficient DO to satisfy the BOD requirements. DO is also limited to 300% of the DO saturation value ($DO_{sat\%}$):

$$DO_{sat\%} = \frac{C_{DO\_WC} \times 100}{Abs_{sat}} \qquad \text{Eqn 129}$$

Where the absolute concentration at which the water column becomes saturated with DO is dependent upon the following relationship with water temperature (ref?):

$$Abs_{sat} = 14.652 - 0.41022 \times Temp_{wc} + 0.0079910 \times {Temp_{wc}}^2 - 0.000077774 \times {Temp_{wc}}^3 \qquad \text{Eqn 130}$$

SI 1.10.1 Atmospheric re-aeration

Inputs of DO through atmospheric re-aeration (DO_{atmox}) can be calculated either by the model, or set using a user-defined parameter. In both cases, re-aeration is a function of the rate that aeration occurs ($DO_{atmox/day}$), the total concentration of DO which the water column can hold (Abs_{sat}) and the DO mass currently within the water column (M_{DOwc}):

$$DO_{atmox} = DO_{atmox/day} \times \left(Abs_{sat} - M_{DO\_WC}\right) \qquad \text{Eqn 131}$$

Where the user defines parameters for input into re-aeration; the $DO_{atmox/day}$ parameter is defined as:

$$DO_{atmox/day} = R_{aer} \times 1.024^{Tempwc^{-20}} \qquad \text{Eqn 132}$$

The model calculated version is dependent upon water temperature, depth and velocity. Where $Temp_{wc}$ is <30°C, the following equations are used:

$$DO_{atmox/day} = 1.7535 \frac{\left(1+Fr^2\right) T_{\emptyset} E^{0.375}}{D \times (0.9+Fr)^{1.5}} \times \frac{1}{tanh\left(\frac{2.751 \times T_{\emptyset 2} \times E^{0.125}}{(0.9+Fr)^{0.5}}\right)} . 12.0 \qquad \text{Eqn 133}$$

Where $Fr = \frac{S}{(D \times 9.806)^{0.5}}$ Eqn 134

And $T_{\emptyset} = 9.68 + 0.054\ (Temp_{wc} - 20)$ Eqn 135

And $E = S \times v \times 9.806$ Eqn 136

And $T_{\emptyset 2} = 0.976 + 0.0137 \times (30 - Temp_{wc})^{1.5}$ Eqn 137

In the event that water temperatures cross the 30°C threshold, equations from one of Owens et al (1964), O'Connor and Dobbins (1958) or Churchill et al (1962) are implemented, dependent upon depth and velocity as per Cox (2002). Where stream depth is less than or equal to 0.74m then :

$$DO_{atmox/day} = 5.3327 \times v^{0.67} \times D^{-1.85} \times 1.0241^{Temp_{wc}-20} \quad \text{Eqn 138}$$

Where stream depth is greater than 0.74m and reach velocity is less than or equal to 1.28m s^{-1} then :

$$DO_{atmox/day} = 3.952 \times v^{0.5} \times D^{-1.5} \times 1.016^{Temp_{wc}-20} \quad \text{Eqn 139}$$

Otherwise :

$$DO_{atmox/day} = 5.014 \times v^{0.969} \times D^{-1.673} \times 1.0241^{Temp_{wc}-20} \quad \text{Eqn 140}$$

SI 1.10.2 Phytoplankton respiration and photosynthesis:

Oxygen contributions from algae (DO_{phytox}) are calculated as a function of O_2 supplied by photosynthesis (DO_{ptsyn}, g O_2 km^2 day^{-1}) and used up through respiration (DO_{ptresp}, g O_2 km^2 day^{-1}):

$$DO_{phtyox} = DO_{ptsyn} - DO_{ptresp} \quad \text{Eqn 141}$$

Thresholds rates of 'low' (DO_{lowSyn}) and 'high' (DO_{hiSyn}) rates of photosynthesis are applied, determined by the phytoplankton concentrations and daylight hours ($Solar_{hours}$); where

$$C_{phyto} < \frac{50ug}{l}: DO_{ptsyn} = Solar_{hours}{}^{0.79} \times DO_{lowSyn} \times 0.0317\, x\, (C_{phyto}) \quad \text{Eqn 142}$$

Otherwise:

$$DO_{ptsyn} = Solar_{hours}{}^{0.79} \times \left(DO_{lowSyn}1.585 + DO_{hiSyn}0.0317 \times \left((C_{phyto} - 50)\right)\right) \quad \text{Eqn 143}$$

$$\text{Where } DO_{lowSyn} = R_{lowSyn}1.08^{Temp_{wc}-20} \quad \text{Eqn 144}$$

$$\text{And } DO_{hiSyn} = R_{hiSyn}1.08^{Temp_{wc}-20} \quad \text{Eqn 145}$$

Removal of DO through phytoplankton respiration (DO_{ptResp}) is determined by phytoplankton concentration (equation 140), and a user defined respiration slope and offset:

$$DO_{ptResp} = \left((R_{phytOffset} + R_{phytoSlope} \times C_{phyto})1.08^{temp_{wc}-20}\right) \quad \text{Eqn 146}$$

SI 1.10.3 Sediment respiration (sediment oxygen demand)

Chemical oxidation of compounds may occur within riverbed sediments where organic matter is incorporated in the channel bed, exerting a significant oxygen demand and influencing the in-stream DO. This 'uptake' of dissolved oxygen is expressed as:

$$DO_{sod} = Sed_{ox} \times \left(\frac{M_{DO\_WC}}{1.4 + M_{DO\_WC}}\right) \quad \text{Eqn 147}$$

Where Sed_{ox} is determined by a user defined rate of oxidation (R_{ox}) and by the temperature of the water column:

$$Sed_{ox} = R_{ox} \times \frac{1}{D}\, 1.08^{temp_{wc}-20} \quad \text{Eqn 148}$$

SI 1.10.4 Biochemical oxygen decay rate

Similarly in the water column, organic compounds may reduce DO concentrations as they decay and oxidise. A key removal mechanism of DO from the watercolumn is therefore associated with the M_{BOD_WC} (Eqn 124).

$$M_{BOD\_decay\_WC} = BOD_{decayCoefficient} \times M_{BOD\_WC} \qquad \text{Eqn 149}$$

The specific quantity of DO removed by the BOD is calculated as being positively related to stream depth and temperature of the water column, where if the water depth is > 2.4m:

$$BOD_{decaycoefficient} = Oxid \times 1.047^{Temp_{wc}-20} \qquad \text{Eqn 150}$$

Otherwise $BOD_{decaycoefficient} = Oxid \left(\frac{D \times 3.28084}{8}\right)^{-0.434} \times 1.047^{Temp_{wc}-20}$ Eqn 151

Where 'oxid' is a user defined parameter, determining the speed at which organic matter is oxidized.

SI 1.11 Phytoplankton

The concentration of phytoplankton in the water column is critical to calculations of DO and BOD mass; considering the influence of phytoplankton respiration, photosynthesis and death on instream processes (Eqns 127 and 141). In INCA-PEco, phytoplankton is represented in units of chlorophyll (µg l^{-1}) :

$$\frac{dC_{phyto}}{dt} = C_{PhytoAdv} + C_{PhytoGrowth} - C_{PhytoDeath} \qquad \text{Eqn 152}$$

Unique to the simulation of phytoplankton in INCA-PEco, but inherent to the aquatic nature of their habitat, there are no direct land phase additions of phytoplankton to the watercolumn. Instead the model is first balanced by calculating the change in phytoplankton inputs from the upstream, minus the outputs from the downstream; referred to here as advection ($C_{phytoAdv}$).

$$C_{Phytoadv} = \left(\frac{C_{phyto\_US} - C_{phyto}}{\frac{S_{reach}}{Q_{outflow} \times 86400}}\right) \qquad \text{Eqn 153}$$

Subsequently, inputs from phytoplankton growth, and outputs of phytoplankton death are then calculated. Due to the lack of land additions of phytoplankton, it is important that at least one upstream reach is set with an initial C_{phyto} >0, in order for phytoplankton growth to proliferate throughout the model.

SI 1.11.1 Phytoplankton growth

Response of phytoplankton to light, temperature and nutrient availability varies by species; e.g. diatoms will flourish in relatively low temperature and light conditions, provided sufficient nutrients are available. It would not be possible to account for this variability without simulating responses of individual species, e.g. in methods such as those applied in PROTECH (Elliot et al 2007). It is not the intention of the INCA-PEco to achieve simulations of the biomass of specific species of algae, but to provide a model capable of providing the BOD and DO concentrations as a result of phytoplankton community growth; as the former are currently the most critical concern regarding the health of waterbodies. The phytoplankton growth ($C_{PhytoGrowth}$) equation has therefore been designed to enable the user to specify the extent to which a community of phytoplankton responds to each driver of light, temperature, nutrients and self-shading; and user-defined thresholds can be set to instigate and cut-off specific drivers. This makes the phytoplankton component of the model particularly adaptable to environments with seasonal blooms, ice-on and ice-off events, extreme light reductions, or extreme heat changes, and particular dominant blooms of specific species. This enables the user to specify where the phytoplankton community within each reach may be particularly sensitive, or not, to the availability of nutrients

$$C_{PhytoGrowth} = R_{growth} \times C_{phyto} \times rT \times rSR \times \frac{C_{SRP}}{R_{SRPmax}} \times \frac{R_{phytoShade}}{R_{phytoShade}+C_{Phyto}} \qquad \text{Eqn 154}$$

Where rT and rSR are designed to activate a threshold below which temperature and light become a limiting factor on algal growth; where:

when $Temp_{air} < RTemp_{air}$; $rT = 1.066^{temp_{wc}-20}$ Eqn 155

Otherwise $rT = 1$

And when $R_{solar} < \frac{SolarRad}{SolarRad\ max} : rSR = \frac{SolarRad}{SolarRad\ max}$ Eqn 156

Otherwise $rSR = 1$

Where *Rsolar* is a user-defined threshold of light required for growth, expressed as a fraction of annual maximum solar radiation (0-1); and where *SolarRad*, and *SolarRadMax* are parameters calculated by INCA-PEco (Eqn 157 - 162). R_{SRPmax} is a user defined threshold concentration of SRP at which phytoplankton growth is uninhibited by nutrient availability; where R_{growth} is a user defined rate of phytoplankton growth, and $R_{phytoShade}$ is the user-defined concentration of chl-a at which phytoplankton growth becomes self-limiting.

Solar radiation is calculated using reach latitude and longitude, using equations from Cox, 2002:

$$Elevation_{solar} = (\sin(lat) \times \sin(dec)) - (\cos(lat) \times \cos(dec) \times \cos(time)) \qquad \text{Eqn 157}$$

Where *dec* is a measure of solar declination, calculated as:

$$dec = 0.39637 - 22.9133 \times \cos(dec1) + 4.02543 \times \sin(dec1) - 0.3872 \times \cos(2 \times dec1) + 0.052 \times \sin(2 \times dec1) \qquad \text{Eqn 158}$$

And where $dec1 = 360 \times \frac{DOY}{days\ in\ year}$ Eqn 159

The solar elevation ($Elevation_{solar}$) can then be used to calculate the photosynthetically active radiation:

$$Solarrad_{30} = Solar_{constant} \times R_{aads} \times (\sin Elevation_{solar}) \qquad \text{Eqn 160}$$

Where R_{aads} is an atmospheric adsorption factor, calculated from Bras, 1990 as:

$$R_{aads} = \exp\left(-3 \times \left(0.12 - 0.054 \times log10 \left(\frac{1}{Elevation_{solar}}\right)\right)\right) \qquad \text{Eqn 161}$$

The 30 minute solar radiation values are then re-sampled to generate average daily solar radiation (Solarrad), using the formula:

$$Solarrad = Mean\ (Solarrad_{30}) \qquad \text{Eqn 162}$$

SI 1.11.2 Phytoplankton death

Death of phytoplankton ($C_{PhytoDeath}$) is controlled by a user defined rate (day):

$$C_{PhytoDeath} = C_{Phyto} \times R_{phytoDeath}$$ Eqn 163

SI2: Parameter table

Location	**Parameter name**	Description	Units	Type
Meteorology	SMD	Soil moisture deficit	mm	input
	SMD_{max}	Maximum value in SMD input time series (must be set slightly higher)	mm	User def
	HER	Hydrologically effective rainfall	mm day^{-1}	input
	PPTN	Precipitation	mm day^{-1}	input
	$Temp_{air}$	Air temperature	^oC	input
	HERf	Flux of hydrologically effective rainfall	$m^3 s^{-1} km^{-2}$	calc
	$PPTN_{r+s}$	Rainfall flux + snowmelt	$m^3 s^{-1} km^{-2}$	calc
	$Snow_{Ppt}$	Precipitation falling as snow	mm day^{-1}	calc
	$R_{RCorrect}$	rainfall correction factor	Unitless	user defined
	$R_{sCorrect}$	Snowfall correction factor	Unitless	user defined
	SM	Snowmelt	mm day^{-1}	calc
	R_{Rtemp}	Temp above which all precipitation falls as rain	^oC	user defined
	R_{Stemp}	Temp below which all precipitation falls as snow	^oC	user defined
	R_{sdeg}	Degree day factor for snowmelt	mm $^oC^{-1}$ day^{-1}	user defined
	R_{melt}	Snow melt temperature	^oC	user defined
	$Snow_{Deqv}$	Snow depth water equivalent	mm	calc
	$Snow_{Deqv0}$	Snow depth water equivalent from previous day	mm	calc
	$Snow_{Evap}$	Water lost to evaporation from snow	mm day^{-1}	calc
	$R_{PotEvap}$	Potential snow evaporation rate	mm day^{-1}	user defined
Terrestrial hydrology General	Q_{SW_land}	Total soil water flow to stream	$m^3 sec^{-1}$	
	LUclasses	Class of landuse to which equation is applied	unitless	Categorical
	Ca	Subcatchment area	km^2	User defined
	LU%	Area of landscape in a subcatchment	%	User defined
	BFI	Baseflow index	Unitless	User defined
	Q_{SW}	Soil water flow (without saturation excess)	$m^3 sec^{-1} km^{-2}$	calc
	Q_{GW_land}	Total groundwater flow to stream	$m^3 sec^{-1}$	Calc
	Q_{GW}	Groundwater flow	$m^3 sec^{-1} km^{-2}$	Calc
	Q_{QF_land}	Total quickzone flow to stream	$m^3 sec^{-1}$	Calc
	Q_{QF}	Quickflow	$m^3 sec^{-1} km^{-2}$	Calc

Soil water	Q_{SW_out}	Soil water flow and saturation excess	m^{3} sec^{-1} km^{-2}	Calc
	T_{SW}	Soil water characteristic time constant	days	User defined
	$Sust_{sw}$	Sustainable soil water flow	m^{3} sec^{-1} km^{-2}	User defined
	V_{SW_drain}	Soil water drainage volume	m^{3} km^{-2}	calc
	Sat_{max}	threshold soil water flow for saturation excess	m^{3} sec^{-1} km^{-2}	User defined
	$V_{SW\_retention}$	Soil water retention volume	m^{3} km^{-2}	calc
	$R_{SWretention}$	Soil water retention volume constant	Unitless	User defined
	V_{SW}	Total soil water volume	m^{3} km^{-2}	calc
Quick flow	Q_{SE}	Saturation excess flow	m^{3} sec^{-1} km^{-2}	calc
	Q_{IE}	Infiltration excess	m^{3} sec^{-1} km^{-2}	calc
	T_{QF}	Quickflow characteristic time constant	days	User defined
	V_{QF}	Quick flow volume	m^{3} km^{-2}	calc
	IE%	IE proportion to quickflow	Unitless	User defined
	$Q_{Infiltration}$	Infiltration rate	m^{3} sec^{-1} km^{-2}	calc
	I_{max}	Max infiltration rate	mm day^{-1}	User def
Groundwater	T_{gw}	Groundwater residence time	days	User defined
	$Sust_{GW}$	Sustainable groundwater flow	m^{3} sec^{-1} km^{-2}	User defined
Soil erosion	$M_{splashDet}$	Mass of sediment mobilised by splash detachment	kg m^{-2} day^{-1}	Calc
	R_{SSP}	Splash detachment scaling parameter	s m^{-1}	User defined
	$R_{SplashPot}$	soil splash erosion potential	kg m^{-2} s^{-1}	User defined
	$R_{vegetation}$	Vegetation cover coefficient	Unitless	User defined
	M_{TC}	Sediment transport capacity	kg km^{-2} day^{-1}	Calc
	R_{TCsf}	Transport capacity scaling factor (a4)	kg m^{-2} day^{-1}	User defined
	R_{TCmax}	Transport capacity Direct runoff threshold (a5)	m^{3} sec^{-1}	User defined
	$R_{TCcoeff}$	Transport capacity non linear coefficient (a6)	unitless	User defined
	L	Reach length	m	input
	$M_{flowErosion}$	Mass of sediment mobilised by flow erosion	kg km^{-2} day^{-1}	Calc
	R_{FEsf}	Flow erosion scaling factor (a1)	s m^{-2}	User defined
	R_{FEpot}	Soil type dependent flow erosion potential	kg km^{-2} s^{-1}	User defined
	R_{FEmax}	Flow erosion direct runoff threshold (a2)	m^{3} s^{-1}	User defined
	$R_{FEcoeff}$	Flow erosion non linear coefficient (a3)	Unitless	User defined
	M_{sedOut}	Mass of sediment transported to the stream (per landuse type)	kg km^{-2} day^{-1}	calc
	$M_{sedStore}$	Store of detached sediment	kg km^{-2}	calc

	$M_{totalSedOut}$	Total mass of sediment transported to the stream from subcatchment	kg day^{-1}	calc
Phosphorus (general)	F_{start}	Start date of fertiliser addition	Julian Day	User defined
	F_{dur}	Duration of fertilizer addition	Days	User defined
	M_{solP}	Sum of solid P inputs	kg ha^{-1} day^{-1}	calc
	R_{SMan}	Solid P manure inputs	kg ha^{-1} day^{-1}	User
	R_{SFert}	Solid P fertiliser inputs	kg ha^{-1} day^{-1}	User
	R_{Res}	Plant residue solid inputs	kg ha^{-1} day^{-1}	user
	R_{sDep}	Annual atmospheric dry P deposition	kg ha^{-1} year^{-1}	user
	M_{LiqP}	Sum of liquid P inputs	kg ha^{-1} day^{-1}	Calc
	R_{lMan}	Liquid P manure inputs	kg ha^{-1} day^{-1}	User
	R_{lfert}	Liquid P fertiliser inputs	kg ha^{-1} day^{-1}	user
	M_{WDep}	Daily atmospheric wet P deposition	kg ha^{-1} day^{-1}	Calc
	$HER_{fannual}$	Annual total hydrologically effective rainfall flux	m^{3} km^{-2} year^{-1}	Calc
	$R_{wdepannual}$	Annual atmospheric wet P deposition	kg ha^{-1} year^{-1}	user
Phosphorus in soil	M_{labile}	Total labile P mass in soil	kg km^{-2}	Calc
	M_{sorbed}	Mass of P sorbed as labile soil P	kg km^{-2}	Calc
	$R_{weather}$	Weathering factor	Day^{-1}	User specified
	C_{t}	Temperature factor	unitless	Calc
	$M_{inactive}$	Inactive P mass in the soil	kg km^{-2}	Calc
	R_{immob}	Immobilisation factor	Day^{-1}	User specified
	M_{soil}	Soil mass	kg km^{-2}	User defined?
	R_{labsat}	Maximum soil labile P content	kg P kg^{-1} sed^{-1}	User defined
	M_{TDP_SW}	Mass of TDP in soil water	kg km^{-2}	Calc
	$M_{TDP\_sw(0)}$	Mass of TDP in soil water from the previous day	kg km^{-2}	Calc
	R_{uptake}	Plant P uptake factor	m day^{-1}	User specified
	P_{uptake}	Plant uptake of P from the soil water	kg km^{-2} day^{-1}	Calc
	C_{TDP_SW}	TDP concentration in soil water	mg l^{-1}	calc
	R_{SSc}	Soil sorption scaling factor	Day^{-1}	User defined
	FI_{SW}	Soil Freundlich isotherm constant	unitless	User defined
	EPC_{0SW}	Soil EPC_{0}	mg l^{-1}	Calculated
	R_{sc_SW}	Soil P sorption coefficient	kg^{-1} soil	User defined
	I_{growth}	Seasonal Plant growth index	unitless	Calculated
	R_{maxup}	Maximum daily plant P uptake	kg ha^{-1} day^{-1}	User specified

	R_{offset}	Plant growth curve vertical offset	Unitless	User defined
	R_{amp}	Plant growth curve amplitude	Unitless	User defined
	DOY	Day of year	Julian day	User defined
	G_{start}	Start date of plant growth period	Julian day	User defined
	$Temp_{Q10}$	Change in rate with a 10^o C change in temperature	Unitless	User defined
	$Temp_{Q10a}$	Temperature at which the rate response is 1	^oC	User defined
	R_{therm}	Soil thermal conductivity	$W\ m^{-1}\ {}^oC^{-1}$	user defined
	$Temp_{soil}$	Current soil temperature	^oC	Calc
	$Temp_{soil0}$	Soil temperature from the previous time step	^oC	Calc
	$Temp_{air0}$	Air temperature from the previous time step	^oC	calc
	R_{damp}	Empirical damping parameter	cm^{-1}	user
	d_{snow}	Snow depth	cm	calculated
	$Heat_{Cap}$	combined specific heat capacity from freezing, thawing and soil	$10^6 Jm^{-3}\ {}^oC^{-1}$	Calc
	K_{soil}	Specific heat capacity of soil	$10^6 Jm^{-3}\ {}^oC^{-1}$	Calc
	R_{freeze}	Specific heat capacity due to freeing and thawing of soil	$10^6 Jm^{-3}\ {}^oC^{-1}$	User specified
	$d_{soilTemp}$	Depth at which soil temperature is calculated	meters	Calculated
	R_{weqv}	Water equivalent factor	Unitless	User defined
Phosphorus in groundwater	$M_{aquifer}$	Aquifer mass	$kg\ km^{-2}$	Calc
	$R_{Maquifer}$	Aquifer mass per m^2 (depth x density)	$10^3\ kg^{-1}\ m^{-2}$	User def
	R_{GW_ssc}	Soil sorption scaling factor	day^{-1}	User defined
	C_{TDP_GW}	TDP concentration in groundwater	$mg\ l^{-1}$	calc
	EPC_{0_GW}	Groundwater EPC_0	$mg\ l^{-1}$	Calculated
	FI_{GW}	Groundwater Freundlich isotherm constant	Unitless	User defined
	$R_{GWcoeff}$	Groundwater P sorption coefficient	kg^{-1} soil	User defined
	$M_{sorbedGW}$	Mass of P sorbed as labile groundwater P	$kg\ km^{-2}$	Calc
	R_{aqsat}	Maximum aquifer solid P content	kg P kg^{-1} sed	User defined
	M_{TDP_GW}	TDP mass in groundwater	$kg\ km^{-2}$	Calc
	$M_{TDP\_GW(0)}$	TDP mass in groundwater of previous day	$kg\ km^{-2}$	Calc
	C_{TDP_GW}	Concentration of TDP in groundwater	$mg\ l^{-1}$	calc
Phosphorus in quickflow	M_{TDP_QF}	TDP mass in quick flow	$kg\ km^{-2}$	Calc
	C_{TDP_QF}	TDP concentration in quick flow	$mg\ l^{-1}$	calc
	$M_{TDPclass}$	Total TDP mass per landuse class	$kg\ km^{-2}$	calc
	$M_{TDPtotal}$	Total TDP mass exported to river reach	$kg\ day^{-1}$	calc

	$M_{PPclass}$	Mass of PP exported to river reach per landuse class	kg km^{-2} day^{-1}	calc
	R_{er}	Soil P enrichment factor	unitless	User defined
	$M_{PPtotal}$	Total PP mass exported to river reach	kg day^{-1}	calc
Biochemical oxygen demand	M_{BOD_SW}	Mass of BOD in Soil water	g km^{-2}	Calc
	$R_{BODfert}$	Mass of BOD in organic materials	g km^{-2}	User defined
	$R_{SW\_BODdecay}$	BOD soil decay rate	g day^{-1}	User defined
	rT	BOD Temperature factor	Unitless	calc
	M_{BOD_GW}	Mass of BOD in the groundwater	g km^{-2}	Calc
	$M_{BODperc}$	Mass of BOD percolating to the GW zone	g km^{-2}	Calc
	$R_{GW\_BODdecay}$	BOD groundwater decay rate	g day^{-1}	User defined
	$M_{BODLUclass}$	Total mass of BOD exported river per landuse class	kg day^{-1} km^{-2}	Calc
	$M_{BODtotal}$	Total mass of BOD exported to river reach	kg day^{-1}	Calc
Dissolved oxygen	R_{DO_SW}	Initial DO soilwater concentration	g l^{-1}	User defined
	M_{DO_sw}	Mass of DO in Soil water	g km^{-2}	Calc
	M_{DO_GW}	Mass of DO in groundwater	g km^{-2}	Calc
	R_{DO_GW}	Initial DO groundwater concentration	g l^{-1}	User defined
	M_{DO_QF}	Mass of DO in quickflow	g km^{-2}	Calc
	R_{DO_QF}	Initial DO quickflow concentration	g l^{-1}	User defined
	$M_{DOLUclass}$	Total mass of DO exported river per landuse class	g day^{-1} km^{-2}	Calc
	$M_{DOtotal}$	Total mass of DO exported to river reach	g day^{-1}	calc
Hydrology	V	Reach velocity	m sec^{-1}	Calc
	N	Manning’s roughness coefficient	unitless	User defined
	HR	Hydraulic radius	m	calc
	S	River bed slope	m m^{-1}	User defined
	A_r	Reach cross sectional area	m^2	Calc
	D	Reach depth	m	calc
	W_{sur}	Reach width at the surface	m	Calc
	W_{sed}	Reach width at the bed sediment	m	calc
	P_r	Reach wetted perimeter	m	calc
	C	$\mathbf{c}Q^f$	Unitless	User defined
	F	$cQ^{\mathbf{f}}$	Unitless	User defined
	A	$\mathbf{a}Q^b$	Unitless	User defined
	B	$aQ^{\mathbf{b}}$	unitless	User defined

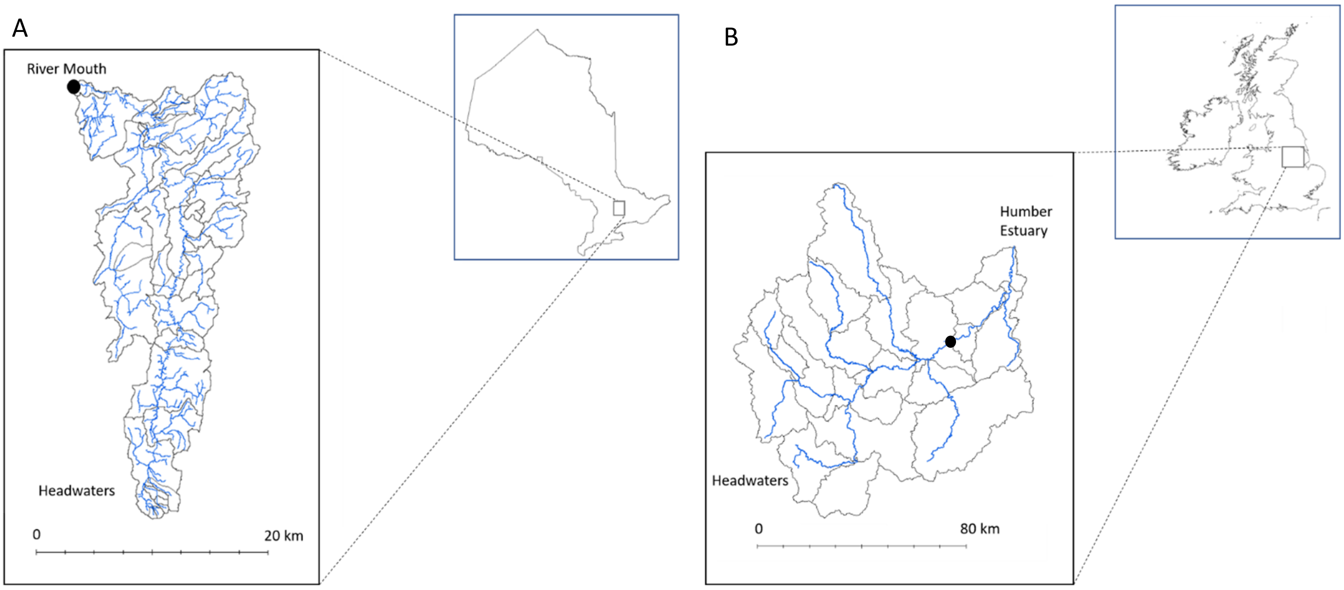
	S_{reach}	Current day’s volume	m^{3}	Calc
	S_{reach0}	Previous day’s reach storage	m^{3}	Calc
	RT_{reach}	Reach residence time	days	calc
	$Q_{reachIn}$	Net reach inflow	m^{3} day^{-1}	calc
	$Q_{reachOut}$	Reach outflow	m^{3} s^{-1}	calc
Instream sediment	G_{max}	Maximum entrainable grainsize	m	Calc
	V_{shear}	Reach shear velocity	m s^{-1}	calc
	G	Acceleration due to gravity	(9.9941) m s^{-2}	constant
	V_{dev}	Deviation from ideal channel characteristics	unitless	User defined
	M_{mov}	Moveable bed mass	kg m^{-3}? Or per day?	Calc
	M_{bed}	Total bed sediment mass	kg m^{-3}? Or per day?	calc
	$Ent_{\%}$	Proportion of bed mass which is entrained	fraction	calc
	M_{ent}	Entrainable sediment mass	kg m^{-3}? Or per day?	Calc
	R_{entsf}	Entrainment Scaling factor (a8 in model)	s^{2} kg^{-1}	User defined
	S_{p}	Stream power per unit area of stream bed	J s^{-1} m^{-2}	Calc
	r_{f}	Bed friction	m^{-1}	calc
	HR_{sed}	Hydraulic radius used in bed friction equation	m	calc
	$Density_{H20}$	Density of water (1000)	kg m^{-3}	constant
	HR_{max}	Hydraulic radius of a pipe	m	Calc
	v_{term}	Terminal velocity	m s^{-1}	Calc
	$Density_{sed}$	Density of sediment (2650)	kg m^{-3}	constant
	G_{med}	Medium particle diameter	m	Calc
	Fv	Fluid viscosity	m s^{-1}	Calc
	M_{sedDep}	Mass of suspended sediment deposited	kg m^{-2} day^{-1}	Calc
	M_{sus}	Mass of suspended sediment available	kg	Calc
	$M_{background}$	Background sediment released within a channel	kg m^{-2} day^{-1}	calc
	Bsf	Background sediment scaling factor	kg m^{-2} m^{-3}	User defined
	Bnl	Background sediment nonlinear coefficient	unitless	User defined
	M_{PS}	Sediment from a point source	kg day^{-1}	User defined
	M_{up}	Mass of suspended sediment from upstream	kg day^{-1}	Calc
In-stream phosphorus (water column)	M_{TDP_WC}	Mass of TDP in the water column	kg	calc
	M_{TP_US}	Mass of TDP input from upstream reach	kg day^{-1}	Calc
	M_{TDPeff}	Mass of TDP effluent	kg day^{-1}	User defined

	M_{TDPex}	Mass of TDP exchanged between pore water and water column	kg day^{-1}	calc
	M_{TDPabs}	Mass of TDP abstraction	kg day^{-1}	User defined
	M_{TDPout}	Mass of TDP output from reach	kg day^{-1}	Calc
	dM_{sorbwc}/dt	Change in mass of P adsorbed to suspended sediment	kg day^{-1}	Calc
	$M_{TDP\_WC(0)}$	Mass of TDP in the water column on previous day	kg	Calc
	M_{PP_WC}	Mass of PP in the water column	kg	Calc
	C_{Psat_WC}	Maximum P:sed ratio in suspended material	kg P kg^{-1} sed	User defined
	C_{TDP_WC}	Concentration of TDP in the water column	mg l^{-1}	Calc
	R_{SF_WC}	Water column sorption scaling factor	day^{-1}	User defined
	FI_{wc}	Water column Freundlich isotherm	Unitless	User defined
	EPC_{O_WC}	Water column equilibrium phosphorus coefficient	mg l^{-1}	calculated
	R_{sc_wc}	Water column sorption coefficient	kg^{-1} day^{-1}	User defined
	M_{TDPex}	Exchange of TDP between water column and bed porewater	Unitless	calc
	$R_{Ex\%}$	Fraction exchanged between the water column and stream bed	day^{-1}	User defined
	C_{TDP_SB}	Concentration of TDP in the stream bed porewater	mg l^{-1}	Calc
	M_{PP_US}	Mass of PP input from upstream	kg day^{-1}	calc
	M_{PPeff}	Mass of PP input from effluent	kg day^{-1}	User defined
	M_{PPent}	Mass of PP entrained from the stream bed	kg day^{-1}	Calc
	M_{PPdep}	Mass of PP deposited on the stream bed	kg day^{-1}	calc
	M_{PPabs}	Mass of PP abstraction	kg day^{-1}	User defined
	M_{PPout}	Mass of PP output from reach	kg day^{-1}	Calc
	M_{PP_SB}	Mass of PP in the stream bed	kg	Calc
	C_{PP_WC}	Concentration of PP in the water column	mg l^{-1}	Calc
Instream phosphorus (stream-bed)	M_{TDP_SB}	TDP mass in stream bed porewater	kg	Calc
	$M_{TDP\_SB(0)}$	Mass of TDP in bed porewater on previous day	kg	Calc
	C_{Psat_SB}	Maximum P:sed ratio in bed material	kg P kg^{-1} sed	User defined
	$dM_{sorbbed}/dt$	Change in mass of P adsorbed to bed sediment	kg day^{-1}	Calc
	R_{SF_SB}	Stream bed sorption scaling factor	day^{-1}	User defined
	FI_{SB}	Stream bed Freundlich isotherm	Unitless	User defined
	R_{SC_SB}	Stream bed sorption coefficient	kg^{-1} day^{-1}	User defined
	EPC_{0SB}	Stream bed equilibrium phosphorus coefficient	mg l^{-1}	calculated
	C_{TDP_SB}	Concentration of TDP in the stream bed porewater	mg l^{-1}	Calc
	V_{pw}	Porewater volume	m^{3}	calc

	$R_{beddepth}$	Bed depth	m	User defined
	$R_{bed\phi}$	Bed sediment porosity	Unitless	User defined
SRP	C_{SRP_WC}	Concentration of SRP in the water column	mg l^{-1}	Calc
	$Grad_{SRP:TP}$	Gradient of the user-defined regression line between SRP and TDP	Unitless	User defined
	$Y_{SRP:TDP}$	Y-intercept of the regression relationship between SRP and TDP	Unitless	User-defined
	$Rat_{SRP:DPC}$	Ratio of dissolved hydrolysable P to DO	Unitless	User defined
	Obs_{DOC}	Mean water column DOC concentration	mg l^{-1}	User-defined
Instream BOD	M_{BOD_WC}	Mass of BOD in water column	g O_2 km^{-2}	calc
	M_{BODus}	Mass of BOD from upstream	g O_2 km^{-2}	Calc
	M_{BODWW}	Mass of BOD from wastewater effluent	g O_2 km^{-2}	User-defined
	BOD_{phyto}	BOD contribution from algae	g O_2 km^{-2}	Calc
	BOD_{settle}	Rate of settling BOD	g km^{2} day^{-1}	calc
	$R_{phytoBOD}$	Contribution of dead phytoplankton to BOD	mg O_2 (μg Chl*a*)$^{-1}$ day^{-1}	User-defined
	$C_{phytoDeath}$	Death rate of phytoplankton	μg Chla l^{-1} day^{-1}	calc
	R_{netv}	Net settling velocity of BOD	m day^{-1}	User-defined
Instream DO	M_{DO_WC}	Mass of DO in water column	g km^{-2}	calc
	M_{DOus}	Mass of DO from upstream	g km^{-2}	Calc
	DO_{phytox}	Rate of phytoplankton oxygen contribution	g O_2 km^{2} day^{-1}	Calc
	DO_{sod}	Rate of chemical oxidation by bed sediments	g O_2 km^{2} day^{-1}	calc
	DO_{atmox}	Rate of oxygen supply from re-aeration	g O_2 km^{2} day^{-1}	calc
	$M_{bod\_decay\_WC}$	BOD water column decay (quantity of BOD from decaying matter)	g km^{-2}	calc
	C_{DO_WC}	Concentration of DO in the water column	mg O_2 l^{-1}	calc
	$Temp_{wc}$	Water column temperature	^{o}C	calc
	Abs_{sat}	Concentration at which DO becomes saturated in the WC	mg O_2 l^{-1}	calc
	$DO_{atmox/day}$	Rate that atmospheric re-aeration occurs	day^{-1}	Calc (or user defined)
	$DO_{sat\%}$	% saturation value for WC DO	Unitless	Calc
	R_{aer}	Rearation rate	day^{-1}	User defined
	DO_{ptSyn}	Phytoplankton photosynthesis	g O_2 km^{2} day^{-1}	Calc
	DO_{ptRep}	Phytoplankton respiration	g O_2 km^{2} day^{-1}	calc
	$Solar_{hours}$	Daylight hours	Hours	calc
	DO_{lowSyn}	Low rate of photosynthesis, when Chla < 50 mg l^{-1}	kg O_2 day^{-1}	calc
	DO_{hiSyn}	High rate of photosynthesis, when Chla >= 50 mg l^{-1}	kg O_2 day^{-1}	calc

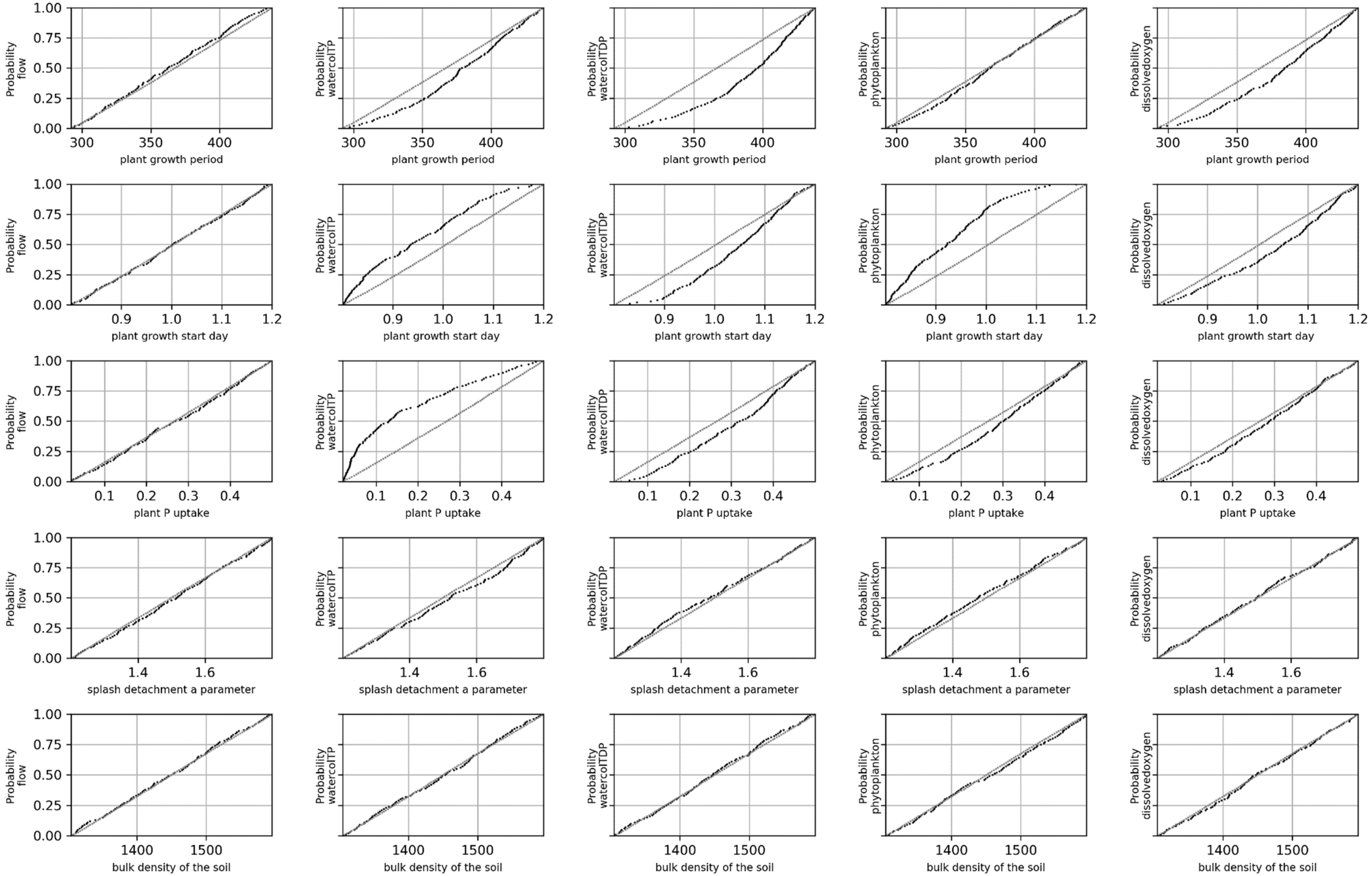
	R_{lowSyn}	Algal photosynthesis rate (low Chla)	mg O_2 (µg Chl*a*)$^{-1}$ day^{-1}	User defined
	R_{hiSyn}	Algal photosynthesis rate (high Chla)	mg O_2 (µg Chl*a*)$^{-1}$ day^{-1}	User defined
	$R_{phytOffset}$	Phytoplankton respiration offset	g O_2 km^{2} day^{-1}	User defined
	$R_{phytoSlope}$	Phytoplankton respiration slope	mg O_2 (µg Chl*a*)$^{-1}$ day^{-1}	User defined
	Sed_{ox}	Proportion of DO oxidised	unitless	Calc
	R_{ox}	Sediment oxidation rate	g m^{-2} day^{-1}	User defined
	$BOD_{decay\_coefficient}$	Rate of water column BOD decay	g O_2 km^{-2} day^{-1}	calc
	Oxid	Speed of oxidation	day^{-1}	User defined
Phytoplankton	$C_{phytoAdv}$	Advection of phytoplankton	µg l^{-1} day^{-1} (Chl-a)	Calc
	$C_{phytoGrowth}$	Growth rate of phytoplankton	µg l^{-1} day^{-1} (Chl-a)	Calc
	C_{phyto_US}	Phytoplankton concentration from upstream reach	µg^{-}l (Chl-a)	Calc
	R_{growth}	Rate of phytoplankton growth	day^{-1}	User defined
	rT	Phytoplankton growth temperature adjustment factor	Unitless	Calc
	rSR	Phytoplankton growth light adjustment factor	unitless	Calc
	R_{SRPmax}	Threshold concentration of SRP at which phytoplankton growth is uninhibited by nutrient availability	µg l^{-1}	User defined
	$R_{phytoShade}$	Concentration of phytoplankton at which self-shading occurs	µg l^{-1}	User defined
	$R_{tempAir}$	Air temperature below which algal growth is limited	^{o}C	User defined
	R_{solar}	Fraction of maximum annual solar radiation required for initiation of algal growth (between 0 and 1)	unitless	User defined
	$Solar_{rad}$	Daily solar radiation	lux	calc
	$Solar_{radMax}$	Maximum annual solar radiation	lux	calc
	$R_{phytoDeath}$	Rate of phytoplankton death	day^{-1}	User defined
	$Elevation_{solar}$	Solar elevation	degrees	calc
	Solarrad30	Solar radiation in 30 minute intervals	lux	calc
	Lat	latitude	degrees	Input parameter
	Dec	Solar declination	degrees	Calc
	dec1	Solar declination parameter	unitless	calc
	Time	Time of day	angle	Calc
	$Solar_{constant}$	Solar constant (1378)	W m^{-2}	Calc
	R_{aads}	Atmospheric solar radiation adsorption factor – Bras 1990	unitless	Calc

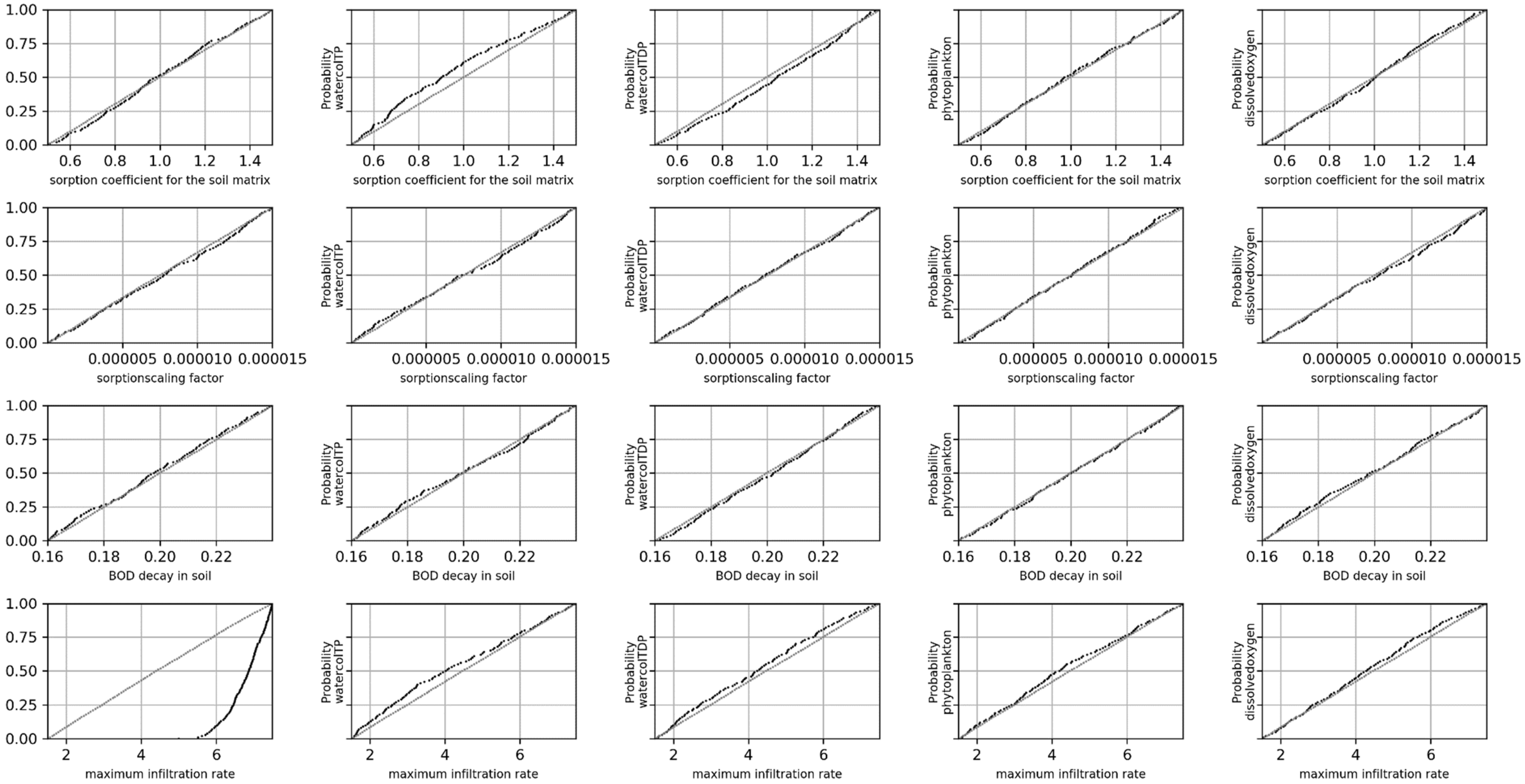
SI 3: Site map of a) Beaver River catchment boundaries, Ontario and b) Trent River catchment boundaries, UK. The black dot indicates the location of INCA PEco application.



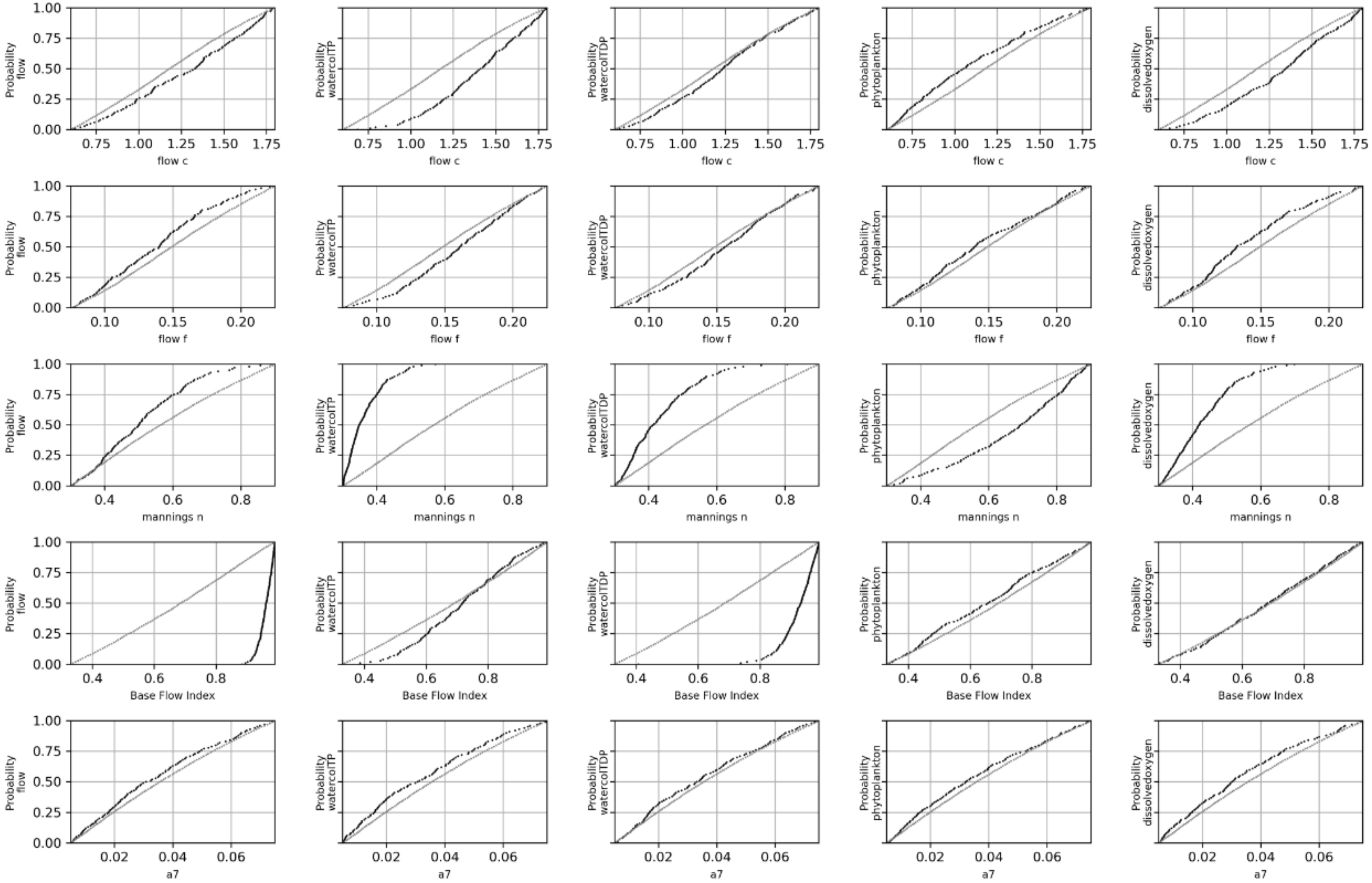
SI.4. Kling and Gupta Efficiency statistics (sensitivity analysis)

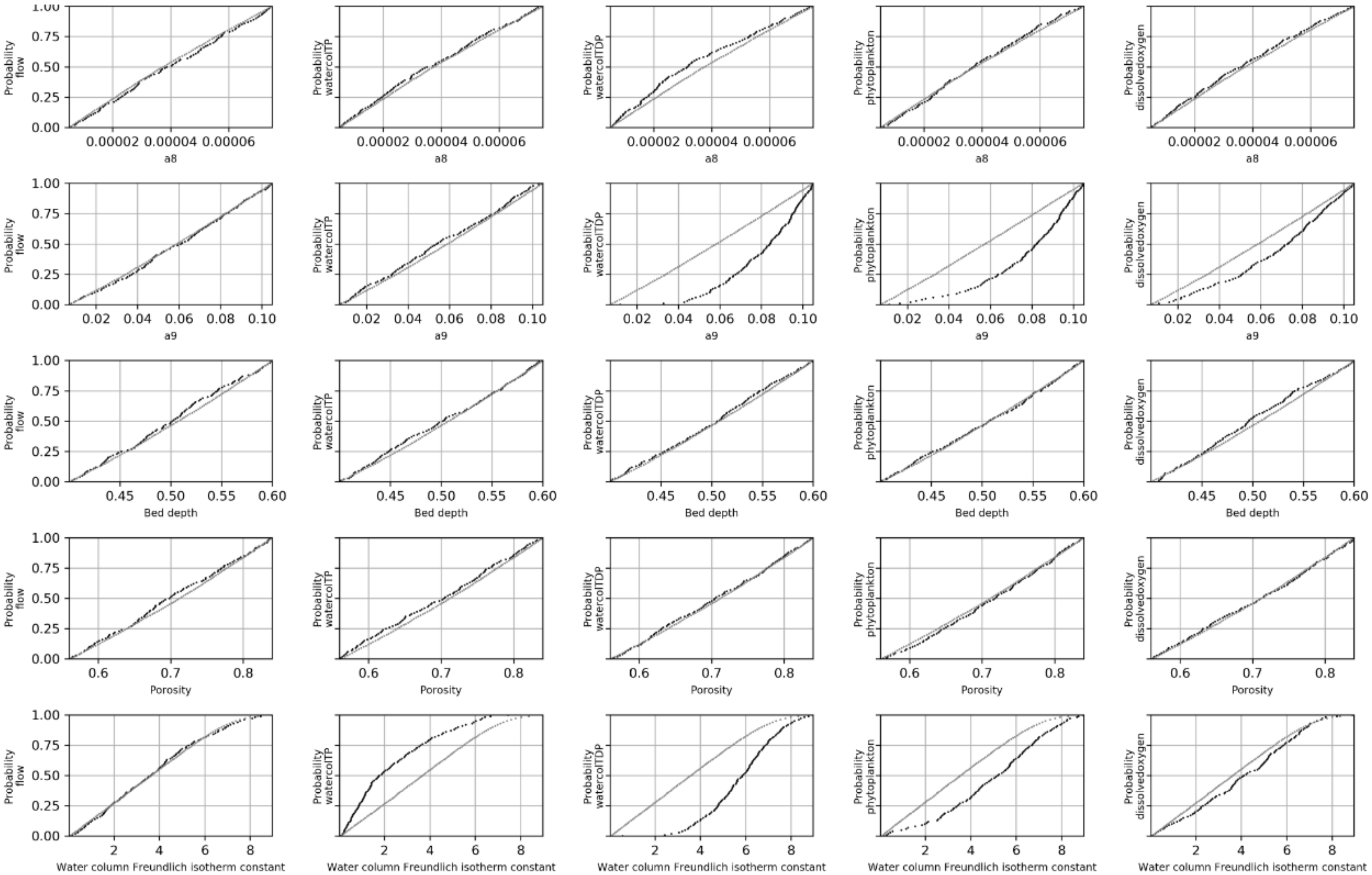
SI 4.1 Beaver Land phase analyses



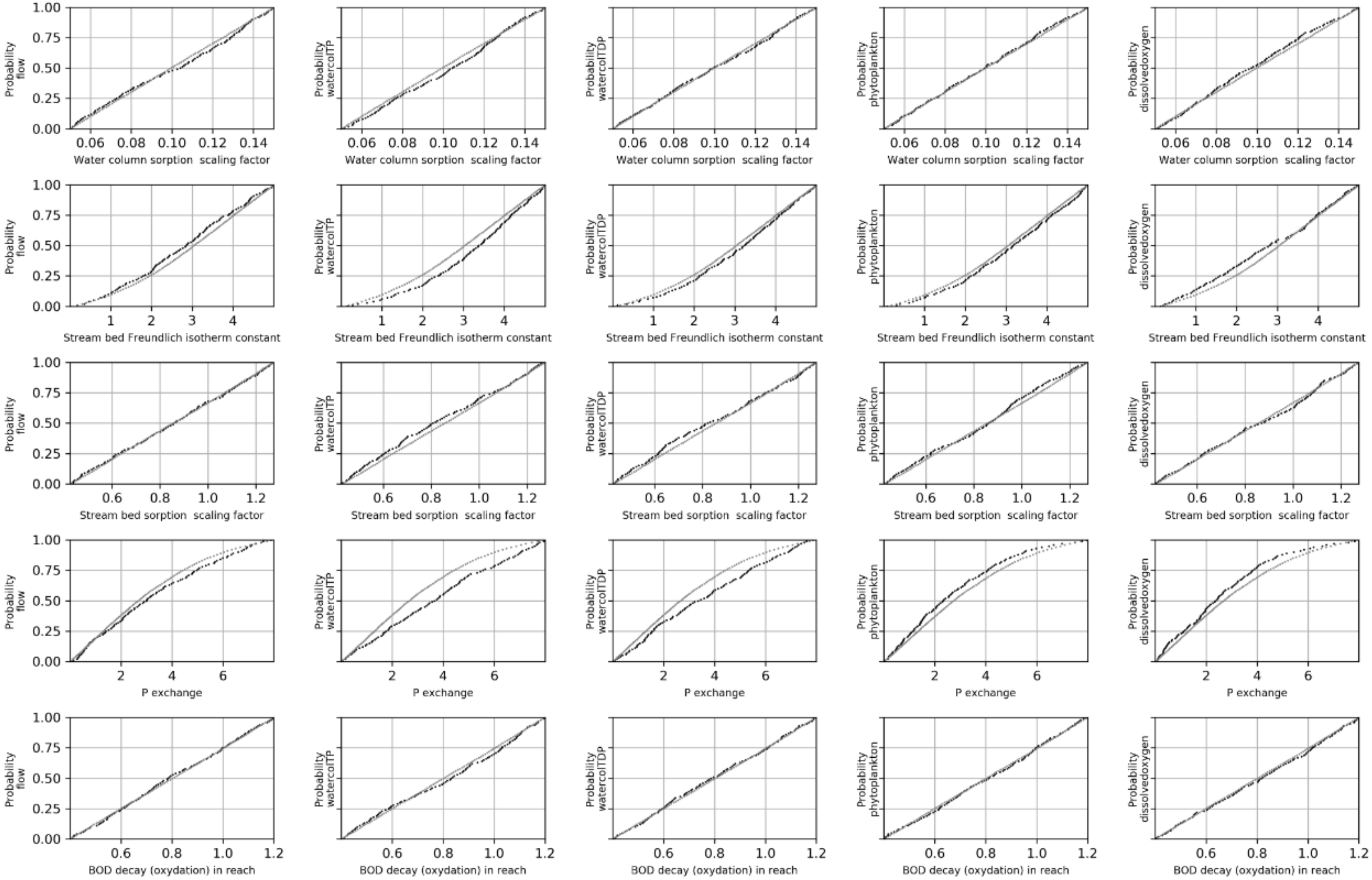

1.00
0.75
0.50
0.25
0.00
Probability watercolTP
Probability watercolTDP
Probability phytoplankton
Probability dissolvedoxygen
0.6
0.8
1.0
1.2
1.4
sorption coefficient for the soil matrix
0.000005
0.000010
0.000015
sorptionscaling factor
0.16
0.18
0.20
0.22
BOD decay in soil
2
4
6
maximum infiltration rate

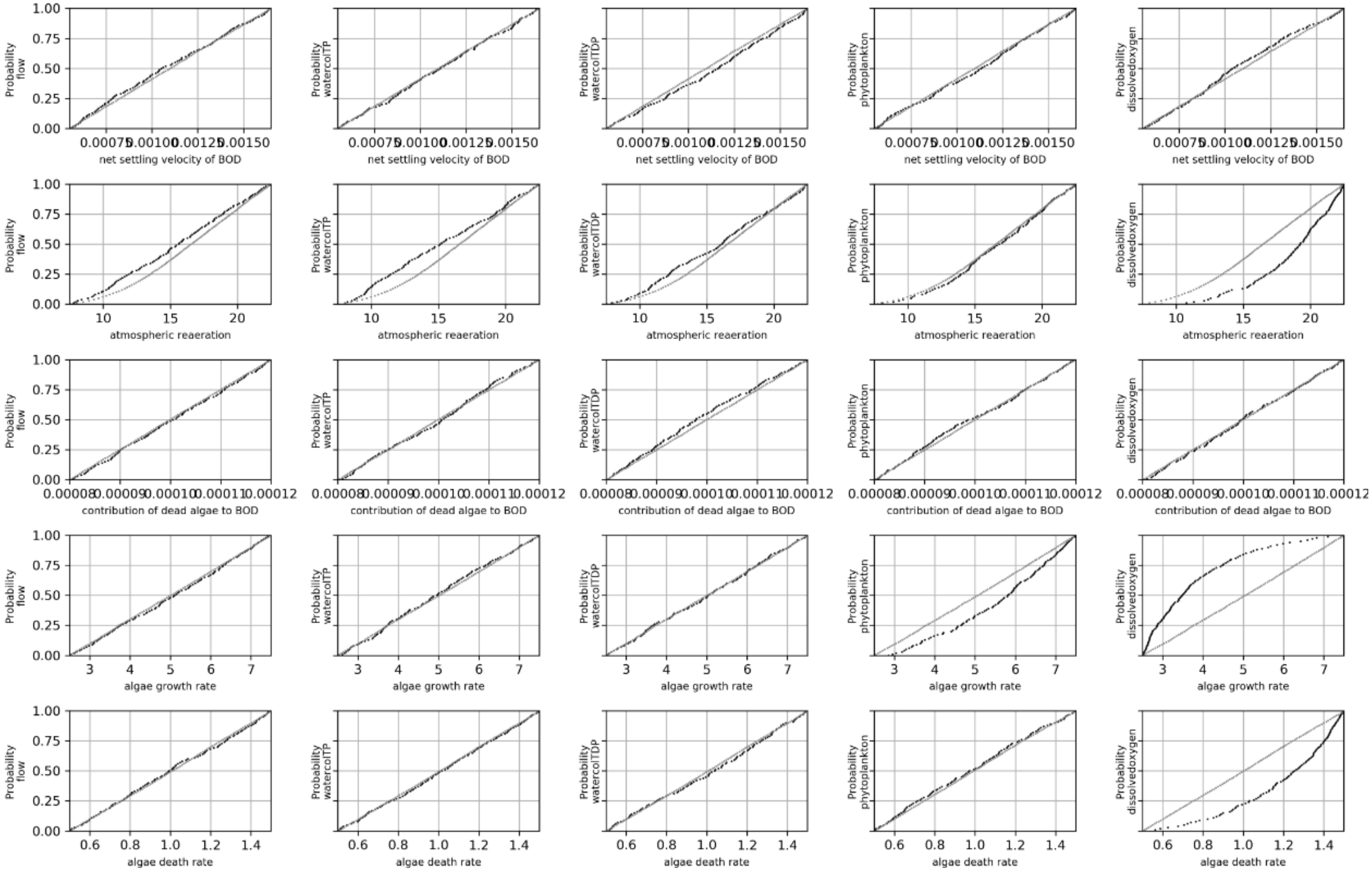
SI 4.2 Beaver River Reach analyses

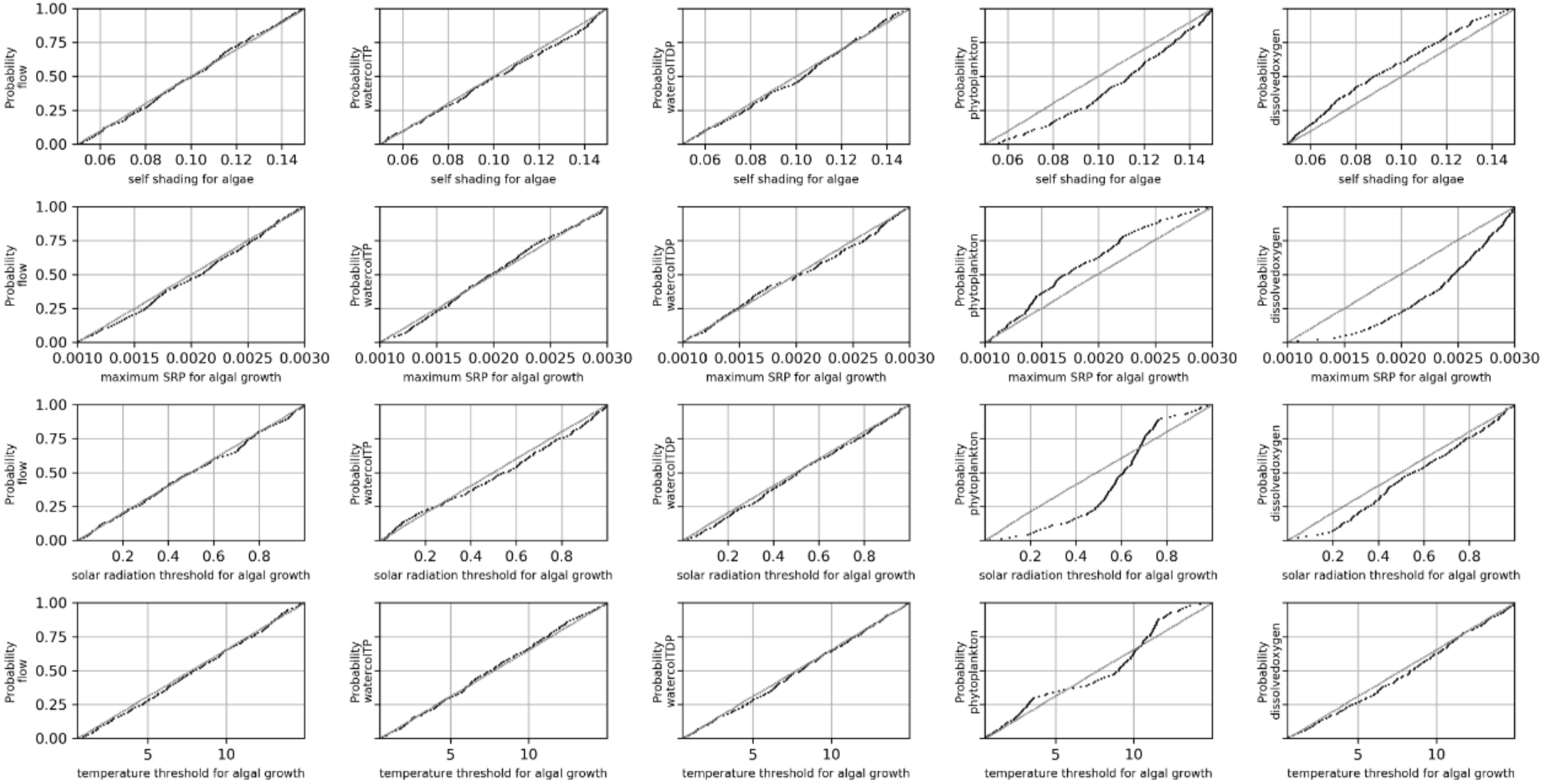




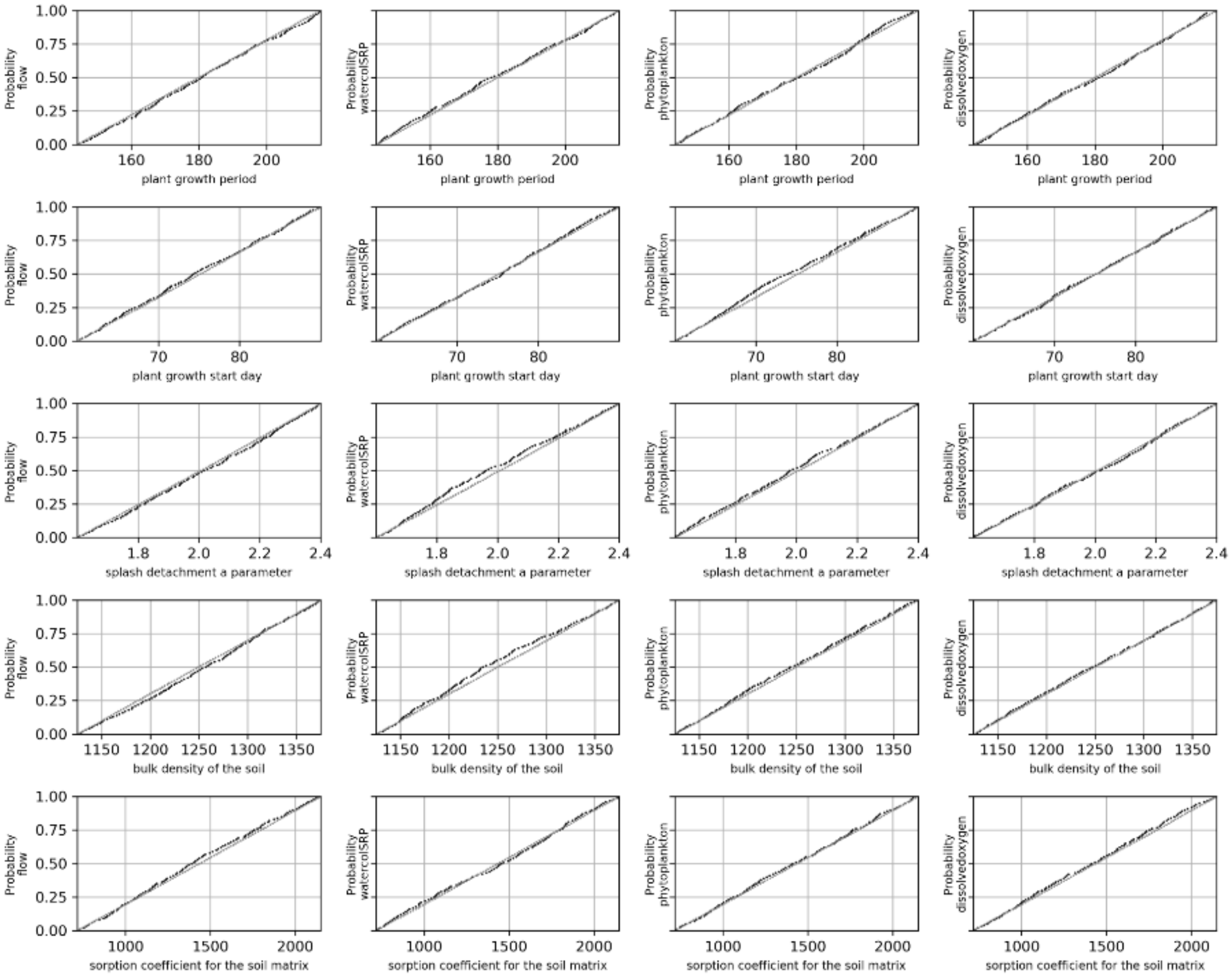
Probability flow
Probability watercolTP
Probability watercolTDP
Probability phytoplankton
Probability dissolvedoxygen
0.00
0.25
0.50
0.75
1.00
0.00002 0.00004 0.00006
a8
0.02 0.04 0.06 0.08 0.10
a9
0.45 0.50 0.55 0.60
Bed depth
0.6 0.7 0.8
Porosity
2 4 6 8
Water column Freundlich isotherm constant

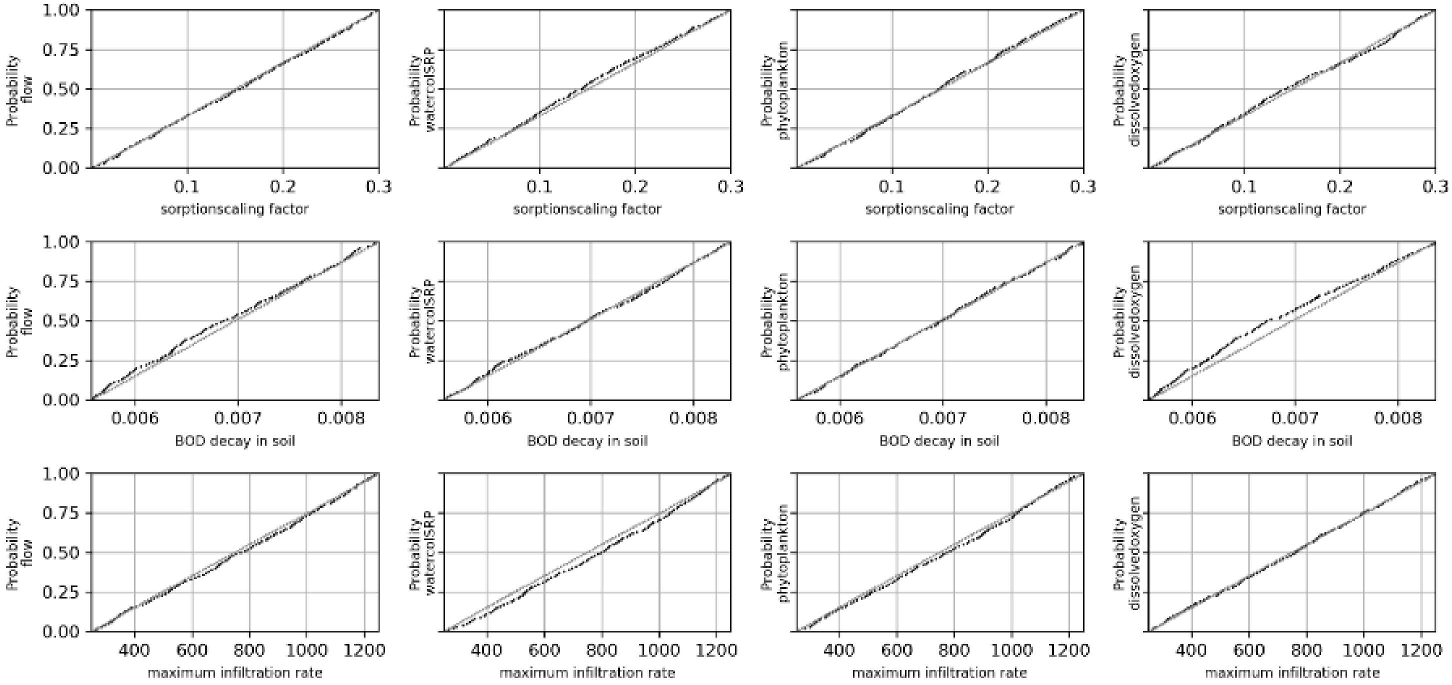

1.00
0.75
0.50
0.25
0.00
Probability
flow
Probability
watercolTP
Probability
watercolTDP
Probability
phytoplankton
Probability
dissolvedoxygen
0.06
0.08
0.10
0.12
0.14
Water column sorption scaling factor
1
2
3
4
Stream bed Freundlich isotherm constant
0.6
0.8
1.0
1.2
Stream bed sorption scaling factor
2
4
6
P exchange
0.6
0.8
1.0
1.2
BOD decay (oxydation) in reach


Probability
flow
watercolTP
watercolTDP
phytoplankton
dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
0.00075
0.00100
0.00125
0.00150
net settling velocity of BOD
10
15
20
atmospheric reaeration
0.00008
0.00009
0.00010
0.00011
0.00012
contribution of dead algae to BOD
3
4
5
6
7
algae growth rate
0.6
0.8
1.0
1.2
1.4
algae death rate

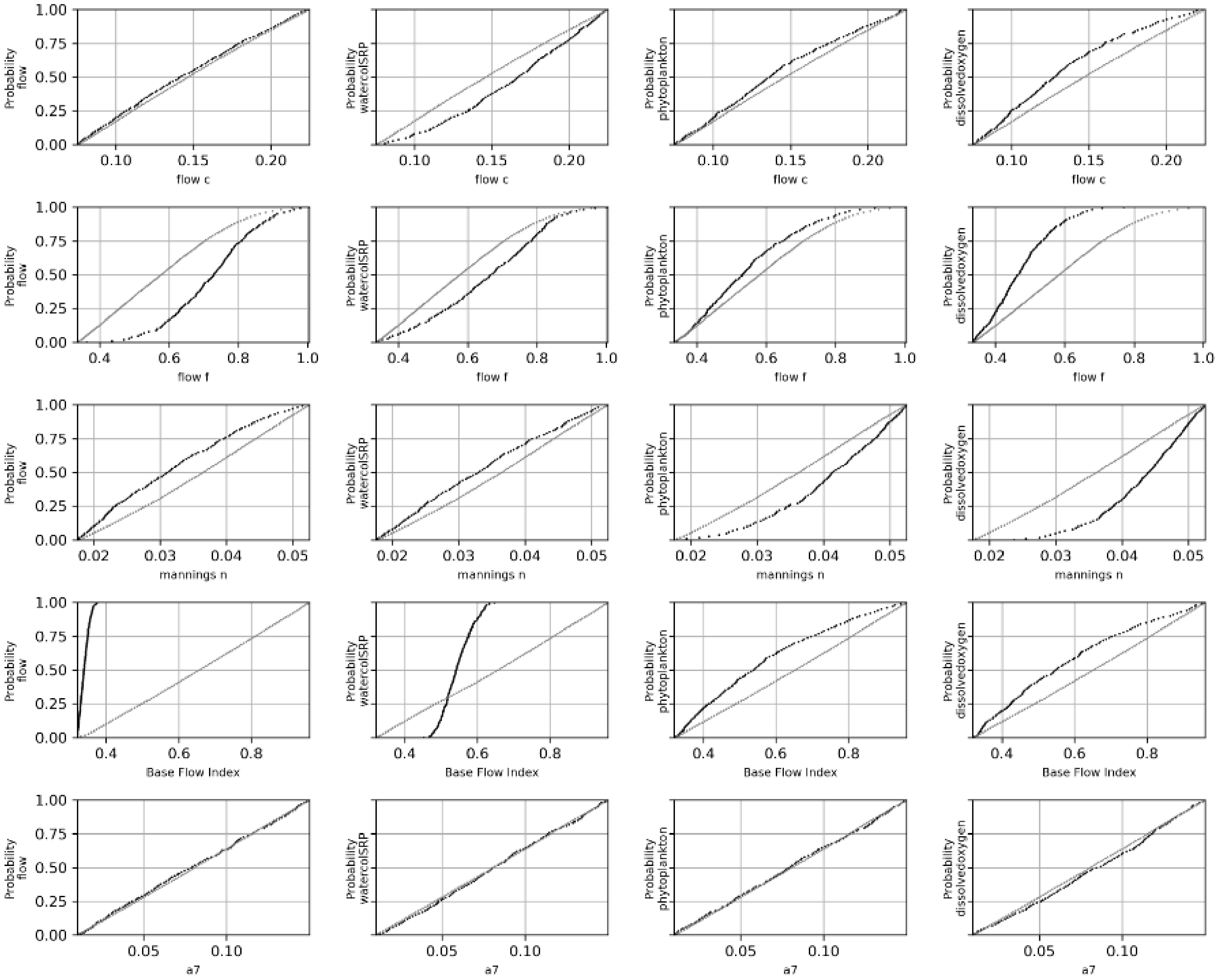

Probability
flow
Probability
watercolTP
Probability
watercolTDP
Probability
phytoplankton
Probability
dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
0.06
0.08
0.10
0.12
0.14
self shading for algae
0.0010
0.0015
0.0020
0.0025
0.0030
maximum SRP for algal growth
0.2
0.4
0.6
0.8
solar radiation threshold for algal growth
5
10
temperature threshold for algal growth

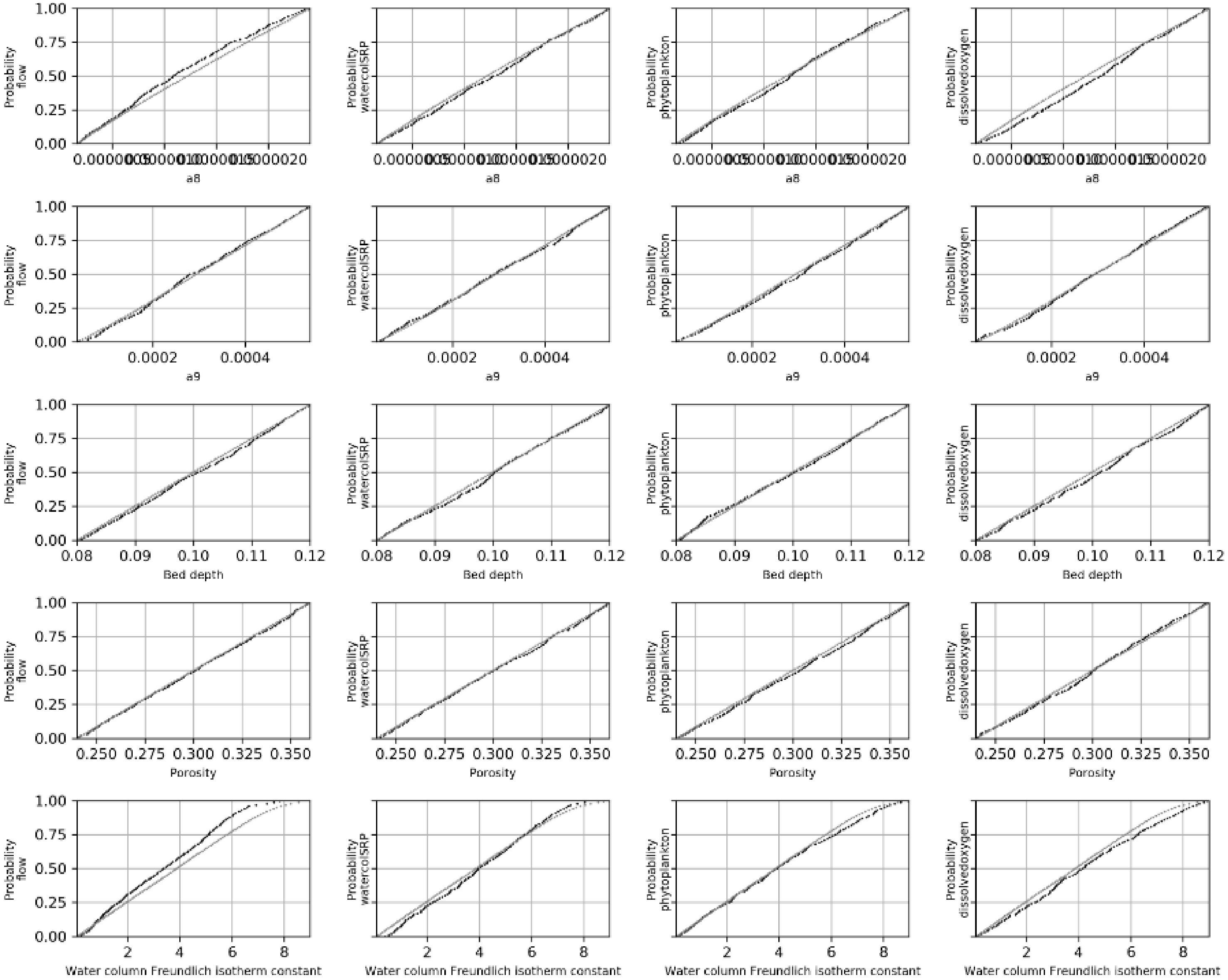
SI 4.3 Trent Land Phase analysis

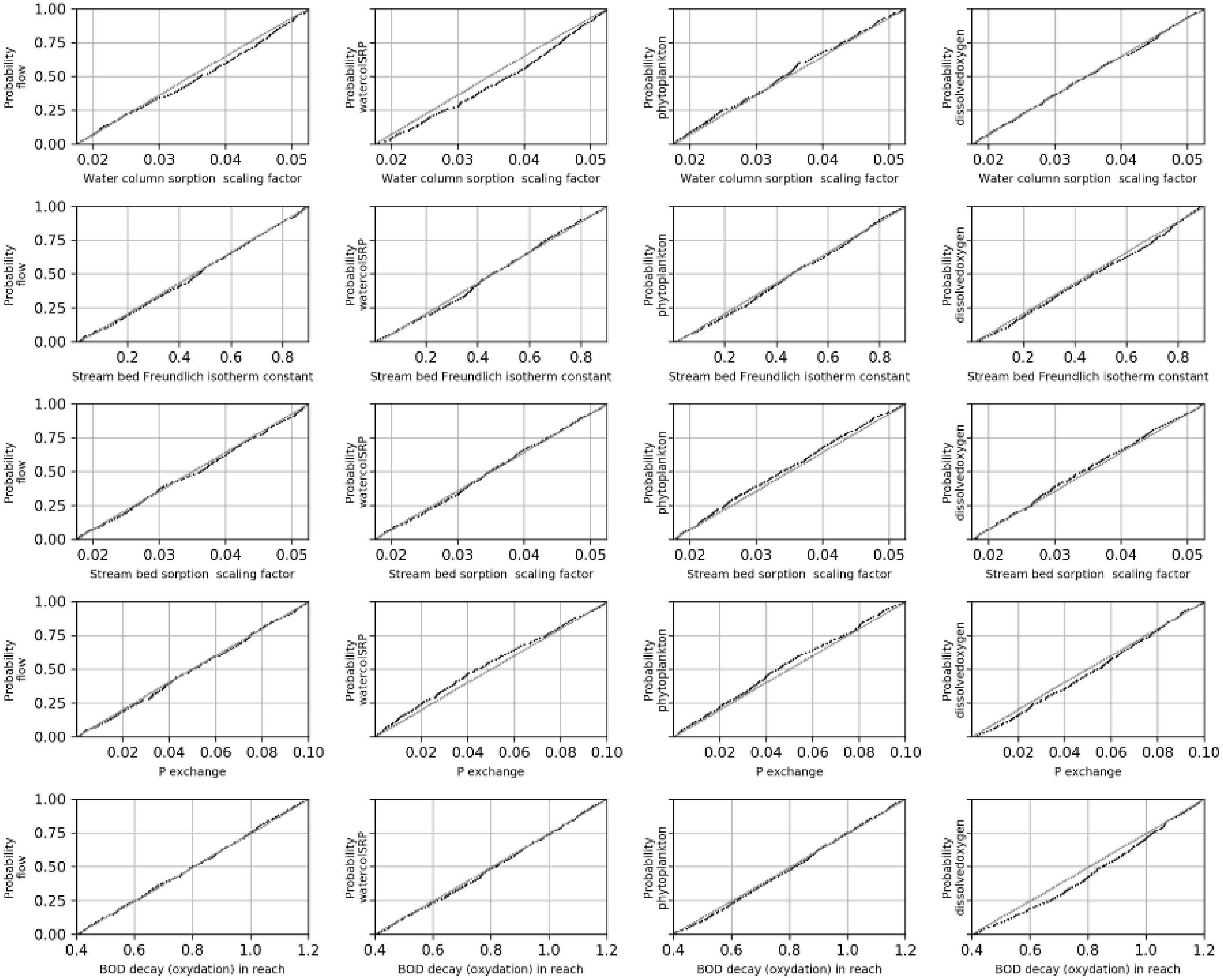


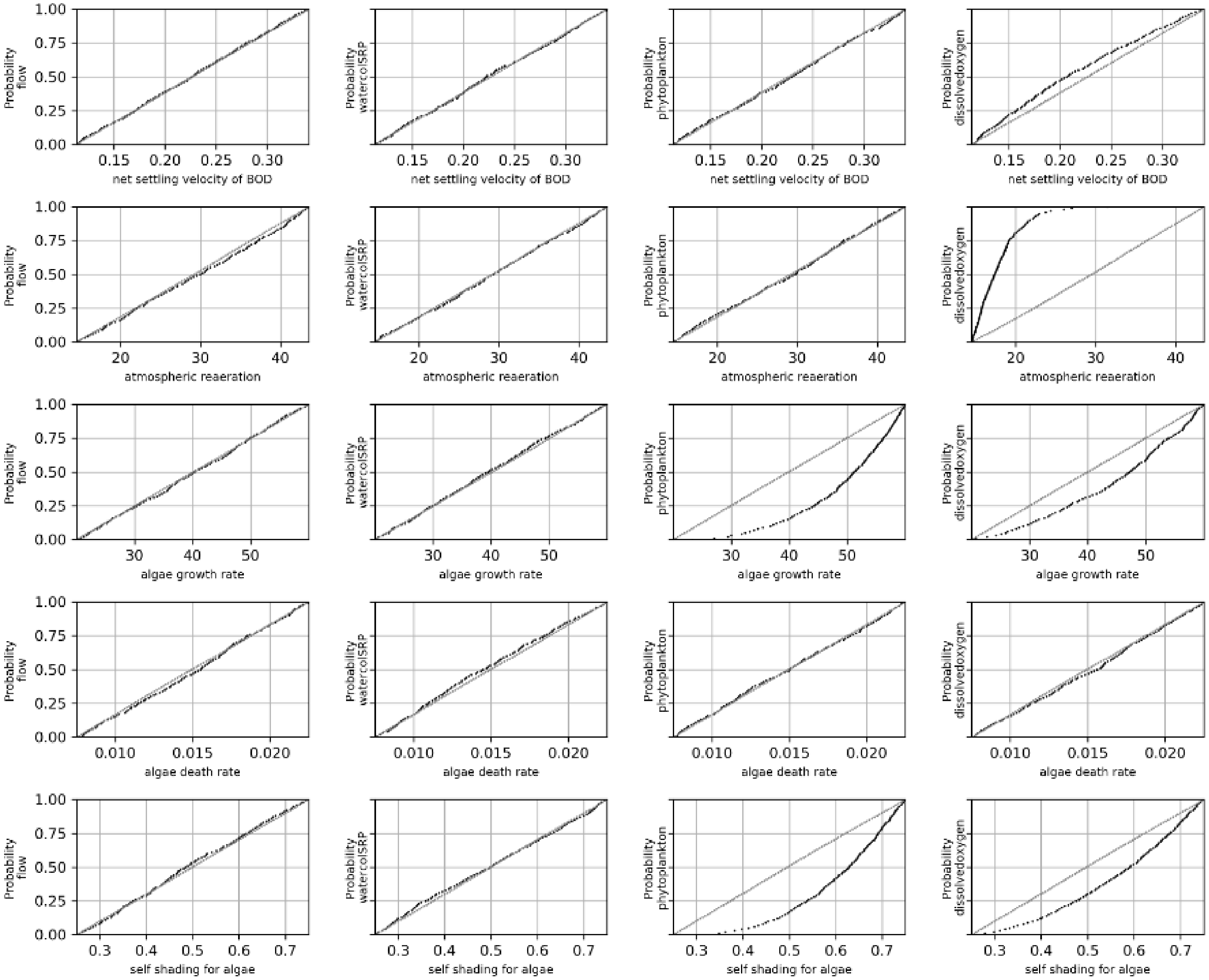

Probability
flow
Probability
watercolSRP
Probability
phytoplankton
Probability
dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.3
sorptionscaling factor
0.006
0.007
0.008
BOD decay in soil
400
600
800
1000
1200
maximum infiltration rate

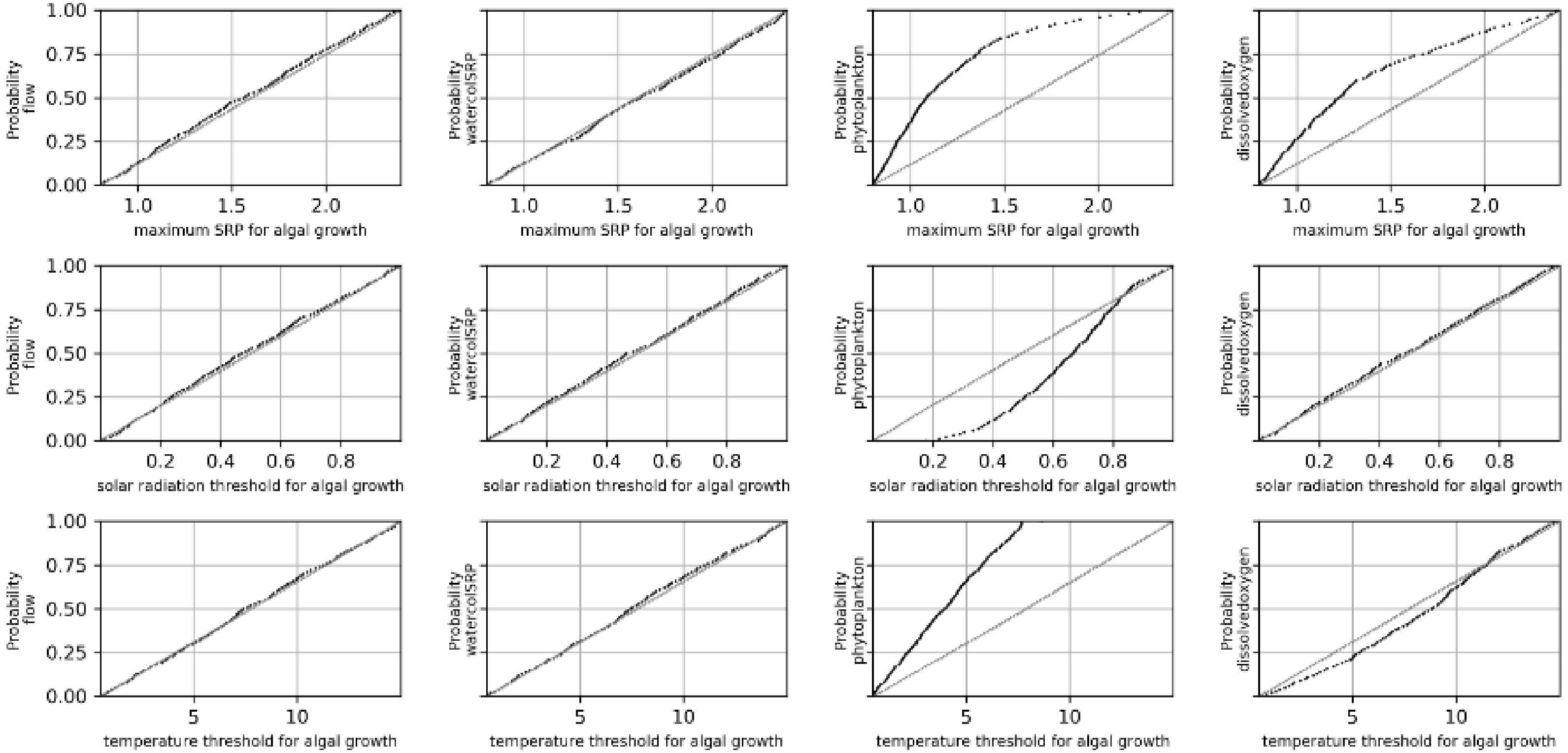
SI 4.4 Trent River Reach Analysis




1.00
0.75
0.50
0.25
0.00
Probability
flow
Probability
watercolSRP
Probability
phytoplankton
Probability
dissolvedoxygen
0.000000
0.000005
0.000010
0.000015
0.000020
a8
0.0002
0.0004
a9
0.08
0.09
0.10
0.11
0.12
Bed depth
0.250
0.275
0.300
0.325
0.350
Porosity
2
4
6
8
Water column Freundlich isotherm constant


Probability flow
Probability watercolSRP
Probability phytoplankton
Probability dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
0.02
0.03
0.04
0.05
Water column sorption scaling factor
0.2
0.4
0.6
0.8
Stream bed Freundlich isotherm constant
Stream bed sorption scaling factor
0.02
0.04
0.06
0.08
0.10
P exchange
0.4
0.6
0.8
1.0
1.2
BOD decay (oxydation) in reach


Probability flow
Probability watercolSRP
Probability phytoplankton
Probability dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
0.15
0.20
0.25
0.30
net settling velocity of BOD
20
30
40
atmospheric reaeration
30
40
50
algae growth rate
0.010
0.015
0.020
algae death rate
0.3
0.4
0.5
0.6
0.7
self shading for algae


Probability flow
Probability watercolSRP
Probability phytoplankton
Probability dissolvedoxygen
1.00
0.75
0.50
0.25
0.00
1.0
1.5
2.0
maximum SRP for algal growth
0.2
0.4
0.6
0.8
solar radiation threshold for algal growth
5
10
temperature threshold for algal growth

References:

Elliott, J.A., Persson, I., Thackeray, S.J., and Blenckner, T. 2007. Phytoplankton modelling of Lake Erken, Sweden by linking the models PROBE and PROTECH *Ecological Modelling* 202 (3-4): 421 – 426

Gustard, A., Bullock, A., Dixon, J. M.. 1992. Low flow estimation in the United Kingdom. Wallingford, Institute of Hydrology, 88pp. (IH Report No.108).