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Comparison of Soil Hydraulic Properties Estimated by Steady- and Unsteady-Flow Methods in the Laboratory

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Abstract: In this study, soil hydraulic conductivity (K) and soil sorptivity (S) values estimated by applying various steady- and unsteady-flow methods using cumulative infiltration data of three disturbed soils (sandy loam, loam, clay) obtained from a disc infiltrometer in the laboratory at various negative pressure heads were compared. The steady-flow methods used were those of Ankeny et al. and Reynolds and Elrick as well as Logsdon and Jaynes, while the unsteady-flow methods were those of Haverkamp et al. (two-term (2T) and three-term (3T) infiltration equations) and Zhang. The method of White et al., which is a steady-flow method but also uses unsteady-flow infiltration data, was also examined. The results showed that the three steady-flow methods, as well as the Zhang equation, for values of the van Genuchten coefficient n > 1.35, tend to give similar values of K. The 2T infiltration equation with β = 0.6 provided hydraulic conductivity values greater than those estimated by the steady-state methods but gave negative K values in some cases. The values of the coefficients C_1 and C_2 of the 2T equation were affected by the infiltration time. The coefficient C_1 increased while C_2 decreased with increasing time when the cumulative linearization method (CL) was applied, but the change in C_1 tended to be smaller than that in C_2. The inverse solution of the 3T equation using the Excel Solver application for β = 0.75 and β = 1.6, when positive values of K were obtained, approached better the K values estimated by the steady-flow methods compared with those estimated using β = 0.6. Regarding the estimation of S from the unsteady-flow equations (2T, 3T, Zhang), comparable S values were obtained by all equations. The differences between the S values of the various methods are smaller compared to those of K, and S is less affected than K in terms of time. The problem of negative estimates of K might be attributed to the fact that the soils used in this study are classified as soils situated in the domain of lateral capillarity or are not completely homogeneous or soil compaction is observed at some depth. In the case where the soils are not completely homogeneous, the Sequential Infiltration Analysis (SIA) method with β = 0.75 corresponding to the soil types studied was proved to be effective in estimating K values.

Keywords: steady-state methods; transient methods; soil hydraulic conductivity; soil sorptivity; laboratory experiments

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1. Introduction

Knowledge of the hydraulic properties of the upper soil layer, which define the water infiltration into the soil, such as hydraulic conductivity (K) and sorptivity (S), is essential for a wide range of applications, e.g., in the design of irrigation systems, water resources management, and hydrology as well as environmental protection.

Many researchers have contributed to the development of methods and apparatuses for measuring and/or estimating soil hydraulic properties and understanding their role in soil physics, hydrology, and water management [1–8]. In the last two decades, the tension disc infiltrometer [9] has been one of the most popular for the indirect estimation of K and S, especially in the field, from cumulative infiltration data [6,10–12]. A tension disc infiltrometer (TI) is a convenient method for characterizing 3D water infiltration. Its use refers to pressure heads less than or equal to zero ($H \leq 0$), i.e., to negative pressure heads close to saturation [2,13–15]. The diameter of the disc ranges from 4.5 to 20 cm [16].

TI has many advantages compared to other apparatuses in the measurement of soil hydraulic properties. During its use, the infiltration soil surface is not disturbed. It is relatively simple to use and can be used in both laboratory and field experiments. Because the experimental procedure is carried out under negative pressure heads or saturation conditions, it simulates the actual infiltration conditions prevailing in the soil. Finally, with TI, it is easy to repeat the experiments, especially in the field, due to limited wetting of the soil, and it provides the ability to carry out infiltration experiments in various soil types (from sandy to clay soils). The duration of an infiltration experiment under TI can vary from a few minutes to several hours depending on the soil type, the number of imposed pressure heads, and the type of flow we want to achieve (steady-state or transient flow).

The methods that have been proposed to study the infiltration data derived from TI are classified into two categories. The first category includes the methods that mainly use steady-flow data. The majority of the proposed methods are based on Wooding's equation [17], which describes the steady flow from a circular source and is expressed as:

$$Q_s = \pi r^2 K_0 + \frac{4K_0}{K_0 - K_i} \Phi_0 r \tag{1}$$

where Q_s ($L^3\ T^{-1}$) is the steady infiltration flux, r (L) is the radius of the disc, K_0 ($L\ T^{-1}$) is the hydraulic conductivity at the applied pressure head, H_0 (L), K_i ($L\ T^{-1}$) is the hydraulic conductivity at the initial pressure head, H_i (L), and Φ_0 is the matric flux potential which is defined as:

$$\Phi_0 = \int_{H_i}^{H_0} K(H)dH \tag{2}$$

In Wooding's equation, the soil is considered to be homogeneous, isotropic, and with uniform initial water content. The first term on the right-hand side of Equation (1) represents the effect of gravitational forces and the second one the effect of capillary forces. The solution of Wooding's equation in unsaturated porous media is achieved using the exponential equation of Gardner [18]:

$$K(H) = K_s e^{aH} \tag{3}$$

where K_s ($L\ T^{-1}$) is the saturated hydraulic conductivity and a (L^{-1}) is a soil texture parameter which expresses the relative importance of the gravity and capillary forces during water movement in unsaturated porous media.

Combining Equations (2) and (3) and assuming that the initial hydraulic conductivity K_i is negligible compared to K_0, the matric flux potential can be expressed as:

$$\Phi_0 = \frac{K_0}{a} \tag{4}$$

Thus, substituting $\Phi_0 = K_0/a$ into Equation (1), the steady-state infiltration rate under a tension disc infiltrometer can be expressed by the following equation:

$$i_s = K_0 \left(1 + \frac{4}{\pi r a}\right) \tag{5}$$

where i_s ($L\ T^{-1}$) is the steady-state infiltration rate ($i_s = Q_s/(\pi r^2)$) for the applied pressure head.

The disadvantages of steady-state methods are that they are time-consuming as they require steady flow to occur at all applied consecutive pressure heads and that long infiltration times require exploring deeper soil profiles, which, under field conditions, probably implies finding more heterogeneities in the soil.

Several methodologies have been proposed to solve Equation (5) and estimate K at the imposed pressure head. Nowadays, the methodology based on a single disc of large radius and multiple pressure heads tends to dominate. The main approaches based on this methodology are those of Ankeny et al. [1], Reynolds and Elrick [19], and Logsdon and Jaynes [20].

The approaches of Ankeny et al. [1] and Reynolds and Elrick [19] give similar estimates of K [21]. The application of these methods requires the imposition of at least two consecutive pressure heads by the TI during the infiltration experiment. Logsdon and Jaynes [20] reported that the hydraulic conductivity values estimated by the method of Ankeny et al. [1] begin to deviate from the actual ones as the pressure head decreases (i.e., becomes more negative). Logsdon and Jaynes [20], instead of piecewise using the infiltration data obtained at successive pressure heads, developed a nonlinear regression technique in which all experimental infiltration data are used simultaneously to calculate K_s and a. Then, K at each applied pressure head is calculated from Equation (3).

The method of White and Sully [22] and White et al. [23], although it is also based on Wooding's equation, does not follow the same philosophy as the previous three approaches mentioned. In this method, it is required to carry out an infiltration experiment under the TI at a pressure head until the steady-state flow condition (Steady-state Single Test, SST) is reached. Also, knowledge of the sorptivity as well as the initial and final soil water content is required to calculate soil hydraulic conductivity. Although the method was considered accurate for estimating K by Logsdon and Jaynes [20], both these researchers and Dohnal et al. [24] observed that in some cases the method was unable to successfully estimate K close to saturation, giving even negative values that have no physical meaning.

The second category of methods is based on the transient or not steady-state flow. The common characteristic of most equations of this category is that their form is similar to the one-dimensional two-term (1D 2T) vertical infiltration equation of Philip [25]:

$$I_{3D} = C_1\sqrt{t} + C_2 t \tag{6}$$

where I_{3D} (L) is the three-dimensional cumulative infiltration, t (T) is the time, and C_1 ($LT^{-1/2}$) and C_2 (LT^{-1}) are coefficients that differ depending on the applied model.

The experimental procedure of these methods is similar to that of White et al. [23], i.e., the application of a pressure head from a tension disc but without the requirement to achieve steady-state flow. The two most widespread equations used to study infiltration data in non-steady flow are those of Zhang [4] and Haverkamp et al. [2], with their common point being the use of Equation (6). However, the two methods correlate the coefficients C_1 and C_2 with sorptivity (S) and hydraulic conductivity (K), respectively, in a different way.

The method of Haverkamp et al. [2] for the estimation of K requires knowledge of the soil sorptivity and the initial and final water content of the soil. However, there are cases where this method is unable to estimate K because it gives even negative values of K as in the method of White et al. [23]. Vandervaere et al. [5] suggested the use of the equations of Haverkamp et al. [2] and White et al. [23] mainly in soils where the water flow is defined by gravity and the following criterion is satisfied:

$$\frac{\gamma {C_1}^2}{r(\theta_0 - \theta_i)} < \frac{C_2}{2} \tag{7}$$

where γ (-) is a constant equal to 0.75 [26], θ_0 ($L^3\ L^{-3}$) is the soil water content at the applied pressure head H_0 (L), r (L) is the radius of the tension disc, and θ_i ($L^3\ L^{-3}$) is the initial soil water content.

Dohnal et al. [24] revised the criterion of Vandervaere et al. [5], as it was found to be stringent, and proposed the following one:

$$\frac{\gamma {C_1}^2}{r(\theta_0 - \theta_i)} < C_2 \tag{8}$$

The method of Zhang [4] for estimating K also requires the knowledge of the soil type or soil water characteristic curve and assumes that the soil hydraulic properties follow the Mualem [27]–van Genuchten [28] model. Dohnal et al. [24] observed that Zhang's method in fine-textured soils, and more specifically in soils where the van Genuchten's [28] coefficient n is less than 1.35, does not reliably estimate K and proposed the following equation which is analogous to Zhang's one:

$$A_2 = \frac{11.65\left(n^{0.36} - 1\right)\exp[6.9(n - 1.3)\alpha H_0]}{\left(\alpha r_0\right)^{0.87}} \quad 1 < n < 1.35 \tag{9}$$

where α and n are van Genuchten's coefficients, r_0 (L) is the radius of the tension disc, and H_0 (L) is the applied pressure head.

The accurate calculation of the coefficients C_1 and C_2 of Equation (6) is of great importance for the application of non-steady-flow methods. It is usually suggested to calculate them through linearization methods of the infiltration data, because it is easy, even visually, for the researcher to detect any failure of experimental points and to decide whether Equation (6) can describe the experimental data in contrast with other methods. Two linearization methods have been proposed. The first method was proposed by Smiles and Knights [29], who suggested the linearization of Equation (6) by dividing both sides with the square root of time (Cumulative Linearization—CL method) as:

$$\frac{I}{\sqrt{t}} = C_1 + C_2\sqrt{t} \tag{10}$$

The second method was proposed by Vandervaere et al. [5] and compared with that of Smiles and Knights [29] has the advantage that it can visually detect the data referring to the layer of contact material between the soil surface and the tension disc of the infiltrometer. The estimation of coefficients C_1 and C_2 can be performed by differentiating the cumulative infiltration data with respect to the square root of time (Differential Linearization—DL method):

$$\frac{dI}{d\sqrt{t}} = C_1 + 2C_2\sqrt{t} \tag{11}$$

Equation (11) is a linear relationship with C_1 equal to the intercept and $2C_2$ equal to the slope of the regression line. In the case where a contact material is used to ensure the hydraulic contact between the disc and the soil, the influence of the contact material is easy to detect since it corresponds to the initially sharply decreasing part of the experimental curve of infiltration data $dI/d\sqrt{t}$ vs. $\sqrt{t}$, deviating from the monotonically increasing part of the experimental curve [21].

Although the two linearization methods derive from the same equation (Equation (6)), several researchers [15,30,31] have shown that there are cases where the coefficient values determined from the two methods differ significantly. Also, Bagarello and Iovino [32] showed that in the DL method, the separation of the experimental data of the contact material from those of the soil could be made easily only when the sorptivity of the contact material was 10 to 12 times higher than the soil sorptivity. In other cases, a transition zone, which complicated the application of the DL method, appeared between the decreasing and increasing portions of the data set. Therefore, the applicability of the DL method required large differences in capillary forces between the contact material and the soil.

Several researchers have also reported that the estimation of the coefficients of Equation (6) is affected by the experimental duration, which will also affect the estimation of S and K [21,33]. Latorre et al. [6] reported that the DL method is valid only for short to medium times.

Haverkamp et al. [2] were based on the study of Parlange et al. [34] for a one-dimensional (1D) equation and using the equation of Smettem et al. [35], which relates three-dimensional (3D) to 1D infiltration, obtained the implicit 3D equation of Haverkamp et al. [2], which is valid for all infiltration times. If the initial hydraulic conductivity is assumed to be negligible, the implicit 3D equation of Haverkamp et al. [2] is converted into the two-term equation (2T) of Haverkamp et al. [2], which is valid for short and medium times.

Given the time limitations of the 2T expansion, Latorre et al. [6] proposed estimating K and S from the numerical solution of the quasi-exact implicit (QEI) analytical Haverkamp et al. [2] equation (Numerical Solution of the Haverkamp equation, (NSH) method), which is valid for the entire infiltration time.

Given the complexity of solving the implicit 3D equation of Haverkamp et al. [2], Moret-Fernadez et al. [36], utilizing the 1D three-term (3T) and four-term (4T) expansion models of the quasi-exact implicit (QEI) analytical Haverkamp et al. [2] equation [37], proposed 3D 3T and 3D 4T cumulative infiltration equations, which are valid for long infiltration times, for estimating S and K and using a nonlinear (weighted) least-squares model implemented in the R software, and estimated the values of K and S in the cases of three synthetic soils and field experiments at 0 cm pressure head. The duration of the experiments ranged from 300 to 10,000 s for synthetic soils, and from 370 to 1800 s for the experiments in the field. The results showed that the most reliable estimations of K and S were obtained for constant values of the parameters β equal to 0.6 and 1.6 and γ equal to 0.75 of the 3T equation and there is a very good correlation of K and S values between 3T and 4T expansion models. Thus, the question of evaluating the 3T equation at other negative pressure heads close to saturation remains open.

Moret-Fernández et al. [38], using the QEI and 4T expansion model, developed the Sequential Infiltration Analysis (SIA) method for analyzing infiltration curves measured on layered soil profiles. The method considers a sequence of increasing time series from the cumulative infiltration data to estimate K and S and its corresponding RMSE characterizing the quality of the fit. Laboratory experiments on layered soils and field measurements showed robust estimates of K and S by applying the SIA method, making it a promising and useful tool for characterizing the hydraulic properties of layered and heterogeneous soils. In addition, no differences were observed between the two models (QEI and 4T) in all cases of the soils examined (synthetic soils, laboratory, and field measurements), in the estimation of K and S values.

Recently, Yilmaz et al. [39] investigated the values of the parameters β and γ of the long-time expansions of the Haverkamp model using numerical infiltration data for different soil types and different initial soil moisture contents when the hydraulic properties of the soils are described by the van Genuchten–Mualem equation (vGM).

From the abovementioned, it appears that among the mentioned methods there are great differences in their characteristics, the number of parameters that must be known in advance, and the form of the equations that describe the relationships K(H) or K(θ). Also, any comparisons among the methods are reported either between steady-flow methods or between non-steady-flow methods, while comparisons among all methods (steady and unsteady) have not been made, except in very few cases [40].

The general purpose of this study is the comparison of steady- and unsteady-flow methods in estimating K and S, in particular, (a) the steady-flow methods of Ankeny et al. [1], Reynolds and Elrick [19], and Logsdon and Jaynes [20] and (b) the unsteady-flow methods of Zhang [4], and the alternative approximate expansions 2T and 3T equations of the quasi-analytical solution of Haverkamp et al. [2], which were proposed by Haverkamp et al. [2] and Moret-Fernadez et al. [36], respectively. The usual values of the parameters β and γ as well as the corresponding ones based on the soil type as proposed by Yilmaz et al. [39] were examined. In addition, the sequential infiltration analysis (SIA) [38] for treating infiltration curves measured on layered soil profiles was applied, and (c) the equation of White et al. [23] was studied. The aforementioned equations and methods were applied to the infiltration data obtained from laboratory experiments at three disturbed soils using a 10 cm radius tension disc infiltrometer by imposing three to four negative pressure heads.

2. Materials and Methods

2.1. Soil Types and Experimental Pressure Heads

Three-dimensional infiltration experiments under negative pressure heads near saturation using a tension disc infiltrometer in three soils of different soil texture were conducted. The soils used were a sandy loam soil (SL), a loam soil (L), and a clay soil (C), and the applied negative pressure heads were −15, −7, −3, and −1 cm. The bulk density (ρ_φ) of the soils was 1.41, 1.17, and 1.05 gcm^{-3}, respectively, and their mechanical composition was 13.2% clay, 8% silt, and 78.8% sand for SL; 20% clay, 28% silt, and 42% sand for L; and 49.48% clay, 24.16% silt, and 26.36% sand for C. The clay soil includes only an aggregated fraction of 0.5–1.2 mm particles. After sampling, the soil samples were air-dried and passed through a 2 mm sieve. The initial water content of all soils ranged from 0.01 to 0.05 cm^3cm^{-3}, i.e., the experiments were carried out under dry initial conditions. For the experimental needs, a container with dimensions 49.5 × 46.4 × 22 cm, which ensured the unhindered three-dimensional infiltration for each type of studied soil, was used in the laboratory. Amounts of soil were gradually put in the container with simultaneous mechanical vibration of the container for achieving uniform bulk density to attain a uniform and homogeneous soil profile. Three repetitions were completed per pressure head and soil type.

2.2. Laboratory Apparatus

The infiltration experiments were carried out using a tension disc infiltrometer (TI—Tension infiltrometer set model 09.09 by Royal Eijkelkamp, Giesbeek, Netherlands). The infiltrometer includes (a) a Mariotte-type bubble tower, which controls the pressure head at the soil surface ($H \leq 0$); (b) a water reservoir, which supplies water to the soil through a porous disc; and (c) a porous disc, which is placed on the soil surface and establishes hydraulic contact with the soil. Tap water was used in the water reservoir of the TI during the experimental procedure. The porous disc had a diameter of 20 cm and can be used for a range of pressure heads $-20 \text{ cm} \leq H \leq 0$ cm.

2.3. Experimental Procedure

The infiltration experiments were conducted by TI at both single and consecutive pressure heads to study steady-state and non-steady-state (transient) flow without using a contact sand layer since the soils were disturbed. Cumulative infiltration data were determined using visual readings of water level drops in the water reservoir of the TI. In the single-pressure head experiments, each pressure head was applied until a steady flow was achieved. The imposed pressure heads were −15, −7, and −1 cm. For each experiment, the separation of the infiltration data referring to steady and non-steady flow was conducted in the way described by Angulo-Jaramillo et al. [21]. Specifically, from the cumulative infiltration curve I(t), the slope of the least-squares regression line corresponding only to the linear part of the curve I(t) was determined. The steady-state flow corresponds to that part of the cumulative infiltration curve where the relative error (RE) between the measured and estimated cumulative infiltration is $RE = \left|\frac{I_i - I_i^*}{I_i}\right| \leq 2\%$ [41]. The I_i is the experimental cumulative infiltration at time t_i, and I_i* is the corresponding estimated cumulative infiltration by applying the linear equation. In this way, we separate the steady-state flow from the transient flow. In the case of consecutive pressure head experiments, the sequence of pressure heads −15, −7, −3, and −1 cm was applied. The experiments started with the −15 cm pressure head, and when steady-state flow was reached the next pressure head of −7 cm was applied until steady flow was again achieved, and then the pressure heads of −3 and −1 cm were applied in the same way. The ascending (dry to wet) sequence (i.e., −15 cm → −7 cm → −3 cm → −1 cm) was selected because drainage occurs close to the disc while wetting continues at the infiltration front, reducing the hysteresis effect of the soil [19,42]. The duration of the experiments did not exceed two hours. The time circles in the initial stages of the experiments were every 10–15 s, while in the final stages (steady state) they reached up to 5 min.

Before and after each experimental procedure in the experiments with the single pressure heads, the initial and final soil water content was measured by the dielectric sensor ML2 (Delta-T Devices Ltd., Burwell, UK) [43]. The initial soil water content was measured by inserting the ML2 sensor vertically into the center of the soil column since the initial water content is uniform with a small value in all soils. On the other hand, the final soil water content (after the end of the experiment) was measured by inserting the ML2 sensor at an angle into the wet soil area formed under each imposed pressure head.

2.4. Methods

2.4.1. Steady-State Methods

Almost all the steady-flow methods that have been proposed are based on the Wooding [17] equation (Equation (1)).

The Ankeny et al. [1] Method

The method of Ankeny et al. [1] is based on applying at least two different consecutive negative pressure heads to the same tension disc (of a certain radius r) and measuring the steady flow at the corresponding pressure heads assuming that the initial hydraulic conductivity is negligible. Specifically, the calculation of K at the two pressure heads is made from the equations:

$$K_1(H_1) = \frac{Q_1}{\pi r^2 + 2\Delta H r\left(\frac{Q_1+Q_2}{Q_1-Q_2}\right)} \quad (12)$$

$$K_2(H_2) = K_1\frac{Q_2}{Q_1} \quad (13)$$

where Q_1 and Q_2 ($L^3\ T^{-1}$) are the steady infiltration fluxes at the corresponding successive pressure heads H_1 and H_2 for which $\Delta H = H_1 - H_2 < 0$ and r (L) is the radius of the infiltrometer tension disc.

So, if an infiltration experiment is conducted with at least two consecutive pressure heads, H_1 and H_2, and the Q_1 and Q_2 are calculated from the steady-flow data then the hydraulic conductivity at the corresponding pressure heads can be estimated from Equations (12) and (13). In the case where the experiments are carried out at more than two consecutive pressure heads, the value of K at the intermediate values of H will be equal to the arithmetic mean of the two individual values.

The Reynolds and Elrick [19] Method

The method of Reynolds and Elrick [19] is partially similar to that of Ankeny et al. [1]. It is based on the application of at least two different consecutive negative pressure heads on a tension disc (of a certain radius r) and the measurement of the steady flow at the corresponding pressure heads assuming that the soil is homogeneous, isotropic, and has negligible initial hydraulic conductivity. Specifically, this method considers that the hydraulic conductivity K(H) can be described by an exponential equation like that of Gardner [18] in the range $-\infty < H \leq 0$:

$$K(H) = K_{s1,2}e^{a_{1,2}H} \quad (14)$$

where $K_{s1,2}$ and $a_{1,2}$ are the saturated hydraulic conductivity and the a parameter, respectively, between a small range of pressure heads, H_1 and H_2, and can be calculated from the equations:

$$a_{1,2} = \frac{\ln\frac{Q_1}{Q_2}}{H_1 - H_2} \quad (15)$$

$$K_{s1,2} = \frac{Ga_{1,2}Q_1}{r(1 + Ga_{1,2}\pi r)\left(\frac{Q_1}{Q_2}\right)^p} \tag{16}$$

where G = 0.237 and $p = \frac{H_1}{H_1 - H_2}$.

So, if an infiltration experiment is conducted with at least two consecutive pressure heads, H_1 and H_2, and the $K_{s1,2}$ and $a_{1,2}$ are calculated from the steady-flow data using Equations (15) and (16) then the hydraulic conductivity at the range of corresponding pressure heads can be estimated from Equation (14). As in the case of the Ankeny et al. [1] method, if the experiments are carried out at more than two consecutive pressure heads, the value of K at the intermediate values of H will be equal to the arithmetic mean of the two individual values.

The Logsdon and Jaynes [20] Method

The method of Logsdon and Jaynes [20] is also based on the equation of Wooding [17], and its application requires the execution of experiments at more than two consecutive pressure heads as well as the measurement of the steady flow i_s at the corresponding pressure heads. The main difference between this method and those of Ankeny et al. [1] and Reynolds and Elrick [19] is that this one utilizes the steady-flow data from all pressure heads simultaneously while the other two methods calculate the corresponding values from a pair of pressure heads. Logsdon and Jaynes [20] introduced the exponential relationship K(H) of Gardner (Equation (3)) into the Wooding equation (Equation (5)) resulting in Equation (17):

$$i_s = K_s e^{aH}\left(1 + \frac{4}{\pi r a}\right) \tag{17}$$

Logsdon and Jaynes [20] developed a nonlinear regression technique for fitting simultaneously all the data sets to calculate the values of K_s and a. The Microsoft Excel Solver tool was used to calculate these values. The fitted a parameter and K_s were then used to calculate K(H) at each negative pressure head by applying Equation (3).

In addition to the Solver tool to calculate K_s and a, the natural logarithm function was also applied in Equation (17) to compare the a and K_s values of the two techniques:

$$\ln i_s = \ln\left[K_s\left(1 + \frac{4}{\pi r a}\right)\right] + aH \tag{18}$$

Equation (18) is a linear equation of the form y = ax + b, where y = lni_s, a = a, x = H, and b = ln[K_s(1 + 4/(πra)]. So, from the diagram lni_s vs. H, the slope of the line is the parameter a, and from the y-intercept of the line, which is b = ln[K_s(1 + 4/(πra)], can be calculated the only unknown parameter K_s. If the method of Logsdon and Jaynes [20] is used only for two successive pressure heads, it is considered similar to that of Reynolds and Elrick [19].

The White et al. [23] Method

The method of White et al. [23] is also based on the equation of Wooding [17] but differs from the aforementioned methods because the steady flow i_s is measured at one applied pressure head. It also utilizes both unsteady-flow data to estimate sorptivity, S, and steady flow.

Initially, White and Sully [22] proposed an alternative expression of Φ_0:

$$\Phi_0 \approx \frac{K_0}{a} = \frac{bK_0{S_0}^2}{(\theta_0 - \theta_i)(K_0 - K_i)} \tag{19}$$

Inserting Equation (19) into Wooding's equation (Equation (1)) and assuming that the initial hydraulic conductivity (K_i) is negligible and the parameter b = 0.55 [22,44] yields the following equation:

$$i_s = K_0 + \frac{2.2{S_0}^2}{\pi r(\theta_0 - \theta_i)} \quad (20)$$

where $i_s = Q_s/(\pi r^2)$ (L T^{-1}) is the steady-state infiltration rate.

If an infiltration experiment is conducted using a tension disc infiltrometer by applying one pressure head, the value of i_s is obtained from the steady-state flow experimental data, the values of θ_0 and θ_i are calculated using a dielectric device, and the value of sorptivity S can be estimated from the infiltration data i(t) at the initial experimental times or from a linearization method. Due to the great uncertainty in the selection of the appropriate initial time interval for the estimation of the S parameter, its value is usually calculated with the linear fitting techniques, the so-called CL or DL (Equation (10) or Equation (11)) where C_1 = S. In our study, S was calculated both ways. The value of K at each pressure head is calculated from Equation (20).

2.4.2. Unsteady-Flow Methods

The Haverkamp et al. [2] Expansion Models

The complete Haverkamp et al. 3D model [2] is given by the following equation, which is valid for the whole infiltration time:

$$\frac{2(K_0 - K_i)^2}{{S_0}^2} t = \frac{2}{1-\beta} \frac{(K_0 - K_i)\left[I_{3D} - K_i t - \gamma {S_0}^2/((\theta_0 - \theta_i)r)t\right]}{{S_0}^2} - \frac{1}{1-\beta} \ln\left[\frac{1}{\beta}\exp\left(\frac{2\beta(K_0 - K_i)\left[I_{3D} - K_i t - \gamma {S_0}^2/((\theta_0 - \theta_i)r)t\right]}{{S_0}^2}\right) + \frac{\beta + 1}{\beta}\right] \quad (21)$$

where S_0 ($LT^{-0.5}$) and K_0 (LT^{-1}) are the sorptivity and hydraulic conductivity, respectively, at the applied pressure head H_0 (L), K_i (LT^{-1}) is the initial hydraulic conductivity corresponding to the initial soil water content θ_i (L^3L^{-3}), β (−) is a dimensionless integral shape parameter that depends on the diffusivity of the porous medium and ranges between 0.6 and 1.7 [45], and γ (−) is a dimensionless parameter equal to 0.75 [26,35].

Several researchers showed that fixing the β parameter at a constant value (0.6 or 1.1) can satisfactorily predict the parameters K_0 and S [6,16,37,46]. Recently, Yilmaz et al. [39] suggested a value β = 0.9 for coarse-textured soils, such as sand and loamy sand. For the rest of the soil types, they suggested values of 0.75 for sandy loam and loam soils and 1.5 for silty loam and silty soils. The γ parameter is a dimensionless parameter for which the value 0.75 is usually used [26,35]. Lassabatere et al. [45] showed that the value of the γ parameter depends on the soil type and ranges between 0.75 and 1. Yilmaz et al. [39] suggested γ = 0.9 for sand and loamy sand soils and γ = 0.75 for the rest of the soils.

If the initial hydraulic conductivity is assumed to be negligible, the Equation (21) is converted into the 2-term equation (2T) of Haverkamp et al. [2], which is valid for short and medium time:

$$I_{3D} = S_0\sqrt{t} + \left[\frac{2-\beta}{3}K_0 + \frac{\gamma {S_0}^2}{r(\theta_0 - \theta_i)}\right]t \quad (22)$$

The first term on the right-hand side of Equation (22) represents the vertical capillary flow that dominates at the initial times; the second term corresponds to the vertical flow of water affected by gravity; and the third term describes the lateral capillary flow as presented by Smettem et al. [35]. The 2T expansion model has the advantages of simple analysis and interpretation of 3D infiltration but has the disadvantage that it is only valid for short infiltration times. According to Haverkamp et al. [2], Equation (22), for initially dry soil conditions where K_i can be neglected, satisfactorily describes the water flow through a tension disc infiltrometer for a time less than or equal to t_{grav} (T), where

$$t_{grav} = \left(\frac{S_0}{K_0}\right)^2 \quad (23)$$

However, Rahmati et al. [47] reformulated the t_{grav} using the analytic implicit model proposed by Parlange et al. [34] valid for all times:

$$t_{grav} = F(\beta)\left(\frac{S_0}{K_0 - K_i}\right)^2 \quad (24)$$

where $F(\beta)$ is a β-dependent function. The reformulated t_{grav} is about three times larger than the classical t_{grav}.

Equation (22) has a similar form to Philip's equation (Equation (6)), and the coefficients C_1 and C_2 can be related to sorptivity, hydraulic conductivity, and lateral capillary flow as follows:

$$C_1 = S_0 \quad (25)$$

$$C_2 = \frac{2-\beta}{3}K_0 + \frac{\gamma {S_0}^2}{r(\theta_0 - \theta_i)} \quad (26)$$

To estimate the values of K and S at each pressure head, the coefficients C_1 and C_2 must first be calculated. Estimation of coefficients C_1 and C_2 can be obtained by fitting Equation (10) (CL method) or Equation (11) (DL method) to the experimental data. The adequacy of Equation (10) or Equation (11) can be checked by observing the linearity of the inputted data set.

The value of K_0 at the applied pressure head H_0 can be calculated from Equation (26) and the value of S from Equation (25). Because the accuracy of the estimating value of hydraulic conductivity from the equation of Haverkamp et al. [2] depends on the relative magnitude of the individual terms of Equation (22), the criteria of Vandervaere et al. [5] (Equation (7)) and Dohnal et al. [24] (Equation (8)) are taken into consideration.

In addition to the 2T equation of Haverkamp et al. [2] (Equation (22)), recently, Moret-Fernández et al. [36] presented corresponding 3-term (3T) and 4-term (4T) equations of the complete Haverkamp et al. 3D model. The 3T equation is valid for longer times compared to the 2T equation, which is valid for short and medium infiltration times. In our study, the following 3-term equation of Haverkamp et al. [2] (Equation (27)) was examined by applying an inverse solution analysis using the Solver tool in Excel for the estimation of K and S:

$$I_{3D} = S_0\sqrt{t} + \left[\frac{2-\beta}{3}K_0 + \frac{\gamma {S_0}^2}{r(\theta_0 - \theta_i)}\right]t + \frac{{K_0}^2}{9S_0}\left(\beta^2 - \beta + 1\right)t^{3/2} \quad (27)$$

In this case, the procedure for estimating K and S was repeated three times, once considering the parameter β equal to 0.6, the second equal to 1.6, and the third equal to the value 0.75 proposed by Yilmaz et al. [39] in order to examine the effect of the parameter β on the estimated values.

The Zhang [4] Method

The method of Zhang [4] correlates the coefficient C_1 with the sorptivity S_0 and the C_2 with the hydraulic conductivity K_0, as follows:

$$C_1 = A_1 S_0 \quad (28)$$

$$C_2 = A_2 K_0 \quad (29)$$

where A_1 and A_2 are dimensionless parameters that depend on soil water content, soil water retention, and infiltrometer parameters. Zhang's hypothesis that C_2 is correlated with gravitational forces (i.e., K_0) has been doubted by several researchers [48]. For soils with the van Genuchten [28] type retention function, the mathematical expressions of these parameters were derived from numerical simulations of infiltration data [4]:

$$A_1 = \frac{1.4b^{0.5}(\theta_0 - \theta_i)^{0.25}\exp[3(n-1.9)aH_0]}{(ar)^{0.15}} \tag{30}$$

$$A_2 = \frac{11.65(n^{0.1}-1)\exp[2.92(n-1.9)aH_0]}{(ar)^{0.91}} \text{ for } n \geq 1.9 \tag{31}$$

$$A_2 = \frac{11.65(n^{0.1}-1)\exp[7.5(n-1.9)aH_0]}{(ar)^{0.91}} \text{ for } n < 1.9 \tag{32}$$

where α (-) and n (L^{-1}) are the van Genuchten's coefficients, and b (-) is a dimensionless parameter equal to 0.55. The van Genuchten equation is given as follows:

$$\theta = \theta_r + (\theta_s - \theta_r)\left((1+\alpha H)^n\right)^{-m} \tag{33}$$

The coefficients C_1 and C_2 can be calculated from the linearization methods (CL and DL) of Equation (6). On the other hand, the A_1 and A_2 can be calculated taking into consideration van Genuchten's coefficients α and n as suggested by Carsel and Parrish [49] for each soil textural class (Table 1). Then, the hydraulic conductivity at the applied pressure head (K_0) can be estimated from Equation (29).

Table 1. Values of the parameters α and n of the van Genuchten [28] equation according to Carsel and Parrish [49] for different soil textural classes.

Soil	Van Genuchten Parameters	
	α (cm^{-1})	**n (-)**
Loam	0.036	1.56
Sandy Loam	0.075	1.89
Clay	0.008	1.09

3. Results and Discussion

3.1. Comparison of the Results of Steady-Flow Methods

Table 2 presents the results of K estimates at four pressure heads obtained by applying steady-flow methods for the three soils studied.

As shown in Table 2, K increased by 2.5 (loam soil) to 9 (clay soil) orders of magnitude in the pressure head range −15 to −1 cm. Such large increases in near-saturated hydraulic conductivity should be taken into consideration by models describing soil hydraulic properties even for disturbed soils. Large increases in K (by three to four orders of magnitude in the finer textured soils) were also found by Jarvis and Messing [42] in six tilled field soils from TI measurements in the pressure head range −10 cm to zero.

The methods of Ankeny et al. [1], Reynolds and Elrick [19], and Logsdon and Jaynes [20] gave K values in good agreement among them in all applied pressure heads except in the case of H = −1 cm for the loam and sandy loam soils studied (Table 2). The difference observed between the estimated K values from the Logsdon and Jaynes [20] method and those from the other methods at pressure head −1 cm is probably related to the inability to describe the K(H) relationship by a single exponential relationship over the entire pressure head range −15 to −1 cm [42]. Similar results regarding the estimation of K values of the methods of Ankeny et al. [1], Reynolds and Elrick [19], and Logsdon and Jaynes [20] have been presented by Angulo-Jaramillo et al. [21] and Bagarello et al. [50] for the estimation of the unconfined hydraulic conductivity of a sandy loam soil in the range of pressure heads (H) between −12 and −3 cm. In contrast, Logsdon and Jaynes [20], in infiltration experiments at 12 different field sites on Nicollet Clay Loam soil using TI at pressure heads of −3, −6, and −15 cm, found that the method of Ankeny et al. [1] underestimates K while their method gave larger K values compared to experimental values. In fact, the differences increased at lower pressure head values. In general, it can be reliably said that the abovementioned three methods will be in agreement to the extent that the K(H) relationship of the porous medium is described by a single exponential equation over the entire range of negative pressure heads near saturation.

Table 2. Hydraulic conductivity values estimated by the steady-flow methods of Ankeny et al. [1], Reynolds and Elrick [19], Logsdon and Jaynes [20] (using the Solver tool), and White et al. [23] for loam, sandy loam, and clay soils at pressure heads of −15, −7, −3, and −1 cm.

H (cm)	**K (cm min^{-1})**			
	White et al. [23]	**Logsdon and Jaynes [20]**	**Ankeny et al. [1]**	**Reynolds and Elrick [19]**
		Loam Soil		
−15	0.0289	0.0089	0.0099	0.0097
−7	0.0369	0.0139	0.0129	0.0126
−3	(no data)	0.0175	0.0151	0.0145
−1	0.0631	0.0195	0.0255	0.0247
		Sandy loam soil		
−15	0.0496	0.0469	0.0428	0.0436
−7	<0	0.1282	0.1289	0.1292
−3	(no data)	0.2119	0.2041	0.1998
−1	<0	0.2724	0.2220	0.2155
		Clay soil		
−15	0.0152	0.0116	0.0131	0.0131
−7	0.0434	0.0411	0.0349	0.0348
−3	(no data)	0.0771	0.0783	0.0773
−1	0.0547	0.1057	0.1087	0.1066

In addition, if the natural logarithm function (Equation (18)) is applied in the method of Logsdon and Jaynes [20], a strong linearity of the equations is observed for each soil with R^2 greater than 0.97 (Figure 1). From the coefficients of the linear equations, the values of the parameters K_s and a can be derived for each soil, and then the values of K at each pressure head can be calculated from Equation (3). The results showed similar K values between the different approaches for the Logsdon and Jaynes [20] method (Table 3).

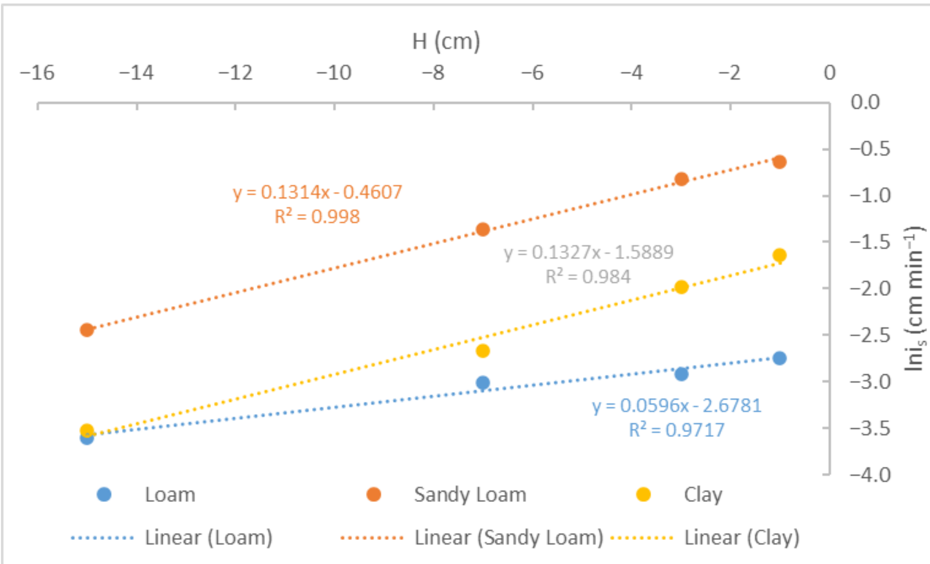

Figure 1. Presentation of the linear equations of the Logsdon and Jaynes [20] method (Equation (18)) for loam, sandy loam, and clay soils at pressure heads of −15, −7, −3, and −1 cm.

Table 3. Hydraulic conductivity (K) values estimated by the Logsdon and Jaynes [20] method using the Solver tool and the linearization/logarithmization method for loam, sandy loam, and clay soils at pressure heads of −15, −7, −3, and −1 cm.

H (cm)	K (cm min^{-1})					
	Linearization/Logarithmization Method			Solver Tool		
	Loam	Sandy Loam	Clay	Loam	Sandy Loam	Clay
−15	0.0090	0.0446	0.0142	0.0089	0.0469	0.0116
−7	0.0144	0.1277	0.0412	0.0139	0.1282	0.0411
−3	0.0183	0.2160	0.0700	0.0180	0.2119	0.0771
−1	0.0206	0.2809	0.0912	0.0195	0.2724	0.1057

3.2. Unsteady-Flow Methods

From the application of the method of Angulo-Jaramillo et al. [21], the times (t_0) referred to the unsteady flow were calculated (Table 4). In Figure 2, the I(t) diagrams and the separation of unsteady from steady flow (given by arrows) are depicted. These t_0 times are the same order of magnitude as the transient flow times reported by Latorre et al. [6] in 266 field infiltration experiments at pressure head H = 0 cm (t_0 ranged from 8 to 15 min) with the exception of loam soil at H = −15 cm where t_0 = 47 min. The hydraulic parameter estimations of the non-steady-flow equations were evaluated based on the aforementioned t_0 times at each pressure head.

Table 4. Experimental times and unsteady-flow times according to the method of Angulo-Jaramillo et al. [21].

H (cm)	Experimental Time (t_{exp}) (min)	Unsteady-Flow Time (t_0) (min)
	Loam Soil	
−15	103.0	47.00
−7	40.00	19.00
−1	19.50	11.00
	Sandy loam soil	
−15	29.00	12.00
−7	52.00	16.00
−1	29.00	14.00
	Clay soil	
−15	29.00	11.00
−7	33.00	11.00
−1	37.00	14.00

3.2.1. The Two-Term Equation (2T) of Haverkamp et al. [2]

The results from the CL and DL linearization methods by applying Equations (10) and (11) are presented in Figure 3, and the corresponding values of the coefficients C_1 and C_2 are given in Table 5. As shown in Figure 3, in all soils and pressure heads, Equations (10) and (11) had high R^2 values ranging from 0.8772 to 0.988 for the CL method and from 0.7202 to 0.9746 for the DL method with the exception of the clay soil at the pressure head −15 cm where R^2 = 0.3435. Comparing the two linearization methods, the CL method gave higher R^2 values than the DL method in all soils and pressure heads. The same results were reported by Moret-Fernadez et al. [51] in field experiments. Although the DL method had a smaller R^2, its values were much higher than those presented by Latorre et al. [6] in 266 field infiltration experiments where only in 21 (8%) infiltration experiments was the $R^2 \geq 0.6$. This difference can probably be attributed to the fact that our experiments were conducted on disturbed soil samples with homogeneous initial soil moisture and the absence of bio-pores and cracks, in contrast to the field experiments of Latorre et al. [6]. In the latter case, abnormal changes in I(t) are very probable to occur, which, due to the derivation process in the DL method, will lead to small R^2 values.

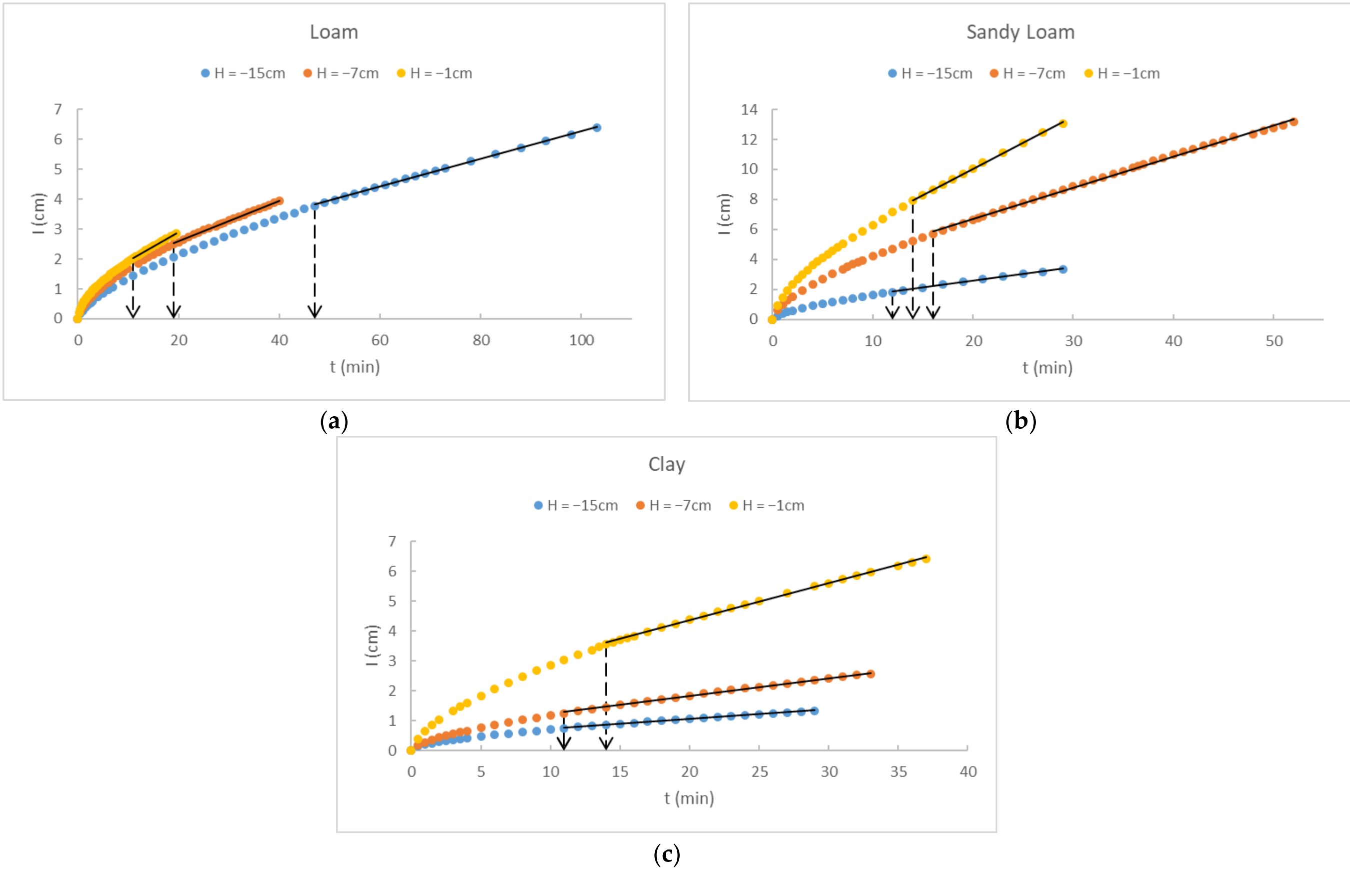

Figure 2. Diagrams of cumulative infiltration versus time, I(t), for loam (**a**), sandy loam (**b**), and clay (**c**) soils at pressure heads of −15, −7, and −1 cm. The lines show that the water flow is steady, and the arrows show the time (t_0) where the unsteady flow prevails.

Angulo-Jaramillo et al. [21] reported that for given experimental conditions, the CL and DL methods should yield identical estimates of coefficients C_1 and C_2. The results showed that the two methods at each pressure head and soil gave quite different values of C_1 and C_2 even though they were applied to the same infiltration times. The DL method gave larger C_1 values and smaller C_2 values compared to the corresponding values of the CL method (Table 5). Specifically, the DL method compared to the CL method gave higher values of up to 41.1% for the coefficient C_1, while correspondingly it gave lower values of up to 52.5% for the coefficient C_2. From these results, it appears that the DL method will probably estimate values of K that are not acceptable since smaller values of C_2 and larger values of C_1 will lead to estimating negative values of K through Equation (26). Similar results were presented by Bagarello et al. [52] for the soils classified in the lateral capillarity domain after numerical infiltration experiments in sandy loam and clay soils applying the DL method to initially dry soils, where an underestimation of K was observed as C_2 was underestimated while C_1, and thus S, was overestimated. For these cases, it is recommended to conduct experiments with higher initial soil moisture and relatively long experimental duration [52]. In contrast, for soils that were situated in the domain of gravity, Bagarello and Iovino [32] showed that the coefficients C_1 and C_2 calculated from the DL method, as well as the K estimations, were affected by the duration of the experiment and the time intervals of the water level drop measurements in the water reservoir of TI. In these soil types, the change in K depends mainly on the change in C_2, while the change in C_1, which is larger, has no significant effect on the K estimation.

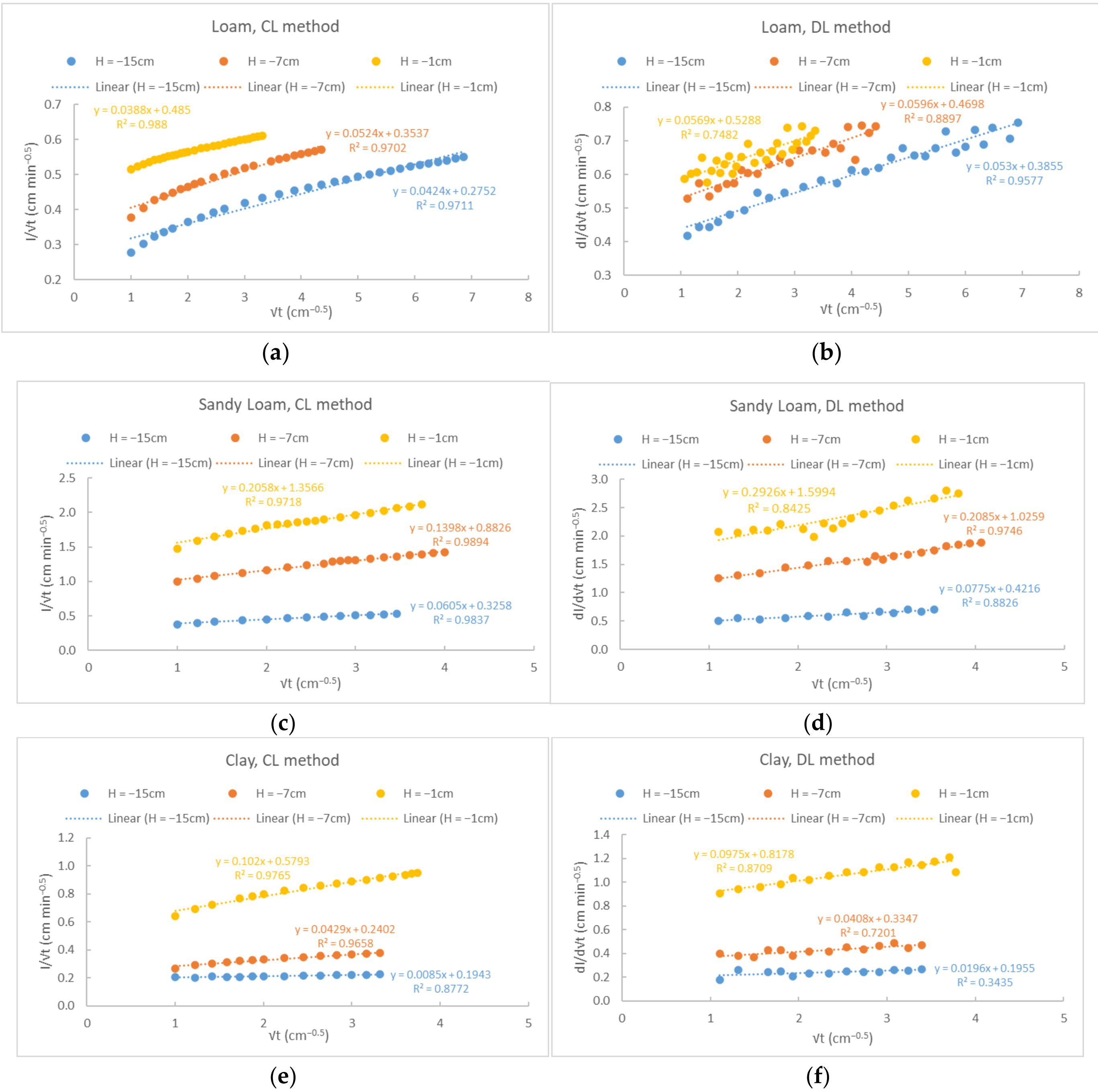

Figure 3. Applying the CL and DL linearization methods for loam, sandy loam, and clay soils, at pressure heads of −15, −7, and −1 cm at corresponding times t_0.

Table 5. The values of coefficients C_1 and C_2 using the cumulative linearization (CL) and differential linearization (DL) methods for loam, sandy loam, and clay soils, at pressure heads of −15, −7, and −1 cm at corresponding times t_0.

H (cm)	CL Method		DL Method	
	C_1 (cm $min^{-0.5}$)	C_2 (cm min^{-1})	C_1 (cm $min^{-0.5}$)	C_2 (cm min^{-1})
		Loam Soil		
−15	0.2752	0.0424	0.3855	0.0265
−7	0.3537	0.0524	0.4698	0.0298
−1	0.4850	0.0388	0.5288	0.02845
		Sandy loam soil		
−15	0.3258	0.0605	0.4216	0.03875
−7	0.8826	0.1398	1.0259	0.10425
−1	1.3566	0.2058	1.5994	0.1463
		Clay soil		
−15	0.1943	0.0085	0.1955	0.0098
−7	0.2402	0.0429	0.3347	0.0204
−1	0.5793	0.1020	0.8175	0.04875

To investigate the effect of time on the values of C_1 and C_2, these were calculated in the cases of the loam soil at a pressure head of −15 cm and for the sandy loam soil at −7 cm applying the CL linearization method. Two experimental times were selected for each soil: for the loam soil, the times t = 10 min (≈ $t_0/5$) and t = $t_{0,}$ and for the sandy loam soil, the times t = 5 min (≈ $t_0/3$) and the t = t_0 (Table 6). The results showed that the duration of the experiment significantly affected the values of C_1 and C_2. Specifically, it was found that the value of C_1 decreased while the value of C_2 increased with the decrease in time, and the change in C_2 was much greater than that in C_1 for both CL and DL methods. These results are contrary to those of Bagarello and Iovino [32], who found a greater change in the coefficient C_1. Factors that may contribute to this difference are whether the soils are classified in the lateral capillary or gravity domain, and also the initial soil water content. It is obvious that increasing C_2 values and decreasing C_1 values with decreasing infiltration time will affect the estimation of K and S values when the 2T equation of Haverkamp et al. [2] is used. Overall, the K estimations obtained from the non-steady methods varied with the duration of the experiment in a rather complex way [52].

Table 6. The values of coefficients C_1 and C_2 using the CL linearization method at different times.

Coefficients C_1, C_2	Loam, H = −15 cm		Sandy Loam, H = −7 cm	
	t = 10 min (≈ t_0/5)	t_0 = 47 min	t = 5 min (≈ t_0/3)	t_0 = 16 min
C_1 (cm $min^{-0.5}$)	0.2752	0.0424	0.3855	0.0265
C_2 (cm min^{-1})	0.4850	0.0388	0.5288	0.02845

As shown in Table 7, the 2T equation of Haverkamp et al. [2] estimated negative values of K mainly in the case where the DL linearization method was applied. A similar phenomenon in field experiments was observed by Latorre et al. [6], where only 40% of experimental infiltration curves could be analyzed by the DL method and a small part of them led to acceptable K values. In the case of the CL method, negative values of K were estimated for the loam soil at H = −1 cm and for the sandy loam soil at pressure heads −7 and −1 cm.

Table 7. Estimated hydraulic conductivity values (K) from unsteady-flow methods for loam, sandy loam, and clay soils, at pressure heads of −15, −7, and −1 cm. In the case of the Haverkamp et al. [2] method, the K values were obtained considering the constant value β = 0.6.

H (cm)	K (cm min^{-1})				
	Haverkamp (2T) CL Method	Haverkamp (2T) DL Method	Haverkamp (3T) Using Solver Tool	Zhang CL Method	Zhang DL Method
			Loam Soil		
−15	0.0569	<0	0.0288	0.0080	0.0050
−7	0.0581	<0	0.0369	0.0205	0.0117
−1	<0	<0	0	0.0264	0.0193
			Sandy Loam soil		
−15	0.0710	<0	0.0509	0.0559	0.0358
−7	<0	<0	0	0.1351	0.1007
−1	<0	<0	0	0.2057	0.1462
			Clay soil		
−15	0.0037	0.0063	0.0052	0.0041	0.0047
−7	0.0724	0.0057	0.0539	0.0304	0.0145
−1	0.1116	<0	0.0752	0.0968	0.0454

Vandervaere et al. [5] argued that the 2T equation of Haverkamp et al. [2] gives reliable values only when K is a dominant term in the coefficient C_2, i.e., gravity plays a decisive role, whereas in the opposite case where lateral capillary forces play a dominant role, the estimated values of K are not reliable.

The satisfaction of the criteria proposed by Vandervaere et al. [5] (Equation (7)) and Dohnal et al. [24] (Equation (8)) is presented in Table 8 to establish the reliability of these criteria on the estimated K values presented in Table 7. All soils examined, except the clay soil at the pressure head −15 cm for the DL method, may be situated in the "lateral capillarity domain" due to the non-satisfaction of the Vandervaere et al. criterion. It is worth noting that the criterion of Vandervaere et al. is very strict compared to the criterion of Dohnal et al. since the latter was satisfied in several cases resulting in positive values of K. Specifically, in loam soil at pressure head −7 cm and clay soil at −15 and −1 cm, even though the criterion of Vandervaere et al. was not satisfied for the CL method, the Haverkamp et al. method estimated positive values of K. In contrast, the criterion of Dohnal et al. [24] includes all positive values of hydraulic conductivity, which shows that it is more representative of the real situation and is therefore recommended.

Table 8. Application of the criteria of Vandervaere et al. [5] and Dohnal et al. [24] for loam, sandy loam, and clay soils at pressure heads of −15, −7, and −1 cm. The values of C_1 and C_2 used were derived from the CL and DL linearization methods. The symbols Y and N are used when the criteria are satisfied and not satisfied, respectively.

	Criteria for CL Method					
H (cm)	Loam Soil		Sandy Loam Soil		Clay Soil	
	Vandervaere et al. [5]	Dohnal et al. [24]	Vandervaere et al. [5]	Dohnal et al. [24]	Vandervaere et al. [5]	Dohnal et al. [24]
−15	Y	Y	Y	Y	N	Y
−7	N	Y	N	N	Y	Y
−1	N	N	N	N	N	Y
	Criteria for DL method					
H (cm)	Loam soil		Sandy Loam soil		Clay soil	
	Vandervaere et al. [5]	Dohnal et al. [24]	Vandervaere et al. [5]	Dohnal et al. [24]	Vandervaere et al. [5]	Dohnal et al. [24]
−15	N	N	N	N	Y	Y
−7	N	N	N	N	N	Y
−1	N	N	N	N	N	N

In the case of soils with the abovementioned characteristics, i.e., the Dohnal criterion (Equation (8)) is not satisfied, for the reliable estimation of K, the use of steady-flow methods with multiple pressure heads or unsteady-flow experiments with higher initial soil moisture and relatively long duration of infiltration should be preferred, as recommended by Bagarello et al. [52]. Note that in the latter case, the condition of the very small value of the initial K must be satisfied.

3.2.2. The Three-Term Equation (3T) of Haverkamp et al. [2]

The inverse solution method of the three-term (3T) equation of Haverkamp et al. [2] (Equation (27)) with $\beta = 0.6$ and $\gamma = 0.75$, using the Solver tool, presented the same characteristics as the 2T equation of Haverkamp et al. [2] (Table 7) in estimating K. In the cases where it estimated positive values of K, these values were significantly higher than those of steady flow for loam and sandy loam soils, while the results for the clay soil ($\beta = 0.6$) were contradictory. Regarding the comparison with the 2T equation for the CL method, the 3T equation presents, in all cases of pressure heads, smaller values except for the pressure head −15 cm in clay soil.

Additionally, the value $\beta = 1.6$ was also used [36] to study its effect on the estimation of K and S parameters (Table 9). It was observed that the estimated K values, taking $\beta = 1.6$, were higher than those estimated by using $\beta = 0.6$ and much higher than those estimated by steady-flow methods. Only in clay soil for H = −1 cm is there a satisfactory correlation with the steady-flow method. However, in the cases where K values are negative with $\beta = 0.6$, no changes were observed with $\beta = 1.6$. Regarding the parameter S, the changes observed were very small due to a change in the value of β. A future in-depth study is needed to investigate the results of the 3T equation in more soil types considering the criteria of Equations (7) and (8) as well as the effect of experimental infiltration time.

Table 9. Estimated hydraulic conductivity values from the inverse solution method and Equation (27) for loam, sandy loam, and clay soils, at pressure heads of −15, −7, and −1 cm considering the values of $\beta = 0.6$ and $\beta = 1.6$, and for the loam and sandy loam soils the value of $\beta = 0.75$ [39].

H (cm)	K (cm min^{-1})		
	β = 0.6	**β = 0.75**	**β = 1.6**
		Loam Soil	
−15	0.0288	0.0307	0.0354
−7	0.0369	0.0398	0.0518
−1	0	0	0
		Sandy Loam soil	
−15	0.0509	0.0547	0.0685
−7	0	0	0
−1	0	0	0
		Clay soil	
−15	0.0052	-	0.0126
−7	0.0539	-	0.0692
−1	0.0752	-	0.1025

Additionally, for loam and sandy loam soils, the values $\beta = 0.75$ and $\gamma = 0.75$ were used as proposed by Yilmaz et al. [39] for these soil types. As expected, the values of K ranged between those estimated using $\beta = 0.6$ and $\beta = 1.6$ since the γ value is constant in all scenarios examined and equal to 0.75. The results showed that in the cases of pressure heads where the estimated K values by the 3T equation were negative, the re-estimated K values using $\beta = 0.75$ were also negative, and therefore the problem remains.

To further investigate the problem of negative K values, the method of Moret-Fernadez et al. [38] was also applied to examine whether there is a compact deep layer that affects the estimation of K and S. In this case, the 4T expansion model of Haverkamp et al. [2] should be used. So, the values of the RMSE must be calculated and checked for any changes with infiltration time, which indicate the existence of a different soil layer. For the loam soil at pressure head H = −1 cm, the RMSE variation over time is presented in Figure 4. As shown, the RMSE values are essentially constant with time except for very small values of the infiltration time. Therefore, the scenario that the soil studied is situated in the domain of lateral capillarity could be considered as the more likely cause of the weakness of calculating K. For sandy loam soil at pressure head H = −1 cm, different behavior of RMSE is observed as a function of time (Figure 5). The RMSE value increases with time except for the period of 200–250 s where a decrease is observed. If we consider the infiltration time equal to 240 s, then positive K values are estimated. The corresponding values of K for β = 0.6 and pressure heads −7 and −1 cm are 0.0848 and 0.13 cm min^{-1}, respectively, while for β = 0.75, they are 0.0923 and 0.1405 cm min^{-1}, respectively, which are correlated with the K values estimated by the steady-state equations as well as Zhang's equation. Similar values of K, at the corresponding pressure heads, are also obtained by applying the 3T equation instead of the 4T for the various values of β. Thus, the SIA method can also be applied using the 3T equation for laboratory experiments with these disturbed soils. However, considering the results presented in Figure 5, it is not always easy to select the appropriate infiltration time for the calculation of parameters K and S, because there is no clear descending and ascending part of the curve as observed in the diagrams of Moret-Fernadez et al. [38]. Such situations may appear in soils where there is no clear soil stratification, e.g., in soils where there is little soil compaction at some depth. In these cases, it is recommended that the measurements be very frequent in the initial stages of the infiltration so that the appropriate selection of the first part of the curve can be made easier. However, the short time intervals make it difficult to apply the method, especially in the case of visual readings.

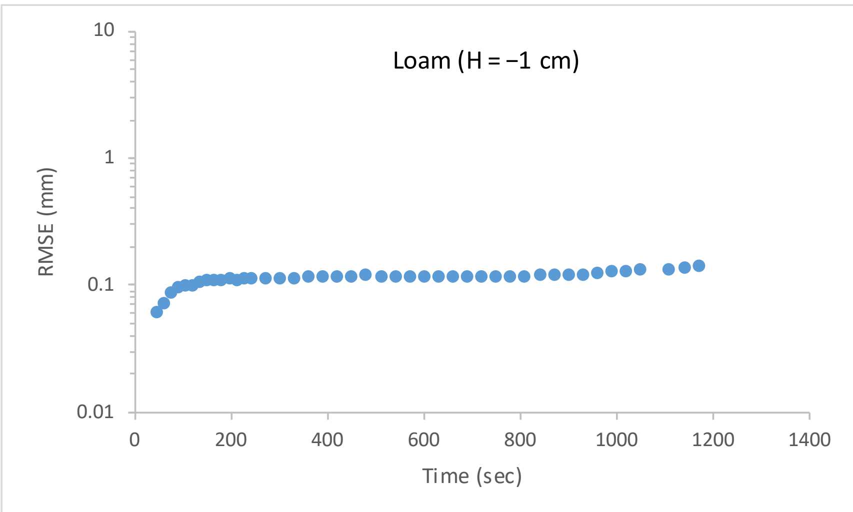

Figure 4. RMSE variation with time for the loam soil at pressure head −1 cm.

In conclusion, it can be said that the selection of the appropriate infiltration time is an important factor for the estimation of K values, especially in the cases of heterogeneous soil profiles. The SIA method using the 3T equation is an efficient tool to estimate K and S of the upper layer of heterogeneous soils. Thus, the phenomenon of negative values of K may be due to the fact that the soils may be situated in the domain of lateral capillarity or to the inappropriate selection of infiltration time in layered soils. It should also be mentioned that the criterion of Dohnal et al. [24] (Equation (8)) could be a safe guide for the application of steady-flow methods with multiple pressure heads or the SIA method.

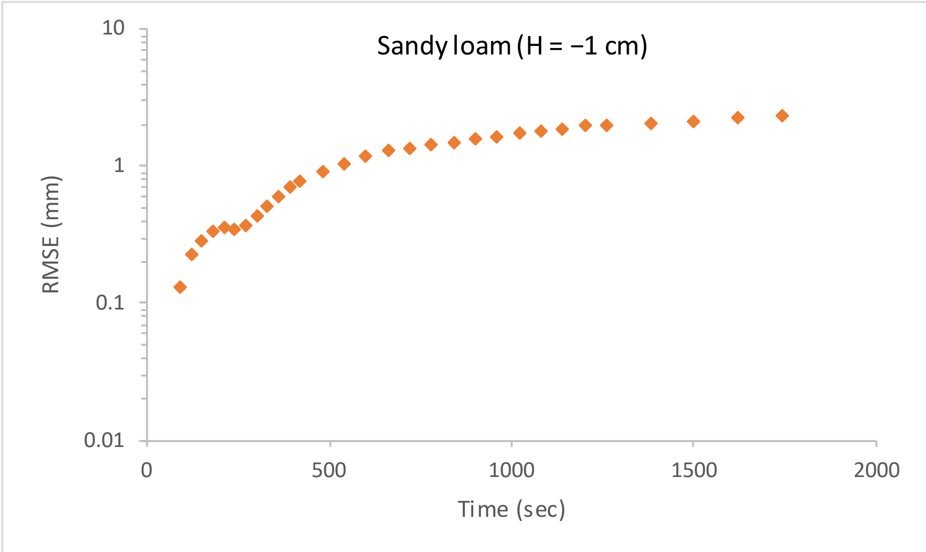

Figure 5. RMSE variation with time for the sandy loam soil at pressure head −1 cm.

3.2.3. The Zhang [4] Method

The method of Zhang [4] was applied using both linearization methods, CL and DL, to calculate the coefficients C_1 and C_2, and positive values of K were estimated in all soils and pressure heads studied (Table 7). In this respect, although it has been criticized for its physical adequacy, it is a very useful method. The estimated K values from the CL method were in almost all cases greater than those estimated from the DL method. The method of Zhang, when the CL method was used to calculate C_1 and C_2, gave comparable values of K to those of the methods of Ankeny et al. [1] and Reynolds and Elrick [19] for all soils and pressure heads, except for the clay soil at pressure head −15 cm. The Zhang CL method compared with the Logsdon and Jaynes [20] method presented differences that are mainly focused on pressure head −1 cm for all soils and on −15 cm for clay soil (Figure 6). The differences vary by a factor of between 0.4 and 2 and may be considered acceptable for practical purposes [52]. These differences can be attributed to the fact that the Zhang model assumes that the hydraulic properties are described by the Mualem [27]-van Genuchten [28] model, while the Logsdon and Jaynes [20] method assumes that the K(H) relationship is described by the Gardner [18] equation. It is worth noting that the van Genuchten's coefficients (α and n) used in the Zhang method were related to the soil texture class and were not obtained by fitting laboratory-determined soil water retention data.

Because the Zhang [4] and Logsdon and Jaynes [20] methods had the largest differences in the clay soil, which has $n < 1.35$, the application of both Equation (9) to calculate the A_2 parameter of Dohnal et al. [24] and the CL linearization method to calculate the C_2 parameter were also examined.

The hydraulic conductivity values estimated from Equation (9) of Dohnal et al. [24] (Table 10) deviated even further from the K values estimated by the steady-flow methods (Table 2). A comprehensive assessment of the large differences observed in K values from these methods, for the category of soils studied, requires an accurate estimation of the value of the parameter n and consequently an appropriate description of the relationship K(H) or K(θ) from the corresponding models. Such an analysis is beyond the scope of this study.

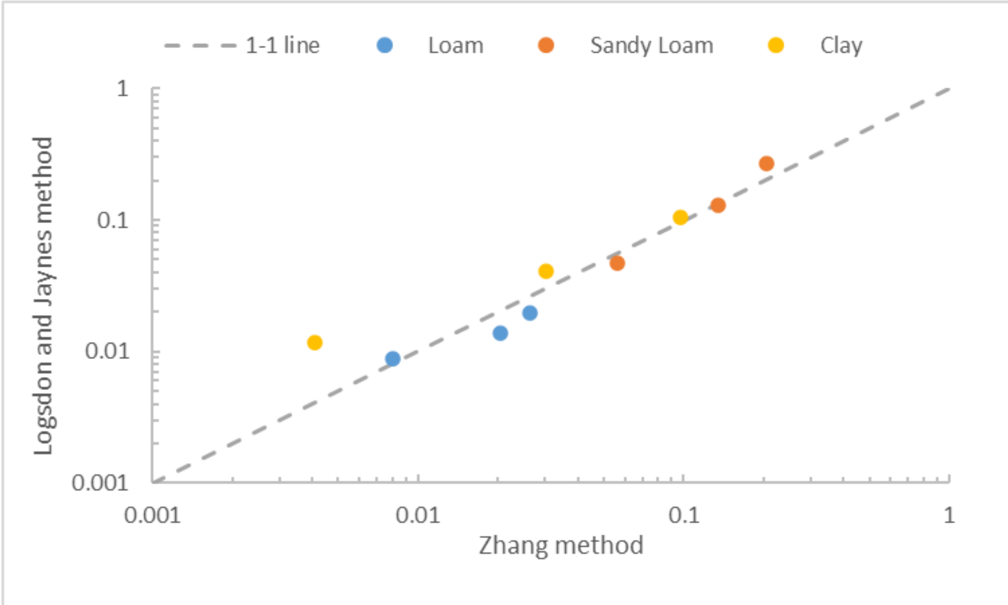

Figure 6. Graphical correlation of Logsdon and Jaynes [20] (with Solver) and Zhang [4] (with CL linearization method) methods for loam, sandy loam, and clay soils at pressure heads of −15, −7, and −1 cm.

Table 10. Comparison of soil hydraulic conductivity values between the methods of Zhang [4] and Dohnal et al. [24] for n < 1.35 at the corresponding times t_0.

H (cm)	**Clay Soil**		
	K (cm min^{-1})		
	Zhang [4] CL Method	**Zhang [4] DL Method**	**Dohnal et al. [24]**
−15	0.0041	0.0047	0.0022
−7	0.0304	0.0145	0.0120
−1	0.0968	0.0454	0.0305

3.2.4. The White et al. [23] Method (Steady-State Single Test Method—SST Method)

The method of White et al. [23], although derived from Wooding's equation, gave K values that differ from those estimated by steady-flow multiple pressure head methods (Table 2). Specifically, for the loam soil, at all pressure heads, it gave 2–3 times higher hydraulic conductivity values than the other steady-flow methods. For the sandy loam soil, it gave negative K values, except for the pressure head −15 cm where the K values approached those of the steady-flow methods. Finally, for the clay soil, the K values were similar except for the pressure head −1 cm.

Overall, the K values estimated from the White et al. [23] method, in almost all cases of positive K values, were greater than those of steady-flow methods.

A similar behavior was observed when it was compared with the Zhang [4] method, while it seems to suffer from similar problems (negative K values), especially in the soils situated in the "lateral capillary domain", with the method of Haverkamp et al. [2] due to the common term $S^2/(r_0\Delta\theta)$ in their equations [24]. This was especially observed in the case of the sandy loam soil where negative K values were obtained at pressure heads −1 and −7 cm.

Similar trends of overestimation of hydraulic conductivity values, as well as estimation of negative values, have been reported by Dohnal et al. [24] and Bagarello et al. [52] in experiments conducted on 12 synthetic soils using the Mini Disc Infiltrometer (disc radius

2.25 cm) and numerical experiments on two soils (clay and sandy loam) using a tension infiltrometer (disc radius 12 cm), respectively.

3.3. Estimation of Soil Sorptivity from Unsteady-Flow Data

In Table 11, the values of sorptivity estimated from the methods of Zhang [4] (Equation (28)) and the 2T equation of Haverkamp et al. [2] (Equation (22)) as well as the 3T equation using the Solver tool (Equation (27)) are presented. Sorptivity was also calculated from the slope of the cumulative infiltration curve $I(t^{0.5})$ using the early-time infiltration data. In the Zhang method [4] and the 2T equation of Haverkamp et al. [2], both linearization methods (CL and DL) were used to calculate the coefficients C_1 and C_2. Note that in the method of Haverkamp et al. $S = C_1$, while in Zhang's method $S = C_1/A_1$.

Table 11. Values of soil sorptivity (S) estimated from unsteady-flow methods for loam, sandy loam, and clay soils at −15, −7, and −1 cm pressure heads. In the case of the Haverkamp et al. [2] method using the Solver tool, the S values were estimated considering the constant value β = 0.6.

H (cm)	**S (cm min$^{-0.5}$)**					
	Early Stage	**Haverkamp (2T) CL Method**	**Haverkamp (2T) DL Method**	**Haverkamp (3T) Using Solver Tool**	**Zhang CL Method**	**Zhang DL Method**
				Loam soil		
−15	0.2825	0.2752	0.3855	0.3126	0.1695	0.2374
−7	0.3812	0.3537	0.4698	0.3732	0.2896	0.3847
−1	0.4750	0.4850	0.5288	0.4822	0.4743	0.5172
				Sandy Loam soil		
−15	0.3790	0.3258	0.4216	0.3394	0.3956	0.5119
−7	1.0013	0.8826	1.0259	0.8415	1.0420	1.2111
−1	1.4964	1.3566	1.5994	1.1436	1.6176	1.9071
				Clay soil		
−15	0.2054	0.1943	0.1955	0.1930	0.1190	0.1198
−7	0.2731	0.2402	0.3347	0.2530	0.1666	0.2321
−1	0.6523	0.5793	0.8178	0.6070	0.4446	0.6277

The DL method compared with CL gave higher S values in both the 2T equation of Haverkamp et al. and Zhang's method. Regardless of the linearization method, the Haverkamp et al. [2] equation gave higher S values than those of the Zhang [4] method in the two soils (loam and clay), while in the sandy loam soil, the opposite happened.

The estimation of S from the early infiltration times in the specific experiments gave similar values with the CL method for the equation of Haverkamp et al. [2]. Nevertheless, the ambiguity in the selection of early times for the calculation of S could lead to its overestimation or underestimation. Such ambiguities and uncertainties in the selection of time for the calculation of S could also lead to large errors in the calculation of K, especially using Equations (20) and (26), because in these equations the term S^2 is included. Additionally, for the safe estimation of the linearity of the relationship $I(t^{0.5})$ in the early times a sufficient amount of data is required that allows taking measurements in short time intervals, which is not always possible, especially in the field. Thus, the application of linearization methods over the entire range of measurements to calculate S is preferred.

The inverse solution method of the 3T equation of Haverkamp et al. [2] (Equation (28)) using the Solver tool appears to be in good agreement with the CL method of the 2T equation (Equation (22)) for all soils and pressure heads. In the case where β = 0.75 was used, the estimated S values are almost the same as those estimated using β = 0.6. Also, the same results were obtained in the case of the two pressure heads (−7 and −1 cm) of the sandy loam soil where the SIA method was applied for both the 3T and 4T expansion models.

Among all the methods, the maximum difference in S values varies by approximately 50% and mainly concerns the difference between the DL method of Haverkamp et al. and the CL method of Zhang. Among the other methods, the differences are smaller. Also, if taking into account that the effect of time on C_1 is smaller than that on C_2, it could be said that S is less affected than K in terms of time [6].

Thus, in the cases of infiltration experiments where contact material between the infiltrometer and the soil infiltration surface is not used, the application of the CL method in combination with the 2T equation of Haverkamp et al. and the estimation of S from the value of C_1 or inverse solution method of the 3T equation are preferred due to both the simplicity of the method and because no additional soil data are required, as in the case of the Zhang method, for the calculation of S.

4. Conclusions

In this study, the K and S values of three laboratory-disturbed soils were estimated using cumulative infiltration data from a tension infiltrometer at various negative pressure heads by applying different steady- and unsteady-flow methods.

The steady-flow methods of Ankeny et al. and Reynolds and Elrick gave similar results in the estimation of K. The method of Logsdon and Jaynes deviated from the two aforementioned methods, mainly at the pressure head −1 cm for loam and sandy loam soils. This difference can be attributed to the assumption of the single exponential equation of K(H) over the entire range of pressure heads applied by the Logsdon and Jaynes method. Also, the steady-flow methods have the inability to calculate S.

The unsteady-flow methods, which are applied to shorter time-duration separate infiltration experiments for each pressure head (Tension Single Test), enable the estimation of both S and K from the same infiltration experiment. The equation of Zhang has the great advantage that in all cases studied it gave positive values of K. In particular, the CL method of Zhang gave K values similar to those of the steady-flow methods of Ankeny et al. and Reynolds and Elrick, in the case of soils with $n > 1.35$. The deviation of the method for soils with $n < 1.35$, even with the method of Dohnal et al., may be related to the different K(θ) relationship assumed by each method. Thus, despite the differences in the assumptions between the steady and unsteady-flow methods, the number of soil parameters that must be known in advance and the form of the equations of K(H) or K(θ) assumed by each method, there is a good agreement of the estimated K values in the cases where $n > 1.35$.

The coefficients C_1 and C_2 of the 2T infiltration equation are affected by the duration of the infiltration experiment. The coefficient C_1 increases while C_2 decreases with increasing time. From the experimental data, for the category of soils studied, the change in C_2 is more important than that of C_1 for the CL method. Comparing the two linearization methods, the CL method gave smaller C_1 values and larger C_2 values than those calculated from the DL method, as well as higher R^2 values in all soils and pressure heads. Therefore, in this category of soils, when no contact material is used, the CL method is considered more appropriate due to its simplicity.

Also, the inverse solution method of the 3T equation of Haverkamp et al. gives similar S values to the CL method of the 2T equation.

As regards the estimation of K, the 3T equation of Haverkamp et al. with $\beta = 0.75$, in general, when estimating positive values, approaches better the K values obtained by the steady-state methods and the method of Zhang. In clay soil, $\beta = 1.6$ gives K values that are close to those obtained from steady-flow methods. In the case of heterogeneous and layered soils, the SIA method can be a useful tool for estimating reliable K values.

The equation of White et al. seems to suffer from similar problems (negative K values) for the soils classified in the "lateral capillarity domain" to the method of Haverkamp et al. due to the common term $S^2/(r_0\Delta\theta)$ in their equations. The problem of negative K values of the equation of White et al. and the 2T equation of Haverkamp et al. will increase as the disc radius of the infiltrometer decreases and the lateral flow increases.

In general, the differences between the S values of the various methods are smaller compared to those of K, and S is less affected than K in terms of time.

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