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Enhancing Decision Fusion for Wastewater Treatment System Selection Using Monte Carlo Simulation and Gray Analytic Hierarchy Process

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Abstract: This research presents an innovative data fusion model that utilizes Monte Carlo simulations (MC) and the Gray Analytic Hierarchy Process (G-AHP) to address the complexity and uncertainty in decision-making processes, particularly in selecting sustainable wastewater treatment systems. The study critiques and extends the Dempster–Shafer and Yager's theories by incorporating a novel MC algorithm that mitigates the computational challenges of large numbers of experts and sensors. The model demonstrates superior performance in synthesizing diverse expert opinions and evidence, ensuring comprehensive and probabilistically informed decision-making under uncertainty. The results show that the combined MC algorithm produces satisfactory results, and thus, offers wide applicability in decision-making contexts. To determine its effectiveness, an extensive empirical study was conducted to identify an appropriate wastewater treatment system for the busy city of Tehran, incorporating the insights and perspectives of respected experts in the field. The selection was based on three technical, economic, and environmental–social criteria. Due to the large dimensions of each of the defined criteria, sub-criteria were also defined to achieve better results for each of the criteria. The in-depth analysis conducted revealed that enhanced aeration activated sludge (EAAS) emerged as the best choice for Tehran's most urgent needs among various competitors, with a remarkable priority rating of 34.48%. Next, the Gray Analytic Hierarchy Process (G-AHP) was used to determine the most important sub-criterion, based on which resistance to hydraulic shock is most important in the enhanced aeration activated sludge system. Due to its versatility in different fields and industries, this method is a powerful tool for managers to optimize system efficiency and identify defects and risks and eventually to minimize costs.

Keywords: data fusion; decision-making; Dempster–Shafer; knowledge management applications; methodologies and tools; Gray Analytic Hierarchy Process

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Citation: Zhian, T.; Hashemi Monfared, S.A.; Rashki, M.; Azizyan, G. Enhancing Decision Fusion for Wastewater Treatment System Selection Using Monte Carlo Simulation and Gray Analytic Hierarchy Process. *Water* **2024**, *16*, 1709. https://doi.org/10.3390/w16121709

Academic Editor: Christos S. Akratos

Received: 17 April 2024
Revised: 25 May 2024
Accepted: 28 May 2024
Published: 16 June 2024

1. Introduction

The decision fusion approach using Monte Carlo simulation and Gray Analytic Hierarchy Process (G-AHP) has shown promising results in enhancing the selection of wastewater treatment systems. Decision-making involves identifying and selecting the best course of action, based on one's own values and preferences, that has the highest chance of success and is most consistent with stated goals. The presence of a large number of criteria to be taken into account, both quantitative and qualitative, increases the complexity of the decision-making process [1]. Consequently, numerous decision-making methodologies and tools have emerged, including multidisciplinary approaches, multi-objective techniques, hierarchical systems, and data fusion strategies, particularly in the last few decades. However, it is worth noting that the latter method has garnered considerable popularity due to its heightened dependability as well as its ability to reduce interference from extraneous factors while significantly decreasing unreliable data occurrences. Having been introduced and documented in America in 1984 [2], a comprehensive framework has emerged that has found application across various scientific disciplines. This framework is widely known for its capabilities in water pollution quality assessments, climate change analysis, decision-making and risk analysis, image processing and tracking, accounting and auditing practices, data mining endeavors, artificial intelligence development, facial recognition studies, and medical research, among many others. To solve the intricate problems concerning data fusion within these fields several methods have been employed, including the Dempster–Shafer evidence approach, Yager theory utilization, the use of averaging techniques, as well as weighted average fusion methods along with employing tools such as Kalman filters backed by assessments derived from MC algorithms [2]. Notably, it is concluded that the most practical method when addressing complex data fusion issues on a decision-making level remains the employment of the aforementioned Dempster–Shafer evidence approach. In 1967, Dempster introduced the concept which would later be known as evidence theory through his publication on upper and lower probability bounds. It was not until 1976 that Shafer further refined and expanded this theory, addressing its limitations and enabling analysis of incomplete and ambiguous information. Consequently, it was identified as the "Dempster–Shafer evidence theory" [3]. However, one obstacle encountered within this theoretical framework lies in its failure to adequately incorporate uncertainties when conflicting expert opinions are present. The issue of inter-evidence conflict was initially highlighted by Professor Lotfizadeh using an illustrative example. His demonstration revealed a significant dependence of the Dempster–Shafer fusion law on inter-evidence consistency while disregarding any conflicts in the evidence. When multiple pieces of evidence are harmonious, the uncertainty associated with combined results can be reduced. Yet if this evidence clashes in a substantial manner, the outcomes become void of logic and cannot be accepted due to their inconsistency. Among the numerous proposed solutions put forth to address this quandary, one is known as the TBM (Transferable Belief Model). In this approach, masses remain unaltered and the issue regarding conflicting mass allocations to the empty set is stored [4]. Another suggestion comes from Dubois and Prade, who propose generating masses through opposing pair foci. Lastly, we have Yager's theory that tackles the problems associated with Dempster–Shafer evidence theory; the method was presented in 1987 [5]. Although Yager's method successfully resolves uncertainty concerns within the Dempster–Shafer theory, it is not without its fundamental drawbacks such as computational complexity and limited capability for accurate deductions when evidence indicates a lack of information about the system. To refine and enhance fusion rules within Dempster–Shafer theory, researchers have presented various methods. Among these approaches is Wang et al.'s hybrid evidence validity-based method, which remains unaffected by the quantity of evidence provided while addressing irrational cases. In a revolutionary approach to decision-making, the novel method underwent thorough evaluation as it aimed to select wind turbines. It carefully assessed the validity of the evidence and employed a ranking technique predicated on its likeness to the ideal solution. The outcomes astoundingly demonstrated its effectiveness in identifying the optimal design from an array of offshore wind turbines [6]. Addressing the vexatious issue of conflicting evidence, Liu et al. introduced an ingenious reliability estimation method rooted in failure modes and effects analysis (FMEA). This remarkable approach not only incorporated experts' distinctive personality traits but also accounted for inter-factor dependencies while effectively managing contradictory pieces of information. Armed with this proposed discount methodology, they triumphantly resolved a critical quandary associated with the supercritical water-to-gas (SCWG) conversion system during an illuminating case study. Through extensive discourse and meticulous analysis, their work undeniably showcased both its efficiency and unparalleled superiority [7]. Lia and her colleagues unveiled an enhanced fusion algorithm, which hinges upon the weighted average of the evidence conflict probability. This novel approach was employed to forecast levels of risk pertaining to water infiltration during various phases of subaqueous tunnel excavation. While conventional means yielded lower estimations of risk, this algorithm impressively anticipated a heightened degree of hazard in the twelfth stage of the boring process—a prognosis that aligned remarkably well with the grave seepage ardently witnessed in experimental trials. Succinctly put, this algorithm possesses the capacity to yield more precise predictions concerning calamitous flooding events while serving as an invaluable point of reference for comparable engineering predicaments [8]. Due to its inability to provide a satisfactory mathematical framework for effectively managing uncertainties when evaluating risk parameters and prioritizing failure modes, the Dempster–Shafer evidence theory was combined with failure mode effects and critical analysis (FMECA) by Sazer et al. in order to assess potential system failures and their underlying causes [9]. Similarly, Wang et al. utilized this approach to identify groundwater zones in arid basins using GIS information. Their study showed that assessment methods derived from this integration were able to reliably and successfully predict the presence of groundwater in the region, thereby serving as a solid scientific basis for ensuring groundwater security and effective management [10]. In the area of data analysis and prediction, one can find comfort in the Dempster–Shafer evidence theory. This remarkable theory has a special feature: it allows all relevant parameters involved in a given problem to be considered simultaneously, without any restrictions. In stark contrast to prevailing statistical methods and machine learning algorithms, which unfortunately do not take into account all relevant factors or have limited accuracy, this theory proves to be a highly viable alternative. Regrettably, few studies have ventured into exploring the combined impacts that may arise from garnering data from multiple sources—be they sensors or expert opinions—on predicting various phenomena. It is deeply disheartening to observe that these crucial aspects have not undergone meticulous investigation, despite their potential for enhancing comprehensive understanding and accurate forecasting. Jiang et al. presented a profound paradigm of learning that seamlessly integrates diverse data sources in order to analyze and predict the quality of urban drainage water in the southern region of China. This pioneering approach merges environmental and social indicators with measurements of water quantity and quality, employing advanced techniques that fuse multiple sources of data. When compared to linear methods such as multiple linear regression, as well as traditional algorithms like multilayer perceptron, it is clear from the researchers' findings that the deep learning algorithm possesses exceptional predictive capabilities. Through incorporating recurrent neural networks and mechanisms known as short long-term memory, Jiang et al.'s methodology surpasses alternative models in performance [11]. To predict the water depths of lakes in São Paulo, Brazil, Manzione and Catrignano employed remote sensors and the co-Kriging data fusion method. Through a process of comparison and cross-validation they demonstrated that the uncertainty associated with estimating water depth using the data fusion method was significantly lower than that of the univariate method. Additionally, it is noteworthy that the data fusion method allows for an examination of vegetation and soil characteristics, while this capability is lacking in the univariate approach [12]. To predict potential underground water reserves accurately and facilitate optimal management strategies, Obeidavi et al. recommended utilizing both the Dempster–Shafer learning model and remote sensing data. They implemented this methodology in the northern Khuzestan region for precise measurement and effective administration of underground water resources [13]. Chen et al. employed the Bayesian inference technique to integrate data from remote sensing and on-site observations, with the aim of determining water quality. In comparing various methods of data fusion such as linear regression, nonlinear regression, cumulative distribution function, and Bayesian approaches, it was found that the latter yielded lower errors and higher correlations. Consequently, this method exhibited promise in determining drinking water quality [14]. Additionally, apart from employing the Dempster–Shafer evidence method for data fusion mentioned earlier, there exist alternative methodologies like the Kalman filter approach. The Kalman filter is a potent tool that enables information synthesis in situations characterized by uncertainty; serving as an estimator leveraging previous state estimates along with current state observations to calculate estimates of the present state. The astonishing capability of the Kalman filter to extract precise information has ensured its longstanding use as an optimal solution for a multitude of data-tracking and prediction tasks [15]. Utilizing the second-order Kalman filter error method and employing soft algorithms known for probabilistic analysis of multi-sensor data, Gunia et al. merged various sensor inputs for water quality monitoring at the esteemed Finnish Environmental Institute. They meticulously calculated uncertainties and spatiotemporal correlations within observational data, yielding accurate and realistic results. Importantly, their findings demonstrated that the implementation of the Kalman filter presented a suitable approach to enhancing water-quality monitoring programs while adhering to established environmental standards [16]. Other techniques for data fusion include the illustrious Monte Carlo (MC) algorithm. Initially introduced in 1988 by Kampke and Pearl, this method was specifically designed to address the challenges presented by the Dempster–Shafer–Yager evidence theory. Their novel approach involved employing a straightforward MC algorithm to assess uncertainty within the Dempster–Shafer evidence theory while acknowledging its intertwining connection with classical probability theory; however, it fell short of achieving satisfactory convergence [17,18]. In subsequent years, Moral and Wilson made significant contributions to refining the utilization of MC algorithms within this context. In 1994, they ingeniously incorporated Markov chains into their MC algorithms as a means to calculate the Dempster–Shafer belief function more efficiently. Remarkably, their groundbreaking research demonstrated that this methodology remained effective even when faced with high levels of inter-evidence conflicts [19]. Ultimately, these scholars continued advancing their proposed technique and solidifying its efficacy through empirical analysis in 1996. Through an insightful experimental example, Moral and Wilson compared their refined method against both the simple MC algorithm and Markov chain variants [20]. Salehy and Okten devised the Monte Carlo and pseudo-Monte Carlo methodologies as a means to tackle the intricate dilemma of the combined Dempster–Shafer rule with regard to time. They successfully demonstrated that through this research, precisely by incorporating techniques such as variance reduction and low-discrepancy sequence, the MC algorithms possess the potential to broaden the scope of application for the Dempster–Shafer theory in relation to obstacles previously deemed insurmountable. Furthermore, they presented empirical data showcasing both the efficacy and convergence rate of select algorithms. Notably, their investigation revealed that utilizing the Dempster rule allows one to obtain a satisfactorily accurate approximation of a merged belief function within a mere half-minute timeframe—an achievement made all-the-more impressive considering that obtaining an exact solution for this particular problem previously demanded over six days [21]. In addition to the importance of choosing the best wastewater treatment system, determining the most effective and important sub-criteria is also of particular importance. To determine the effect of different factors in a system, methods such as TOPSIS, AHP, and G-AHP are used. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a multi-indicator decision-making method like AHP for evaluating and prioritizing options based on criteria according to their distance from positive and negative ideals. This method was proposed by Huang and Yun in 1981 and soon found its place in multi-criteria decision-making. Anaokar et al., in a report, evaluated the effective indicators in urban wastewater treatment using TOPSIS multi-criteria decision-making models. In this research, the relative importance of the criteria were determined by the decision makers. The performance of six urban sewage treatment plants was evaluated using the multi-criteria decision-making technique for ranking with similarity to the ideal solution. Their efficiency was based on the characteristics of the wastewater and their performance was based on the parameters of temperature, total suspended solids, total dissolved solids, biological oxygen consumption (BOD), chemical oxygen consumption (COD), and pH [22]. In another study, Golfam et al. presented a method to select the best alternative for the reuse of treated wastewater based on the gray system. The criteria are selected by the Analytic Hierarchy Process (AHP) method. The results show that reuse of wastewater in the environmental sector has the highest priority among several alternative applications [1]. Perez et al. discussed the risk assessment method to reduce the operational risk of facilities in a domestic wastewater treatment plant system based on constructed wetlands. The approach used in this research is a three-dimensional risk matrix, which is a simplified version of the probabilistic risk assessment method that makes it more accessible and allows for wider application. The results show that human factors appear significantly as the main risk factors related to wetland operations [23]. The aim of this research is to significantly advance the Dempster–Shafer evidence theory and Yager theory by introducing the innovative application of the Monte Carlo (MC) algorithm. While existing methods require experts to reach a common consensus while simultaneously selecting a system in order to calculate the associated probability, they falter when confronted with an increasing amount of evidence or sensors. This increase requires an excessive number of random samples for accurate probability estimation, resulting in increased computation time and higher sampling requirements. Additionally, it reduces the likelihood that all evidence or sensors will relate to a single option at any given time. While the Dempster–Shafer evidence theory is effective in many contexts, it often has problems dealing efficiently with uncertainty and conflicting expert opinions. Existing methods are computationally intensive and may not integrate large amounts of evidence or sensor data well. Furthermore, a notable gap exists in the methodologies that combine advanced decision algorithms with structured frameworks for comprehensive assessment and prioritization of decision criteria.

The approach presented in this study ingeniously solves this dilemma by constructing a robust probability space. This space not only accounts for the uncertainty inherent in each individual expert's opinion, but also facilitates decision-making in scenarios involving a larger number of experts. By integrating the MC algorithm, this research not only overcomes the computational challenges associated with large-scale evidence or sensor inputs, but also improves the reliability and efficiency of decision-making processes in complex scenarios. To further highlight the novelty and importance of our approach, we also use the Gray Analytic Hierarchy Process (G-AHP) to rank the sub-criteria. This method complements the MC algorithm by providing a structured framework for evaluating and prioritizing the various factors that contribute to decision-making processes.

Incorporating the Monte Carlo algorithm helps overcome the computational challenges associated with large-scale evidence or sensor data inputs, thereby improving the reliability and efficiency of decision-making processes. This approach also constructs a robust probability space that accounts for the uncertainty inherent in each expert's opinion, making it easier to handle scenarios involving numerous experts. The Gray Analytic Hierarchy Process (G-AHP) is employed to rank sub-criteria, offering a structured framework for evaluating and prioritizing various factors that influence decision-making. This complements the MC algorithm by providing a clear and systematic approach to criteria evaluation, ultimately enhancing the decision-making framework's comprehensiveness and reliability. Through this combined approach, our research provides a comprehensive solution to the challenges of selecting optimal wastewater treatment systems, filling a critical gap in current methodologies, and paving the way for more effective decision-making in complex areas.

2. Materials and Methods

This section provides a summary of the data fusion and describes in detail the Dempster–Shafer and Yager theories, the Monte Carlo simulation, sensitivity analyses, and the Gray Analytic Hierarchy Process.

2.1. Wastewater Treatment Plant

Wastewater treatment refers to performing any physical, chemical, biological, or combined treatment on raw sewage. The purpose of this process is to reduce or eliminate polluting parameters that cause water pollution and turn it into wastewater, so that the quality of the treated effluent reaches a level that meets the existing standards. According to the information of the Iran Water and Sewerage Company, currently the most common urban wastewater treatment methods in Iran include four activated sludge processes, stabilization pond, aeration lagoon, and trickling filter method, which more than 90% of urban wastewater treatment plants in Iran use. They use one of these four processes. Due to the climatic conditions of the study area, which are explained further in the full case study section, and considering that the price of land in the study area is very high, the population is large and growing, and the temperature changes in different seasons are large, the activated sludge method is a more suitable option for wastewater treatment than other methods. Among the advantages of this method are the use of less land area, no unpleasant smell (important for refineries built near cities), low sensitivity of the process to temperature changes, low construction cost compared to some purification methods, high purification efficiency.

2.2. Data Fusion

Data fusion combines the data obtained from sensors and various other evidence to accurately predict the properties and states of a system, with decision-making being the highest level that not only has the highest accuracy and lowest error, but also the most complex and the largest volume of calculations. In fact, data fusion is a knowledge management application. The decision level integrates different types of data—quantitative, waveform, multidimensional, etc.—and uses certain tools such as the Dempster–Shafer law (the most common) and Yager's method, explained below, to solve the data fusion problem [24].

2.3. Dempster–Shafer Theory

The belief function theory, also known as Dempster–Shaffer's or evidence theory, was first introduced by Arthur P. Dempster in 1967, and then, developed and popularized by Glenn Shaffer [25]. It is a generalization of the Bayesian approach and provides a general framework for quantification, representation, and management. The Dempster–Shafer theory involves three very important functions—basic probability mass function, trust function, and acceptability function—that form the basis of the equations and calculations. The former, the most important function in the theory of evidence, is in fact the evidence belief mapping for the existence of state A and is defined by a number in the range 0–1 [26].

$$m : P\ (X) \qquad m = [0.1] \tag{1}$$

If

$$m(\varnothing)\ \neq 0 \tag{2}$$

$$\sum_{A \epsilon P} m(A) = 1 \tag{3}$$

The other two (acceptability and trust), which are, respectively, the upper and lower limits of the occurrence of a subject, are defined based on the mass function and calculated accordingly. The trust function, which is the lowest bound of probabilities for a state to occur, is the most pessimistic probability estimate of a subject A, and the trust level for set A to occur is calculated from the sum of the mass function of all the sets that share A. The acceptance rate calculations assume that sharing with A will occur in any case and the other part of the estimated probability will not occur. P(A), the actual probability of an event A, is a value between Bel(A) and PI(A) of that event. The belief function (Bel) and plausibility (PI) are two important concepts in Dempster–Shafer theory. The belief of a set represents the amount of evidence that fully supports that set, while the plausibility represents the amount of evidence that does not negate that set at all. These two are defined as follows:

$$Bel\ (A) \leq P\ (A) \leq PI\ (A) \tag{4}$$

$$Bel\ (A) = \sum_{B|B \subseteq A} m\ (B) \tag{5}$$

$$PI\ (A) = \sum_{B|B \cap A \neq \varnothing} m\ (B) \tag{6}$$

When the upper and lower confidence limits of the occurrence of a state are equal, its occurrence probability will also be equal to them; this usually occurs when the evidence has a definite estimate of the occurrence of an event.

$$If\ PI\ (A) = Bel\ (A) \tag{7}$$

Then,

$$Bel\ (A) = P\ (A) = PI\ (A) \tag{8}$$

In the Dempster–Shafer theory, the combination law is in the form of Equation (9).

$$m_{12}\ (A) = m_1\ (B) \oplus\ m_2\ \ (C) = \frac{1}{1-K} \sum_{B \cap C = A \neq \varnothing} m_1\ (B) \times m_2\ (C) \tag{9}$$

where

$$k = \sum_{B \cap c = \varnothing} m_1(B) \times m_2(C) \tag{10}$$

where k indicates the contrast between two mass sets [27].

Figure 1 shows four different modes to estimate the subject evidence.

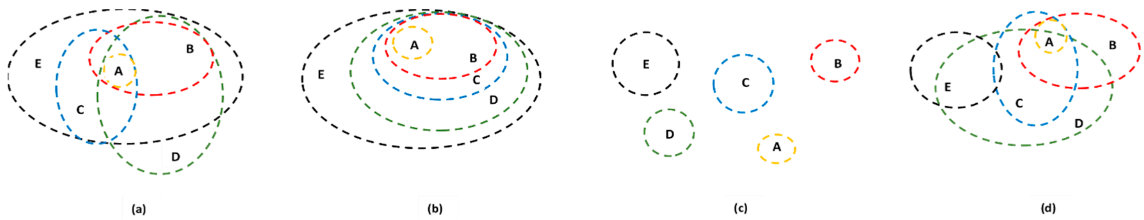

Figure 1. Different modes of estimating evidence from the system. (**a**) involves an integrated inclusion–sharing relationship between the estimation of evidence, in (**b**) only the inclusion relation prevails, (**c**) exhibits a kind of complete contradiction, and in (**d**) the dominant inter-evidence relation is communal; the latter two are more important because they are faced more often [28].

2.4. Yager Theory

As the theory of evidence involves inter-evidence conflicts that may result in a totally wrong estimation, the Dempster–Shafer theory needs to be modified even more. Among many methods introduced to justify and fix this defect, the most efficient one, was formulated by Yager in 1987. Yager, in his theory known as Yager's rule, deforms the mass function (m) and introduces a new function called the "ground probability mass assignment" (q). This method is based on the idea that the value of q can be greater than zero ($q(\varnothing) \geq 0$), which means that the second condition of the definition equation of the mass function has been violated [29]. In many real applications, the evidence performance usually involves many errors; hence, it is logical to reduce percent detections by a confidence factor to make them more realistic. Accordingly, Yager introduced the "important factor" (α_i) parameter, which is the confidence level in a witness and explains its weight against other evidence. If $Q_i(A)$ is the ith estimate witness of the event of state A and α_i is its weight, the new values of the mass function are found as Equation (11) [30].

$$m_i(A) = \alpha_i \times Q_i(A) \tag{11}$$

Yager classified the possible errors and contradictions among evidence in a group θ. If the probability of the null state is positive, the probability that the witness may not choose any state that conflicts with others, or make an error, is greater than zero. Using the important factor, θ is defined as $\theta_i = 1 - \alpha_i$. Parameter θ shows that a witness does not know the subject's state, making it likely that it is either state; that is, in the diagnosis of each state, the estimation of θ of each witness shares with that of other evidence. Mathematical interpretation means that the evidence shares the least uncertainty in the diagnosis of a situation. Therefore, the value of θ is effective in deducing the composition formulas because it affects the diagnosis of each piece of evidence. In addition, since the index θ has something in common with the estimate of other evidence, it cannot be in the denominator of the composition subtraction rules since it does not satisfy the related conditions [31]. According to Equation (11), the probability of evidence and Equation (13), the combination of evidence estimate in Yager theory is calculated.

$$q\ (A) = \sum_{\bigcap A_i = A} [m_1(A_1) \times m_2(A_2) \times m_3(A_3) \times \ldots \times m_i(A_i) + \theta_i \times m_i] \tag{12}$$

$$m\ (A) = \frac{q\ (A)}{1 - q(\varnothing)} \tag{13}$$

2.5. Monte Carlo Algorithm

Contrary to deterministic methods that rely on a realistic view of how a process is carried out over time, Monte Carlo simulation, first presented in 1949 by Metropolis and Yolam, considers possible future events based on a probability distribution and is, in fact, a very widely used accurate simulation method for reliability evaluations of many engineering problems. In a general and rather imprecise division, Monte Carlo simulation is a simple random sampling method applied in mathematics, physics, chemistry, engineering, and so on, by creating a sequence of random samples where each variable Xi is randomly sampled to finally check the limit state function [32]. If g(Xi) ≤ 0, the generated sample of random data is placed in the failure area; otherwise, it lies in the safe area. This process of simulating the random variables by controlling the limit state function involves many repetitions, in each of which the Xi vector is randomly selected to generate several points in the failure zone. This method is advantageous because the type and shape of fx(X) and the limit state function g(X) ≤ 0 are not constrained. The probability in the failure area is calculated as follows [33]:

$$P_f = P[g(x) \leq 0] = \int_{-\infty}^{+\infty} I(x) f(x) dx \tag{14}$$

where f(x) and I(x) are the density and counter functions; the latter is as follows:

$$I(x) = \begin{cases} 1 & if \quad g \leq 0 \\ 0 & if \quad g > 0 \end{cases} \tag{15}$$

Since solving the above integral and finding the failure probability in a general mode is not very easy, the Monte Carlo simulation estimates its average value as Equation (16).

$$E(I(x)) = \int_{-\infty}^{+\infty} I(x) f(x) dx = \frac{\sum_{i=1}^{N} I(x)}{N} \approx \frac{N_f}{N} \tag{16}$$

2.6. Sensitivity Analysis

Sensitivity analysis is a fundamental modeling step that determines the variability in model outputs by varying the variables. Considering the variety of simulation models and their complexity due to numerous variables, sensitivity analysis is an essential tool to understand the role and importance of variables in the modeling process. Methods used for sensitivity analysis and the ranking of random variables are mostly based on (1) first- and second-order reliability methods, and (2) simulation methods, where sensitivity analysis is performed by evaluating the rate of change in the probability of the damage to each statistical feature of each variable by simply evaluating its mean or standard deviation. Estimating the sensitivity by the Monte Carlo simulation method is explained below [34,35], and the failure probability in the Monte Carlo method is calculated as follows [36,37]:

$$P_f = \int_{g(x)\leq 0} f_x(x)dx = \int_x I_{g\leq 0}(x) f_x(x)dx = E_f\left(I_{g\leq 0}(x)\right) \tag{17}$$

where $E_f(0)$, f_x, and x, g(x) $\leq$ 0 are, respectively, the expectation operator, probability density function (PDF) of random variables, and failure set ($I_{g\leq 0}(x)$ was explained in the previous section). An MCS provides an accurate failure probability for any reliability problem by evaluating $E_f\left(I_{g\leq 0}(x)\right)$ and simulating independent and identically distributed samples based on their PDF f_x. By this approach, the partial derivative of the failure probability with respect to the ith component of ξ is as follows:

$$\frac{\partial P_f(\xi)}{\partial \xi_i} = \frac{\partial}{\partial \xi_i}\int_x I_F(x)\partial f_x(x.\xi)dx. \tag{18}$$

where ξ_i is a statistical parameter (mean value or standard deviation of the ith random variable). Using the Lebesgue-dominated convergence theorem and importance sampling (IS), the equation can be written as follows [36,37]:

$$\begin{aligned} \frac{\partial P_f(\xi)}{\partial \xi_i} &= \int_x I_F(x)\frac{\partial f_x(x.\xi)}{\partial \xi_i}dx = \int_x I_F(x)\frac{\partial f_x(x.\xi)}{\partial \xi_i}\frac{f_x(x.\xi)}{f_x(x.\xi)}dx \\ &= \int_x I_F(x)\frac{\frac{\partial f_x(x.\xi)}{\partial \xi_i}}{f_x(x.\xi)} f_x(x.\xi) = \int_x I_F(x)\frac{\partial log f_x(x.\xi)}{\partial \xi_i} f_x(x.\xi)dx \\ &= E_x[I_F(x)\cdot K_i(x.\xi)] \end{aligned} \tag{19}$$

where the so-called score function is introduced as follows:

$$K_i(x.\xi) = \frac{\partial log f_x(x.\xi)}{\partial \xi_i} \tag{20}$$

Hence, using only MCS samples (no additional sampling) the partial derivative of the failure probability can be found as follows [38]:

$$\widehat{\frac{\partial P_f(\xi)}{\partial \xi_i}} = \frac{1}{N}\sum_{j=1}^{N} I_F\left(X^{(j)}\right)\cdot K_i\left(X^{(j)}.\xi\right) \tag{21}$$

Using the proposed partial derivatives, a dimensionless sensitivity measure with respect to a non-zero parameter ξ is defined as Equation (22) [39].

$$e_{\xi_i} = \widehat{\frac{\partial P_f(\xi)}{\partial \xi_i}}\cdot\frac{\xi_i}{P_f} \tag{22}$$

2.7. Decision Theory in Gray Environment

Gray systems theory is a new approach in the uncertainty environment that focuses on problems that use small samples and incomplete information [40]. Gray systems theory is one of the leading methods in the mathematical analysis of systems with uncertain information. Considering the amount of uncertainty in the study of systems, colors can be used to name and display the system. A black system indicates that the relevant data, the internal relationships between them, and the system's structure are completely unknown. In the same way, a white system is a system that people know about and the information about it is complete. From this point of view, and in a general classification, systems that are neither completely unknown nor fully known can be called gray systems [41]. The gray system theory was first proposed by Professor Ju Long Deng in 1982 [42].

G-AHP Method

The Analytic Hierarchy Process has been proposed to model the multi-branch mental decision-making process in a hierarchical system. In a short period of time, this method has been widely used in company planning, portfolio selection, cost–benefit analysis by government agencies, and location, etc. The Gray Analytic Hierarchy Process is basically similar to AHP. In the G-AHP (Gray Analytic Hierarchy Process), gray numbers are used instead of definite numbers [43]. The calculation steps of G-AHP after identifying the problem (objectives, criteria and options) and forming the hierarchical structure are as follows:

The first step: the matrix of paired comparisons with gray numbers obtained from the opinions of decision makers is formed as follows:

$$\otimes X = \begin{bmatrix} \otimes x_{11} & \otimes x_{12} & \cdots & \otimes x_{1n} \\ \otimes x_{21} & \otimes x_{22} & \cdots & \otimes x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m1} & \otimes x_{m1} & \cdots & \otimes x_{mn} \end{bmatrix} \tag{23}$$

The second step: the matrices of pairwise comparisons of expert opinions are integrated using Equation (24):

$$\otimes x_{ii} = \sqrt[k]{\prod_{i=1}^{k} \otimes x_{ii}^{k}} \tag{24}$$

The third step: normalize the columns of the matrix using relations (25) and (26):

$$\underline{n}_{ij} = \left(\underline{x}_{ij} - min_j \underline{x}_{ij}\right) / \Delta_{min}^{max} \tag{25}$$

$$\overline{n}_{ij} = \left(\overline{x}_{ij} - min_j \underline{x}_{ij}\right) / \Delta_{min}^{max} \tag{26}$$

$$\Delta_{min}^{max} = max\overline{x}_{ij} - min\underline{x}_{ij} \tag{27}$$

The fourth step: calculating the degree of gray importance is obtained through the linear average and the relationship:

$$\frac{\sum_{j=1}^{n} \otimes n_{ij}}{n} \tag{28}$$

Fifth step: After determining the importance of the sub-criteria, the rank of each option is obtained from the relation

$$\sum_{j=1}^{n} w_j \otimes n_{ij} \tag{29}$$

All the criteria and sub-criteria are selected based on the facts, compatible with the real conditions. In determining the main criteria of wastewater treatment it is of great importance that their selection and evaluation was performed based on the experiences gained from the design, implementation, and operation of these processes in selected cities. These criteria are introduced in the form of three main criteria, which are technical criteria, economic criteria, and environmental–social criteria. Due to the large dimensions of each of the defined criteria, sub-criteria were also defined to achieve better results for each of the criteria. The sub-criteria that were taken into consideration in the technical evaluation of the processes are resistance to organic and hydraulic shocks, continuous operation, simple operation, the possibility of upgrading, BOD removal, equipment construction and installation costs, maintenance, energy consumption, sludge disposal, odor production, reaching purification degree required, worker safety, and sludge production [1,22]. Figure 2 shows the criteria and sub-criteria of the EAAS system.

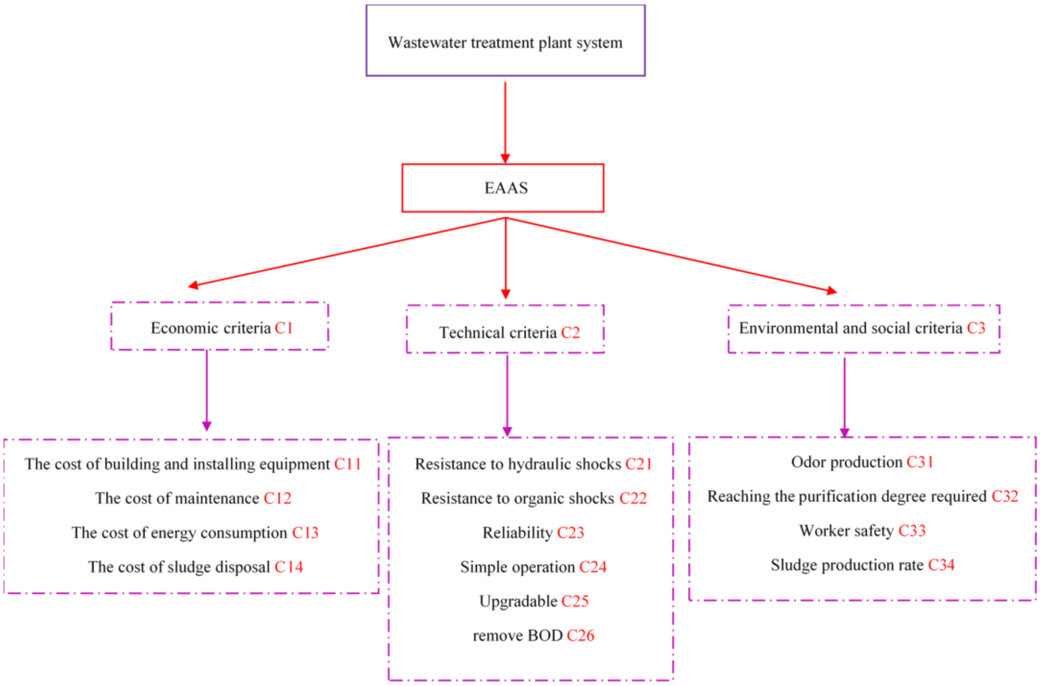

Figure 2. Criteria and sub-criteria for the EAAS system.

In order to collect data (opinion of experts), first, questionnaires should be designed according to the instructions of each method, based on which experts can express their opinion. These questionnaires are prepared based on linguistic variables, and then, they should be converted into gray numbers. Linguistic variables are converted to gray numbers in the G-AHP using Table 1, as shown below.

Table 1. Converting linguistic variables to gray numbers by G-AHP method.

Linguistic Variable	Gray Numbers
Equal importance	[1, 2]
A bit important	[2, 4]
Important	[4, 6]
Very important	[6, 8]
Absolutely important	[8, 9]

The experts were selected based on their experience and work experience in the field of sewage treatment plants. Among them, some are university professors who have a long history of teaching courses related to water and sewage in universities and research centers and in various studies in this field. Some were experienced people from wastewater treatment plant staff, and some of them were working in the Parand Treatment Plant and South Tehran Treatment Plant.

3. Proposed Method

Data fusion (Figure 3) means moving towards the main goal (system selection/evaluation, troubleshooting, etc.) using evidence (estimators, classifiers, decision makers, etc.) or information sources (sensors, satellite data, etc.), and is the joint analysis of multiple related datasets that provide complementary views of a phenomenon. The process of correlating and integrating information from multiple sources generally enables more accurate inferences than using information obtained from the analysis of only one dataset.

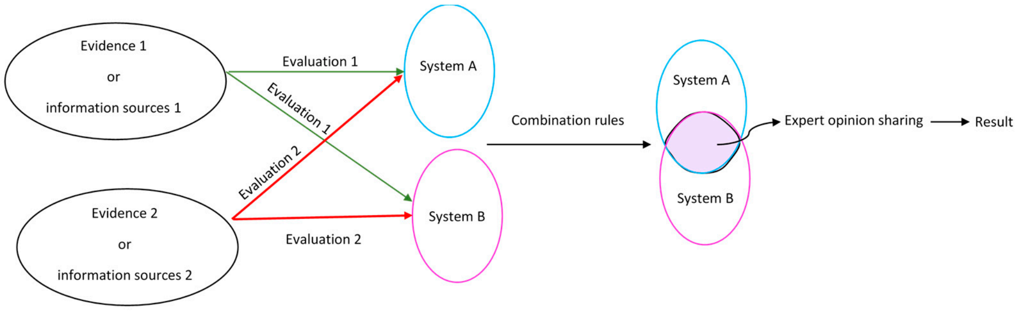

Figure 3. Schematic view of the data fusion process.

Information sources do not perform any decision-making, estimation, or data processing; they only provide information from the system to the evidence and decision makers. Since this research examines data fusion at the decision-making level, the information sources are experienced experts with decision-making power. Although data fusion is a multilateral concept with clear benefits, it involves challenges that need to be considered carefully. One problem of these methods is that when the number of experts increases, calculations become complicated. In the Dempster–Shafer and Yager methods, since increasing the input space may easily reduce the accuracy, effort has been made in this study to solve the problems of the previous methods by presenting a new MC algorithm-based method, where the first step is to map the decision-making space in the probability space. Now, suppose one system is to be selected as the best of several systems.

According to the Monte Carlo rules, a system probability is calculated by dividing the number of random samples that lie in this common area by the total number of samples; finally, the best system is determined by comparing the obtained possibilities. In Figure 4, when the number of experts increases, the calculations do not become complicated and the results are not confusing; rather, the accuracy of the calculations and probability estimations increases. This method has been so designed as to successfully summarize the evidence's belief values at different levels.

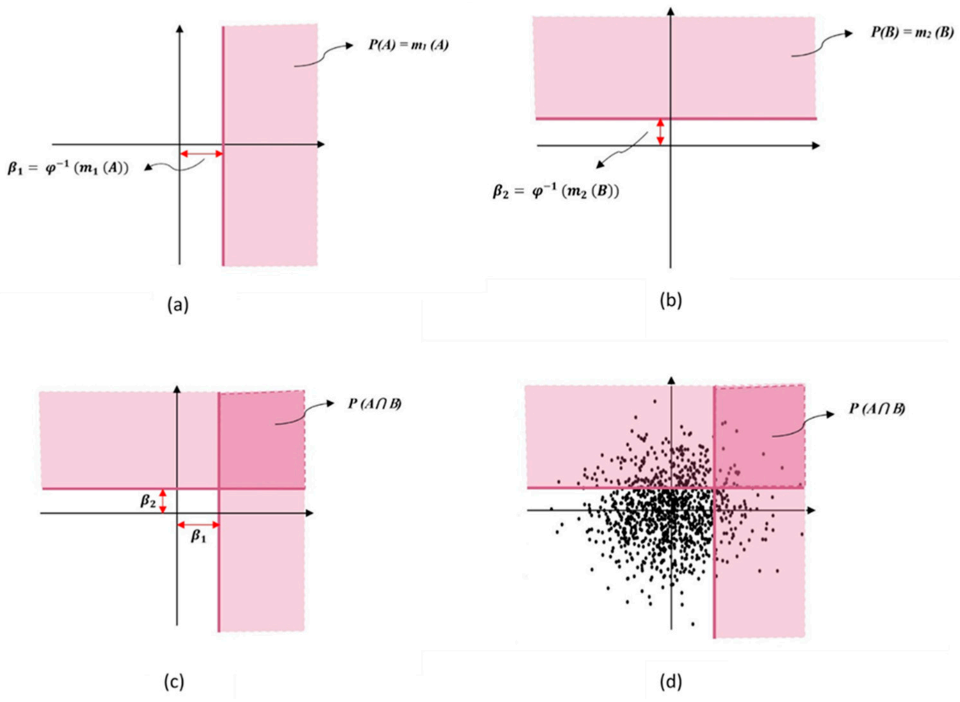

Figure 4. Decision fusion steps in the Monte Carlo algorithm. (**a**,**b**) the mapping of the opinions of experts A and B in the probability space, in (**c**) a common area has been calculated, which is equivalent

to the probability estimation in the MC algorithm, (**d**) random samples was created based on the standard normal distribution function, and (β) is the reliability index, which, by definition, is the average distance of the limit state function from the probability region.

Another problem of the Dempster–Shafer method is ignoring uncertainties; if there is a probability that experts do not know the system, the results will differ from reality, and if their opinions are uncertain, two situations may occur, according to Algorithm 1, in the defined probability space by comparing the generated samples (r) and the uncertainty in each expert's opinion (θ): (1) if $r > \theta$, the probability can be found by the fusion rules; and (2) if $r < \theta$, random samples are regenerated, some are selected randomly and the probability is calculated. The proposed MC algorithm enables the probability to be estimated in any situation, that is, if the experts' opinions are alike (i.e., have something in common) or are in conflict, the probability is found with high accuracy, which is another ability of the MC algorithm.

In the following, the MCS pseudo-code for decision fusion is shown.

Algorithm 1: Monte Carlo Algorithm for Decision fusion

```
procedure Monte Carlo Decision Fusion
Inputs: Expert Opinion (Xij), Uncertainty of expert opinion (θi), number of Monte Carlo samples (N)
Output: An estimate of the probability of system selection (m1, m2, …, mn)
for a large number of trials n = 1: N do
    for i= 1: m do
        Randomly sample generation (ri) and probability calculation (Pi)
    end for
    let θ = (θ1, θ2, …, θm)
    if θ = 0
        m = Pi/sum(Pi)
    end if
    if θ ≠ 0
        if r > θ
            m = Pi/sum(Pi)
        else
            Randomly sample generation (w)
            m = Pwi/sum(Pwi)
        end if
    end if
end for
end procedure
```

Figure 5 shows the MC algorithm with and without considering the uncertainty in the experts' opinions.

Next, the MCM is compared with the existing decision-making methods (Dempster–Shafer and Yager evidence theory) through an example to check the correctness of the method presented in this research.

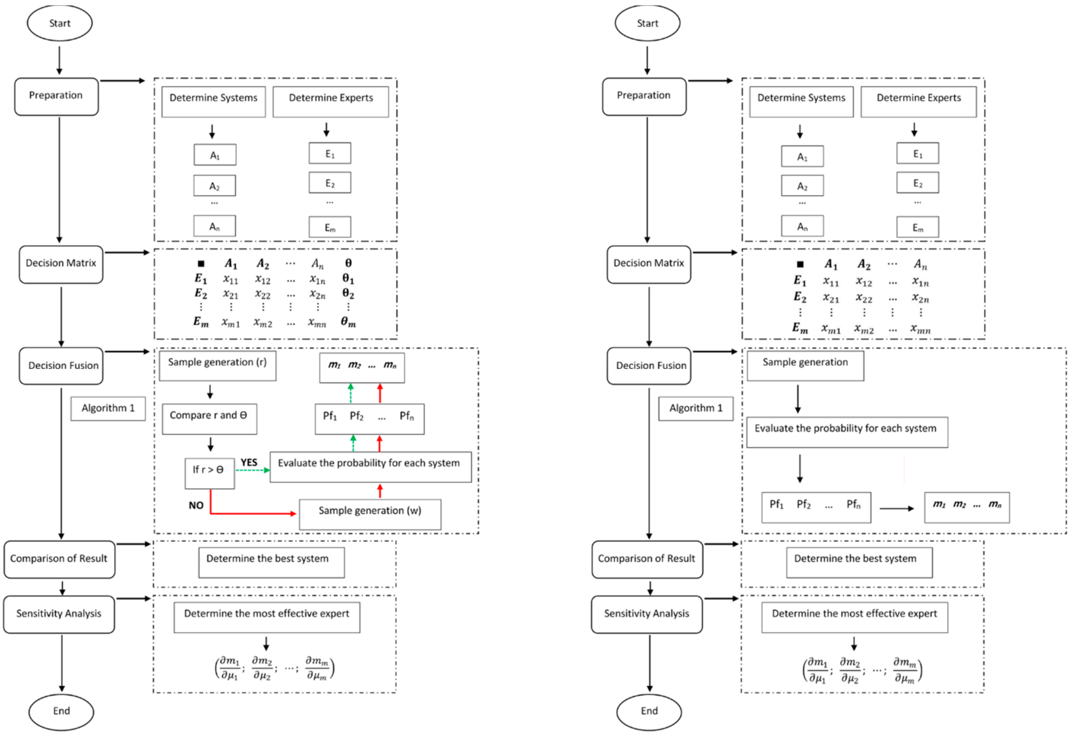

Figure 5. Flowchart of the algorithm of the present study with/without uncertainty ($\theta \neq 0/\theta = 0$).

3.1. Example

Table 2 shows the opinions of two experts E1 and E2 asked to choose the appropriate one of two systems A and B, and the uncertainty they considered for their opinion (θ).

Table 2. Opinions of the experts and the related uncertainties (θ is the uncertainty they considered for their opinion).

Expert	System (A)	System (B)	θ
E1	0.4	0.6	0.1
E2	0.45	0.55	0.05

3.1.1. Dempster–Shafer Evidence Theory

First, the multiplication matrix C of the two pieces of evidence is formed.

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 0.18 & 0.27 \\ 0.22 & 0.33 \end{pmatrix} \tag{30}$$

According to the definition of the Dempster–Shafer theory of evidence, stated in the previous section, the main diameter values are actually equivalent to the fractions in the related equations. Other values of the matrix have other specific meanings, showing the difference between the evidence when they do not have a common opinion. The sum of the values of the dimensions, except for the main diameter, helps calculate the value of k, which is the integration error; according to matrix C, this value is calculated as follows and added with those of the main diameter to combine the results:

$$k = c_{12} + c_{21} = 0.27 + 0.22 = 0.49 \tag{31}$$

$$E_{12}(A) = \frac{c_{11}}{1-k} = 0.3529 \tag{32}$$

$$E_{12}(B) = \frac{c_{22}}{1-k} = 0.6471 \tag{33}$$

where E12 (A) and E12 (B) are the opinion estimates of E1 and E2 about systems A and B, respectively. As shown, since the probability of selecting system A is ≈35% and that of system B is ≈65%, the latter is selected as the right one, concluding that the theory of evidence has weakened the weak probabilities and strengthened the stronger ones in terms of the real system reliability. These values reveal that if the final obtained values are added together, they will equal 1, which has an important mathematical interpretation—the evidence has selected the system accurately and not partially, that is, the state assumed in the mass function equation has occurred.

3.1.2. Yager Theory

First, the weight of each witness is determined according to the value of Θ, and then, the above-mentioned multiplication matrix C is formed.

$$\theta_1 = 0.1 \ \rightarrow \alpha_1 = 0.9 \tag{34}$$

$$\theta_2 = 0.05 \ \rightarrow \alpha_2 = 0.95 \tag{35}$$

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = \begin{pmatrix} 0.1539 & 0.2309 & 0.0427 \\ 0.1881 & 0.2822 & 0.0522 \\ 0.018 & 0.027 & 0.005 \end{pmatrix} \tag{36}$$

The last row and column of the matrix (values in red) are the results of multiplying θ. Next, using the stated equations and calculating the values of the probability level function for each system, the consolidated results of the evidence are found as follows:

$$q(\varnothing) = c_{12} + c_{21} = 0.2309 + 0.1881 = 0.419 \rightarrow 1 - q(\varnothing) = 0.581 \tag{37}$$

$$E_{12}(A) = \frac{c_{11} + c_{13} + c_{31}}{1 - q(\varnothing)} = 0.3694 \tag{38}$$

$$E_{12}(B) = \frac{c_{22} + c_{23} + c_{32}}{1 - q(\varnothing)} = 0.6220 \tag{39}$$

As shown, since the probability of selecting system A is 36.94% and that of system B is 62.2%, like before, system B is selected as the right one according to the witnesses. In the Yager method, if the final obtained values are added together, they will not equal 1. This residual value, which is the probability of the null set, is called the ignore factor (IF), which is equal to 0.86% for the mentioned example, and its mathematical interpretation is that no guess can, with a probability of 0.86%, be made about the state of the system. This value has a direct relationship with the difference between the acceptability and trust functions, that is, the greater this difference is, the higher the waiver factor will be and vice versa.

$$IF = 1 - (0.3694 + 0.6220) = 0.0086 \tag{40}$$

3.1.3. Monte Carlo Algorithm

Now, the Monte Carlo algorithm is used to create a probability space based on the two experts' opinions, and to consider their uncertainties in calculating the probability of each system by generating a random sample; the results of the Monte Carlo algorithm, Dempster–Shafer theory, and Yager theory are listed in Table 3.

Table 3. Results of the MC algorithm, Dempster–Shafer theory, and Yager theory.

System (B)	System (A)	Method
0.6471	0.3529	Dempster–Shafer
0.622	0.3694	Yager
0.6374	0.3626	MC algorithm

As shown, since the probability of selecting system A is 36.26% and that of system B is 63.74%, like before, system B is selected as the right one according to the experts (Table 3).

3.1.4. Sensitivity Analysis

As stated before, the MCM is advantageous over other data fusion methods because it can be used in sensitivity analyses to estimate the influence and importance of each expert in the selected result. The results of the sensitivity analyses performed for the example are listed in Table 4.

Table 4. Results of the sensitivity analyses performed for the example.

Expert	System (A)	System (B)
E1	0.9482	1.3664
E2	0.0569	0.0694

As shown, in selecting system B as the superior one, the opinion of expert 1 is more important and more influential than that of expert 2 and should, hence, be considered in the next steps. To show the efficacy of the MC algorithm and its application, a real case study has been conducted as follows to select the best sewage treatment system in Tehran.

4. Case Study

Spreading on the southern slopes of the Alborz Mountain range, Tehran (51°23′20″ E, 35°41′21″ N), capital of Iran, with a population of 9,259,009 (most populous) and area of 730 km^2 (Figure 6) has been selected for this case study. It has a semi-arid climate with an average rainfall of 316 mm/year and an average annual temperature of 29 °C. In most years, half of its total annual precipitation is provided in winter, and summer is its least rainy season [44]. In Tehran, per capita drinking water consumption is 240 L/s, which is 70 L more than the national per capita (170 L/s). Here, 70% of the drinking water is supposed to be supplied from surface water and 30% from underground water, but a 64% decrease in rainfall has reduced the share of the former and increased that of the latter, which means using the water reserves of the future generation [45]. One of the efficient and practical ways to provide potable/non-potable water is to treat wastewater and reuse it in various industrial, agricultural, and domestic sectors to save water and reduce environmental pollution [46]. Due to the scarcity of water resources, treated wastewater is increasingly reused and is seen as a valuable resource that requires effective management with special care for human health, environmental protection, and water security. Due to limited technical and economic support and underdeveloped monitoring systems, many cities have inadequate wastewater treatment infrastructure. Therefore, the construction of suitable sewage treatment plants is very important because of its connection with the quality of surface and underground water [47]. According to the latest statistics published by the Water and Wastewater Engineering Company, Iran has a total of 237 treatment plants, of which 20, operating mainly under the activated sludge process (Figure 7), are in the Tehran province [48].

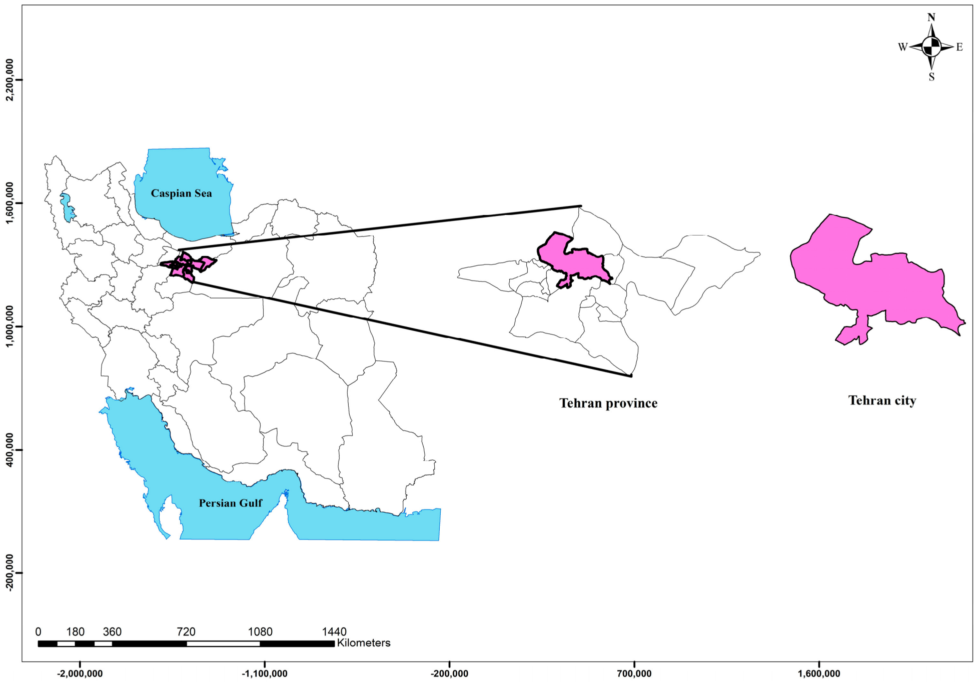

Figure 6. Case study area (Tehran).

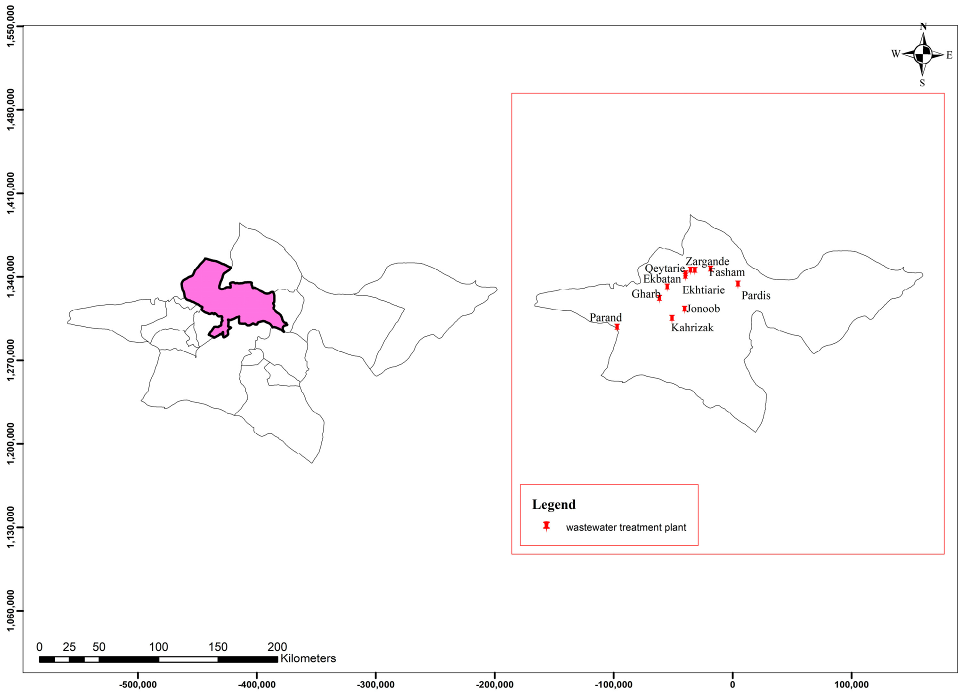

Figure 7. The location of the most important wastewater treatment plant in the case study area (Tehran).

5. Results

5.1. Problem Solving

The Dempster–Shafer and Yager theories are two important methods in combining experts' opinions and estimating probability. While the former theory is weak in considering uncertainties in the experts' opinions, the latter considers them in the data fusion process, but still has a problem; it works pairwise, that is, it can only fuse the opinions of two experts and cannot proceed when their number increases. To solve this problem and combine the uncertain opinions of several experts, this study has proposed an MC algorithm-based method in the standard normal space that not only considers the uncertainties in the experts' opinions, but also combines their related opinions, considering the mapping of the decision space to the probability space. Through sensitivity analyses, this method also examines the effects of the opinion of each expert on the final results.

5.2. Result of the Proposed Method for Experts with Uncertainty

The ever-increasing growth of the world's population and the significant progress of industry have enormously increased wastewater production in agriculture, industry, and domestic areas. The associated pollution, on the one hand, and the increased need for clean water, on the other hand, have made the construction of wastewater treatment plants a very serious environmental priority, with protection, planning, and management being quite important aspects when deciding on suitable treatment systems due to their high construction costs and their key role in providing services to citizens and protecting the environment; their incorrect selection will significantly increase the associated costs and prevent the achievement of the goals. The criteria for selecting a proper treatment system were determined considering existing research and the progress of the current research on selecting wastewater treatment plants, and then, the experts discussed them one by one based on the required objectives and principles. Five wastewater treatment systems were considered. (1) Conventional activated sludge (CAS). (2) Completely mixed activated sludge (CMAS). (3) Enhanced aeration activated sludge (EAAS). (4) Modified Ludzack–Ettinger (MLE); in the MLE method, a set of biological methods is used; these methods are a combination of aerobic and anaerobic treatment methods known as MLE activated sludge. (5) A2O; the A2O process is one of the methods used to treat industrial and sanitary wastewaters, which uses a combination of anaerobic, anaerobic, and aerobic conditions. This method, which is improved by AO, is used in order to provide conditions for the removal of phosphorus and nitrogen. These methods were considered based on (a) technical conditions (resistance against organic/hydraulic shocks, simple/continuous operation, upgradability, and BOD removal), (b) economic conditions (costs of construction, equipment, installation, maintenance, energy consumption, and sludge disposal), and (c) environmental conditions (smell release, required degree of treatment, worker safety, and sludge production). In the data fusion process, related experts were asked to rank the mentioned systems according to their importance. Table 5 shows their opinions about the systems proposed for Tehran; the last column shows the experts' degree of uncertainty about their opinion (Θ). For instance, 0.1 for expert 1 indicates that they are 10% unsure about their opinion; this is 5% for expert 2, 15% for expert 4, 10% for expert 5, 5% for expert 6, and 0% for expert 3 (they are quite confident and not uncertain about their opinion).

Table 5. Experts' opinions for the case study. (θ is the uncertainty they considered for their opinion).

Expert	CAS	CMAS	EAAS	MLE	A2O	θ
1	0.19	0.21	0.21	0.18	0.2	0.1
2	0.22	0.2	0.22	0.18	0.18	0.05
3	0.23	0.2	0.21	0.17	0.19	0
4	0.23	0.19	0.22	0.18	0.18	0.15
5	0.23	0.21	0.2	0.17	0.19	0.1
6	0.23	0.19	0.22	0.18	0.19	0.05

Experts' opinions were reviewed based on the fusion rules and fused in the proposed method; the decision-making fusion process is shown in Figure 8.

After the decision-making fusion process, systems should be prioritized based on the probabilities found from the MC algorithm and the one with the highest probability is ranked first. The probability (P) of selecting a system was found using Figure 8 and Table 6, where P(EAAS) > P(CAS) > P(A2O) and P(CMAS) > P(MLE). The EAAS, which is actually a modified process, with a priority of 34.48%, was ranked first and selected by experts as the most suitable system.

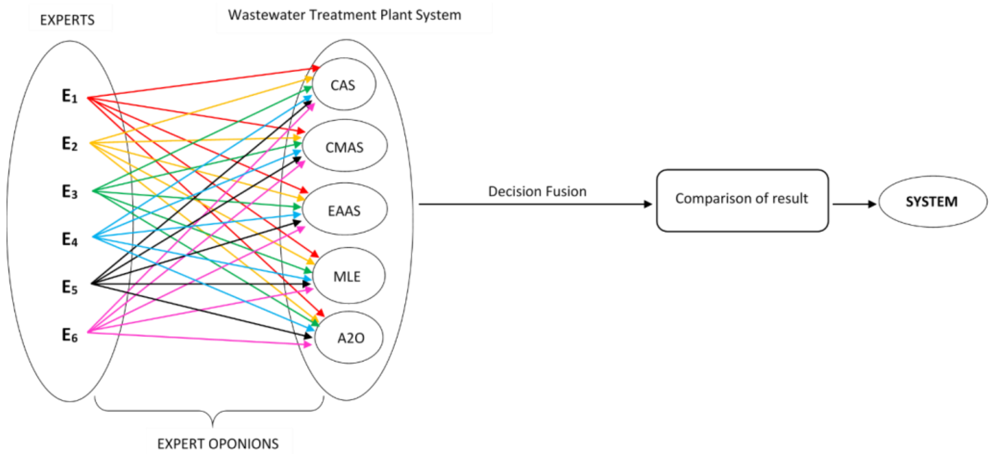

Figure 8. Decision steps to choose a wastewater treatment system for Tehran.

Table 6. Results of the decision fusion case study.

System	CAS	CMAS	EAAS	MLE	A2O
P(i)	0.2069	0.1724	0.3448	0.1034	0.1724
Rank	2	3	1	4	3

The Merits of the selected system, which show its acceptable efficiency in all places, are (1) removing up to 95% of the BOD5, (2) clarity of the treated effluent, (3) no odor during operation, (4) ease of operation and installation, (5) low operating costs, (6) high efficiency against organic-load wastewater, (7) high flow fluctuations due to high retention time, (8) usability in all areas/places, (9) low environmental effects/losses, (10) low sludge production due to long sludge life, and (11) nitrate removability in the process with no need for a primary clarifier. The adaptability of the selected system to the climate of the study area is another proof of its importance and appropriateness compared to other activated sludge treatment methods. Currently, the EAAS system is used in important wastewater treatment plants in Sahebqaranieh, Mahallati, Qaitrieh, Zargandeh, southern Tehran, and so on.

5.3. Results of Sensitivity Analyses

Decision-making generally involves not only probability estimates but also the importance of each variable and the associated impacts. In making decisions to select the best system among many, the most important expert can be identified based on their opinion and sensitivity analyses results, which can be found by evaluating the probability variation rate (∂Pf)/∂p, determined by the statistical features of each variable (p can be the mean or standard deviation (SD) of each variable). Table 7 shows the results of the sensitivity analyses (probability) estimated for each expert according to the average variations.

Table 7. Case study sensitivity analyses result.

Expert	$\frac{\partial P_f}{\partial \mu}$				
	CAS	CMAS	EAAS	MLE	A2O
Expert 1	0.1795	0.1944	0.2043	0.1717	0.1875
Expert 2	0.2341	0.2154	0.2332	0.1933	0.1955
Expert 3	0.5426	0.4711	0.4960	0.4093	0.4469
Expert 4	0.6189	0.5286	0.5903	0.5009	0.5054
Expert 5	0.4276	0.3901	0.3710	0.3219	0.3627
Expert 6	0.8485	0.7044	0.7787	0.6769	0.7236

As shown, in all five systems, since expert 6 is ranked first in terms of probability, their opinion should be considered in the final decision because changing it has the highest effect on selecting the priority of the wastewater treatment system for Tehran; experts 4, 3, 5, 2, and 1, respectively, stand next. As expert 1, with the least probability, is the least effective person in selecting the system, changing their opinion has the least effect on the presented results.

5.4. Monte Carlo Simulation Result with Random Uncertainty

In Section 5.2, the results, analysis, and system selection were performed based on the opinions of experts and the level of uncertainty they considered for their decision. But the amount of uncertainty that the expert raises may not be correct and they may have not been able to express the appropriate uncertainty for their decision correctly, which could affect the ranking of the systems, meaning the selection of the system would not have been performed correctly. To eliminate this doubt, according to the rules and statistical relationships between the mean, and standard deviation, the value of Θ is generated randomly. The range of variables considered for the standard deviation is between 0.01 and 0.09. Now, we recalculate the value of Θ for each expert based on different standard deviations and perform the data integration operation on the new basis, the results of which are shown in Figure 9.

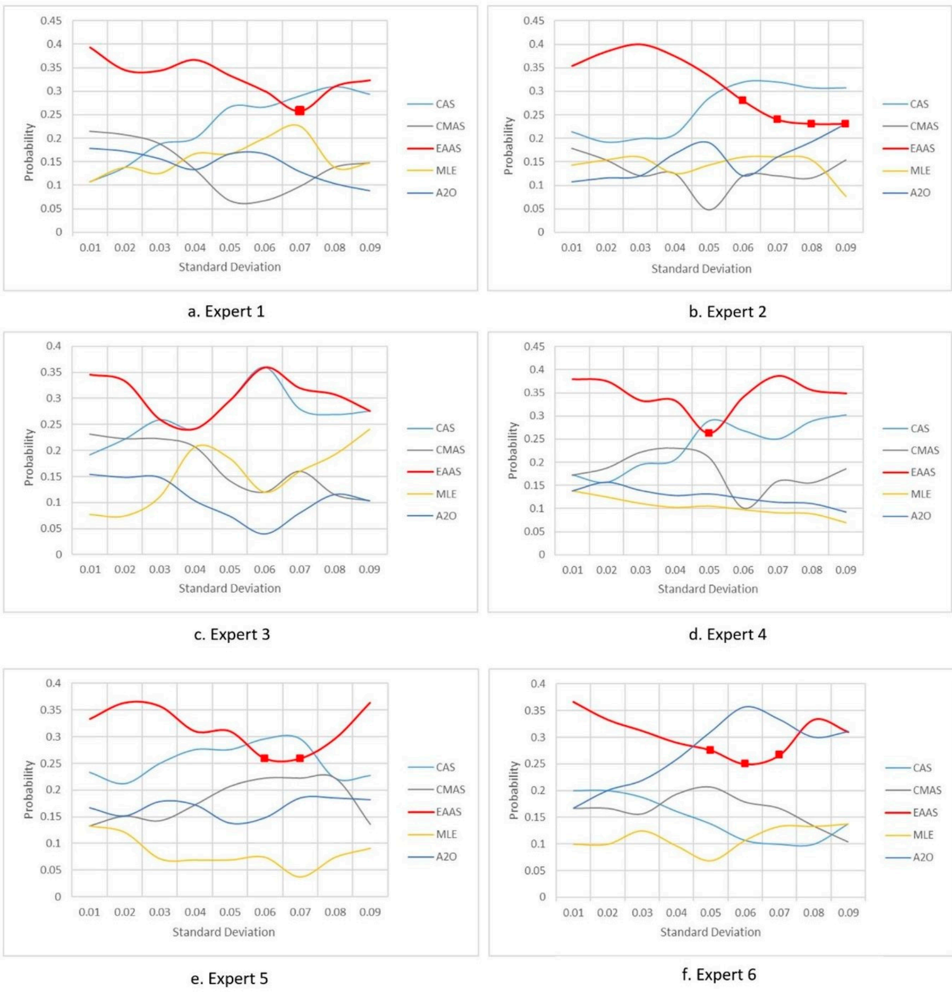

Figure 9. The effect of random Θ on system selection.

As can be seen in Figure 9, as the value of the standard deviation changes, the EAAS system is selected as the superior system in most cases. In Figure 9c, it can be seen that according to the change in the value of the standard deviation, the EAAS system was selected as the superior system in all cases, so it can be concluded that the third expert provided an estimate of the systems with great accuracy. But in the case of other experts, the priorities may have changed in some values, which is more the case for the second expert than other experts. As we know, if the standard deviation of a set of data is close to zero, it is a sign that the data are close to the mean and have little dispersion. While a large standard deviation indicates significant dispersion of the data. In all the figures, the change in priorities has occurred at a standard deviation above 0.05, that is, when we have moved away from the initial uncertainty and the uncertainty has increased more than the initial state. As is clear in Figure 8, these changes for expert 1 occurred at the standard deviation of 0.07, for expert 2 in the interval (0.06–0.09), for expert 4 at the standard deviation of 0.05, for expert 5 and expert 6 in the interval (0.06–0.07). To estimate the amount the probabilities changed in this part compared to the initial probability, based on which the EAAS system was selected as the superior system, the growth rate (GR) can be used. Looking at the calculated probabilities for the EAAS system in Figure 8 compared to the initial state, it can be concluded that the reduction in the estimated probability of expert 1 at SD = 0.07 is 25.14%, expert 2 at SD = (0.06, 0.07, 0.08, 0.09) corresponds to 18.79%, 30.39%, 33.06%, and 33.06%, respectively. For expert 4 at SD = 0.05 is 23.67%, expert 5 at SD = (0.06, 0.07) is 24.79% and 24.79%, respectively, and finally, for expert 6 at SD = (0.05, 0.06, 0.07) the reductions in the estimated probability are equal to 19.98%, 27.49%, and 22.65%, respectively. Table 8 shows the effect of a random Θ on the reduction in choice percentage for the EAAS system.

Table 8. Effect of random Θ on the reduction in choice percentage for EAAS system compared to the initial state (EAAS = 0.3448).

Expert	SD	EAAS (Random θ)	Reduction in Results (Percent)
1	0.07	0.2581	25.14
2	0.06	0.28	18.79
	0.07	0.24	30.39
	0.08	0.2308	33.06
	0.09	0.2308	33.06
4	0.05	0.2632	23.67
5	0.06	0.2593	24.79
	0.07	0.2593	24.79
6	0.05	0.2759	19.98
	0.06	0.25	27.49
	0.07	0.2667	22.65

5.5. Results of G-AHP

After forming the hierarchical model based on the opinions of the experts and according to Table 8, the weight of each criterion was calculated in relation to the goal and the weight of sub-criteria in relation to the corresponding criterion. Table 9 shows the final weights of the sub-criteria and their ranking.

As the results of Table 9 show, sub-criterion C21 (resistance to hydraulic shocks) is known as the most important sub-criterion, followed by resistance to organic shocks, reliability, simple operation, the cost of building and installing, upgradable, the cost of maintenance, remove BOD, the cost of energy consumption, odor production, reaching the degree of purification required, the cost of sludge disposal, worker safety, and sludge production rate.

By integrating the Monte Carlo algorithm with the Gray Analytic Hierarchy Process (G-AHP), managers can significantly increase the efficiency of their decision-making processes. This integration enables faster processing of large volumes of evidence and data, which is particularly valuable in industries that rely on timely and accurate decisions, such as wastewater treatment. By providing a more reliable and comprehensive decision-making framework, managers can also enhance their strategic planning and risk management. The ability to consider multiple criteria and uncertainties allows for more robust scenario planning and risk assessment. This leads to better preparation and more strategic resource allocation to mitigate potential risks.

Table 9. The final scores of the sub-criteria and their rank.

Criterion	Criterion	Criterion Name	Score	Rank
Economic (C1)	C11	The cost of building and installing	0.2672	5
	C12	The cost of maintenance	0.1415	7
	C13	The cost of energy consumption	0.0558	9
	C14	The cost of sludge disposal	0.0135	12
Technical (C2)	C21	Resistance to hydraulic shocks	0.6111	1
	C22	Resistance to organic shocks	0.4549	2
	C23	Reliability	0.2927	3
	C24	Simple operation	0.2769	4
	C25	Upgradable	0.11815	6
	C26	Remove BOD	0.0944	8
Environmental–social (C3)	C31	Odor production	0.0276	10
	C32	Reaching the degree of purification required	0.0151	11
	C33	Worker safety	0.0083	13
	C34	Sludge production rate	0.0027	14

6. Conclusions

Technological advances and the increasing complexity of systems and organizations have led managers to search for tools, devices, and processes that minimize risks and costs while optimizing system efficiency and identifying deficiencies. Competent managers equipped with the necessary knowledge are essential for effective decision-making in such contexts. Decision fusion enabled by the Monte Carlo algorithm (MC) proves to be a powerful tool for managers, planners, and system designers due to its versatility in various fields and industries. These include applications in non-destructive testing, robotics, geography, economics, system operating criteria determination, medical diagnostics, and more. This study presents a novel method based on the combined MC algorithm that aims to address the challenges caused by the Dempster–Shafer and Yager evidence theories. Validation of the proposed method included comparison with the Dempster–Shafer and Yager methods. The results showed the consistency of all three approaches, and thus, confirmed the validity of the new method. Consequently, decision fusion using MC simulation proves to be a promising approach to alleviate the limitations inherent in the Dempster–Shafer and Yager theories. Furthermore, the effectiveness of the proposed method in selecting a suitable wastewater treatment system for Tehran was demonstrated. University professors and experts who are working in the wastewater treatment plants are familiar with treatment issues took part in the evaluation and evaluated the systems according to technical, ecological, and economic criteria. The results showed that the enhanced aeration activated sludge (EAAS) system emerged as the most suitable option for Tehran due to its commendable performance and compatibility with the region, with a priority score of 34.48%. In addition, a sensitivity analysis of expert opinions was carried out to estimate their impact on the decision-making process. These analyzes revealed that expert 6 had the greatest influence on the selection of the EAAS system, while expert 1 had the least influence. In addition, the G-AHP gray hierarchical method was used to prioritize sub-criteria, identifying hydraulic shock resistance as the most important sub-criterion within the technical criteria. In summary, using the MC algorithm for decision fusion provides a more robust and effective approach to addressing the challenges associated with traditional evidence theories. The application of this approach, as demonstrated in the selection of a wastewater treatment system, highlights its potential to improve decision-making processes in various areas. the proposed method in this paper covers the defects of the Dempster–Shafer and Yager methods and can provide accurate analysis. But it has a limitation and that is in the amount of evidence, that is, considering that more opinions can be used in this method than the other two methods, but when the number of experts is too large, the accuracy of the method decreases, the authors suggest. To solve this problem, other methods of reliability and random data generation such as the form method or line sampling should be used for the problem and the results should be compared with the obtained results.

Author Contributions: Software, T.Z. and S.A.H.M.; Validation, M.R. and G.A.; Formal analysis, S.A.H.M.; Investigation, T.Z., S.A.H.M. and M.R.; Data curation, S.A.H.M.; Writing—original draft, T.Z. and S.A.H.M.; Supervision, S.A.H.M.; Project administration, S.A.H.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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