

Index S1: Theory

S1.1. Coupled hydromechanical model

In the following, the key elements of the hydromechanical modelling approach are briefly described and readers are referred to Beck et al. [32] for more details. In a comprehensive multi-phase flow system [e.g., 23,45], the mass balance equation for each phase is described by

$$\frac{\partial(\phi S_i Q_i)}{\partial t} + \nabla \cdot (Q_i \mathbf{v}_i) + W_i = 0 \quad (S1)$$

where ϕ [-] is porosity, t [T] is time, S_i [-], Q_i [ML⁻³], \mathbf{v}_i [LT⁻¹], and W [ML⁻³T⁻¹] are the effective saturation, density, velocity, and possible sink/source of phase i , respectively. In the case of variably saturated hillslopes, a two-phase flow system consisting of water (wetting phase) and air (non-wetting phase) is considered. Here, bold symbols indicate the tensorial nature of these parameters. In the case of laminar flow behaviour for infiltration and subsurface flow, \mathbf{v}_i can be defined by Darcy's law. The extended Darcy's law for multiphase flow e.g., [46] is:

$$\mathbf{v}_i = \mathbf{K}_i (S_i) \nabla H_i \quad (S2)$$

where H_i [L] is the total hydraulic head and \mathbf{K}_i [LT⁻¹] is the hydraulic conductivity tensor, which depends on the saturation of phase i , S_i [-]. In case of a two-phase flow system, the saturation of each phase is the fraction of the pore space occupied by that specific fluid phase, so that the sum of both phase saturations (air, S_a and water S_w) is equal to one, as:

$$\sum_i S_i = S_w + S_a = 1 \quad (S3)$$

In such a flow system, the pressure of the wetting phase (water), p_w [ML⁻¹T⁻²], and the non-wetting phase (air), p_a [ML⁻¹T⁻²], are related via the capillary pressure, p_c [ML⁻¹T⁻²] as:

$$p_c = p_a - p_w \quad (S4)$$

The unknowns of the balance equations (S3) and (S4) need to be reduced to two primary variables for two-phase flow systems. In this study, the pressure of the wetting phase, p_w , and the saturation of the non-wetting phase, S_a , were selected. In the case of a simplified single-phase (water) flow system, only one primary variable is required. In this study, the water pressure, p_w , was selected as the primary variable of the Richards' model. Following van Genuchten [47] model, the water retention curve is given as

$$S_w = \frac{\theta(p_c) - \theta_r}{\theta_s - \theta_r} \quad (S5)$$

in which

$$p_c = \begin{cases} \frac{Q_w g}{\alpha} [S_w^{-1/m} - 1]^{1/n} & S_w < 1 \\ 0 & S_w = 1 \end{cases} \quad (S6)$$

where θ_s [-] and θ_r [-] are the saturated and residual soil water content, respectively, Q_w [ML⁻³] is the density of water, g [LT⁻²] is the gravitational constant, and n [-], $m (=1 - \frac{1}{n})$ [-], and α [L⁻¹] are the so-called van Genuchten parameters that depend on the pore size distribution and the inverse of the air entry suction, respectively. The hydraulic conductivity for the water, K_w [LT⁻¹], and air phase, K_a [LT⁻¹], can be defined using the commonly used approach of van Genuchten [47] and Mualem [48] as:

$$K_w(S_w) = K_{sw} S_w^l \left[1 - \left(1 - S_w^{1/m} \right)^m \right]^2 \quad (S7)$$

$$K_a(S_a) = K_{sa} (1 - S_w)^l \left[1 - S_w^{1/m} \right]^{2m} \quad (S8)$$

where K_{sw} [LT^{-1}] and K_{sa} [LT^{-1}], are the saturated hydraulic conductivity for the water and air phase, respectively and l [-] is a tortuosity parameter that is commonly set to 0.5 [48].

In the hydromechanical model formulation used here, the soil is considered to be a linear-elastic medium. This implies that no plastic behaviour and deformation occurs due to the stress state. The stress distribution within a linear-elastic material is described by a momentum balance equation. After ignoring the inertia term due to the quasi-static conditions, the conservation of momentum can be written as e.g., [49,50]:

$$\nabla \cdot (\boldsymbol{\sigma}) + \gamma \mathbf{F}_v = 0 \quad (S9)$$

where $\boldsymbol{\sigma}$ [$ML^{-1}T^{-2}$] is the total stress tensor, γ [$ML^{-2}T^{-2}$] is the wet unit weight of the soil, and \mathbf{F}_v [-] is the body force vector. The generalized effective stress for variably saturated soils is given by Bishop [51] as:

$$\boldsymbol{\sigma}' = (\boldsymbol{\sigma} - p_a \mathbf{I}) + \chi (p_a - p_w) \mathbf{I} \quad (S10)$$

where $\boldsymbol{\sigma}'$ [$ML^{-1}T^{-2}$] is the effective stress tensor, χ is the Bishop's parameter or the effective stress parameter which is defined to be 1 for saturated and less than 1 for unsaturated soils, and \mathbf{I} is the unit vector. The second term on the right-hand side of equation (S10) represents the suction stress, σ^s [$ML^{-1}T^{-2}$], which is described by Lu and Likos [52] as:

$$\sigma^s = -S_w (p_a - p_w) \quad (S11)$$

Accordingly, the effective stress can be described in terms of effective pressure, p_{eff} [$ML^{-1}T^{-2}$], as:

$$\boldsymbol{\sigma}' = (\boldsymbol{\sigma} - p_{eff} \mathbf{I}) \quad (S12)$$

while

$$p_{eff} = S_w p_w + S_a p_a \quad (S13)$$

Once the effective stress term is defined, the resulting elastic deformation of the porous media can be obtained from the elastic moduli [53]. For an isotropic linear elastic material, only two elastic moduli are needed to define the behaviour of the material: Young's modulus, E [$ML^{-1}T^{-2}$], and Poisson's ratio, ν [-], in which

$$E = \frac{\sigma_x}{\epsilon_x} \quad (S14)$$

and

$$\nu = -\frac{\epsilon_x}{\epsilon_y} \quad (S15)$$

with σ_x [$ML^{-1}T^{-2}$], the normal stress along the x-axis of the coordinate system, and ϵ_x [-] and ϵ_y [-], the strains along the x and y axis, respectively. The strain tensor, $\boldsymbol{\epsilon}_v$ [-], can be defined based on the displacement tensor, \mathbf{u} [L] as:

$$\boldsymbol{\epsilon}_v = \frac{1}{2} (\text{grad } \mathbf{u} + \text{grad}^T \mathbf{u}) \quad (S16)$$

Displacement, u , is considered to be the primary variable in the momentum balance equation for the mechanical problem. If it is assumed that the solid matrix is rigid, the changes in bulk volume due to deformation will affect the pore volume. In order to consider this, an effective porosity, φ_{eff} [-], is defined as:

$$\varphi_{eff} = \frac{\varphi_0 - \epsilon_v}{1 - \epsilon_v} \approx \varphi_0 - \epsilon_v \quad (S17)$$

where φ_0 [-] is the initial porosity. Moreover, the volumetric strain can be replaced by its equivalent, $\text{div } \mathbf{u}$ [-]:

$$\varphi_{\text{eff}} = \left(\frac{\varphi_0 - \text{div } \mathbf{u}}{1 - \text{div } \mathbf{u}} \right) \quad (\text{S18})$$

The change in effective porosity will induce changes in hydraulic conductivity. Different empirical equations have been proposed to describe the relationship between the porosity and the hydraulic conductivity e.g., [54,55]. In this study, the modified version of the empirical relationship of Davies and Davies [56] proposed by Rutqvist and Tsang [57] is used:

$$K_{\text{eff}} = K_0 \exp(22.2 \left(\frac{\varphi_{\text{eff}}}{\varphi_0} - 1 \right)) \quad (\text{S19})$$

where K_0 [LT^{-1}] is the initial hydraulic conductivity, and K_{eff} [LT^{-1}] is the effective hydraulic conductivity for both phases as a result of a change in porosity. It is important to note that this empirical equation provides approximate estimate of permeability, and is initially developed for rock material and therefore should be used with caution.

S1.2. Coupling strategies

Various strategies exist for solving the set of coupled equations defined previously (equations S1-S19). In a so-called fully coupled approach, the unknowns of the flow equation (here, p_w and S_a) are solved simultaneously with those of the mechanical problem (here, \mathbf{u}) in each time step [34] (Figure S1a). This approach enables a comprehensive interaction between variable hydraulic and mechanical parameters, offering stability and accuracy e.g., [37]. However, it often comes with higher computational costs, particularly challenging for complex model domains requiring high spatial and temporal resolution. A more commonly used strategy is the sequentially coupled approach, which breaks the coupled problem into sub-problems with varying levels of interfaces. These flexible coupling interfaces may result in different levels of accuracy and computational efficiency [27]. In this approach, sub-problems are solved sequentially. First, the hydrological problem is solved for each time step (Figure S1b and c). Then, the water pressure distribution is used to solve the mechanical problem's momentum balance equation [32,58], constituting one coupling step. This approach does not consider the transient impact of mechanical processes on hydrological properties within the same time step. To address this limitation, it's possible to iterate between the hydrological and mechanical models within a single time step. The main advantage of the sequentially coupled model is its flexibility in using different simulators with potentially varying spatial and temporal resolutions to solve the sub-problems [59].

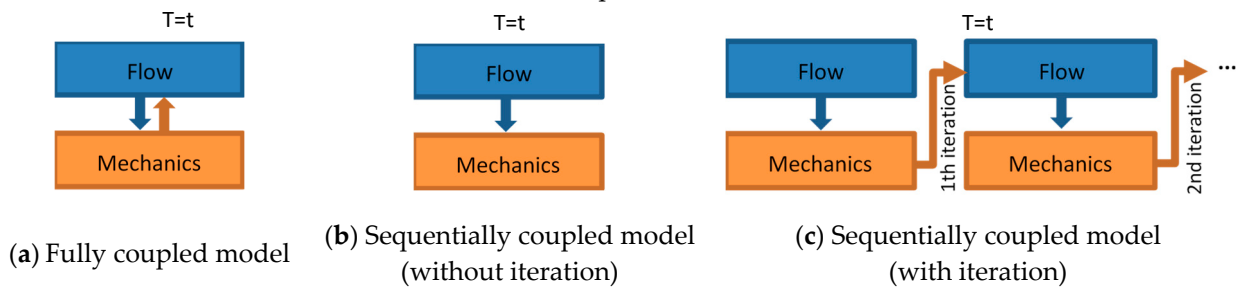


Figure S1. Illustration of different coupling strategies and the considered interactions between sub-problems: (a) a fully coupled model, (b) a sequentially coupled without iterations, and (c) a sequentially coupled model with iterations within each time step.

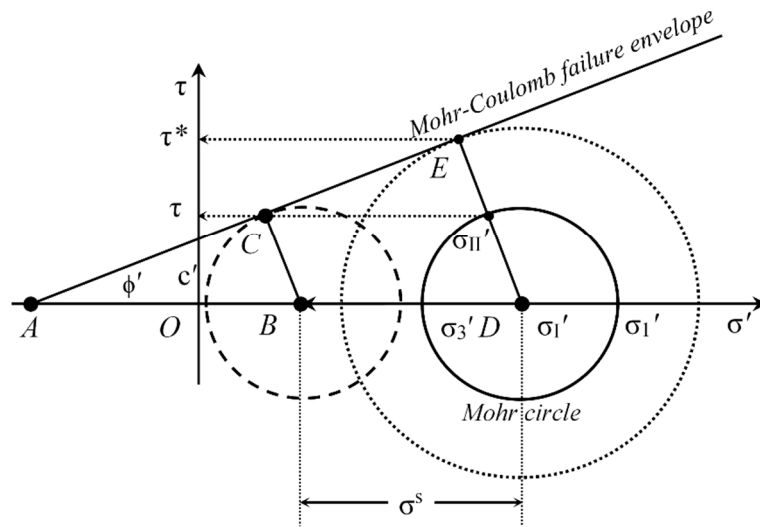


Figure S2. Illustration of the local factor of safety (LFS) concept using the Mohr circle (adapted from Lu et al. [7]).

Index S2: Hydraulic parameters

Table S1. Hydraulic and mechanical parameters of the simulated slope (based on Lu et al. [7]).

| Symbol | Parameter name | Unit | Value |
|----------------|---|-------------------------------|---------|
| θ_s | Saturated water content | $\text{cm}^3 \text{ cm}^{-3}$ | 0.46 |
| θ_r | Residual water content | $\text{cm}^3 \text{ cm}^{-3}$ | 0.034 |
| K_{sw} | Saturated hydraulic conductivity of water phase | m s^{-1} | 1.39E-6 |
| K_{sa} | Saturated hydraulic conductivity of air phase | m s^{-1} | 1.0E-12 |
| α | van Genuchten fitting parameter | m^{-1} | 1.6 |
| n | van Genuchten fitting parameter | -* | 1.37 |
| γ_{dry} | Dry unit weight | kN m^{-3} | 20 |
| E | Young's modulus | MPa | 10 |
| ν | Poisson's ratio | -* | 0.33 |
| ϕ' | Friction angle | $^\circ$ | 30 |
| c' | Effective cohesion | kPa | 15 |

*dimensionless