

Article

A Hybrid Fuzzy Evaluation Method for Quantitative Risk Classification of Barrier Lakes Based on an AHP Method Extended by D Numbers

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Abstract: The risk classification of barrier lakes is the key to conducting emergency treatment in a scientific manner. However, risk classification faces difficulties such as a short time for risk evaluation, complex evaluation indicators, difficulty in obtaining information quickly, and quantifying index weights. Based on this, this paper constructs a quantitative risk classification model for barrier lakes based on D-AHP. On the basis of studies on nearly 100 cases of barrier lakes, an eight-factor evaluation index system and quantitative classification are proposed. The methods of rapid calculation of reservoir capacity curve of barrier lakes and intelligent identification of particles on the surface of barrier bodies were developed, which realized the rapid acquisition of eight-factor evaluation index information in an emergency environment. The D-AHP method dealt with inconsistent weight assignment to evaluation factors by experts, which helped achieve weight quantification of eight factors. The risk assessment on 15 barrier lakes such as Tangjiashan barrier lake shows that the conclusions drawn for the risk classification method proposed in this paper are basically consistent with those of the traditional table-lookup method. However, the table-lookup method ignores cumulative loss impacts on the risk level of barrier lakes and considers the extremely severe loss of barrier lakes as a sufficient condition for the evaluation level to be grade I, and thus a deviation in the evaluation. The risk classification method proposed in this paper is more reasonable and reliable.

Keywords: barrier lake; risk classification; weight; D-AHP; short window; information acquisition



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1. Theoretical Background

A barrier lake is a natural damming of a river by collapse or slide of the bank slope of the river valley, which happens worldwide [1] (Figure 1). Such dammed lakes flood upstream areas and damage the critical infrastructures as well as lives and properties of residents downstream in cases of collapse. In 2000, a large landslide occurring along Yiong Tsangpo River created a barrier lake 54 m in length, 2500 m in width, $288 \times 10^8 \text{ m}^3$ in storage capacity, and $2.8 \times 10^8 \sim 3.0 \times 10^8 \text{ m}^3$ in volume, endangering several million residents downstream [2,3]. The Wenchuan Earthquake in 2008 (Figure 2a) induced 257 barrier lakes, of which the largest was Tangjiashan barrier lake, threatening 1.3 million residents downstream. As a result, 275,000 residents were relocated [4,5]. Another case was the barrier lake dammed by a landslide on Bailong River caused by the sudden extraordinary rainstorm in the northern hilly area of Zhouqu County, Gansu Province, in 2010, killing 1799 people. The landslide (Figure 2b) in Baige village upstream of the Jinsha River in 2018 shaped a large barrier lake with a maximum reservoir volume of 760 million m^3 and resulted in the relocation of 85,000 residents and direct economic loss of CNY 13.5 billion [6–8].

Studies on 73 barrier lakes by Costa and Schuser from the United States Geological Survey (USGS) demonstrated that 85% of the lakes lasted less than 1 year. A case study

was done on 352 barrier lakes which have collapsed, with statistics showing 84.4% of the lakes existed less than 1 year, 68.2% of them with lifespans shorter than 1 month and 29.8% collapsing within one day. Peng and Zhang [9] and Shen et al. [10] also found similar statistical results based on 204 and 352 cases, respectively. Given such diversity in the duration of existence of barrier lakes and short time for emergency relief, scientific risk classification is critical for targeted and well-organized emergency response. Risk Management-Risk Assessment Techniques [11] defines risk as uncertainties confronted by an organization in accomplishing its targets. Identification of risks indicates evaluation of risk types, probability, and impact. Generally, the risk can be expressed as the risk probability (P) of an incident multiplied by the consequences (C) of such an incident, based on which the risks of barrier lakes can be expressed as $R = PC$, where R indicates risks of barrier lakes, P indicates the probability of dam failure, and C indicates the consequences of dam failure. P is mainly about the hazards of the barrier lakes; the larger the hazards, the higher the probability of dam failure. C mainly includes the damages caused by the barrier lake as well as floods due to dam failure.



Figure 1. Worldwide distribution of barrier lakes (incomplete statistics).



Figure 2. (a) Tangjiashan barrier lake; (b) Baige barrier lake.

The stability of a barrier dam is subject to several factors. Based on a database of 70 barrier dam cases collected in the northern Apennines, Casagli et al. [12] proposed the blockage index (BI) to evaluate barrier dam stability with barrier dam volume V_d and catchment area A_L as input parameters. Based on the BI methodology, Ermini et al. [13] further introduced barrier dam height H_d into the BI method and proposed a new dimensionless blockage index (DBI). Based on 43 barrier dam cases in Japan, Dong et al. [14] proposed a quantita-

tive risk assessment method for barrier dams with inflow water volume from upstream and barrier dam height, width, and length selected as input parameters. Subsequently, based on 300 cases in Italy, Stefanelli et al. [15] identified the hydromorphological dam stability index (HDSI), with barrier dam volume and catchment area as input parameters. Based on 79 barrier dam occurrences, Shi et al. [16,17] proposed a quantitative method to evaluate barrier dam stability in which five parameters were adopted as input parameters: barrier dam height, width, and length; dammed lake volume; and backwater length. All the parameters selected by the above-mentioned scholars fall into two categories, i.e., lake volume relevant and barrier dam relevant, as shown in Table 1.

Table 1. Hazard assessment parameters for barrier dams worldwide.

List of Scholars	No. of Samples	Lake Volume Relevant Parameters					Barrier Dam Relevant Parameters				
		A_L	V_L	L_L	Q	V_d	H_d	W_d	L_d	S_d	I
Casagli et al. [12].	70	Yes				Yes					
Ermini et al. [13].	84	Yes				Yes	Yes				
Dong et al. [14].	43				Yes	Yes	Yes	Yes	Yes		
Stefanelli et al. [15].	300	Yes				Yes					
Shan et al. [1].	115	Yes				Yes				Yes	Yes
Shi et al. [16,17].	79		Yes	Yes			Yes	Yes	Yes		

Breaches of barrier lake dams bring a long disaster chain and induce disasters of a large scope. Studies show that the loss caused by failure of landslide barriers is similar to that of regular dams, including loss of lives, economy, and ecology. Assessments of life loss take into account such factors as the population at risk, population density, level of flooding, understanding of residents, timing of alarm, rate of young adults to the elderly and kids, time of dam failure, weather, distance to dam site, emergency response plan, dam height, reservoir volume, downstream river slope, topography, impact resistance of structures, temperature, rescue capability, etc. [3,18,19]. Major factors for economic loss include duration of floods, velocity of floods, sediment concentration, flood water temperature, depreciation of properties, timing of alarm, pollutant concentration, etc. [20–23]. Ecological loss is mainly assessed based on factors including geomorphology of the river channel; water environment; human ecology; natural reserves; damage to animal species; soil environment; vegetation coverage; reduction in agricultural, forestry, and fishery production; air quality; and dirty industries [24–26]. The factors for assessment of barrier dam failure are demonstrated in Table 2.

Table 2. The factors for assessment of barrier dam failure.

Authors	Type of Loss	Factors
Zhou et al. [18]; Wu et al. [19]	Life loss	Population at risk, population density, level of flooding, understanding of residents, timing of alarm, rate of young adults to the elderly and kids, time of dam failure, weather, distance to dam site, emergency response plan, dam height, reservoir volume, downstream river slope, topography, impact resistance of structures, temperature, rescue capability
Xiao et al. [20]; Yang [21]; Wang et al. [22]; Liu et al. [23]	Economic loss	Duration of floods, velocity of floods, sediment concentration, flood water temperature, depreciation of properties, timing of alarm, pollutant concentration
Wang et al. [24]; Li et al. [25]; Wu et al. [26]	Ecological loss	Geomorphology of river channel; water environment; human ecology; natural reserves; damage to animal species; soil environment; vegetation coverage; reduction in agricultural, forestry, and fishery production; air quality; dirty industries

Proper allocation of weight to each factor is the key to risk assessments of barrier lakes [27–29]. AHP is a multi-criteria decision-making method combining qualitative and quantitative analyses while remaining simple and practical. However, there are still

some deficiencies and limitations when applying this methodology. First, the comparative judgments are subjective because they rely heavily on expert opinion, which may sometimes cause inconsistency. Furthermore, AHP lacks the ability to adequately cope with any inherent uncertainty and imprecision in the data. Finally, in a real situation, an expert may have limited knowledge of and experience with alternatives; the preferred information may contain fuzziness and incompleteness, and AHP is unable to handle this incomplete information. In the actual risk classification of barrier lakes, the following situations often occur. Occasion 1: All 10 experts consider factor 1 more important than factor 2. Eight of them assigned a weight score of 0.8 to factor 1. However, the other two experts assign a weight score of 0.7 to factor 1. Occasion 2: Seven experts out of ten consider factor 1 more important than factor 2 and allocate a weight score of 0.6 to factor 1. However, the other three regard both factors as equally important. Occasion 3: Eight experts out of ten consider factor 1 more important than factor 2 and allocate a weight score of 0.7 to factor 1. However, the other two give no comment on either factor since they do not have a deep understanding of them. The D-AHP method can represent uncertain information more effectively because it overcomes the shortcomings and deficiencies of the traditional AHP and Dempster–Shafer theories. First, the D-AHP method uses a D numbers preference relation instead of a pairwise comparison; the D numbers preference relation is the classical fuzzy preference relation extended by D numbers. Although the preference relations of the alternatives or criteria given by the experts are imprecise, fuzzy, and incomplete, the D numbers preference relation can effectively express this uncertain information without causing inconsistency. Furthermore, the sum of all focal elements in a D numbers preference relation need not equal 1; i.e., if the assessment information given by experts is incomplete, this value may be less than 1. In view of these advantages of D-AHP, this paper uses the D-AHP method instead of the traditional AHP method to determine the weight distribution of the evaluation index [30–32].

Based on previous studies globally on the risk assessment of barrier lakes, in Section 2, a mathematical model is proposed for quantitative risk classification of barrier lakes based on D-AHP with a set of risk evaluation factors, a quick information acquisition method, and risk classification quantification functions. Section 3 presents the set of risk evaluation factors as well as standards for classification based on studies on about 100 barrier lake cases and domestic studies in China, solving the problem of complex evaluation indicators on the risk evaluation of barrier lakes. The proposed set of factors identifies proper ones amid a huge pool of factors and has been included in the Code for Risk Classification and Emergency Measures of Barrier Lake [33]. In Section 4, an elaboration is presented on the calculation method of the storage capacity curve and intelligent identification of particles on the surface of the dam. Such a method would allow for quick acquisition of information during an emergency rescue. A topographic database was constructed through data overlay and dynamic checking based on information obtained through topographic mapping, IEM modeling, multispectral data, RADAR data, and 3D topographic mapping, and the storage capacity curve of a barrier lake can be calculated based on this topographic database. A qualitative analysis was carried out on material components of barriers through the provenance methodology. Surface particles in a barrier dam were identified through the intelligent identification methodology. The diameters of particles in a barrier dam were calculated based on longitudinal profile data through a natural source surface wave. A grading curve for particles in a barrier dam was produced based on the above-mentioned data. Sections 5 and 6 elaborate in details on the preference relation matrix and the calculation of a weight indicator based on D-AHP, solving the problem of quantifying weights for evaluation factors. In Section 7, the application of such a method on 15 barrier lake cases demonstrates that the proposed method is more scientific and reliable and shall be promoted for further application.

2. Mathematical Model for Quantitative Risk Classification of Barrier Lakes Based on D-AHP

2.1. The Mathematical Model

The mathematical model for quantitative risks classification of barrier lakes based on D-AHP is shown in Figure 3. The model comprises three parts:

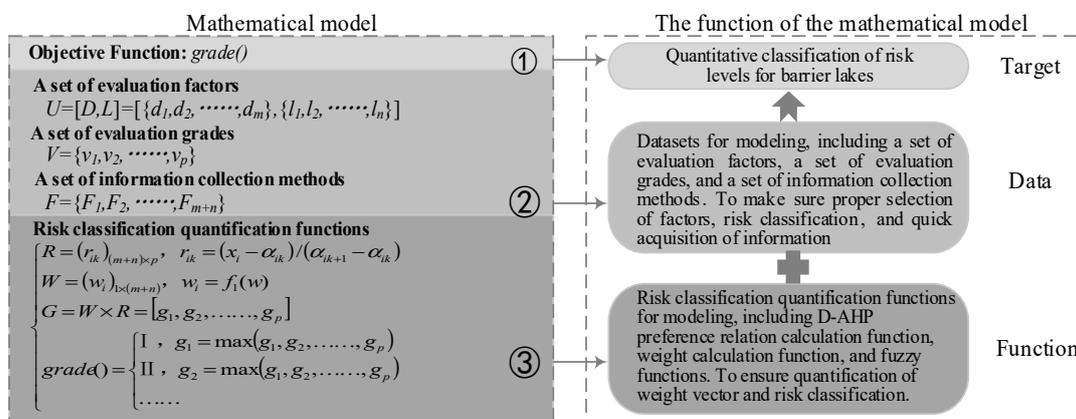


Figure 3. Mathematical model for quantitative risks classification of barrier lake based on D-AHP.

- (1) Objective function;
- (2) Datasets for modeling, including a set of evaluation factors, a set of evaluation grades, and a set of information collection methods ensuring proper selection of factors, risk classification, and quick acquisition of information.
- (3) Risk classification quantification functions for modeling, including D-AHP preference relation calculation function, weight calculation function, and fuzzy functions, ensuring quantification of the weight vector and risk classification.

Explanation of symbols in the mathematical model in Figure 3:

1. U is the set of factors for risk classification of barrier lakes; D is the set of factors for hazards evaluation of the barrier with m elements, namely, d_1, d_2, \dots, d_m ; and L is the set of factors for the assessment of loss by dam failure with n elements, namely, l_1, l_2, \dots, l_n .
2. V is the set of evaluation grades with P elements, namely, v_1, v_2, \dots, v_p .
3. F is the set of information collection methods, with $m+n$ elements, F_1, F_2, \dots, F_{m+n} , which is in compliance with the m elements in set D and information collection methods for number of n in set L .
4. R is the reference matrix of set U to set V . r_{ik} indicates the preference relation of the i -th parameter to the k -th evaluation grade; the range $[\alpha_{ik}, \alpha_{ik} + 1]$ means the range corresponding to the k -th evaluation grade; x_i is the i -th parameter. W is the weight vector of U for the set of factors for the risk classification of barrier lakes. $f_1()$ is the weight calculation function on the basis of D-AHP; G indicates the vector of evaluation grades, with p elements, g_1, g_2, \dots, g_p ; and $max()$ is the function for maximum value.

2.2. Solution Using the Model

The solution using the model follows five procedures (see Figure 4) as follows:

- (1) To establish set U , set D was the factors for risk classification, and set L was the factors for loss assessment. (2) Set V was built on evaluation grades to identify risk levels (grade I, II, III, etc.) of barrier lakes. (3) The fuzzy preference relation matrix R was established. Elements in the matrix indicate the preference relation of a certain parameter to its risk grade. (4) The weight vector W was calculated. The calculation of the weight vector on evaluation parameters was based on the following procedure: ① Establishment of the D numbers preference relation matrix (D Matrix); ② conversion of the D matrix into a crisp matrix; ③ establishment of a probability matrix based on the crisp matrix; ④ ranking of

the parameters with the triangle method; and ⑤ calculation of the weight vectors of the parameters. (5) The risk grade evaluation function *grade()* was calculated based on *R*, the fuzzy preference relation matrix, and weight vector *W*, thus resulting in the risk grade of barrier lakes.

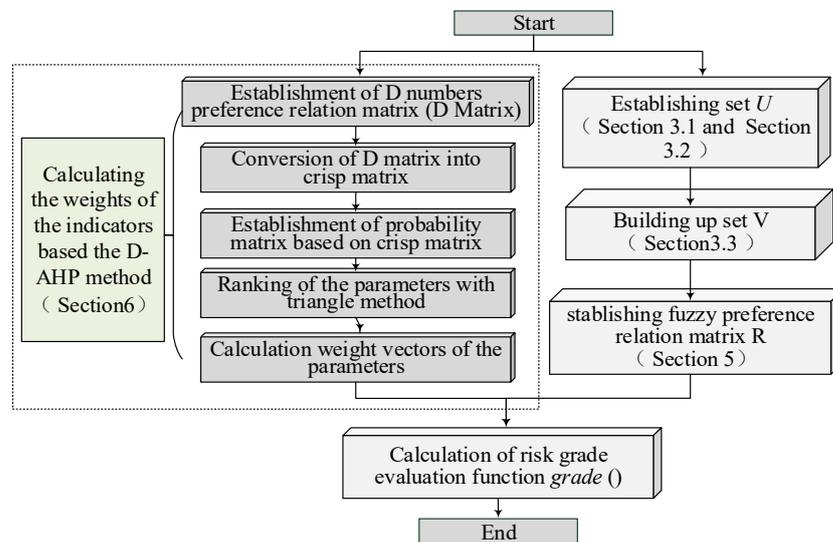


Figure 4. Procedure for barrier lake risk grading through mathematical model.

3. Selection and Grading of Risk Evaluation Factors

3.1. Selection and Grading of Risk Evaluation Factors for Barrier Dams

In accordance with the studies worldwide, the risk evaluation factors can be categorized into two groups: reservoir-volume-related factors and dam-body-related factors (see Table 1). The former group mainly includes the reservoir volume and inflow from upstream, both of which directly influence the damage to the barrier body by floods in case of breaching. The latter group mainly comprises the material component and geometry of the dam body. The larger the particles in the dam, the better it works against flushing; the lower the dam, the longer distance the water flows, the smaller potential energy the water flow takes, and the less risk. These four factors, reservoir capacity, inflow from upstream, material component, and geometry of the barrier, are reasonable and feasible since they cover most risk evaluation factors adopted in current studies.

$$D = [d_1, d_2, d_3, d_4] = [\text{reservoir capacity, inflow from upstream, material component, and geometry of the barrier}] \quad (1)$$

This paper analyzes the relation between such factors and the grading of barrier risks based on studies of about 100 barrier lake cases.

3.1.1. Relation Between Reservoir Volume (d_1) and Risk Grades

The paper studies the relation between reservoir volume and peak flooding upon dam failure based on the statistics of 86 cases showing a linear relation in which the larger the reservoir volume, the higher the flood peak upon dam failure, the more destructive the flood to the barrier, and the higher the risks of the barrier (see Figure 5). When the reservoir volume is smaller than 1 million m^3 , the peak flow at the breach is normally less than 1000 m^3/s . When the reservoir volume is 1 million–10 million m^3 , the peak flow at the breach is normally 1000–3000 m^3/s . When the reservoir volume is 10 million–100 million m^3 , the peak flow at the breach is normally 3000–10,000 m^3/s . When the reservoir volume is large than 100 million m^3 , the peak flow at the breach is normally larger than 10,000 m^3/s . Therefore, barrier lakes and their risk grades can be categorized into four groups regarding their reservoir volume: less than 1 million m^3/s (low risk), 1 million–10 million m^3/s (moderate risk), 10 million–100 million m^3/s (high risk), and more than 100 million m^3/s (extra high risk).

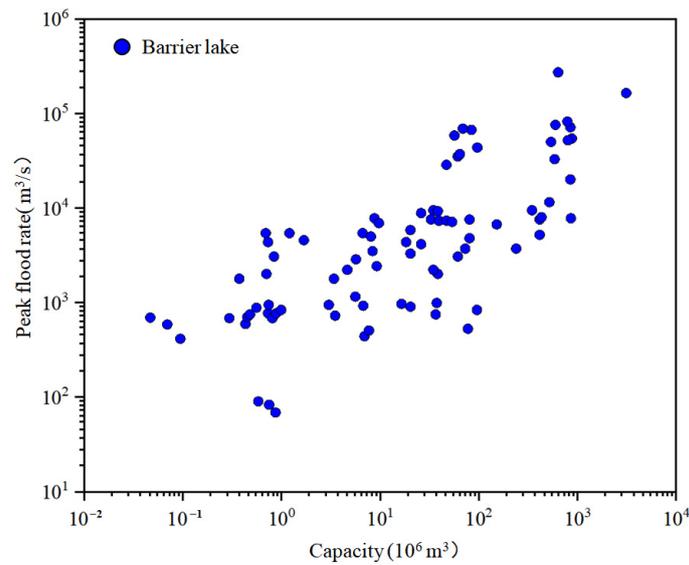


Figure 5. Relation between lake capacity and peak flood upon collapse.

3.1.2. Relation Between Inflow from Upstream (d_2) and Risk Grades

This paper studies the relation between reservoir volume and peak flood upon dam failure based on statistics of 86 cases (see Figure 6). Generally, the inflow from upstream can be categorized into four ranges, and the risks of the barrier are categorized into four corresponding grades: (1) 17 cases had an upstream inflow of less than $10 \text{ m}^3/\text{s}$, 12 of which were evaluated as low risk with a risk probability of 70.5%; (2) 17 cases had an upstream inflow of $10\text{--}50 \text{ m}^3/\text{s}$, 11 of which were evaluated as medium risk with a risk probability of 62.5%; (3) 15 cases had an upstream inflow of $50\text{--}150 \text{ m}^3/\text{s}$, 10 of which were evaluated as high risk with a risk probability of 64.7%; and (4) 23 cases had an upstream inflow of more than $150 \text{ m}^3/\text{s}$, 15 of which were evaluated as extra high risk with a risk probability of 65.2%.

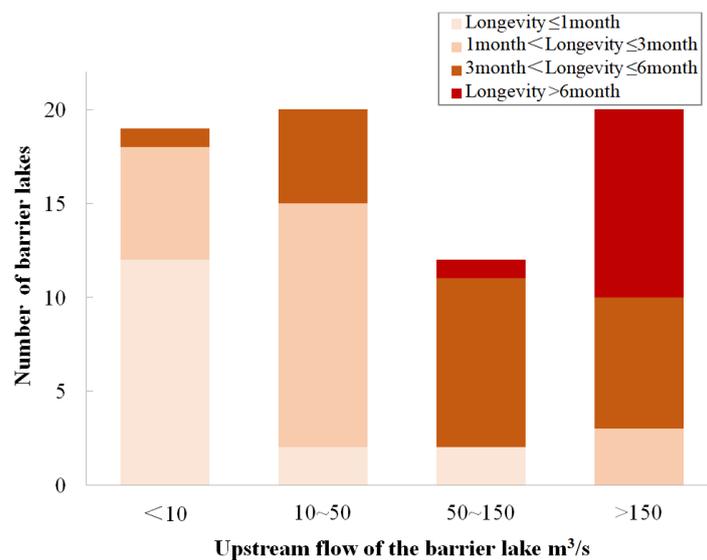


Figure 6. Relation between inflow from upstream and duration of barrier lake.

3.1.3. Relation Between Material Components of Barrier (d_3) and Risk Grades

Studies on the breach and dam failure process show the following: (1) When the discharge is less than $10 \text{ m}^3/\text{s}$, the velocity is less than $1\text{--}2 \text{ m/s}$, and the flow is able to flush clay particles and sand. (2) When the discharge is $10\text{--}50 \text{ m}^3/\text{s}$, the velocity is about $1\text{--}2 \text{ m/s}$, and the water flow is able to flush rubble. (3) When the discharge is $50\text{--}150 \text{ m}^3/\text{s}$,

the velocity is about 2–3 m/s, and the flow is able to flush gravel. (4) When the discharge is 150–1000 m³/s, the velocity is about 3–6 m/s, and the flow is able to flush stones of a small size. (5) When the discharge is larger than 1000 m³/s, the velocity is about 6–10 m/s, and the flow is able to flush stones of all sizes. A study by Wang et al. [34] from the Institute of Mountain Hazards and Environment, Chinese Academy of Science, reveals that the average grain size of the barrier is explicitly influential on the features of dam failure. The larger the average grain size, the more capable the barrier is against flood flushing, and the smaller the probability of risk. Based on the above-mentioned studies and classification according to the Code for investigation of Geotechnical Engineering [35], the mid-value (d_{50}) of the grain distribution curve of the barrier was selected as the eigenvalue to judge the barrier’s capability to resist flushing. The risk probability was graded as extra high, high, medium, and low when the eigenvalue was less than 2 mm, 2–20 mm, 20–200 mm, and higher than 200 mm, respectively.

3.1.4. Relation Between Geometry of Barrier (d_4) and Risk Grades

We collected data on the length/height ratio (L/H) of 54 landslide barriers and studied its relation to the duration of barrier lakes (see Figure 7), showing the following: (1) When $L/H \leq 5$, the barrier collapses after several days of overtopping. (2) When $5 < L/H < 20$, the barrier lasts until it has been flushed for tens of days to several months. (3) When $L/H \geq 20$, the barrier lasts for more than 1 year. Furthermore, the height of the barrier decides the potential energy of the water flushing. With reference to Design Code for Rolled Earth-rock Fill Dams [36], barriers can be categorized into four groups in terms of their height: less than 15 m, 15–30 m, 30–70 m, and more than 70 m. In consideration of the two factors above, the relation between geometry and risk grading of barriers can be seen in Figure 8.

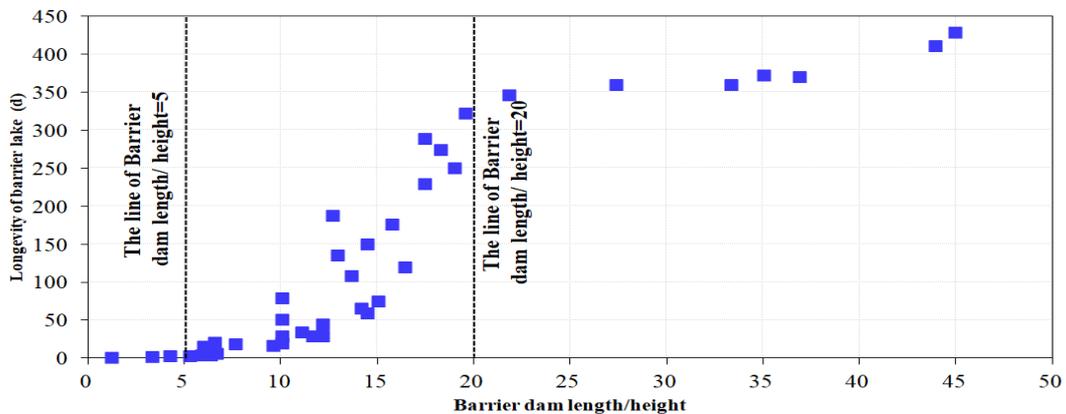


Figure 7. Relation between L/H and the duration of barrier lakes.

	$L/H \leq 5$	$5 < L/H < 20$	$L/H > 20$
$H \leq 15$	Low	Low	Moderate
$15 \leq H < 30$	Low	Moderate	High
$30 \leq H < 70$	Moderate	High	Extra high
$H \geq 70$	High	Extra high	Extra high

Figure 8. Geometry of barriers and their risk grading.

3.2. Selection and Grading of Loss Evaluation Factors for Barrier Dams

Previous studies show that there are three main factors for loss evaluation due to dam failure: life loss, economic loss, and ecological loss (see Table 2). Life loss mainly indicates the population under barrier risks [37]. For example, Tangjiashan barrier lake threatens

a population of 1.3 million downstream [5]. Economic loss mainly includes the loss of cities and towns downstream and loss of public facilities and infrastructures. For example, due to the collapse of the Baige barrier lake, 16 downstream villages and towns in four counties of Diqing and Lijiang were lost, 8051 houses collapsed, and there was damage to 18189 rooms. Loss in public facilities and infrastructures included damage to 632.12 km of road, 13 bridges flushed and destroyed, and 13 bridges damaged [6]. Ecological loss mainly included loss of ecological diversity, human ecology, the river channel, and the water environment [24]. Based on previous studies and a case study of about 100 barriers, evaluation factors for loss due to dam failure and grading methods (Table 3) were decided.

$$L = [l_1, l_2, l_3, l_4] = [\text{population at risk, impacted cities and towns, impacted public facilities and infrastructures, impacted ecological environment}] \quad (2)$$

Table 3. Evaluation factors for loss due to dam failure and grading methods.

Grades of Loss Due to Flooding and Dam Failure	Evaluation Factors			
	l_1	l_2	l_3	l_4
Extremely severe	$\geq 10^5$	Prefecture-level city	Important state-level infrastructures: transportation, power transmission, oil and gas transmission, large water resources and hydropower projects, cascade development, large-scale chemical industries, pesticide plants, highly toxic chemical industries, heavy metals, etc.	Cultural relics and rare animals/plants of the world. Water sources for urban areas involved. Major geological disasters can lead to river blocking, impacting a population of more than 1000.
Severe	10^4-10^5	County-level city	Important provincial-level infrastructures: transportation, power transmission, oil and gas transmission, medium-sized water resources and hydropower projects, relatively large chemical industries, pesticide plants, highly toxic chemical industries, heavy metals, etc.	Cultural relics and rare animals/plants at the state level. Water sources for counties involved. Geological disasters can lead to river narrowing, impacting a population of 300–1000.
Relatively severe	10^3-10^4	Villages and towns	Important municipal infrastructures: transportation, power transmission, oil and gas transmission, mining industries, ordinary chemical industries, heavy metals.	Cultural relics and rare animals/plants at the township level. Water sources for counties involved. Geological disasters can lead to river narrowing or impact a population of 100–300.
Moderate	$< 10^3$	Residential areas within villages	Infrastructure of a smaller size than those in the relatively severe level.	Cultural relics and rare animals/plants at the county level. Water sources for villages involved. Geological disasters can lead to river narrowing or impact a population of less than 100.

The final eight selected factors are all based on Cole’s analysis of more than 100 barrier lakes globally (SL/T 450-2021) [33], which is the only official code for risk assessment of barrier lakes in China. Therefore, the reliability and validity are ensured. In Section 3.1, the hazards of barriers are graded using four levels: extra high, high, moderate, and low. In Section 3.2, the loss induced by dam failure is graded as extremely severe, severe, relatively severe, and moderate. Based on these two grading methods, the risk evaluation grades can be classified as grade I (extremely high), II (high), III (medium), and IV (low), as shown in Equation (3).

$$V = [v_1, v_2, v_3, v_4] = \text{grade I, II, III, and IV} \quad (3)$$

4. Information Acquisition for Risk Evaluation Factors

Quick acquisition of information is key for risk grading of barrier lakes under emergency circumstances within a short time frame. The information acquisition method for risk grading with eight factors is demonstrated in Table 4. (1) Capacity of barrier lake (d_1) data can be acquired dynamically through predication of possible highest water level based on the capacity curve of the barrier lake (see Section 4.1). (2) Inflow from upstream (d_2) data can be calculated based on runoff-yielding rules [38–40]. (3) Material components (d_3) data can be calculated dynamically from multiple dimensions, including intelligent identification of surface particles, geophysical investigation of space-equivalent particles, tracing provenance analysis, etc. (see Section 4.2). (4) Geometry of the barrier (d_4) data can be obtained through Boolean calculation based on oblique photography with UVA, LiDAR, satellite images, and multi-dimensional 3D modeling with DEM [41,42]. (5) Data on the population at risk (l_1) can be acquired through quick identification technology based on LBS (Location-Based Services). (6) Data on impacted towns and cities (l_2), impacted public facilities and infrastructures (l_3), and impacted ecological environment (l_4) can be acquired from corresponding government authorities based on a risk map of flooding induced by dam failure.

Table 4. Information acquisition method for risk grading with eight factors.

Factors	Methods of Data Acquisition	Factors	Methods of Data Acquisition
d_1	Capacity curve of the barrier lake	l_1	Acquisition through quick identification technology based on LBS (Location-Based Services)
d_2	Calculated based on runoff yielding in barrier lake area	l_2	
d_3	Intelligent identification of surface particles, geophysical investigation of space-equivalent particles, tracing provenance analysis, etc.	l_3	Acquisition from corresponding government authorities based on risk map of flooding induced by dam failure
d_4	Oblique photography with UVA, LiDAR, satellite images, and multi-dimensional 3D modeling with DEM	l_4	

4.1. Acquisition of Information on the Capacity of a Barrier Lake (d_1)

This paper builds up a topographic database for the Tangjiashan barrier lake and produces its capacity curve through overlaying; dynamic checking; elevation unification based on 1:50,000, 1:2000, and 1:5000 topographic maps acquired; 1:50,000 DEM data through remote sensing technology; multispectral data (8 m resolution, Beichuan county) and RADAR data (3 m resolution, barrier lake area); and a 3D topographic map acquired through an airborne LIDAR system. The highest water level for the Tangjiashan barrier lake is an elevation of 752 m, and its capacity (d_1) is 316 million m^3 , as shown in the capacity curve. Figure 9 shows the process of the acquisition of d_1 .

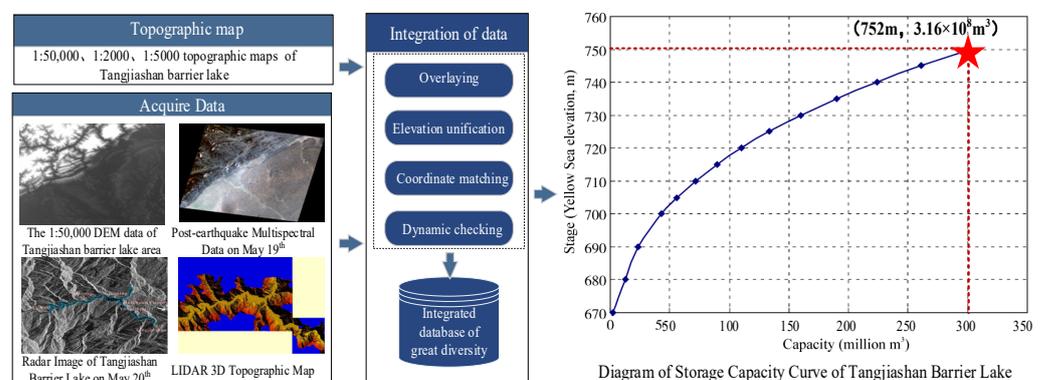


Figure 9. The process of the acquisition of d_1 (case study of Tangjiashan).

4.2. Acquisition of Data on the Material Components of a Barrier Dam (d_3)

A qualitative analysis was carried out on the material components of a barrier dam using the provenance methodology. Surface particles in a barrier dam were identified through an intelligent identification methodology. The diameter of particles in a barrier dam was calculated based on the longitudinal profile data through a natural source surface wave. A grading curve for particles in a barrier dam was produced based on the above-mentioned data. In accordance with such a calculation, the mid-value of the particle diameters for Tangjiashan's material components was 83 mm, and the mid-value of the particle diameter for Baige's material components was 4.3 mm. Figure 10 shows the process of the acquisition of d_3 .

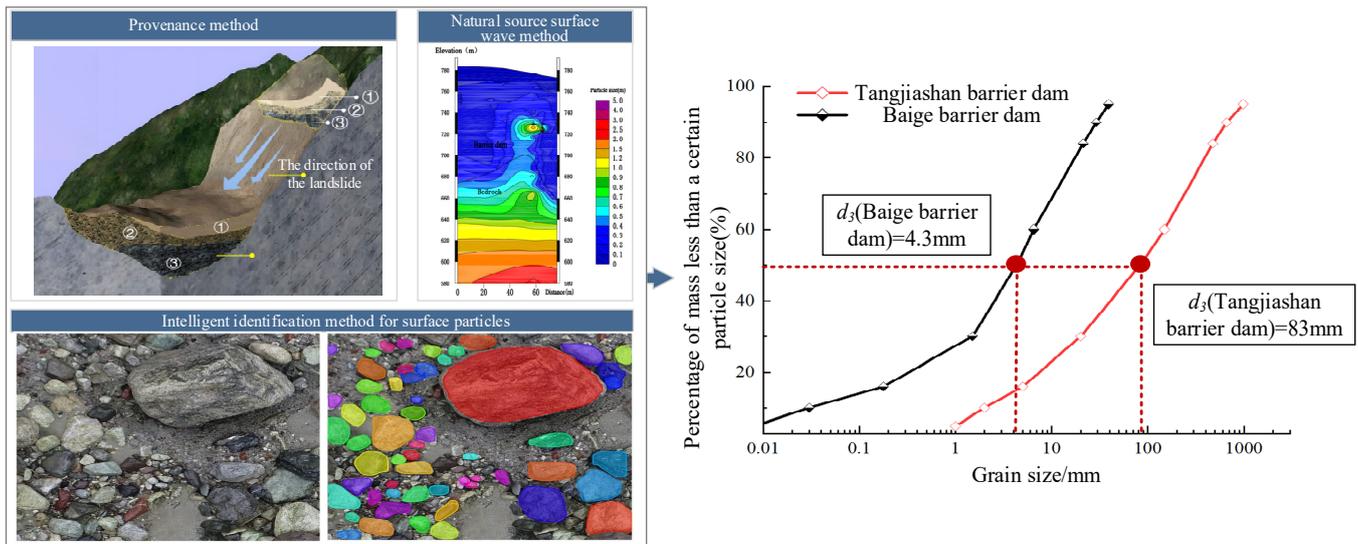


Figure 10. The process of the acquisition of d_3 .

5. Solution to Preference Matrix (R)

5.1. The Range for Evaluation and Values of Parameters

As has been listed in Section 3, there are eight parameters for risk evaluation for barrier lakes, and each parameter evaluated has four ranges: $[\alpha_{i1} = 0, \alpha_{i2} = 25]$, $[\alpha_{i2} = 25, \alpha_{i3} = 50]$, $[\alpha_{i3} = 50, \alpha_{i4} = 75]$, and $[\alpha_{i4} = 75, \alpha_{i5} = 100]$. Parameter d_1 (capacity), d_2 (inflow from upstream), d_3 (geometry), d_4 (material components of barrier dam), and l_1 (population at risk) can be calculated through linear interpolation. This paper will demonstrate the calculation process of d_1 (capacity) as an example. Parameter l_2 (impacted cities and towns), l_3 (impacted public facilities and major infrastructures), and l_4 (impacted ecological environment) can be valued through quantifying the number of impacted cities/towns, facilities, and the environment. This paper will demonstrate the calculation process of l_2 .

(1) Calculation of d_1 (capacity)

- 1) When $0 < d_1 \leq 100$, $x_1 = 25 \times d_1 / 100$;
- 2) When $100 < d_1 \leq 1000$, $x_1 = 25 + (50 - 25) \times (d_1 - 100) / (1000 - 100)$;
- 3) When $1000 < d_1 \leq 10,000$, $x_1 = 50 + (75 - 50) \times (d_1 - 1000) / (10,000 - 1000)$;
- 4) When $10,000 < d_1 \leq 100,000$, $x_1 = 75 + (100 - 75) \times (d_1 - 10,000) / (100,000 - 10,000)$;
- 5) When $100,000 > d_1$, $x_1 = 100$.

(2) Calculation of l_2 (impacted cities and towns)

- 1) When the impacted area is residential areas within a village, $x_6 = 3 \times l_{21}$ and $x_6 \leq 25$, l_{21} indicates the number of impacted villages and towns;
- 2) When the impacted area is villages and towns, $x_6 = 25 + 3 \times l_{21}$ and $x_6 \leq 50$;
- 3) When the impacted area is county-level cities, $x_6 = 50 + 6 \times l_{22}$ and $x_6 \leq 75$, l_{21} indicates the number of county-level cities and prefecture-level cities;

- 4) When the impacted area is prefecture-level cities, $x_6 = 75 + 6 \times I_{22}$ and $x_6 \leq 100$.

5.2. Function for Calculation of the Preference Relation

Calculation of the preference relation (r_{ik}) of i -th parameter to the k -th evaluation grade in the preference relation matrix R (8×4) is shown in Figure 11.

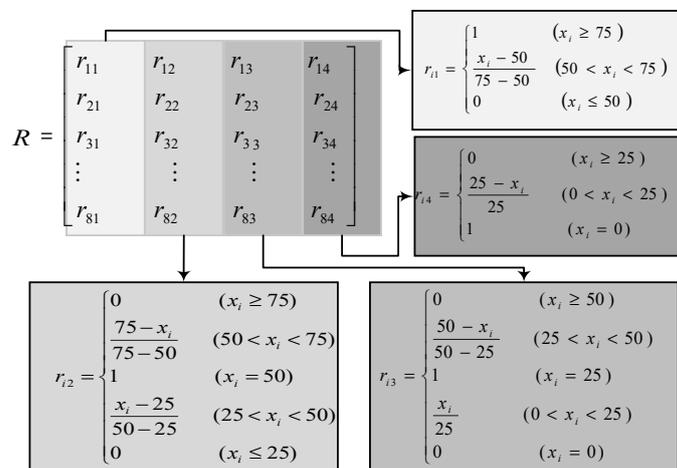


Figure 11. Calculation function of preference relation r_{ik} .

6. Calculating the Weights of the Indicators Based on the D-AHP Method

6.1. Definition of D Number

D numbers, first developed by Deng [43], are a good representation of uncertain information. They are widely used in many fields, such as supplier selection problems [44] and fault analysis [45].

Definition 1. Let Ω be a finite nonempty set. A D number is a mapping formulated by

$$D : \Omega \rightarrow [0, 1] \tag{4}$$

$$\sum_{B \subseteq \Omega} D(B) \leq 1 \text{ and } D(\Theta) = 0 \tag{5}$$

where Θ is an empty set and B is a subset of Ω .

From this definition, we notice that the completeness constraint is released if D numbers are used. If $\sum_{B \subseteq \Omega} D(B) = 1$, then the information is complete; and if $\sum_{B \subseteq \Omega} D(B) < 1$, the information is incomplete.

Suppose that the set $\Omega = \{b_1, b_2, \dots, b_i, \dots, b_n\}$, where $b_i \in \mathbb{R}$ and $b_i \neq b_j$ if $i \neq j$. Then, a special form of D numbers can be expressed as: $D = \{(b_1, v_1), (b_2, v_2), \dots, (b_i, v_i), \dots, (b_n, v_n)\}$, where $v_i > 0$ and $\sum_{i=1}^n v_i \leq 1$.

Definition 2. Let $D = \{(b_1, v_1), (b_2, v_2), \dots, (b_i, v_i), \dots, (b_n, v_n)\}$ be a D numbers. The integration representation of D is defined as:

$$I(D) = \sum_{i=1}^n b_i v_i \tag{6}$$

where, $v_i > 0$, and $\sum_{i=1}^n v_i \leq 1$.

6.2. D Numbers Extended Fuzzy Preference Relation

The fuzzy preference relation is provided to construct pairwise comparison matrices based on expert judgment and is described by a fuzzy pairwise comparison with an additive reciprocal ($r_{ij} + r_{ji} = 1$) that is different from the multiplicative preference relation. r_{ij} denotes the preference degree of an alternative A_i over another alternative A_j and can be expressed as follows:

$$r_{ij} = \begin{cases} 0 & A_j \text{ is absolutely preferred to } A_i \\ \in (0, 0.5) & A_j \text{ is preferred to } A_i \text{ to some degree} \\ 0.5 & \text{indifference between } A_i \text{ and } A_j \\ \in (0.5, 1) & A_i \text{ is preferred to } A_j \text{ to some degree} \\ 1 & A_i \text{ is absolutely preferred to } A_j \end{cases} \tag{7}$$

There are some shortcomings when using the fuzzy preference relation to represent certain situations. For example, if the expert assessments are uncertain or incomplete, it is difficult to construct the fuzzy preference relation. To overcome these shortcomings, Deng et al. [43] extended the classical fuzzy preference relation by using D numbers. The derived relation is called a D numbers preference relation, and the corresponding matrix is called a D numbers preference matrix, which can be abbreviated as a D matrix. The D matrix is defined as follows.

Definition 3. A D numbers preference relation R_D on a set of alternatives A is represented by a D matrix on the product set $A \times A$, whose elements are formulated by

$$R_D : A \times A \rightarrow D \tag{8}$$

The D numbers preference relation in matrix form is

$$R_D = \begin{matrix} & & A_1 & A_2 & \dots & A_n \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_n \end{matrix} & \begin{bmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \\ \dots & \dots & \dots & \dots \\ D_{n1} & D_{n2} & \dots & D_{nn} \end{bmatrix} \end{matrix} \tag{9}$$

where $D_{ij} = \{ (b_1^{ij}, v_1^{ij}), (b_2^{ij}, v_2^{ij}), \dots, (b_m^{ij}, v_m^{ij}) \}, \{1, 2, \dots, n\}$,
 $D_{ji} = \{ (1 - b_1^{ij}, v_1^{ij}), (1 - b_2^{ij}, v_2^{ij}), \dots, (1 - b_m^{ij}, v_m^{ij}) \}, \forall i, j \in$ and $b_k^{ij} \in [0, 1]$,
 $\forall \in \{1, 2, \dots, m\}$.

Consequently, with the help of the D numbers preference relation, the preference relations of the three situations presented in Section 1 as an example are shown in Equations (10)–(12), respectively.

$$R_{D1} = \begin{matrix} & & A_1 & & A_2 \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} \{(0.5, 1.0)\} & \{(0.8, 0.8), (0.7, 0.2)\} \\ \{(0.2, 0.8), (0.3, 0.2)\} & \{(0.5, 1.0)\} \end{bmatrix} \end{matrix} \tag{10}$$

$$R_{D2} = \begin{matrix} & & A_1 & & A_2 \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} \{(0.5, 1.0)\} & \{(0.6, 0.7), (0.5, 0.3)\} \\ \{(0.4, 0.7), (0.5, 0.3)\} & \{(0.5, 1.0)\} \end{bmatrix} \end{matrix} \tag{11}$$

$$R_{D3} = \begin{matrix} & A_1 & A_2 \\ A_1 & \{(0.5, 1.0)\} & \{(0.7, 0.8)\} \\ A_2 & \{(0.2, 0.8)\} & \{(0.5, 1.0)\} \end{matrix} \quad (12)$$

6.3. Calculating Procedure of the Weights of Alternatives Using the D-AHP Method

The calculation process includes five steps: ① establish the D numbers preference matrix (D matrix); ② convert the D matrix to a crisp matrix; ③ construct a probability matrix based on the crisp matrix; ④ rank the alternatives using the triangularization method; and ⑤ calculate the relative weights of alternatives.

7. Case Application

Based on the D-AHP method, this paper calculates the risk level of 15 barrier lakes (see Figure 12): Jiguanling in Chongqing, Yigong in Tibet, Qingyandong in Chongqing, Houziyan in Dadu River, Hongshiyuan in Niulan River, Tangjiashan in Sichuan, Jiala in Tibet, Baige in Jinsha River, Yankou in Guizhou, Shaziba in Hubei, Xiaojiangqiao in Sichuan, Tanggudong in Yalong River, Zhouqu in Gansu, Xiaogangjian in Sichuan, and Xujiaba in Sichuan.

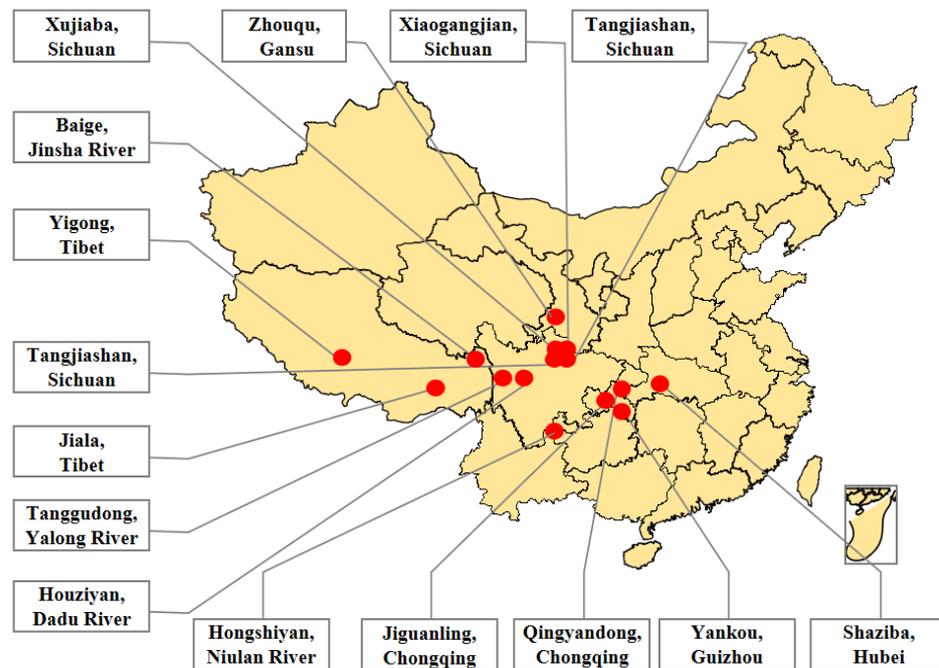


Figure 12. Location of the 15 barrier lakes.

7.1. Calculation of Matrix R

Based on the calculation formula in Section 5, eight evaluation indicators (see Table 5) were assigned, and the preference relation matrixes (see Table 6) were obtained for the 15 cases.

Table 5. Assignment results of eight evaluation indicators for the 15 cases.

Barrier Lake		d_1	d_2	d_3	d_4	d_1	l_2	l_3	l_4
Jiguanling	Data	12,000	1010	90	H = 10 m L/H = 11	65,000	Baitao Town	G319	Water source for villages
	Results	75.05	86.62	40.28	16.67	65.28	28	79	29
Yigong	Data	260,000	88.5	30	H = 100 m L/H = 25	6000	Yigong Village	8 bridges	Same as Jiguanling
	Results	81.31	59.63	48.61	59.38	38.89	34	24	29

Table 5. Cont.

Barrier Lake		d_1	d_2	d_3	d_4	d_1	l_2	l_3	l_4
Qingyanlo-ng	Data	150	34.4	80	H = 30 m L/H = 7	6000	9 villages and towns	S201	Same as Jiguanling
	Results	26.39	40.25	41.67	50	38.89	4	49	29
Houziyan	Data	6000	2570	85	H = 40 m L/H = 7.5	20,000	3 counties and cities	S306	Water source for cities
	Results	63.89	100	40.97	56.25	52.78	68	54	54
Hongshiyuan	Data	26,000	360	9.44	H = 89 m L/H = 10.22	30,000	10 villages and towns	33,000 mu of farmland	Same as Jiguanling
	Results	75.4	77.84	64.67	80.94	55.56	49	33	29
Tangjiasha-n	Data	24,700	85	83	H = 89 m L/H = 9.67	1,303,500	Beichuan County	S302, S105	Same as Houziyan
	Results	75.37	58.75	41.25	79.06	80.94	74	54	54
Jiala	Data	55,000	1600	35	H = 60 m L/H = 36.7	16,000	7 villages and towns	2 bridges	Same as Jiguanling
	Results	87.5	100	43.06	54.69	51.67	46	4	29
Baige	Data	57,800	700	4.3	H = 64 m L/H = 20.31	76,000	11 villages and towns	G214	Same as Jiguanling
	Results	76.21	82.43	71.80	46.25	68.33	49	79	29
Yankou	Data	6400	13	70	H = 54 m L/H = 4.67	50,000	Yinjiang County, 1 village, 1 town	7050 houses	Same as Houziyan
	Results	65	26.88	43.06	90	61.11	56	33	54
Shaziba	Data	692	151	3	H = 43 m L/H = 16.28	8397	Tunpu Village	Multiple houses	Same as Jiguanling
	Results	41.44	75	73.61	58.13	45.55	28	4	29
Xiaojiqiao	Data	2000	11	201	H = 65 m, L/H = 5.54	114,000	6 towns	National factories	Same as Houziyan
	Results	52.78	25.63	24.92	71.88	75.04	43	79	54
Tanggulon-g	Data	68,000	1500	10	H = 170 m L/H = 11.53	1102	Bayirong Village	hydrological stations	Same as Jiguanling
	Results	76.46	93.24	63.89	100	25.28	34	33	29
Zhouqu	Data	150	128.33	8.65	H = 9 m L/H = 166	69,400	Zhouqu County	2/3 of Zhouqu County	Same as Houziyan
	Results	26.39	69.58	65.76	15	66.5	74	37	54
Xiaogangji-an	Data	1200	15	378	H = 70 m L/H = 4.26	47,188	Hanwang	Hanqing Highway	Same as Jiguanling
	Results	50.56	28.13	10.17	75	60.33	46	29	29
Xujiaba	Data	980	8	201	H = 150 m L/H = 4.67	44,000	Qingping Village, Hanwang Town	Factories and mines	Same as Jiguanling
	Results	49.44	20.02	24.92	100	59.44	31	8	29

Table 6. Calculation results for preference relation for the 15 cases.

Jiguanling				Yigong				Qingyandong				Houziyan				Hongshiyuan			
1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.056	0.944	0.000	0.556	0.444	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.611	0.389	0.000	0.385	0.615	0.000	0.000	0.000	0.610	0.390	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.000	0.667	0.333	0.000	0.944	0.056	0.000	0.000	0.667	0.333	0.000	0.000	0.639	0.361	0.000	0.587	0.413	0.000	0.000
0.611	0.389	0.000	0.000	0.375	0.625	0.667	0.333	0.000	1.000	0.000	0.000	0.250	0.750	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.120	0.880	0.000	0.000	0.556	0.444	0.000	0.000	0.056	0.944	0.000	0.111	0.889	0.000	0.000	0.222	0.778	0.000	0.000
1.000	0.000	0.000	0.000	0.000	0.360	0.640	0.000	0.000	0.000	0.160	0.840	0.720	0.280	0.000	0.000	0.000	0.940	0.040	0.000
0.000	0.160	0.840	0.000	0.000	0.000	0.960	0.040	0.000	0.960	0.040	0.000	0.160	0.840	0.000	0.000	0.000	0.320	0.680	0.000
				0.000	0.160	0.840	0.000	0.000	0.160	0.840	0.000	0.160	0.840	0.000	0.000	0.000	0.160	0.840	0.000
Tangjiashan				Jiala				Baige				Yankou				Shaziba			
1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.600	0.400	0.000	0.000	0.000	0.658	0.342	0.000
0.350	0.650	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.075	0.925	0.000	1.000	0.000	0.000	0.000
0.000	0.650	0.350	0.000	0.000	0.722	0.278	0.000	0.872	0.128	0.000	0.000	0.000	0.722	0.278	0.000	0.944	0.056	0.000	0.000
1.000	0.000	0.000	0.000	0.188	0.812	0.000	0.000	0.000	0.850	0.150	0.000	1.000	0.000	0.000	0.000	0.325	0.675	0.000	0.000
1.000	0.000	0.000	0.000	0.067	0.933	0.000	0.000	0.733	0.267	0.000	0.000	0.444	0.556	0.000	0.000	0.000	0.822	0.178	0.000
0.960	0.040	0.000	0.000	0.000	0.840	0.160	0.000	0.000	0.960	0.040	0.000	0.240	0.760	0.000	0.000	0.000	0.120	0.880	0.000
0.160	0.840	0.000	0.000	0.000	0.000	0.160	0.840	1.000	0.000	0.000	0.000	0.000	0.320	0.680	0.000	0.000	0.000	0.160	0.000
0.160	0.840	0.000	0.000	0.000	0.160	0.840	0.000	0.000	0.160	0.840	0.000	0.160	0.840	0.000	0.000	0.000	0.160	0.840	0.000

Table 6. Cont.

Jiguanling				Yigong				Qingyandong				Houziyan				Hongshiyuan			
0.011	0.889	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.056	0.944	0.000	0.022	0.978	0.000	0.000	0.000	0.978	0.022	0.000
0.000	0.025	0.975	0.000	1.000	0.000	0.000	0.000	0.783	0.217	0.000	0.000	0.000	0.125	0.875	0.000	0.000	0.000	0.801	0.199
0.000	0.000	0.997	0.003	0.556	0.444	0.000	0.000	0.630	0.370	0.000	0.000	0.000	0.000	0.407	0.000	0.000	0.000	0.997	0.003
0.875	0.125	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.600	0.400	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
1.000	0.000	0.000	0.000	0.000	0.011	0.989	0.000	0.660	0.340	0.000	0.000	0.413	0.587	0.000	0.000	0.378	0.622	0.000	0.000
0.000	0.720	0.280	0.000	0.000	0.360	0.640	0.000	0.960	0.040	0.000	0.000	0.000	0.840	0.160	0.000	0.000	0.240	0.760	0.000
1.000	0.000	0.000	0.000	0.000	0.320	0.680	0.000	0.000	0.480	0.520	0.000	0.000	0.160	0.840	0.000	0.000	0.000	0.320	0.680
0.160	0.840	0.000	0.000	0.000	0.160	0.840	0.000	0.160	0.840	0.000	0.000	0.000	0.160	0.840	0.000	0.000	0.160	0.840	0.000

7.2. Calculation of Weight Vectors

The D numbers preference matrix (i.e., D matrix) must be constructed before calculating the weights of the indicators using the D-AHP method. The weight ratings of eight indicators by 10 experts are shown in Tables 7 and 8. All 10 experts worked with Changjiang Water Resources Commission (CWRC), Hohai University, Power Construction Corporation of China, etc., and all of them were senior engineers with master’s degrees. They had been working in the area of emergency treatment for barrier lakes and other water disasters, hydropower, and water resources. They all contributed to emergency treatment for the Tangjiashan and Baige barrier lakes. The following four indicators of the risks of barrier lakes were taken as an example to construct the D matrix based on the D numbers preference relation as follows:

- (1) Ten experts were asked to score the importance of the four indicators and then construct the D matrix based on the D numbers preference relation, as shown in Equation (13):

$$R_D = \begin{matrix} & d_1 & d_2 & d_3 & d_4 \\ \begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{matrix} & \left[\begin{array}{cccc} \{ (0.50, 1.0) \} & \{ (0.45, 0.2), (0.55, 0.3), (0.60, 0.3), (0.65, 0.1), (0.90, 0.1) \} & \{ (0.40, 0.2), (0.55, 0.1), (0.60, 0.4), (0.65, 0.1), (0.70, 0.2) \} & \{ (0.35, 0.1), (0.60, 0.2), (0.65, 0.3), (0.80, 0.4) \} \\ \{ (0.55, 0.2), (0.45, 0.3), (0.40, 0.3), (0.35, 0.1), (0.10, 0.1) \} & \{ (0.50, 1.0) \} & \{ (0.35, 0.2), (0.40, 0.1), (0.45, 0.1), (0.50, 0.1), (0.55, 0.2), (0.60, 0.2), (0.65, 0.1) \} & \{ (0.40, 0.2), (0.45, 0.1), (0.50, 0.1), (0.60, 0.2), (0.65, 0.2), (0.70, 0.2) \} \\ \{ (0.60, 0.2), (0.45, 0.1), (0.40, 0.4), (0.35, 0.1), (0.30, 0.2) \} & \{ (0.65, 0.2), (0.60, 0.1), (0.55, 0.1), (0.50, 0.1), (0.45, 0.2), (0.40, 0.2), (0.35, 0.1) \} & \{ (0.50, 1.0) \} & \{ (0.45, 0.1), (0.50, 0.1), (0.55, 0.4), (0.60, 0.2), (0.65, 0.1), (0.70, 0.1) \} \\ \{ (0.65, 0.1), (0.40, 0.2), (0.35, 0.3), (0.20, 0.4) \} & \{ (0.60, 0.2), (0.55, 0.1), (0.50, 0.1), (0.40, 0.2), (0.35, 0.2), (0.30, 0.2) \} & \{ (0.55, 0.1), (0.50, 0.1), (0.45, 0.4), (0.40, 0.2), (0.35, 0.1), (0.30, 0.1) \} & \{ (0.50, 1.0) \} \end{array} \right] \end{matrix} \quad (13)$$

- (2) The D matrix was converted to a crisp matrix R_C using the integration representation of D numbers as follows:

$$R_C = \begin{matrix} & d_1 & d_2 & d_3 & d_4 \\ \begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{matrix} & \left[\begin{array}{cccc} 0.500 & 0.590 & 0.580 & 0.670 \\ 0.410 & 0.500 & 0.495 & 0.565 \\ 0.420 & 0.505 & 0.500 & 0.570 \\ 0.330 & 0.435 & 0.430 & 0.500 \end{array} \right] \end{matrix} \quad (14)$$

- (3) According to the rules proposed to generate the probability matrix by Deng et al. [43], the probability matrix was constructed as below:

$$R_P = \begin{matrix} & d_1 & d_2 & d_3 & d_4 \\ \begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \tag{15}$$

- (4) Using the triangularization method, the ranking of the indicators was calculated as $d_1 \gg d_3 \gg d_2 \gg d_4$, where the symbol “ \gg ” indicates preference.
- (5) Then, we calculated the relative weights of the indicators. First, based on the ranking of the indicators, the matrix R_c was converted to a triangulated crisp matrix R_c^T :

$$R_C^T = \begin{matrix} & d_1 & d_3 & d_2 & d_4 \\ \begin{matrix} d_1 \\ d_3 \\ d_2 \\ d_4 \end{matrix} & \begin{bmatrix} 0.500 & 0.580 & 0.590 & 0.670 \\ 0.420 & 0.500 & 0.505 & 0.570 \\ 0.410 & 0.495 & 0.500 & 0.565 \\ 0.330 & 0.430 & 0.435 & 0.500 \end{bmatrix} \end{matrix} \tag{16}$$

- (6) Using the weight relation of the indicators represented in the matrix, the weight equations were constructed by incorporating necessary constraints:

$$\begin{cases} \lambda(w_1 - w_3) = 0.580 - 0.500 \\ \lambda(w_3 - w_2) = 0.505 - 0.500 \\ \lambda(w_2 - w_4) = 0.570 - 0.500 \\ w_1 + w_2 + w_3 + w_4 = 1 \\ \lambda > 0 \\ w_i \geq 0, \forall i \in \{1, 2, 3\} \end{cases} \tag{17}$$

where w_i refers to the weight of the i -th indicator, and λ indicates the granular information about the pairwise comparison and is associated with the cognitive ability of the experts.

- (7) According to Deng et al. [43], a feasible scheme of λ is:

$$\lambda = \begin{cases} [\lambda] = 1, & \text{The information is with high credibility} \\ n, & \text{The information is with mudium credibility} \\ n^2/2, & \text{The information is with low credibility} \end{cases} \tag{18}$$

Table 7. The weight ratings of four indicators of barrier lake risks by 10 experts.

Expert	d_1/d_2	d_1/d_3	d_1/d_4	d_2/d_3	d_2/d_4	d_3/d_4
1	0.55	0.65	0.65	0.6	0.6	0.5
2	0.55	0.6	0.8	0.55	0.65	0.55
3	0.55	0.6	0.65	0.55	0.6	0.55
4	0.45	0.4	0.35	0.45	0.4	0.45
5	0.45	0.6	0.65	0.65	0.7	0.55
6	0.9	0.7	0.8	0.35	0.4	0.6
7	0.6	0.7	0.8	0.55	0.65	0.55
8	0.6	0.6	0.8	0.5	0.7	0.7
9	0.65	0.55	0.6	0.4	0.45	0.6
10	0.6	0.4	0.6	0.35	0.5	0.65

Table 8. The weight ratings of four indicators of barrier lake loss by 10 experts.

Expert	l_1/l_2	l_1/l_3	l_1/l_4	l_2/l_3	l_2/l_4	l_3/l_4
1	0.55	0.65	0.65	0.6	0.6	0.5
2	0.55	0.6	0.8	0.55	0.65	0.55
3	0.55	0.6	0.65	0.55	0.6	0.55
4	0.45	0.4	0.35	0.45	0.4	0.45
5	0.45	0.6	0.65	0.65	0.7	0.55
6	0.9	0.7	0.8	0.35	0.4	0.6
7	0.6	0.7	0.8	0.55	0.65	0.55
8	0.6	0.6	0.8	0.5	0.7	0.7
9	0.65	0.55	0.6	0.4	0.45	0.6
10	0.6	0.4	0.6	0.35	0.5	0.65

The 10 experienced experts had strong cognition towards various indicators. The information reliability was high, thus $\lambda = 1$. After calculation, the weights of the four indicators d_1, d_2, d_3 , and d_4 were 0.33, 0.245, 0.25, and 0.175, respectively. Similarly, the weights of the four indicators of loss evaluation factors for barrier dams l_1, l_2, l_3 , and l_4 were calculated to be 0.393, 0.268, 0.228, and 0.113, respectively. The weight indicators for the danger of barrier dam D and the dam break loss L were 0.525 and 0.475. After integrating the weights of various levels, eight indicator weight value vectors $W = [0.173, 0.129, 0.131, 0.092, 0.186, 0.127, 0.108, 0.053]$ were obtained.

7.3. Calculation of Risk Level of Barrier Lakes

For comparison, this paper refers to the hybrid fuzzy evaluation method for quantitative risk classification based on D-AHP as Method A. The risk levels of 15 barrier lakes after calculation are shown in Table 9. The table-lookup method is referred to as Method B, and the corresponding risk level calculation method is shown in Table 10. The comparison of the two methods for calculating the level of barrier lakes risk is shown in Figure 13.

Table 9. Evaluation result for the risk levels of 15 barrier lakes (Method A).

Barrier Lake	g_1	g_2	g_3	g_4	Grade()	Risk Level
Jiguanling	0.524	0.177	0.269	0.031	0.524	I
Yigong	0.257	0.418	0.320	0.004	0.418	II
Qingyandong	0.000	0.483	0.410	0.107	0.483	II
Houziyan	0.386	0.567	0.047	0.000	0.567	II
Hongshiyuan	0.512	0.364	0.123	0.000	0.512	I
Tangjiashan	0.644	0.310	0.046	0.000	0.644	I
Jiala	0.332	0.459	0.119	0.091	0.459	II
Baige	0.661	0.275	0.064	0.000	0.605	I
Yankou	0.318	0.453	0.229	0.000	0.453	II
Shaziba	0.282	0.360	0.266	0.091	0.360	II
Xiaojiqiao	0.403	0.305	0.292	0.000	0.403	I
Tanggudong	0.467	0.149	0.384	0.000	0.467	I
Zhouqu	0.437	0.251	0.275	0.037	0.437	I
Xiaogangjian	0.173	0.427	0.322	0.078	0.427	II
Xujiaba	0.162	0.324	0.414	0.100	0.414	III

Table 10. Calculation table for the risk levels of barrier lakes (Method B).

Risk Level of Barrier Dam	Severity of Losses Due to Barrier Lake	Risk Level of Barrier Lake
Extra high risk, high risk	Extremely severe	I
Extra high risk	Severe, relatively severe	
High risk	Severe	
Moderate risk	Extremely severe, severe	II
Low risk	Extremely severe	

Table 10. Cont.

Risk Level of Barrier Dam	Severity of Losses Due to Barrier Lake	Risk Level of Barrier Lake
Extra high risk	Moderate	III
High risk	Relatively severe, moderate	
Moderate risk	Relatively severe	
Low risk	Severe, relatively severe	
Moderate risk, low risk	Moderate	IV

Notes: 1. Risk Level of Barrier Dam: When $S \geq 3.0$, it is considered an extremely high risk. When $2.25 \leq S < 3.0$, it is considered a high risk. When $1.5 \leq S < 2.25$, it is considered a moderate risk. When $S < 1.5$, it is considered a low risk. $S = 0.25 (S_1 + S_2 + S_3 + S_4)$. S_1, S_2, S_3 , and S_4 are the assigned values for the four grading indicators d_1, d_2, d_3 , and d_4 , with extra high risk, high risk, moderate risk, and low risk assigned values of 4, 3, 2, and 1, respectively. 2. Severity of Losses: The level of severity of losses due to the formation of a barrier lake is based on the highest level of loss severity among the single grading indicators l_1, l_2, l_3 , and l_4 .

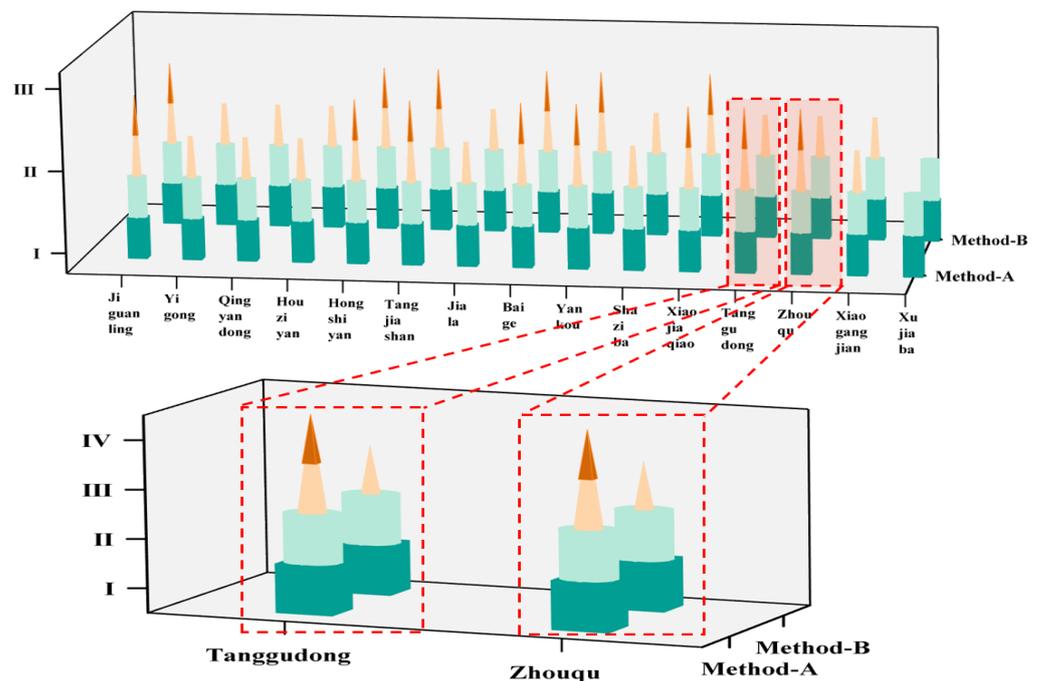


Figure 13. Comparison between Method A and Method B calculation results on risk level of barrier lakes.

The following conclusions can be drawn from Figure 13:

- (1) After calculation, the risk level calculation results of the two methods for 13 barrier lakes were the same, accounting for 86.7%. Overall, the evaluation conclusions of the two methods showed good consistency.
- (2) Analysis of reasons for inconsistent calculation results of risk evaluation levels for two barrier lakes:
 - 1) Tanggudong Barrier Lake: From the preference relation matrix R of Tanggudong barrier lake, the preference relation degrees r_{11}, r_{21} , and r_{41} corresponding to d_1, d_2 , and d_4 were all 1, indicating that the barrier dam is extremely risky. From the perspective of dam break losses, the downstream population at risk of Tanggudong barrier lake exceeds 1000. The regions and facilities at risk include Bayirong Village, Yayihe Village, Bosihe Town, three hydrological stations, eight bridges, 51 km of highway, and large amounts of farmland and township water sources, indicating severe losses. Method B indicates that the extremely severe loss of the barrier lake is a sufficient condition for the risk evaluation level to be level I; however, based on the scores given by 10 experts, the weight of the risk indicator of the barrier dam is greater than the weight of

the dam break loss, indicating that it is unreasonable to consider the extremely severe losses due to the barrier lake as a sufficient condition for the evaluation level to be level I. Therefore, it is recommended to supplement Method B with the sufficient condition that “the risk level of the barrier lake is extremely high, and the losses due to the barrier lake are more than relatively severe” for the barrier lake risk level to be classified as Level I.

- 2) Zhouqu Barrier Lake: The loss indicators l_1 , l_2 , and l_4 of the Zhouqu Barrier Lake have all reached severe level, but Method B uses the level with the highest loss severity among the l_1 , l_2 , l_3 , and l_4 single grading indicators as the level of loss severity for the barrier lake, failing to reflect cumulative losses. Meanwhile, due to the different weights of l_1 , l_2 , l_3 , and l_4 , there are differences in the social impacts brought by the same level of loss. Only using the highest-level loss of a certain indicator as the severity level of the barrier lake is one-sided. Method A considers both cumulative losses and weight differences, resulting in a more objective evaluation conclusion.
- 3) Based on the above analysis, both Method A and Method B are relatively reliable in evaluating the risk level of barrier lakes. However, Method B has certain deviations in evaluating the risk level of individual cases. It is recommended that Method B supplement “the risk level of the barrier lake is extremely high, and the losses due to the barrier lake are more than relatively severe” as a sufficient condition for classifying the risk level of the barrier lake into Level I, while considering the impact of cumulative losses on the risk level of barrier lakes.

8. Conclusions

This paper addresses the problems faced by the risk classification of barrier lakes, including a short evaluation window period, complex evaluation indicators, difficulty in obtaining information quickly, and difficulty in quantifying index weights. For the first time, a hybrid fuzzy evaluation model for quantitative risk classification of barrier lakes based on D-AHP is constructed, and an eight-factor evaluation index system and quantitative weight indicators are proposed to achieve the rapid acquisition of eight-factor evaluation index information for emergency rescue conditions. The specific conclusions are as follows:

- (1) This paper proposed a risk classification method for barrier lakes based on D-AHP, which solved the problem of difficult quantification of evaluation index weights. The D-AHP method proposed in this article has three advantages over the AHP method: Firstly, AHP’s comparative judgments are subjective because they heavily rely on expert experience and professionalism, which may sometimes lead to inconsistencies. Secondly, AHP lacks the ability to adequately cope with any inherent uncertainty and imprecision in the data. Finally, the preferred information may contain fuzziness and incompleteness, and AHP is unable to handle this incomplete information. The risk evaluation results of 15 barrier lakes, including Tangjiashan Barrier Lake, show that the proposed barrier lake risk classification method in this paper has good consistency with the results using the traditional table-lookup method. The risk classification conclusions of 13 barrier lakes are consistent, but the table-lookup method considers that the extremely severe loss of barrier lakes is a sufficient condition for the evaluation level to be level I and does not consider the impact of cumulative loss on the risk level of barrier lakes, resulting in deviations in the risk level classification of some individual barrier lakes. Further correction is needed to the table-lookup method.
- (2) This paper, on the basis of international and domestic research of risk assessments of barrier lakes and studies on about 100 barrier lake cases, proposed a set of risk classification factors and grading criteria, which is $U = [D, L] = [d_1, d_2, d_3, d_4, l_1, l_2, l_3, l_4] = [\text{reservoir capacity, inflow from upstream, material component and geometry of the barrier, population at risk, impacted cities and towns, impacted public facilities}$

and infrastructures, and impacted ecological environment], solving the problem of complex evaluation indicators on the risk assessment of barrier lake. The proposed set of factors is included in the Code for Risk Classification and Emergency Measures of Barrier Lake (SL/T 450-2021).

- (3) Rapid acquisition of information in a short time period and extremely dangerous conditions are the conditions for risk evaluations of barrier lakes. This paper developed the methods of rapid calculation of the reservoir capacity curve of barrier lakes and intelligent identification of particles on the surface of barrier dams, which realized the rapid acquisition of an eight-factor evaluation index of information, thus solved the problem of acquiring information within a short time period.
- (4) The hybrid fuzzy evaluation method for quantitative risk classification of barrier lakes based on D-AHP proposed in this paper is reasonable in evaluation index's systems and classification, feasible for information acquisition methods, and scientific regarding weight evaluation indicators, thus generating reliable risk level evaluation results.

The limitation of the method is as follows. For the application of such a method, experts/scholars with rich experience on emergency treatment or risk analysis of barrier lakes are preferred. Therefore, the professionalism and experience of the expert team will impact the outcomes of the study.

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