water

MDPI

Review

Current State of Advances in Quantification and Modeling of Hydrological Droughts

Tribeni C. Sharma and Umed S. Panu *

Department of Civil Engineering, Lakehead University, Thunder Bay, ON P7B 5E1, Canada; tcsharma@lakeheadu.ca
* Correspondence: uspanu@lakeheadu.ca

Abstract: Hydrological droughts may be referred to as sustained and regionally extensive water shortages as reflected in streamflows that are noticeable and gauged worldwide. Hydrological droughts are largely analyzed using the truncation level approach to represent the desired flow condition such as the median, mean, or any other flow quantile of an annual, monthly, or weekly flow sequence. The quantification of hydrologic droughts is accomplished through indices, such as the standardized streamflow index (SSI) in tandem with the standardized precipitation index (SPI) commonly used in meteorological droughts. The runs of deficits in the SSI sequence below the truncation level are treated as drought episodes, and thus, the theory of runs forms an essential tool for analysis. The parameters of significance from the modeling perspective of hydrological droughts (or tantamount to streamflow droughts in this paper) are the longest duration and the largest magnitude over a desired return period of T-year (or month or week) of the streamflow sequences. It is to be stressed that the magnitude component of the hydrological drought is of paramount importance for the design and operation of water resource storage systems such as reservoirs. The time scales chosen for the hydrologic drought analysis range from daily to annual, but for most applications, a monthly scale is deemed appropriate. For modeling the aforesaid parameters, several methodologies are in vogue, i.e., the empirical fitting of the historical drought sequences through a known probability density function (pdf), extreme number theorem, Markov chain analysis, log-linear, copulas, entropy-based analyses, and machine learning (ML)-based methods such as artificial neural networks (ANN), wavelet transform (WT), support vector machines (SVM), adaptive neuro-fuzzy inference systems (ANFIS), and hybrid methods involving entropy, copulas, and machine learning-based methods. The forecasting of the hydrologic drought is rigorously conducted through machine learning-based methodologies. However, the traditional stochastic methods such as autoregressive integrated moving average (ARIMA), seasonal autoregressive integrated moving average (SARIMA), copulas, and entropy-based methods are still popular. New techniques for flow simulation are based on copula and entropy-based concepts and machine learning methodologies such as ANN, WT, SVM, etc. The simulated flows could be used for deriving drought parameters in consonance with traditional Monte Carlo methods of data generation. Efforts are underway to use hydrologic drought models for reservoir sizing across rivers. The ML methods whilst combined in the hybrid form hold promise in drought forecasting for better management of existing water resources during the drought periods. Data mining and pre-processing techniques are expected to play a significant role in hydrologic drought modeling and forecasting in future.

Keywords: copula; extreme number theorem; entropy; Markov chains; machine learning; standardized streamflow index; theory of runs; truncation level

Citation: Sharma, T.C.; Panu, U.S. Current State of Advances in Quantification and Modeling of Hydrological Droughts. *Water* **2024**, *16*, 729. https://doi.org/10.3390/w16050729

Academic Editor: Athanasios Loukas

Received: 8 January 2024
Revised: 22 February 2024
Accepted: 26 February 2024
Published: 29 February 2024

1. Introduction to Hydrological Droughts

Several definitions have been proposed for hydrological droughts. Yevjevich [1] defined the hydrological drought as a period with water content below the average water content in streams, reservoirs, aquifers, lakes, and soils. This period is associated with the effects of precipitation shortfall on surface and subsurface water supply, rather than with direct shortfalls in precipitation. In recent years, Tallaksen and van Lanen [2] defined the hydrological drought as a sustained and regionally extensive occurrence of below-average water availability. Hydrological droughts can cover extensive areas and can last for months to years, with devastating impacts on the ecological system and many economic sectors such as drinking water supply, crop production (irrigation), waterborne transportation, electricity production (hydropower or cooling water), and recreational activities (rowing, boating, etc., due to low water levels in lakes, rivers, reservoirs, etc.). The other deleterious impact of the hydrologic drought is the deterioration of the water quality as a consequence of the decline in water quantity in the surface water bodies. Almost all studies on hydrologic droughts have used streamflows as the drought variable primarily due to the wide availability of recorded streamflow time series across the globe. It is for this reason that while dealing with droughts based on streamflows, the term streamflow drought is also commonly used.

1.1. Time Scales of Hydrological Droughts

The hydrological drought analysis began with the annual scale (annual droughts) as pioneered by Yevjevich [1] and associates [3,4]. This work on an annual scale was extended by Dracup et al. [5], Sen [6], Guven [7], Yevjevich [8], Lee et al. [9], Horn [10], Fernandez and Salas [11], Sharma [12], Salas et al. [13], Panu and Sharma [14], Akyuz et al. [15], Sen [16], among others. The yearly analysis offered theoretical simplification as the annual flow sequences can be perceived as stationary in the statistical and stochastic sense. The stationarity requirement eased the application of the existing theories of probability and stochastic processes, which paved the way for further analysis on a short time basis such as month and week. Although the annual scale is rather long, it can be used to abstract information on the regional behavior of hydrologic droughts and the assessment of equivalent deficit volumes that need to be stored in the reservoirs on a long-term basis. An appropriate time scale for the analysis of hydrological droughts can be deemed as a month [17–29] because of the consideration that a month is a reasonable time unit for monitoring drought effects in situations related to water supply, groundwater abstraction, crop irrigation, management of reservoirs for hydropower generation, etc. However, it is to be noted that Wu et al. [27] used multiple time scales (month to year) using non-standardized streamflow indices. Also, for the hydrologic design of reservoirs in the context of dams, a monthly scale of flows is considered adequate [17,25], and thus, the drought magnitude-based analysis with a monthly scale is very apt. In recent years, Sharma and Panu [24,30], and Gurrapu et al. [31] have extended the analysis to a weekly scale for assessing duration and associated water shortages within a year or a season, while reckoning the strong persistence characteristics of hydrologic droughts on a short time basis. It is well known that the operational drought forecasts are traditionally issued on a weekly basis such as Palmer drought severity indices and Palmer moisture anomaly indices, which make weekly analysis very desirable. Further, weekly analysis is a more precise way of assessing the water needs during drought periods on an emergency and crisis basis. Additionally, weekly analysis provides a complementary way of testing the validity of drought models developed on a monthly scale, as the periodic or non-stationary effects on flow sequences are imbued on both time scales [24]. For studying the behavior of short-term droughts within a year or a season, the daily scale has also been used [2,23,32–41].

1.2. Parameters of Hydrological Droughts

Hydrological drought is characterized by a multitude of parameters but chiefly by (a) duration; (b) magnitude (earlier called severity); (c) time of initiation and cessation; and (d) areal spread. At times, the term magnitude is complemented by the intensity, which is defined as magnitude/duration by Dracup et al. [5]. Conceptually, the above drought parameters can be illustrated in terms of an untransformed (or historical) flow sequence (Figure 1A) and a transformed (or standardized) flow sequence (Figure 1B).

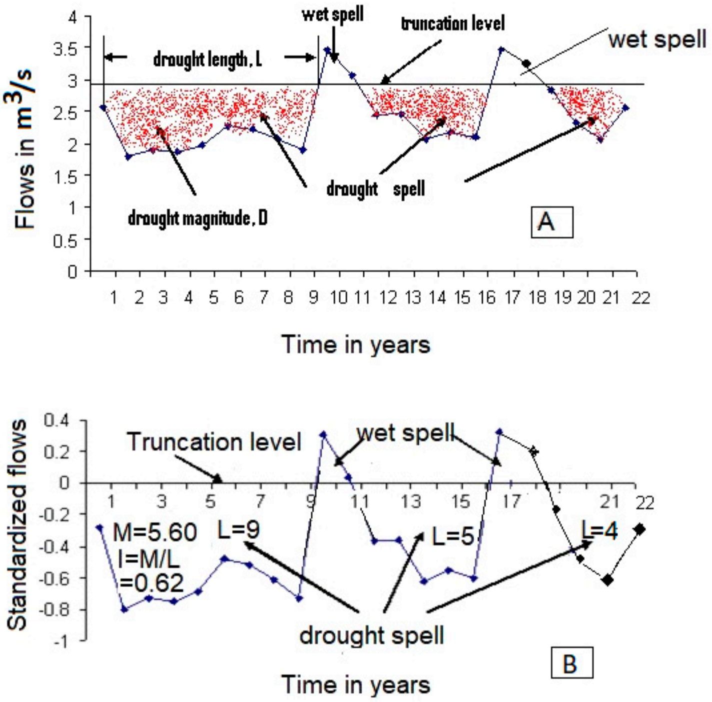

Figure 1. Definition sketch of hydrological drought parameters: (**A**) natural (or untransformed) flow sequence and (**B**) standardized (or transformed) flow sequence.

Natural (or untransformed) form means a sequence of annual flows $\{x_i\ (m^3/s)\}$ as they are observed and recorded in the field, and later on reported in the hydrological yearbooks or related websites, whereas the standardized form means $\{x_i\}$ is transformed to $\{e_i\}$ such that $\{e_i = (x_i - \mu)/\sigma\}$, where μ and σ are mean and standard deviation of x_i sequence, and thus, $\{e_i\}$ sequence has mean = 0 and standard deviation = 1.

It should be mentioned that in the earlier literature from the 1960s until recently (say the early part of the first decade of the 21st century, i.e., 2000–2006) on hydrologic droughts, the term severity was used to denote the cumulative deficit. The cumulative deficit is named as magnitude in the context of meteorological droughts or when the drought variable is precipitation [42–44]. The usage of the term magnitude in contrast to the term severity is increasing over time [19,24,45]. This transition seems to be motivated by ambiguity in the meaning of the term "severity". In the context of meteorological drought, severity is expressed to indicate the rigor or the category of the drought such as mild, moderate, severe, or extreme. For example, Palmer [46] suggested an index that now is popularly known as the "Palmer drought severity index "(PDSI)" to indicate the severity of drought from mild (PDSI < −1.0) to extreme (PDSI < −4). Likewise, another index termed the standardized precipitation index (SPI) suggested by Mckee et al. [42,43] is being currently used to denote the category (mild, moderate, severe, extreme) of the severity of meteorological droughts. In the context of hydrological droughts, the severity denotes a deficit in volumetric or depth units, and therefore, its usage contradicts the index connotation associated with meteorological droughts. The term magnitude alleviates this anomaly and is also in sync with the volume (essentially magnitude) connotation associated with the cumulative deficit. Given the foregoing discussion, the term magnitude has been used to denote the cumulative deficit in this paper.

The most basic element for deriving the above parameters is the truncation level which divides the time series (or sequence) of the desired drought variable such as streamflow into sections named as "deficit" and "surplus". The parameters of a drought such as duration, magnitude, and intensity are the properties of the deficit sections. Consequently, the duration (L) has the unit of time such as year, month, week, or day, depending on the time unit of variable manifesting the drought. For instance, when dealing with monthly streamflows, drought duration will be registered/recorded in months. The term deficit (D) refers to the cumulative shortage (sum of individual deficit epochs in a drought episode) below the chosen demand level (or the so-called truncation level), and it has the unit of volume, i.e., m^3. It should be borne in mind that the truncation level in the parlance of hydrologic drought is representative of the demand level of water to be met by a river in the equivalent form. For example, in a river, if the water needs are assessed as equivalent to the mean flow of the river; then, the truncation level can be taken as the long-term mean flow. In reservoir design, the demand level is taken preferably as 75% of the mean annual flow (also known as a draft) [25,47]; so, under such a situation, the truncation level may correspond close to the median (Q50, i.e., flows exceeding or equal to 50% of the time in a flow duration curve). In most cases, the median (Q50) is considered to be a better choice as the mean annual flow imposes a significantly high value of draft, resulting in exceptionally large size of reservoirs. The drought duration and deficit can be analyzed in the standardized form for the ease of interpretation and the possible inter-comparison of drought scenarios in varied environments. For a standardized flow sequence, the deficit is termed as magnitude and denoted by M (=D/σ; Yevjevich [1]), which is a dimensionless quantity. However, the unit of duration (L) remains unchanged and consequently would have a time unit of month or year depending on whether a monthly or annual flow sequence is being analyzed as the drought variable. Therefore, a value of L = 9 (Figure 1) in the standardized domain is tantamount to L = 9 years for the case when the analysis is carried out on a yearly sequence of the drought variable. The quantity M/L is termed as the drought intensity (I). The longest run length for a sample size of T years (or equivalently of the return period of T-year) is denoted as L_T and the corresponding deficit as D_T (or M_T in the standardized form of the sequence of a drought variable). Similar connotations apply when the analysis is being carried out on a monthly or weekly basis, i.e., T would represent the return period in months or weeks. In mathematical terms, $M_T = I \times L_T$; $D_T = \sigma \times M_T$, with T being the time in year(s), month(s), or week(s) of the streamflow sequences considered for analysis. The term deficit (D) is tantamount to reservoir volume.

1.3. A Note on the Choice of Truncation Level for Identification of Hydrological Droughts

Yevjevich [1] introduced the concept of the truncation level approach and statistical theory of runs in the analysis and modeling of hydrologic droughts. The truncation level specifies some statistics of the drought variable, which may be a constant or a function of time. Several investigators have considered it as a long-term mean or a median flow [1,5–7,12,14]. At times, it can be construed as a very relaxed criterion, simply because droughts are unlikely to be tangible at such a level of truncation. In general, a drought is perceived when the flows in a river are low, rocks in the riverbed may protrude in the air, lake and groundwater levels are low, and wells tend to run dry. It is for this reason that truncation levels such as Q70 (flows exceeding or equal to 70% of the time in a flow duration curve), Q90, Q95, etc., are used for identifying the droughts in flow series. It is because of this consideration that recent researchers [23,32,33,38] have preferred other percentile levels from the flow duration curve ranging from Q50 to Q95. In a statistical sense, the truncation level at the mean flow simplifies the analysis on a seasonal (monthly) scale, meaning that the drought is considered to occur when the flow drops below the mean of the respective month. Likewise, when one takes the median (Q50) or Q70 as the truncation level, the flow is deemed to drop below the Q50 or Q70 flow of the respective month. These threshold values of flow are not uniform even though the probability of occurrence of drought is uniform throughout 12 months. When the analysis is conducted in terms of the standardized streamflows (named SSI), the truncation level becomes uniform and equal to zero only for the situation in which the truncation level is the mean. At other flow levels, the truncation level will feature as a curve with respect to time, yet the probability of drought would remain the same for all months. Thus, the non-uniform flow cut-offs can easily absorb the climate change and/or land use changes for the hydrologic drought analysis. The same argument applies to the impact of truncation levels on the reservoir storage requirements. Specifically, at high drafts (high truncation levels), the storage requirements would increase, whereas at low drafts, the storage needs would be minimized with a greater risk of the reservoir running dry. This is the reason that an optimal truncation level for reservoir analysis is taken as 75% of the mean annual flow [25,47]. Further, the truncation level and the corresponding drought probability will be governed by the pdf of monthly flow sequences, and the linkage relationships are documented by Sharma [12]. On an annual basis, the truncation level on plotting will be a straight line, and the corresponding drought probability will be dictated by the pdf of the annual flow sequence [12]. The truncation level (Q50, Q70, etc.), in terms of changed flows in the wake of climate change or land use considerations in a catchment, can be moved up or down appropriately.

A flow duration curve could be constructed based on annual, monthly, weekly, or daily flow sequences. For the near-normal probability distribution function (pdf) of streamflow, the mean serves the purpose of a truncation level, whereas, for skewed distributions, the median should be used as a better measure of truncation level [24]. For the design and planning of a water storage system on a permanent or a long-term basis for ameliorating drought events, the use of a truncation level corresponding to the mean or median level of flow, for example, would result in a conservative design alluding to the need for a desirable drought mitigation scenario. In a regional drought frequency analysis, on the contrary, a value of the truncation level such as Q70 or Q80 would portray more tangible drought impacts over the region. However, in a short-term contingency planning for drought amelioration where drought impacts are vividly tangible, one could conduct drought investigations even at Q90 to allow for the mobilization of resources on a cost-effective basis. In the design of reservoirs, the draft is normally taken as 75% of the mean annual flow [47]; therefore, the truncation level of 75% of the mean annual flow is a wise consideration [25]. Irrespective of the truncation level, some anomalies persist in the process of delineating the dry (or drought) and wet periods. For example, if two drought spells, each of five months are separated by a wet spell, say of one month, then a natural question arises whether this one month has eliminated the drought or the drought is still persisting even during this (wet) month. It is obvious that although the flows during this month are above the cut-off level, it does not state that drought impacts on the river ecology, fish habitat, and other environmental and water use concerns are over during such a short period of a month. Likewise, when two long wet spells are broken by a dry spell of one or two months, then the dry spell is unlikely to be perceived as a drought. Under such a situation, the effects of wet spells are still rampant. The concerns arising from the above issues on the hydrological drought identification were recognized by Zelenhasic and Salvai [32], which led them to improvise the identification procedure as follows. They used Q90 or Q95 as the truncation level and observed that when the flow exceeds the threshold level for a short period of time, a long drought tends to be segmented into several small droughts, which were also found to be mutually dependent. Such an observation led to the consideration of some pooling procedures where small drought events could be pooled together to define an independent sequence. Therefore, they devised a method named inter-event-time and volume-criterion (IC) for pooling together small drought events. Also, two other methods, i.e., a moving average procedure (MA) and a sequent peak algorithm (SPA), have been suggested by Tallaksen et al. [33] for resolving the above problem while using truncation levels such as Q50, Q70, and Q90. Further, a comparative analysis of the aforesaid three methods, i.e., IC, MA, and SPA, led [33] to the inference that IC and MA methods were comparable, whereas the SPA method tended to be restrictive in terms of drought durations and less satisfactory at higher truncation levels. The inter-event-time of 5 days (IC method, meaning that any contiguous drought events separated by an inter-event-time of 5 days or less) were pooled together, and the averaging period of 11 days (MA method) was found satisfactory for two Danish catchments [33]. Tate and Freeman [36] found that the IC method was satisfactory for Zimbabwean rivers with an inter-event time of 6 days. On daily flows, Fleig et al. [38]

tested the above three methods on 16 rivers across the globe and found the MA procedure to be the most flexible and worthy of recommendation.

1.4. Indices of Hydrological Droughts

The indices in the realm of hydrological droughts are still in the phase of development, and none can be said to be widely acceptable unlike the Palmer drought severity index (PDSI) [46] and standardized precipitation index (SPI) [22,42–44,48,49] in the arena of meteorological droughts. Since hydrological droughts are more related to the declining levels of surface water resources and primarily to streamflow, the interest has been more on the quantification of drought magnitudes for assessing shortages of water supplies during the extended periods of droughts. The truncation level approach advanced by Yevjevich [1] has therefore been rigorously applied for quantifying drought magnitudes of varying return periods. Consequently, the efforts have focused on developing conservative estimates of drought magnitudes based on the long-term mean or median as the truncation level of streamflow time series, even though droughts at this truncation level may not be tangible in the river basin, and hence, lower truncation levels may be more desirable such as Q70. For water supply situations, an index named as surface water supply index (SWSI) was developed as a measure of surface water status for the state of Colorado by Shafer and Dezman [50] to complement the PDSI by integrating snowpack, reservoir storage, streamflow, and precipitation at high elevations. Since SWSI has a scale similar to PDSI, both SWSI and PDSI are used together to trigger the drought assessment and response plan for the state of Colorado. The SWSI has been modified and adopted by other western states in the USA (Oregon, Montana, Idaho, and Utah) and is computed primarily for river basins [51,52] situated in these states. Another prominent index that falls under the purview of hydrological droughts is the PHDI (Palmer hydrological drought index) [53].

Recently, some attempts have been made to develop the indices for hydrological droughts in tandem with meteorological droughts such as SRI (standardized runoff index) [54], SDI (streamflow drought index) [55,56], and SHI (standardized hydrological index) [24,25,57]. Indices such as SDI and SRI are essentially standardized and normalized to characterize the hydrological droughts with respective time units (i.e., annual or monthly scales). It should be borne in mind that SRI and SDI are derived from the lognormal pdf of the monthly flow sequences. Nalbantis and Tsakiris [55] have defined four states for the hydrological droughts, i.e., mild drought ($-1 \leq$ SDI < 0); moderate drought ($-1.5 \leq$ SDI < -1.0); severe drought ($-2 \leq$ SDI < -1.5); and extreme drought (SDI < -2.0), in sync with drought states defined by SPI for meteorological droughts.

In the context of hydrological droughts, the standardized streamflow is named a standardized hydrological index (SHI) by Sharma and Panu [24,25], which is not a normalized index but follows a gamma pdf. It should be noted that SPI is standardized and normalized, whereas SHI is only standardized but not normalized. On monthly and weekly scales, the standardization implies the month-by-month or week-by-week standardization in the aforesaid indices. WMO [49] has used the term standardized streamflow index (SSI), which can be regarded as a general term with offshoots such as SRI, SDI, and SHI. Zalokar and Kobold [28] have used the term SSI, which is a term synonymous to SHI used by Sharma and Panu [24]. The SSI has been derived (normalized) for several pdfs of streamflow sequences other than gamma and lognormal [21]. Since SSI is an analogous term to SPI, therefore, it seems to have gained better acceptance in the parlance of the hydrologic drought [20,26,58,59]. A review highlighting the merits and shortcomings of various indices concerning meteorological, hydrological, and agricultural droughts is provided by Mishra and Singh [60].

2. Hydrological Drought Modeling—Relevant Preliminaries

For predicting the aspects of frequency, duration, and magnitude of hydrological droughts, a variety of models based on the probability theory and the data simulation approach are in vogue, and the following subsections describes their common components.

2.1. Identification of the Probability Distribution of Streamflow Sequence as a Drought Variable

The modeling of droughts (or more aptly the drought parameters) begins with the identification of the underlying probability distribution (i.e., pdf) of the streamflow sequence, where streamflow acts as the drought variable. For identifying the underlying pdf of the drought variable, the product moment and L-moment (the linear combination of product moments) are the most popular tools. The mathematical aspects of L-moments are described well by Hosking and Wallis [61]. In the product moment method, the plot between skewness (γ) and coefficient of variation (cv) provides a clue on the probable underlying pdf. For a normal pdf, the γ is 0 irrespective of cv. To reaffirm the hypothesis of the underlying normality of a sequence of size N, the standard statistical test $[0 \pm 1.96 \times (6/N)^{0.5}]$ corresponding to a 95% confidence level for a normal distribution, vis à vis sample values of γ can be used. In the case of a gamma pdf, the scatter of points corresponding to the sampling estimates of γ and cv should lie around a straight line defined by γ = 2cv. Likewise, for a lognormal pdf, the points related to the estimated γ and cv should scatter around the theoretical curve defined by $\gamma = 3cv + cv^3$. In the ambit of the L-moments, the plots of L-skewness and L-kurtosis, respectively, are designated as τ-3 and τ-4, which tend to be better descriptors of the pdf of data. There are characteristic plots of the L-skewness and L-kurtosis for pdfs such as normal, gamma, lognormal, etc., which can be matched with the sampling estimates of these parameters. The product moment and L-moment diagrams are complementary to each other in the process of identifying pdf. It should be noted that the stable estimates of L-moments require a large sample size, which is generally available for monthly, weekly, or daily time series of a drought variable. On the other hand, an annual sequence, commonly, has a small sample size, typically ($N < 100$), such that a good amount of useful information is unlikely to result from the L-moment analysis. In such a case, the product moment method can be regarded as sufficient, or other methods of identification such as a plot on a normal probability paper or significance test on $\gamma = 0$ should be considered.

2.2. Identification of Dependence Structure in the Streamflow Sequence

The dependence structure of the data can be ascertained by computing autocorrelations at lags 1, 2, 3, ---,10. For a Markov process, the autocorrelations at increasing lags tend to decay exponentially. The tests for the significance of autocorrelations at various lags are well described by Box et al. [62], which can be used to infer the order of dependence (random, Markov order-1 or Markov order-2, etc.). To validate the hypothesis of independence or randomness of a sequence (size N), the standard statistical tests $[0 \pm 1.96 \times (1/N)^{0.5}$ corresponding to 95% confidence limits for randomness vis à vis sample values of autocorrelations] can be employed. In addition, Box et al. [62] have suggested the use of a chi-square statistic (called as Portmanteau statistic) based on the initial (16 to 25) values of the autocorrelations to infer independence in the sequence.

2.3. A Note on the Theory of Runs as Used in Drought Modeling

The theory of runs is well developed in the ambit of statistics and probability, which can be transposed for analyzing the drought phenomenon [4,8]. The segments (or sequels) of wet (surplus) or drought (deficit) values as shown in part-A, and part-B (Figure 1) can be regarded as runs. The wet values can be designated as "w", whereas drought values as "d". Therefore, the flow sequence shown in part-A or part-B (Figure 1) can be written as dddddddddwwdddddwwdddd. This sequence comprises five runs of which three runs represent drought spells and two runs represent wet spells. It is obvious and also stated for clarity that each drought event or episode is tantamount to a run. The length of a run is equivalent to the drought duration and a run sum is equivalent to the drought magnitude. The first run has a length of nine, implying the drought duration is of 9 years. The area under the red dots is the run sum. When the analysis is conducted in the standardized domain, the sequence of d's and w's of the drought variable remains the same with the length of the first run as nine (i.e., drought duration of 9 years) and the run sum (i.e., standardized drought magnitude, M of 5.60, with no units attached). On the yearly scale, 9 means 9 years, and hence, M = 5.60 × (σ × 365 × 24 × 60 × 60) m^3 (i.e., σ in m^3/s is converted into m^3/year). When one is dealing with monthly streamflows, then the drought duration is 9 months, and the standardized drought magnitude M = 5.60 × (σ × 30 × 24 × 60 × 60) m^3. The sequential occurrences of d's could be random or may follow some dependence structure (the simplest being the Markov dependence). In simple terms, the number of runs (or the number of drought events) can be modelled using the Poisson probability law, the length of run (i.e., the drought duration) can be modelled using geometric probability law, and likewise, the drought intensity (= magnitude/duration) can be modelled by the truncated normal pdf. Thus, the theory of runs provides a powerful tool for the analysis of parameters of hydrological droughts.

One major requirement for the application of the theory of runs is that the time series of a drought variable must be at least weakly stationary in the statistical sense. The requirement of stationarity is generally met for the annual precipitation (or streamflow) sequence of the drought variable. The sequence of monthly or weekly streamflows is non-stationary and must be transformed into a stationary time series. The month-by-month or week-by-week standardization renders the non-stationary series into a stationary one which can readily be subjected to hydrological drought analysis by the theory of runs. Once a suitable probability distribution fitting a monthly or weekly flow sequence is ascertained, the underlying dependence structure of such a flow sequence can be investigated as briefly described below.

For a flow sequence $\{x_{y,t}\}$ comprising of the t^{th} month (t = 1, 2, 3,. . . . . .,12) in the y^{th} year ($1 \leq y \leq N$), the standardized series $\{u_{y,t} = (x_{y,t} - \mu_t)/\sigma_t\}$ with μ_t and σ_t, respectively, being the mean and standard deviation of the t^{th} month is rendered as a stationary sequence with mean zero and variance one. Since $\{u_{y,t}\}$ series is non-periodic and stationary stochastic, it can be designated as $\{e_i$, $1 \leq i \leq n$ (=12 × N$\}$. In the case of a weekly sequence, "t" ranges from 1 to 52, and n = 52 × N. Hence, there are 52 μ's and 52 σ's for the weekly flows, whereas 12 sets of μ's and σ's stand for the monthly flows. It is recognized that $\{e_i\}$ is a standardized series but not a normalized one because its generic source $\{x_{y,t}\}$ is generally non-normal for a monthly or a weekly flow sequence. For an annual sequence, $\{e_i\}$ can be obtained by standardizing an annual sequence $\{x_i\}$ by a set of μ and σ (i.e., $e_i = (x_i - \mu)/\sigma$, $1 \leq i \leq N$). The $\{e_i\}$ series being stationary (with mean 0 and standard deviation 1) can be analyzed using the theory of runs. Since $\{e_i\}$ series is standardized, it is referred to as the Standardized Hydrologic Index (SHI) series in the ensuing text. When SHI is normalized, it is called SSI, developed by WMO and GWP [49].

3. Major Modeling Methodologies

There are four dominant approaches for analyzing and predicting the drought parameters, i.e., duration and magnitude, and are described as follows:

3.1. Empirical or Frequency Analysis of the Historical Drought Data

The first data can be said to be an empirical one, in which, the annual maximum or partial duration series of drought durations and magnitudes are fitted to the suitable pdfs and the corresponding return periods of the aforesaid parameters are estimated [2,23,32,33,37,39]. The durations and deficits are identified by choosing a suitable truncation level such as Q70, Q90, etc., and the duration and magnitudes are determined by a counting technique on the historical streamflow flow data. The method offers the advantage that the streamflow data can be analyzed in a non-standardized form. The drawback of the method is that it relies heavily on the recorded data series, which at times, could be short in length. The method, therefore, offers the advantage of being independent of standardization of data or hydrologic drought index calculations for the analysis and prediction. On a short time scale, such as a day during summer months, this method can be used for predicting the drought lengths and estimating the deficit volumes for combating the summer droughts. However, on a monthly or annual scale, its applicability is limited because the sample size

(N) generated by truncating the monthly and annual flow sequences turns out to be very small, particularly in data-deficient situations. Therefore, on a monthly or annual basis, the other methods discussed below are used.

3.2. Experimental or Time Series Simulation by Monte Carlo Method

This method relies on the synthetic streamflows generated by the Monte Carlo method of data generation. The techniques of generating independent and dependent normal, gamma, and lognormal flows are well documented in the literature on stochastic hydrology [63,64]. In recent years, considerable advances have been made in streamflow synthesis using the artificial neural network (ANN) methodology [65,66], non-parametric simulation methods [67,68], and copula [69] and entropy-based methods [70]. Thus, the methodology of generating annual and monthly streamflows is well developed, which can aid in simulating the long series of streamflow sequences. The stochastically generated streamflow series can be broken into several non-overlapping segments of the size N (year or month) and truncated at the desired flow level. By an enumeration technique, the drought duration and corresponding magnitude are counted. Each segment of size N will yield the longest length and the largest deficit, which would form the basis for successive analysis. It is desirable to have the size N to be long to approximate the equivalence of N to return period, T (i.e., N = T). For example, if interest lies in determining the $L_{T=75}$ (drought length with a return period of 75 years), several segments (say 10 samples) of synthetic flows of the size N = 75 can be constructed. Each sample is truncated at the desired flow level and runs of drought (below the truncation level) are counted. From each sample, the run with the longest size is used to count the longest run representing $L_{T=75}$. Such values of $L_{T=75}$ are counted for all 10 samples, which can be averaged out. The same exercise is to be carried out for deficit $D_{T=75}$. For example, if the length of such longest negative runs is found to range from say 4 to 6 years with a mean value of 5.2 years, then $L_{T=75}$ = 5 years. Likewise, the deficit volume (D_T) in each of the 10 samples can be calculated (by summing up the individual deficits) to the chosen truncation level. The averaged-out value of D_T can be worked out based on the 10 samples and is designated as $D_{T=75}$. The relationship ($M_T = D_T/\sigma$) is used to determine the magnitude $M_{T=75}$. These longest (L_T) and largest (M_T) values are tantamount to critical duration and magnitude as are used in the context of reservoir design [25]. Using the above procedure, the critical duration and magnitude corresponding to a desired combination of the return period (say T = 50, 100, 200, 500 or 1000 years) and the truncation level (say Q50, Q60, ..., Q95, etc.) can be obtained. The simulation of streamflow data can be carried out on several rivers in a region, and drought duration and corresponding magnitude can be mapped on an area basis along with their return periods.

Based on such simulation experiments, Millan and Yevjevich [3] discovered that the distribution of L_T and M_T can be represented by the lognormal probability function, and the relationships for logarithms of L_T and M_T are expressed in terms of T (return period), q (drought probability), ρ (lag-1 autocorrelation), and γ (coefficient of skewness) of the annual flow sequences. Horn [10] found that the regression relationships worked well for rivers in Idaho for annual droughts. Panu and Sharma [14] noted that the aforementioned relationships were satisfactory in predicting the drought parameters (L_T and M_T) on an annual basis for Canadian rivers and compared well with the analysis based on the extreme number theorem introduced by Sen [6].

It is prudent to discuss the procedures to estimate the drought probability q. A simple and distribution-free procedure is used to rank the streamflow data (size N) in ascending order, i.e., the highest value at the top. The ranked data can be chopped at the desired cut-offs or truncation level (i.e., mean, Q50, Q70, etc.), and all the values below this level are counted (say N_d). An estimate of drought probability can be expressed as $q = N_d/N$. The other more precise procedure is based on using the pdf of the streamflow sequence. In this procedure, the pdf of the streamflow sequence is integrated from 0 to the cut-off level say x_0 ($q = \int f(x)dx$), where f(x) stands for the mathematical form of the pdf of the streamflow sequence under consideration. In other words, this integrand is the area under the probability curve from the cut-off level to the left until the least value = 0. For normal pdf, standard normal tables can be used to determine this area or a polynomial function approximating the standard normal function can be used. It is noted that a normally distributed flow sequence yields q = 0.5 when truncated at the mean or median flow level. In addition, a positively skewed flow sequence following either a gamma or a lognormal pdf when truncated at the mean flow level will yield q > 0.5, and at the median level, q = 0.5. The probability function-based procedure for determining the drought probability (q) for normal, gamma, and lognormal streamflow sequences is documented by Sharma [12]. The algorithms for estimating ρ and γ are documented in statistical books such as [62].

Though the time series simulation method is a powerful procedure, it is likely to suffer from the limitation that simulation should be performed with a knowledge of the precise pdf of flow sequences. Even though the technology for simulating annual and monthly flow is reasonably well developed, the uncertainty level of simulated flows is high in arid environments, which are more prone to hydrological droughts. Further, the technology for simulating the weekly and daily flows is still in its nascent stage, meaning that drought analysis and prediction are limited only up to months, whereas the weekly scale is more desired for drought monitoring and forecasting purposes. Nevertheless, the monthly scale is very useful in terms of the design and management of water resources systems on a long-term basis as a viable strategy for combating droughts on a risk-based approach.

3.3. Probability-Based Analytical Methodologies

The third approach can be said to be analytical in which the axioms of probability are used to derive the drought parameters using statistics (mean, variance, skewness, autocorrelation, etc.) of the drought variable. This approach is named hereafter the Probability Theory Based Approach [14]. By and large, the statistical theory of runs has been a major tool for analysis since the 1960′s. Further, the identification of a pdf of the drought variable and its dependence structure is a prerequisite for the analysis and prediction of the drought parameters, i.e., L_T and M_T. The basic axioms of probability are used to derive expressions for L_T and M_T [6,7,12]. The approach is shown to be tractable for normal, lognormal, and gamma distributions [12]. Unlike the graphical or empirical procedure used in the time series simulation approach, q at a given truncation level is obtained based on the basic laws of probability. For Markovian drought spells, the conditional probability is estimated using the information of lag-1 autocorrelation through a set of analytical relationships or some counting procedure [16]. There are other offshoots of the analytical approach such as those built on the hazard functions [9], the low-order discrete autoregressive moving average processes [71], the compound renewal processes [72], etc. The methods which are adapted and pursued vigorously for further research are discussed as follows.

3.3.1. Extreme Number Theorem-Based Method

In the realm of the analytical approach, the prediction of drought length was based on the extreme number theorem [6], which was applied to Canadian streamflows [24]. The prediction of the drought magnitude was accomplished by coupling the drought length to the drought intensity proposed by Dracup et al. [5]. The performance of the extreme number theorem was found satisfactory for annual droughts and weakly persistent monthly droughts. For strongly persistent monthly and weekly droughts, the extreme number theorem was found less satisfactory [24]. Further, the extreme number theorem requires the flow sequences to be standardized or the analysis to be performed in terms of the SSI. The extreme number theorem is truly titled as a theorem of extremes of random numbers of random variables [6,14]; therefore, it is most suited for randomly evolving entities such as annual SSI sequences. For month-based drought analysis for hydrologic drought parameters, it is necessary to have weak autocorrelations in the SSI sequences.

3.3.2. Markov Chain-Based Method

The Markov Chain methodology was applied in hydrological drought analysis by Sen [73], Nalbantis and Tsakiris [55], Akyuz et al. [15], Sharma and Panu [24], Tabari et al. [56], Yeh [57], Yang et al. [74], among others. The methodology works on standardized flow sequences, and T-year drought duration can be estimated. For the estimation of the drought magnitude, a simple linkage relationship, i.e., magnitude = intensity $\times$ duration is used [24]. The intensity function is represented by a truncated normal distribution. The Markov chain method (MC) worked very well for modeling the Canadian streamflow droughts for the time scales ranging from week to year [24]. On the weekly scale, second-order MC were found more appropriate, whereas first-order MC was adequate on the monthly scale and zero-order MC on the annual scale [24]. While comparing the extreme number theorem with Markov chain methodology, the MC method was found to be much simpler and capable of handling the random-to-highly persistent flow sequences. The results based on MC methodology were applied to the sizing of reservoirs for Canadian rivers, which were found comparable [25] to prevalent methods of reservoir sizing documented in McMahon and Adeloye [46].

An offshoot of the Markov chain-based method is the log-linear modeling approach, which was first introduced by Paulo et al. [75] for SPI sequences under the agro-climatic conditions of Portugal. The method was adopted by Li et al. [76], where the SRI sequences were converted into drought class intervals bearing the discrete numbers. Using such discrete class numbers, contingency tables were formed, and from the statistics of various drought level transitions, the short-term hydrological droughts on a monthly scale were predicted. The method was applied to the Luahne River basin in China [76] with encouraging results in terms of reliable forecasts.

3.3.3. Copula-Based Method

Another emerging technique under the analytical approach is that of "Copulas". The copula is a function that links two or more univariate marginal distributions to form bivariate or multivariate distributions. Since the drought duration and drought magnitude exhibit significant correlation, a bivariate distribution is used to jointly model the drought duration and magnitude. Generally, the drought duration and drought magnitude are modelled by different distributions; the copula offers an attractive technique of modeling them jointly by constructing a suitable bivariate distribution [18,77–83]. Bivariate return periods are also established to explore the drought characteristics of the historically observed droughts. The methodology has been successfully employed to model the drought characteristics on a monthly scale, i.e., the duration and magnitude jointly of the Yellow River in China [18]. The work was further extended by involving daily flows from the East River in China [81]. There are multiple copula classes and families such as Archimedean, meta-elliptic, extreme value, Plackett, Gumbel Hoggard, Frank, Clayton, etc., and each of them needs to be tested for a particular situation. Song and Singh [78] found the Plackett copula most suitable for hydrologic drought modeling with periodic streamflows, from three rivers in China. In a study of the Karkheh River basin in Iran, Archimedean, Clayton, Frank, and extreme value copulas were used by involving daily flows from three gauging stations [82]. The study was aimed at predicting the return periods of hydrologic droughts at the Q75 level of truncation in the daily flow duration curve. The results show that the stations located in tributaries of the main river have smaller return periods for drought events than the main tributary for identical drought duration and magnitude. In general, it is noted that the copula-based methodology offers a unique opportunity for its use in reducing uncertainties in the estimates of frequency distribution parameters [83]. One special merit of the copula method is that it can be used to model the drought parameters up to a daily scale, thus rendering the analysis workable for non-stationary or periodic streamflows. This attribute of the method makes it very attractive for short-term drought monitoring and forecasting such as on a weekly scale.

3.3.4. Entropy-Based Method

Entropy may be interpreted as an indirect measure of the information content of a process. It has applications in many areas of water resources and hydrology such as interaction between hydrological processes in terms of information transfer, planning of the measurement network, optimization, and decision-making theory. In terms of drought studies, it has found applications to synthesize flow series [69,70]. The simulated series can be analyzed using the desired truncation levels to estimate the hydrologic drought lengths and magnitudes corresponding to the desired return periods. Further, Hao and Singh [84] applied the entropy-based method exclusively to model the drought duration and magnitude to determine their corresponding return periods on monthly streamflow data for the Brazos River at Waco, Texas. There are few other studies using the entropy-based method for catchments in Turkey [85] and in China [86] using SPI as a basis for modeling.

In a recent study, Yang et al. [87] applied a copula-based entropy method for the analysis of hydrological droughts in the Kaidu River China. Almost all the studies tend to suggest that the entropy-based method is a good alternative to traditional methods for hydrological drought analysis and prediction. Despite the good capability of entropy-based methods in drought modeling, they exhibit some limitations. For many types of entropy models, there is no analytical solution for optimization and parameter estimation. This calls for numerical solutions or sampling approaches for estimating the parameters. For high-dimensional problems, as the number of parameters grows, parameter estimation becomes more computationally demanding [88]. Nevertheless, the method is still in a nascent stage, and considerable work is warranted before commenting on its merits and limitations.

3.3.5. Wavelet Transform (WT) Method

Wavelet transform is a powerful mathematical signal processing tool like Fourier transform with the ability to analyze stationary and non-stationary data as encountered in hydrologic scenarios such as monthly or weekly streamflows. The advantage of WT over a Fourier transform is the use of a shifting window of variable width that enables enhanced resolution along all frequencies. In the WT method, the signal can be decomposed into a series of wavelet functions for analysis at various resolutions, thus capturing both time and frequency information. This property makes the wavelet transform valuable for examining non-stationary signals or data that are common in the wake of climate change and other environmental changes.

There are some applications for meteorological drought prediction using PMDI (Palmer modified drought index) in Texas [89] and using SPI on a monthly basis in the Awash basin, Ethiopia [90]. In another study, using wavelet packet transform (WPT; an extension of wavelet transform), Das et al. [91] used SPI sequences for forecasting droughts in a district in Karnataka state, India. They found that the hybrid model WPT-ANN performed better than the stand-alone model, i.e., ANN. In all such studies, the incorporation of the wavelet analysis significantly improved the prediction results.

Recently, using SDI for a month ahead hydrologic drought forecast (prediction) for the Ysilirmak River in Turkey, Katipoglu [92] found the method to be optimal, while comparing it with discrete wavelet transform, support vector machines, Gaussian process regression, regression tree, and ensemble tree models. These models were used not only alone but also in combination; however, the effectiveness of the wavelet transform can be restricted due to the choice of the wavelet basis function, as different wavelets may be more suitable for specific types of data. For example, Katipoglu [92] found that the db10 main wavelet was more accurate in predicting short-term droughts than other wavelets.

3.4. Machine Learning-Based Methods

In recent years, new methods based on the concept of machine learning (ML) have been introduced in the arena of hydrological drought modeling. The basis of the evolution of all ML algorithms emanates from the concept that records/data, relevant patterns, and their relationships can be developed to predict the behavior of a specific system or process (such as floods and droughts) in future. Thus, past weather data inclusive of floods and droughts represents a great treasure trove for developing predictive tools involving ML techniques. Drought is the least understood natural disaster because it emanates from complex relationships of a multitude of contributing factors. The initiation and termination of droughts are difficult to assess with certainty and may last for months or even years.

The ML models can handle complex data relationships and interactions and can identify non-linear and higher-order dependencies, as well as capture spatial and temporal correlations in drought data. This is particularly useful for capturing the spatial coherence and propagation of drought events across different locations. Further, these models can adapt and learn from new data such as changes in draft rates for reservoirs caused by human needs either due to climate change or agricultural cropping patterns and/or increasing municipal water requirements. They can continuously update their internal parameters to improve their performance as new observations become available. This adaptability is particularly valuable in drought forecasting, where environmental conditions and climate patterns can change over time in view of impending climate change. Another main advantage of ML models is that they can effectively integrate data from multiple sources, such as satellite imagery, climate models, ground-based observations, and remote sensing. By incorporating diverse data types and sources, ML models can provide a more comprehensive and accurate understanding of drought dynamics. For drought modeling and forecasting methodology, the data sets primarily comprise various drought indices such as SSI, SRI, and SDI. Commonly known ML algorithms are artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS), support vector machine (SVM), random forest (RF), gradient boosting (GB), k-nearest neighbor (k-NN), decision tree (DT), among others. One of the noteworthy contributions in terms of machine learning is that of Shamshirband et al. [93], who used SPI and SPEI to predict SSI by using three MLs for the hydrologic drought in Iran. A comprehensive review of the ML-based drought analysis and prediction models is provided by Nandgude et al. [94]. The salient features of the prevalent ML methodologies such as ANN, ANFIS, and SVM are discussed below.

3.4.1. ANN-Based Method

In essence, ANN is an advanced mathematical tool utilizing the concept of the biological neuro-system. The function of ANN can be compared to the human brain, having nodes connected to one another. Thus, ANN consists of several interlinked nodes called neurons, which are arranged into different layers, namely, input layer, hidden layer, and output layer. The nodes in one layer are linked in the subsequent layers. Each node is assigned a weight that measures the strength of the nodes. During model training, these weights are updated such that the predicted output closely resembles the observed values. The computation within each neuron involves two main steps, a linear combination of inputs and an activation function. The linear combination involves multiplying the input signals by their corresponding weights and summing them. The activation function introduces non-linearity to the output of neurons, thus enabling the network to learn complex relationships and patterns in the data.

Based on the aforesaid features of the ANN methodology, drought analysis has also been attempted by researchers, and one of the pioneering attempts in this field is the regional analysis of droughts by Shin and Salas [95]. Also, the methodology was applied by Morid et al. [96] and Mishra and Singh [83] on SPI sequences for drought forecasting, among others. For hydrological drought, SRI sequences were involved in forecasting daily inflows to a reservoir in Iran for the impending drought periods by Rezaeianzadeh et al. [97]. In brevity, ANNs are a subset of ML algorithms that are designed to learn and recognize patterns from data. Because of this property, ANN tends to become the popular choice among ML methods for drought forecasting.

Fung et al. [98] interpret that the ANN tends to outperform other traditional non-ML-based models with the advantages of less statistical training and its non-linear property. The availability of different variants of ANN is another advantage to cope with different needs and situations compared with the other methods. The major limitation of the ANN method lies in its black-box nature, which impedes the interpretation of the functional behavior of the model. The model performance can be unsatisfactory when the data size is small, while the model can become unwieldy and computationally expensive when the input data is large. It may also suffer from model complexity and overfitting with large input data size.

3.4.2. Adaptive Neuro-Fuzzy Inference System (ANFIS)

The fuzzy-based modeling approach is gaining pace in most of the areas of hydrology and water resources, as it is capable of accounting for uncertainties in processes and interactions among them. Due to its ability to consider uncertainty and vagueness, it works efficiently in real-time forecasting applications. This property was harnessed by Bacanli et al. [99] using SPI as the base for drought forecasting on a monthly basis for a catchment in Turkey. They found that ANFIS resulted in higher accuracy and reliability compared to feed-forward neural networks and multiple regression.

In a recent study, six advanced ML models such as ANFIS, ANN, deep learning neural network (DLNN), fuzzy rule-based system (FRBS), SVM, and decision tree (DT) were applied to the Han River basin in South Korea to calculate the SRI by Jehanzaib et al. [100]. The result revealed that the fuzzy rule-based network (ANFIS) provided better performance compared to neural networks. Similar findings have also been reported in the review paper by Nandgude et al. [94]. Fung et al. [98] list the positive attributes of the method, chiefly its ability to model imprecise data and non-linear functions of arbitrary complexity. The fuzzy rules can be interpreted using natural languages. On the negative side, the method can become computationally expensive with an increase in the number of fuzzy rules. It also requires expert knowledge to define the rules.

3.4.3. Support Vector Machine

Support vector machine (SVM) or support vector regression (SVR) is a popular ML algorithm used for both classification and regression tasks. It is effective in solving both linearly as well as non-linearly separable problems. SVMs are based on the concept of finding an optimal hyperplane that maximally separates different classes or approximates a regression line with the maximum margin. In short, SVMs are different from the traditional regression methods as they find a hyperplane that best fits the data points in a continuous space, instead of fitting a line to the data points. SVMs can handle non-linearly separable data by using the kernel technique. The kernel technique involves the use of a kernel function, which computes the inner product between pairs of transformed data points in the higher-dimensional feature space. The key idea is that this inner product can be computed directly in an original feature space without explicitly transforming the data. One important feature of the SVM is its ability to handle noisy data.

In a recent study on hydrologic drought prediction, Achite et al. [101] studied various ML techniques, i.e., ANN, ANFIS, SVM, and DT to construct a hydrological drought forecasting model for the Wadi Ouahrane basin in Algeria. The results showed that the SVM model outperformed the other models. The performance of the ANFIS and DT was found to be somewhat lower as they overpredicted the drought. The other advantage of SVM is that its convergence rate is faster and thus requires less computational resources and effort.

It has the capability to avoid overfitting while offering the choice of kernels. It can be computationally expensive in the validation stage, as a large number of iterations are needed to tune the parameters [98].

3.4.4. Data Sources, Data Pre-Processing Techniques, and Validation Methods for Assessing the Accuracy and Reliability of Models

A major source of data for hydrologic drought modeling is the gauged or simulated streamflows. The gauged flows are available on a daily scale, which can be converted to weekly, monthly, and annual scales. The available length of data on annual and monthly scales can be extended by techniques of synthetic data generation that are well documented in the literature [63,64]. At times, data can also be generated using distributed hydrological models such SWAT (soil water assessment tool) model [102]. For such models, the input data requirement is high as data on meteorological variables (precipitation, temperature, humidity, wind speeds, solar radiation, etc.), soil water properties, land use, etc., are required. Whatever the data set, it must be pre-processed in terms of quality, homogeneity, and consistency. At times, a considerable amount of data infilling is also needed as gaps are always found in the observed data sets. These data require normalization through Box–Cox transformation [62] and Wilson–Hilferty transformation [19,24] to transform them into SSI, SDI, or SRI sequences. Thus, based on the available data, suitable sequences such as SSI are fed into ML-based algorithms to produce the desired output. Of late, new ML-based techniques of data pre-processing have emerged such as discrete or continuous wavelet transform methods. One major advantage of ML-based techniques for data pre-processing is that it removes the noise in the input data sequences, which results in the improved forecasting ability of ANN, AFNIS and other similar models.

The performance of a particular model is validated through statistical measures such as root mean square (RMSE), Nash Sutcliffe efficiency (NSE), coefficient of determination (R^2), mean relative error (MRE), mean absolute error (MAE), coefficient of correlation (R), etc. The exact details of relationships in terms of observed and predicted values for aforesaid statistics are well documented by Alwasi et al. [103] and Sundararajan et al. [104]. These performance measures can be used solo or in combination to choose a superior model. For example, NSE statistics should be used while being accompanied by the mean relative error (MER), whereas the coefficient of correlation (R) or the coefficient of determination (R^2) can be used solo. The models can be differentiated based on the values of the above statistics, with the notion that a better model should yield a more appropriate value of the performance statistic.

4. Forecasting of Hydrological Droughts

Forecasting of hydrological droughts has vast applications and thus provides ample opportunities in terms of averting and/or combating drought effects. The knowledge of the occurrence, frequency, duration, and magnitude of hydrological droughts could foretell the need for drastic changes in the supply of urban water and hydropower, and hence ameliorate the usual widespread economic impacts of sudden reduction in water availability. The management of irrigation water supply could be enhanced to better meet the expected future needs if it was known that a very low proportion of the normal reservoir inflow is expected to occur in the succeeding months. Multipurpose storage could be managed appropriately by temporarily re-allocating the available and expected storage for irrigation, power generation, urban water supply, and flood control.

There are two major routes for forecasting hydrological droughts. The first one deals with forecasting a drought index based on the historical values of the drought index using the streamflow time series [105]. The linear stochastic models such as the autoregressive integrated moving average (ARIMA) and seasonal autoregressive integrated moving average (SARIMA) class of models [81,105], non-linear artificial neural network (ANN) type models [95], a mixture of linear ARIMA and ANN class of models [81], models based on adaptive neuro-fuzzy inference system (ANFIS) [99], log-linear regression model [75], and a mixture of Markov chain and ANN models [97] fall in this category. In recent years, Jahanzaib et al. [100] and Achite et al. [101] have suggested ML-based models for forecasting hydrological droughts on a short-term basis. The capability of these models is limited to forecasting, say for 1–3 months ahead. Although some of the aforesaid models are applied to values of a drought index, derived from precipitation time series (mainly SPI), they can easily be adapted to streamflow time series by forming a suitable series of hydrological drought index such as SSI or SHI.

The second route deals with the tele-connections of streamflows to ENSO (El Nino-southern oscillation). Suitable drought indices can be derived from the streamflows which can be forecasted using the indices related to ENSO, SST (sea surface temperature), SOI (southern oscillation index), and NAOI (North Atlantic oscillation index). This route of drought forecasting is more apt for a longer time scale such as a year or a season (3 months, 6 months, etc.). Studies on drought forecasting using this route have been attempted in Australia [106] and the USA [107] by correlating the PDSI to ENSO-based indices. Some studies in this realm also exist for European countries [108], Canadian watersheds [109], Iranian catchments [110], and Turkish hydrologic conditions [111].

To improve the accuracy of drought forecast, some studies have explored hybrid models that combine multiple ML techniques or integrate ML with other modeling approaches such as the investigations by Belayneh et al. [90] using SPI on an Ethiopian catchment and by Katipoglu [92] and using SDI for a Turkish catchment. A review of hybrid models used for drought forecasting has been reported by Alawsi et al. [103], describing briefly data pre-processing and the advantages and disadvantages of different hybrid models. In general, the results based on hybrid models have been found to yield better results than the stand-alone models. The review by Alawsi et al. [103] can serve as a good basis for selecting an appropriate algorithm for a specific drought forecasting task, while understanding the characteristics of the data, considering the necessary assumptions and requirements of the models, and conducting a thorough experimentation.

Succinctly hydrological drought forecasting technology appears to be in a relatively advanced stage with the introduction of multiple ML-based methods. Yet, there lies a vast scope and potential to rigorously test the ML-based forecasting methods in terms of reliability at varying lead times. The identification of a suitable index of hydrological drought such as SSI or PDHI is the key consideration in this endeavor, which will be used as a variable for forecasting. In a nutshell, the ML-based methodology holds promise but needs comprehensive testing and validation before reliable and robust methods are adapted for practical usage. While a plethora of literature has used data-driven approaches for meteorological drought forecasting [104], very few studies have attempted data-driven models for hydrological drought forecasting.

5. Challenges in Hydrological Drought Research—Some Future Directions

Though considerable efforts have been put into hydrologic drought research in recent times, the emphasis has been to investigate the duration aspects that too on an annual scale, which is already fairly developed in terms of models and methodologies. The hydrologic drought requires understanding and estimating the water needs that are necessary to meet the water demands during short drought periods such as months and weeks. Therefore, efforts should be directed to launch the research efforts vigorously on the magnitude aspect, while keeping in mind the time frame such as month and week. It is opportune that such analyses could be undertaken in relevant parts of Spain where numerous prospects and data exist for conducting such investigations. Such studies are crucial for dealing with shortages for the summer season when the water resources experience maximum stress. In northern environments such as the Canadian north, winter droughts are not uncommon. The winter droughts play a significant role in influencing the water supplies for industries, municipalities, navigation, agriculture, forestry, hydropower generation, and pollution control and/or abatement. Furthermore, they impact the ecology and environmental conditions in the forthcoming spring and summer seasons. It is prudent to mention here that abnormal little snowpack occurrences during the winter reflect either below-normal winter precipitation (i.e., snow drought) or a lack of snow accumulation despite near-normal precipitation caused by warm temperatures that prevent precipitation from falling as snow and/or caused by unusually early snowmelt or warm snow drought [112]. Collectively, this type of drought is referred to as winter drought. Unusually warm departures during the winter of 2015 in the Western United States led to a classic example of how warm temperatures can cause snow/winter drought. [112]. Geographical regions that receive precipitation in the form of snow may face significant challenges when snow/winter droughts occur. The impacts of snow drought are often widespread across ecosystems, water resource management, operation of reservoirs, and winter recreation. Effects of winter droughts have been reported by Saavedra et al. [113] on nitrate concentrations in the streamflow of German catchments where significantly lower nitrate concentrations were attributed to winter drought or post-winter droughts.

Therefore, studies related to the hydrological aspects of winter droughts are of prime relevance for Canada and other northern countries in Europe. It is also prudent to keep a record of past hydrological droughts to be aware of the likelihood of occurrence of extreme future droughts [45] for the planning and development of long-term amelioration measures, such as building new dams and developing adequate-sized reservoirs.

There is a need to standardize the terms associated with various drought indices. For instance, virtually the same entity has been named a standardized hydrological index (SHI), streamflow drought index (SDI), or standardized runoff index (SRI). Likewise, confusion persists on the usage of the term severity and magnitude. Even the most recent papers are addressing the shortage of water as a severity, while others are attempting to popularize the term magnitude. Literally, severity denotes some kind of qualitative attribute, whereas magnitude refers explicitly to some kind of quantity, meaning that it is more apt to signify the shortage or the deficiency in volumetric units.

Although the analysis and modeling of the hydrological drought in terms of duration and magnitude for a particular river (rivers) are progressing at a reasonable pace, the analysis and modeling of the regional behavior of hydrologic drought are very deficient. Parallel to the pattern of the regional flood frequency analysis, hydrologic drought frequency analysis on a regional basis should be carried out. Such regional analyses should portray the relevant information in the form of maps as an aid in the planning and development of measures for mitigating water shortages in the aftermath of droughts. In the past, ARIDE (Assessment of the Regional Impacts of Droughts in Europe) project was a successful attempt, in which the frequency, magnitude, and regional spread of droughts at different temporal and spatial scales were studied [36]. This effort has been further advanced by Zaidman et al. [37] and Fleig et al. [39] for European conditions and by Hosseinzadeh et al. [110] for Iranian conditions. This is a praiseworthy endeavor and should be promoted in the domain of hydrological drought research.

Hydrological droughts deal with water shortages emanating from rivers, surface reservoirs, and groundwater sources; therefore, their occurrences not only impact the general availability of water to various users but also affect the environment, the ecology, and their interaction with human beings. This calls for a need to develop commonly acceptable, comprehensible, and easily adaptable indicators (aptly known as indices) to predict the occurrence and seriousness of impending droughts. Information on the significance of the impending droughts is of vital importance for developing and organizing effective drought amelioration measures [114,115] on a crisis basis. Particularly, if the additional water resources are to be organized or the rationing of existing water is to be implemented, the forewarning should come through easily comprehensible indices.

The general public perception of droughts relates to the lack of precipitation, which is reflected in the drier vegetation on the ground and low water levels in streams, lakes, and water wells. Such conditions only arise when a particular drought has already reached an advanced stage. This calls for an early warning system for public awareness based on the simple methods of drought forecasting. The present methods of drought forecasting are riddled with mathematical jargon with a modest level of reliability. The need still exists for reliable forecasts based on simple and easily comprehensible methods. It is noted that, of late, there has been a spate of research papers on ML-based methods for drought forecasting. However, there is a paucity of papers on the analysis and design applications of these methods. For example, if interest lies in finding the drought magnitude and duration of a "T" year drought, there are few publications that can provide the methodology for such information. The traditional MC-based methodology provides such information readily, which can be used for the design of reservoirs [25]. The bright side in this domain is the accessibility of the published material on the use of ML-based methods for drought forecasting during the last 5–7 years in computer-oriented journals such as Sundararajan et al. [104]. However, the majority of models use the SPI as the basis for forecasting, whereas SSI has been used to a limited extent, which is very crucial for hydrologic droughts.

Another perception emerging vehemently refers to climate change, which is likely to impact all phases of the hydrological cycle, and thus, hydrologic droughts are no exception. However, conclusive evidence is yet to emerge on the exact nature of impacts that hydrologic droughts are expected to undergo in the wake of the impending climate change. In other words, which component of the hydrologic drought, i.e., frequency, magnitude, persistence, or duration, is being or likely to be disturbed and/or intensified? Intensive research, in a timely manner, is needed and desired to investigate the issues and concerns about the above problems.

Most investigations in the form of models and information in the realm of hydrological droughts are still confined to technical and/or research journals. Research journals have become relatively expensive, thus restricting their use to only a handful of researchers and practitioners. A majority of developing countries cannot even afford them in their libraries [115]. There is a need to make drought research amenable to practitioners engaged in drought monitoring, forecasting, and water management operations, through monograms, books, or web networks. Fortunately, of late, since the second decade of the 21st century, there has been an emergence of a spate of new online journals such as Water, Hydrology, Water Resources and Protection, Sustainability, etc., where substantial numbers of papers on hydrological drought aspects are being published. This new form of publication merely represents a transitory stop-gap measure, which is bound to improve or be replaced by a more equitable publication system for researchers across the globe to freely share and debate ideas.

6. Concluding Remarks

In tandem with SPI in the arena of meteorological droughts, the recently developed index by WMO and GWP [49] for hydrological drought is SSI. The other offshoots of SSI are SRI, SDI, and SHI. The criteria for the identification of hydrological droughts are also in the stage of standardization. The truncation level approach initiated by Yevjevich [1] is mainly used for the identification of droughts with long-term mean or median as the truncation level of the annual or monthly streamflow time series. There have been suggestions that hydrological droughts are more tangible at the truncation level of Q70–Q75, when analyses are based on monthly and weekly scales. While identifying the hydrological droughts on a daily time frame, truncation levels such as Q90–Q95, etc., have also been used. The important parameters of hydrological droughts are duration, magnitude, time of initiation, time of cessation, and regional spread.

The monthly scale can be regarded as optimal for the analysis and prediction of hydrologic droughts. Major approaches for modeling hydrological drought are (i) empirical; (ii) experimental or time series simulation approach; (iii) analytical that include extreme number theorem, Markov chains, log-linear model, copula, and entropy-based analyses; and (iv) machine learning (ML)-based methodologies, i.e., artificial neural network (ANN), wavelet transform (WT), support vector machines (SVM), the adaptive neuro-fuzzy inference system (ANFIS), etc. The models currently available for hydrological drought essentially address mainly the issues concerning the duration and magnitude aspects.

The regional analysis of hydrological droughts is an important aspect that needs intensive efforts to develop water resources for drought-prone regions. Likewise, there is a need to apply hydrologic drought models to design and manage reservoirs for various applications. The major developments after the year 2000 in modeling the hydrological droughts have occurred through the introduction of new techniques involving the concepts of copula, entropy, and ML-based methods. However, the promise of these approaches needs exhaustive testing before they are recommended for practical use in water management and drought amelioration scenarios.

Though considerable work has been performed in the domain of forecasting and early warning technology of meteorological droughts (SPI) based on the emerging concepts of machine learning (ML), copula, entropy, and hybrid techniques involving remote sensing, and traditional ARIMA models, similar work on hydrologic droughts (SSI) is still pick up the pace. The other important component of hydrological drought modeling is to inculcate the impact of climate change on the behavior of drought patterns. Although ML-based methods are more focused on the forecasting and early warning of droughts, there is an outstanding need for these methods to develop the information for the design aspects of drought amelioration measures such as reservoirs, drilling new groundwater wells, etc.

Author Contributions: T.C.S. and U.S.P. collaboratively accomplished various tasks related to this manuscript, including draft preparation and finalization of the manuscript for journal publication such as conceptualization and development of the methodology, data collection, and data analysis. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Acknowledgments: The partial financial support of the Natural Sciences and Engineering Research Council of Canada for this paper is gratefully acknowledged.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Yevjevich, V. *An Objective Approach to Definitions and Investigations of Continental Hydrologic Drought*; Hydrology Paper 23; Colorado State University: Fort Collins, CO, USA, 1967.
2. Tallaksen, L.; van Lanen, H. Hydrological drought. In *Processes and Estimation Methods for Streamflow and Groundwater*; Elsevier: Amsterdam, The Netherlands, 2004.
3. Millan, J.; Yevjevich, V. *Probabilities of Observed Droughts*; Hydrology Paper 50; Colorado State University: Fort Collins, CO, USA, 1971.
4. Salazar, P.G.; Yevjevich, V. *Analysis of Drought Characteristics by the Theory of Runs*; Hydrology Paper 80; Colorado State University: Fort Collins, CO, USA, 1975.
5. Dracup, J.A.; Lee, K.S.; Paulson, E.G., Jr. On identification of droughts. *Water Resour. Res.* **1980**, *16*, 289–296. [CrossRef]
6. Sen, Z. Statistical analysis of hydrological critical droughts. *J. Hydraul. Eng.* **1980**, *106*, 99–115.
7. Guven, O. A simplified semi-empirical approach to the probability of extreme hydrological droughts. *Water Resour. Res.* **1983**, *19*, 441–453. [CrossRef]
8. Yevjevich, V. Methods for determining statistical properties of droughts. In *Coping with Droughts*; Yevjevich, V., da Cunha, L., Vlachos, E., Eds.; Water Resources Publications: Littleton, CO, USA, 1983; pp. 22–43.
9. Lee, K.S.; Sadeghipour, J.; Dracup, J.A. An approach for frequency analysis of multiyear drought durations. *Water Resour. Res.* **1986**, *22*, 655–662. [CrossRef]
10. Horn, D.R. Characteristics and spatial variability of droughts in Idaho. *J. Irrig. Drain. Eng.* **1989**, *115*, 111–124. [CrossRef]
11. Fernandez, B.; Sales, J.D. Return period and risk of hydrologic events. II. applications. *J. Hydrol. Eng.* **1999**, *4*, 308–316. [CrossRef]
12. Sharma, T.C. Drought parameters in relation to truncation level. *Hydrol. Proc.* **2000**, *14*, 1279–1288. [CrossRef]
13. Salas, J.; Fu, C.; Cancelliere, A.; Dustin, D.; Bode, D.; Pineda, A.; Vincent, E. Characterizing the severity and risk of droughts of the Poudre River, Colorado. *J. Water Resour. Plan. Manag.* **2005**, *131*, 383–393. [CrossRef]
14. Panu, U.S.; Sharma, T.C. Analysis of annual hydrologic droughts: A case of the northwest Ontario, Canada. *Hydrol. Sci. J.* **2009**, *54*, 29–42. [CrossRef]
15. Akyuz, D.E.; Bayazit, M.; Onoz, B. Markov chain models for hydrological drought characteristics. *J. Hydrometeorl.* **2012**, *13*, 298–309. [CrossRef]
16. Sen, Z. *Applied Drought Modelling, Prediction and Mitigation*; Elsevier Inc.: Amsterdam, The Netherlands, 2015.
17. Burn, D.H.; Wychreschul, J.; Bonin, D.V. An integrated approach to the estimation of streamflow drought quantiles. *Hydrol. Sci. J.* **2004**, *49*, 1011–1024. [CrossRef]
18. Shiau, J.T.; Feng, S.; Nadarajah, S. Assessment of hydrological droughts for the Yellow River China using copulas. *Hydrol. Proc.* **2007**, *21*, 2157–2163. [CrossRef]

19. Lopez-Moreno, J.I.; Vicente-Serrano, S.M.; Beguria, S.; Garcia-Ruiz, J.M.; Portela, M.M.; Almeida, A.B. Dam effects on drought magnitude and duration in a transboundary basin: The lower River Tagus, Spain and Portugal. *Water Resour. Res.* **2009**, *45*, W02405. [CrossRef]
20. Lorenzo-Lacruz, J.; Vicente-Serrano, S.M.; Gonzalez-Hidalgo, J.C.; Lopez-Moreno, J.I.; Cortesi, N. Hydrological drought response to meteorological drought in the Iberian Peninsula. *Clim. Res.* **2013**, *58*, 117–131. [CrossRef]
21. Vicente-Serrano, S.M.; Lopez-Moreno, J.I.; Beguria, S.; Lorenzo-Lacruz, J.; AzorinMolin, C.; Moran-Tejeda, E. Accurate computation of streamflow drought index. *J. Hydrol. Eng.* **2012**, *17*, 318–322. [CrossRef]
22. Vicente-Serrano, S.M.; Dominguez-Castro, D.C.; McVicar, T.R.; Thomas-Burguera, M.; Pena-Gallardo, M.; Noguera, M.; Lopez-Moreno, J.I.; Pena, D.; Kenaway, A.E. Global characterization of hydrological and meteorological droughts under future climate change: The importance of time scale, vegetation-CO_2 feedbacks and change to distribution functions. *Int. J. Clim.* **2020**, *40*, 2557–2567. [CrossRef]
23. Sung, J.H.; Chung, E.-S. Development of streamflow drought severity-duration-frequency curves using the threshold level method. *Hydrol. Earth Syst. Sci.* **2014**, *18*, 3341–3351. [CrossRef]
24. Sharma, T.C.; Panu, U.S. Modeling of hydrological droughts: Experiences on Canadian streamflows. *J. Hydrol. Reg. Stud.* **2014**, *1*, 92–106. [CrossRef]
25. Sharma, T.C.; Panu, U.S. A drought magnitude-based method for reservoir sizing: A case of annual and monthly flows from Canadian rivers. *J. Hydrol. Reg. Stud.* **2021**, *36*, 100829. [CrossRef]
26. Rad, A.M.; Ghahraman, B.; Khalili, D.; Ghabremani, Z.; Ardakani, S.A. Integrated meteorological and hydrological drought model: A management tool for proactive water resources planning in semi-arid regions. *Adv. Water Resour.* **2017**, *107*, 336–353. [CrossRef]
27. Wu, J.; Chen, X.; Love, C.A.; Yao, H.; Chen, X.; AghaKouchak, A. Determination of water required to recover from hydrological drought: Perspective for drought propagation and non-standardized indices. *J. Hydrol.* **2020**, *590*, 125227. [CrossRef]
28. Zalokar, L.; Kobold, M. Investigation on spatial and temporal variability of hydrological droughts in Slovenia using SSI. *Water* **2021**, *13*, 3197. [CrossRef]
29. Botai, C.M.; Botai, J.O.; dewit, J.P.; Ncongwane, K.P.; Murambadoro, M.; Barasa, P.M.; Adelo, A.M. Hydrological drought assessment based on the standardized streamflow index: A case study of the three cape provinces of South Africa. *Water* **2021**, *13*, 3498. [CrossRef]
30. Sharma, T.C.; Panu, U.S. A procedure for estimating drought duration and magnitude at a uniform cutoff of streamflow: A case of weekly flows from Canadian rivers. *Hydrology* **2022**, *9*, 109. [CrossRef]
31. Gurrapu, S.; Sauchyan, D.J.; Hodder, K.R. Assessment of hydrological drought risk in Calgary, Canada using weekly river flows of the past millennium. *J. Water Clim. Chang.* **2022**, *13*, 1920–1935. [CrossRef]
32. Zelenhasic, E.; Salvai, A. A method of streamflow drought analysis. *Water Resour. Res.* **1987**, *23*, 156–168. [CrossRef]
33. Tallaksen, L.M.; Madsen, S.; Clausen, B. On definition and modelling of streamflow deficits and volume. *Hydrol. Sci. J.* **1997**, *42*, 15–33. [CrossRef]
34. Kjeldsen, T.R.; Lundroff, A.; Rosbjberg, D. Use of two-component exponential distribution in partial duration modelling of hydrological droughts in Zimbabwean rivers. *Hydrol. Sci. J.* **2000**, *45*, 285–298. [CrossRef]
35. Tate, E.I.; Freeman, S.N. Three modelling approaches for seasonal streamflow droughts in southern Africa: The use of censored data. *Hydrol. Sci. J.* **2000**, *45*, 27–42. [CrossRef]
36. Demuth, S.; Stahl, K. (Eds.) *ARIDE-Assessment of the Regional Impact of Droughts in Europe*; A Final Report; Institute of Hydrology, University of Freiburg: Freiburg im Breisgau, Germany, 2001.
37. Zaidman, M.D.; Rees, H.G.; Young, A.R. Spatio-temporal development of streamflow droughts in northwestern Europe. *Hydrol. Earth Syst. Sci.* **2002**, *6*, 733–751. [CrossRef]
38. Fleig, A.K.; Tallaksen, L.M.; Hisdal, H.; Demuth, S. A global evaluation of streamflow drought characteristics. *Hydrol. Earth Syst. Sci.* **2006**, *10*, 532–552. [CrossRef]
39. Fleig, A.K.; Tallaksen, L.M.; Hisdal, H.; Hannah, D.M. Regional hydrological drought in north-western Europe: Linking a new regional drought area index with the weather type. *Hydrol. Proc.* **2011**, *25*, 1163–1179. [CrossRef]
40. van Huijgevoort, M.H.J.; Hazenburg, P.; van Lanen, H.A.J.; Uijlenhoet, R. A generic method for hydrological drought identification across different climate regions. *Hydrol. Earth Syst. Sci.* **2012**, *16*, 2437–2451. [CrossRef]
41. van Loon, A.F.; Laaha, G. Hydrological drought severity explained by climate and catchment characteristics. *J. Hydrol.* **2015**, *26*, 3–14. [CrossRef]
42. McKee, T.B.; Doesen, N.J.; Kleist, J. The relationship of drought frequency and duration to time scales. In Proceedings of the 8th Conference on Applied Climatology, Anaheim, CA, USA, 17–22 January 1993; pp. 179–184.
43. McKee, T.B.; Doesen, N.J.; Kleist, J. Drought monitoring with multiple time scales. In Proceedings of the 9th conference on Applied Climatology, Dallas, TX, USA, 15–20 January 1995; pp. 233–236.
44. Heim, H.M., Jr. A review of twentieth-century drought indices used in the United States. *Bul. Am. Meteorol. Soc.* **2002**, *83*, 1149–1165. [CrossRef]
45. Timilsena, J.; Piechota, T.C.; Hidalgo, H.; Tootle, G. Five hundred years of hydrological drought in the upper Colorado River basin. *J. Am. Water Resour. Asso.* **2007**, *43*, 798–812. [CrossRef]

46. Palmer, W.C. *Meteorological Droughts*; Paper No. 45; Weather Bureau, U.S. Dept. of Commerce: Washington, DC, USA, 1965; pp. 1–58.
47. McMahon, T.A.; Adeloye, A.J. *Water Resources Yield*; Water Resources Publications: Littleton, CO, USA, 2005.
48. Guttman, N.B. Accepting the standardized precipitation index: A calculation algorithm. *J. Am. Water Resour. Asso.* **1999**, *30*, 311–322. [CrossRef]
49. World Meteorological Organization (WMO); Global Water Partnership (GWP). *Handbook of Drought Indicators and Indices (M. Svoboda and B.A. Fuchs)*; Integrated Drought Management Programme (IDMP), Integrated Drought Management Tools and Guidelines Series 2: Geneva, Switzerland, 2016.
50. Shafer, B.A.; Dezman, E.A. Development of a surface water supply index (SWSI) to assess the severity of drought conditions in the snowpack runoff area. In *Proceedings of the 50th Annual Western Snow Conference*; Colorado State University: Fort Collins, CO, USA, 1982; pp. 164–175.
51. Dosken, N.J.; Garen, D. Drought monitoring in the western United States using a surface water supply index. In Proceedings of the Seventh Conference on Applied Climatology, Salt Lake City, UT, USA, 10–13 September 1991; American Meteorological Society: Boston, MA, USA, 1991; pp. 266–269.
52. Garen, D.C. Revised surface water supply index for the western United States. *J. Water Resour. Plan. Manag.* **1993**, *119*, 437–454. [CrossRef]
53. Keyantash, J.; Dracup, J.A. The quantification of drought: An evaluation of indices. *Bul. Am. Meteorol. Soc.* **2002**, *83*, 1167–1180. [CrossRef]
54. Shukla, S.; Wood, A.F. Use of standardized runoff index for characterizing hydrologic drought. *Geophys. Res. Let.* **2008**, *35*, L02405. [CrossRef]
55. Nalbantis, I.; Tsakris, G. Assessment of hydrological drought revisited. *Water Resour. Manag.* **2009**, *23*, 881–897. [CrossRef]
56. Tabari, H.; Nikbakht, J.; Talaee, P.H. Hydrological drought assessment in northeastern Iran based on streamflow drought index (SDI). *Water Resour. Manag.* **2013**, *27*, 137–151. [CrossRef]
57. Yeh, C.F.; Wang, J.; Yeh, H.F.; Lee, C.H. SDI and Markov chain for regional drought characteristics. *Sustainability* **2015**, *7*, 10789–10808. [CrossRef]
58. Tijdeman, E.; Stahl, K.; Tallaksen, L.M. Drought characteristics derived based on the standardized streamflow index: Large sample comparison for parametric and non-parametric methods. *Water Resour. Res.* **2020**, *5610*, 20119WR026315. [CrossRef]
59. Teutschbein, C.; Montano, B.Q.; Todorovic, A.; Grabs, T. Streamflow droughts in Sweden: Spatiotemporal patterns emerging from six decades of observations. *J. Hydrol. Reg. Stud.* **2022**, *42*, 101171. [CrossRef]
60. Mishra, A.K.; Singh, V.P. A review of drought concepts. *J. Hydrol.* **2010**, *391*, 202–216. [CrossRef]
61. Hosking, J.R.M.; Wallis, J.R. *Regional Frequency Analysis: An Approach Based on L-Moments*; Cambridge University Press: Cambridge, UK, 1997.
62. Box, G.E.P.; Jenkins, G.M.; Reinsel, G.C.; Ljung, G.M. *Time Series Analysis, Forecasting and Control*, 5th ed.; Wiley: Hoboken, NJ, USA, 2015.
63. Loucks, D.P.; Beck, E.; Stedinger, J.R.; Dijkman, J.P.M.; Villars, M.T. *Water Resources Systems Planning and Management: An Introduction to Methods, Models, and Application*; UNESCO: London, UK, 2005.
64. Fiering, M.B. *Streamflow Synthesis*; Harvard University Press: Cambridge, MA, USA, 2014.
65. Ochoa-Riveria, J.C.; Garcia-bartal, R.; Andreau, J. Multivariate synthetic streamflow generation using a hybrid model based on artificial neural network. *Hydrol. Earth Syst. Sci.* **2002**, *6*, 641–654. [CrossRef]
66. Ahmad, J.A.; Sarma, A.K. Artificial neural network model for synthetic streamflow generation. *Water Resour. Manag.* **2007**, *21*, 1015–1029. [CrossRef]
67. Prairie, J.; Nowak, K.; Rajagopalan, B.; Lall, U.; Fulp, T. A stochastic nonparametric approach for streamflow generation combining observational and paleo-constructed data. *Water Resour. Res.* **2008**, *44*, w06423. [CrossRef]
68. Salas, J.D.; Lee, T. Non-parametric simulation of single site seasonal streamflows. *J. Hydrol. Eng.* **2010**, *15*, 284–296. [CrossRef]
69. Hao, Z.; Singh, V.P. Single site monthly streamflow simulation using entropy theory. *Water Resour. Res.* **2011**, *47*, W09528. [CrossRef]
70. Hao, Z.; Singh, V.P. Entropy-copula method for single site monthly streamflow simulation. *Water Resour. Res.* **2012**, *48*, 1–8. [CrossRef]
71. Chung, C.; Salas, J.D. Drought occurrence probabilities and risk of dependent hydrologic processes. *J. Hydrol. Eng.* **2000**, *5*, 259–268. [CrossRef]
72. Loaiciga, H.A. On the probability of droughts: The compound renewal model. *Water Resour. Res.* **2005**, *41*, 41W01009. [CrossRef]
73. Sen, Z. Critical drought analysis by second-order Markov chain. *J. Hydrol.* **1990**, *120*, 183–202. [CrossRef]
74. Yang, J.; Wang, Y.; Chang, J.; Huang, Q. Integrated assessment for hydro-meteorological drought based on Markov chain model. *Nat. Hazard* **2016**, *84*, 1137–1160. [CrossRef]
75. Paulo, A.A.; Ferreira, E.; Coelho, C.; Pereira, L.S. Drought class transition analysis through Markov and log-linear models, an approach to early warning. *Agric. Water Manag.* **2005**, *17*, 59–81. [CrossRef]
76. Li, J.; Zhou, S.; Hu, R. Hydrological drought class transition using SPI and SRI time series by log-linear regression. *Water Resour. Manag.* **2015**, *30*, 669–684. [CrossRef]
77. Kuo, S.C.; Govindaraju, R.S. A copula-based joint deficit index for droughts. *J. Hydrol.* **2010**, *380*, 121–134. [CrossRef]

78. Song, S.; Singh, V.P. Frequency analysis of droughts using the Plackett copula and parameter estimation by genetic algorithm. *Stoch. Environ. Res. Risk Assess.* **2010**, *24*, 783–805. [CrossRef]
79. Lu, C.; Singh, V.P.; Guo, S.; Mishra, A.K.; Guo, J. Drought analysis using Copula. *J. Hydrol. Eng.* **2012**, *18*, 797–808. [CrossRef]
80. Borgomeo, E.; Pflug, G.; Hall, J.W.; Hochrainer-Stigler, S. Assessing water resources system vulnerability to understand hydrological drought using copulas to characterize drought duration and deficit. *Water Resour. Res.* **2015**, *51*, 8927–8948. [CrossRef]
81. Zhang, Q.; Xiao, M.; Singh, V.P.; Chen, X. Copula based risk evaluation of hydrologic droughts in the East River basin, China. *Stoch. Environ. Res. Risk Assess.* **2013**, *27*, 1397–1406. [CrossRef]
82. Dodangeh, E.; Shahedi, K.; Shiau, J.T.; Mirakbari, M. Spatial hydrological characteristics in Karkheh basin, southwest Iran using copula. *J. Earth Syst. Sci.* **2017**, *126*, 80. [CrossRef]
83. Mishra, A.K.; Singh, V.P. Drought modelling—A review. *J. Hydrol.* **2011**, *403*, 157–175. [CrossRef]
84. Hao, Z.; Singh, V.P. Entropy-based method for bivariate drought analysis. *J. Hydrol. Eng.* **2013**, *18*, 780–786. [CrossRef]
85. Bacanli, U.G.; Baran, T.; Dikbas, F. Identification of drought parameters through the use of entropy. *Int. J. Res. Earth Environ. Sci.* **2016**, *4*, 1–13.
86. Zuo, D.; Hao, W.; Hu, J. An entropy-based investigation into bivariate drought analysis in China. *Water* **2017**, *9*, 632. [CrossRef]
87. Yang, X.; Li, P.; Huang, P.H. A maximum entropy copula-based frequency analysis method for assessing bivariate drought risk: A case study of Kaidu River Basin. *Water Clim. Chang.* **2022**, *13*, 175–189. [CrossRef]
88. AghaKouchak, A. Entropy-copula in hydrology and climatology. *J. Hydromet.* **2014**, *15*, 2176–2189. [CrossRef]
89. Ozger, M.; Mishra, A.K.; Singh, V.P. Long lead time drought forecasting using a wavelet and fuzzy logic combination model: A case study in Texas. *J. Hydromet.* **2012**, *13*, 284–297. [CrossRef]
90. Belayneh, A.; Adamowski, J.; Khalil, B.; Ozga-zielinski, B. Long-term SPI drought forecasting in the Awash River Basin in Ethiopia using wavelet network and wavelet support vector regression models. *J. Hydrol.* **2014**, *508*, 418–429. [CrossRef]
91. Das, P.; Naganna, S.R.; Deka, P.S.; Puspraj, J. Hybrid wavelet packet machine learning approaches for drought modelling. *Environ. Earth Sci.* **2020**, *79*, 221. [CrossRef]
92. Katipoglu, O.K. Prediction of streamflow index for short-term hydrological drought in semi-arid Yesilirmak basin using wavelet transform and artificial intelligence techniques. *Sustainability* **2023**, *15*, 1109. [CrossRef]
93. Shamshirband, S.; Hashemi, S.; Salimi, H.; Samadianfard, S.; Asadi, E.; Shadkarni, S.; Kargar, K.; Mosavi, A.; Nabipour, N.; Chau, K.-W. Predicting standardized streamflow index for hydrological drought using machine learning model. *Eng. Appl. Comput. Fluid Mech.* **2020**, *14*, 339–350. [CrossRef]
94. Nandgude, N.; Singh, T.P.; Nandgude, S.; Tiwari, M. Drought prediction: A comprehensive review of different drought prediction models and adopted technologies. *Sustainability* **2023**, *15*, 11684. [CrossRef]
95. Shin, H.; Salas, J.D. Regional drought analysis based on neural network. *J. Hydrol. Eng.* **2000**, *5*, 145–155. [CrossRef]
96. Morid, S.; Smakhtin, V.; Bagherzadeh, K. Drought forecasting using artificial neural networks and time series of drought indices. *Int. J. Clim.* **2007**, *27*, 2103–2111. [CrossRef]
97. Rezaeianzadeh, M.; Stein, A.; Cox, J.P. Drought forecasting using Markov chain model and artificial neural networks. *Water Resour. Manag.* **2016**, *30*, 2245–2259. [CrossRef]
98. Fung, K.F.; Huang, Y.F.; Kao, C.H.; Soh, Y.W. Drought forecasting: A review of modelling approaches. *J. Water Clim. Chang.* **2020**, *11*, 3. [CrossRef]
99. Bacanli, U.G.; Firat, M.; Dikbas, F. Adaptive neuro-fuzzy inference system for drought forecasting. *Stoch. Environ. Res. Risk Assess.* **2009**, *23*, 1143–1154. [CrossRef]
100. Jahanzaib, M.; Idrees, M.B.; Kim, D.; Kim, T.-W. Comprehensive evaluation of machine learning techniques for hydrological drought forecasting. *J. Irrig. Drain. Eng.* **2021**, *147*, 04021022. [CrossRef]
101. Achite, M.; Jehanzaib, M.; Elashaboury, N.; Kim, T.-W. Evaluation of machine learning techniques for hydrological drought modelling: A case study of the Wadi Ouahrane basin in Algeria. *Water* **2022**, *14*, 431. [CrossRef]
102. Liang, Z.; Su, X.; Feng, K. Drought propagation and construction of a comprehensive drought index based on the Soil and Water Assessment Tool (SWAT) and empirical Kendall distribution function ($K_{c'}$): A case study for the Jinta River basin in north-eastern China. *Nat. Hazards Earth Syst. Sci.* **2021**, *21*, 1323–1335. [CrossRef]
103. Alwasi, M.A.; Zubaidi, S.L.; Al-Bdairi, N.B.; Al-Ansari, N.; Hasim, K. Drought forecasting a review and assessment of the hybrid techniques and data pre-processing. *Hydrology* **2022**, *9*, 115. [CrossRef]
104. Sundararajan, K.; Garg, L.; Srinivasan, K.; Bashir, A.K.; Kaliappan, J.; Ganapathy, G.P.; Selvaraj, S.K.; Thiruvadi, M. A contemporary review on drought modelling using machine learning approaches. *Comput. Model. Eng. Sci.* **2021**, *128*, 447–487. [CrossRef]
105. Modarres, R. Streamflow drought time series forecasting. *Stoch. Environ. Res. Risk Assess.* **2007**, *21*, 223–233. [CrossRef]
106. Cordery, I.; McCall, M.A. A model for forecasting drought from teleconnections. *Water Resour. Res.* **2000**, *36*, 763–768. [CrossRef]
107. Pichota, T.C.; Dracup, J.A. Drought and regional hydrologic variation in the United States: Association with the El-Nino-Southern Oscillation. *Water Resour. Res.* **1996**, *32*, 1359–1373. [CrossRef]
108. Pongracz, R.; Bogardi, I.; Duckstein, L. Climatic forcing of droughts: A central European example. *Hydrol. Sci. J.* **2003**, *48*, 39–50. [CrossRef]
109. Bonsal, D.R.; Wheaton, E.L.; Chipangi, A.C.; Lin, C.; Sauchyn, D.J.; Lei, W. Drought research in Canada: A review. *Atmosphere-Ocean* **2011**, *49*, 303–319. [CrossRef]

110. Hosseinzadeh Talee, P.; Tabari, H.; Ardakani, S.A. Hydrological drought in the west of Iran and possible association with large-scale atmospheric circulation patterns. *Hydrol. Proc.* **2014**, *28*, 763–773. [CrossRef]
111. Abdelkander, M.; Yerddelen, C. Hydrological drought variability and its teleconnections with climate indices. *J. Hydrol.* **2022**, *605*, 127290. [CrossRef]
112. Harpold, A.; Dettinger, M.; Rajagopal, S. Defining snow drought and why it matters. *Eos Trans. Amer. Geophys. Union* **2017**, *98*, 15–17. [CrossRef]
113. Saavedra, F.; Musolff, A.; Freyberg, J.A.; Merz, R.; Knoller, K.; Muller, C.; Brunner, M.; Tarasova, L. Winter post-droughts amplify extreme concentrations in German rivers. *Environ. Res. Lett.* **2024**, *19*, 024007. [CrossRef]
114. Wilhite, D.A. *Drought: A Global Assessment*; Routledge: London, UK, 2000; Volume 1.
115. Panu, U.S.; Sharma, T.C. Challenges in drought research: Some perspectives and future directions. *Hydrol. Sci. J.* **2002**, *47*, S19–S30. [CrossRef]