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Generalized Storage–Yield–Reliability Relationships for Analysing Shopping Centre Rainwater Harvesting Systems

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Abstract: The objective of this study was to develop guidelines for analysing rainwater harvesting (RWH) systems of shopping centres in South Africa. A model consisting of three dimensionless relationships relating rainwater supply and demand to storage capacity, yield and reliability was formulated. Data from daily simulation of potential RWH systems of 19 shopping were used to obtain the relationships. The simulations revealed within-year storage behaviour with considerable variation of annual yield. By applying the Weibull plotting position formula, yield–reliability relationships were derived. The aim to maximize yield and reliability whilst minimizing storage identified Pareto-optimal combinations of the three variables and these combinations were used to develop two dimensionless relationships. An additional relationship based on the dependence of the slope of the yield–reliability plots on yield was formulated to enable analysis of hydrologically non-optimal systems. Verification tests using four RWH systems obtained results that matched those from simulation and the model could therefore be applied for RWH feasibility analysis and preliminary design. This study highlights the need to incorporate inter-annual variability in RWH analysis and shows how reliability can be used to quantify this. This study further demonstrates how reliability can be fully integrated into regression relationships for generalized RWH analysis.

Keywords: rainwater harvesting; yield; reliability; storage; inter-annual variability; generalization; Pareto optimum

1. Introduction

The escalating global demand on finite water resources is of great concern [1] and is likely to constrain future economic growth and development [2]. Diverse approaches to deal with this challenge are applied in different regions of the world including rolling water resources planning [3]. With large population growth and imprudent water use habits, there is an ever-increasing demand for water in urban areas [4] and rainwater harvesting (RWH) could significantly complement centralized urban water supply [5]. Although RWH life cycle cost analyses (LCCA) sometimes obtain long payback periods [6–8], water supply is subsided in many regions of the world [9–11] and the tariffs used in RWH LCCA are likely to be lower than the actual costs of centralized water supply. Rebates are provided in some cities for installation of rainwater harvesting systems [6] indicating that RWH systems are valuable water sources. RWH systems provide other benefits such as stormwater attenuation [12] and also reduce water supply energy usage [13] and carbon footprint [5]. RWH could also lead to a feeling of independence from centralized water supply [9]. Although rainwater sometimes fails to meet drinking water standards [14–18], the quality of rainwater is usually superior to that from surface water and groundwater that may have been contaminated [19]. Sazakli et al. [20] report that rainwater from rooftops generally meets international drinking water quality standards. Rainwater is always soft unlike groundwater and can therefore be readily used for laundry and in hot water systems [5].

South Africa is a water-scarce country with a mean annual precipitation (MAP) of 465 mm which is unevenly distributed in time and space. In South Africa, RWH is given credence as a valuable water source [3]. Mwenge Kahinda et al. [21] inform that over 26,500 rural households in South Africa use rooftop rainwater harvesting as the main source of drinking water. RWH has also been implemented in urban areas and, in one project, this led to a 10% reduction in bulk water demand for 500 low-income households [3]. Ndiritu et al. [22] analysed the rainwater harvesting potential of 32 schools in South Africa and found that, in every year, between 42 and 132 days of the daily school demand could be provided at a reliability of 90%. As there are many shopping centres in South Africa (stated as 1785 with areas larger than 2000 m^2 in 2010 [23]), the potential contribution of shopping centre RWH systems to water security could be significant. RWH is being promoted as a green building technology [24] and unpublished information reveals increased installation of RWH systems for buildings and other structures with large roofs. Since no guidelines for the hydrologic analysis of shopping centre RWH systems in South Africa were found in the literature, this study set out to formulate them. The aim was to obtain generalized guidelines that would be applicable for RWH feasibility analysis and preliminary design. This would help to forestall wrong investment decisions and inappropriate sizing of RWH systems.

Comprehensive models for detailed RWH analysis have been developed [5,25] and Campisano et al. [9] provide a review of many others. Acquiring detailed models may however not be free and applying them is likely to take more time than using simple generalized models. Generalized models are applicable for regional assessment of RWH potential [26] and may be preferred to detailed ones especially where time and other resources are scarce. Generalized RWH storage–yield–reliability relationships have been developed in other regions of the world [26–28] as regression equations of dimensionless ratios of the variables involved (with few exceptions). In three recent studies [26–28], the data for generalizing were obtained from multiple simulations of RWH systems. In these studies, the regression equations were developed at specified reliabilities and reliability itself was not included as a variable in the equations. The dimensionless ratios that have been applied are the yield, the demand and the storage fraction. The yield fraction has been defined as the ratio of yield to demand and the demand fraction as the ratio of demand to supply [26–28]. The storage fraction has been specified as storage capacity divided by annual rainfall volume [26,28] or as this ratio multiplied by the ratio of rain days to dry days [27]. For RWH analysis, reliability has been commonly defined as volumetric reliability (volume supplied to that demanded over the simulation period) [6,28–31] or as the ratio of days of full supply to the total days of simulation [26]. However, volumetric reliability does not capture inter-annual variability of rainfall, and statistical analyses of the yield (or volumetric reliabilities) obtained in each year of simulation have been used to incorporate this variability [25,27]. The effect of inter-annual rainfall variability has also been assessed by analysing RWH performance for dry, normal and wet years [29] or for a typical dry year [32]. Since South Africa experiences large inter-annual variability of rainfall [3], the RWH guidelines to be developed in the current study needed to incorporate inter-annual rainfall variability and to also fully integrate reliability (the quantifier of variability) into the regression equations of the model.

2. Materials and Methods

A dependable generalized model needs to be based on the expected characteristics and performance of actual RWH systems. Since no long-term data on installed RWH system behaviour were available, daily time-step simulation was used to provide empirical data for development of the generalized model—an approach that has been used in other generalization studies [26–28]. The generalized model was formulated by developing regression equations using data from the simulation of 19 potential RWH systems located in four South African provinces. Verification was then done by comparing results from the model with those from simulation. Four potential RWH systems located far from those used in model formulation (in four other provinces) were used for verification. A case study RWH design and assessment of a single shopping centre was then used to illustrate the application of the model.

2.1. Selection and Acquisition of Data

Rainfall distribution in South Africa exhibits high temporal and spatial variability. Robust generalized model development therefore needs to be based on long rainfall time series from different regions zones. Additionally, the guidelines also need to be applicable to the all classes of shopping centres. Prinsloo [33] classified shopping centres according to their floor area as: Neighbourhood centres (5000–12,000 m^2), Community centres (12,000–30,000 m^2), Large community/small regional centres (30,000–50,000 m^2), Regional centres (50,000–100,000 m^2) or Super regional centres (>100,000 m^2). For the development of the model, it is decided to select one shopping centre from each category from four South African provinces (Kwa Zulu Natal, Gauteng, Limpopo and Western Cape). Selecting five shopping centre categories in four regions would provide 20 shopping centres but, since no super regional centre is located in Limpopo, a total of 19 shopping centres are therefore selected. For model verification, two regional and two small regional centres located in four other provinces (North West (NW), Eastern Cape (EC), Free State (FS) and Mpumalanga (MP) provinces) are used. All the selected shopping centres are located in cities.

The rainfall database developed by Lynch [34] is used to find the rain gauge station with a long and reliable daily rainfall record closest to each shopping centre. Table 1 shows the shopping centres selected for analysis, the respective rain gauge stations and the distances from the shopping centres to the rain gauge stations. Table 2 provides additional information on the selected rainfall stations. The average length of data is 117 years with 67% of this consisting of observed measurements. The rest of the data were in-filled using Expectation Maximization, Median ratio or the Inverse distance weighting method in the development of the database [34]. As Table 2 shows, a small proportion of the unobserved data could not be in-filled and was categorized as missing. Lynch [34] did not provide quantitative information on the possible errors in the observed data and from infilling but informed that the data had been checked for consistency. Graphical plots of the selected data for the current study revealed only one inconsistency (for site 0436495 W) and 16 years of questionable data (that was all patched) was discarded. Figure 1 shows the locations of the rainfall stations used in model development and verification. Figure 2 shows the annual time series, rainfall–duration and rain days–duration curves for representative rainfall stations and highlights the large temporal variability of annual rainfall in various regions of South Africa.

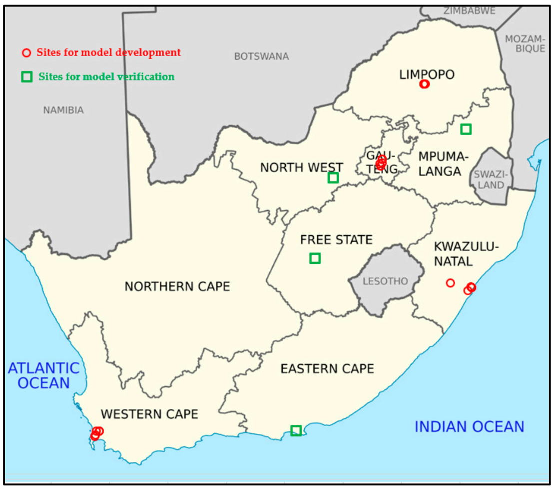

Figure 1. Location of selected rainfall stations.

No information on demand was availed by the shopping centres and CSIR [35] proposed a demand of 4 m^3/m^2 of floor area/year for South Africa. A study on 40 shopping centres of varying sizes in Western Australia [36] obtained highest demands of 2.828, 3.141, 1.347, and 1.383 m^3/m^2/per year for neighbourhood centres, large community/small regional centres, regional centres, and super regional centres, respectively. Since these demands were based on detailed field measurements and the demands in Western Australian malls are not likely to be substantially different from those in South Africa, they were adopted for analysis in this study. For community centres, the average overall demand of 2.18 m^3/m^2/per year obtained by Saunders [36] was adopted. It was assumed that the RWH systems would supply non-potable water use (cooling towers for air conditioning, toilets, urinals and cleaning) which was assumed as 45% of the total demand as found by Saunders [36]. Daily demand on weekends was assumed to be twice that on weekdays and the monthly demand for December was assumed to be twice the demand in the other months of the year.

Table 1. Selected shopping centres and rainfall stations.

Region	Mall	Retail Area (m^2)	Roof Area (m^2)	Rainfall Station No.	Distance from Mall (km)
Gauteng	Sandton City	128,000	83,472	0476093 W	6.67
	South Gate Mall	89,700	45,349	0476044 W	6.24
	Norwood Mall	32,344	32,194	0476129 W	0.43
	Braamfontein Centre	21,309	3416	0475881 W	0.97
	Grayston Centre	5000	4198	0476093 W	5.68
Cape Town	Canal Walk	141,000	26,082	0020896 W	4.38
	Tygervally Centre	90,000	55,403	0021230 W	4.74
	Willow Bridge	40,051	23,390	0021230 W	3.95
	Howard Centre	15,000	14,052	0021055 w	2.88
	Capricon Square	5889	6374	0020839 W	13.11
Limpopo	Mall of the North	75,000	35,199	0678023 W	1.26
	Savanah Mall	37,000	17,880	0677834 W	2.15
	Limpopo Mall	27,766	7446	0677834 W	1.81
	Cycad Shopping Centre	12,000	5267	0677834 W	0.94
Kwa Zulu Natal	Gateway Mall	180,000	123,498/73,313 *	0241103 W	0.97
	Liberty Midlands Mall	75,000	74,702/55,241 *	0239605 P	3.21
	Musgrave Centre	39,886	20,058	0240738 W	8.36
	Phoenix Plaza	24,162	29,307/18,070 *	0241042 W	2.72
	Granada Square	5818	2097	0241103 W	2.15
NW	*Matlosana*	65,000	50,100/40,000 !	0436495 W	8.03
FS	*Mimosa*	25,000	5297	0261368 W	2.30
MP	*Riverside*	49,529	45,000	0556088 W	9.02
EC	*Baywest*	90,000	45,351	0035209 W	14.85

Note: * Reduced area to ensure within-year storage behaviour, centres in *italics* were used for model verification; ! Reduced area to diversify supply to demand ratios in verification.

Table 2. Rainfall station information.

Rainfall Station No.	MAP (mm/Year)	Length of Data (Years)	Percentage of Observed of Data	Percentage of In-Filled Data	Percentage of Missing Data
0476093 W	552	107	55.2	40.9	3.9
0476044 W	727	107	75.7	20.4	3.9
0476129 W	752	107	73.7	22.4	3.9
0475881 W	788	107	84.4	11.7	3.9
0020896 W	563	149	44.3	54.7	1.0
0021230 W	586	150	47.8	52.1	0.1
0021055 W	483	149	60.6	38.4	1.0
0020839 W	1183	149	44.4	54.6	1.0
0678023 W	464	96	83.1	16.9	0.0
0677834 W	485	96	92.9	7.1	0.0
0241103 W	1144	125	59.6	37.5	2.9
0239605 P	925	107	67.9	31.2	0.9
0240738 W	876	127	45.7	52.4	1.9
0241042 W	1072	125	49.1	48.0	2.9
0436495 W	588	82	97.9	2.1	0.0
0556088 W	718	98	69.4	30.6	0.0
0261368 W	550	97	91.9	8.1	0.0
0035209 W	590	124	54.0	45.8	0.2

Note: Rainfall stations in italics were used for model verification.

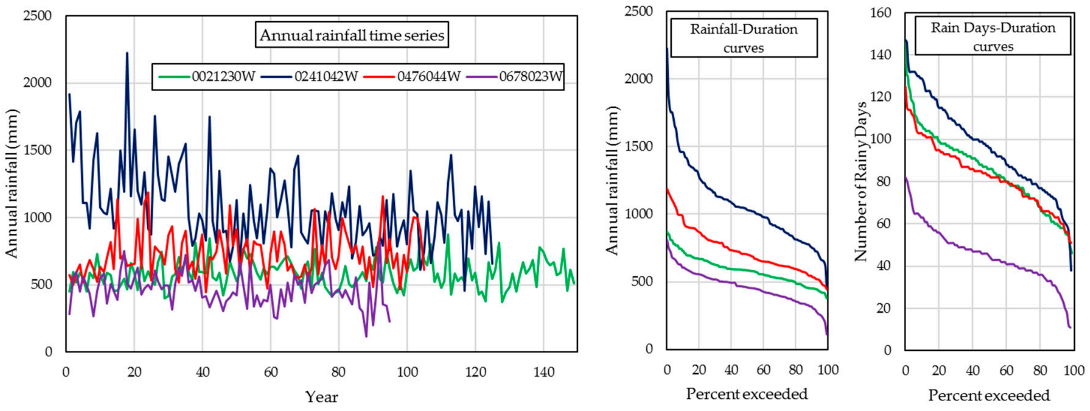

Figure 2. Time series, Rainfall–Duration and Rain Days–Duration curves of annual rainfall.

2.2. Simulation Analysis

Daily time step simulation of RWH systems has been used to provide data for generalizing storage–yield–reliability relationships [26–28] and for other analyses [22,37–40]. Simulation also forms the basis of most RWH modelling tools [9]. A typical roof RWH system consists of a roof catchment, a conveyance arrangement, a storage, a conduit to the demand and an opening for spillage. A RWH system could also include a first flush device [41] or a water filtration mechanism [42]. Rain falling on the roof catchment is conveyed in to the storage and this water becomes available to supply the demand. The storage spills if the inflow exceeds the outflow while the storage is full. The water balance fluxes of RWH systems are sub-daily but a daily analysis has been found to be sufficiently accurate [43]. Figure 3 show the components of a RWH system as implemented here, and Equations (1) and (2) describe the yield-after-spillage mass balance computations that were applied.

$$R_e(t) = \begin{array}{c} D(t)\ if\ S(t) \geq D(t) \\ S(t)\ if\ S(t) < D(t) \end{array} \tag{1}$$

$$S(t+1) = \quad min \begin{array}{c} C - R_e(t) \\ S(t) + \eta R(t) A - R_e(t) \end{array} \tag{2}$$

where $R_e(t)$ is the volume of water released to meet the demand in period t, $D(t)$ is the demand in period t, $S(t)$ is the volume of water in storage at the start of period t, C is the live storage capacity of the tank, η is the water collection efficiency, $R(t)$ is the rainfall intensity in period t and A is the vertical projection of the effective roof area.

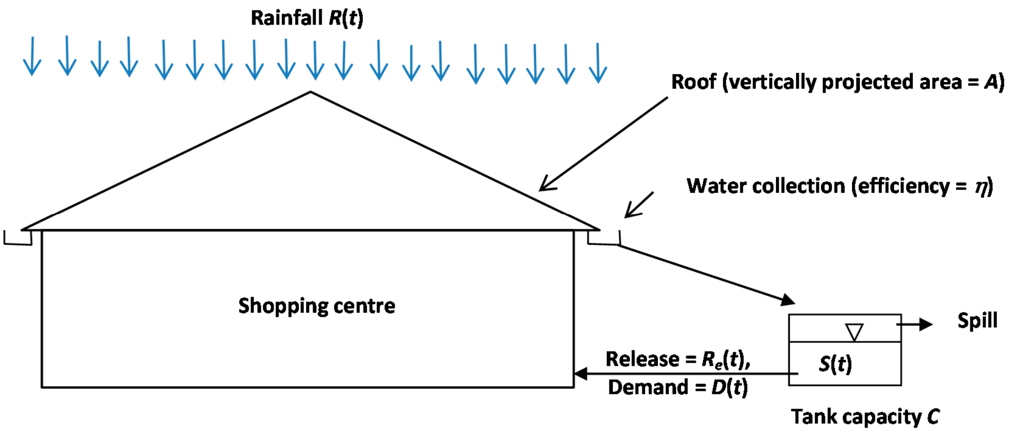

Figure 3. Rainwater harvesting system simulation components.

The simulation analysis does not explicitly model first-flush, evaporation and other losses, and it is assumed that these could be adequately effected in the selection of the water collection efficiency. This approach has been used by Berwanger and Ghisi [44] and Melville-Shreeve et al. [45] inform that a constant first-flush loss of 5 L/day could be applied for a typical UK house. Water collection efficiencies of 80 and 82% have been obtained from field experiments in South Africa [46] and Taiwan [30]. An efficiency of 80%, which has been used in other studies [22,44,47], is adopted.

2.3. Formulating Generalized Storage–Yield–Reliability Relationships

The formulation of the generalized relationships is carried out intuitively and iteratively using the data from the simulation and graphical presentation of simulation results. Regression analysis on spreadsheet is used to help obtain robust relationships. Trial simulation runs revealed that the RWH systems mainly exhibited within-year storage behaviour (Figure 4) with the storage emptying in all (or most) of the years during the dry seasons. For three shopping centres (Gateway, Liberty and Phoenix) where the supply to demand ratio exceeds unity, over-year storage behaviour occurred if large storages were used. Reliability-based analysis for over-year storage is well developed and widely applied in South Africa [48–50] and it is decided to confine the current analysis to within-year storage behaviour. This is accomplished by reducing the effective roof areas for the three centres (Table 1) to obtain supply to demand ratios less than unity.

The number of days that the RWH system fully supplied the demand in a given year is used as a practical measure of within-year yield. The yield ratio could also be specified as the proportion of the year for which demand is met. As expected, the number of days of full supply in each year varies highly given the high inter-annual rainfall variability (Figure 2). This confirms the need to include reliability (exceedance probability) of the days of supply in the formulation. To determine the reliabilities, an empirical plotting position approach is preferred to a subjectively selected probability distribution. Several plotting position formulae including the Weibull formula [51] and the alternatives proposed by Cunnane [52] are tried and all give similar exceedance probabilities. The Weibull formula is considered to have a better theoretical basis than other plotting position formulae [53] and is selected. The numbers of days of full supply in each year are ranked in descending order the exceedance probability is obtained as:

$$p = m/(n+1) \tag{3}$$

where p is the exceedance probability (reliability) of the days of full supply ranked m, and n is the total number of years of simulation.

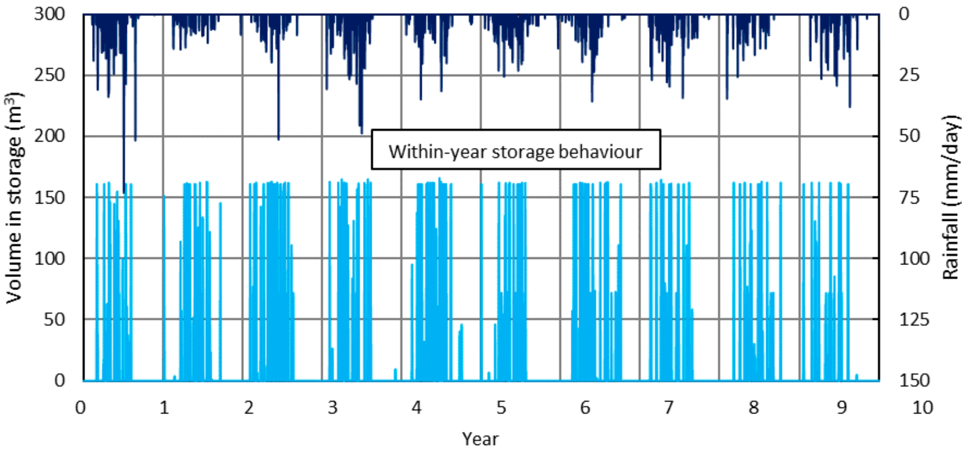

Figure 4. Illustration of within-year storage behaviour.

Resource optimization is a common objective of water resource design and operation including RWH [9,27,30,54]. Minimizing life-cycle cost is often the RWH optimization objective although yield maximization for set levels of reliability has also been used [25]. Yield and reliability considerations are some of the main basis for planning and operational decisions for reservoir systems in South Africa [48]. For this study, it is decided to develop guidelines that would maximize RWH yield and reliability but also enable analysis of non-optimal systems. Maximization of both yield and reliability are conflicting objectives because the yield that a system can provide at low reliabilities cannot be achieved at much higher reliabilities. The generalization therefore needed to use Pareto-optimal data relating to these two objectives. Trial simulation runs revealed that, for specified reliabilities, the yield could be increased by increasing storage up to a limit (Figure 5a). Beyond this, the yield could only be increased by reducing the reliability. Likewise, for specified yields (the number of days that demand is met in a year), the reliability could be increased by increasing storage up to a limit (Figure 5b). Beyond this, increasing reliability could only be achieved by a reduction in yield. The sets of yield, reliability and storage at which these limits occurred were concurrent and defined a Pareto front (Figure 5c) between yield and reliability. All combinations of storage, yield and reliability at the Pareto front were considered as hydrologically optimum.

Simulations were carried out for increasing tank sizes until the yield levelled off to the highest value for a range of reliabilities in order to locate the Pareto front. Probabilistic water resource systems analysis in South Africa applies reliabilities in the range 90–99% [48] and this study applied a slightly wider range of 85–99%.

Dimensionless ratios now needed to be defined from the Pareto optimal data. It was perceived that working with dimension ratios defined so as to take values not exceeding unity (1.0) could help obtain stable relationships. Generalized relationships between the supply to demand ratio (Equation (4)) and proportion of days supplied per year (Equation (5)) had been obtained for school RWH systems in South Africa [22] and these two ratios were adopted here. Although storage fraction has been defined as the ratio of storage to rainfall volume previously [26–28], the storage at the Pareto front was expected to relate more to the demand than to rainfall. This is because "providing water" is perceived as a more active role of storage than "receiving rainfall". The storage ratio was therefore defined as the ratio of storage to annual demand (Equation (6)). The search for relationships between $\boldsymbol{S_{P\text{-}r}}$ and the other two dimensionless ratios was then carried out by regression analysis. This was done for reliabilities of 85%, 90%, 95%, 98% and 99%. To incorporate reliability into the regression equations, additional regression analysis seeking to define the parameters of the regression equations as functions of reliability was carried out.

$$R_{SD} = \frac{\eta\, A\, \overline{P}}{\overline{D_t}} \tag{4}$$

$$S_{P-r} = \frac{Nr}{365.25} \tag{5}$$

$$R_{TD-r} = \frac{Cr}{\overline{D_t} \times 365.25} \tag{6}$$

where R_{SD} is the ratio of average supply to average demand, η is the efficiency of rainwater collection into storage, A is the vertical projection of the roof area, $\overline{P}$ is the average daily rainfall, $\overline{D_t}$ is the average daily demand, r is the reliability of supply, $S_{P\text{-}r}$ is proportion of the year fully supplied, N_r is the expected number of days that the demand is fully met in a year at reliability r, $R_{TD\text{-}r}$ is the ratio of storage capacity at reliability r to the volume of annual demand, and C_r is the storage capacity that is optimal at reliability r.

The generalization analysis described so far used data obtained at the Pareto front and would therefore be applicable to hydrologically optimal systems. In reality, a potential RHW user may be constrained financially [6,21], by availability of space [9] and by other factors. Furthermore, even if an optimal system were initially installed, changes in demand or supply would render it non-optimal. There was therefore the need to formulate generalization for the analysis of hydrologically non-optimal systems. Graphical plots between yield and reliability for the 85–99% range of reliabilities were found to be approximately linear for a given storage (Figure 6) and these slopes generally increased as the proportion of year supplied ($S_{P\text{-}r}$) increased.

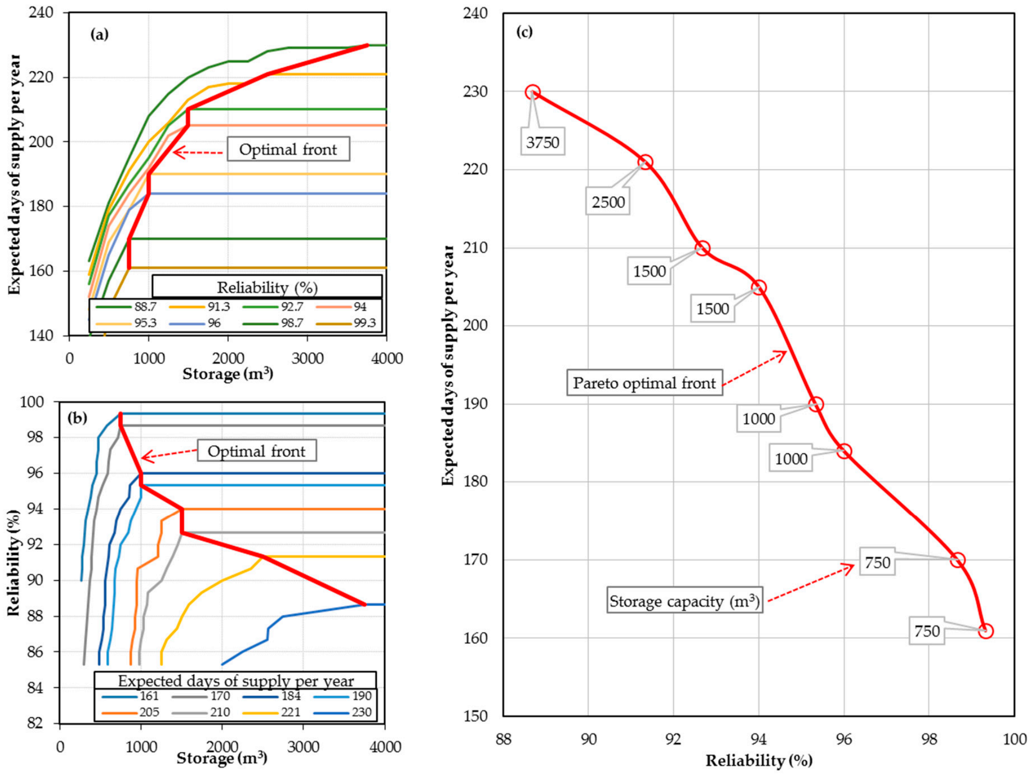

Figure 5. Illustration of the hydrologic optimality of storage, yield and reliability using the Capricon Square RWH system.

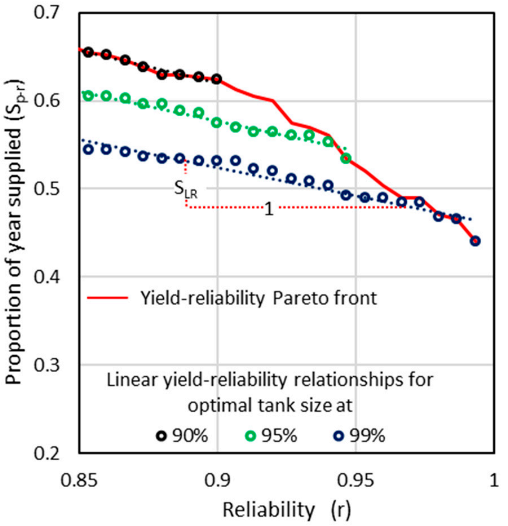

Figure 6. Linear yield–reliability plots for specified tank size.

The search for a relationship between the slope of this line (S_{LR} in Figure 6) and $S_{P\text{-}r}$ was then carried out, as this would extend the generalization to the non-optimal space below the Pareto front. Slopes S_{LR} were therefore obtained for four ranges of reliability: 85–90%, 85–95%, 85–98% and 85–99% for the simulation runs of all 19 shopping centres using least squares fitting. Regression analysis between S_{LR} and $S_P\text{-}r$ was used to search for a generalized relationship.

3. Results

3.1. Simulation Analysis

Table 3 presents the results of the hydrologically optimum combinations of storage and yield for five levels of reliability. At 85% reliability, the number of days of full supply per year varies from 12 to 243 days, while the storage capacities range from 130 to 32,000 m^3. At 99% reliability, the respective ranges are 6–161 days per year and 90–6000 m^3. From a hydrological perspective, RWH could be a viable source of water for some but not all the shopping centres. This viability could however be constrained by the high cost and space required to install large storages.

Table 3. Yield and storage capacities of hydrologically optimum systems at various reliabilities.

Shopping Centre	Reliability (%)	85	90	95	98	99
South Gate	Yield *	142	132	122	101	72
	Storage **	4750	4250	2750	1500	1500
Braamfontein	Yield	12	11	8	7	6
	Storage	130	160	90	130	90
Grayston	Yield	70	61	55	45	40
	Storage	240	165	240	150	105
Norwood	Yield	116	107	95	84	76
	Storage	1800	3200	2000	1600	1600
Sandton	Yield	120	112	104	69	68
	Storage	6800	6800	5600	2000	1600
Capricon	Yield	242	228	195	172	161
	Storage	3000	3750	1250	1000	750
Howard	Yield	91	84	77	69	63
	Storage	800	550	500	350	300
Willow Bridge	Yield	35	32	28	26	25
	Storage	1000	600	450	450	450
Tyger Valley	Yield	139	125	117	108	101
	Storage	5100	3300	3000	2400	2400
Canal Walk	Yield	21	19	16	12	8
	Storage	750	1125	750	450	300
Mall of North	Yield	65	58	54	44	30
	Storage	4000	3250	3000	2000	1250
Savanah	Yield	23	22	18	13	11
	Storage	525	825	825	525	525
Limpopo	Yield	16	15	11	10	8
	Storage	240	300	240	390	270
Cycad	Yield	23	22	18	13	12
	Storage	160	240	240	160	180
Gateway-Reduced area	Yield	140	131	108	101	85
	Storage	32,000	24,000	10,000	8000	6000
Liberty-reduced area	Yield	243	196	162	142	138
	Storage	15,300	9000	4500	2700	1800
Musgrave	Yield	41	37	30	27	18
	Storage	1100	1000	600	600	400
Phoenix-reduced area	Yield	153	143	121	104	90
	Storage	7500	8500	2500	1500	1500
Granada	Yield	46	39	33	27	26
	Storage	255	135	120	90	105

Note: * expected number of days of full supply per year; ** Storage capacity (m^3).

3.2. Generalized Storage–Yield–Reliability Relationships

The non-linear power law model was found to fit the relationships between R_{SD} and $S_{P\text{-}r}$ best at the five reliabilities of 85%, 90%, 95%, 98% and 99%. Figure 7 shows the fits for four of these. Figure 8 shows the relationships between the parameters of the power law models and reliability. These models are themselves highly correlated power law models. Figures 9 and 10 show the respective relationships between $S_{P\text{-}r}$ and $R_{TD\text{-}r}$ and the parameters of the power models with reliability. The correlations between $S_{P\text{-}r}$ and $R_{TD\text{-}r}$ were lower than those between R_{SD} and $S_{P\text{-}r}$ but they were still considered satisfactory. The generalized models of the RWH system could therefore be summarized as:

$$S_{p-r} = aR_{SD}{}^{b} \; a = 1.1428(1-r)^{0.1514} \; b = 1.2416(1-r)^{-0.037} \; 0.85 \leq r \leq 0.99 \tag{7}$$

$$R_{TD-r} = cS_{p-r}{}^{d} \; c = 1.4365(1-r)^{0.5703} \; d = 2.0065(1-r)^{0.2131} \; 0.85 \leq r \leq 0.99 \tag{8}$$

where $S_{P\text{-}r}$ is proportion of the year fully supplied at reliability r, R_{SD} is the ratio of average supply to average demand, $R_{TD\text{-}r}$ is the ratio of storage capacity at reliability r to the volume of annual demand, a and c are coefficients, and b and d are indices of the regression models.

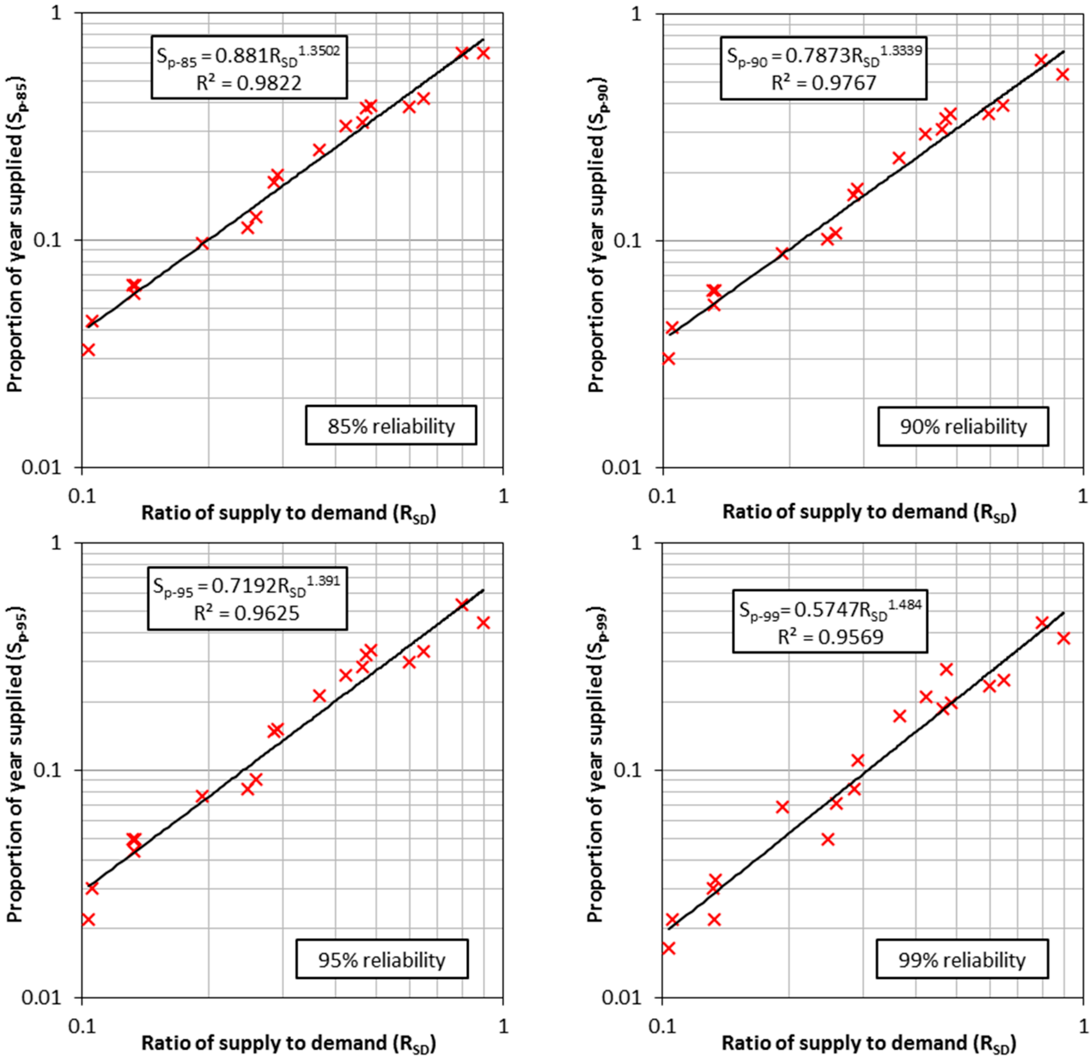

Figure 7. Generalized relationships among supply, demand and level of supply.

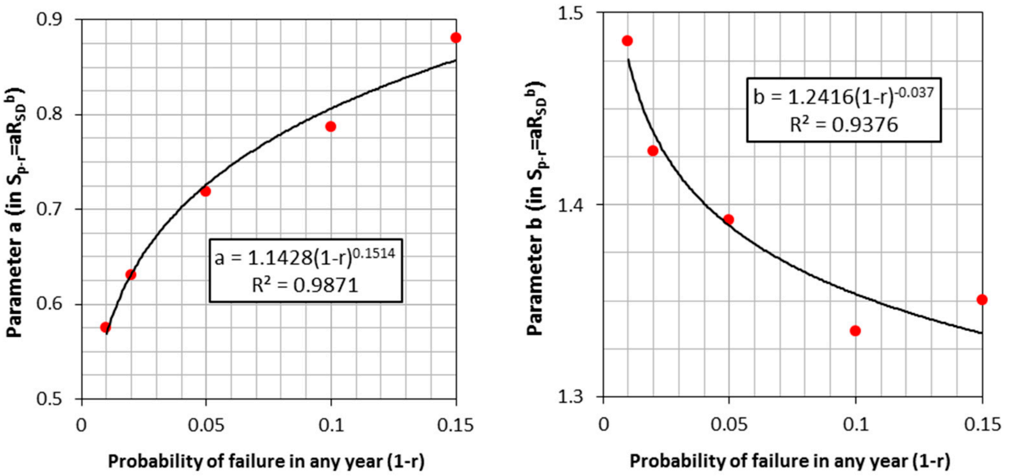

Figure 8. Relationships between supply level and supply-to-demand ratio model parameters with reliability.

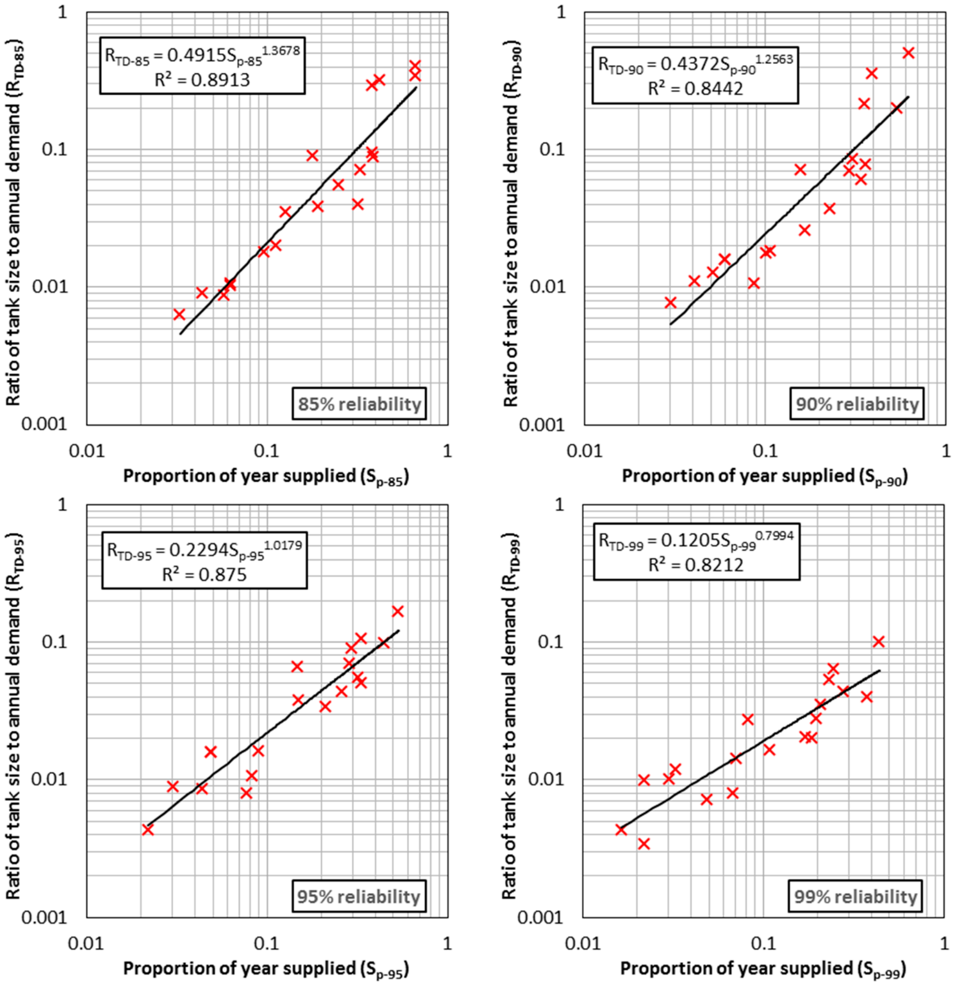

Figure 9. Generalized relationships among level of supply, demand and tank size.

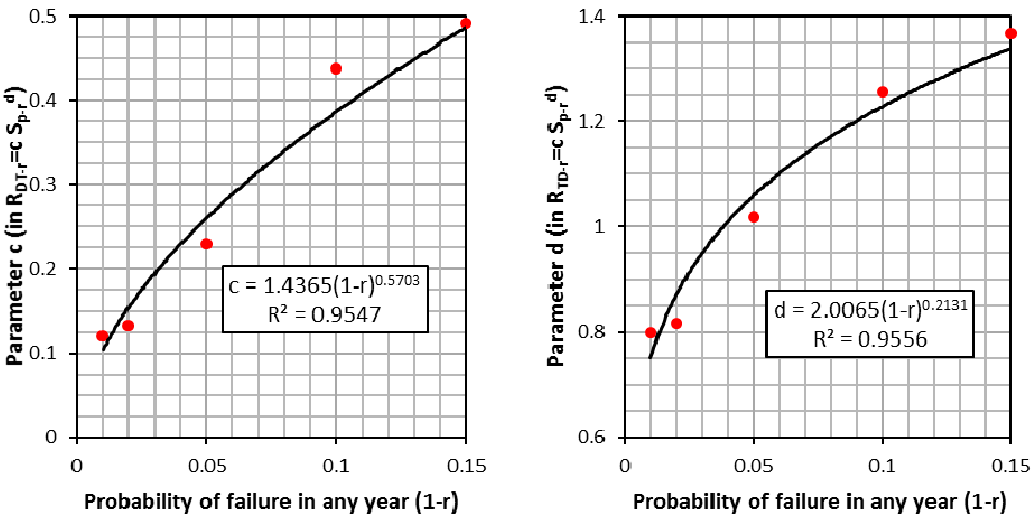

Figure 10. Relationships between tank size are demand ratio model parameters with reliability.

These equations and their graphical form in Figure 11 can be used to size hydrologically optimum RWH systems and to assess existing hydrologically optimum ones. For the generalization to be applicable to hydrologically non-optimal systems, a relationship between slope S_{LR} (Figure 6) and $S_{P\text{-}r}$ needs to be found. Figure 12 shows the best fitting power law models between the S_{LR} and $S_{p\text{-}r}$ while Figure 13 shows the relationships obtained between the parameters of the power law model and the reliability at the Pareto front. The generalized model for the slope is defined as:

$$S_{LR} = eS_{p-r}{}^{f} \; e = 0.6629(1-r)^{-0.184} \; f = -1.7615r + 2.3725 \; 0.85 \leq r \leq 0.99 \tag{9}$$

where S_{LR} is the slope of the yield–reliability plot, $S_{P\text{-}r}$ is proportion of the year fully supplied at reliability *r*, *e* is a coefficient, and *f* an index of the regression model.

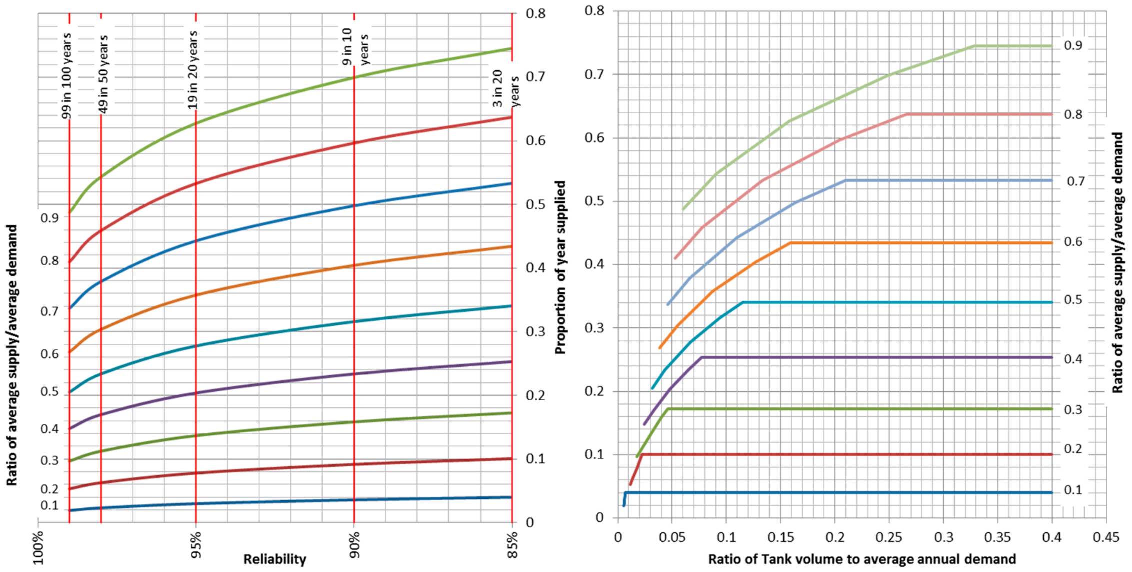

Figure 11. Charts of generalized model for analysing hydrologically optimal RWH systems.

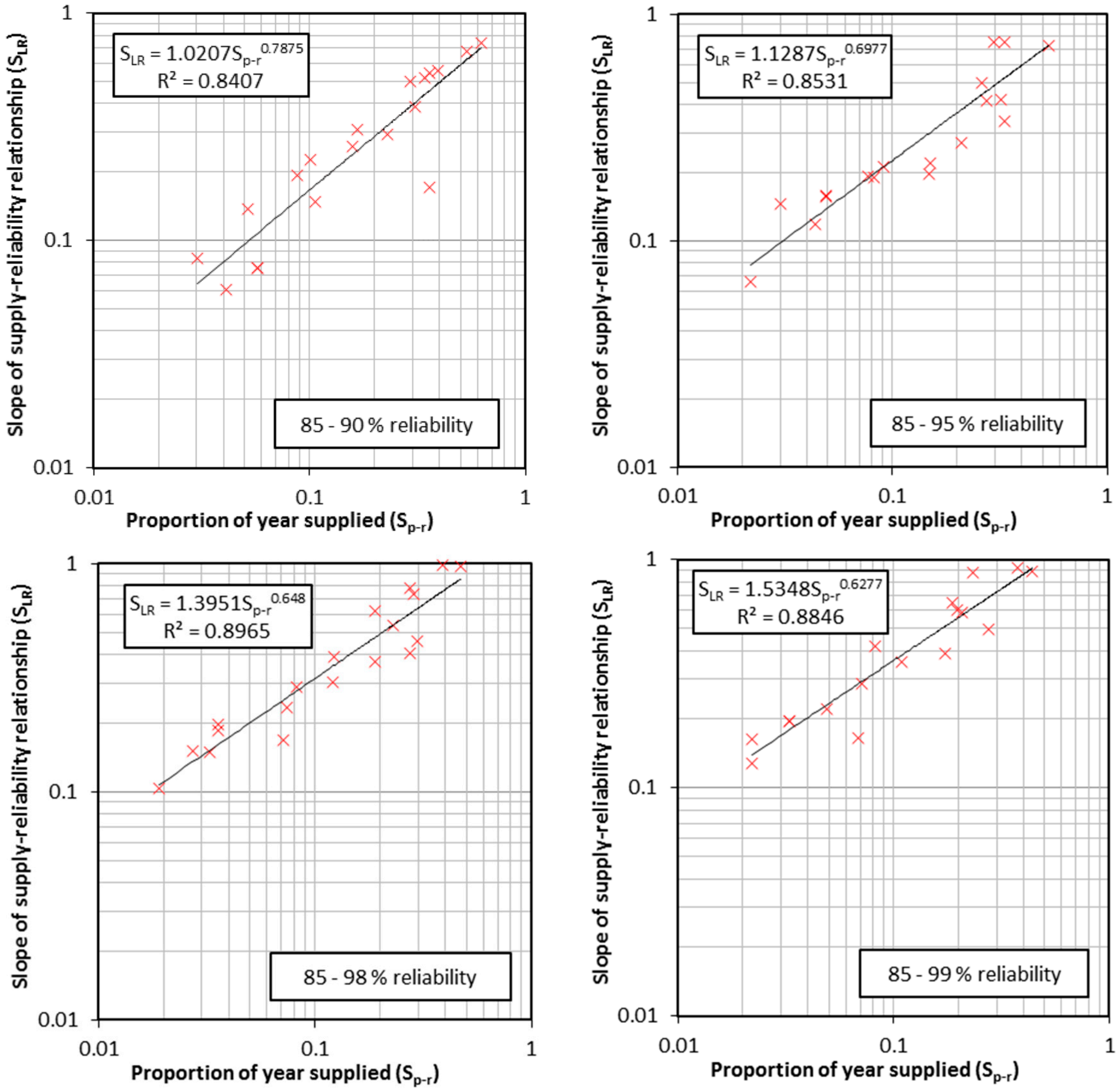

Figure 12. Relationships between reliability-level of supply slope and proportion of supply.

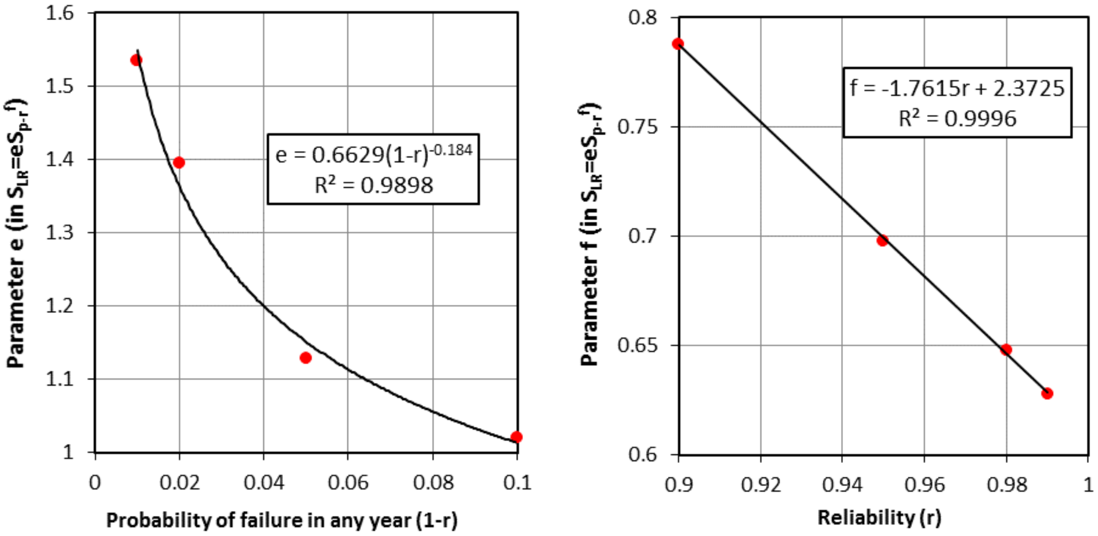

Figure 13. Relationship between parameters of slope-supply level model and reliability.

The generalized slope (S_{LR}) is now applied to obtain the yield–reliability relationship below the Pareto front, as illustrated on Figure 14. For a RWH system whose storage is optimal at reliability r, the yield (proportion of year supplied) for reliability r_t is obtained as:

$$S_{p-r_t} = S_{p-r} + (r - r_t)S_{LR} \; r \geq r_t \tag{10}$$

where $S_{p\text{-}rt}$ is the proportion of full supply for reliability r_t, r is the reliability at the Pareto front and S_{LR} is the slope of the yield–reliability plot for the storage capacity that is optimal (located at the Pareto front) for reliability r.

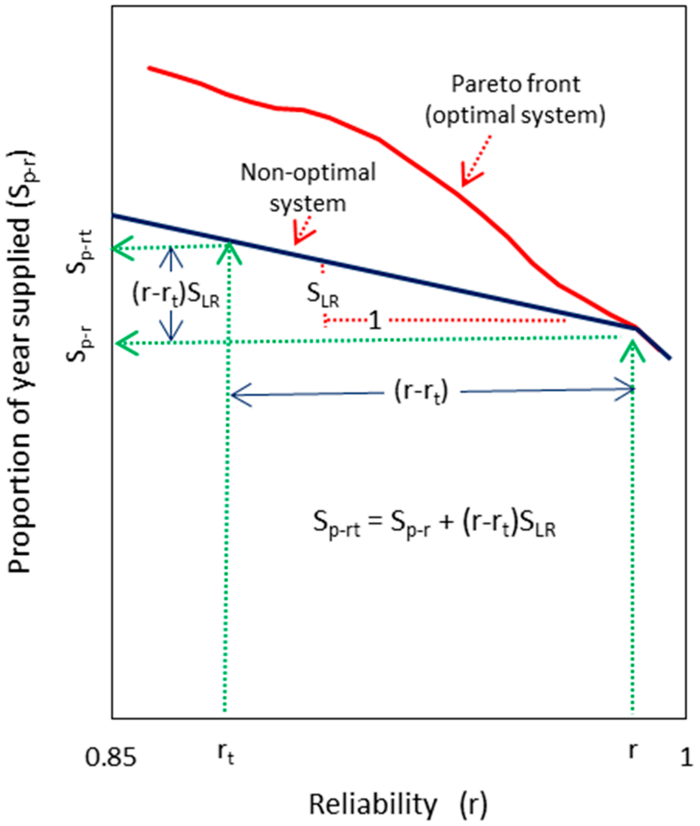

Figure 14. Illustration of yield–reliability modelling for non-optimal RWH systems.

3.3. *Verification of Generalized Model*

The verification of the model involved comparison of the storage–yield–reliability relationships from the generalized model with those from daily RWH simulation. This was done using RWH systems located in different provinces and far from those used in model formulation (Figure 1). The effective roof area of one of the systems (Matlosana mall) was reduced from the estimated 50,100 to 40,000 m^3 to diversify the supply to demand (R_{SD}) ratios to use in verification. With this change, the supply to demand ratios were 0.146, 0.302, 0.698 and 0.862, respectively, for Mimosa, Baywest, Matlosana and Riverside mall. Figures 15 and 16 compare the simulated and modelled storage–reliability and the yield–reliability relationships for hydrologically optimum configurations of the four RWH systems. The storages obtained as optimal at 98% reliability by the generalized model were then used to verify the modelling of hydrologically non-optimal systems. Simulation was carried out using these storages and the resulting yield–reliability plots were compared with those from the generalized model. These yield–reliability plots are compared on Figure 17. Figures 15–17 reveal satisfactory verification performance of the generalized model. Figure 15 also indicates that the generalizing could probably be used to smooth the large scatter of the simulated storage–reliability relationships.

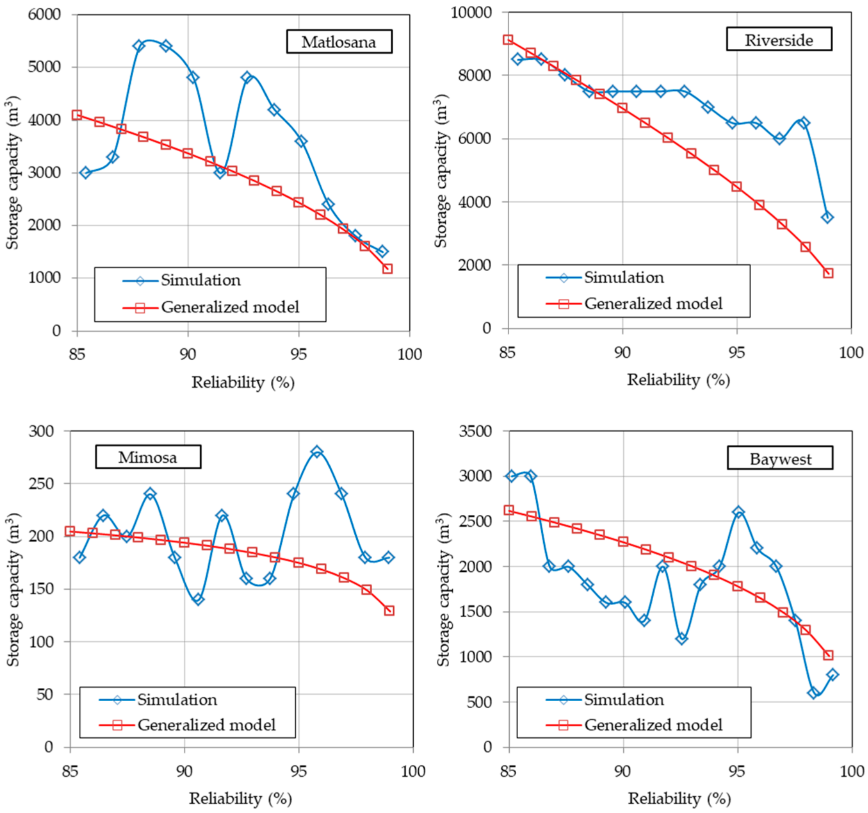

Figure 15. Generalized model and daily simulation storage–reliability relationships.

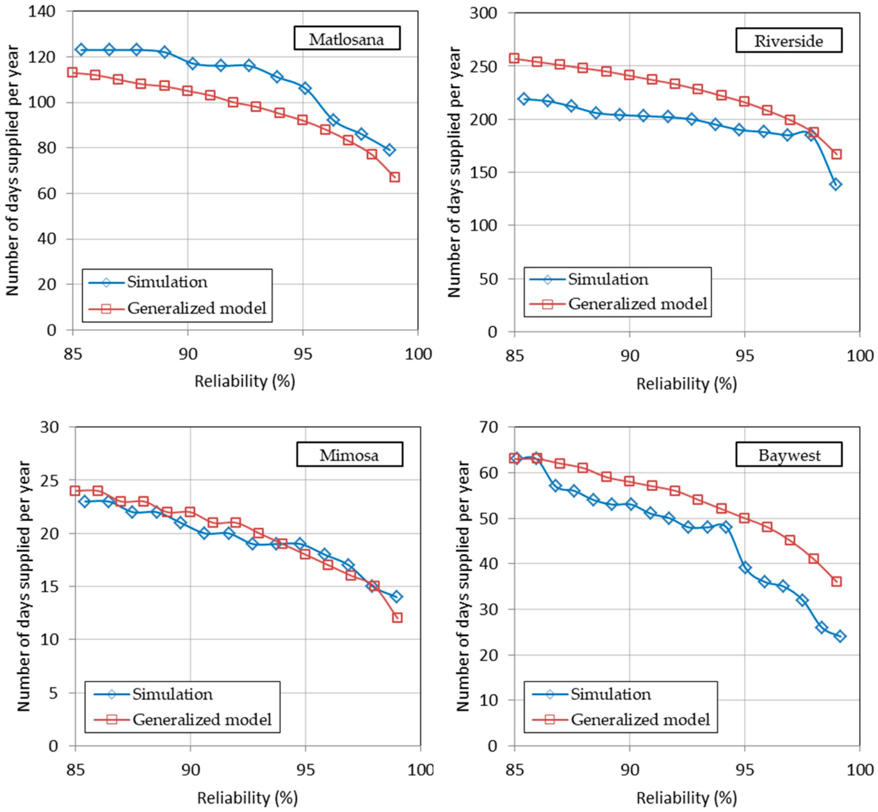

Figure 16. Generalized model and simulation yield–reliability relationships for hydrologically optimal systems.

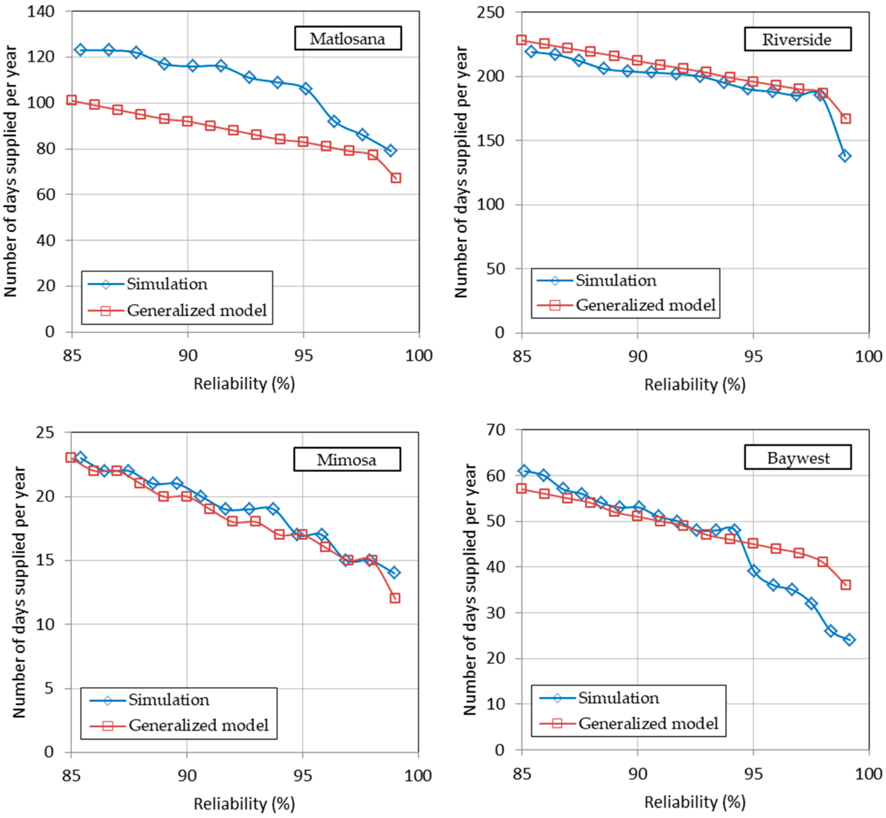

Figure 17. Generalized model and simulation yield–reliability relationships for hydrologically non-optimal systems.

4. Case Study: RWH System for Maponya Mall

Maponya mall is a regional shopping centre located in Soweto, Johannesburg. It has a retail area of 70,000 m^2 and a roof area of 60,000 m^2. The generalized model was used to determine the hydrologically optimum RWH storage to supply the non-potable demand at 95% reliability. The probable change in the performance of this system due to climate change was then assessed. Daily rainfall was sourced from Lynch [34] and gauging station 0475736 W located 1.90 km from the mall provided 107 years of daily rainfalls. Overall, 72.5% of the rainfall was observed, 23.9% was patched and 3.6% could not be patched and was classified as missing. The station has an MAP of 655 mm/year. This would increase to 753 mm/year assuming the projected 18% increase in rainfall for Johannesburg in the climatic "near-future" (2046–2065) [55]. The current average temperature of Johannesburg is 16 °C and is projected to increase by an average of 2.4 °C in the "near future" [55]. Unpublished analysis by the first author shows that HVAC (air conditioning) water demand in Johannesburg varies in direct proportion to the temperature (expressed in °C). The 2.4 °C rise in temperature would therefore increase the HVAC demand by 15%. Assuming that HVAC demand takes 64% of the non-potable demand, as found by Saunders [36], the total non-potable water demand currently estimated as 0.606 m^3/m^2/year would increase to 0.665 m^3/m^2/year in the climatic "near future".

The supply to demand ratio (R_{SD}) for the current climate is obtained as 0.741 using Equation (4). By using Equation (8), a proportion of full supply ($S_{P\text{-}95}$) of 0.479 is obtained for a reliability of 95%. By Equation (5), the expected number of days of full supply (N_r) is obtained as 175 days per year at 95% reliability. Using Equation (9), the ratio of storage capacity to annual demand ($R_{TD\text{-}95}$) comes to 0.119 and the storage capacity (C_r) is then obtained as 5062 m^3 by Equation (6).

For the climatic "near future" the new ratio of supply to demand ratio (R_{SD}) is 0.797 (Equation (4)) and the new storage to annual demand ratio ($R_{DR\text{-}r}$) is 0.109 (Equation (6)). Because the supply and demand have changed, the capacity (5062 m^3) that was hydrologically optimal at 95% reliability is now optimal at some other reliability. This new reliability is obtained by setting R_{SD} as 0.797 in Equation (8) and $R_{TD\text{-}r}$ as 0.109 in Equation (9), and then determining r simultaneously using both equations. This obtains a reliability (r) of 0.965 (96.5%). The proportion of year supplied $S_{P\text{-}96.5}$ comes to 0.500 (Equation (8)) obtaining an expected full supply of 183 days per year at 96.5% reliability (Equation (5)). At reliabilities exceeding 96.5%, the proportion of year supplied ($S_{P\text{-}r}$) is obtained using Equation (8) and, at lower reliabilities, Equations (10) and (11) are used to obtain $S_{P\text{-}rt}$. The yield–reliability relationships for the current and the climatic "near future" are shown on Figure 18.

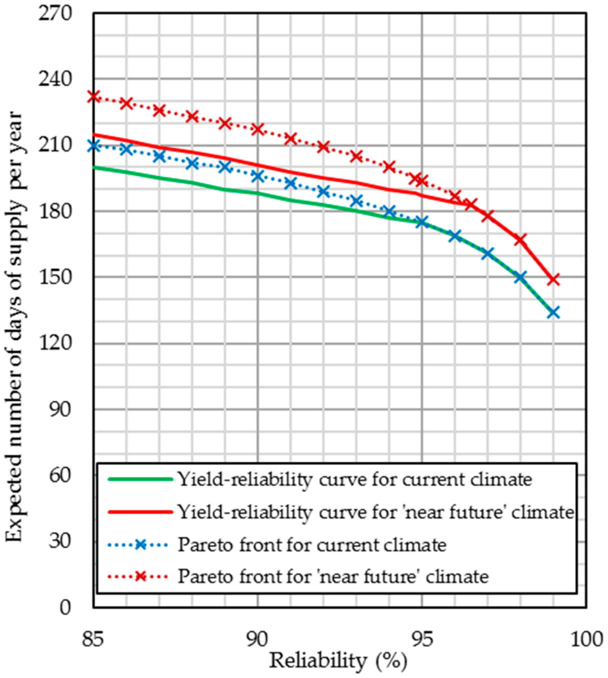

Figure 18. Maponya mall RWH system yield–reliability relationships for current and the climatic "near future" condition.

5. Discussion and Conclusions

This study aimed to develop guidelines for sizing and the assessing the rainwater harvesting (RWH) potential of shopping centres in South Africa as none of these were known to exist. A generalized model has been developed using data from the simulation of potential RWH systems of 19 shopping centres. The 19 centres are located in four South African provinces and verification of the model has been done using four RWH systems located in four other provinces. The generalized model consists of three regression equations of dimensionless ratios. These ratios are derived from the variables that characterize RWH systems and are defined as: the supply to demand ratio, the yield ratio, the storage capacity to annual demand ratio, and reliability. The yield was defined as proportion of the year that the RWH system meets the demand and reliability was defined as the probability that this yield would be met in any year of the operational life of the system. These definitions enabled inter-annual variability of rainfall to be integrated into the model and the Weibull plotting position formula was used to determine the probabilities of exceedance of yield.

Maximizing yield and reliability is a common objective of storage design and, in this study, a hydrologically optimum system was considered as one that maximizes yield and reliability with minimum storage. Since maximizing both yield and reliability are conflicting objectives, the Pareto front of the two objectives was used to identify hydrologically optimum combinations of yield, reliability and storage. These data were then used to formulate two of the three regression equations of the model. One equation is between the supply to demand ratio, the yield ratio and reliability. The other is between the ratio of optimal tank size to annual demand, the yield ratio and reliability.

RWH systems may however not be hydrologically optimum as financial considerations and space limitations often constrain design. Furthermore, the rainwater collection area is likely to be set and use of commercially available tanks may be more economical than on-site construction of storage. To enable the design and analysis of hydrologically non-optimal systems, an additional regression equation was formulated. This equation expresses the slope of the yield–reliability plot as a function of the optimal yield for a specified storage. All the model fits have high correlation coefficients that exceed 0.8 and average 0.92. The generalized model is found to perform well in verification for both hydrologically optimum and non-optimum systems (Figures 15–17). A case study RWH design using the generalized model is carried out for Maponya mall, a regional shopping centre located in Soweto, Johannesburg. The case study includes an assessment of the effect of the projected climate change in the "near future" (2046–2065) on the performance of the RWH system.

In contrast to several RWH generalization studies [26,28,31,43], within-year yield and inter-annual variability have been incorporated comprehensively into the modelling here. The generalization by Compisano and Modica [27] however also included probability of exceedance considerations in the generalization. Several generalization studies [26–28,31,43] have obtained relationships at specified reliabilities while the model developed here includes reliability as a variable within the regression equations—an aspect that improves its applicability. The model has very low data requirements and uses mean annual precipitation (MAP) as the only hydrological input. The satisfactory verification results, low data requirements and the high level of model parsimony are considered as indicators of appropriate choice and combination of variables and. apt selection of regression equations. In its current form, the generalized model is considered applicable for feasibility analysis and preliminary design of RWH systems but not as a replacement of detailed analysis.

For the future, generalization with a larger number of rainfall stations could obtain more dependable relationships that include statistical confidence intervals. The influence of the distribution of demand within the year could be investigated and first-flush losses could also be included explicitly. Although the simplicity of the current model is favourable, it is probable that use of additional rainfall statistics (as done in other studies [26,27]) could improve modelling performance. This aspect will therefore be pursued in the future.

Author Contributions: J.N. conceived the concepts; J.N., M.G. and Y.M. designed the experiments; J.N., Y.M. and M.G. performed the experiments; J.N., M.G. and Y.M. analysed the data; and J.N. wrote the paper.

Conflicts of Interest: The authors declare no conflicts of interest.

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