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The Variety of 7-Dimensional 2-Step Nilpotent Lie Algebras

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Abstract: In this note, we consider degenerations between complex 2-step nilpotent Lie algebras of dimension 7 within the variety \mathcal{N}_7^2 . This allows us to obtain the rigid algebras in \mathcal{N}_7^2 , whose closures give the irreducible components of the variety.

Keywords: nilpotent Lie algebras; variety of Lie algebras; degenerations

MSC: 17B30; 17B56

1. Introduction

The study of degenerations (also called contractions in Physics) and the geometric classification of varieties of different kinds of structures is an active research field. There are several works on degenerations, orbit closures and geometric classifications of different algebras. We provide here a list of references on many of them: Lie algebras (see, for example, [1–10]), Jordan algebras (see for example [11–14]), Leibniz algebras (see [15–17]), pre-Lie algebras in [18], Novikov algebras in [19], Filippov algebras in [20], binary Lie and nilpotent Malcev algebras in [21], and also for different superalgebras (see [22–24]).

In particular for Lie algebras, the problem has been completely solved in dimension ≤ 4 and for nilpotent Lie algebras in dimension ≤ 6 . In dimension 7, the cases of 5-step and 6-step nilpotent Lie algebras have been studied by Burde in [2]. Another related and interesting problem is the study of rigid 2-step nilpotent within the variety of 2-step nilpotent Lie algebras. This has been done in [25–27]. The aim of this work is to consider complex 2-step nilpotent Lie algebras of dimension 7. This is a step forward in order to obtain the complete picture of degenerations of nilpotent Lie algebras in dimension 7. In this variety, there are three rigid Lie algebras.

2. The Variety \mathcal{N}_n^2

Let V be a complex n -dimensional vector space with a fixed basis $\{e_1, \dots, e_n\}$. Identify a Lie algebra structure on V , \mathfrak{g} , with its set of structure constants $\{c_{ij}^k\} \in \mathbb{C}^{n^3}$, $\left([e_i, e_j] = \sum_{k=1}^n c_{ij}^k e_k\right)$. Since every set of structure constants must satisfy the polynomial equations given by the skew-symmetry and the Jacobi identity: $c_{ij}^k + c_{ji}^k = 0$ and $\sum_{l=1}^n \left(c_{j,k}^l c_{i,l}^r + c_{k,i}^l c_{j,l}^r + c_{i,j}^l c_{k,l}^r\right) = 0$, the set of n -dimensional Lie algebras is an affine variety in \mathbb{C}^{n^3} , denoted by \mathcal{L}_n . The group $G = \text{GL}(n, \mathbb{C})$ acts on \mathcal{L}_n via change of basis:

$$g \cdot [X, Y] = g \left([g^{-1}X, g^{-1}Y] \right), \quad X, Y \in \mathfrak{g}, g \in G.$$

The variety \mathcal{N}_n^2 is the closed subset of \mathcal{L}_n given by all at most 2-step nilpotent Lie algebras. This variety is endowed with the Zariski topology.

3. Degenerations of Lie Algebras

Given two Lie algebras \mathfrak{g} and \mathfrak{h} , we say that \mathfrak{g} degenerates to \mathfrak{h} , and denoted by $\mathfrak{g} \rightarrow \mathfrak{h}$, if \mathfrak{h} lies in the Zariski closure of the G -orbit $O(\mathfrak{g})$. An element $\mathfrak{g} \in \mathcal{N}_n^2$ is called rigid, if its orbit $O(\mathfrak{g})$ is open in \mathcal{N}_n^2 . Since each orbit $O(\mathfrak{g})$ is a constructible set, its closures relative to the Euclidean and the Zariski topologies are the same (see [28], 1.10 Corollary 1, p. 84). As a consequence, the following is obtained:

Lemma 1. *Let $\mathbb{C}(t)$ be the field of fractions of the polynomial ring $\mathbb{C}[t]$. If there exists an operator $g_t \in \text{GL}(n, \mathbb{C}(t))$ such that $\lim_{t \rightarrow 0} g_t \cdot \mathfrak{g} = \mathfrak{h}$, then $\mathfrak{g} \rightarrow \mathfrak{h}$.*

The previous Lemma gives the definition of a Lie algebra contraction. In general, a contraction is a particular type of degeneration.

A nice result concerning degenerations is the fact that every n -dimensional (Lie) algebra degenerates to the trivial one. Consider the operator $g_t = t^{-1}I_n$, i.e., $g_t(e_k) = t^{-1}e_k$ for $1 \leq k \leq n$, then

$$\lim_{t \rightarrow 0} g_t \cdot [e_i, e_j] = \lim_{t \rightarrow 0} g_t \left([g_t^{-1}(e_i), g_t^{-1}(e_j)] \right) = \lim_{t \rightarrow 0} g_t \left([te_i, te_j] \right) = \lim_{t \rightarrow 0} t^2 \sum_{k=1}^n c_{i,j}^k g_t(e_k) = \lim_{t \rightarrow 0} t \left(\sum_{k=1}^n c_{i,j}^k e_k \right) = 0$$

There are many invariants for the orbit closure of a Lie algebra. Among them, we mention the ones that will be used in this work.

Lemma 2. *Let $\mathfrak{g}, \mathfrak{h} \in \mathcal{N}_n^2$. If $\mathfrak{g} \rightarrow \mathfrak{h}$, then the following relations must hold:*

- $\dim O(\mathfrak{g}) > \dim O(\mathfrak{h})$,
- $\dim [\mathfrak{g}, \mathfrak{g}] \geq \dim [\mathfrak{h}, \mathfrak{h}]$,
- $\dim \mathfrak{z}(\mathfrak{g}) \leq \dim \mathfrak{z}(\mathfrak{h})$, where $\mathfrak{z}(\mathfrak{g})$ is the center of \mathfrak{g} ,
- $\dim H^k(\mathfrak{g}) \leq \dim H^k(\mathfrak{h})$ for $0 \leq k \leq n$, where $H^k(\mathfrak{g})$ is the k -th trivial cohomology group for \mathfrak{g} ,
- $\dim \mathfrak{a}(\mathfrak{g}) \leq \dim \mathfrak{a}(\mathfrak{h})$, where $\mathfrak{a}(\mathfrak{g})$ is the maximal abelian subalgebra of \mathfrak{g} .

Proof. The first relation follows from the Closed Orbit Lemma (see [29], I. Lemma 1.8, p. 53). The remaining follow by proving that the corresponding sets are closed, using in some cases the upper semi continuity of appropriated functions (see Ref. [30], §3, Theorem 2, p. 14). See also [2,5] for the proofs of (d) and (e), respectively. \square

The Classification of Complex 7-Dimensional 2-Step Nilpotent Lie Algebras

We consider here the classification of indecomposable 2-step nilpotent Lie algebras over \mathbb{C} of dimension 7, given by Gong in [31]. For Lie algebras of dimension ≤ 6 , we follow the book of Šnobl and Winternitz [32].

In Table 1, we provide the isomorphism classes of Lie algebras in \mathcal{N}_7^2 and the dimension of every orbit.

By using Lemma 2, we can discard possible degenerations. We show in Table 2 the non-degeneration reasons. To clarify this table, consider the Lie algebras (37B) and (27A). Since the dimension of the orbits of (37B) and (27A) are 29 and 28, respectively, (37B) may degenerate to (27A) according to Lemma 2(a). The centers of (37B) and (27A) are $\mathfrak{z}((37B)) = \langle e_5, e_6, e_7 \rangle$ and $\mathfrak{z}((27A)) = \langle e_6, e_7 \rangle$, respectively. Thus, we obtain that $\dim \mathfrak{z}((37B)) = 3 > 2 = \dim \mathfrak{z}((27A))$, which contradicts Lemma 2(c); therefore, (37B) $\not\rightarrow$ (27A).

Table 1. 7-dimensional Lie algebras.

g	Lie Product	dim O(g)
(17)	$[e_1, e_2] = e_7, [e_3, e_4] = e_7, [e_5, e_6] = e_7$	21
(27A)	$[e_1, e_2] = e_6, [e_1, e_4] = e_7, [e_3, e_5] = e_7$	28
(27B)	$[e_1, e_2] = e_6, [e_1, e_5] = e_7, [e_3, e_4] = e_6, [e_2, e_3] = e_7$	30
(37A)	$[e_1, e_2] = e_5, [e_2, e_3] = e_6, [e_2, e_4] = e_7$	24
(37B)	$[e_1, e_2] = e_5, [e_2, e_3] = e_6, [e_3, e_4] = e_7$	29
(37C)	$[e_1, e_2] = e_5, [e_2, e_3] = e_6, [e_2, e_4] = e_7, [e_3, e_4] = e_5$	27
(37D)	$[e_1, e_2] = e_5, [e_1, e_3] = e_6, [e_2, e_4] = e_7, [e_3, e_4] = e_5$	30
$\mathfrak{n}_{6,1}$	$[e_4, e_5] = e_2, [e_4, e_6] = e_3, [e_5, e_6] = e_1$	24
$\mathfrak{n}_{6,2}$	$[e_3, e_6] = e_1, [e_5, e_4] = e_1, [e_4, e_6] = e_2$	25
$\mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}$	$[e_2, e_3] = e_1, [e_5, e_6] = e_4$	26
$\mathfrak{n}_{5,1}$	$[e_3, e_5] = e_1, [e_4, e_5] = e_2$	22
$\mathfrak{n}_{5,3}$	$[e_2, e_4] = e_1, [e_3, e_5] = e_1$	20
$\mathfrak{n}_{3,1}$	$[e_2, e_3] = e_1$	15
\mathbb{C}^7	$[\cdot, \cdot] = 0$	0

Table 2. Non-degenerations.

g ↘ h	Reason Lemma 2
$\mathfrak{n}_{5,1} \not\rightarrow \mathfrak{n}_{5,3}; \mathfrak{n}_{6,1} \not\rightarrow (17), \mathfrak{n}_{5,3}; (37A) \not\rightarrow (17); \mathfrak{n}_{6,2} \not\rightarrow (17); \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1} \not\rightarrow (17); (37C) \not\rightarrow (17); (27A) \not\rightarrow (17); (37B) \not\rightarrow (27A); (27B) \not\rightarrow (17); (37D) \not\rightarrow (27A), (17)$	(c)
$(17) \not\rightarrow \mathfrak{n}_{5,1}; \mathfrak{n}_{6,2} \not\rightarrow \mathfrak{n}_{6,1}; \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1} \not\rightarrow (37A), \mathfrak{n}_{6,1}; (27A) \not\rightarrow (37C), \mathfrak{n}_{6,1}, (37A); (27B) \not\rightarrow (37B), (37C), (37A), \mathfrak{n}_{6,1}$	(b)
$(37A) \not\rightarrow \mathfrak{n}_{6,1}$	(d)
$(37A) \not\rightarrow \mathfrak{n}_{5,3}$	(e)

Next, we prove that the remaining possible degenerations are in fact degenerations. Since the relation of degeneration is transitive, we consider only essential degenerations. For every essential degeneration $g \rightarrow h$, we provide in Table 3, a parametrized basis of g .

To explain Table 3, consider the degeneration $\mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1} \rightarrow \mathfrak{n}_{6,2}$. Lie brackets of $\mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}$ are, in our parametrized basis, given by:

$$[x_3, x_5] = tx_1, \quad [x_3, x_6] = x_1, \quad [x_5, x_4] = x_1, \quad [x_4, x_6] = x_2.$$

When $t \rightarrow 0$, we obtain the Lie brackets of $\mathfrak{n}_{6,2}$.

Table 3. Degenerations.

$\mathfrak{g} \rightarrow \mathfrak{h}$	Parametrized Basis						
$\mathfrak{n}_{5,3} \rightarrow \mathfrak{n}_{3,1}$	$x_1 = e_1,$	$x_2 = e_2,$	$x_3 = e_4,$	$x_4 = e_3,$	$x_5 = te_5,$	$x_6 = e_6,$	$x_7 = e_7$
$\mathfrak{n}_{5,1} \rightarrow \mathfrak{n}_{3,1}$	$x_1 = e_1,$	$x_2 = e_3,$	$x_3 = e_5,$	$x_4 = te_4,$	$x_5 = e_2,$	$x_6 = e_6,$	$x_7 = e_7$
$(17) \rightarrow \mathfrak{n}_{5,3}$	$x_1 = e_7,$	$x_2 = e_1,$	$x_3 = e_3,$	$x_4 = e_2,$	$x_5 = e_4,$	$x_6 = te_6,$	$x_7 = e_5$
$\mathfrak{n}_{6,1} \rightarrow \mathfrak{n}_{5,1}$	$x_1 = e_3,$	$x_2 = e_1,$	$x_3 = e_4,$	$x_4 = e_5,$	$x_5 = e_6,$	$x_6 = \frac{1}{t}e_2,$	$x_7 = e_7$
$(37A) \rightarrow \mathfrak{n}_{5,1}$	$x_1 = e_6,$	$x_2 = e_7,$	$x_3 = -e_3,$	$x_4 = -e_4,$	$x_5 = e_2,$	$x_6 = te_1,$	$x_7 = e_5$
$\mathfrak{n}_{6,2} \rightarrow \mathfrak{n}_{5,1}$	$x_1 = e_1,$	$x_2 = e_2,$	$x_3 = e_3,$	$x_4 = e_4,$	$x_5 = e_6,$	$x_6 = te_5,$	$x_7 = e_7$
$\mathfrak{n}_{6,2} \rightarrow \mathfrak{n}_{5,3}$	$x_1 = e_1,$	$x_2 = e_3,$	$x_3 = e_5,$	$x_4 = e_6,$	$x_5 = e_4,$	$x_6 = \frac{1}{t}e_2$	$x_7 = e_7$
$\mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1} \rightarrow \mathfrak{n}_{6,2}$	$x_1 = -\frac{1}{t}e_4,$	$x_2 = e_1 + \frac{1}{t^2}e_4,$	$x_3 = e_6,$	$x_4 = e_2 - \frac{1}{t}e_6,$	$x_5 = e_5,$	$x_6 = \frac{1}{t}e_5 + e_3,$	$x_7 = e_7$
$(37C) \rightarrow \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}$	$x_1 = e_5,$	$x_2 = t^{-1}e_1,$	$x_3 = te_2,$	$x_4 = t^2e_7,$	$x_5 = te_3 + t^2e_2,$	$x_6 = e_4,$	$x_7 = e_6$
$(37C) \rightarrow (37A)$	$x_1 = e_1,$	$x_2 = e_2,$	$x_3 = te_3,$	$x_4 = e_4,$	$x_5 = e_5,$	$x_6 = te_6,$	$x_7 = e_7$
$(37C) \rightarrow \mathfrak{n}_{6,1}$	$x_1 = e_5,$	$x_2 = e_6,$	$x_3 = e_7,$	$x_4 = e_2,$	$x_5 = e_3,$	$x_6 = e_4,$	$x_7 = te_1$
$(27A) \rightarrow \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}$	$x_1 = e_6,$	$x_2 = e_1,$	$x_3 = e_2,$	$x_4 = e_7,$	$x_5 = e_3,$	$x_6 = e_5,$	$x_7 = te_4$
$(37B) \rightarrow (37C)$	$x_1 = e_3,$	$x_2 = e_1 + \frac{i}{\sqrt{t}}e_2 + \frac{i}{\sqrt{t^3}}e_3 - \frac{1}{t}e_4,$	$x_3 = -\frac{i}{\sqrt{t}}e_3 + e_4,$	$x_4 = e_2 + \frac{1}{t}e_3,$	$x_5 = -\frac{1}{t}e_7,$	$x_6 = \frac{1}{t}e_6,$	$x_7 = e_5 + \frac{1}{t^2}e_7$
$(27B) \rightarrow (27A)$	$x_1 = e_3,$	$x_2 = e_4,$	$x_3 = te_1,$	$x_4 = -e_2,$	$x_5 = \frac{1}{t}e_5$	$x_6 = e_6,$	$x_7 = e_7$
$(37D) \rightarrow (37B)$	$x_1 = e_2,$	$x_2 = e_4,$	$x_3 = -e_3,$	$x_4 = te_1,$	$x_5 = e_7,$	$x_6 = e_5,$	$x_7 = te_6$

Table 3 allows us to draw the Hasse diagram for essential degenerations (see Figure 1).

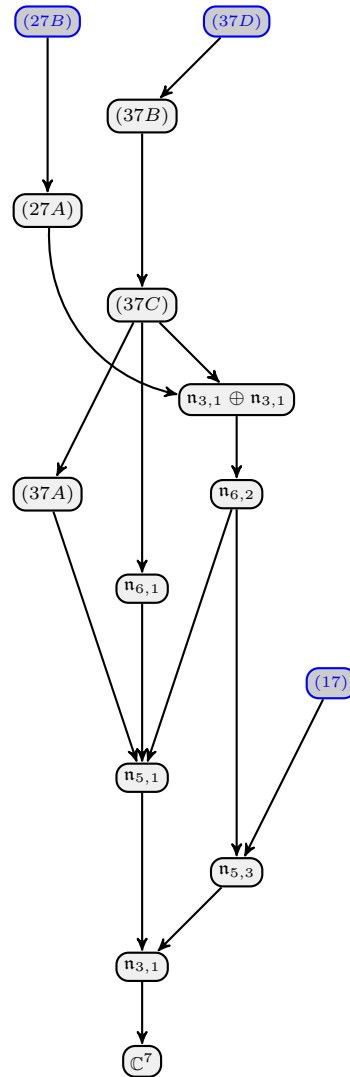


Figure 1. Degenerations in \mathcal{N}_7^2 .

4. The Irreducible Components

It is known (see for example [28]) that the irreducible components of a variety are closures of single orbits or closures of infinite families of orbits. Since there are no infinite families of 7-dimensional 2-step nilpotent Lie algebras, the irreducible components of \mathcal{N}_7^2 are the orbit closures of the rigid Lie algebras: if \mathfrak{g} is a rigid Lie algebra in \mathcal{N}_7^2 , then there exists an irreducible component C of \mathcal{N}_7^2 , such that $C \cap O(\mathfrak{g}) \neq \emptyset$ is open; then, $C \subset \overline{O(\mathfrak{g})}$. In the Hasse diagram, one can identify the rigid algebras as those that have no entering arrows. Another proof of this fact can be found in [27].

With all this, we can state:

Theorem 1. *The variety \mathcal{N}_7^2 , of at most 2-step nilpotent Lie algebras of dimension 7, has three irreducible components:*

1. $C_1 = \overline{O((17))}$,
2. $C_2 = \overline{O((27B))}$,
3. $C_3 = \overline{O((37D))}$.

Moreover, the Lie algebras (17), (27B), (37D) are rigid in \mathcal{N}_7^2 .

Proof. It follows from Tables 2 and 3 and the Hasse diagram. Moreover, one obtains:

- $\overline{O((17))} = \{(17), \mathfrak{n}_{5,3}, \mathfrak{n}_{3,1}, \mathbb{C}^7\}$,
- $\overline{O((27B))} = \{(27B), (27A), \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}, \mathfrak{n}_{6,2}, \mathfrak{n}_{5,1}, \mathfrak{n}_{5,3}, \mathfrak{n}_{3,1}, \mathbb{C}^7\}$,
- $\overline{O((37D))} = \{(37D), (37B), (37C), \mathfrak{n}_{3,1} \oplus \mathfrak{n}_{3,1}, (37A), \mathfrak{n}_{6,2}, \mathfrak{n}_{5,1}, \mathfrak{n}_{5,3}, \mathfrak{n}_{3,1}, \mathbb{C}^7\}$.

□

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