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Topological Gravity Motivated by Renormalization Group

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Abstract: Recently, we have proposed models of topological field theory including gravity in *Mod. Phys. Lett. A* **2016**, *31*, 1650213 and *Phys. Rev. D* **2017**, *96*, 024009, in order to solve the problem of the cosmological constant. The Lagrangian densities of the models are BRS (Becchi-Rouet-Stora) exact and therefore the models can be regarded as topological theories. In the models, the coupling constants, including the cosmological constant, look as if they run with the scale of the universe and its behavior is very similar to the renormalization group. Motivated by these models, we propose new models with an the infrared fixed point, which may correspond to the late time universe, and an ultraviolet fixed point, which may correspond to the early universe. In particular, we construct a model with the solutions corresponding to the de Sitter space-time both in the ultraviolet and the infrared fixed points.

1. Introduction

In *Mod. Phys. Lett. A* **2016**, *31*, 1650213 [1] and *Phys. Rev. D* **2017**, *96*, 024009 [2], models of topological field theory including gravity have been proposed in order to solve the cosmological constant problem. The accelerating expansion of the present universe may be generated by the small cosmological constant. Although the cosmological constant could be identified with a vacuum energy, the vacuum energy receives very large quantum corrections from matters and therefore in order to obtain a realistic very small vacuum energy, very fine-tuning of the counter term for the vacuum energy is necessary (The discussion about the small but non-vanishing vacuum energy is given in [3], for example.) Motivated by this problem of large quantum corrections to the vacuum energy, models of unimodular gravity [4–30] have been proposed. There have been also proposed many scenarios, such as the sequestering mechanism [31–38]. Among of the possible scenarios, we have proposed the models of the topological field theory including gravity in [1] and the cosmology described by these models has been discussed in [2].

The large quantum corrections from matter appear not only in the cosmological constant but other coupling constants. Even if we include the quantum corrections only from matter, the following coupling constants α, β, γ, and δ include large quantum corrections,

$$\mathcal{L}_{qc} = \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu} + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} . \tag{1}$$

The coefficient α diverges quadratically and β, γ, and δ diverge logarithmically. We should note that if we include the quantum corrections from the graviton, there appear infinite numbers of divergent quantum corrections, which is one of the reasons why the general relativity is not renormalizable. By using the formulation for the divergence in the cosmological constant proposed in [1,2], these divergences can be tuned to be finite [2,39]. In this formulations, the coupling constants, α, β, γ, δ, and other coupling constants including the cosmological constant are replaced by the scalar fields. Then the divergences coming from the quantum corrections can be absorbed into a redefinition of the

scalar fields. The fields depend on the cosmological time, or the scale of the universe. In this sense, the scalar fields, which corresponds to the coupling constants, run with a scale as in the renormalization group. Motivated by the above observation, in this paper, we propose new models where there appear an infrared fixed point, which may correspond to the late time universe, and an ultraviolet fixed point, which may correspond to the early universe. Especially we construct a model with solutions connecting two asymptotic de Sitter space-times, which correspond to the ultraviolet and the infrared fixed points.

In the next section, we review the models of topological gravity presented in [1,2,39]. In Section 3, we propose new models where there appear an infrared fixed point, which may correspond to the late time universe, and an ultraviolet fixed point, which may correspond to the early universe. Especially we construct a model, where the solutions expresses the flow from the de Sitter space-time corresponding to the ultraviolet fixed point to the de Sitter space-time both in the infrared fixed point. The last section is devoted to the summary, where we mention on the problems which have not been solved in this paper and some possibilities to solve them are shown in Appendix A.

2. Review of the Models of Topological Field Theory Including Gravity

We start to review the model proposed in [1]. The action of the model is given by

$$S' = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\text{gravity}} + \mathcal{L}_{\text{TP}} \right\} + S_{\text{matter}}, \quad \mathcal{L}_{\text{TP}} \equiv -\lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c. \tag{2}$$

Here $\mathcal{L}_{\text{gravity}}$ is the Lagrangian density of gravity, which may be arbitrary. The Lagrangian density $\mathcal{L}_{\text{gravity}}$ may include the cosmological constant. In the action (2), S_{matter} is the action of matters, λ and φ are ordinary scalar fields while b is the anti-ghost field and c is the ghost field. The (anti-)ghost fields b and c are fermionic (Grassmann odd) scalar. (The action without c and b has been proposed in [40] in order to solve the problem of time. The cosmological perturbation in the model motivated in the model (2) has been investigated in [41]). Please note that no parameter or coupling constant appear in the action (2) except in the parts of S_{matter} and $\mathcal{L}_{\text{gravity}}$.

We separate the gravity Lagrangian density $\mathcal{L}_{\text{gravity}}$ into the sum of some constant Λ, which corresponds to the cosmological constant and may include the large quantum corrections from matter, and the remaining part $\mathcal{L}_{\text{gravity}}^{(0)}$ as $\mathcal{L}_{\text{gravity}} = \mathcal{L}_{\text{gravity}}^{(0)} - \Lambda$. By shifting the scalar field λ by a constant Λ as $\lambda \to \lambda - \Lambda$, the action (2) can be rewritten as

$$S' = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\text{gravity}}^{(0)} - \lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c \right\} - \Lambda \int d^4x \sqrt{-g} \nabla_\mu \partial^\mu \varphi + S_{\text{matter}}. \tag{3}$$

Since the cosmological constant Λ appears as a coefficient of total derivative in the action (3), there is no contribution from the constant Λ to any dynamics in the model. Thus we have succeeded to tune the large quantum corrections from matter to vanish.

As a quantum field theory, the action (2) generates negative norm states [1], The negative norm states can be, however, removed by defining the physical states which are annihilated by the BRS (Becchi-Rouet-Stora) charge [42]. Please note that the action (2) is invariant under the following infinite number of BRS transformations,

$$\delta\lambda = \delta c = 0, \quad \delta\varphi = \epsilon c, \quad \delta b = \epsilon (\lambda - \lambda_0). \tag{4}$$

Here ϵ is a Grassmann odd fermionic parameter and λ_0 should satisfy,

$$0 = \nabla^\mu \partial_\mu \lambda_0, \tag{5}$$

which is just equation for λ: $\left(0 = \nabla^\mu \partial_\mu \lambda\right)$ obtained by the variation of the action (2) with respect to φ. (The existence of the BRS transformation where λ_0 satisfies Equation (5) was pointed out by R. Saitou.) In the BRS formalism, the physical states are BRS invariant and the unphysical states including the

negative norm states are removed by the quartet mechanism proposed by Kugo and Ojima in the context of the gauge theory [43,44]. (We can assign the ghost number, which is conserved, 1 for c and -1 for b and ϵ. The four scalar fields λ, φ, b, and c are called a quartet [43,44].) Because $\lambda - \lambda_0$ in (4) is given by the BRS transformation of the anti-ghost b, however, the BRS invariance breaks down spontaneously when $\lambda - \lambda_0$ does not vanish and therefore it becomes difficult to remove the unphysical states and keep the unitarity of the model. In the real universe, we find $\lambda - \lambda_0 \neq 0$ in general because λ plays the role of the dynamical cosmological constant and therefore BRS symmetry is spontaneously broken in general. We should note, however, that in the real universe, one and only one λ satisfying the equation $0 = \nabla^\mu \partial_\mu \lambda$ is realized. Then if we choose λ_0 to be equal to the λ in the real universe, one and only one BRS symmetry in the infinite number of the BRS symmetries given in (4) remains [2]. The remaining BRS symmery is enough to eliminate the unphysical states. and the unitarity is guaranteed.

We can regard the Lagrangian density $\mathcal{L}_{\mathrm{TP}}$ in the action (2) as the Lagrangian density of a topological field theory proposed by Witten [45]. In a topological field theory, the Lagrangian density is given by the BRS transformation of some quantity. We may consider the model where only one scalar field φ is included but the Lagrangian density of the model vanishes identically and therefore the action is trivially invariant under any transformation of φ. Then the transformation of φ can be regarded as a gauge symmetry. We now fix the gauge symmetry by imposing the following gauge condition,

$$1 + \nabla_\mu \partial^\mu \varphi = 0\,. \tag{6}$$

By following the procedure proposed by Kugo and Uehara [46], we can construct the gauge-fixed Lagrangian with the Fadeev-Popov (FP) ghost c and anti-ghost b by the BRS transformation (4) of $-b\left(1 + \nabla_\mu \partial^\mu \varphi\right)$ by choosing $\lambda_0 = 0$,

$$\delta\left(-b\left(1 + \nabla_\mu \partial^\mu \varphi\right)\right) = \epsilon\left(-\left(\lambda - \lambda_0\right)\left(1 + \nabla_\mu \partial^\mu \varphi\right) + b\nabla_\mu \partial^\mu c\right) = \epsilon\left(\mathcal{L} + \lambda_0 + (\text{total derivative terms})\right)\,. \tag{7}$$

Then we confirm that the Lagrangian density $\mathcal{L}_{\mathrm{TP}}$ in (2) is given by the BRS transformation of $-b\left(1 + \nabla_\mu \partial^\mu \varphi\right)$ and the model is surely topological. Because λ does not vanish in the real universe, the BRS invariance is broken. In this sense, the model (2) is not topological in the real universe, which could be the reason why this model gives physical contributions.

The above mechanism can be applied to the divergences in (1) or more general divergences as shown in [2]. When we consider the model in (1), the model in (2) is generalized as follows,

$$\begin{aligned}
\mathcal{L} = {}& -\Lambda - \lambda_{(\Lambda)} + \left(\alpha + \lambda_{(\alpha)}\right) R + \left(\beta + \lambda_{(\beta)}\right) R^2 + \left(\gamma + \lambda_{(\gamma)}\right) R_{\mu\nu} R^{\mu\nu} + \left(\delta + \lambda_{(\delta)}\right) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\
& + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\
& + \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} + \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)}\,.
\end{aligned} \tag{8}$$

We now shift the fields $\lambda_{(\Lambda)}$, $\lambda_{(\alpha)}$, $\lambda_{(\beta)}$, $\lambda_{(\gamma)}$, and $\lambda_{(\delta)}$ as follows,

$$\lambda_{(\Lambda)} \to \lambda_{(\Lambda)} - \Lambda\,, \quad \lambda_{(\alpha)} \to \lambda_{(\alpha)} - \alpha\,, \quad \lambda_{(\beta)} \to \lambda_{(\beta)} - \beta\,, \quad \lambda_{(\gamma)} \to \lambda_{(\gamma)} - \gamma\,, \quad \lambda_{(\delta)} \to \lambda_{(\delta)} - \delta\,, \tag{9}$$

then the Lagrangian density (8) has the following form,

$$\begin{aligned}
\mathcal{L} = {}& -\lambda_{(\Lambda)} + \lambda_{(\alpha)} R + \lambda_{(\beta)} R^2 + \lambda_{(\gamma)} R_{\mu\nu} R^{\mu\nu} + \lambda_{(\delta)} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\
& + \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\
& + \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} \\
& - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} + \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)} \\
& + (\text{total derivative terms})\,.
\end{aligned} \tag{10}$$

Except the total derivative terms, the obtained Lagrangian density (10) does not include the constants Λ, α, β, γ, and δ, which include the divergences from the quantum corrections. Therefore we can absorb the divergences into the redefinition of the scalar fields $\lambda_{(i)}$, $(i = \Lambda, \alpha, \beta, \gamma, \delta)$ and the divergences becomes irrelevant for the dynamics.

In the initial model (1), the parameters are coupling constants but in the new models, (8) or (10), the parameters are replaced by dynamical scalar fields. This is one of the reasons why the divergence coming from the quantum corrections can be absorbed into the redefinition of the scalar fields. Furthermore because the scalar fields are dynamical, as we will see later, the scalar fields play the role of the running coupling constant.

The Lagrangian density (10) is also invariant under the following BRS transformations

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0\,, \quad \delta\varphi_{(i)} = \epsilon c\,, \quad \delta b_{(i)} = \epsilon\left(\lambda_{(i)} - \lambda_{(i)0}\right)\,, \quad (i = \Lambda, \alpha, \beta, \gamma, \delta)\,, \tag{11}$$

where $\lambda_{(i)0}$'s satisfy the equation,

$$0 = \nabla^\mu \partial_\mu \lambda_{(i)0}\,, \tag{12}$$

as in (5). The Lagrangian density (10) is also given by the BRS transformation (11) with $\lambda_{(i)0} = 0$,

$$\delta\left(\sum_{i=\Lambda,\alpha,\beta,\gamma,\delta}\left(-b_{(i)}\left(\mathcal{O}_{(i)} + \nabla_\mu \partial^\mu \varphi_{(i)}\right)\right)\right) = \epsilon\left(\mathcal{L} + (\text{total derivative terms})\right)\,. \tag{13}$$

As mentioned, due to the quantum correction from the graviton, an infinite number of divergences appear. Let $\mathcal{O}_i$ be possible gravitational operators; then a further generalization of the Lagrangian density (10) is given by

$$\mathcal{L} = \sum_i \left(\lambda_{(i)}\mathcal{O}_{(i)} + \partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)}\right)\,. \tag{14}$$

Then all the divergences are absorbed into the redefinition of λ_i. The Lagrangian density (14) is invariant under the BRS transformation and given by the the BRS transformation of some quantity and therefore the model can be regarded as a topological field theory, again.

Well-known higher derivative gravity can be renormalizable, but the ghosts appear and therefore the higher derivative gravity model is not unitary. Although our model may be renormalizable because the divergence does not appear, the problem of the unitarity remains because the Lagrangian density (14) includes the higher derivative terms. In the viewpoint of string theory, for example, we may expect that if we include the infinite number of higher derivative terms, the unitarity could be recovered but this is out of scope in this paper.

Usually the problem of the renormalizability in quantum field theory is the predictability. Even if we consider the quantum theory of gravity starting from the general relativity, if we include an infinite number of the counterterms, the theory becomes finite but due to the infinite number of the counter terms, the model loses predictability. In the model of (14), there could not be the problem of the divergence but because λ_i's become dynamical, we need infinite number of the initial conditions or somethings and therefore even in the model (14), the predictability could be lost. If the λ_i's have infrared fixed points, however, the predictability could be recovered. In the original model (14), however, we have not obtained non-trivial fixed points, which is one of the motivation why we considered the model in next section, where we try to construct the models with the fixed points.

3. Model Motivated by Renormalization Group

We assume that the space-time is given by the FRW (Friedmann-Robertson-Walker) universe with flat spacial part and a scale factor $a(t)$

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} \left(dx^i \right)^2 . \tag{15}$$

Equation (12) tells that the scalar fields $\lambda_{(i)}$ depend on the scale factor $a(t)$ and then become time-dependent. Because $\lambda_{(i)}$ correspond to the coupling with the operator $\mathcal{O}_{(i)}$, Then the scale factor dependence of $\lambda_{(i)}$ is similar to the scale dependence of the renormalized coupling $\lambda_{(i)}$ Motivated by this observation, we consider the models with an infrared fixed point, which may correspond to the late time universe, and an ultraviolet fixed point, which may correspond to the early universe.

We now assume the following BRS transformations instead of (4),

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0 , \quad \delta\varphi_{(i)} = \epsilon c , \quad \delta b_{(i)} = \epsilon\lambda_{(i)} , \tag{16}$$

and consider the Lagrangian density which is given by the BRS transformation (16) of some quantity,

$$\delta \left(\sum_{i=\Lambda,\alpha,\beta,\gamma,\delta} \left(b_{(i)} \left(\mathcal{O}_i + \nabla_\mu \partial^\mu \varphi_{(i)} + f_i \left(\lambda_{(j)} \right) \varphi_{(i)} \right) \right) \right) = \epsilon \left(\mathcal{L} + (\text{total derivative terms}) \right) . \tag{17}$$

Here $\mathcal{O}_i$ are possible gravitational operators as in (14). and $f_i \left(\lambda_{(j)} \right)$'s are functions of $\lambda_{(j)}$. Then we obtain

$$\mathcal{L} = \sum_i \left(\lambda_{(i)} \mathcal{O}_{(i)} + \partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} + \lambda_{(i)} f_i \left(\lambda_{(j)} \right) \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)} - f_i \left(\lambda_{(j)} \right) b_{(i)} c_{(i)} \right) . \tag{18}$$

The obtained model (18) is different from the original model (1), (8) or (10). We are using a different gauge fixing and the background solution is not BRS invariant. Then, in this background, the model (18) is not topological.

By the variation with respect to $\varphi_{(i)}$, we obtain the following equations,

$$- \nabla_\mu \nabla^\mu \lambda_{(i)} = \lambda_{(i)} f_i \left(\lambda_{(j)} \right) . \tag{19}$$

In the FRW space-time with flat spacial part (15), Equation (19) can be written as follows,

$$\frac{d^2\lambda_{(i)}}{dt^2} + 3H \frac{d\lambda_{(i)}}{dt} = \lambda_{(i)} f_i \left(\lambda_{(j)} \right) . \tag{20}$$

Here H is the Hubble rate defined by using the scale factor in Equation (15) as $H \equiv \dot{a}/a$. By defining τ by $a = e^\tau$, we find

$$\frac{d}{dt} = H \frac{d}{d\tau} , \quad \frac{d^2}{dt^2} = H^2 \frac{d^2}{d\tau^2} + \dot{H} \frac{d}{d\tau} , \tag{21}$$

and therefore we obtain

$$H^2 \left\{ \frac{d^2\lambda_{(i)}}{d\tau^2} + \left(3 + \frac{\dot{H}}{H^2} \right) \frac{d\lambda_{(i)}}{d\tau} \right\} = \lambda_{(i)} f_i \left(\lambda_{(j)} \right) . \tag{22}$$

Because the change of a can be identified with the scale transformation, we may compare (20) with the renormalization group equation,

$$\frac{d\lambda_{(i)}}{d\tau} = g_i\left(\lambda_{(j)}\right). \tag{23}$$

In cosmology, the Hubble rate H is usually used as energy scale but an analogy with the renormalization group in the quantum field theory, suggest the possibility to use the scale factor a as the energy. From

$$\frac{d^2\lambda_{(i)}}{d\tau^2} = \sum_k \frac{\partial g_i\left(\lambda_{(j)}\right)}{\partial\lambda_{(k)}} g_k\left(\lambda_{(j)}\right), \tag{24}$$

we find

$$f_i\left(\lambda_{(j)}\right) = \frac{H^2}{\lambda_{(i)}}\left\{\sum_k \frac{\partial g_i\left(\lambda_{(j)}\right)}{\partial\lambda_{(k)}} g_k\left(\lambda_{(j)}\right) + \left(3+\frac{\dot{H}}{H^2}\right) g_i\left(\lambda_{(j)}\right)\right\}. \tag{25}$$

The interpretation of Equation (20) as a renormalization group equation requires $f_i\left(\lambda_{(j)}\right)$ to be time independent. Therefore the above identification (25) can have any meaning only if H is a constant at least near the fixed points, that is, the space-time should be, at least asymptotically, the de Sitter space-time. Later we consider the model where two fixed points are connected by the renormalization group. The two fixed points correspond to the ultraviolet (UV) and infrared (IR) limits. Between the two fixed points, H cannot be a constant because H takes different values in the two fixed points. As we will see later, the scale dependence of H can be absorbed into the redefinition of $f_i\left(\lambda_{(j)}\right)$ or $g_i\left(\lambda_{(j)}\right)$. We may assume that the renormalization equations (23) has a ultraviolet or infrared fixed point. If the universe asymptotically goes to the de Sitter universe in the early time or late time. Then if we choose $f_i\left(\lambda_{(j)}\right)$ by (25), the early universe corresponds to the ultraviolet (UV) fixed point and the late time universe to the infrared (IR) fixed point. Because the shift of τ corresponds to the change of the scale and τ is defined by using scale factor as $a = e^\tau$, the UV limit corresponds to $\tau \to -\infty$ and therefore $a \to 0$ and the IR limit to $\tau \to \infty$, that is, $a \to \infty$. In the neighborhood of the UV fixed point λ_{UV}^*, we now assume,

$$\frac{dg_{(i)}\left(\lambda_{(j)}\right)}{d\lambda_{(i)}} > 0. \tag{26}$$

Then $g_{(i)}\left(\lambda_{(j)}\right)$ can be expressed as,

$$g_{(i)}\left(\lambda_{(j)}\right) \approx k_{(i)\mathrm{UV}}(\lambda_{(j)})\left(\lambda_{(i)} - \lambda_{(i)\mathrm{UV}}\right), \tag{27}$$

where $k_{(i)\mathrm{UV}}(\lambda_{(j)})$ is a function of $\lambda_{(j)}$ and $k_{(i)\mathrm{UV}}(\lambda_{(j)\mathrm{UV}}) > 0$. By using the approximation that $k_{(i)\mathrm{UV}}(\lambda_{(j)})$ could be regarded as a constant when $\lambda_{(i)} \approx \lambda_{(i)\mathrm{UV}}$, that is, $k_{(i)\mathrm{UV}}(\lambda_{(j)}) \approx k_{(i)\mathrm{UV}}(\lambda_{(j)\mathrm{UV}})$, the solution of (23) with (27) is given by

$$\lambda_{(i)} \approx \lambda_{(i)\mathrm{UV}} + \lambda_{(i)\mathrm{UV}0} a(t)^{k_{(i)\mathrm{UV}}(\lambda_{(j)\mathrm{UV}})}. \tag{28}$$

Here $\lambda_{(i)\mathrm{UV}0}$ is a constant of the integration. On the other hand, near the IR fixed point, we replace $k_{(i)\mathrm{UV}} \to -k_{(i)\mathrm{IR}}$ and $\lambda_{(i)\mathrm{UV}} \to \lambda_{(i)\mathrm{IR}}$ in (27) and (28) as follows,

$$g_{(i)}\left(\lambda_{(j)}\right) \approx -k_{(i)\mathrm{IR}}(\lambda_{(j)})\left(\lambda_{(i)} - \lambda_{(i)\mathrm{IR}}\right). \tag{29}$$

Then we find

$$\lambda_{(i)} \approx \lambda_{(i)\text{IR}} + \lambda_{(i)\text{IR}0} \left(\frac{1}{a(t)} \right)^{k_{(i)\text{IR}}\left((\lambda_{(j)\text{IR}}) \right)} . \tag{30}$$

Here $\lambda_{(i)\text{IR}0}$ is a constant of the integration. When $a(t) \to 0$ in (28), and $a(t) \to \infty$ in (30), $\lambda_{(i)}$ goes to $\lambda_{(i)\text{UV}}$ and $\lambda_{(i)\text{IR}}$, respectively. Thus, as long as the above condition in the neighborhood of UV (IR) fixed point is satisfied, $\lambda_{(i)} = \lambda_{(i)\text{UV}}$ ($\lambda_{(i)} = \lambda_{(i)\text{IR}}$) is surely the UV (IR) fixed point. When $g_i \left(\lambda_{(j)} \right)$ behaves as (27) near the UV fixed point, Equation (25) tells that $f_i \left(\lambda_{(j)} \right)$ behaves as

$$f_i \left(\lambda_{(j)} \right) = \tfrac{H^2}{\lambda_{(i)\text{UV}}} \left(k_{(i)\text{UV}}(\lambda_{(j)\text{UV}}) + 3 \right) k_{(i)\text{UV}}(\lambda_{(j)\text{UV}}) \left(\lambda_{(i)} - \lambda_{(i)\text{UV}} \right) + \mathcal{O}\left(\left(\lambda_{(i)} - \lambda_{(i)\text{UV}} \right)^2 \right) . \tag{31}$$

On the other hand, when $g_i \left(\lambda_{(j)} \right)$ behaves as (29) near the IR fixed point, $f_i \left(\lambda_{(j)} \right)$ behaves as

$$f_i \left(\lambda_{(j)} \right) = \frac{H^2}{\lambda_{(i)\text{IR}}} \left(k_{(i)\text{IR}}(\lambda_{(j)\text{IR}}) - 3 \right) k_{(i)\text{IR}}(\lambda_{(j)\text{IR}}) \left(\lambda_{(i)} - \lambda_{(i)\text{IR}} \right) + \mathcal{O}\left(\left(\lambda_{(i)} - \lambda_{(i)\text{IR}} \right)^2 \right) . \tag{32}$$

When we consider the Einstein gravity with cosmological constant, the action is given by,

$$S = \int d^4x \sqrt{-g} \left[\lambda_{(\alpha)} R - \lambda_{(\Lambda)} + \Sigma_{i=\Lambda,\alpha} \left(\partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)} + \lambda_{(i)} f_{(i)}(\lambda_{(j)}) \varphi_{(i)} \right) \right] + S_{\text{matter}} . \tag{33}$$

Here S_{matter} is the action of matters. Varying the action (33) with respect to the metric $g^{\mu\nu}$, we obtain the following equation,

$$\lambda_{(\alpha)} G_{\mu\nu} + \tfrac{1}{2}\lambda_{(\Lambda)} g_{\mu\nu} - (\nabla_\mu \nabla_\nu - \nabla^2) \lambda_{(\alpha)} + \Sigma_{i=\Lambda,\alpha} \left[\tfrac{1}{2} g_{\mu\nu} \left(\partial_\rho \lambda_{(i)} \partial^\rho \varphi_{(i)} + \lambda_{(i)} f_{(i)}(\lambda_{(j)}) \varphi_{(i)} \right) + \partial_\mu \lambda_{(i)} \partial_\nu \varphi_{(i)} \right] = T_{\mu\nu} . \tag{34}$$

We should note that if the FP ghost and anti-ghost has any classical value, which may correspond to the vacuum expectation value, superselection rule or ghost number conservation is violated and therefore we put them vanish. In (34), $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy momentum tensor of matters. In the spatially flat FRW background if we assume that $\lambda_{(i)}$ and $\varphi_{(i)}$ depend only on the cosmological time t, the $(0,0)$-component of Equation (34) has the following form,

$$H^2 = \frac{1}{6\lambda_{(\alpha)}} \left\{ \lambda_{(\Lambda)} - 3H\dot{\lambda}_{(\alpha)} - \sum_{i=\Lambda,\alpha} \left(\dot{\lambda}_{(i)} \dot{\varphi}_{(i)} - \lambda_{(i)} f_i(\lambda_j) \varphi_{(i)} \right) \right\} \tag{35}$$

In the the neighborhood of the UV fixed point, substituting (25) and (28) into the above expression, we obtain,

$$\begin{aligned}
H^2 \approx \frac{1}{6\lambda_{(\alpha)}} & \left(\lambda_{(\Lambda)\text{UV}} + \lambda_{(\Lambda)\text{UV}0} a(t)^{k_{(\Lambda)\text{UV}}(\lambda_{(j)\text{UV}})} - 3H\dot{\lambda}_{(\alpha)} \right. \\
& \left. + \sum_{i=\Lambda,\alpha} k_{(i)}(\lambda_{(j)}) H a(t)^{k_{(i)}(\lambda_{(j)})} \lambda_{(i)\text{UV}0} \dot{\varphi}_{(i)} \right) \\
& + \frac{H^2}{6\lambda_{(\alpha)}} \sum_{i=\Lambda,\alpha} \left\{ \sum_k \frac{\partial g_i\left(\lambda_{(j)} \right)}{\partial \lambda_{(k)}} g_i \left(\lambda_{(j)} \right) + \left(3 + \frac{\dot{H}}{H^2} \right) g_i \left(\lambda_{(j)} \right) \right\} \varphi_{(i)} ,
\end{aligned} \tag{36}$$

Then in the UV limit

$$a(t) \to 0, \quad g_{(i)} \to 0, \quad \lambda_{(i)} \to \lambda_{(i)\text{UV}} , \tag{37}$$

we obtain the de-Sitter solution, where H is a constant,

$$H = H_{\text{UV}} = \sqrt{\frac{\lambda_{(\Lambda)\text{UV}}}{6\lambda_{(\alpha)\text{UV}}}} = \text{const.} \tag{38}$$

On the other hand, near the IR fixed point, instead of (36), we obtain

$$
\begin{aligned}
H^2 \approx \frac{1}{6\lambda_{(\alpha)}} &\left(\lambda_{(\Lambda)\text{IR}} + \lambda_{(\Lambda)\text{IR0}} a(t)^{-k_{(\Lambda)\text{IR}}\left(\lambda_{(j)\text{IR}}\right)} \right. \\
&- 3H\dot{\lambda}_{(\alpha)} - \sum_{i=\Lambda,\alpha} k_{(i)}\left(\lambda_{(j)}\right) Ha(t)^{-k_{(i)}\left(\lambda_{(j)}\right)} \lambda_{(i)\text{IR0}}\dot{\varphi}_{(i)} \Bigg) \\
&+ \frac{H^2}{6\lambda_{(\alpha)}} \sum_{i=\Lambda,\alpha} \left\{ \sum_k \frac{\partial g_i\left(\lambda_{(j)}\right)}{\partial \lambda_{(k)}} g_i\left(\lambda_{(j)}\right) + \left(3 + \frac{\dot{H}}{H^2}\right) g_i\left(\lambda_{(j)}\right) \right\} \varphi_{(i)} ,
\end{aligned} \tag{39}
$$

Then in the IR limit

$$a(t) \to \infty, \quad g_{(i)} \to 0, \quad \lambda_{(i)} \to \lambda_{(i)\text{IR}}, \tag{40}$$

we obtain the de-Sitter solution, where

$$H = H_{\text{IR}} = \sqrt{\frac{\lambda_{(\Lambda)\text{IR}}}{6\lambda_{(\alpha)\text{IR}}}} = \text{const.} \tag{41}$$

We now try to construct a model, where the IR fixed point is connected with the UV fixed point by the renormalization flow. As an example, we may consider the following model

$$f_{(i)}\left(\lambda_{(j)}\right) = C_{(i)}\left(\lambda_{(j)}\right)\left(\lambda_{(i)} - \lambda_{(i)\text{UV}}\right)\left(\lambda_{(i)} - \lambda_{(i)\text{IR}}\right), \tag{42}$$

Here $C_{(i)}\left(\lambda_{(j)}\right)$ is a positive function. By using (38) and comparing (31) and (42), we find

$$\frac{\lambda_{(\Lambda)\text{UV}}}{6\lambda_{(\alpha)\text{UV}}\lambda_{(i)\text{UV}}}\left(k_{(i)\text{UV}}(\lambda_{(j)\text{UV}}) + 3\right) k_{(i)\text{UV}}(\lambda_{(j)\text{UV}}) = C_{(i)}\left(\lambda_{(j)\text{UV}}\right)\left(\lambda_{(i)\text{UV}} - \lambda_{(i)\text{IR}}\right), \tag{43}$$

which can be solved with respect to $k_{(i)\text{UV}} > 0$, as follows,

$$k_{(i)\text{UV}} = -\frac{3}{2} + \frac{1}{2}\sqrt{9 + \frac{24\lambda_{(\alpha)\text{UV}}\lambda_{(i)\text{UV}}C_{(i)}\left(\lambda_{(j)\text{UV}}\right)}{\lambda_{(\Lambda)\text{UV}}}\left(\lambda_{(i)\text{UV}} - \lambda_{(i)\text{IR}}\right)}. \tag{44}$$

On the other hand, by using (41) and comparing (32) and (42), we find

$$\frac{\lambda_{(\Lambda)\text{IR}}}{6\lambda_{(\alpha)\text{IR}}\lambda_{(i)\text{IR}}}\left(k_{(i)\text{IR}}(\lambda_{(j)\text{IR}}) - 3\right) k_{(i)\text{IR}}(\lambda_{(j)\text{IR}}) = -C_{(i)}\left(\lambda_{(j)\text{IR}}\right)\left(\lambda_{(i)\text{UV}} - \lambda_{(i)\text{IR}}\right), \tag{45}$$

which can be solved with respect to $k_{(i)\text{IR}} > 0$, as follows,

$$k_{(i)\text{IR}} = \frac{3}{2} \pm \frac{1}{2}\sqrt{9 - \frac{24\lambda_{(\alpha)\text{IR}}\lambda_{(i)\text{IR}}C_{(i)}\left(\lambda_{(j)\text{IR}}\right)}{\lambda_{(\Lambda)\text{IR}}}\left(\lambda_{(i)\text{UV}} - \lambda_{(i)\text{IR}}\right)}, \tag{46}$$

which requires

$$9 \geq \frac{24\lambda_{(\alpha)\text{IR}}\lambda_{(i)\text{IR}}C_{(i)}\left(\lambda_{(j)\text{IR}}\right)}{\lambda_{(\Lambda)\text{IR}}}\left(\lambda_{(i)\text{UV}} - \lambda_{(i)\text{IR}}\right). \tag{47}$$

Therefore, as long as we choose $C_{(i)}\left(\lambda_{(j)}\right)$ to satisfy the constraint (47), the model (42) surely connect the IR fixed point with the UV fixed point by the renormalization flow.

4. Summary

Motivated with the model in [1,2,39], we have proposed models of topological field theory including gravity. In those models, the coupling constants are replaced by scalar fields, which run as in the renormalization group following the scale of the universe. As an example, we have constructed a model which connects the inflation in the early universe and the accelerating expansion of the present universe or late time. The de Sitter space-times corresponding to the inflation and the late time accelerating expansion appear as the ultraviolet and infrared fixed points, respectively. There remains, however, several problems, which violate the good properties in the original models in [1,2,39].

1. Because the shift symmetry as in (8) is lost, the models in this paper do not solve the problem of the large quantum correction.
2. Because $\lambda_{(i)}$ in (11) has a non-trivial value, the BRS symmetry in (11) should be broken.
3. Although the original model in [1,2,39] has no parameters, the models proposed in this paper should have several parameters.

Therefore it could be interesting if we constructed any model which solves some of the above problems by keeping the structure similar to the renormalization group. Some ideas that to try to solve these problems are given in Appendix A.

In summary, we have not succeeded to solved all the problems but we may have shown that there might be possibilities to solve them. In this paper, we have considered models where the scalar fields $\lambda_{(i)}$'s play the role of the running coupling constants as in the renormalization group. We have treated the scalar fields classically although the renormalization group, of course, comes from the quantum corrections. Therefore the models proposed in this paper might be realized by an effective field theory connecting the low energy region with the high energy regions. If the models are really given as effective theories, the models need not always to satisfy all the unitarity conditions.

We have anyway succeeded in constructing such models and we have shown that we can construct the model with fixed point. The models have, however, arbitrariness, which could be removed by the constraints from the observations and/or the consistencies of the models. We like to pursue the problem in the future work.

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Appendix A. Some Propositions to Improve the Models

In this appendix, we consider models, which may solve the problem given in summary section. We believe the models in this section may give some clues to solve the problems.

An example of the model, which may solve the second problem, could be

$$\delta\left(\sum_{i=\Lambda,\alpha,\beta,\gamma,\delta}\left(b_{(i)}\left(\mathcal{O}_i+\nabla_\mu\partial^\mu\varphi_{(i)}\pm k_{(0)i}\varphi_{(i)}\right)\right)\right)=\epsilon(\mathcal{L}+(\text{total derivative terms})),$$
$$\mathcal{L}=\sum_i\left(\lambda_{(i)}\mathcal{O}_{(i)}+\partial_\mu\lambda_{(i)}\partial^\mu\varphi_{(i)}\pm k_{(0)i}\lambda_{(i)}\varphi_{(i)}-\partial_\mu b_{(i)}\partial^\mu c_{(i)}\mp k_{(0)i}b_{(i)}c_{(i)}\right).$$
(A1)

Then $\lambda_{(i)} = 0$ is a ultraviolet (infrared) fixed point for $+k_{(0)i}$ $\left(-k_{(0)i}\right)$. By the variation of $\varphi_{(i)}$, we obtain

$$0 = -\nabla^{\mu}\partial_{\mu}\lambda_{(i)} \pm k_{(0)i}\lambda_{(i)}\,. \tag{A2}$$

Let a solution of (A2) be $\lambda_{(i)} = \lambda_{(i)}^{\mathrm{cl}}$. Then the action given by the Lagrangian density $\mathcal{L}$ in (A1) is invariant under the following BRS transformation instead of (16),

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0\,, \quad \delta\varphi_{(i)} = \epsilon c\,, \quad \delta b_{(i)} = \epsilon\left(\lambda_{(i)} - \lambda_{(i)}^{\mathrm{cl}}\right)\,, \tag{A3}$$

Then because one of the solutions in $\lambda_{(i)}^{\mathrm{cl}}$ is realized in the real world, the BRS symmetry corresponding to the solution $\lambda_{(i)}^{\mathrm{cl}}$ is not broken and the unitarity can be preserved.

Another kind of the solution may be given by the following kind of model,

$$S = \int d^4x \sqrt{-g}\left\{\frac{R}{2\kappa^2} - \lambda + \mathcal{L}\left(g_{\mu\nu}, X, Y_{\mu\nu}\right)\right\}\,, \quad X \equiv -\partial_{\mu}\lambda\partial^{\mu}\lambda\,, \quad Y_{\mu\nu} \equiv \nabla_{\mu}\partial_{\nu}\lambda\,. \tag{A4}$$

Here $\mathcal{L}$ could be the Lagrangian density of the *k*-essence or the Galileon model. Because $\mathcal{L}$ is invariant under the shift of λ by a constant λ_0: $\lambda \to \lambda + \lambda_0$, the vacuum energy can be absorbed into the definition of λ and the first problem could be solved. Then if we choose $\mathcal{L}$ to give a unitary model, we need not to consider the second problem. When we consider $\mathcal{L}$ of the *k*-essence, $\mathcal{L} = \mathcal{L}(X)$, for simplicity, by the variation of λ, we obtain

$$0 = 1 - 2\nabla^{\mu}\left(\partial_{\mu}\lambda\mathcal{L}'\left(X\right)\right)\,. \tag{A5}$$

In the FRW universe with the flat spacial part (15), Equation (A5) has the following form,

$$0 = 1 + 2a(t)^{-3}\frac{d}{dt}\left(a(t)^3\dot{\lambda}\mathcal{L}'\left(\dot{\lambda}^2\right)\right)\,, \tag{A6}$$

which tells that the fixed point, where $\dot{\lambda} = 0$ is not the solution.

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